Decision by majority and the right to vote

Quesada, Antonio

17 December 2009

Decision by majority and the right to vote

Antonio Quesada[†]

Departament d'Economia, Universitat Rovira i Virgili, Avinguda de la Universitat 1, 43204 Reus, Spain

17th December 2009

161.2

Abstract
The (relative) majority rule is a benchmark collective decision norm. This paper provides a simple characterization of the majority rule, for the two-alternative case, that relies on the following property: the choice prescribed by the rule to a group *I* of individuals must be the one that would be prescribed in at least 50% of the strict subgroups that can be formed in *I*. This property means if some subgroup is denied the right to participate in the collective decision, the most likely event is that the exclusion of the subgroup will have no effect on the decision.

Keywords: Social welfare function, majority rule, axiomatic characterization, two alternatives, manipulation.

JEL Classification: D71

[†] E-mail address: aqa@urv.cat. Financial support from the Spanish *Ministerio de Educación y Ciencia*, under research project SEJ2007-67580-C02-01, is gratefully acknowledged.

1. Introduction

One of the probably most accepted social norms asserts that the default rule to make collective decisions is the (relative) majority rule. The rationale for the acceptance of this norm is even more compelling when the collective decision involves just two options. There is some generality in considering decisions involving two options since a decision over *n* options can be transformed into a sequence of dual decisions. For instance, if the options are α, β, and γ, it may be first decided whether the chosen option lies in the set $\{\alpha\}$ or in the set $\{\beta, \gamma\}$ and, if it lies in $\{\beta, \gamma\}$, then it must be simply decided whether to choose β or γ.

What justifies the pre-eminence of the majority rule as a norm for collective decision-making? One possible justification is that the majority rule is the only rule satisfying a certain set of properties considered desirable. This paper identifies one such set of properties for the case in which the collective decision problem involves two options.

Specifically, the problem considered has a group of individuals who must collectively choose between two possible alternatives, α and β. For instance, the group may decide whether to maintain the status quo concerning some given issue or to accept a certain change. The opinion of each member *i* of the group is represented by a preference over the set $\{\alpha, \beta\}$: *i* may prefer α to β, may prefer β to α, or may be indifferent between α and β. A social welfare function is a way of generating a collective preference using the individuals' preferences as inputs. Once the collective preference has been established, a collective choice between α and β can be made by choosing the most preferred alternative, if there is one, or any of the two, if the collective preference declares indifference.

Viewed as a mechanism, a social welfare function may not be self-enforcing, because individuals dissatisfied with the output of the mechanism may have an incentive to follow strategies susceptible of inducing the mechanism to produce a more desirable result. Lying about one's preference is a simple way to try to manipulate the outcome of a social welfare function. Fortunately, it is easy to neutralize this form of manipulation by letting the social welfare function respond monotonically to preferences: if an individual increases his preference for an alternative, then the collective cannot decrease its preference for that alternative. Hence, with just two alternatives and a sensible social welfare function, giving less support to the favoured alternative cannot make its selection more probable.

The preference aggregation problem considered has two inputs: the set of individuals and their preferences. Since the scope of manipulation by lying about preferences appears to be limited, it may be worth considering attempts to manipulate the outcome of the social welfare function by modifying the set of individuals. In particular, the idea is that some individuals may try to exclude some other individuals from the preference aggregation process by, for instance, denying them the right to have their preferences taken into account.

The criteria determining who is entitled to be listed on an electoral roll could be viewed as forms of manipulating the result of an election by excluding some individuals from the decision process: although universal suffrage has become the norm, not every resident in a country is always given the right to vote in all the elections called in that country. Gerrymandering also illustrates this kind of manipulation: people can be excluded from an election by redistricting.

Unfortunately, only constant social welfare functions are absolutely robust to the exclusion of some group. Given that asking for absolute robustness is asking too much, the problem becomes one of selecting a reasonable amount of robustness in order to ascertain the social welfare functions that can match that level. One natural level is 50%: when any subgroup of individuals is removed with the same probability, the social welfare function must remain unchanged in at least the 50% of the cases. When this property holds, groups of individuals trying to deny other groups the right to vote (that is, to state their preference) will find that the most likely result of their strategy is that the social welfare function will output the same preference. This fact severely limits the incentive to react against the output of the social welfare function by trying to exclude some group of individuals from the aggregation process.

The main axiom of the paper formalizes the above requirement as follows. Let I be a group of individuals, with given preferences over two alternatives, α and β. Compute the value of the social welfare function for any strict subgroup of I. For $a \in \{\alpha, \beta\}$, let n_a be the number of subgroups for which the social welfare function declares a to be the most preferred alternative. When $n_\alpha = n_\beta$, both alternatives obtain the same support among subgroups. In that case, the collective preference associated with the group I must be indifference. If $n_\alpha \neq n_\beta$ and the proportion of cases in which α is the most preferred is at least 50%, then α must be declared the preferred alternative by the group I. And if $n_\alpha \neq n_\beta$ and the proportion of cases in which β is the most preferred is at least 50%, then it is β that has to be declared the preferred alternative by the group I. In view of this, that $a \in \{\alpha, \beta\}$ is the choice that group I makes can be justified on the grounds that most subgroups of I would also choose a.

This paper shows that the (relative) majority rule is, essentially, the only social welfare function satisfying this property. This result provides another rationale for the majority rule to keep its role as the benchmark collective decision rule. The predominant framework for the axiomatic analysis of the majority considers the possibility that only preferences may change or that both preferences and the set of individuals may change; see, for instance, the characterizations in May (1952, p. 682), Fishburn (1973, p. 58), Aşan and Sanver (2002, p. 411), Woeginger (2003, p. 91; 2005, p. 9), and Miroiu (2004, p. 362). Xu and Zhong (2009) suggest another framework in which the majority rule is investigated by holding preferences fixed and considering the preference aggregation problem for different subgroups. The characterization of the majority rule presented in this paper (Proposition 3.2) applies to both frameworks.

2. Definitions and axioms

Members of the set $\mathbb{N}$ of positive integers will be names for individuals. A society is a finite non-empty subset of $\mathbb{N}$. There are two alternatives, α and β. A preference over $\{\alpha, \beta\}$ is represented by a number from the set $\{-1, 0, 1\}$. If the number is 1, α is preferred to β; if -1, β is preferred to α; if 0, α is indifferent to β. A preference profile for a society I is a function $x_I : I \to \{-1, 0, 1\}$ assigning a preference over $\{\alpha, \beta\}$ to each member of I.

For preference profile x_I and society $J \subset I$, x_J is the restriction of x_I to J, that is, the preference profile x_J such that, for all $i \in J$, $x_J(i) = x_I(i)$. For preference profile x_I and $i \in I$, x_i will abbreviate $x_I(i)$. For $n \in \mathbb{N}$, X_n is the set of all preference profiles for societies with exactly n members. The set X is the set of all preference profiles for all societies. For $X' \subseteq X$ and $n \in \mathbb{N}$, $X_n' = X' \cap X_n$.

Definition 2.1. A social welfare function on $X' \subseteq X$ is a mapping $f : X' \to \{-1, 0, 1\}$.

A social welfare function on a subset X' of profiles of preferences over $\{\alpha, \beta\}$ transforms each such profile into a collective preference over $\{\alpha, \beta\}$. Hence, for $x_I \in X'$, $f(x_I)$ is the preference over $\{\alpha, \beta\}$ ascribed to society I.

Definition 2.2. The majority rule on $X' \subseteq X$ is the social welfare function $\mu : X' \to \{-1, 0, 1\}$ such that, for all $x_I \in X'$: (i) if $\sum_{i \in I} x_i > 0$, then $\mu(x_I) = 1$; (ii) if $\sum_{i \in I} x_i < 0$, then $\mu(x_I) = -1$; and (iii) if $\sum_{i \in I} x_i = 0$, then $\mu(x_I) = 0$.

For any $x_I \in X'$, the majority rule on X' just compares the number n_1 of members of society I preferring α to β with the number n_{-1} of members of society I preferring β to α. If $n_1 > n_{-1}$, α is declared to be preferred to β; if $n_1 < n_{-1}$, it is β that is declared preferred to α; otherwise, α and β are considered indifferent.

Definition 2.3. A social welfare function f on $X' \subseteq X$ satisfies SING (unanimity for singleton societies) if, for all $x_I \in X_1'$, $f(x_I) = \mu(x_I)$.

When a social welfare function satisfies SING, the collective preference associated with a society having just one member coincides with the preference of that member. SING is a mere consistency requirement between collective and individual preferences: when the collective consists of a single individual, collective and individual preferences must be the same.

For a given welfare function f on some $X' \subseteq X$, preference profile $x_I \in X'$ and $a \in \{-1, 0, 1\}$, define: (i) $n_a(x_I)$ to be the number of societies J such that $J \subset I$ and $f(x_J) = a$; and (ii) $\pi_a(x_I) = n_a(x_I) \,/\, [n_{-1}(x_I) + n_0(x_I) + n_1(x_I)]$ to be the proportion of societies strictly included in I in which the social welfare function yields value a when the preferences of the societies are obtained from x_I.

Definition 2.4. A social welfare function f on $X' \subseteq X$ satisfies LIKE (most likely decision in a subsociety) when, for each $x_I \in X' \backslash X_1'$:

(i) $f(x_I) = 0$ if $\pi_1(x_I) = \pi_{-1}(x_I)$;
(ii) $f(x_I) = 1$ if $\pi_1(x_I) \neq \pi_{-1}(x_I)$ and $\pi_1(x_I) \geq ½$; and
(iii) $f(x_I) = -1$ if $\pi_1(x_I) \neq \pi_{-1}(x_I)$ and $\pi_{-1}(x_I) \geq ½$.

A social welfare function f satisfying LIKE determines the value $f(x_I)$ as follows, where I has at least two members. First, for every society $J \subset I$, the value $f(x_J)$ is computed. Next, for $a \in \{-1, 0, 1\}$, the number $n_a(x_I)$ of societies $J \subset I$ such that $f(x_J) = a$ is determined. This could be considered a rough measure of the support that outcome a has in society I: how many subsocieties would have their collective preference represented by a. And finally, define $f(x_I)$ to be the member of $\{1, -1\}$ having more support and, if both have the same support, then declare $f(x_I)$ equal to 0. Specifically, if both 1 and −1 have the same support among subsocieties of I, so $n_{-1}(x_I) = n_1(x_I)$ or, equivalently, $\pi_1(x_I) = \pi_{-1}(x_I)$, then indifference is the outcome: $f(x_I) = 0$. If 1 and −1 do not have the same support and, moreover, $a \in \{1, -1\}$ has at least the support of 50% of the subsocieties, then a is the outcome: $f(x_I) = a$. Accordingly, $f(x_I)$ declares one of the alternatives preferred to the other when one of them has the support of at least the 50% of the subsocieties. In this respect, if the members of I chose at random a subsociety J so that $f(x_I)$ is to be defined equal to $f(x_J)$, LIKE would imply that $f(x_I)$ coincides with the most likely preference of a subsociety.

3. Result

For a finite set S, let $|S|$ designate the number of members of S. For society $I \subset \mathbb{N}$ having at least two members, define $S(I)$ to be the set of non-empty, strict subsets of I. A subset $X' \subseteq X$ is closed if, for all $x_I \in X'$ and $J \in S(I)$, $x_J \in X'$.

Lemma 3.1. Let f be a social welfare function on a closed $X' \subseteq X$, $k \geq 2$, and $x_I \in X_k'$. If,

$$\text{for all } J \in S(I), f(x_J) = \mu(x_J), \tag{1}$$

then:

(i) $\mu(x_I) = 0$ implies $n_1(x_I) = n_{-1}(x_I)$;
(ii) $\mu(x_I) = 1$ implies $n_1(x_I) \neq n_{-1}(x_I)$ and $\pi_1(x_I) \geq ½$; and
(iii) $\mu(x_I) = -1$ implies $n_1(x_I) \neq n_{-1}(x_I)$ and $\pi_{-1}(x_I) \geq ½$.

Proof. (i) It will be first shown that (2) holds.

$$\text{For all } J \in S(I) \text{ such that } x_J \in X_2' \text{ and } \mu(x_J) = 0, n_1(x_J) = n_{-1}(x_J). \tag{2}$$

Let $x_J \in X_2'$ satisfy $J \in S(I)$ and $\mu(x_J) = 0$. Then, with $J = \{i, j\}$, either $x_i = x_j = 0$ or $\{x_i, x_j\} = \{1, -1\}$. In the first case, by (1), $n_1(x_J) = n_{-1}(x_J) = 0$. In the second case, by (1), $n_1(x_J) = n_{-1}(x_J) = 1$. In both cases, $n_1(x_J) = n_{-1}(x_J)$, which proves (2).

Taking (2) as the base case of an induction argument, choose $r \in \{3, \dots, k\}$, $x_K \in X_r'$ such that $K \in S(I)$ and $\mu(x_K) = 0$, and assume (3).

$$\text{For all } J \in S(I) \text{ such that } x_J \in X_2' \cup \dots \cup X_{r-1}' \text{ and } \mu(x_J) = 0, n_1(x_J) = n_{-1}(x_J). \tag{3}$$

It must be shown that $n_1(x_K) = n_{-1}(x_K)$. Case 1: for some $i \in K$, $x_i = 0$. With $i \in K$ such that $x_i = 0$, let $|S(K\backslash\{i\})| = t$. Then $|S(K)| = 2t + 2$, the members of $S(K)$ being $K\backslash\{i\}$, $\{i\}$ and, for each $J \in S(K\backslash\{i\})$, both J and $J \cup \{i\}$. By (1), $f(x_{K\backslash\{i\}}) = \mu(x_{K\backslash\{i\}}) = \mu(x_K) = 0$ and $f(x_i) = \mu(x_i) = 0$. By (1), for all $J \in S(K\backslash\{i\})$, $f(x_{J\cup\{i\}}) = \mu(x_{J\cup\{i\}}) = \mu(x_J) = f(x_J)$. Hence, for all $J \in S(K\backslash\{i\})$ and $a \in \{1, -1\}$, $f(x_J) = a$ if and only if $f(x_{J\cup\{i\}}) = a$. Consequently, $n_1(x_K) = 2n_1(x_{K\backslash\{i\}})$ and $n_{-1}(x_K) = 2n_{-1}(x_{K\backslash\{i\}})$. By (3), $n_1(x_{K\backslash\{i\}}) = n_{-1}(x_{K\backslash\{i\}})$. As a result, $n_1(x_K) = n_{-1}(x_K)$.

Case 2: for all $i \in K$, $x_i \neq 0$. Since $\mu(x_K) = 0$, the sets $K_1 = \{i \in K\colon x_i = 1\}$ and $K_{-1} = \{i \in K\colon x_i = -1\}$ have the same number of elements. It is then possible to define a bijection $\beta : K_1 \to K_{-1}$. For $J \in S(K)$, define $\beta(J)$ to be the member of $S(K)$ obtained from J by replacing each $i \in J \cap K_1$ by $\beta(i)$ and each $i \in J \cap K_{-1}$ by $\beta^{-1}(i)$. Letting $|K| = k$, $|S(K)| = 2^k - 2$, which is an even number. It then follows that $S(K)$ can be partitioned into two sets S_1 and S_2 such that $J \in S_1$ if and only if $\beta(J) \in S_2$. As a consequence, for all $J \in S_1$, $\mu(x_J) = 1$ if and only if $\mu(x_{\beta(J)}) = -1$. By (1), for all $J \in S_1$, $f(x_J) = 1$ if and only if $f(x_{\beta(J)}) = -1$. In consequence, $n_1(x_K) = n_{-1}(x_K)$.

(ii) It will be first shown that (4) holds.

$$\text{For all } J \in S(I) \text{ such that } x_J \in X_2' \text{ and } \mu(x_J) = 1,\ n_1(x_J) \neq n_{-1}(x_J) \text{ and } \pi_1(x_J) \geq ½. \quad (4)$$

Let $x_J \in X_2'$ satisfy $J \in S(I)$ and $\mu(x_J) = 1$. Then, with $J = \{i, j\}$, either $x_i = x_j = 1$ or $\{x_i, x_j\} = \{1, 0\}$. In the first case, by (1), $n_1(x_J) = 2$, $n_0(x_J) = n_{-1}(x_J) = 0$, and $\pi_1(x_J) = 1$. In the second case, by (1), $n_1(x_J) = n_0(x_J) = 1$, $n_{-1}(x_J) = 0$, and $\pi_1(x_J) = ½$. In both cases, $n_1(x_J) \neq n_{-1}(x_J)$ and $\pi_1(x_J) \geq ½$, which proves (4).

Taking (4) as the base case of an induction argument, choose $r \in \{3, \ldots, k\}$, $x_K \in X_r'$ such that $K \in S(I)$ and $\mu(x_K) = 1$, and assume (5).

$$\text{For all } J \in S(I) \text{ such that } x_J \in X_2' \cup \ldots \cup X_{r-1}' \text{ and } \mu(x_J) = 1,$$
$$n_1(x_J) \neq n_{-1}(x_J) \text{ and } \pi_1(x_J) \geq ½. \quad (5)$$

It must be shown that $n_1(x_K) \neq n_{-1}(x_K)$ and $\pi_1(x_K) \geq ½$. Case 1: for some $i \in K$, $x_i = 0$. With $i \in K$ such that $x_i = 0$, let $|S(K\backslash\{i\})| = t$. Then $|S(K)| = 2t + 2$, the members of $S(K)$ being $K\backslash\{i\}$, $\{i\}$ and, for each $J \in S(K\backslash\{i\})$, both J and $J \cup \{i\}$. By (1), $f(x_{K\backslash\{i\}}) = \mu(x_{K\backslash\{i\}}) = 1$ and $f(x_i) = \mu(x_i) = 0$. By (1), for all $J \in S(K\backslash\{i\})$, $f(x_{J\cup\{i\}}) = \mu(x_{J\cup\{i\}}) = \mu(x_J) = f(x_J)$. Hence, for all $J \in S(K\backslash\{i\})$ and $a \in \{-1, 0, 1\}$, $f(x_J) = a$ if and only if $f(x_{J\cup\{i\}}) = a$. In sum, $n_{-1}(x_K) = 2n_{-1}(x_{K\backslash\{i\}})$, $n_0(x_K) = 2n_0(x_{K\backslash\{i\}}) + 1$, and $n_1(x_K) = 2n_1(x_{K\backslash\{i\}}) + 1$. By

(5), $n_1(x_{K\setminus\{i\}}) \neq n_{-1}(x_{K\setminus\{i\}})$ and $\pi_1(x_{K\setminus\{i\}}) \geq ½$ imply $n_1(x_{K\setminus\{i\}}) > n_{-1}(x_{K\setminus\{i\}})$, so $n_1(x_K) > n_{-1}(x_K)$.

Since, by (5), $\pi_1(x_{K\setminus\{i\}}) \geq ½$, $n_1(x_{K\setminus\{i\}}) \geq n_{-1}(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}})$. In view of this, $2n_1(x_{K\setminus\{i\}}) \geq 2n_{-1}(x_{K\setminus\{i\}}) + 2n_0(x_{K\setminus\{i\}})$. Therefore, $2n_1(x_{K\setminus\{i\}}) + 1 \geq 2n_{-1}(x_{K\setminus\{i\}}) + 2n_0(x_{K\setminus\{i\}}) + 1$, which is equivalent to $n_1(x_K) \geq n_{-1}(x_K) + n_0(x_K)$, and this to $\pi_1(x_K) \geq ½$.

Case 2: for all $i \in K$, $x_i \neq 0$. As $\mu(x_K) = 1$, choose $i \in K$ such that $x_i = 1$. Then either $\mu(x_{K\setminus\{i\}}) = 1$ or $\mu(x_{K\setminus\{i\}}) = 0$. By (1), either $f(x_{K\setminus\{i\}}) = 1$ or $f(x_{K\setminus\{i\}}) = 0$. Case 2a: $f(x_{K\setminus\{i\}}) = 1$. By (1), $f(x_i) = \mu(x_i) = 1$ and, for all $J \in S(K\setminus\{i\})$ such that $f(x_J) \in \{0, 1\}$, $f(x_{J\cup\{i\}}) = \mu(x_{J\cup\{i\}}) = 1$. Hence, $n_1(x_K) = n_1(x_{K\setminus\{i\}}) + 2 + n_1(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}})$. Let $|S(K\setminus\{i\})| = t = n_1(x_{K\setminus\{i\}}) + n_{-1}(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}})$. Since $|S(K)| = 2t + 2 = n_1(x_K) + n_{-1}(x_K) + n_0(x_K)$, it follows that $n_{-1}(x_K) + n_0(x_K) = 2t + 2 - n_1(x_K)$. As a result, $n_{-1}(x_K) + n_0(x_K) = 2n_{-1}(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}})$. Given that $\pi_1(x_K) \geq ½$ if and only if $2n_1(x_K) \geq n_1(x_K) + n_0(x_K) + n_{-1}(x_K)$, and given that the latter is equivalent to $2[2n_1(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}}) + 2] \geq [2n_1(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}}) + 2] + 2n_{-1}(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}})$, it must be that $\pi_1(x_K) \geq ½$ if and only $n_1(x_{K\setminus\{i\}}) \geq n_{-1}(x_{K\setminus\{i\}}) - 1$. By (5), $\pi_1(x_{K\setminus\{i\}}) \geq ½$, so $n_1(x_{K\setminus\{i\}}) \geq n_{-1}(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}})$. This implies $n_1(x_{K\setminus\{i\}}) \geq n_{-1}(x_{K\setminus\{i\}}) - 1$, for which reason $\pi_1(x_K) \geq ½$.

To show that $n_1(x_K) \neq n_{-1}(x_K)$, either $n_0(x_K) > 0$ or $n_0(x_K) = 0$. If $n_0(x_K) > 0$, then $n_1(x_K) \neq n_{-1}(x_K)$ follows from $\pi_1(x_K) \geq ½$. If $n_0(x_K) = 0$, then, for all $j \in K$, $x_j = 1$, in which case $n_{-1}(x_K) = 0 < n_1(x_K)$.

Case 2b: $f(x_{K\setminus\{i\}}) = 0$. By (1), $f(x_i) = \mu(x_i) = 1$ and, for all $J \in S(K\setminus\{i\})$ such that $f(x_J) \in \{0, 1\}$, $f(x_{J\cup\{i\}}) = \mu(x_{J\cup\{i\}}) = 1$. Thus, $n_1(x_K) = n_1(x_{K\setminus\{i\}}) + 1 + n_1(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}})$. Letting $|S(K\setminus\{i\})| = t = n_1(x_{K\setminus\{i\}}) + n_{-1}(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}})$, then $|S(K)| = 2t + 2 = n_1(x_K) + n_{-1}(x_K) + n_0(x_K)$. Since $n_1(x_K) = 2n_1(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}}) + 1$, $n_{-1}(x_K) + n_0(x_K) = 2n_{-1}(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}}) + 1$. As (i) has been proved, (3) holds. By (3), $f(x_{K\setminus\{i\}}) = 0$ implies $n_1(x_{K\setminus\{i\}}) = n_{-1}(x_{K\setminus\{i\}})$. Hence, $\pi_1(x_K) = n_1(x_K)/[n_1(x_K) + n_0(x_K) + n_{-1}(x_K)] = [2n_1(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}}) + 1]/[2n_1(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}}) + 1 + 2n_{-1}(x_{K\setminus\{i\}}) + n_0(x_{K\setminus\{i\}}) + 1] = ½$. It follows from $f(x_{K\setminus\{i\}}) = 0$ that $n_0(x_K) > 0$, so $\pi_1(x_K) = ½$ and $n_0(x_K) > 0$ imply $n_1(x_K) > n_{-1}(x_K)$.

(iii). By the symmetry of 1 and −1 under the majority rule, the proof is analogous to the proof of (ii).■

Proposition 3.2. Let $X' \subseteq X$ be closed. A social welfare function $f : X' \rightarrow \{-1, 0, 1\}$ satisfies LIKE and SING if and only if f is the majority rule on X'.

Proof. "$\Leftarrow$" The majority rule on X' obviously satisfies SING. As regards LIKE, choose $x_I \in X'$. Case 1: $\pi_1(x_I) = \pi_{-1}(x_I)$. For LIKE to hold, it must be that $\mu(x_I) = 0$. Suppose not: $\mu(x_I) \neq 0$. By Lemma 3.1, $n_1(x_I) \neq n_{-1}(x_I)$, so $\pi_1(x_I) \neq \pi_{-1}(x_I)$: contradiction. Case 2: $\pi_1(x_I) \neq \pi_{-1}(x_I)$ and $\pi_1(x_I) \geq ½$. It has to be shown that $\mu(x_I) = 1$. If $\mu(x_I) = 0$, then, by Lemma 3.1, $n_1(x_I) = n_{-1}(x_I)$, which is equivalent to $\pi_1(x_I) = \pi_{-1}(x_I)$: contradiction. If $\mu(x_I) = -1$, then, by Lemma 3.1, $\pi_{-1}(x_I) \geq ½$. Since $\pi_1(x_I) \neq \pi_{-1}(x_I)$, $\pi_{-1}(x_I) \geq ½$ implies $\pi_1(x_I) < ½$: contradiction. Case 3: $\pi_1(x_I) \neq \pi_{-1}(x_I)$ and $\pi_{-1}(x_I) \geq ½$. Replace "1" by "−1" and "−1" by "1" in the proof of case 2.

"$\Rightarrow$" By SING, $f = \mu$ on X_1'. Taking this result as the base case of an induction argument, choose $n > 1$ and suppose that $f = \mu$ on $X_1' \cup \ldots \cup X_{n-1}'$. The proof amounts to showing that $f = \mu$ on X_n'. To this end, choose $x_I \in X_n'$. Case 1: $\mu(x_I) = 0$. By Lemma 3.1, $n_1(x_I) = n_{-1}(x_I)$. Therefore, $\pi_1(x_I) = \pi_{-1}(x_I)$. By LIKE, $f(x_I) = 0$. Case 2: $\mu(x_I) = 1$. By Lemma 3.1, $n_1(x_I) \neq n_{-1}(x_I)$ and $\pi_1(x_I) \geq ½$. Since $n_1(x_I) \neq n_{-1}(x_I)$ implies $\pi_1(x_I) \neq \pi_{-1}(x_I)$, by LIKE, $f(x_I) = 1$. Case 3: $\mu(x_I) = -1$. By Lemma 3.1, $n_1(x_I) \neq n_{-1}(x_I)$ and $\pi_{-1}(x_I) \geq ½$. As $n_1(x_I) \neq n_{-1}(x_I)$ implies $\pi_1(x_I) \neq \pi_{-1}(x_I)$, by LIKE, $f(x_I) = -1$.■

By Proposition 3.2, the majority rule on any closed set X' of preference profiles is, among those satisfying the arguably indispensable requirement SING, the only social welfare function on X' that satisfies LIKE. In this respect, LIKE can be viewed as a property essentially characterizing the majority rule.

4. Comments

LIKE has been motivated for the context in which it is possible to exclude subgroups. LIKE requires that, even if this possibility is allowed, the most likely outcome when some group is excluded is that the collective preference does not change. This robustness may also be an attractive feature when the original society is altered by exogenous reasons. For instance, if some individual accidentally dies after the collective decision has been made, is there any need to rethink the decision? When the decision is more likely to persist than to be changed, it is reasonable to maintain the original decision. LIKE is motivated by this presumption.

Another illustrative example is given by decisions that take into account the preferences of present and future generations (like the decision to fight against climate change). When the future generation will be alive, the present generation, the one that made the decision, will be dead. Would this fact be enough reason for the future generation to rethink the decision made by the past generation? If the present generation makes the decision applying the majority rule to the preferences of all generations, LIKE guarantees that the most probable event is that the future generation would not modify the decision inherited from the past generation.

There is another interesting context in which LIKE and Proposition 3.2 can be reinterpreted. This context can be termed “representative democracy”: the group I of individuals chooses a subgroup that is entrusted to make the decision on behalf of I. LIKE is then not motivated by the possibility of excluding a group but by that of selecting a group as representative. Roughly speaking, postulating LIKE means that the decision made by I corresponds to the decision that most subgroups would made. Consequently, if a subgroup is chosen at random and asked to make a decision, agreement with the decision of the whole group is the most likely event.

To interpret Proposition 3.2 in this context, compute, for every alternative, the number of representative subgroups favouring that alternative. Assuming SING means that singleton subgroups favour the alternative favoured by the only member of the subgroup. Let group I choose as representative subgroup one favouring the most favoured alternative. Then Proposition 3.2 says that the group I is actually making decisions by majority.

References

Aşan, G. and Sanver, M. R. (2002): “Another characterization of the majority rule”, *Economics Letters* 75, 409–413.

Fishburn, P. C. (1973): *The Theory of Social Choice*, Princeton University Press, Princeton, New Jersey.

May, K. O. (1952): “A set of independent, necessary and sufficient conditions for simple majority decision”, *Econometrica* 20, 680–684.

Miroiu, A. (2004): “Characterizing majority rule: From profiles to societies”, *Economics Letters* 85, 359–363.

Woeginger, G. J. (2003): “A new characterization of the majority rule”, *Economics Letters* 81, 89–94.

Woeginger, G. J. (2005): “More on the majority rule: Profiles, societies, and responsiveness”, *Economics Letters* 88, 7–11.

Xu, Y. and Zhong, Z. (2009): “Single profile of preferences with variable societies: A characterization of simple majority rule”, to appear in *Economics Letters*.