The Predictive Power of Conditional Models: What Lessons to Draw with Financial Crisis in the Case of Pre-Emerging Capital Markets?

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Abstract. The uncertainty plays a central role in most of the problems which addressed by the modern financial theory. For some time, we know that the uncertainty under the speculative price varies over the time. However, it is only recently that a lot of studies in applied finance and monetary economics using the explicit modelling of time series involving the second and the higher moments of variables. Indeed, the first tool appeared in order to model such variables has been introduced by Engel (1982). This is the autoregressive conditional heteroskedasticity and its many extensions. Thus, with the emergence and development of these models, Value-at-Risk, which plays a major role in assessment and risk management of financial institutions, has become a more effective tool to measure the risk of asset holdings. Following the current financial debacle, we give the simple question about the progress and some achievements made in the context of emerging and pre-emergent financial markets microstructure which can sustain and limit the future fluctuations. Today, we know that the crisis has no spared any financial market in the world. The magnitude and damage of the crisis effects vary in the space and time. In the Moroccan stock market context, it was found that the effects were not so harmful and that the future of these markets faces a compromise or at least a long lethargy. Indeed, inspired by these events, our study attempts to undertake two exercises. In first, we are testing the ability of the nonlinear ARCH and GARCH models (EGARCH, TGARCH, GJR-GARCH, QGARCH) to meet the number of expected exceedances (shortfalls) of VaR measurement. In second, we are providing a forecasting volatility under the time-varying of VaR.

Keywords. Market Microstructure, ARCH Models, VaR, Time-Varying Volatility, Forecasting Volatility, Casablanca Stock Exchange.

JEL-Classification. G14, G18, C22, C52, C53.

1. Introduction

Volatility measures the amplitude of variations in the assets, the derivative securities and commodities and in the financial market indexes. This is a parameter quantifying the risky returns and prices. The Volatility is also used in calculations in order both to optimize the portfolio diversification of financial assets and to valuate the financial derivative contracts such as call and put options.

The monetary and financial data is characterized by volatility clustering, i.e. the periods of high volatility interspersed with the periods of low volatility. This phenomenon, which we usually called the conditional heteroskedasticity, is particularly common in the stock market data, the exchange rates or the other determined prices of capital markets. For such temporal series, the linear models already used in the past, such as autoregressive moving average process (ARMA), are inefficient to describe better the behavior of variables: they do not allow taking into account the phenomena of variability of volatility over the time. The phenomena of asymmetry and flattening are not addressed by such models. The ARMA(p,q) models can not generate the squared autocorrelations. Indeed, in the ARMA model, the conditional mean varies over the time while the conditional variance does not change.

In order to compensate for the shortcomings of ARMA(p,q) model, Engle (1982) have been proposed a new classes of models. It are an autoregressive conditionally heteroskedastic (ARCH) models, which are able to capture the behavior of volatility over the time.

The ARCH model consists of two equations. The first connects the return and some explain variables and the second models capture the conditional variance of residuals. The main principle proposed by Engle is to introduce the dynamics in the volatility determining which assume that the variance is conditional to available information.

Through this study, we try to show the robustness of non linear ARCH and GARCH models to estimate the portfolio return in the Moroccan financial market case. In addition, we are implement the forecasting of Value-at-Risk associated with this portfolio.

In the first part of this article, we discuss the theoretical overview of ARCH and GARCH models and estimate the historical, semi-parametric and parametric VaR under these models. In the second part, we will take the portfolios given by some historical indexes of Casablanca Stock Exchange. We try in fact to estimate the performance of these portfolios and the VaR that is linked. Moreover, in the ultimate goal of forecasting, by using backtesting tests, we note, among the ARCH and GARCH nonlinear classes, the model which can estimate the volatility and predict it in the best way.

2. The ARCH and GARCH literature Review

The conditional volatility measurement, which resulting from the econometric ARCH models, allows us to extract the anticipated volatility by discarding aside the influence of unpredictable shocks. So, this measurement permits to seize the volatility such as it is *ex ante* expected by the market under the available relevant information.

Similarly, it measures the stock prices effect that can have a persistent behavior, over the time: a period of strong fluctuations, upward or downward, whose behavior is repeated over the time and in the amplitude, is followed by a period of relative calm.

2.1. The Linear ARCH and GARCH Models

In his founding paper[1], Engle (1982) refers to a stochastic process in discrete time (ε_t) with the following type:

$$\varepsilon_t = z_t h_t$$
$$E(z_t) = 0$$
$$Var(z_t) = 1$$

Where h_t is a measurable function, positive, and non-constant with available information in the (t-1). By definition, (ε_t) is an uncorrelated process with zero mean. But his (its?) conditional variance which is h_t^2 can vary over the time.

In the main econometric applications, (ε_t) describe the innovation of another stochastic process y_t, such as:

$$y_t = g(x_{t-1}, b) + \varepsilon_t \quad (1)$$

Where, $g(x_{t-1}, b)$ is a function of (x_{t-1}) and of a vector of parameters *b*. (x_{t-1}) belonging to the all available information, in the time of $(t-1)$.

2.1.1. The linear ARCH Model

The ARCH model (Autoregressive Conditional Heteroskedasticity), which introduced by Engle[2] (1982) is considered as a processes treating the conditional Heteroskedasticity.

Unlike the linear models, which are interest only to the first order of moment (the mean), ARCH model introduces a large study including the second order of moment (conditional and non conditional variance). The main objective of the ARCH model is to mitigate the inability of linear ARMA models to describe, in the best way, the behavior of financial series. The financial returns data are indeed characterized by a volatility changes over the time and by an asymmetric phenomena, which cannot be considered in the ARMA process. The ARCH model is based on an endogenous parameterization of conditional variance. Indeed, one can express h_t^2 linearly with the square of innovation past values:

$$h_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 = a_0 + a(L)\varepsilon_t^2$$

$$h_t = \sqrt{\left[a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 \right]} \quad (2)$$

Where, $a_0 > 0$, and $a_i \geq 0$ for all i; L is the lag operator.

It is possible, then, to present ε_t^2 under the process of $AR(q)$; so, we assume:

$$v_t = \varepsilon_t^2 - h_t^2 \quad (3)$$

Hence:

$$\varepsilon_t^2 = a_0 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 + v_t \quad (4)$$

v_t has the mean and the covariance equal to zero, but its variance is no constant. According to the AR(q) formulation, the regression model with ARCH is obtained by assuming that the average of ε_t is a linear

[1] Engle, R.F. (1982), "AutoRegressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation", Econometrica, 50, n°1, pp. 987-1008.

[2] Engle, R.F., Ibid.

combination of exogenous and lagged dependent variables (X_t), multiplied by a vector of unknown parameters (β):

$$\begin{cases} (\varepsilon_t | I_{t-1}) \to N(X_t\beta, h_t^2) \\ h_t = h(u_{t-1}, u_{t-2}, ..., u_{t-p}, a) \\ u_t = \varepsilon_t - X_t\beta \end{cases} \quad (5)$$

The last expression has very interesting properties to the econometric modeling. McNees[3] (1980) has been showed that uncertainty should be varying according to the scale of periods in relation both with the forecasting horizon and the residuals regression. The residuals are often clustered around to the high errors followed by the weak errors. The ARCH model, in which the variance depends to the time and to the lag of errors, allows us to describe this phenomenon.

Besides, the asymmetrical character of ARCH process is one of the reasons that its application is more interesting in the finance area because the majority of financial assets present this main feature[4].

2.1.2. The linear GARCH model

It is often necessary, in practice, when one tries to identify a linear ARCH(q) model, to retain a large lag number of *q*. The generalized ARCH (Bollerslev (1986)) presents a big alternative in this field. Indeed, the GARCH(q,p) model holds an advantage because it can retain a more flexible lag structure[5].

$$h_t^2 = a_0 + \sum_{j=1}^{q} b_j h_{t-j}^2 + \sum_{i=1}^{p} a_i \varepsilon_{t-i}^2 \quad (6)$$

In order that the process will be properly defined, we need that all model parameters are not negative. In the case of GARCH(1,1) model, this amounts to ensure that a_1 and b_1 are positive or null. Also, one concluded that, in order to assume that ε_t is stationary in its covariance, it suffices that we have $1 + b_1 < 1$. In these circumstances, the GARCH(p,q) is equivalent to a linear ARCH model with infinite order.

2.2. The No-Linear ARCH and GARCH Models

In the GARCH(p,q), the variance should be depend only on the absolute value of ε_t, but not in its own sign. This should not be consistent with the empirical behavior of stock market prices for which there probably exists a leveraged effect. In the EGARCH(p,q)[6], which is presented by Nelson (1990), h_t^2 is considered as an asymmetric function of the lagged ε_t:

$$\log h_t^2 = a_0 + \sum_{i=1}^{q} a_i (\phi z_{t-1} + \gamma(|z_{t-i}| - E|z_{t-i}|)) + \sum_{i=1}^{p} b_i \log h_{t-i}^2 \quad (7)$$

In contrast to the GARCH(p,q) case, there is no restriction in the a_i and b_i parameters to ensure the positivity of the conditional variance. Thus, the representation model is mainly closer to the ARMA(p,q) model. If $\varphi < 0$, the variance tends to increase (decrease), when ε_{t-i} is negative (positive). As noted above, it is largely consistent with the empirical findings in the stock market returns case. If z_t is a normal iid, then ε_t is stationary in the covariance.

Indeed, a many formulations of parametric ARCH types have been proposed in the literature, such as the power transformations of ε_t^2 which it can be expressed in the Higgins and Bera (1990) and Bera and Higgins (1997) non-linear ARCH models, in the McCurdy and Morgan (1990a) model which study the trigonometric transformations of ε_t, or in the TARCH models which are developed by Zakoian in 1990. In the threshold model, h_t^2 is a piecewise function which allows finding different functions of volatility which depending on the sign and values of the shocks. A closer model, which is based on an approach in terms of Markov chain, was developed by Gourieroux and Monfort (1990). In addition, Harvey, Ruiz and Sentana (1992) have recently proposed a model in which the unobservable components of ARCH disturbances are presented in both state equations and update equations.

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3 McNees, S. K., "A Critique of Alternative Methods of Comparing Macroeconomic Models", in Ramsey, J. and Kmenta, ed. *Methodology of Macroeconomic Models*, North-Holland, 1980.

4 This assumption is discovered and dealt with by Fama in the 1963s.

5 Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, 31, pp. 307-327.

6 Nelson, D.B. (1990), "ARCH Models as Diffusion Approximation", *Journal of Econometrics*, 45, 7-38.

2.3. The ARCH-M model

In the ARCH-in-Mean (ARCH-M) model, which has been introduced by Engle, Lilien and Robins (1987), the conditional mean depends explicitly on the conditional variance process. The model is given by:

$$y_t = g(x_{t-1}, h_t^2; b) + \varepsilon_t \quad (8)$$

In this model, the variability of conditional variance will be necessary accompanied by a change in the conditional mean of y_t. The changes in direction depending on the sign of partial derivative of g function to h_t^2. A many financial theories have explicitly linking the risk to the expected returns. The ARCH-M models are perfectly adapted to such issues in a dynamic framework, i.e., where the conditional variance changes over the time. The most commonly form which adopted for $g(x_{t-1}, h_t^2; b)$ is a linear or logarithmic function in h_t^2 or in h_t.

In fact, we begin with the simplest GARCH(1,1) specification:

$$y_t = x_t^{'}\theta + \varepsilon_t \quad (9)$$

$$h_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + b_1 h_{t-1}^2 \quad (10)$$

in which the mean equation, given in (9), is written as a function of exogenous variables with an error term. Since h_t^2 is the one-period ahead forecast variance based on the past information, it is called the *conditional variance*. The conditional variance equation specified in (10) is a function of three terms:

- A constant term: a_0;
- News about volatility from the previous period, measured as the lag of the squared residual from the mean equation: ε_{t-1}^2 (the ARCH term);
- Last period's forecast variance: h_{t-1}^2 (the GARCH term).

The x_t in equation (10) represents exogenous or predetermined variables that are included in the mean equation. If we introduce the conditional variance into the mean equation, we get the ARCH-M model:

$$y_t = x_t^{'}\theta + \lambda h_t^2 + \varepsilon_t \quad (11)$$

The ARCH-M model is often used in financial applications where the expected return on an asset is related to the expected asset risk. The estimated coefficient on the expected risk is a measure of the risk-return trade-off. Two variants of this ARCH-M specification use the conditional standard deviation or the log of the conditional variance in place of the variance in equation (11).

$$y_t = x_t^{'}\theta + \lambda h_t + \varepsilon_t$$

$$y_t = x_t^{'}\theta + \lambda \log(h_t^2) + \varepsilon_t \quad (12)$$

3. The VaR Estimating by using GARCH models:

The GARCH models are firstly used to model and to forecast the conditional variance of the returns distribution. Then, it allows deriving a model or a forecasting of Value-at-Risk under some conditional distribution assumptions of yields. We are, actually, will present all these models in the next section.

Under the normality assumption of conditional distribution of profits and losses which may predict the VaR and associated with $\alpha\%$ coverage rate, is defined as following[7] :

$$VaR_{t+1/t}(\alpha) = -\mu - \sqrt{h_{t+1}}\,\Phi^{-1}(\alpha), \quad (13)$$

where h_{t+1}^2 denotes the conditional variance of returns.

4. The empirical investigation: An equity portfolio case, based by the Casablanca stock market

4.1. Data and Methodology

Our study will focus on a hypothetical portfolio composed by four market sectoral indices: real estate sector, building and public works sector, banks sector and Holding companies sector. The choice of these four indices is due to two reasons: First of all, these indices are representative of the most powerful and most attractive among of the Moroccan economy, and secondly, these are indexes that have been created recently and the availability of their data coincides with the period of major reform of Casablanca stock exchange. This allows us to have a history of data that covers a period between 03/01/1994 and 31/12/2008.

Figure 1 shows the evolution of these four indices during this period compared to the MASI and MADEX indices[8]:

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[7] Engle, R.F. (2001), "The Use of ARCH/GARCH Models in Applied Econometrics", *Journal of Economic Perspectives*.

[8] The Moroccan All Shares Index and the Moroccan Active index.

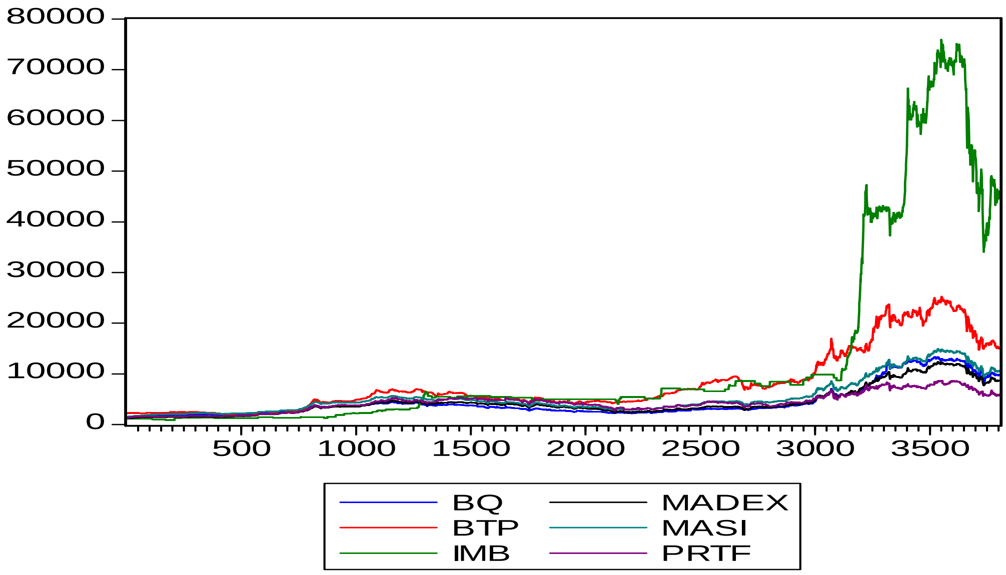

Figure 1: Evolution of four sectoral indices and two market indices during the period: 1994/03/02-2008/12/31

From the figure, we can see that the four indexes follow the market MASI and MADEX indices changes during this period, except the index of real estate which has undergone a major changes from January 2006 (3000th observation).
The market has not experienced high volatility during the period 1994/01/03 - 2001/12/31. However, from January 2002 (2250th observation), the index volatility has become increasingly important. Indeed, the market has been, therefore, an upward trend until early 2006 and a downward trend from January 2008, due to the impact of global financial crisis.
The evolution of these indices requires us to conduct two studies, in two different periods: the first start in 2002/01/02 and finish in 2008/12/31, and the second start in 2006/01/02 and finish in 2008/12/31.
In a first step, we calculate the portfolio returns in logarithmic. After, we try to model the performance of such portfolio by using, in first, the ARMA-GARCH models with normal distributions, and then by using the Student law. Subsequently, we will estimate the Value-at-Risk at a significance level of 5% and of 1%, by using the appropriate selected models for each period.
At the end of these studies, we will conduct a Backtesting process which it allow us relevantly to judge the validity and power of the selected models in order to accurately forecast a VaR model.

4.2. The study of the period between 2002/01/02 and 2008/12/31
4.2.1. The Preliminary Finding
4.2.1.1. Descriptive Statistics
The returns serie has been a significant change from early 2002 (1000th observation). There is the atypical appearance of a stationary series with a volatility clustering.
The descriptive statistics of returns series show that portfolio performance is asymmetrical with a Skewness equal to -0.43 and a kurtosis equal to 8.73 confirming that a series have a Leptokurtic distributions. The Jarque and Bera statistic value is very high proving that the series distributions are not Gaussian. These aspects allow us to include the series of that returns in the "family of ARCH and GARCH models" by providing of course that the distributions that would otherwise be asymmetric, must have tails thicker than those of the normal law.

4.2.1.2. The stationarity of series
Our study confirms that the returns series has been stationary: The ADF test with a value of -29.83, which is lower than critical values, provide this fact. The probabilities of lag return and the constant are significant at the 5% threshold. The time returns allow a constant.

4.2.1.3. ARMA Modelling
The autocorrelations and the partial autocorrelations correlograms show that the serial returns can admit the ARMA process. In fact, it is precisely based on the one of the following models: AR(1), MA(1) or ARMA(1,1). The estimation of these three models and with the based help of the residual autocorrelations correlograms and either with Akaike and Schwartz criteria, we can conclude that the serie of returns follows the AR(1) process.

4.2.2. The ARCH and GARCH Estimation

As we saw above, the data descriptive statistics and the moments' analysis, with three and four orders show that the daily returns present some ARCH effect. The ARCH test output confirms this assumption with zero probability of both tests.

The models AR(1)-ARCH(1), AR(1)-ARCH(2), AR(1)-ARCH(3), AR(1)-ARCH(4), AR(1)-GARCH(1,1), AR(1)-GARCH(2,1), AR(1)-GARCH(3,1), AR(1)-GARCH(1,2), AR(1)-GARCH(1,3), AR(1)-GARCH(2,2), AR(1)-EGARCH(1,1) AR(1)-TARCH(1,1) and AR(1)-PARCH (1.1) will be estimated with assuming, in first, that the residual distribution is log-normal and, in the second, that the residual distribution follows a Student law.

Indeed, based on the AIC and SCHWARTZ information criteria, and the log of likelihood and the R^2 determination coefficient, we have consider these models: the AR(1)-GARCH(1,2), AR(1)-GARCH (1,1) and AR(1)-ARCH(4) for the log-normal distribution, and the AR(1)-GARCH (1,1) and AR(1)-GARCH(2,1) models for the Student law.

4.2.2.1. The VaR Estimation with 5% of threshold

We estimate the Value-at-Risk under the log-normal distribution and under a law student, with 5% of threshold. The two following figures show us the results of these estimations:

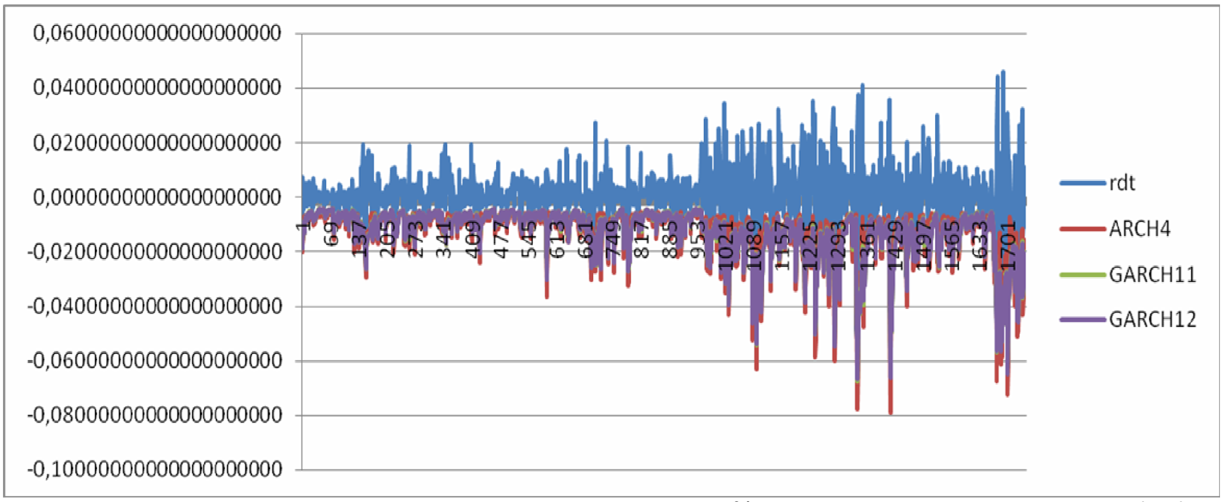

Figure 1: The VaR Estimation under the normal distribution with 5% of threshold over the period 2002/01/02 - 2008/12/31.

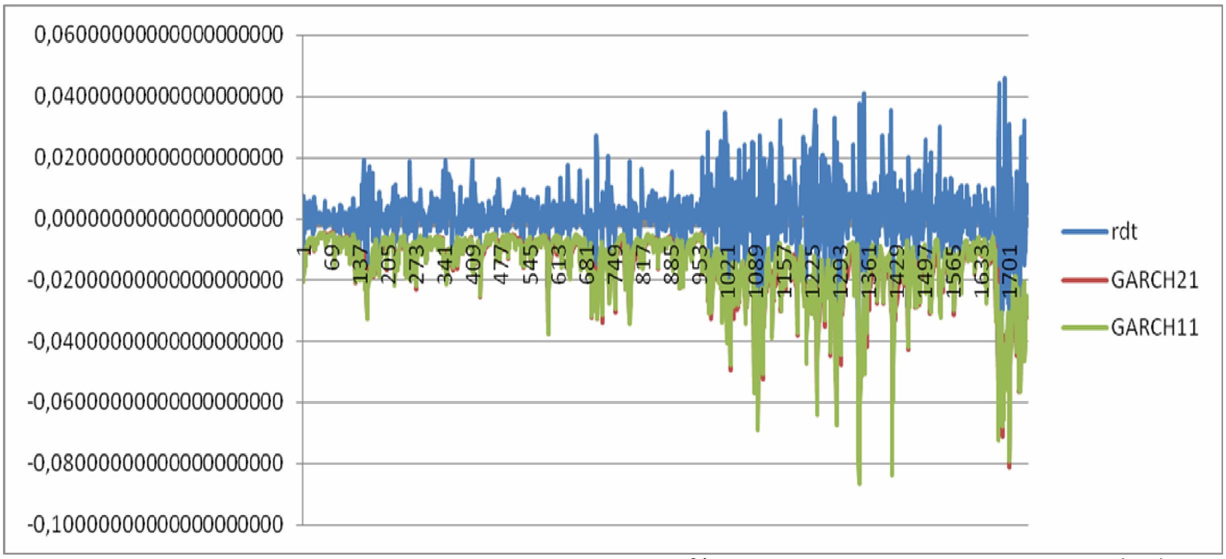

Figure 3: The VaR Estimation under the Student law with 5% of threshold over the period 2002/01/02 - 2008/12/31.

4.2.2.2. The VaR Estimation with 1% of threshold

We estimate the Value-at-Risk under the log-normal distribution and under a law student, with 1% of threshold. The two following figures show us the results of these estimations:

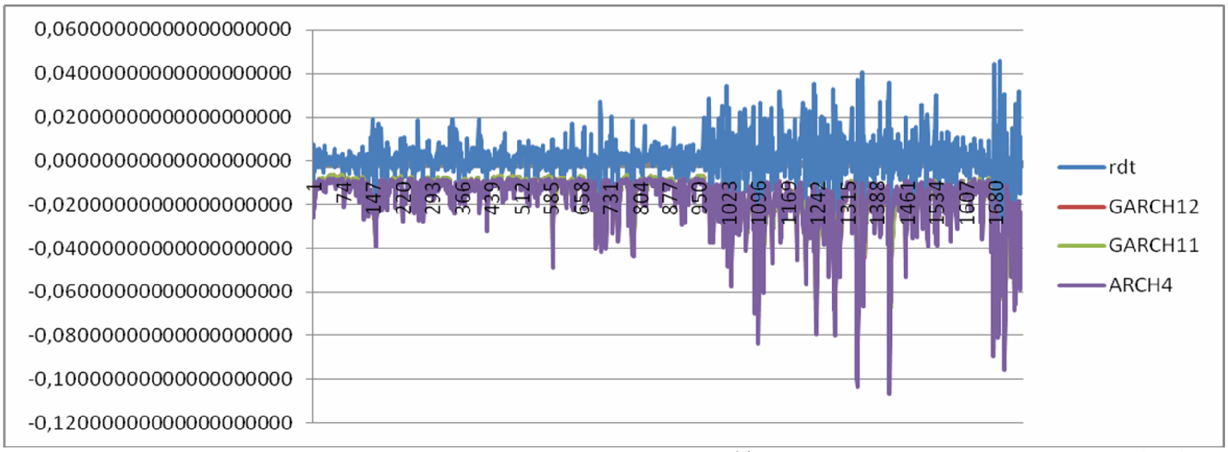

Figure 4: The VaR Estimation under the normal distribution with 1% of threshold over the period 2002/01/02 - 2008/12/31.

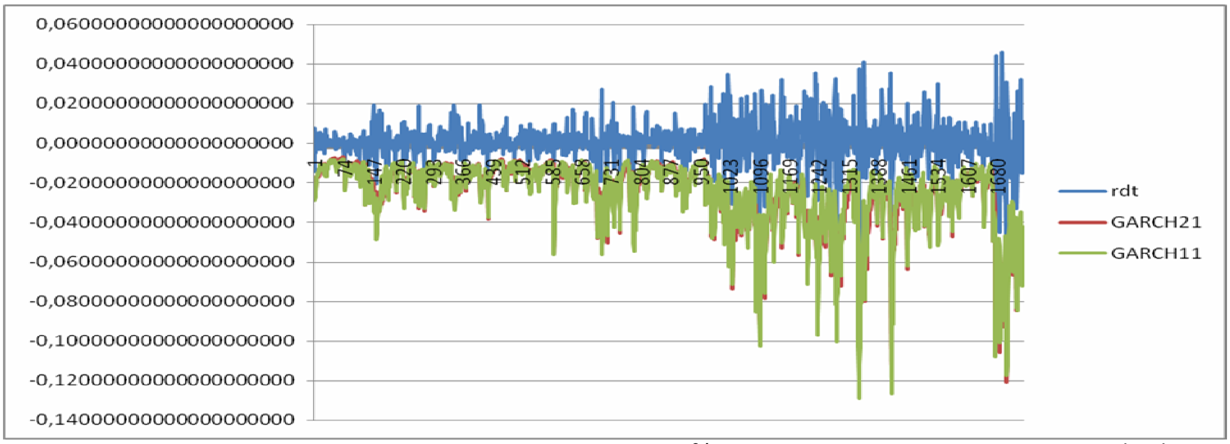

Figure 5: The VaR Estimation under the Student law with 1% of threshold over the period 2002/01/02 - 2008/12/31.

4.3. Study of the period between 2006/01/02 and 2008/12/31

4.3.1. The Preliminary Finding

4.3.1.1. Descriptive Statistics

The returns serie has been a significant volatility during this period. We can observe then some stationary of series, with volatility clustering.

The descriptive statistics of returns series show that portfolio performance is asymmetrical with a Skewness equal to -0.49 and a kurtosis equal to 5.66 confirming that a series have a Leptokurtic distributions. The Jarque and Bera statistic value is very high proving that the series distributions are not Gaussian.

4.3.1.2. The stationarity of time returns

The study shows us a stationarity of time returns: The ADF test value (-19.57) is lower than the critical value, and the lag return and trend and constant probabilities are significant with 5% of threshold. The time returns output allows a constant and a trend model.

4.3.1.3. The ARMA Modeling

The autocorrelations and the partial autocorrelations correlograms show that the serial returns can admit the ARMA process. In fact, it is precisely based on the one of the following models: AR(1), MA(1) or ARMA(1,1).

The estimation of these three models and with the based help of the residual autocorrelations correlograms and either with Akaike and Schwartz criteria, we can conclude that the serie of returns follows the MA (1) process.

4.3.2. The ARCH and GARCH Estimation

As we saw above, the data descriptive statistics and the moments' analysis, with three and four orders show that the daily returns present some ARCH effect. The ARCH test output confirms this assumption with zero probability of both tests.

The models AR(1)-ARCH(1), AR(1)-ARCH(2), AR(1)-ARCH(3), AR(1)-ARCH(4), AR(1)-GARCH(1,1), AR(1)-GARCH(2,1), AR(1)-GARCH(3,1), AR(1)-GARCH(1,2), AR(1)-GARCH(1,3), AR(1)-GARCH(2,2), AR(1)-EGARCH(1,1)

AR(1)-TARCH(1,1) and AR(1)-PARCH (1.1) will be estimated with assuming, in first, that the residual distribution is log-normal and, in the second, that the residual distribution follows a Student law.
Indeed, based on the AIC and SCHWARTZ information criteria, and the log of likelihood and the R^2 determination coefficient, we have consider these models: the MA(1)-GARCH(1,1), MA(1)-ARCH(3) and MA(1)-ARCH(2) for the log-normal distribution, and the MA(1)-GARCH(1,1) and MA(1)-ARCH(2) for the Student law.

4.3.2.1. The VaR Estimation with 5% of threshold

We estimate the Value-at-Risk under the log-normal distribution and under a law student, with 5% of threshold. The two following figures show us the results of these estimations:

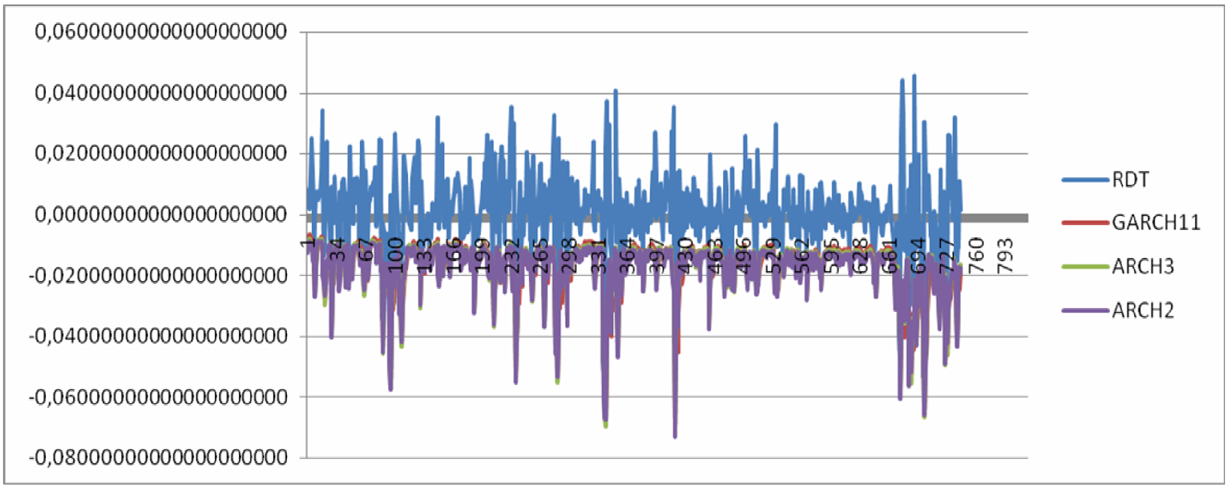

Figure 2 : The VaR Estimation under the normal distribution, with 5% of threshold over the period 2006/01/02-2008/12/31.

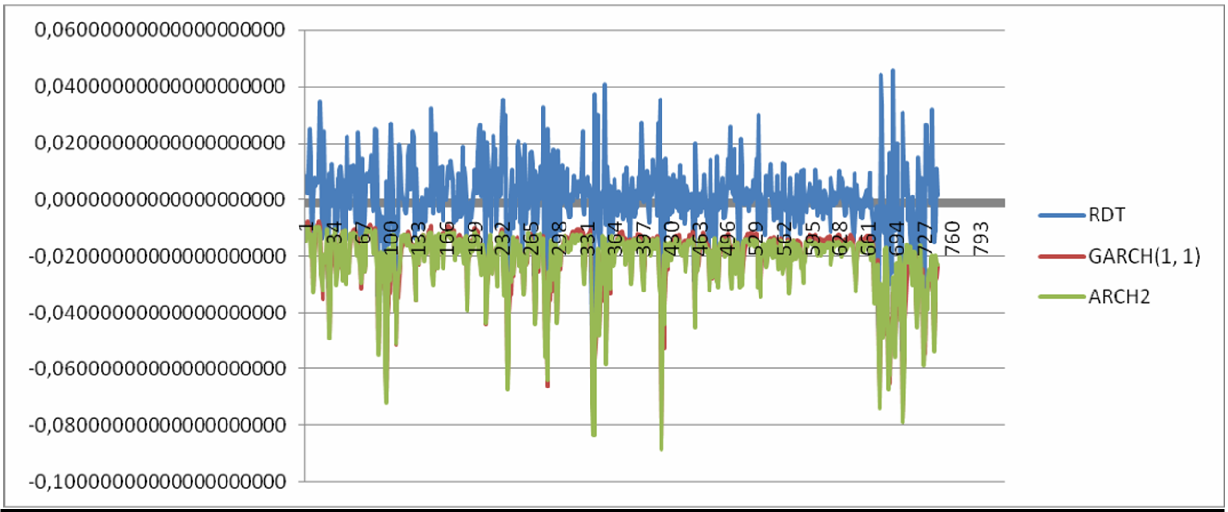

Figure 3: The VaR Estimation under the Student law, with 5% of threshold over the period of 2006/01/02 - 2008/12/31.

4.3.2.2. The VaR Estimation with 1% of threshold

We estimate the Value-at-Risk under the log-normal distribution and under a law student, with 1% of threshold. The two following figures show us the output estimations:

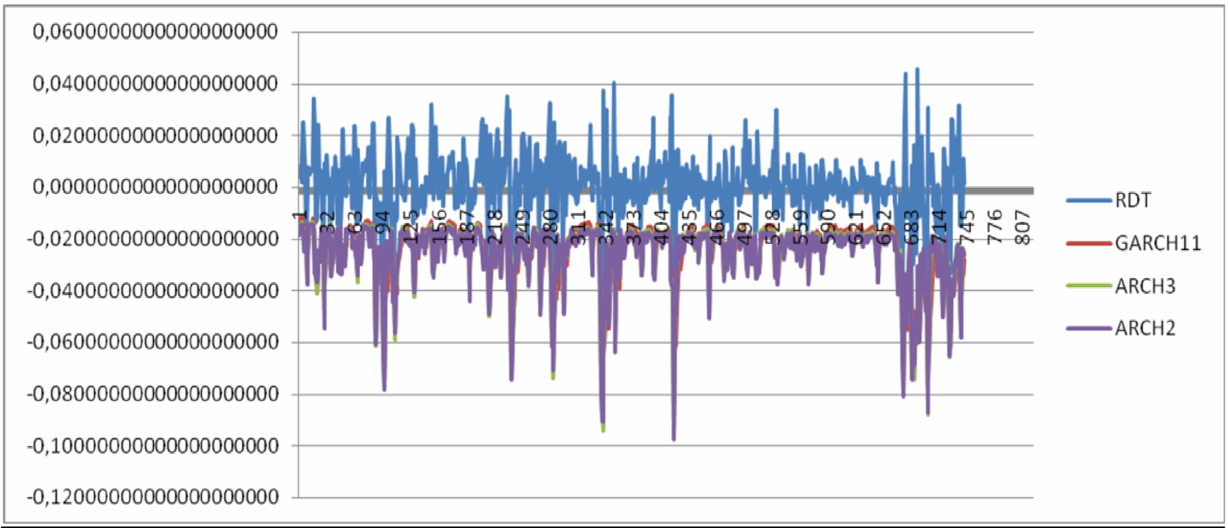

Figure 4 : The VaR Estimation under the normal distribution with 1% of threshold over the period 2006/01/02-2008/12/31.

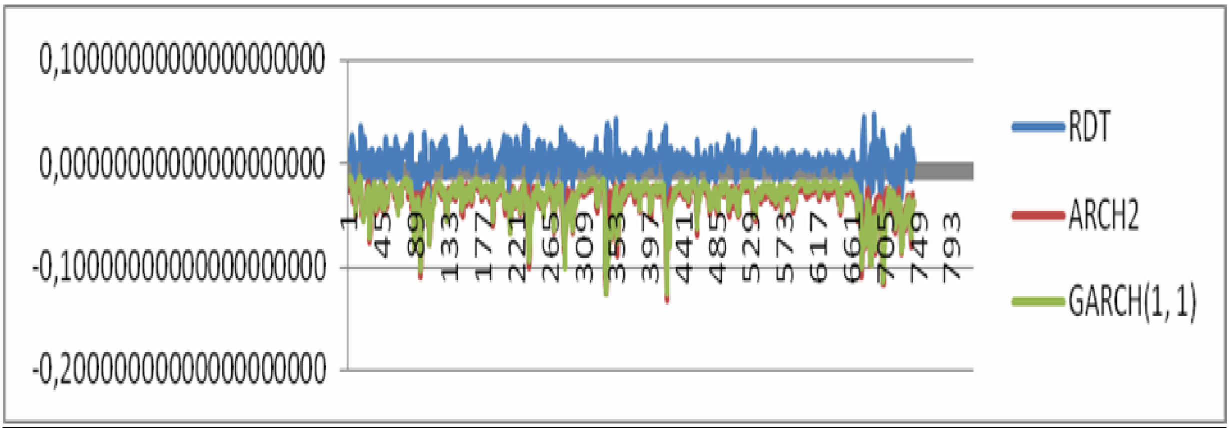

Figure 5 : The VaR Estimation under the Student law with 1% of threshold over the period 2006/01/02-2008/12/31.

4.4. The Backtesting

4.4.1. A definition

The backtesting is a set of statistical procedures. Its purpose is to verify that the actual losses observed ex-post is in line with expected losses. This involves systematically comparing the historical forecasts of the Value-at-Risk with the observed portfolio returns[9].

Traditionally, the prediction validity of any economic variable is measured by comparing its *ex-post* realization with the predicted value which is expected *ex ante*. The comparison of the different forecasting models is then made through the use of a criterion based on the difference between the predicted value and the realized value (or the loss function).

The VaR evaluation is generally based on the statistical tests which are based on two main assumptions that the processes associated with violations of the VaR should satisfy. The both assumptions are: the hypothesis of unconditional coverage and independence assumption[10].

4.4.2. The empirical results

Tables 3 and 4 summarize the results of tests applied to backtesting of VaR forecasts for two series of returns over the two periods.

4.4.2.1. The First Period: From January 2002 to December 2008

Three quantitative types of tests will be implemented in order to compute the difference (i.e., the deviation) between the VaR models and the average risk. It may also compare the observed frequency of exceptions with the frequency of expected exceptions of VaR models (the Frequency of Tail Losses) and the time dynamics of those exceptions (Conditional Coverage of Frequency and Independence of Tail Losses).

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[9] Jorion, P. (2007), *Value-at-Risk*, Third edition, McGraw-Hill.

[10] Christoffersen, P. F. (1998), “Evaluating Interval Forecasts”, *International Economic Review*, 39, pp. 841-862.

4.4.2.1.1. The Kupiec test

The Kupiec's test[11] attempts to determine whether the observed frequency of exceptions is consistent with the frequency of expected exceptions according to the VaR model and the chosen confidence interval. Under the null hypothesis that the model is "correct", the number of exceptions should follow a binomial distribution. The probability of experiencing *x* or more exceptions, if the model is correct, is given by:

$$P\left(x \middle| n, p\right) = \binom{n}{x} p^{x} \left(1-p\right)^{n-x} \quad (14)$$

Where *x* is the number of exceptions, *P* is the probability of an exception for a given confidence level and n is the number of trials. If the estimated probability is above the desired "null" significance level (usually 5% - 10%), we accept the model. If the estimated probability is below the significance level, we reject the model and conclude that it is not correct. We can conduct this test for loss and gain exceptions to determine how well the model predicts the frequency of losses and gains beyond VaR numbers.

The Kupiec calculated test, with using the expected VaRs' and under a confidence level of 5% and 1%, shows that the all models (with a Gaussian distribution or according to the Student law) are significantly good. These models can be then used to predict the conditional variance of returns and therefore provide better estimates of the VaR at the 5% and 1% thresholds. The value of the test is less than that of Chi-square with one degree of freedom (5%: 3.8414, 1%: 6.63). In conclusion, according to the Kupiec test, all models are valid.

4.4.2.1.2. The Christoffersen test

Presentation

The Kupiec test focuses solely on the frequency of exceptions, and ignores the temporal dynamics of these (those?) exceptions. The VaR models assume that exceptions should be distributed independently over time. If the exceptions were of a certain type of clustering, then the VaR model may not capture the variability of the exceptions (losses).

The main contribution of this approach is its ability to test sub-hypothesis regarding the frequency and independence of exceptions, and the joint hypothesis that the VaR model has the right frequency of independent exceptions. This test was in first proposed by Christoffersen[12].

An additional benefit of achieving this type of testing is that it generates useful information such as the conditional probability of experiencing an exception followed by an exception in the risk model, and the average number of days between exceptions.

Method of interval prediction

Define Rt as a return and Dt as a dichotomous variable that takes the value 1 when an exception occurs, i.e., when the VaR does not cover losses.

Construct the characteristic variable of exceptions such as:

$$I_t = (R_t < VaR_t)(D_t) + (R_t \geq VaR_t)(1 - D_t) \quad (15)$$

Once I_t serie has been built, we define T_{ij} as the number of observations in the j state which being in state I, in the previous period. For example,

T_{00} is the number of times that VaR covers the losses in t since those covered by VaR in t-1.

Let:

$$\pi_{01} = T_{01} / (T_{00} + T_{01}) \text{ et } \pi_{11} = T_{11} / (T_{10} + T_{11})$$

The likelihood ratio test of Christoffersen is calculated as:

$$LR = LR_{kupiec} + LR_{ind} \approx \chi^2(2) \quad (16)$$

The likelihood ratio test jointly whether the proportion of failures is consistent with that anticipated and whether the exceptions (failures) are independent of each other. LR_{ind} is the likelihood ratio for the null hypothesis of serial independence. In this case, if one rejects the null hypothesis that there is dependence of first-order Markov:

The likelihood ratio test jointly whether the proportion of failures is consistent with those already anticipated and whether the exceptions (failures) are independent of each other. LR_{ind} is the likelihood ratio for the null hypothesis of serial independence. In this case, if we reject the null hypothesis, there is Markov dependence in the first-order:

$$LR_{ind} = 2(\log(L_A) - \log(L_0)) \quad (17)$$

[11] Kupiec, Paul H., 1995, "Techniques for verifying the accuracy of risk measurement models", *Journal of Derivatives* 3, 73 – 84.

[12] Christoffersen, P. (1998), "Evaluating Interval Forecasts", *International Economic Review*, 39, 841-862.

where L_A is the likelihood function of Markov dependence in first order:

$$L_A = (1-\pi_{01})^{T_{00}} \pi_{01}^{T_{01}} (1-\pi_{11})^{T_{00}} \pi_{11}^{T_{11}}$$

and L_0 is the likelihood function when there is serial dependence, i.e. $\pi_{01} = \pi_{11} = \pi$:

$$L_0 = (1-\pi)^{(T_{00}+T_{10})} \pi^{(T_{01}+T_{11})} \text{ où } \pi = (T_{01}+T_{11})/N .$$

In our finding and under a normal distribution at 5% of threshold, we conclude that the values of conditional coverage tests (i.e., independence tests) show just the AR(1)-GARCH(1,1) model have the values significantly lower than the chi-squared value with one degree of freedom (3.8414). So, if we assume that the residual distribution follows a normal distribution, the model AR(1)-GARCH (1,1) will be used to forecast the conditional variance and therefore may provide a Value-at-Risk of the portfolio which is reliable within the meaning of Christoffersen.
Under a normal distribution at 1% of threshold, all conditional coverage tests are not significantly lower than the value of chi-squared with one degree of freedom (6.63). The predictions from these models are not reliable under the Christoffersen meaning. This may be due to the assumption of normality of the residual returns.
Under a Student law at 5%, all tests of conditional coverage are not significantly lower than the value of chi-squared with one degree of freedom (3.84). The predictions of these models are not reliable under the Christoffersen meaning.
Under a Student law at 1%, the values of the coverage tests or of the conditional independence show that only the AR(1)-GARCH (2,1) has a value significantly below compared with the value of chi-squared at one degree of freedom (6.63). So, if we assume that the residuals distribution follow the Student law, the model AR(1)-GARCH (2,1) will be used to forecast the conditional variance and therefore may provide a Value-at-Risk of the portfolio which is reliable within the meaning of Christoffersen.

	02/01/2002 - 31/12/2008				02/01/2002 - 31/12/2008					
	Student VaR at 5%		Student VaR at 1%		Normal VaR distribution at 5%			Normal VaR distribution at 1%		
	GARCH21	GARCH11	GARCH21	GARCH11	GARCH12	GARCH11	ARCH4	GARCH12	GARCH11	ARCH4
Exceptions number	49	49	13	10	79	80	74	30	30	33
In Pourcentage	0,02811245	0,02811245	0,00745841	0,005737235	0,04532415	0,04589788	0,04245554	0,0172117	0,0172117	0,01893287
LRtuff	2,885730141	2,88573014	0,05024614	0,050246144	2,88573014	2,88573014	2,88573014	0,05024614	0,05024614	0,05024614
LRind	0,12098021	0,12098021	2,98972836	4,015219125	4,55883983	2,67407692	2,28262337	0,37015826	0,37015826	0,19863325
LRcc	20,81829851	20,8182985	4,23163694	7,786420396	5,37619425	3,29965464	4,46326392	7,916852	7,916852	11,346457
RMB	-0,00033257	0,00033257	-0,00117702	0,001177018	-0,01052004	-0,01009769	0,02061773	-0,01075889	-0,00985438	0,02061328
RMSRB	0,081846659	0,08184666	0,07995951	0,079959512	0,07074452	0,06245333	0,12948322	0,06877867	0,06024242	0,12561538

Table 1: Summary of Backtesting: 2002/01/02 - 2008/12/31 period.

	02/01/2006 - 31/12/2008				02/01/2006 - 31/12/2008					
	Student VaR at 5%		Student VaR at 1%		Normal VaR distribution at 5%			Normal VaR distribution at 1%		
	garch11	ARCH2	garch11	ARCH2	GARCH11	ARCH3	ARCH2	GARCH11	ARCH3	ARCH2
Exceptions number	30	31	5	5	36	40	39	18	17	19
In Pourcentage	0,040376851	0,04172275	0,00672948	0,006729475	0,04845222	0,0538358	0,05248991	0,02422611	0,02288022	0,02557201
LRtuff	0,681248088	0,00244372	0,47356749	0,681248088	0,68124809	0,68124809	0,68124809	1,17878705	1,17878705	3,32272249
LRind	0,042993812	0,06784313	0,06784313	0,362831443	2,432859	3,17359468	0,43540241	0,56750692	0,71078523	0,44311249
LRcc	1,569093323	0,96839538	0,96839538	1,47842202	2,46751642	3,40654061	0,53621308	11,4635625	9,86343492	13,1965048
RMB	-0,023038648	0,02303865	-0,02071604	0,020716042	-0,0079959	-0,00263581	0,0106317	-0,00697414	-0,00208388	0,00905802
RMSRB	0,098182458	0,09818246	0,09430022	0,094300216	0,09815271	0,03985359	0,07704019	0,09449183	0,03847547	0,07384023

Table 4: Summary of Backtesting: 2006/01/02 - 2008/12/31 period.

4.4.2.1.3. The Mean Relative Bias (MRB) and the Root Mean Squared Relative Bias (RMSRB)

The MRB examines whether different VaR models produce similar forecasts. We first calculate VaR under each VaR models on each sample date, and then compute the average VaR over the forecast sample. Given *h* forecasting periods and *N* VaR models, the MRB of model *i* is computed as:

$$MRB_i = \frac{1}{h}\sum_{t=s+1}^{s+h}\frac{VaR_{t,t+T}^{(i)} - \overline{VaR_{t,t+T}}}{\overline{VaR_{t,t+T}}}, \quad (18)$$

where $\overline{VaR_{t,t+T}} = \frac{1}{N}\sum_{i=1}^{T} VaR_{t,t+T}^{(i)}$,

The RMSRB, which is proposed by Hendricks[13], measures the relative size of VaR variability, i.e., it measures the degree to which the risk measures tend to vary around the all-model average risk for a given date. It captures two effects: the effect of the extent to which the estimated average risk under a given model systematically differs from the average risk measure, and the effect of variability in risk estimation of each model. The RMSRB is simple but more perfect for an effective measure of deviations around the average risk. The RMSRB is a negative indicator, with smaller RMSRB indicating greater conservativeness. The RMSRB is computed as:

$$RMSRB_i = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left[\frac{VaR_{it} - \overline{VaR_t}}{\overline{VaR_t}}\right]^2}, \quad (19)$$

where $\overline{VaR_t} = \frac{1}{N}\sum_{i=1}^{T} VaR_{it}$, VaR_{it} is i^{th} model's VaR at time *t*, *T* is the time periods, and *N* is the number of VaR models.

The AR(1)-GARCH (1,1) Model under the normal distribution at 5%

The MRB value shows us that the VaR provided by this model is at 1% smaller than the average VaR obtained by the three above models. This implies that it underestimates the risk. This risk underestimation may be explained by the fact that the approach is based on the assumption of normality of returns and we know that this assumption can be misleading and lead us to underestimate the risk because of the leptokurtic true nature of the returns distribution of our portfolio.

The RMSRB test shows that this model tends to underestimate the risk in the 6.24% cases. This shows the superiority of AR(1)-GARCH (1,1) Model among the other two models: AR(1)-GARCH(1,2), which underestimates the risk in the 7.07% of cases and the AR(1)-ARCH(4), which overestimates the risk in the 12.94% of cases.

The AR(1)-GARCH(2,1)Model under the Law of Student at 1%

The MRB value shows us that VaR provided by this model is 0.1% smaller than the average VaR obtained by the other two models. This implies that it underestimates the risk, but in a way less important than the model AR(1)-GARCH(1,1) under a normal distribution with a threshold at 5%.

The RMSRB test shows that this model tends to underestimate the risk in the 7.99% cases.

4.4.2.2. The Second Period: From January 2006 to December 2008

4.4.2.2.1. The Kupiec test

The Kupiec calculated test, with using the expected VaRs' and under a confidence level of 5% and 1%, shows that the all models (with a Gaussian distribution or according to the Student law) are significantly good. These models can be then used to predict the conditional variance of returns and therefore provide better estimates of the VaR at the 5% and 1% of thresholds. The value of the test is less than that of Chi-square with one degree of freedom (5%: 3.8414, 1%: 6.63). In conclusion, according to the Kupiec test, all models are valid.

4.4.2.2.2. The Kupiec test The Christoffersen Test

Under a normal distribution at 5% of threshold, the values of conditional coverage tests show that the three models: GARCH(1,1), ARCH(2) and ARCH(3) have values significantly below comparatively with the value of the chi-squared law at one degree of freedom (3.8414). So, if we assume that the residuals distribution follows a normal distribution, the three models used to predict the conditional variance and therefore to provide a Value at Risk portfolio are reliable under the Christoffersen meaning.

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[13] Hendricks, D. (1996), "Evaluation of Value-at-risk Models Using Historical Data," *Federal Reserve Bank of New York Economic Policy Review*, 2, 39-70.

Under a normal distribution at 1% of threshold, all conditional coverage tests are not significantly lower than the value of chi-squared with one degree of freedom (6.63). The prediction of these models is not reliable under the Christoffersen meaning. This may be due to the normality assumption of residual returns.
Under a Student law at 5%, all tests of conditional coverage are not significantly lower than the value of chi-squared with one degree of freedom (3.84). The predictions of these models are not reliable under the Christoffersen meaning.

Under a Student law at 1%, the values of the conditional coverage tests show that both two models (GARCH(1,1) and ARCH(2))have values significantly less than the value of the law of chi-squared with one degree of freedom (6.63). So, if we assume that the residuals distribution follow the Student law, the two models will be used to forecast the conditional variance and therefore may provide a Value-at-Risk of the portfolio which is reliable with the meaning of Christoffersen.

4.4.2.2.3. The Mean Relative Bias (MRB) and the Root Mean Squared Relative Bias (RMSRB)

The normal distribution case at 5%

The MRB value shows us that VaR provided by AR (1)-ARCH (3) is at 0.26% smaller than the average VaR obtained by the three models, as well as those provided by the AR(1)-GARCH(1,)model is at 0.7% smaller than the average VaR obtained by the three models, while the AR(1)-ARCH(3) gives an estimate of VaR greater at 1.06% than the average value. This implies that the model AR(1)-ARCH(2) is the best model for forecasting because it gives the closest results compared to the average of the three approaches.
The RMSRB test confirm this result because the AR(1)-ARCH(3) model underestimates the VaR in 3.9% of cases compared with the AR(1)-GARCH(1,1) model, which underestimates the VaR in 9.8% of cases, while the third model overestimates the VaR in 7.7% of cases. The superiority of the AR(1)-ARCH(3) is proved if we assume that the residual returns distribution is normal.

The Student law case at 5%

The MRB value of shows us that the VaR model provided by the AR(1)-GARCH(1,1) is at 2.3% smaller than the average VaR obtained by the two models. Similarly, the VAR model provided by the AR(1)-ARCH(2) is at 2.3% greater than the average VaR obtained by the two models. This implies that the arbitrage between the two models is difficult to do because it depends to the investor risk aversion. An investor who takes some risk prefers the second model. In contrast, an investor with high risk aversion will prefer the first model.
The RMSRB test confirm this result because the AR(1)-GARCH(1,1) model underestimates the VaR in the 9.8% of cases compared to the AR(1)-ARCH (2) model, which overestimates the VaR in the 9.8% of cases.

The Student law at 1%

The MRB values show that the VaR provided by the AR(1)-GARCH(1,1) model is 2.07% smaller than the VaR average obtained by the two models. However, that provided by the AR(1)-ARCH(2) model is 2.07% greater than the VaR average obtained by the two models. This implies that the arbitration between the two using models is difficult to do because it depends to the risk aversion of investor. An investor who takes the risk prefers the second model. In contrast, the risk-averse investor will prefer the first model. The RMSRB test confirms this result because the model AR(1)-GARCH(1,1) underestimates the VaR in 9.43% cases compared to the model AR(1)-ARCH(2) which over-estimates the VaR in 9.43% cases.

4.5. The forecasting VaR portfolio under a period between January 2009 and end May 2009

We will now make forecasts of the portfolio VaR using the AR(1)-GARCH(1,1). This model was being chosen because the backtesting has proved its superiority to predict the VaR of portfolio at 5%.
Indeed, the portfolio return is written as:

$$
\begin{aligned}
& rdt(t) = 0{,}000587 + (0{,}297548 * rdt(-1)) + \varepsilon_t \\
& \varepsilon_t = z_t \sqrt{h_t} \\
& h_t = (2.40\mathrm{E}\text{-}06) + (0.272599 * \varepsilon_{t-1}) + (0.732055 * \mathrm{h}_{t-1}) \quad (20)
\end{aligned}
$$

With:
rdt = return
εt = residual
ht = Conditional Variance.
Figure 10 shows the evolution of the actual return and the expected VARs of portfolio at 5% and 1% of threshold during the period between January 2009 and end May 2009.

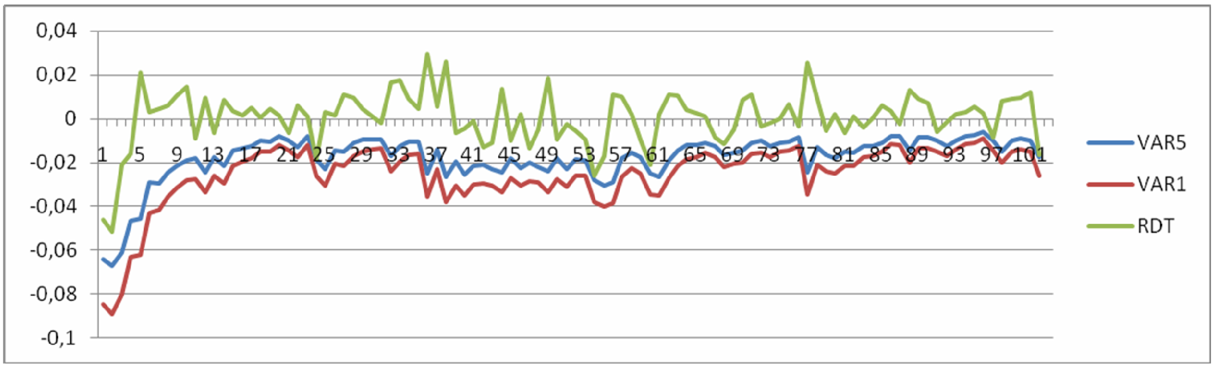

Figure 10: Forecast of the VAR (Period 02/02/2009 - 31/05/2009

The estimated VAR at 5% has been violated only once in comparison with the actual return. This shows some robustness of this model to forecast the VaR under a 5 months following the end-period of our studied sample. The VaR under 1% has never been violated by the actual return. The AR(1)-GARCH(1,1) is a very good model to predict the portfolio Value-at-Risk.

5. Conclusion

Through this investigation, we could demonstrate the robustness of the ARMA-GARCH model in the Value-at-Risk forecasting of a portfolio composed by four sectoral indices of Moroccan economy. We have verified that the number of VaR's violations decreases in the case of student law at the expense of the normal distribution. We have shown, by using the Backtesting findings, that the Value-at-Risk is a very good measurement of risk for our portfolio, even in the financial crisis situation (i.e., in the 2008 crisis case).

Nevertheless, the VaR remains a powerful tool of risk management, but it should be used with a high caution in order to interpret the given results with some efficiency. The VAR is a technique that appears scientific and precise, but it remains, in the several its own assertions, as a simply subjective method which is widely based on the personal judgments. Assessing the risk of maximum losing reflects both the acceptance of a number of assumptions and highlighting the value judgments which based on the intuition. Indeed, the calculation of VaR and its forecasting can sometimes be an excellent tool of decision, in the case where the estimated risk is in its "true value", sometimes an inefficient forecasting mean, when the risky value is overestimated.

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