The Measurement of Employment Inequality Between Population Subgroups: Theory and Application

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1. Introduction

A major preoccupation of public policy is to ensure that people — of different sexes, ethnic backgrounds, religions, colours — are treated 'fairly' when they participate, either as job-seekers or as employees, in the labour market. There are two aspects to this concern. The first, is the treatment of persons *already* in employment: here the concern is that persons from different groups are rewarded differently and the moot point is whether such differences can be justified by their differences in productivity, or whether such differences in pay are the result of 'discrimination'. Borooah *et al*. (1995) and Harkness (1996) are examples of analysis which focus on this question. The second aspect relates to persons *seeking* employment: here the concern is whether the different degrees of success, which persons from different groups, meet with in obtaining jobs, is justified by inter-group differences in worker attributes or whether it is the result of prejudice, either for, or against, job-seekers from certain groups.

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This paper is concerned with the second, that is the job-seeking, aspect. It asks four broad questions. First, in the next section, what is a 'good' way of measuring inequality in employment outcomes between population sub-groups? Second, in Section 3, does it make a difference whether one bases the measure of inequality on the proportion of the working-age population, or on the proportion of the labour force, that is employed? Third, in Section 4, how do conventional indicators of employment inequality compare to this 'good' measure, assuming, of course, that such a measure exists? Fourth, in Section 5, how might the ideas developed in the previous sections be applied to 'real-world' instances of inter-group employment inequality? These applications are provided firstly, in the context of employment inequality between Catholics and Protestants in Northern Ireland and then in the context of employment inequality between the regions of the United Kingdom. Section 6 then concludes the paper.

2. Measuring inter-group inequality in employment outcomes

One way of measuring income inequality is by the natural logarithm of the ratio of the arithmetic mean income to the geometric mean income.[1] As Bourguignon (1979) demonstrates, such a measure is differentiable and income-homogenous of degree zero;[2] it also satisfies the symmetry axiom for population[3] and the Pigou-Dalton condition.[4] This idea translates very naturally, from its usual application to income inequality, to measuring the degree of inequality associated with labour market outcomes in which people in different population groups meet with different degrees of success in securing employment. This latter inequality is referred to, hereafter, as 'employment inequality' or as 'inequality of employment outcomes' or, simply, as 'inequality'. The purpose of this section is to develop measures for such inequality.

Consider a partition of a population into K mutually exclusive and collective exhaustive groups: group k contains N_k persons of working-age, M_k persons who are in the labour force,[5] E_k persons who are employed, U_k $(= M_k - E_k)$ persons who are unemployed and T_k $(= N_k - E_k)$ persons who are non-employed (jobless), $k = 1, 2 \ldots K$. Let the corresponding totals be represented by: $N = \sum N_k$; $M = \sum M_k$; $E = \sum E_k$, $U = \sum U_k$, and $T = \sum T_k$.

Define the (population-based) employment rate for group k as $e_k = E_k/N_k$ and denote by e and e^G, the arithmetic and geometric means, respectively, of e_k, $k = 1, 2 \ldots K$, where:

$$e \sum_{k=1}^{K} e_k n_k \qquad \text{and} \qquad e^G = \prod_{k=1}^{K} (e_k)^{n_k}$$

$$\text{where} \qquad n_k = N_k/N, \; \sum_{k=1}^{K} n_k = 1 \qquad [1]$$

Similarly, define $\upsilon_k = E_k/M_k$ as the (labour force-based) employment rate for group k ($u_k = (1 - \upsilon_k)$ being the unemployment rate) and denote by υ and υ^G, the arithmetic and geometric means, respectively, of υ_k, $k = 1, 2 \ldots K$, where:

$$\upsilon = \sum_{k=1}^{K} \upsilon_k m_k \qquad \text{and} \qquad \upsilon^G = \prod_{k=1}^{k} (\upsilon_k)^{m_k}$$

$$\text{where} \qquad m_k = M_k/M, \; \sum_{k=1}^{K} m_k = 1 \qquad [2]$$

Define the measures of inequality as:

$$J_e = \log(e/e^G) = \log(e) - \sum_{k=1}^{K} n_k \log(e_k)$$

$$J_\upsilon = \log(\upsilon/\upsilon^G) = \log(\upsilon) - \sum_{k=1}^{K} m_k \log(\upsilon_k) \qquad [3]$$

Since the inequality measures, J_e and J_υ, are defined as the natural logarithm of the ratio of the arithmetic mean employment rate to the geometric mean employment rate they: (i) satisfy the Pigou-Dalton condition, in that a transfer of employment from an 'employment-rich' to an 'employment-poor' group would reduce employment inequality; (ii) satisfy the symmetry condition, in that the evaluation of employment inequality does not depend upon the identity of the group; (iii) are differentiable, so that changes in inequality, consequent upon changes in employment rates, can be evaluated; and (iv) are homogenous of degree zero, in e_k and υ_k respectively, so that equi-proportionate changes in all e_k and υ_k leave J_e and J_υ, respectively, unchanged. This last property implies that inequality will remain unchanged if the pair-wise ratio of employment rates, across all the groups, does not alter.[6]

In addition to these attractive properties, the inequality measures J_e and J_v also have, along the lines suggested by Bourguignon (1979), an appealing interpretation. If social welfare is the sum of identical and concave group utility functions whose arguments are e_k (or v_k), then social welfare is maximized when e_k (or v_k) is the same for every group. If the utility functions are of the logarithmic form, then J_e represents the distance between the social welfare that would result from a given total of employment being distributed between the groups according to their shares in the working age population[7] and the actual distribution of employment; J_v represents the distance between the social welfare that would result from a given total of employment being distributed between the groups according to their shares in the labour force[8] and the actual distribution of employment. On this interpretation, therefore, reducing (employment) inequality and increasing social welfare are equivalent: social welfare is maximized when inequality is minimized.

Since, $e_k = E_k/N_k = (E_k/N_k)(N/E)(E/N) = (s_k/n_k)e$ and $v_k = E_k/M_k = (E_k/M_k)(M/E)(E/M) = (s_k/m_k)v$, J_e and J_v can, from equation [3], be also written as:

$$J_e = \log(e/e^G) = \log\left(\prod_{k=1}^{K} (e/e_k)^{n_k}\right) = \log\left(\prod_{k=1}^{K} (n_k/s_k)^{n_k}\right) = \sum_{k=1}^{K} n_k \log(n_k/s_k)$$

and [4]

$$J_v = \log(v/v^G) = \log\left(\prod_{k=1}^{K} (v/v_k)^{m_k}\right) = \log\left(\prod_{k=1}^{K} (m_k/s_k)^{m_k}\right) = \sum_{k=1}^{K} m_k \log(m_k/s_k)$$

where $s_k = E_k/E$, is the employment share of k, $\sum_{k=1}^{K} s_k = 1$.

From equation [4], $J_e = 0$ (inequality, defined on the basis of population shares, is minimized) when each group's share in total employment is equal to its population share; otherwise $J_e > 0$. Correspondingly, $J_v = 0$ (inequality, defined on the basis of labour force shares, is minimized) when each group's share in total employment is equal to its labour force share; otherwise, $J_v > 0$.

In order to examine the effects of changes in employment and labour force shares on changes in inequality differentiate J_v from equation [4], with respect to s_k and m_k, to obtain:[9]

$$\partial J_v/\partial s_k = -(m_k/s_k) \qquad \text{and} \qquad \partial J_v/\partial m_k = 1 + \log(m_k/s_k) \quad [5]$$

If $\Delta s_k = \Delta m_k = 0$, for $k \neq i$ and $k \neq j$ then,[10] from equation [5]:

$$\begin{aligned}\Delta J_v &\approx -(m_i/s_i)\Delta s_i - (m_j/s_j)\Delta s_j + \log(m_i/s_i)\Delta m_i \\ &\qquad + \log(m_j/s_j)\Delta m_j \\ &= [(m_j/s_j) - (m_i/s_i)]\Delta s_i + [\log(m_i/s_i) - \log(m_j/s_j)]\Delta m_i \quad [6]\end{aligned}$$

Equation [6] suggests that if group i is relatively disadvantaged, compared to group j, (that is, $m_i/s_i > m_j/s_j$), then, with no change in the labour force shares of either group ($\Delta m_i = \Delta m_j = 0$), an increase in the employment share of the disadvantaged group ($\Delta s_i > 0$) would lead to a fall in inequality ($\Delta J_v < 0$); conversely, with no change in employment shares ($\Delta s_i = \Delta s_j = 0$), an increase in the labour force share of the disadvantaged group ($\Delta m_i > 0$) would cause inequality to rise ($\Delta J_v > 0$).

If the employment and labour force shares of group i increased by the same number of percentage points[11] (that is, Δs_i, Δm_i, > 0, $\Delta s_i = \Delta m_i$) then inequality would increase (decrease) if the logarithmic difference, between groups i and j, in their labour force to employment ratios was greater (less) than the arithmetic difference in these ratios. More formally, if $\Delta s_i = -\Delta s_j = \Delta m_i = -\Delta m_j$, then:

$$\begin{aligned}&\Delta J_v \geqslant (\leqslant) 0 \\ &\text{if} \quad [\log(m_1/s_1) - \log(m_2/s_2)] \geqslant (\leqslant)[m_1/s_1 - m_2/s_2] \quad [7]\end{aligned}$$

3. Population or labour force shares as a basis for measuring employment inequality?

The previous section defined two measures of inequality: J_e, based on population shares and J_v, based on labour force shares.

The question is whether the two measures would yield different values for inequality? In order to answer this question, define the participation rate of group k as $\pi_k = M_k/N_k$ so that the overall participation rate, is given by: $\pi = \sum_{k=1}^{K} \pi_k n_k$. Then:

$$\begin{aligned} J_e - J_v &= \sum_{k=1}^{K} n_k \log(n_k/s_k) - \sum_{k=1}^{K} m_k \log(m_k/s_k) \\ &= \sum_{k=1}^{K} (n_k - m_k)\log(m_k/s_k) + \sum_{k=1}^{K} n_k \log(n_k/m_k) \\ &= \sum_{k=1}^{K} n_k[1 - (\pi_k/\pi)]\log(m_k/s_k) + \sum_{k=1}^{K} n_k \log(\pi/\pi_k) \end{aligned} \quad [8]$$

and a sufficient condition for $J_e = J_v$ is that the participation rates of all the groups are the same, that is: $\pi_1 = \ldots \pi_K = \pi$.

In general, however, $J_e \neq J_v$ and the sign of $J_e - J_v$ cannot be predicted. In order to see this, consider the case where $K = 2$, with the participation rate for group 1 being lower than the average participation rate ($\pi_1 < \pi$) and with this group having a higher share in the labour force than in employment ($m_1 > s_1$). By definition, for group 2, $\pi_2 > \pi$ and $m_2 < s_2$. Since[12] $m_k = (\pi_k/\pi)n_k$, $m_1 < n_1$ and $m_2 > n_2$ and, therefore, $m_1/s_1 < n_1/s_1$ and $m_2/s_2 > n_2/s_2$. Hence, from equation [8]: $\sum_{k=1}^{2} n_k[1 - (\pi_k/\pi)]\log(m_k/s_k) > 0$. However, the sign of $\sum_{k=1}^{2} n_k \log(\pi/\pi_k) = n_1 \log(\pi/\pi_1) + n_2 \log(\pi/\pi_2) = \log(\pi/\pi_2) + n_1 \log(\pi_2/\pi_1)$ cannot be predicted,[13] leaving the sign of $J_e - J_v$ indeterminate. However, if n_1 is sufficiently large — so that, $n_1 > -\,[\log(\pi/\pi_2)/\log(\pi_2/\pi_1)]$ — then one may expect that, under the conditions of this example, $J_e > J_v$.

Of course, the appropriateness of using population or labour force shares in computing employment inequality depends upon *why* people are outside the labour force. If most people are jobless because they are *discouraged* workers (they want work but are not seeking work because they believe that there no jobs available for them) then it would be appropriate to use population shares because to use labour force shares would mean excluding persons who are 'quasi-unemployed'. On the other hand, if most people are jobless because they do not want to work (homemakers; students) then it would be appropriate to use labour force shares because to use population shares would mean ignoring the preference of those outside the labour force not to work. In practice, a satisfactory compromise might be to use labour force shares, *but only after extending the definition of the labour force to include discouraged workers*.

What can be predicted, however, is the direction, and magnitude, of changes in J_e and J_υ. Using the methodology of Theil and Sorooshian's (1979) analysis of regional income inequality, J_e and J_υ can, respectively, be viewed as functions of e_k and N_k, and of υ_k and M_k, with the following logarithmic derivatives, obtained, firstly, by using equation (4) to differentiate J_e and J_υ, with respect to, respectively, $\log e_k$ and $\log \upsilon_k$:

$$\partial J_e/\partial \log e_k = s_k - n_k \qquad \text{and} \qquad \partial J_\upsilon/\partial \log \upsilon_k = s_k - m_k \qquad [9]$$

Then differentiating J_e and J_υ, with respect to, respectively, $\log N_k$ and $\log M_k$:

$$\begin{aligned} \partial J_e/\partial \log N_k &= \{s_k - n_k + n_k[\log(n_k/s_k) - J_e]\} \\ \partial J_\upsilon/\partial \log M_k &= \{s_k - m_k + m_k[\log(m_k/s_k) - J_\upsilon]\} \end{aligned} \qquad [10]$$

By equation [9], if the (population-based) employment rate for a group equals the (corresponding) average employment rate ($e_k = e$), then its employment share equals its population share ($s_k = n_k$) and a small change in its employment rate, e_k, will leave inequality, as measured by J_e, unchanged. If the (labour force-based) employment rate for a group equals the (corresponding) average employment rate[14] ($\upsilon_k = \upsilon$), then its employment share equals its labour market share ($s_k = m_k$) and a small change in its employment rate, υ_k, will leave inequality, as measured by J_υ, unchanged. If $e_k > (<) e$ (that is, $s_k > (<) n_k$), or $\upsilon_k > (<) \upsilon$ (that is, $s_k > (<) m_k$) then inequality would increase (decrease) consequent upon an increase in group k's employment rate.

By equation [10], a small increase in the population of group k, N_k, or in its numbers in the labour force, M_k, would, if $s_k = n_k$ or $s_k = m_k$ cause inequality to *fall* by, respectively, $n_k J_e$ and $m_k J_v$. However, inequality would increase with increasing numbers in group k when group k was sufficiently 'employment rich' ($e_k > e$) or when it was sufficiently 'employment poor' ($e_k < e$).

Bearing in mind that that $M_k = \pi_k N_k$ and that $e_k = E_k/N_k = (E_k/M_k)(M_k/N_k) = \pi_k v_k$, changes in J_e and J_v are related since:

$$\begin{aligned}
\partial J_\upsilon/\partial \log e_k &= (\partial J_\upsilon/\partial \log \upsilon_k)(\partial \log \upsilon_k/\partial \log e_k) \\
&= (s_k - m_k)(e_k/\upsilon_k)[\pi_k + e_k(\partial\pi_k/\partial e_k)]\} \\
&= (s_k - m_k)[1 + (e_k/\pi_k)(\partial\pi_k/\partial e_k)] \\
&\geqslant \partial J_\upsilon/\partial \log \upsilon_k \qquad \text{since} \qquad \partial\pi_k/\partial e_k \geqslant 0 \\
\partial J_e/\partial \log \upsilon_k &= (\partial J_e/\partial \log e_k)(\partial \log e_k/\partial \log \upsilon_k) \\
&= (s_k - n_k)(\upsilon_k/e_k)(\partial e_k/\partial \upsilon_k) \\
&= (s_k - n_k)\pi_k\{([\pi_k - \upsilon_k(\partial\pi_k/\upsilon_k)]/\pi_k^2\} \\
&= (s_k - m_k)[1 - (\upsilon_k/\pi_k)(\partial\pi_k/\partial\upsilon_k)] \\
&\leqslant \partial J_e/\partial \log e_k \qquad \text{since} \qquad \partial\pi_k/\partial\upsilon_k \geqslant 0 \\
\partial J_\upsilon/\partial \log N_k &= \partial J_e/\partial \log M_k
\end{aligned} \tag{11}$$

From equation [11] it follows that, if $\partial\pi_k/\partial e_k = 0$ and $n_k = m_k$, then:

$$\partial J_e/\partial \log e_k - \partial J_\upsilon/\partial \log e_k = [m_k - n_k] - (s_k - m_k)[(e_k/\pi_k)(\partial\pi_k/\partial e_k)] = 0 \tag{12}$$

In other words, in an echo of equation [8], equation [12] says that the change in inequality, whether measured by population (J_e) or by labour force (J_υ) shares, consequent upon a change in the (population-based) employment rate of group k, is the same, provided that the participation rate of group k is insensitive to its (population-based) employment rate and is also equal to the average participation rate.[15]

An application of equation [11] may be seen by considering a group whose population share is larger than its share of the labour force (that is, $n_k > m_k$ or, equivalently, $\pi_k < \pi$) and whose share of employment is less than its population share ($s_k < n_k$) but is equal to its labour force share ($s_k = m_k$).[16] Then, by equation [11], for a small increase in the (population based) employment rate of group k:

$$\partial J_v / \partial \log e_k = 0 \qquad \text{but} \qquad \partial J_e / \partial \log e_k = s_k - n_k < 0.$$

Hence, under this scenario, consequent upon an increase in the (population based) employment rate of group k, inequality, as measured by J_e, will fall, even though inequality, as measured by J_v, remains unchanged.

Conversely, consider a group whose share of the population is smaller than its share of the labour force (that is, $n_k < m_k$ or, equivalently, $\pi_k > \pi$) and whose share of employment is greater than its population share ($s_k > n_k$) but is equal to its labour force share ($s_k = m_k$).[17] Then, by equation [11]:

$$\partial J_v / \partial \log e_k = 0 \qquad \text{but} \qquad \partial J_e / \partial \log e_k = s_k - n_k > 0.$$

Hence, under this scenario, consequent upon an increase in the (population-based) employment rate of group k, inequality, as measured by J_e will rise, even though inequality, as measured by J_v, remains unchanged.

From equations [9] and [10], the change in inequality can be expressed as:

$$\begin{aligned} \Delta J_e &= \sum_{k=1}^{K} a_k \Delta \log e_k + \sum_{k=1}^{K} (a_k + b_k) \Delta \log N_k \\ \Delta J_v &= \sum_{k=1}^{K} c_k \Delta \log v_k + \sum_{k=1}^{K} (c_k + d_k) \Delta \log M_k \end{aligned} \qquad [13]$$

where:

$$a_k = s_k - n_k, \qquad b_k = n_k[\log(n_k/s_k) - J_e]$$

$$c_k = s_k - m_k, \qquad d_k = m_k[\log(m_k/s_k) - J_v]$$

If, in equation [13], the employment rates for the different groups changed at the same rate ($\Delta \log e_k = g_e$ and $\Delta \log v_k = g_v$, $\forall k = 1 \ldots \mathrm{K}$) and all the group populations/labour forces grew at the same rate ($\Delta \log N_k = g_N$ and $\Delta \log M_k = g_M, \forall k = 1 \ldots \mathrm{K}$), then:

$$\Delta J_e = g_e \sum_{k=1}^{K} (s_k - n_k) + g_N \sum_{k=1}^{K} (s_k - n_k) + g_N \sum_{k=1}^{K} n_k \log(n_k/s_k) - g_N J_e \sum_{k=1}^{K} n_k = 0$$

$$\Delta J_\upsilon = g_\upsilon \sum_{k=1}^{K} (s_k - m_k) + g_M \sum_{k=1}^{K} (s_k - m_k) + g_M \sum_{k=1}^{K} m_k \log(m_k/s_k) - g_M J_\upsilon \sum_{k=1}^{K} m_k = 0 \quad [14]$$

Equation [14] reinforces the point made earlier that equiproportionate changes in the employment rates, with unchanged population/labour force shares, would leave the values of the inequality indices, J_e and J_υ, unchanged.

4. Good and bad indicators of employment inequality

The previous section established measures of employment inequality that were 'good' in that they possessed a number of desirable properties; in addition, they were also capable of interpretation in terms of the distance between the optimal level of social welfare and the level which actually existed. However, discussion of inter-group employment inequality is usually conducted in terms of more 'rough-and-ready' inequality indicators, in the belief that movements in such indicators reflect movements in underlying inequality. It is important, therefore, to examine instances where this belief does, and does not, have support, in terms of the inequality measures set out earlier, and, through such examination, to separate inequality indicators into those that do ('good' indicators), and those that do not ('bad' indicators), mirror movements in J_e and J_υ.

This section evaluates some of the indicators of employment inequality used when there are only two groups, in terms of the measure J_υ of equation [4]. These indicators, which have been collated by Gudgin and Breen (1994), are, usually, defined in terms of inter-group (labour force-based) employment, or unemployment, rates and expressed either as ratios, or as differences, of these rates. Some of the more commonly used of such indicators are discussed below:

(A) The employment rate ratio, $(E_1/M_1)/(E_2/M_2) = \upsilon_1/\upsilon_2$

This ratio will be constant for equi-proportionate changes in υ_1 and υ_2, that is when $\Delta \log \upsilon_1 = \Delta \log \upsilon_2$. Now, if $\Delta \log M_1 = \Delta \log M_2 = g_M$, then, from equation [13]:

$$\Delta J_\upsilon = \sum_{k=1}^{2} (s_k - m_1)\Delta \log \upsilon_k$$

$$= (s_1 - m_1)[\Delta \log \upsilon_1 - \Delta \log \upsilon_2] \qquad [15]$$

since, when $K = 2$, $s_1 + s_2 = m_1 + m_2 = 1$, so that: $s_2 - m_2 = -(s_1 - m_1)$.

If group 1 is the disadvantaged group, so that $s_1 < m_1$, then:

$$\Delta J_\upsilon = 0, \text{ if } \Delta \log \upsilon_2, \text{ that is, } \upsilon_1/\upsilon_2 \text{ constant}$$

$$\Delta J_\upsilon > (<) 0, \text{ if } \Delta \log \upsilon_1 < (>) \Delta \log \upsilon_2, \text{ that is, } \upsilon_1/\upsilon_2 \text{ falls(rises)} \qquad [16]$$

The employment rate ratio is thus a *good* indicator of employment inequality since, from equation [16], movements in this ratio mirror changes in J_υ, *provided labour force shares remain unchanged*. In the face of changes in labour force shares ($\Delta \log M_1 \neq \Delta \log M_2$), it would not be possible to deduce changes in inequality from movements in the employment rate ratio, leaving no alternative but to calculate the values of the inequality index, J_υ.

(B) The employment rate difference, $(E_1/M_1) - (E_2/M_2) = \upsilon_1 - \upsilon_2$

Since, $\Delta \log \upsilon_k/\upsilon_k$, from equation [15]:

$$\Delta J_\upsilon = \sum_{k=1}^{2} (s_k - m_k)(\Delta \upsilon_k/\upsilon_k) = (s_1 - m_1)[(\Delta \upsilon_1/\upsilon_1) - (\Delta \upsilon_2/\upsilon_2)] \qquad [17]$$

The employment rate difference is unaltered if $\Delta v_1 = \Delta v_2$, in which case equation [17] becomes:

$$\Delta J_v = \sum_{k=1}^{2} (s_k - m_k)(\Delta v_k / v_k) = (s_1 - m_1)[(1/v_2)]\Delta v \qquad [18]$$

If group 1 is the disadvantaged group (so that $s_1 < m_1$), then $v_1 < v_2$ and $\Delta J_v < 0$. The employment rate difference is, therefore, a *bad* indicator of employment inequality since one cannot infer from its constancy that inequality — as measured by J_v — remains unchanged. The same absolute increase, Δv, in the employment rates of the two groups, gives, with $v_1 < v_2$, a higher percentage increase to group 1, than to group 2, and hence, through a rise in the employment rate ratio, v_1/v_2, leads to a fall in inequality.

(C) The unemployment rate ratio, $(U_1/M_1)/(U_2/M_2 = u_1/u_2 = (1 - v_1)/(1 - v_2)$

Differentiating J_v with respect to log u_k, and using equation [9] yields:

$$\partial J_v / \partial \log u_k = (\partial J_v / \partial \log v_k)(\partial \log v_k / \partial \log u_k)$$
$$= -(s_k - m_k)(u_k / v_k) \qquad [19]$$

Consequently, when $K = 2$:

$$\Delta J_v = -(u_1/v_1)(s_1 - m_1)\Delta \log u_1 + (u_2/v_2)(s_1 - m_1)\Delta \log u_2$$
$$= (s_1 - m_1)[(u_2/v_2)\Delta \log u_2 - (u_1/v_1)\Delta \log u_1] \qquad [20]$$

If $s_1 < m_1$, so that group 1 is the disadvantaged group, then:

$\Delta J_v = 0$, if, and only if, $\Delta \log u_2 / \Delta \log u_1 = u_1 v_2 / u_2 v_1$

$\Delta J_v >(<) 0$, if, and only if, $\Delta \log u_2 / \Delta \log u_1 <(>) u_1 v_2 / u_2 v_1$

[21]

Since, by virtue of group 1 being the more disadvantaged group, $u_1 v_2 / u_2 v_1 > 1$, equation [21] implies that for inequality, as measured by J_v, to remain unchanged, the unemployment rate ratio, u_1/u_2, must fall, since the percentage change in u_2 ($\Delta \log u_2$) must exceed the percentage change in u_1 ($\Delta \log u_1$). Conversely, if the ratio of unemployment rates remained unchanged

$(\Delta \log u_2 = \Delta \log u_1 = g_u)$ then:

$$\Delta J_v = g_u(s_1 - m_1)[(u_2/v_2) - (u_1/v_1)]$$
$$= g_u(s_1 - m_1)(v_1 v_2)^{-1}[v_1 u_2 - v_2 u_1] \quad [22]$$

Since, the fact that group 1 is disadvantaged, relative to group 2, implies that the term $[v_1 u_2 - v_2 u_1]$, in equation [22] above, is negative, $\Delta J_v > 0$ if $g_u > 0$ and $\Delta J_v < 0$ if $g_u < 0$. In other words, if the constancy of the unemployment rate ratio is achieved through an equi-proportionate *rise* in group unemployment rates $(g_u > 0)$, then inequality will *increase*; on the other hand, if this constancy is achieved through an equi-proportionate *fall* in group unemployment rates $(g_u < 0)$, then inequality will *decrease*; if there is no change in group unemployment rates $(g_u = 0)$, then inequality will not change.

Movements in the unemployment rate ratio are, therefore, a *bad* indicator of movements in the level of inequality since one cannot, from the fact that this ratio might have remained unchanged over a period of time, infer that inequality levels, also, were unchanged over that period: as the above discussion indicates, inequality may have increased, decreased, or remained unchanged, depending upon how the constancy of the ratio was obtained. From a policy point of view this is an important finding since, in most discussions of fair employment, the unemployment rate ratio occupies primacy as an indicator of the 'fairness' of inter-group employment outcomes.[18]

(D) The unemployment rate difference, $u_1 - u_2$

From equation (20):

$$\Delta J_v = -(u_1/v_1)(s_1 - m_1)\Delta \log u_1 + (u_2/v_2)(s_1 - m_1)\Delta \log u_2$$
$$= (s_1 - m_1)[(\Delta u_2/v_2) - (\Delta u_1/v_1)] \quad [23]$$

The unemployment rate difference is unaltered if $\Delta u_1 = \Delta u_2$, in which case equation [23] becomes:

$$\Delta J_v = (s_1 - m_1)[(1/v_2) - (1/v_1)]\Delta u \quad [24]$$

If group 1 is the disadvantaged group (so that $s_1 < m_1$), then $v_1 < v_2$ and $\Delta J_v > 0$. The unemployment rate difference (like the employment rate difference) is, therefore, a *bad* indicator of inequality since one cannot infer from its constancy that inequality — as measured by J_{υ} — remains unchanged. The same absolute increase, Δu, in the unemployment rates of the two groups, leads to an increase in inequality and this should be contrasted with an earlier result, from equation [18], where the same absolute increase, $\Delta\upsilon$, in the employment rates of the two groups, led to a decrease in inequality. It should be emphasized that the employment rate ratio, which, as the previous discussion has shown, is a 'good' indicator of employment inequality, is only capable of use in a two-group context and, then, only under the assumption of constant labour force shares. When labour force shares are changing, or if the number of groups to be analysed exceeds two, then what is needed is an inequality measure, with desirable properties, that maps the vector of employment outcomes for the different groups into a scalar statistic. The inequality measures, J_e and J_{υ}, proposed in this paper, are designed to do precisely that.

5. Application to inter-community and inter-regional employment inequality

The analysis of the previous sections was applied to two separate areas of employment inequality: that between Catholics and Protestants in Northern Ireland and that between the regions of the United Kingdom. The results for both these areas were based on analysis of Labour Force Survey (LFS) data. Data for Northern Ireland, disaggregated by religion, was obtained from Northern Ireland Statistics and Research Agency (1996) for the period 1990–1994; data for the UK, disaggregated by region, was obtained from Office of National Statistics (1996) for the period 1984–96. Turning first to Northern Ireland, Table 1 below shows, for men, participation rates (π_k), unemployment rates (u_k) and shares in: employment (s_k), the labour force (m_k) and the working-age population (n_k) for, respectively, Catholics, Protestants and the entire population. Tables 2 and 3 reproduce the same information for, respectively, women and both sexes.

As Tables 1–3 make clear, the share of Catholics — for men, women and both sexes — in employment (s_1) was consistently less than their share in the labour force (m_1) and, since the Catholic participation rate was lower than the average participation rate ($\pi_1 < \pi$ or, equivalently, $m_1 < n_1$) this shortfall was even larger when compared to their population share (n_1). Conversely, the

Table 1. Participation and unemployment rates (%) and shares (%) in employment, labour force and working-age population Catholics and Protestants (men)

Year	s_1	s_2	m_1	m_2	n_1	n_2	π_1	π_2	u_1	u_2
1990	38	62	41	59	42	58	80	85	20	10
1991	38	62	42	58	44	56	79	87	23	9
1992	34	66	38	62	40	60	77	85	24	10
1993	38	62	41	59	42	58	80	83	23	11
1994	37	63	40	60	43	57	74	83	22	11

1 = Catholic; 2 = Protestant.
Source: NISRA.

Table 2. Participation and unemployment rates (%) and shares (%) in employment, labour force and working-age population Catholics and Protestants (women)

Year	s_1	s_2	m_1	m_2	n_1	n_2	π_1	π_2	u_1	u_2
1990	37	63	38	62	43	57	54	66	9	7
1991	39	61	40	60	44	56	56	65	11	6
1992	35	65	36	64	41	59	54	68	10	7
1993	38	62	39	61	43	57	55	64	11	7
1994	39	61	39	61	43	57	53	63	8	5

1 = Catholic; 2 = Protestant.
Source: NISRA.

Table 3. Participation and unemployment rates (%) and shares (%) in employment, labour force and working-age population Catholics and Protestants (both sexes)

Year	s_1	s_2	m_1	m_2	n_1	n_2	π_1	π_2	u_1	u_2
1990	38	62	40	60	43	57	67	76	16	9
1991	38	62	41	59	44	56	68	76	18	8
1992	35	65	37	63	41	59	66	77	18	9
1993	38	62	40	60	42	58	67	74	18	9
1994	38	62	40	60	43	57	64	73	16	9

1 = Catholic; 2 = Protestant.
Source: NISRA.

share of Protestants in employment (s_2) was consistently greater than their share in the labour force (m_2)and, since the Protestant participation rate was greater than the average participation rate ($\pi_2 > \pi$ or, equivalently, $m_2 > n_2$) this surplus was even larger when compared to their population share (n_2). The information on employment shares was combined with information on labour market, and population, shares (Tables 1, 2 and 3) to calculate, for each year of the period 1990–94, values for, respectively, J_v and J_e and the results from these calculations are shown in Table 4. Table 4 also shows, for each year, the corresponding unemployment rate ratio since, as noted earlier, much of policy discussion about fair employment in Northern Ireland is conducted in the context of movements in this ratio.

Employment inequality in Northern Ireland was higher when measured on the basis of population, rather than labour force, shares: this reflected the fact that the population share of Catholics was greater than their labour force share while, for Protestants, precisely the opposite was true. On both measures, inequality between Catholic and Protestant men rose[19] between 1990 and 1991 because, while in both these years, the employment share of Catholics and Protestants was 38 percent and 62 percent respectively, the Catholic share in the labour force, and in the population, increased, between these years, by, respectively 1 and 2 percent. Between 1991 and 1992, inequality increased only slightly — a fall of 4 percent in the Catholic employment share coincided with an identical fall in its labour force share, and a fall of 3 percent in its population share. Between 1992 and 1993, employment inequality between Catholic and Protestant males fell — the

Table 4. The level of Catholic-Protestant employment inequality in Northern Ireland, by sex

	Men			**Women**			**Both sexes**		
Year	J_e	J_v	u_1/u_2	J_e	J_v	u_1/u_2	J_e	J_v	u_1/u_2
1990	4.19	1.89	2.00	7.15	0.21	1.29	5.35	0.84	1.78
1991	8.44	3.35	2.56	4.44	0.21	1.83	6.82	1.89	2.25
1992	8.76	3.50	2.40	8.99	0.22	1.43	6.89	0.87	2.00
1993	3.18	1.89	2.09	4.54	0.21	1.57	4.07	0.84	2.00
1994	7.04	1.91	2.00	3.63	0.00	1.60	5.64	0.84	1.78

Notes: The values of J_e and J_v are shown as: calculated values $\times$ 1000.
1 = Catholic; 2 = Protestant.

rise of 3 percent in the Catholic employment share coincided with an identical rise in its labour force share, and a rise of 2 percent in its population share.[20] Between 1993 and 1994 male employment inequality changed only slightly on a labour force basis — the fall of 1 percent in the Catholic employment share was matched by an identical fall in its labour force share — but considerably on a population basis since the population share of Catholics, between 1993 and 1994, increased by 1 percent.

The evolution of employment inequality, over 1990–94, between Catholic and Protestant women was very different from that for men. Inter-community female employment inequality, when measured on the basis of labour force shares, was very low for every year of the period and indeed, in 1994, when the Catholic labour force share, at 39 percent, was equal to its employment share,[21] inequality was non-existent.

These low levels of inequality, as measured by J_v, were the result of Catholic women having a lower participation rate than Protestant women (Table 2). This meant that while the average, over the period, proportion of Catholic women in the working age population was 43 percent (Protestants: 57 percent), their labour market share was only 38 percent (Protestants: 62 percent).[22] Consequently, while the labour force share of Catholic women was close to their employment share, because of the low participation rates of Catholic women, their population share was much higher and, as a result, the calculated values of J_e were much greater than those for J_v. If the participation rate of Catholic women had been the same as that for Protestant women, then $\pi_1 = \pi_2 = \pi$, and, by equation [8], $J_v = J_e$ or, in other words, much higher levels of employment inequality between Catholic and Protestant women would have been recorded. Needless to say, an identical conclusion would hold if Catholic men had had the same participation rate as Protestant men.

The second empirical application was to examine the degree of employment inequality, for each year of the period 1984–96, between the twelve standard regions of the UK.[23] The measured levels of this inequality, in terms of J_v, are shown in Table 5.

Table 5 indicates that inter-regional inequality in the UK was much lower for women than it was for men. For both men and women employment inequality was highest in the years 1984–88. Thereafter, it fell quite sharply and though, over 1989–96, there were fluctuations in the level of inequality, the levels observed in 1984–88 were never reached.

Table 5. Inter-regional employment inequality in the UK: 1984–96

Year	J_v: Men	J_v: Women	J_v: both sexes
1984	0.925	0.320	0.620
1985	0.809	0.257	0.533
1986	0.701	0.216	0.451
1987	0.708	0.239	0.451
1988	0.787	0.232	0.501
1989	0.501	0.190	0.346
1990	0.336	0.134	0.223
1991	0.268	0.176	0.148
1992	0.185	0.098	0.128
1993	0.250	0.089	0.232
1994	0.359	0.086	0.198
1995	0.221	0.076	0.121
1996	0.295	0.098	0.176

Note: The values of J_v are shown as: calculated values × 1000.

As the discussion of Section 2 indicated, employment inequality would change because employment rates, v_k, and/or labour force shares, m_k, changed (per-capita changes versus population changes) and an interesting empirical question is to examine the amount of the observed change in inequality that could be ascribed to each of these two forces. Table 6 shows the results of

Table 6. Decomposing the change in employment inequality by changes in employment rate and labour force

	Men			Women			Both sexes		
Year	J_v Agg	J_v Δlgv	J_v ΔlgM	J_v Agg	J_v Δlgv	J_v ΔlgM	J_v Agg	J_v Δlgv	J_v ΔlgM
1985	−0.116	−0.114	−0.002	−0.063	−0.061	−0.002	−0.087	−0.085	−0.002
1986	−0.108	−0.110	0.002	−0.041	−0.041	0.000	−0.082	−0.084	0.002
1987	0.007	0.007	0.000	0.023	0.024	−0.001	0.000	0.000	0.000
1988	0.079	0.077	0.002	−0.007	−0.007	0.000	0.050	0.047	0.003
1989	−0.286	−0.285	−0.001	−0.042	−0.041	−0.001	−0.155	−0.153	−0.002
1990	−0.165	−0.163	−0.002	−0.056	−0.056	0.000	−0.122	−0.122	0.000
1991	−0.068	−0.068	0.000	0.043	0.043	0.000	−0.075	−0.076	0.001
1992	−0.083	−0.083	0.000	−0.080	−0.080	0.000	−0.020	−0.020	0.000
1993	0.065	0.064	0.001	−0.009	−0.009	0.000	0.104	0.103	0.000
1994	0.108	0.110	−0.002	−0.003	−0.003	0.000	−0.033	−0.033	0.000
1995	−0.138	−0.139	0.001	−0.010	−0.010	0.000	−0.077	−0.077	0.000
1996	0.074	0.074	0.000	0.022	0.022	0.000	0.054	0.054	0.000

decomposing, *vide* equation [13], over 1984–96, the change in inequality, between successive years, into that part which was due to changes in the employment rate and that part which was due to population changes.

Table 6 makes clear that practically all of the year-on-year change in employment inequality between the regions of the UK, over 1984–96, was the result of changes in employment rates in the different regions and hardly any of the inequality change was the result of changes in the sizes of the regional labour forces.

6. Conclusions

This paper represents an inquiry into employment inequality between population sub-groups. The first, and most obvious, starting point to this inquiry — and which forms the subject matter of this paper — was to ask how such inequality should be measured. This question was answered in terms of an idea adapted from the area of income inequality. This was to use, as the inequality index, the natural logarithm of the ratio of the arithmetic mean, to the geometric mean, of the employment rate, where the latter was defined as either the proportion of the working-age population, or the proportion of the labour force, that was employed. It was shown that such an index had several attractive properties and also admitted of an appealing interpretation in terms of social welfare. Another advantage of this general measure was that it could be used to evaluate more conventional indicators of employment inequality. When this evaluation was carried out, it was found that most conventional indicators of employment inequality, as used in the two-group case were unsatisfactory: the only one that was not, was the ratio of employment rates. However, this ratio could only be regarded as a 'good' indicator of employment inequality when there were only two groups and then, only under the assumption of constant labour force shares. Under circumstances where labour force shares were changing, or the number of groups to be analysed exceeded two, what was needed was an inequality measure, with desirable properties, that mapped the vector of employment outcomes, for the different groups, into a scalar statistic. The employment inequality measures, J_e and J_v, proposed in this paper, are designed to do precisely that.

Notes

[1] See Bourguignon (1979) and Theil (1967).
[2] That is, equi-proportionate changes in income leave inequality unchanged.
[3] This corresponds to the idea that the identity of the income earner is irrelevant for the measurement of inequality (anonymity rule).
[4] A transfer of income from a richer to a poorer person reduces inequality.
[5] That is, either employed or, if jobless, available and searching for employment.
[6] That is e_i/e_j and/or v_i/v_j, for all $i, j = 1 \ldots K$.
[7] That is: $e_1 = \cdots = e_K = e$.
[8] That is: $v_1 = \cdots = v_K = v$.
[9] Identical conclusions hold for the effects of changes in employment and population shares on changes in J_e.
[10] Bearing in mind that $\Delta s_i + \Delta s_j = \Delta m_i + \Delta m_j = 0$.
[11] By definition, employment and labour force shares of group j would decrease by the same number of percentage points.
[12] $m_k = M_k/M = (M_k/N_k)(N/M)(N_k/N)$.
[13] Since the first term is negative and the second term is positive.
[14] Then the unemployment rate for the group equals the average unemployment rate ($u_k = u$).
[15] That is, $\pi_k = \pi$, since $m_k = (M_k/M) = (\pi_k N_k/\pi N) = (\pi_k/\pi)n_k$.
[16] Catholic women in Northern Ireland provide an example of such a group: see Section 5.
[17] Protestant women in Northern Ireland provide an example of such a group: see Section 5.
[18] See Gudgin and Breen (1994) for a discussion of the importance attached to this ratio in the context of Catholic and Protestant employment outcomes in Northern Ireland.
[19] Note that the inequality indices, J_e and J_v are ordinal, not cardinal measures: while one can say that there was more inequality in 1991 than in 1990, one cannot say how much more.
[20] See equation (7) for an analysis of the effects on inequality of equal (percentage point) changes in employment and labour force/population shares.
[21] By definition, this equality also held for Protestant women.
[22] Remembering that $m_1 = (\pi_1/\pi)/n_1$, this implied that the Catholic female participation (54 percent) was only 88 percent of the overall female participation rate of 61 percent.
[23] These were: Greater London; the South-East (excluding Greater London); the South-West; East Anglia; the West Midlands; the East Midlands; Yorkshire and Humberside; the North-West; Wales; the North; Scotland; and Northern Ireland.

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