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zone**

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**APPROACHING A PROBLEM OF THE LONG-RUN REAL  
EQUILIBRIUM EXCHANGE RATE OF POLISH ZLOTY WHILE  
ENTERING THE ERM-2 AND EURO ZONE**

Abstract:

Taking into account a large number of types of nominal and real exchange rates, while estimating the real equilibrium exchange rate, one should always remember that there is no a single, universal equilibrium exchange rate. A point value or a path of that exchange rate depends on the adopted definitions and assumptions as well as on the method and purpose of the analysis. However, a value added of each estimation of the equilibrium exchange rate is an answer, whether the economic policy causes upset or stabilisation of the economy. Moreover, in the period of discussion on the exchange rate of accession to ERM-2, showing an interval of the exchange rate where all values of the exchange rate ensure at least suboptimal behaviour of the economy may help to make a decision on the date of accession to ERM-2 that will minimise costs of retention of the exchange rate within a definite currency band. For Poland, estimated by the NATREX method the long-run real equilibrium exchange rate ensures the internal equilibrium with annual growth rates of GDP amounting to 4.1%, comprised of growth of consumption by 4% p.a., investment by 8.7%, volume of exports by 8.5% and volume of imports by 8.1% p.a. Estimating on the ground of real exchange rates an approximate value of nominal exchange rates, one can state that the long-term equilibrium in the economy is ensured with the exchange rate of 3.80-3.90 zlotys for 1 euro. The current exchange rate will probably approach the equilibrium exchange rate at the turn of 2010 and 2011, and it will remain near that level over 5-6 quarters. This means that in that period cost of retention of the PLN exchange rate within a narrow band of fluctuations is relatively the least. The next period where the current exchange rate should approach the optimal exchange rate is 2014. Then, also in the medium term, the exchange rate of zloty should be comprised within the interval of 3.80-3.90 (assuming the stable exchange rate of USD/EUR=1.40).\*\*

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\*\* The usual disclaimer applies.

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## Introduction

**The real exchange rate** is composed of the nominal exchange rate and a measure of the real level thereof. The nominal exchange rate covers bilateral foreign currency exchange relations; in a more complex form, it becomes an effective exchange rate, i.e. index of value of the domestic currency being set with consideration of the currency pattern of the country's settlements with foreign countries. The measures of the real level of exchange rate may be cost indices (e.g. unit labour costs) or prices (producers', retail, foreign trade).

Whereas **the real equilibrium exchange rate** is a normative value, impossible to be observed directly; only the real exchange rate is directly observable. The level of the real equilibrium exchange rate is decided both by the fundamental real external and domestic variables, including those dependent on the economic policy, and variables exerting their temporary, a short run, influence.

Let's assume that any economy is in every moment in a short-run equilibrium, being a combination of a temporary equilibrium in the market for goods and in the financial market. The state of temporary equilibrium needs not to be an accepted state; therefore, it needs not to mean either full employment or desired inflation, or a definite deficit on the current account, which would be safely financed. Nevertheless, the short-run equilibrium sets out current values of all endogenous variables for a given economy, including also values of the real exchange rate, irrespective of the adopted definition of that rate. In turn, the current values of endogenous variables are determined by the values adopted by the set of variables that are either exogenous variables for the economy or the variables, which are not a part of the short-run equilibrium, nevertheless, their changes over the time depend both on the current and expected states of economy. The second type of variables is traditionally divided into predetermined variables (e.g. lagged values of endogenous variables) and instrumental variables (e.g. interest rate, tax rates).

The real exchange rate ensuring the long-run equilibrium (LREER) is then determined by the real exchange rate ensuring a short-run equilibrium, which takes place with fixed values of instrumental and exogenous variables.

LREER, though is a stable rate in a long run, needs not to be either the optimal rate or the desired one, as the set of instrumental values not necessarily is a set of optimal values of those variables. If we assume that the optimal vector of values of instrumental variables is dependent on the values being adopted by exogenous variables, then we receive the desired equilibrium real exchange rate, DEER.

Of an approximate to DEER interpretation is the real exchange rate, described by fundamental variables, called in the literature FEER, the fundamental equilibrium exchange rate.

According to the described definitions of the real exchange rate, the concept of the equilibrium exchange rate resolves itself to examination of the existence and constancy of long-term relations between the fundamental variables. The nominal exchange rate is then being adjusted until the real exchange rate reaches the postulated level. In such a situation, examination of the real equilibrium exchange rates resolves itself to:

- seeking for a long-run interdependence between the level of prices or costs (the PPP, purchasing power parity model; the Mundel-Fleming model);
- modelling behavioural long-run interdependences between the fundamental variables (structural models, reduced possibly to single-equation models, e.g. models of the NATREX (natural real equilibrium exchange rate) type, based on the theory of growth, structural models of Edwards and Elbadawi or the model of McDonald, seeking for a long-run interdependence between the chosen variables (without reference to the structural models);
- general equilibrium models (or, possibly, partial equilibrium models), e.g. of the 'macroeconomic balance approach' type, in which there is indicated the desired level of equilibrium on the current account and financial account; models of the CGE type, e.g. the DLR model (the three-sector model of general equilibrium, Devarajan, Lewis, Robinson).

Taking into account a large number of types of nominal and real exchange rates, while estimating the real equilibrium exchange rate, one should always remember that there is no a single, universal equilibrium exchange rate. A point value or a path of that exchange rate depends on the adopted

definitions and assumptions as well as on the method and purpose of the analysis. However, a value added of each estimation of the equilibrium exchange rate is achievement of an answer, whether the carried out economic policy causes upset or stabilisation of the economy. Moreover, in the period of discussion on the exchange rate of accession to ERM-2, showing an interval of the exchange rate where all values of the exchange rate ensure at least suboptimal behaviour of the economy may help to make a decision on the date of accession to ERM-2 that will minimise costs of retention of the exchange rate within a definite currency band.

If we assume that over at least two years (the minimum period of participation in ERM-2) zloty variations should theoretically be comprised within the interval of +/-15%, and, in practice zloty, particularly on the side of depreciation, should not exceed the limits of a narrow band (+/- 2.5%), then, looking at variability of zloty within the last 24 months (from 30.06.2007 to 1.07.2009), we have to state that zloty is one of the least stable currencies in the region (Table 1 and Figure 1). In July 2008, there was crossed the allowable limit of appreciation of zloty (on 27 June, zloty was appreciated by 15.6% as compared with its value at the beginning of the reference period), and in February 2009 the maximum limit of depreciation – by more than 10 percentage points: on 21 February, zloty was depreciated by 25.3%. They were values worse than forint's behaviours (8.6% maximum appreciation, 24.1% maximum depreciation) and definitely worse than Czech koruna (respective values are 9.2 and 3.2%) and Slovak koruna (respectively 9.8 and 2.2%). If, on the other hand, we put back the reference period by two years, to the period from 30.06.2005 to 1.07.2007, then zloty was, at that time, one of the most stable currencies in the region – it was considerably more stable than Slovak koruna and Hungarian forint.

Table 1

Variability of currencies of Poland, Czech Republic, Slovakia and Hungary  
in the conventional reference periods

	PLN	CZK	SKK	HUF
Reference period: 30.06.2003 – 01.07.2005				
Maximum depreciation (w %)	13.7	7.5	4.8	8.7
Maximum appreciation (w %)	10.5	6.4	5.0	3.5
Reference period: 30.06.2005 – 01.07.2007				
Maximum depreciation (w %)	5.9	5.6	6.2	9.1
Maximum appreciation (w %)	3.9	2.1	10.6	5.4
Reference period: 30.06.2007 – 01.07.2009				
Maximum depreciation (w %)	25.3	3.2	2.2	24.1
Maximum appreciation (w %)	15.6	9.2	9.8	8.6

Source: Tables of daily exchange rates of relevant central banks.

It is proper to remind that in the two-year reference period prior to euro acceptance, the mean maximum depreciation for the countries applying to that zone amounted to 2.1% (1.4% excluding Italy), and appreciation to 3.7% (1.3% excluding Ireland and Greece). On the side of depreciation, a significant exception was Italy (7.8%); on the side of appreciation, Ireland (10.9%) and Greece (9.2%).

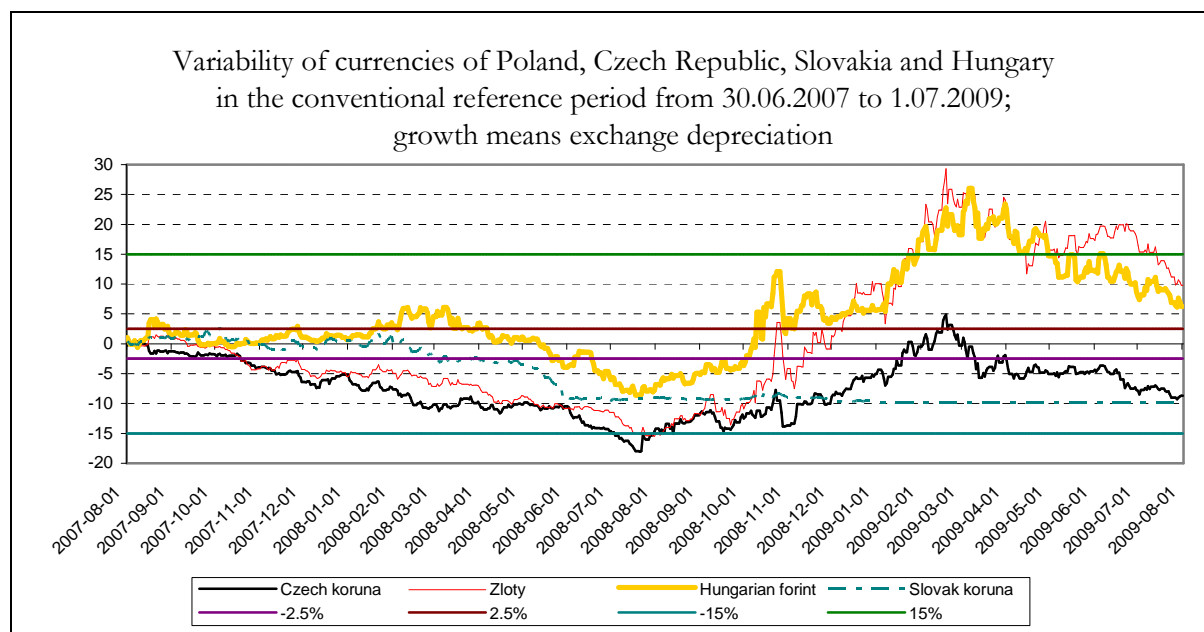
The reasons for appreciation both of zloty and Czech and Slovak koruna, as earlier pound or drachma, not fully can be explained by a relatively faster growth of labour productivity in the sphere of tradable goods than non-tradable ones, though opinions of economists on this issue vary very much. Alberola and Navia<sup>1</sup> as well as Egert<sup>2</sup> believe that an important factor causing appreciation of currency is

<sup>1</sup> Enrique Alberola and Daniel Navia: *Equilibrium Exchange Rates in New EU Members: External Imbalances vs. Real Convergence*, Bank of Spain 2007.

<sup>2</sup> Balázs Egert: *Real Convergence, Price Level Convergence and Inflation Differentials in Europe*, Working Paper 138/2007, Oesterreichische Nationalbank

the Balassa-Samuelson effect. The problem is that results of empirical studies are very much unstable. Depending on the method applied, the B-S effect for the same currency explains either one half or almost zero of appreciation. In this context, there seems to be interesting a survey carried out by Cincibuch and Podpiera<sup>3</sup>, showing a significant importance of foreign direct investment in explanation of currency appreciation – FDI made in modern sectors rises the medium level of productivity and quality of products, increasing competitiveness of domestic products in the home market and abroad what does, through CA, act towards appreciation.

Figure 1



Source: Tables of daily exchange rates of the relevant central banks.

In general, among economists dealing with empirical studies the relation of exchange rate with the real macro or micro sphere is not unanimous. For example, MacDonald and Ricci<sup>4</sup> think that explanation of exchange rate changes in a short and medium period, with application of variables describing the real economy, does not yield better results than the random walk models. If, however, we use respectively current information coming from the market and not past information of statistical offices on the fundamental factors, then results of exchange rate modelling may be surprisingly good (Evans and Lyons<sup>5</sup>).

We dealt with a different situation from August 2008 to March 2009. Over eight months, from the end of July 2008 to the end of March 2009, the exchange rate of zloty as related to dollar was depreciated by 87%. Counted on monthly data, the nominal effective exchange rate of zloty was depreciated by 33% (in the Czech Republic, by 13%, in Hungary by 16%). Appreciation of zloty with a similar scale lasted from the second quarter of 2004 to the third quarter of 2008; i.e. 50 months. A larger scale of depreciation of zloty than Czech koruna and Hungarian forint, despite Hungary's troubles with financing their indebtedness, was caused by anxiety about Poland's macroeconomic stability and, first of all, about our abilities to provide for debt service. Investors placed Poland between the Czech Republic and Hungary, on the one hand, and the Baltic countries and Ukraine, on the other hand, withdrawing only in October 2008 from the Polish market 12 bn zlotys of portfolio capital. Having added to this a permanent transfer abroad of income on direct investment, approx. 3.3 bn PLN per month, and a drop by 1/3 of sales in the exchange market (a significant shallowness of the market facilitates speculative attacks), one should not be surprised to deal with 33% depreciation of the nominal effective exchange rate of zloty.

<sup>3</sup> Martin Cincibuch, Jiri Podpiera, *Beyond Balassa-Samuelson: Real Appreciation in Tradables in Transition Countries*, Economics of Transition vol. 14(3)/2006, The European Bank for Reconstruction and Development.

<sup>4</sup> MacDonald, R. and L. A. Ricci, *Real exchange rates, imperfect substitutability and imperfect competition*, Journal of Macroeconomics, vol. 29(4)/2007

<sup>5</sup> Martin D. D. Evans, R. Lyons, *How Is Macro News Transmitted to Exchange Rates*, Journal of Financial Economics, forthcoming 2008.

Table 2

## Comparison of equilibrium exchange rates

	Purchasing Power Parity	Behavioural Equilibrium Exchange Rate, BEER	Fundamental Equilibrium Exchange Rate, FEER	Desired Equilibrium Exchange Rate, DEER	Permanent Equilibrium Exchange Rate, PEER	Natural Real Equilibrium Exchange Rate, NATREX
Theoretical bases	Convergence of the levels of domestic and foreign prices. The exchange rate is constant over the entire period.	Based on the concept of real uncovered interest parity (UIP) plus the risk premium approximated by fundamental variables.	Real exchange rate ensuring internal equilibrium (real GDP equal to the potential product) and external one (permanent balance on the current account).	Like FEER but with the assumed optimal economic policy	Like BEER but with a separated permanent, determined by the trend, component of explaining variables.	Real exchange rate ensuring internal equilibrium (real GDP equal to the potential product) and external one (permanent level of foreign indebtedness).
Horizon	(Very) long run	Short run	Medium run	Medium run	Medium/long run	Long/medium run
Assumptions	Identical price baskets (identical structure of domestic and foreign consumption). Permanent cointegrating relation, i.e. disregard for impact of productivity changes on the exchange rate.	Expected change of an equilibrium exchange rate depends on disparity of real interest rates and on the set of variables that can affect the real current exchange rate.	Demand gap estimated from the Cobb-Douglas function. Arbitrary set-up of the optimal balance on the current account.	As FEER, plus arbitrarily assumed volumes of instrumental variables.	As BEER.	Demand gap estimated from the Cobb-Douglas function or from the trend. Assumption on equilibrium in the balance of portfolio capital (i.e. assumption of convergence of domestic and foreign real interest rates).
Advantages	Easy determination of the equilibrium exchange rate.	Easy way of determination of the exchange rate. It allows identifying the factors determining the current real exchange rate.	Great transparency in exchange rate determination. Real exchange rate concordant with the medium-term macroeconomic equilibrium.	As FEER.	As BEER.	As FEER but both in medium and long term.
Short-comings	Disregard for specificity of economies and structural changes taking place therein. PPP ensures only achievement of a definite level of prices.	Vague interpretation of BEER. BEER does not guarantee achievement of either internal or external equilibrium.	Results of computations depend on the adopted assumptions. There is the need for an analysis of sensitivity to the adopted assumptions.	As FEER.	As BEER.	Great sensitivity to changes in parameters of the production function. Lack of the possibility to introduce speculative capital.

Source: Own elaboration.

It is proper to pay attention to the fact that the spread over time changes in the exchange rate are accompanied by a drop in its variability what, on the one hand, gives enterprises time to adjust to new terms of competitiveness and, on the other hand, lowers costs of protection against the exchange risk. Each acceleration of changes in the exchange rate cuts the time of adjustments in the real sphere as well as it extorts and increases the cost of protection.

Remembering that the real exchange rate is a combination of the nominal exchange rate and the price (cost) index, definition of the path of the long-run equilibrium exchange rate may help in choice of the optimal date of ERM-2 accession and the suboptimal (as related to a given date) level of the accession exchange rate.

In the hitherto surveys on estimation of the real equilibrium exchange rate in Poland, there have been used the concepts of behavioural equilibrium exchange rate<sup>6</sup> (BEER), the exchange rate assuring equilibrium of the fundamental variables (FEER) and in the version of FEER applied by the International Monetary Fund, called the macroeconomic balance, and the permanent equilibrium exchange rate (PEER), approximate in its concept to BEER. The results of estimation of the exchange rate by the FEER method received by M. Rubaszek, systematically repeated at NBP, show cyclical fluctuations of the current real exchange rate around the equilibrium exchange rate. The highest overestimation of the exchange rate of zloty was observed in the first half of 2000 (more than 20%) and in the first half of 2008 (approx. 10%). On the other hand, the highest underestimation took place at the turn of 2003 and 2004 – a little bit below 20% and in the first half of 2009 – approx. 15%.

The model works carried out by Smidkova and Bulir<sup>7</sup> confirm significantly lower deflections of the national currencies parity from the equilibrium in Greece, Spain and Portugal prior to their accession to ERM-2 than in the Czech Republic, Poland and Hungary as well as dependence of the results on the survey concept adopted<sup>8</sup>. Horvath and Komarek<sup>9</sup> see a similar dependence, emphasising that deflections from the parity, due to uncertainty of estimates, have their information value rather as regards the direction of changes and not the level. Coudert and Couharde<sup>10</sup> show that of the key importance for estimation may be the period adopted for the examination: differentiating the length of sample they receive for zloty in 2003 deflection from the equilibrium exchange rate comprised between -1% and 12.7%, and for 2005, respectively, from 4% to 11%.

Table 2 presents theoretical bases of individual types of the equilibrium exchange rate, showing the assumptions, with which those concepts were built, as well as presenting advantages and shortcomings of each concept, indicating the horizon where the obtained results may be used.

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<sup>6</sup> Rubaszek, M.: *A Model of Balance of Payments Equilibrium Exchange Rate: Application to the Zloty*, Eastern European Economics, 42(3)/2004.

<sup>7</sup> Bulir A., K. Smidkova: *Exchange Rates in the New EU Accession Countries: What Have We Learned from the Forerunners?*, IMF Working Paper, WP/05/27, February 2005, Washington.

<sup>8</sup> In the short run, best proves itself the NATREX and BEER method; in medium: FEER and DEER; in long run: NATREX and PPP.

<sup>9</sup> Horvath R., L. Komarek: *Equilibrium Exchange Rates in EU New Members: Applicable for Setting the ERM II Central Parity?*, Czech National Bank, 2006.

<sup>10</sup> Coudert V, C. Couharde: "Real Equilibrium Exchange Rate in European Union New Member and Candidate", 3<sup>rd</sup> Euroframe Conference on Economic Policy Issues in the European Union, 2 June 2006, Berlin.



## Chapter 1: Concept of the Natural Real Exchange Rate, NATREX

Contrary to models of the PPP (purchasing power parity) type, the macroeconomic balance approach or FEER (fundamental real equilibrium exchange rate) that satisfactorily explain the dynamics of the real exchange rate in a (very) long or medium time horizon, **the concept of NATREX** formulated in 1990 by Stein<sup>11</sup> proves itself in the both periods, **constituting a generalisation of the PPP and FEER hypotheses**. Therefore, in the presented study, the analysis of the long-run equilibrium real exchange rate of zloty will be carried out based on the NATREX method.

If, according to PPP, the real equilibrium exchange rate is constant, then **NATREX explains how that rate evolves depending on changes of the fundamental factors**, such as relation of consumption (private and public) to GDP, productivity of capital (private + public), terms of trade or real interest rates in Poland and in the most important economic areas (USA, EU). If the linear combination of these (or those adapted to the definite model) fundamentals is stationary, then the real equilibrium exchange rate is a constant, i.e. there takes place the PPP hypothesis. NATREX does not require a stationary form of variables, the real equilibrium exchange rate is in this case not a point but a trajectory, along which the fundamental variables approach their steady states.

According to Stein, **NATREX is a real equilibrium exchange rate that ensures an internal equilibrium as well as external equilibrium** in the medium and long term, provided that:

- there are not cyclical fluctuations,
- there are no flows of speculative capital,
- there are no changes in the state of international reserves.

The internal equilibrium is assured if the real gross domestic product is equal to the potential product.

The external equilibrium takes place if the medium- and long-term balance of payments keeps foreign indebtedness at a constant level. Short-term (speculative) flows of capital as well as changes in the official reserves are considered as short-term transactions. If, however, those transactions are maintained in a long run, then they are taken into consideration on long-term accounts of balance of payments. Otherwise, in a long run, their mean value should equal zero. Then surplus of national investments  $I$  (private and public) over national savings  $S$  (private and public) is fully financed by long-term foreign loans. In such a situation, for the real values  $I$  and  $S$  true is the equation:

1.  $S-I=CA$ , where  $CA$  means current account of the balance of payments.

It is proper to note that the expression (1) ensures the medium-run external equilibrium and it does not differ from the standard model of macroeconomic balance approach used by the IMF for computation of the real equilibrium exchange rate or from the FEER model. NATREX does, in the medium term, equal the balance on current account with savings and investments if production capacity is at its long-term mean<sup>12</sup> level, and portfolio investments are balanced. Then the real domestic interest rate tends to the level determined by the real external rate increased by the risk premium. In such a situation, investors are indifferent whether they buy the domestic or foreign securities.

Let's analyse what takes place with the real equilibrium exchange rate if we knock off the economy from the equilibrium, introducing, for example, a demand impulse, consisting in growth of the relation of consumption to GDP, what took place in Poland at the turn of 2003 and 2004 (accession-related shock). Growth of the relation of consumption to GDP should cause in the medium term the so-called Mundell-Fleming effect, i.e. decrease in the result of S-I. This means either decrease in outflow of capital, or increase in inflow thereof. In both cases, it comes up to appreciation of the real exchange rate of the domestic currency what, in turn, should result in increase of deficit on the current account. Deficit affects the growth of foreign indebtedness (F) what does, through increase

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<sup>11</sup> See: Stein, J. L., 1990, The Real Exchange Rate, *Journal of Banking and Finance*, 14 (5), special issue.

<sup>12</sup> This is identical with the assumption of a natural unemployment rate or an approximate to zero level of demand gap.

Figure 2

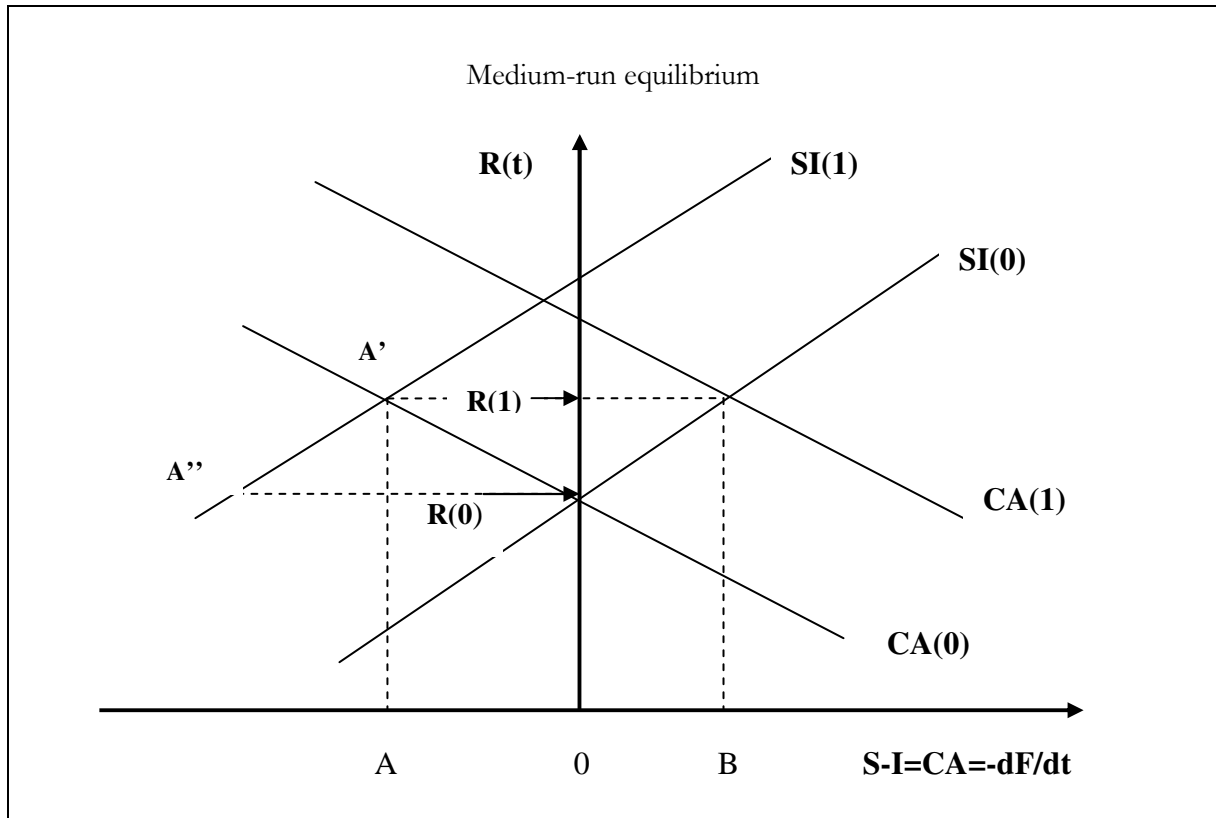
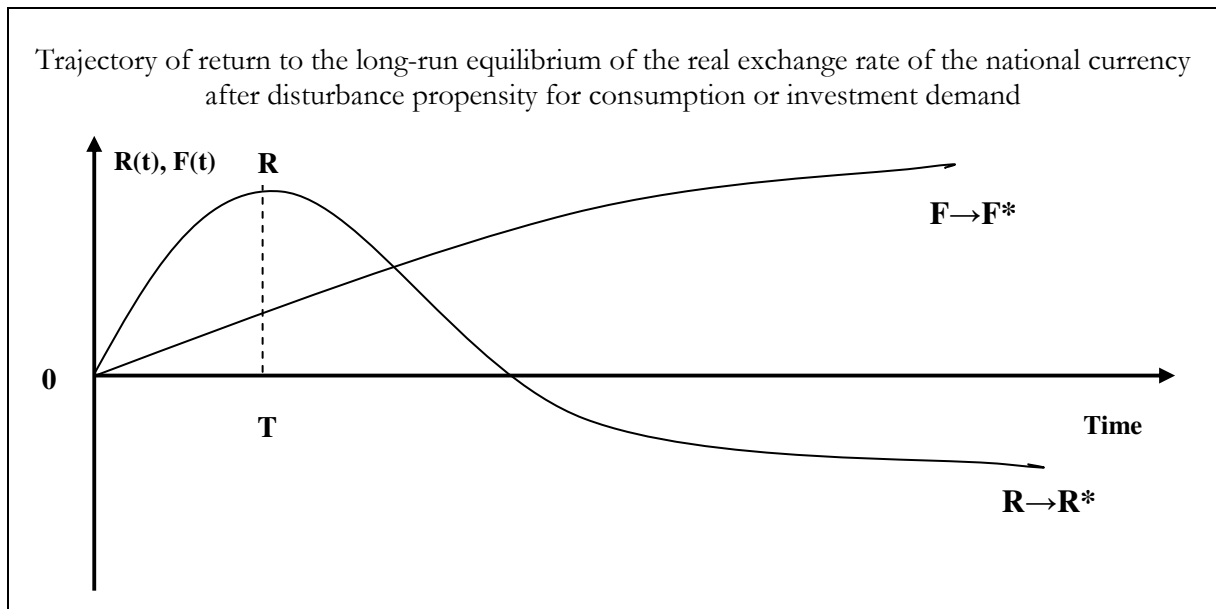


Figure 3



Source: According to the concept described in: Stein J. L., 1990, *The Real Exchange Rate*, Journal of Banking and Finance, 14 (5), special issue.

of value of the interest paid, additionally increase deficit on the current account. In the dynamically stable system, savings should be positively correlated with indebtedness, i.e. growth in indebtedness should force growth of savings (otherwise indebtedness would have been increasing to infinity). In result, there will decline previously induced an additional consumption demand. At the same time, till the moment foreign indebtedness is stabilised, i.e. in a long run, the real exchange rate of the national

currency should be depreciated<sup>13</sup>. Depending on the scale of drop in consumption demand and on the real depreciation of the domestic currency,  $S-I$  may acquire both negative and positive values. If  $I > 0$  and  $S-I < 0$ , then, due to a full mobility of capital, one should expect inflow of long-term capital that will affect appreciation of the rate of exchange, increasing deficit on the current account. One should also remember that dynamics of investment determines changes in the stock of capital  $K$  what does, affecting productivity of the economy, also exert its impact towards appreciation of the exchange rate and affects changes both on the current account and savings. Shocking the real level of the fundamental variable, we receive new levels of foreign indebtedness and productivity, which cause a permanent shift of the earlier trajectory of the real equilibrium exchange rate assuring the external and internal equilibrium in the medium and long run.

The described dependencies may be presented graphically in the following way (Figure 2): growth in propensity for consumption causes that the curve  $S-I$  shifts from the position  $SI(0)$  to the position  $SI(1)$ . Drop in savings in relation to investments induces influx of capital that affects appreciation of the real exchange rate of the national currency from  $R(0)$  to  $R(1)$ . Inflow of capital causes a greater transfer of stocks, and because  $CA = -dF/dt$ , so deficit on the current account causes growth of foreign indebtedness. The curve  $CA$  shifts leftwards, along  $SI(1)$ , from the point A' to the point A'', and the real exchange rate depreciates. Growth of indebtedness causes decrease of wealth – in result, propensity for consumption declines, savings grow, the curve  $S-I$  shifts rightwards, and foreign indebtedness stabilises at a higher level.

The analogical reasoning, hence, the analogical effects are received if there grows demand for investments, e.g. if, in result of liberalisation or privatisation, there grows inflow of foreign direct investment (FDI).

Figure 3 describes the shift from the medium-run to long-run equilibrium. Figure 2 shows that the real exchange rate of the national currency appreciates from the level  $R(0)$  to  $R(1)$ . At the same time, there grows foreign indebtedness  $F(t)$  (Figure 3). After the time  $t=T$ , growing interest (and dividend) on foreign indebtedness aggravates balance on the current account, induce depreciation of the real exchange rate to its long-run level  $R^*$ . This exchange rate level may be below the initial level. At the same time, foreign indebtedness grows till it achieves a new level of the long-run equilibrium  $F^*$ .

As is has been pointed out, in the NATREX concept, there is clearly emphasised both cohesiveness between the medium- and long-run equilibrium as well as differentiation of these equilibriums: in the state of medium-run equilibrium, there are made investment decisions causing change of the stock of capital ( $K$ ) and affecting foreign indebtedness ( $F$ ). Terms and conditions ensuring the medium-run equilibrium, changing over time, cause that the stock of capital and foreign indebtedness strive in a long run for their steady states. We receive, therefore, a structural model of growth basing on a rigorous compliance with the principles of resource flows and combining the real exchange rate with the set of fundamental variables explaining investments and savings as well as the current account. The model, in which the decisions on consumption, production, savings and investments determine equilibrium in the economy.

In a given economy, functioning according to the above-described logic of NATREX, the path of the real exchange rate of the national currency ensures a full equilibrium in the medium and long period if there are met the following assumptions:

- Consumption is understood as a sum of private and public consumption. Investment is also a sum of investment in the private and public sectors.
- Decisions on consumption and investment are made in a decentralised way. Households decide on consumption and savings, firms – on production and investment. Decisions are made in the identical way by private and public persons/entities.

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<sup>13</sup> The scale of depreciation is not determined unanimously. By Stein (in: Stein J.L. and G. Paladino, 1998, *Exchange Rate Misalignments and Crises*, Brown University Working Paper 98/7), the scale of depreciation of the national currency should be higher than that of appreciation. And by Martinez (in: Detken C., C. M. Martinez, 2001, *The Effective Euro Equilibrium Exchange Rate since the 70'ties: A Structural Natrex Estimation*, ECB Conference Paper), the scale of depreciation depends on how fast there is stabilised foreign indebtedness – depreciation may be both symbolic and significantly exceeding the previous appreciation.

- The optimal decisions on consumption and savings are made based on maximisation of the inter-temporal function of utility. Decisions on production and investment are made under conditions of perfect competitiveness (homogenous, small firms) and having assumed the constant returns resulting from the scale of production.
- Produced and consumed goods are homogenous.
- Supply of labour is non-flexible; flexible are wages and salaries.
- Capital is perfectly mobile and the external interest rate is exogenous. Then the national real interest rate is determined in the market for portfolio capital or by the condition to maintain the interest rate parity (taking place continuously), plus possible risk premium. This assumption means that return to equilibrium after a shock is made by adjustment of variables from the real sphere and not by the interest rate.
- Fluctuations of speculative capital are allowable in a short run. Their sum in the medium and long term amounts to zero. A similar reservation is made in relation to international reserves.
- There takes place neutrality of money, and monetary authorities have been maintaining inflation for a long period at the level ensuring the internal equilibrium. Thus, there is no need to model the market for money, and they may have focused on the real sphere.
- NATREX describes inter-temporal behaviours. This means the necessity to eliminate the cyclical components from the time series. It is assumed that the interest rate disparity ought to be the only stationary variable.

## Chapter 2. The NATREX model for zloty: theoretical construction

The models based on the NATREX concept belong to the class of dynamic models of growth describing the open economy and they are constructed for the purpose of determination of a medium- and long-run path of equilibrium of the real exchange rate. In the subject matter literature, there are described two possible ways to estimate the real equilibrium exchange rate. First of them, applied in Stein's studies, consists in estimation of the reduced form of single-equation model with the use of co-integration techniques. The second, popularised by Gandolfo (1998), Detken and Martinez (2001), uses the classic structural multi-equation models (Gandolfo) or the dynamical models of general equilibrium (Detken and Martinez).

In the presented study, the multi-equation model was applied, similar to the NATREX models built for the Italian lira (Gandolfo 1998) and forint (Karadi 2003). The reason for such a decision was sharing of the Clark's critique (1996) of single-equation models. Clark states that in models of this type there is danger that they rather multiply the current trend of the real equilibrium exchange rate than define a path of the long-run equilibrium of the real exchange rate.

The model consists of four dynamic behavioural equations (investment, consumption, trade balance and real interest rate) and of relevant balance equations.

### The investment equation

Investments ( $I$ ) are understood as gross fixed investments in constant prices, being made in the private and public sectors. Inter-temporal investment decisions are made by rationally acting entities, which effectively use the entire information available. The investment function is received by way of use of the so-called closed-loop rule on dynamic programming (Gandolfo 1998) that allows sub-optimal steering in the closed system, guaranteeing convergence with an unknown state of equilibrium<sup>14</sup>. Thus obtained investment is a positive function of the difference between the marginal productivity of capital and the long-term real interest rate. The built-in feedback mechanism guarantees a direct convergence to the steady state, i.e. assuming declining income from capital, any growth of the stock of capital ( $dK/dt \equiv I > 0$ ) leads to a decrease of the marginal productivity of capital and, *ceteris paribus*, to a decline in investment rate of growth. Graphically this is presented in Figure 4.

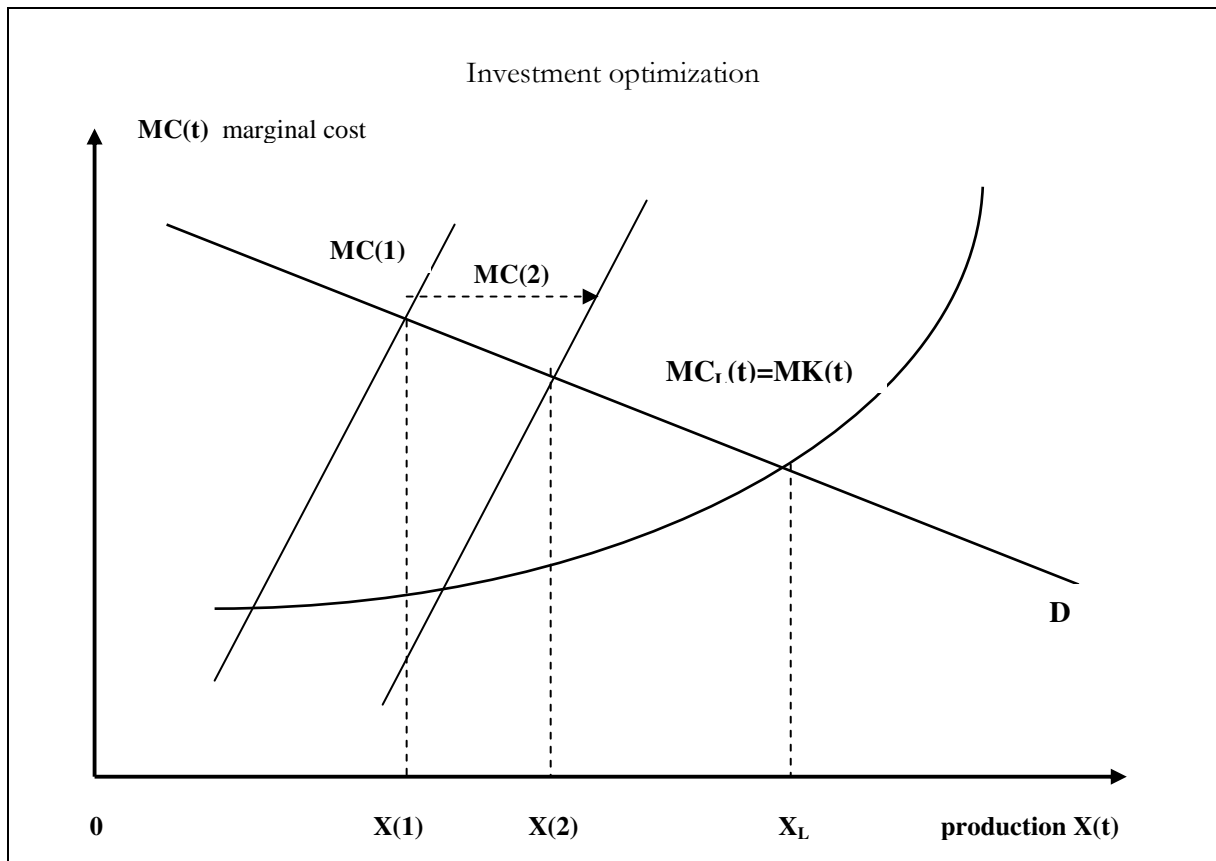
Investment decisions are made if the expected marginal return on capital employed exceeds alternative costs. If the decision is made, then there is expected productivity growth that will affect the marginal cost<sup>15</sup> ( $MC$ ) of production of tradable goods. Volume of production ( $X$ ), depending on  $MC$  and on quality thereof is determined by demand  $D$ . In a short period,  $MC$  is a function of prices of production and productivity; capital  $K$  is constant. In a long run,  $K$  varies, affecting the marginal cost of production, according to the function  $MC_L(t)$ . For the marginal cost  $MC(1)$ , the optimal volume of production amounts to  $X(1)$ . At this point, the short-term  $MC$  exceeds the long-term marginal cost  $MC_L$ , equal to the optimal productivity of capital  $MK_L$ , what means that the stock of capital is below the optimal one, i.e. marginal return on investment exceeds the alternative cost. This induces to make investment decisions what causes an increase in productivity and a decline in marginal costs of production from  $MC(1)$  to  $MC(2)$ . The lower effectiveness at the point  $MC(1)$  causes that more profitable was production of goods of a low quality. The growth in productivity and the shift of the curve of marginal costs to the point  $MC(2)$  causes that production of goods of higher quality becomes profitable. Therefore, growth of production from  $X(1)$  to  $X(2)$  means not only growth in quantity but also growth in quality of production what, in turn, affects improvement of competitiveness of goods being manufactured in foreign markets, improving the trade balance and, thus,  $CA$ . In a long run, this takes effect of a long-term appreciation of the real exchange rate of the national currency  $R(t)$  and of a decrease in the share of foreign indebtedness in GDP  $F(t)$  (Figure 5).

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<sup>14</sup> We deal with the optimal steering in the closed-loop system if we know the state of equilibrium (see, e.g. Infante and Stein, 1973).

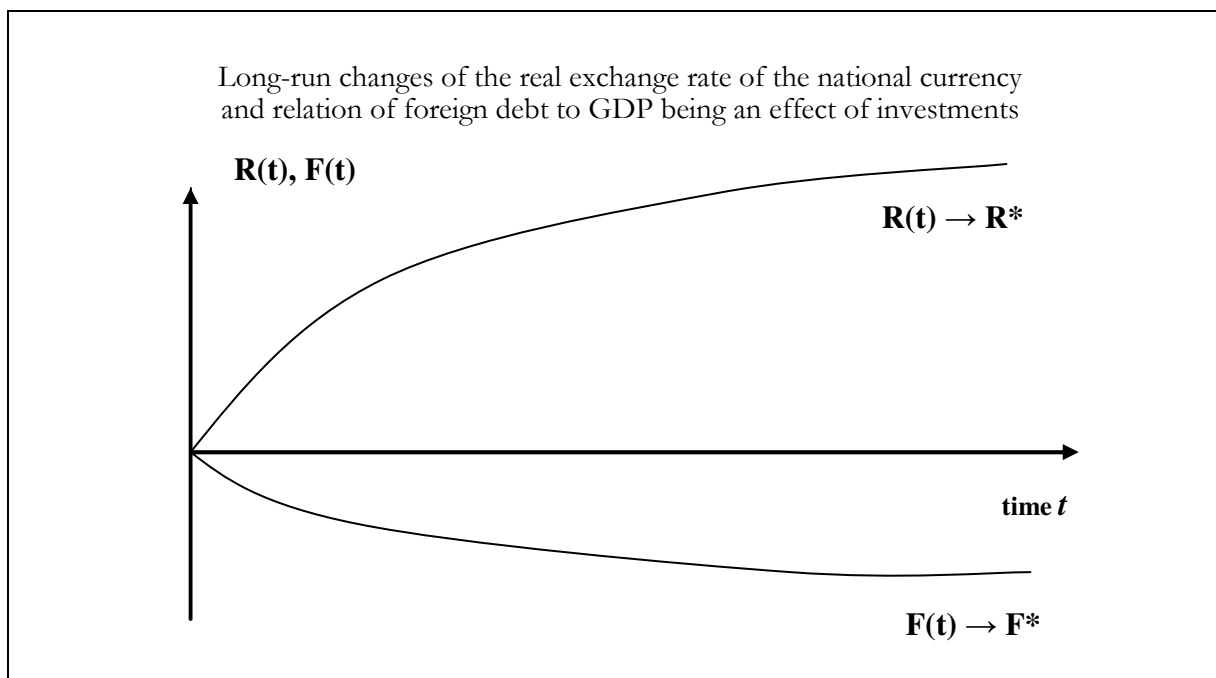
<sup>15</sup> The marginal cost is a relation of prices of production to the marginal productivity.

Figure 4



Source: According to the concept outlined in: Gandolfo G., A. Felettigh *The NATREX: an Alternative Approach. Theory and Empirical Verifications*, CIDEI Working Paper, no. 52, November 1998.

Figure 5



Source: As in Figure 4.

In the one-sector economy, we shall receive the optimal level of capital when the marginal productivity of capital ( $\mathbf{MK}$ ) is equal to the long-run interest rate ( $r$ ). In the multi-sector economy producing tradable and non-tradable goods  $\mathbf{MK}$  is also affected by a relative price of various goods. This price may be approximated by the real exchange rate of the national currency ( $\mathbf{R}$ ). In effect, the marginal productivity of capital is a function of the long-run real interest rate and of the real exchange rate of the national currency:

$$2. \quad \mathbf{MK}_t = f(r_t, \mathbf{R}_t)$$

The investment equation received from resolution of the task of sub-optimal steering (remembering that  $d\mathbf{K}/dt \equiv \mathbf{I}$ ) has the following form:

$$3. \quad \mathbf{I}_t - \mathbf{I}_{t-1} = \alpha_1 (\mathbf{I}^*_{t-1} - \mathbf{I}_{t-1}) \quad 0 < \alpha_1 < 1$$

where  $\mathbf{I}^*$  means the desired investment, defined as a positive function of the difference between the marginal productivity of capital ( $\mathbf{MK}$ ) and the long-run real interest rate ( $r$ ):

$$4. \quad \mathbf{I}^*_t = f_1(\mathbf{MK}_t - r_t)$$

From the equations 3 and 4, we receive:

$$5. \quad \mathbf{I}_t = \beta_1 \mathbf{I}_{t-1} + f_1(\mathbf{MK}_{t-1} - r_{t-1}) \quad 0 < \beta_1 = 1 - \alpha_1 < 1$$

The constant  $\alpha_1$  is included, for simplification, under the function of operator  $f_1$ . In the equation 3, the desired investment  $\mathbf{I}^*$  is delayed by one period due to the time necessary for production of investment goods. Therefore, investment decisions made in time  $t_0$  will be actually carried out in time  $t_1$ . In the equation, there is no absolute term what results from the analysis horizon: in the long-term equilibrium  $d\mathbf{K}/dt = 0$ , i.e. there is only carried out restitution investment.

### The consumption equation

The term ‘consumption’ ( $\mathbf{C}$ ) means private and public consumption in constant prices. The consumption equation is received in the analogous way as the investment equation. The NATREX concept postulates that consumption ought to be positively correlated with the real current GDP ( $\mathbf{Y}_t$ ) and negatively with the level of foreign indebtedness ( $\mathbf{F}_{t-1}$ ). As it is presented in Figures 2 and 3, the variable  $\mathbf{F}$  guarantees convergence of consumption in a long period to its steady state (growth in  $\mathbf{F}$  causes contraction at the part of consumption and, in effect, growth of savings what stabilises  $\mathbf{F}$ ). The assumption of neutrality of money, included into the NATREX method results in that under conditions of incomplete information entities making their decisions on consumption take into account both the nominal and real values of variables of their concern (imperfect money illusion). In practice, this means that the equation should include approximation of the general price level in the economy. Most often there is used deflator of GDP ( $\mathbf{PY}$ ). If  $\mathbf{F}$  means the nominal level of foreign indebtedness deflated by  $\mathbf{PY}$ , then growth  $\mathbf{PY}$  means growth in nominal values of relevant variables. As  $\mathbf{C}$  is correlated positively with  $\mathbf{Y}$  and negatively with  $\mathbf{F}$ , then the sign of  $\mathbf{PY}$  is not determined unanimously and it depends on propensity for consumption related to  $\mathbf{Y}$  and to  $\mathbf{F}$ . The consumption equation takes the following form:

$$6. \quad \mathbf{C}_t - \mathbf{C}_{t-1} = \alpha_2 (\mathbf{C}^*_t - \mathbf{C}_{t-1}) \quad 0 < \alpha_2 < 1$$

The desired consumption  $\mathbf{C}^*$ , according to the earlier description, is defined as:

$$7. \quad \mathbf{C}^*_t = f_2(\mathbf{Y}_t, \mathbf{F}_{t-1}, \mathbf{PY}_t)$$

$$8. \quad \mathbf{C}_t = \beta_2 \mathbf{C}_{t-1} + f_2(\mathbf{Y}_t, \mathbf{F}_{t-1}, \mathbf{PY}_t) \quad 0 < \beta_2 = 1 - \alpha_2 < 1$$

The absolute term is omitted for the same reason as in the investment equation, and the constant  $\alpha_2$  is included under the operator  $f_2$ .

## The trade balance equation

The notion ‘trade balance’ ( $B$ ) means, according to the NATREX terminology, the balance of goods and services in constant prices. The volume of exports ( $XV$ ) in constant prices is a function of the real GDP of the major trade partners of the country in question ( $YEU$ ) (it appears with the plus sign) and the real effective exchange rate of the national currency ( $R$ ), appearing with the minus sign (growth of value of exchange rate means appreciation). The volume of imports ( $MV$ ) depends positively on the real GDP in the country in question ( $Y$ ) and on the real exchange rate ( $R$ ).

In a small economy, which is a price taker, the real exchange rate ( $R$ ) is defined as the nominal effective exchange rate of the national currency (the unit of foreign currency for the unit of national currency  $N$ ) deflated by the index of unit labour costs in manufacturing domestic ( $DULC$ ) and foreign ( $FULC$ ) or deflated by the index of GDP prices domestic ( $DPY$ ) and foreign ( $FPY$ ):

$$9. \quad R = \frac{N}{\frac{FULC}{DULC}} \quad 9a. \quad R = \frac{N}{\frac{FPY}{DPY}}$$

So defined the real exchange rate is a measure of international competitiveness of the sector of economy producing tradable goods<sup>16</sup>.

In the NATREX concept, there is made an assumption that commodity trade is supplemented by exports and imports of services (transport, bank and insurance services, other transaction costs) and, therefore, the latter is proportional to exports and imports of goods. Therefore, there are not needed additional equations describing trade in services. Hence, the trade balance equations are as follows:

$$10. \quad B = XV - MV$$

$$11. \quad B_t - B_{t-1} = \alpha_3(B_{t-1}^* - B_{t-1}) \quad 0 < \alpha_3 < 1$$

where  $B^*$  means difference between the desired volume of exports and imports, and it is described by the following equation:

$$12. \quad B_t^* = f_3(Y_t, R_t, YEU_t)$$

In the equation 11, the variable  $B^*$  appears with a lag of one period (quarter), as trade transactions, particularly in case of durables, are not immediate, and lag results from the process of production and transport. Thus, decisions on carrying out a real transaction are made in the period  $t_0$ , and they yield results in the period  $t_1$ . From the equations 11 and 12 there comes in effect the equation:

$$13. \quad B_t = \beta_3 B_{t-1} + f_3(Y_t, R_t, YEU_t) \quad 0 < \beta_3 = 1 - \alpha_3 < 1$$

Like in the former equations, the absolute term is omitted, and the constant  $\alpha_3$  is included under the sign of function operator  $f_3$ .

## The real interest rate and the real exchange rate disparity equation

The NATREX model assumes convergence of the national and foreign long-run real interest rate. In the situation where investors make decisions based on the real and not nominal values, in the open economy, the equilibrium in the market of portfolio capital requires that the expected real rates of return, corrected for the risk premium, were the same. The expected rate of return on investment in national assets should be, therefore, equal to the sum of the relevant interest rate, the expected real appreciation/depreciation of the foreign currency and risk premium. Assuming that investors, fully using the available information, have stable expectations related to the foreign currency exchange rate, one may come to the conclusion that the long-run real interest rate in the economy in question ( $r$ ) fulfils the condition of interest rate disparity:

<sup>16</sup> The detailed discussion on this index, as related to the Polish economy, can be found in IBRKK’s reports ”Economy and Foreign Trade of Poland”.



$$14. \quad (r + RISK) - rEU = 0$$

where  $rEU$  is the long-run interest rate in the euro zone countries and  $RISK$  is the risk premium.

Introducing the mechanism of a gradual adjustment of real interest rates, we receive:

$$15. \quad (r_t - rEU_t) = \alpha_4((r_{t-1}^* - rEU_{t-1}^*) - (r_{t-1} - rEU_{t-1})) + f_4(RISK) \quad 0 < \alpha_4 < 1$$

In the equilibrium:

$$16. \quad (r_t^* - rEU_t^*) = (r_{t-1}^* - rEU_{t-1}^*) = 0$$

therefore, under the conditions of perfect mobility of capital, we receive:

$$17. \quad (r_t - rEU_t) = \beta_4(r_{t-1} - rEU_{t-1}) + f_4\left(\frac{F_{t-1}}{Y_{t-1}} - \frac{FEU_{t-1}}{YEU_{t-1}}\right) \quad 0 < \beta_4 < 1$$

where function  $f_4$  means the risk premium  $RISK$  understood as difference between relation of Poland's foreign indebtedness to GDP and relation of indebtedness of the euro zone countries to GDP.

In the NATREX method, we are interested in such a real exchange rate of the national currency ( $R$ ), which guarantees maintenance of the internal and external equilibrium. The internal equilibrium is maintained if the real GDP is approximate to the level of potential GDP ( $YPOT$ ). The external equilibrium is defined as the state on the current account of balance of payments, ensuring a full service of foreign indebtedness. Defining  $CA$  as:

$$18. \quad CA_t \equiv B_t + NFIA_t + UT_t$$

where  $NFIA$  means net income from abroad and  $UT$  means foreign transfers. For the steady state where  $CA=0$ , we receive:

$$19. \quad B_t = -(NFIA_t + UT_t)$$

Exogenising  $R$  in the equation 13 we receive the behavioural equation of the real exchange rate of the national currency:

$$20. \quad R_t = -B_{t+1} + f_5(B_t, Y_t, YEU_t)$$

The real equilibrium exchange rate equation is then as follows:

$$21. \quad NATREX_t = -B_{t+1} + f_5(B_t, YPOT_t, YEU_t)$$

It is not possible to close the model without introduction of the Fisher equation, binding the nominal disparity with the real disparity:

$$22. \quad \log(i_t - iEU_t) = \log(PY_t - PYEU_t) + \log(r_t - rEU_t)$$

where  $i$ ,  $iEU$  are nominal long-run rates in Poland and in the euro zone countries; the equation of the real stock of capital:

$$23. \quad K_t = (1 - \delta)K_{t-1} + \frac{I_t}{PY_t}$$

where  $\delta$  means the rate of depreciation (amortisation) of capital; and the balancing equation:

$$24. \quad Y_t \equiv C_t + I_t + B_t + ST_t,$$

where  $ST$  – change in the stock (mean  $ST$  in a long run is equal to zero).

### Chapter 3. Statistical data

A specific feature of the NATREX method is lack of requirements concerning the stationary nature of statistical series. A consequence of this is a high probability of occurrence of co-linearity of variables and autocorrelation of residuals. Trying to eliminate these phenomena, each of the equations presented in Chapter 2 was supplemented with the mechanism of error correction. Equation with error correction does not change the long-term nature of a given category; there is only corrected agents' behaviour in a short period. The presented model was estimated one equation after another by the least square method, with the option of elimination of heteroscedasticity by the White's method. Moreover, autocorrelation of residuals (up to 4 lags) was analysed by Breusch-Godfrey test. Estimation was made with the use of econometrical package EViews 6 for the periods: the first quarter of 1998 – the first quarter of 2009, with a gradual limitation of the length of sample from below with a 1-quarter step from the first quarter of 1998 to the fourth quarter of 2003.

The model was resolved in the WinSolve 3.2 package. Corrections of parameters were made taking into account the results of rolling estimation.

The statistical data relating to quarterly national accounts in Poland from the period of 1998 – the first quarter of 2009 are from the CSO stock, except for the statistics of the balance of payments whose author is the National Bank of Poland, NBP. Statistics related to 12 countries of the euro zone were acquired from Eurostat. Data from the financial market related to exchange rates are taken from NBP, a long-run interest rates – from Eurostat. For Poland (like for the euro-zone countries), the latter are synthetic harmonised indices of benchmark 5- and 10-year bonds. Due to significantly greater liquidity of Polish 5-year bonds than 10-year ones, estimations were carried out for the both interest rates. The results differed so insignificantly that there was made the decision to use the more adequate for the NATREX concept index of 10-year bonds.

For the purpose of computations, there were used statistical series filtered by the Hodrick-Prescott method, having them previously lengthened (forecast) with the AR(1) process by 4 periods. Volume of GDP components in prices of 2000 was calculated dividing the value in current prices by relevant indices of the volume – thus, there were obtained series with dynamic features identical as their corresponding indices.

The two key economic categories in the NATREX method require a separate discussion: stock of capital and index of marginal productivity of capital.

Value of the stock of capital at current prices is calculated from the formula (23), ignoring in the second element of the formula (i.e.  $I_t / PY_t$ ) the deflator GDP  $PY_t$ , and with assumption of the coefficient of depreciation of capital  $\delta$  equal to 4.3% on average for the entire period in question<sup>17</sup>. Volume of the stock of capital in constant prices of 2000 was received dividing the value of computed capital by a GDP unibasal price index previously standardised in such a manner that the mean price in GDP in 2000 was equal to 1.00.

The marginal productivity of capital is calculated from the Cobb-Douglas production function, assuming that this productivity is equal to the partial derivative of production relative to capital, i.e.:

$$25. \quad \frac{\delta Y}{\delta K} = \alpha \frac{Y}{K}$$

Value of the coefficient  $\alpha$  in the production function is estimated at the level  $\alpha = 0.26$

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<sup>17</sup> Coefficient of depreciation of capital, which was adopted in the presented study, roughly corresponds to the so-called golden rule of capital derived from the Solow model with exogenous growth of employment and technology. The golden rule describes the state, where the level of savings guarantees the maximum level of consumption, with the fixed dynamics of employment and technology. Then the marginal productivity of capital = rate of depreciation of capital + exogenous growth of employment + exogenous growth of technology. In the period in question, this equation is made good in case of the following values: 2.5%=4.3%+(-2.7%)+1.6%.

## Chapter 4. Estimation of the NATREX model and discussion of the results

### Equation of the marginal productivity of capital (2) and long-run real interest rate (17)

The equation (2) describing the marginal productivity of capital, estimated in the primary version for the multi-sector economy (comprising a relative price for tradable goods and non-tradable ones, approximated by the real exchange rate) shows a gradual marginalisation of the importance of real exchange rate together with shortening of the sample. For the sample beginning from the third quarter 2000, the real exchange rate is not significant, and solutions for the whole model are more stable. It may mean that the multi-sector nature of the economy loses its importance and the relative price for goods stops determining the exchange rate of currency. In other words, **the Balassa-Samuelson effect, significant in a rapidly transforming economy, in a more stable economy stops to work.** Taking into account the observed process, it was assumed that entering the EU means that the Polish economy began to operate as a typical small, open, stable market economy, where we shall receive the optimal level of capital when the marginal productivity of capital is equal to the long-run interest rate increased by the risk premium.

The equation describing the marginal productivity of capital is of the following form:

$$MK=0.9879*MK(-1)+0.0117*R+0.3765*(MK(-1)-(0.9879*MK(-2)+0.0088*R(-1)))$$

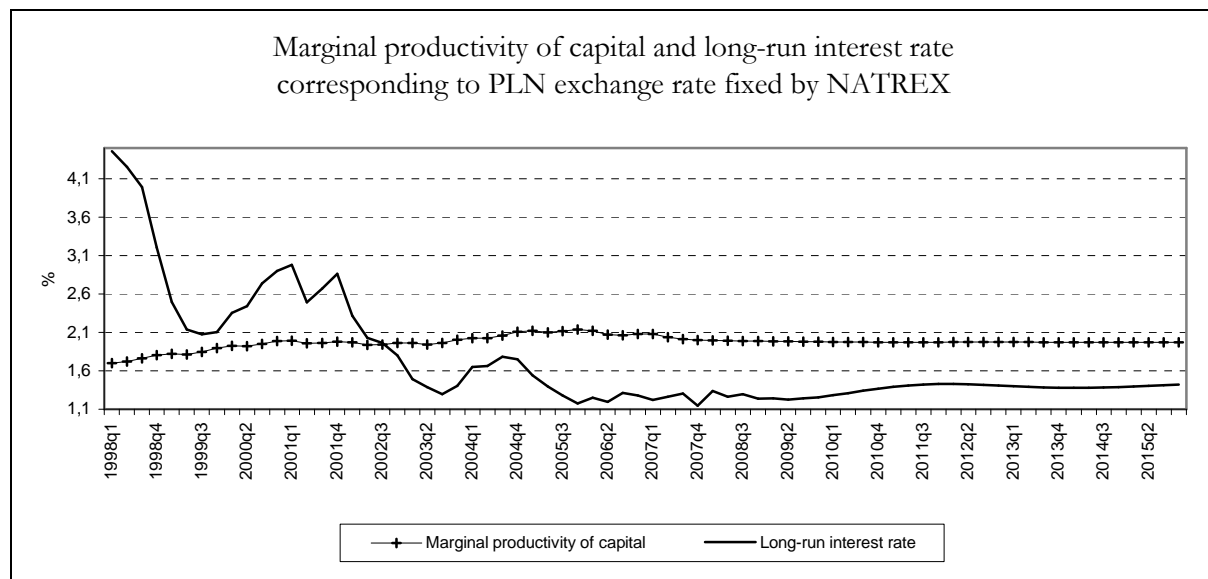
Designations are identical as in Chapter 2. The numbers in the brackets next to a given variable mean lags (if they are with minus).

The equation of the long-run real interest rate is fully concordant with the theoretical expectations described in Chapter 2; only the risk premium is defined not as relation of the Poland's indebtedness and of the EU indebtedness but in a more unilateral way – as relation of the Poland's indebtedness do GDP<sup>18</sup>:

$$R=REU-1.3486*(R(-1)-REU(-1))+0.044*(F(-1)/Y(-1)+B/Y)+1.2537*(R(-1)-(REU(-1)-0.2486*(R(-2)-REU(-2))+0.043*(F(-2)/Y(-2)+B(-1)/Y(-1))))$$

The long-term solution (simulation on 100 quarters ahead) shows that quite quickly, as early as in 2010, the risk premium will be stabilised. Its level is estimated for 0.6 percentage point, with the real interest rate ensuring the long-run equilibrium equalling 1.3% (in Figure 6, there is presented a fragment of the simulation – till the end of 2015).

Figure 6

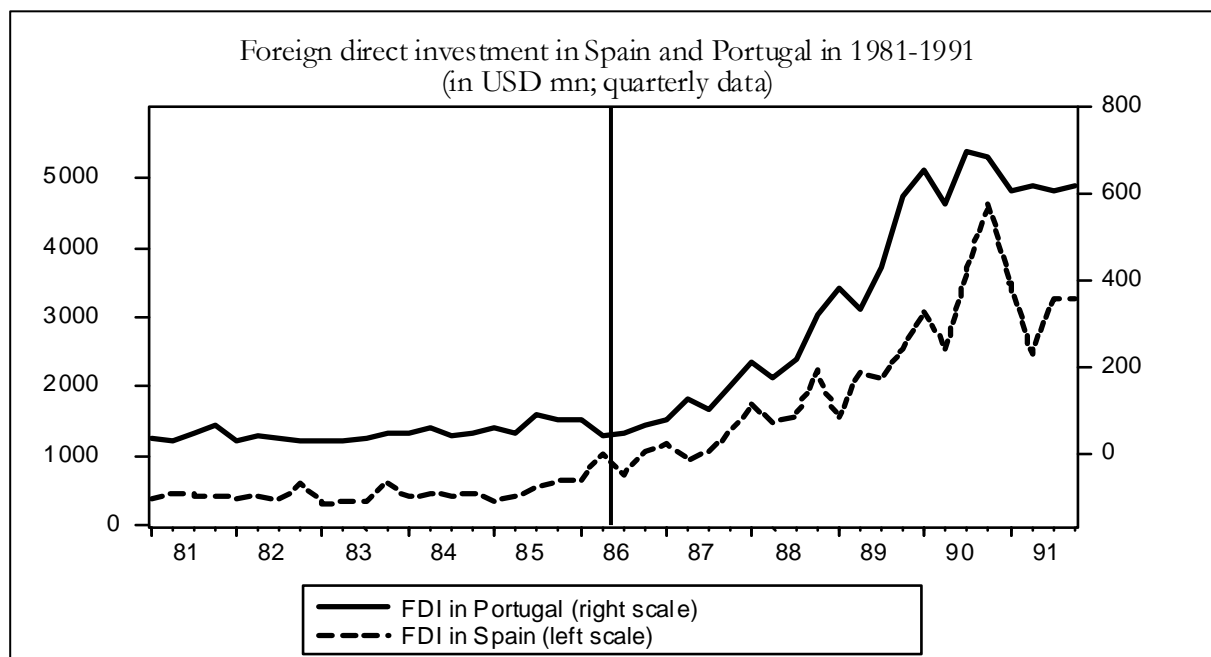


Source: Author's own calculations.

<sup>18</sup> The mechanism of stabilisation of indebtedness is described in Chapter 2.

This means that, assuming the above-described mechanism, there is a good climate for investing in a long-term perspective. Taking into account the investment needs (infrastructure) and transfers from the EU, and, on the other hand, foreign capital seeking for profitable long-term investment, the thesis of a long-term investment attractiveness of Poland does not seem to be exaggerated – see, as an example, the countries perceived in the period of their accession to the EU as Poland today (Spain and Portugal) where for almost two decades there had been observed an investment boom connected, first of all, with inflow of funds from the EU and a quick growth of foreign direct investment after the year 1986 (Figure 7).

Figure 7



Source: IFS 1995.

### The investment equation (5)

The equation (5) states that investment directly depends on the marginal productivity of capital and on the real long-run interest rate. There was mentioned above that, depending on the degree of substitutionality of labour and capital characterising the economy, the desired level of investments may be different. Observing investment outlays in Poland since 2001, it seems that they go on the path fixed by the marginal real productivity of capital, with the coefficient  $\alpha = 0.26$ . On the other hand, before 2000, the relation between the marginal productivity and investments was definitely weaker. Hence, the investment equations estimated for various sub-periods cannot be homogenous.

None equation of volume of investment, which would be estimated for the entire period in accordance with the NATREX concept, i.e.  $I_t = \beta_1 I_{t-1} + f_1(MK_{t-1} - r_{t-1})$ , cannot be accepted from the econometrical point of view. In the first decade of transformation, the dynamics of investments was defined by the internal demand. The only, acceptable from the econometrical point of view, estimated for the entire period, equation of volume of investment, specified in accordance with the NATREX concept, where there is taken into consideration impact of the demand approximated by the sum of share of consumption and trade balance in the gross domestic product, cannot be accepted from the theoretical point of view. Namely, the parameter estimated for the variable  $(MK_{t-1} - r_{t-1})$  has the sign minus, what means that investment is declining with growth of the marginal productivity of capital or with a decrease in the real interest rate. At the same time, the internal demand is important, and it affects investment in accordance with expectations.

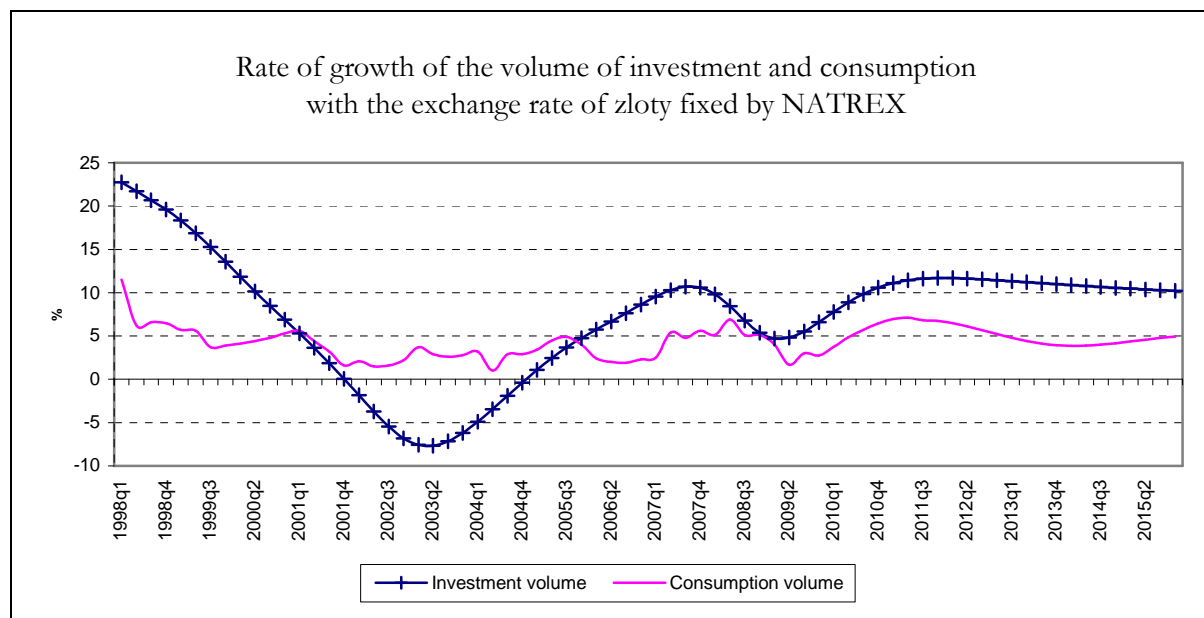
Estimating the investment equation for the period from the second quarter of 2000 to the first quarter of 2009, we receive all signs in line with the expectations, and demand is an unimportant

variable. Further estimations, where there was eliminated the final demand, give the following equation for coefficient  $\alpha = 0.26$ :

$$I = 0.3259 * I(-1) + 110.6428 * (MK026(-1) - R) + 1.0214 * (I(-1) - (0.3259 * I(-2) + 110.6428 * (MK026(-2) - R(-1))))$$

A fragment of resolution of a dynamic model (till the end of 2015) is presented in Figure 8.

Figure 8



Source: Author's own calculations.

As expected, the rate of growth of investment is relatively high: in the entire period of forecast, it amounts to almost 10% per year, and in the third decade, it stabilises at the level of approx. 8.7%. It is proper to pay attention to the declining but positive in the entire period, rate of growth of volume of capital, what may mean a continuous drop of coefficient ( $\alpha$ ) in the Cobb-Douglas function of production, entailing growth in wages and salaries and, in effect, growth in dynamics of consumption.

### The consumption equation (8)

In the consumption equation, estimated both for the entire period and for the period from the second quarter of 2001 to the first quarter of 2009, all variables are included in equations with the expected signs, i.e. the volume of consumption is positively correlated with the volume of income and negatively with the volume of foreign indebtedness. The parameter at the variable determining the general price level in the economy is greater than zero what means that growth in propensity for consumption is dependent on income and stronger than drop of propensity for consumption caused by growth of indebtedness. At the same time, this parameter is statistically significant and stable, disregarding the equation version<sup>19</sup>.

The consumption volume equation is as follows:

$$C = -0.5754 * C(-1) + 0.6039 * Y(-1) - 0.0073 * F(-1) + 10.8029 * PY(-1) + 1.0074 * (C(-1) - (-0.5754 * C(-2) + 0.6039 * Y(-2) - 0.0073 * F(-2) + 10.8029 * PY(-2)))$$

Dynamic forecast of consumption received from the model resolution is shown in Figure 8 (together with the investment volume). The medium rate of growth of volume of consumption in

<sup>19</sup> Insignificance of the parameter would mean rejection of the thesis of money neutrality, incorporated in the NATREX method.

a long period amounts to 4.5%; at the end of the period, the forecast stabilises at the level a little bit higher than 4%. These rates seem to be high but complying with the expected growth in wages and salaries whose estimation is based on results of the above-specified equations of marginal productivity of capital and investment.

### The export and import volume equation and the trade balance equation (13)

In the NATREX method, there is important to explain the foreign trade with the domestic final demand and foreign demand as well as with the real exchange rate of the national currency.

The export volume equation should include the external demand, approximated in the equation by GDP of the EU countries, with the plus sign, and the real exchange rate – with the minus sign. In the import equation, the domestic final demand (**FD**), approximated by the sum of consumption, investment and export, as well as the real exchange rate appear with the plus sign. The estimations of export and import equations fulfil these requirements.

The trade balance, which is estimated in the original versions of the NATREX model (omitting the export and import equations), in the presented survey is not estimated; it is an algebraic difference of the volume of export and import.

In turn, volume of foreign indebtedness, since the second quarter of 2009, has been calculated as a sum of indebtedness from the period t-1, interest on capital borrowed in the period t and the trade balance (volume) from the period t:

$$F_t = F_{t-1}/(1-R_t) + B_t/(1-R_t)$$

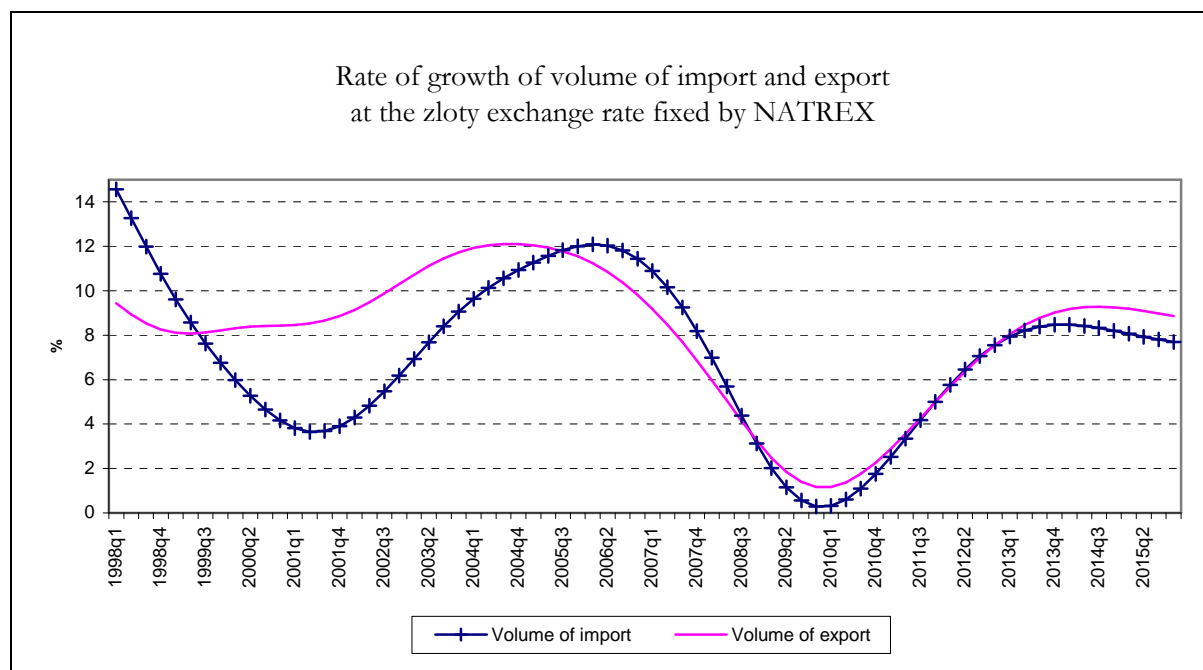
$$B_t = X_t - M_t; \text{ where } X - \text{volume of export, } M - \text{volume of import.}$$

The export and import volume equations have the following forms:

$$X = 0.9483 * X(-1) + 0.0034 * YEU(+1) - 2.4474 * RER + 0.2984 * (X(-1) - (0.9562 * X(-2) + 0.0028 * YEU - 2.4474 * RER(-1)))$$

$$M = 0.6106 * M(-1) + 0.0738 * FD + 3.4603 * RER + 1.0254 * (M(-1) - (0.6133 * M(-2) + 0.0734 * FD(-1) - 3.5253 * RER(-1)))$$

Figure 9



Source: Author's own calculations.

In the export equation, there pays attention a high coefficient occurring with exports lagging by one quarter. In the equation, estimated at IBRKK, besides the above-presented variables, there is appearing direct foreign investment. Their role in explaining the growths in export volume is systematically growing. Since 2001, foreign investors have been launching production designed, first of all, for export what causes an ever growing inertia of export. In case of lack of that investment, in specification of the export equation their role is taken over by lagging export – hence, the high coefficient at this variable.

The results of dynamical solution of the model for export and import equations are presented in Figure 9. The mean annual rate of growth of export volume, amounting to 8.5% in the period of model solution, correlated with 8.1% growth in import volume since 2014, allows for stabilisation of foreign indebtedness (including interest paid in the period t).

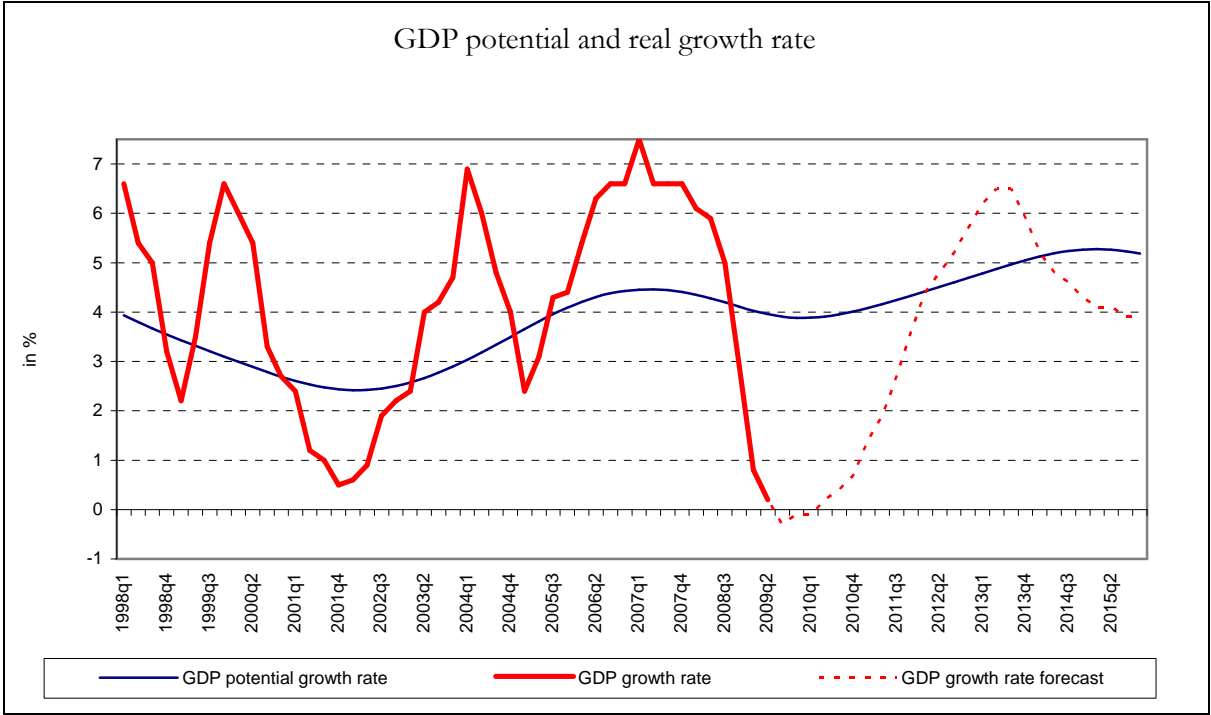
**The PLN real equilibrium exchange rate equation estimated on the basis of NATREX method**

The equations of the marginal productivity of capital and the real long-run interest rate were indispensable for computation of the desired level of investments. The PLN real exchange rate served estimation of the volume export and import. The equation of desired consumption does not directly refer to the interest rate and exchange rate but to foreign indebtedness whose servicing is decided by the state of current accounts, i.e. the export and import equation.

Long-run equilibrium exchange rate of the NATREX type should assure the internal equilibrium understood as equalisation of the real GDP with the potential GDP, and the external equilibrium understood as such a surplus in the trade balance, which ensures foreign indebtedness service, making no mention of transfers and net income.

Taking into account the results of model solution, discussed so far, concerning individual components of GDP, and assuming that, in a long run, the sum of increase in stock equals zero, the path of potential GDP, ensuring the internal equilibrium (the zero demand gap), is presented in Figure 10.

Figure 10

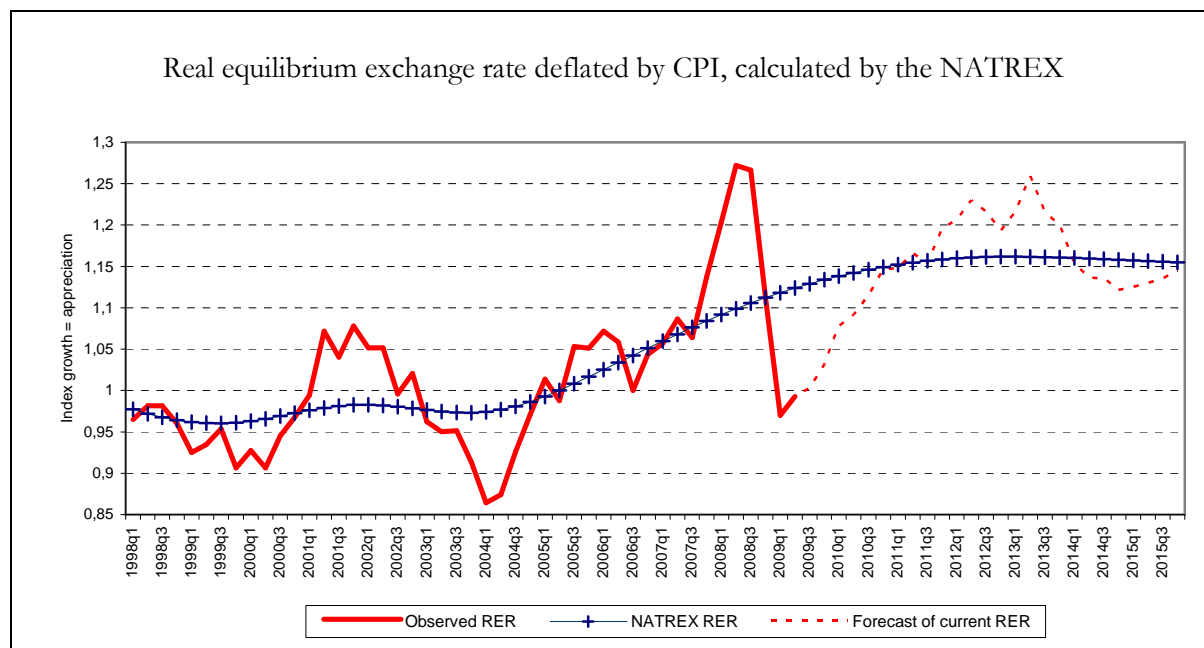


Source: Author’s own calculations.

At the end of the period of simulation, the rate of growth of potential GDP stabilises at the level of 4.1%. Putting on the optimal path of GDP the most probable path (forecast of the real GDP, Figure 10) shows that oscillations around the potential GDP will not end with going out from the current crisis; however, their amplitude should decline. Equation of the potential rate of growth with the real rate will probably take place in 2011. This means that also in that time the real current exchange rate of zloty will be, over several quarters, approximate to the long-run equilibrium exchange rate (Figure 11) calculated from the RER equation presented below:

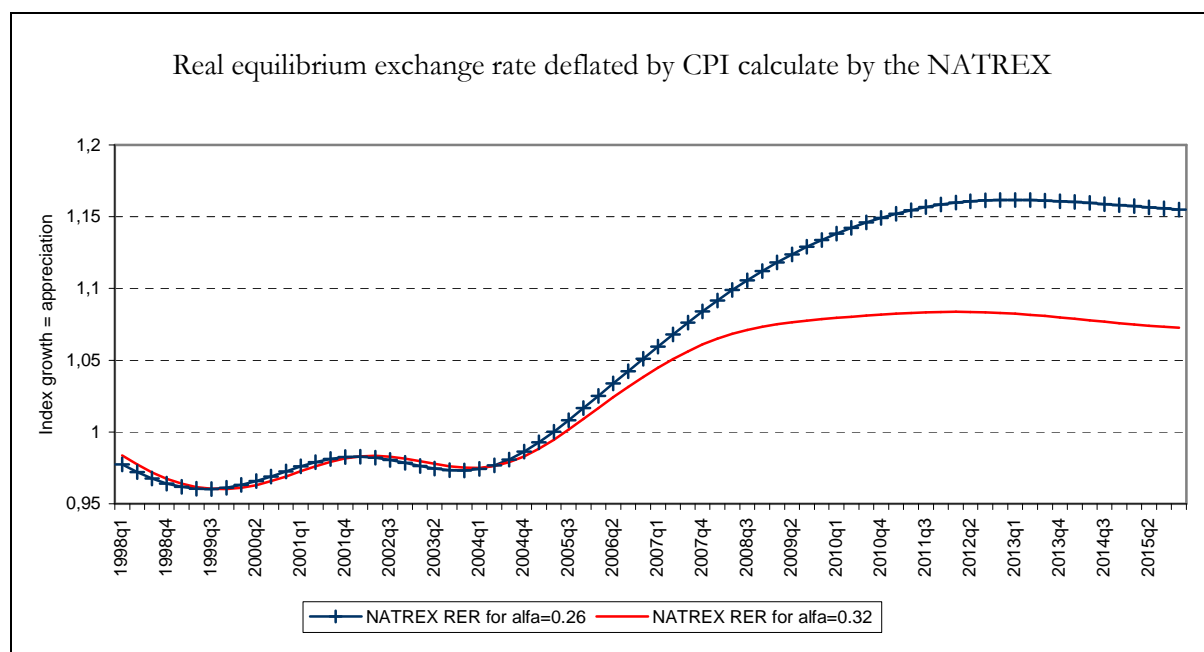
$$\text{RER} = -0.0061 \cdot \text{B}(+1) - 0.08 \cdot \text{B} - 0.0129 \cdot \text{Y}(+1) + 0.0039 \cdot \text{YEU} + 0.1836 \cdot (\text{B}(-1) - (-0.0339 \cdot \text{B} - 0.1779 \cdot \text{B}(-1) - 0.0129 \cdot \text{Y} + 0.0016 \cdot \text{YEU}(-1)))$$

Figure 11



Source: Author's own calculations.

Figure 12



Source: Author's own calculations.



All the values discussed in this chapter take place with the real exchange rate of zloty, calculated, according to the NATREX logic, for capital and labour distribution in the Cobb-Douglas production function in proportion of 0.26 to 0.74 corresponding to the situation shown in Figure 4, i.e. when making investment decisions that cause increase in productivity and decrease of the marginal costs of production what, in turn, prompts to produce goods of higher quality.

Let's finally take into consideration the situation where there are developed labour-consuming sectors, related, for example, with investments in infrastructure, requiring a relatively quicker growth of employment than that of capital, and, in the model sense, increasing the coefficient  $\alpha$  in function of production to  $\alpha = 0.32$ . In such a situation, the rate of growth of investments would be, in the whole forecast horizon, lower by 3.5 percentage points *per annum* than the rate shown in Figure 8. The PLN exchange rate would be as early as in 2010 by almost 7% more depreciated (Figure 12); nevertheless, in case of labour-consuming growth, the volume of exports would grow by 3 percentage points slower than in case of development of sectors with a greater intensity of capital. The rate of growth of imports would be by 2.7 percentage points lower, causing that the trade balance volume would remain at an almost unchanged level. At the same time, greater labour intensity of the economy would result in minimally higher (by 0.1 percentage points) growth of consumption. On the whole, the potential GDP, in the economy based more on labour ( $\alpha = 0.32$ ), would grow by approx. 0.3 percentage points slower than in the economy with a more intense use of capital.

## Chapter 5. Analysis of the real long-run equilibrium exchange rate of zloty. Comparison with the forecasted path of zloty exchange rate till 2015

In the previous chapters, there was shown the method of calculation of such a real exchange rate of zloty that ensures the optimal development of the economy. It was also emphasised that the level of long-run equilibrium exchange rate depends on the research method and on the period taken for the purpose of analysis. In Table 3, there are shown the results of estimations carried out by various methods in various analytical centres.

Table 3

Deviation from the long-run equilibrium exchange rate  
(%; the sign [+] means exchange rate overevaluation, the sign [-] means its underevaluation)

Institution	Method	II q. 2008 (period of max appreciation of the current exchange rate of zloty)	I q. 2009 (period of max depreciation of the current exchange rate of zloty)
IBRKK	NATREX	+17.3	-15.0
IMF	Macroeconomic balance approach (approx. to FEER)	+11	-5
	Method of the external equilibrium (approximate to BEER)	+5	-13
	Equilibrium exchange rate from a model of the DSGE type	+15	-12
NBP	FEER	+5.0	-14
	BEER	+3.1	-
	PEER	+4.0	-
NBCz	FEER	+6	-18

Source: Author's own calculations based on studies of IBRKK, IMF, NBP, NBCz.

For the purpose of comparison, there were chosen two the sorest periods: the second quarter of 2008, the period of the greatest appreciation of the nominal exchange rate of zloty, and the first quarter of 2009, the period of the greatest depreciation of the nominal exchange rate of zloty. The two facts attract attention:

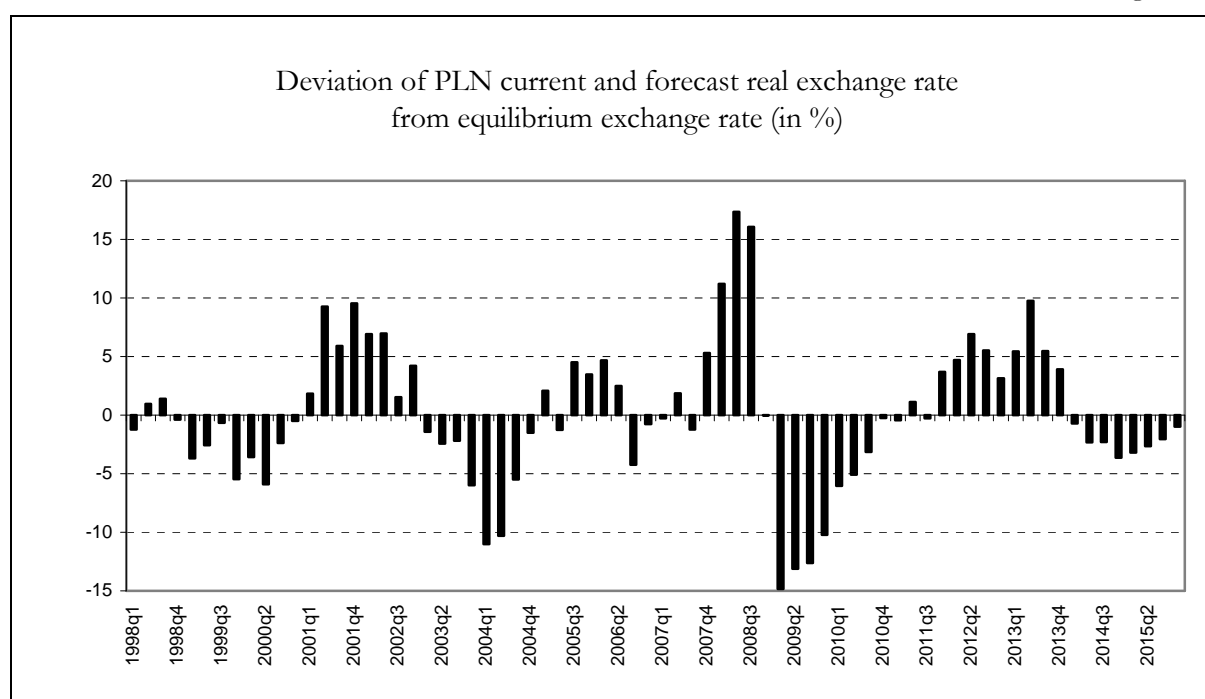
- 1) the degree of overevaluation and underevaluation of the exchange rate of zloty calculated by the IMF from the general equilibrium model is similar to the results received with use of the NATREX method (respectively, +15%, -12% and +17.3%, -15%). It may suggest a similar worth of estimations made by methods less sensitive to a subjective nature of experts' assumptions, and it can confirm the validity of choice of the NATREX method for this study's purposes;
- 2) differentiation of results in the period of excessive appreciation is definitely greater than in the period of excessive depreciation. Rejecting the extreme values, estimations of overevaluation of zloty are contained within the interval of (+4;+15), the spread amounts to 11 percentage points; estimations of underevaluation of zloty – within the interval of (-15;-12), the spread amounts to 3 percentage points (the spread difference may testify against the methods requiring subjective experts' assumptions).

Let's assume, according to the estimations of long-run equilibrium exchange rate made by the NATREX method, that in the first quarter of 2009 overevaluation of the current real exchange rate of zloty amounted to 15%. In subsequent periods, the current exchange rate, in line with improvement of the macroeconomic situation, will undergo a gradual appreciation at the rate of expected changes on the difference between the potential and real GDP (demand gap) and changes in disparity of the foreign

and domestic interest rate (Figure 11). At the same time, the real long-run equilibrium exchange rate will also be appreciated (Figure 11) for the reasons discussed in Chapters 1 and 2. Difference in dynamics will cause that the current exchange rate and the equilibrium exchange rate will match each other probably in the first half of 2011. Related to the first quarter of 2009, the current exchange rate will then be appreciated by approx. 17% and the equilibrium exchange rate by 3.5% (Figure 13).

In the first half of 2011, demand gap should quickly be narrowed and become positive at the turn of 2011/2012 (Figure 9). The positive gap spread will be increasing over at least 6 subsequent quarters. As a whole, the gap can be positive over approx. 12 quarters, causing a continuous appreciation of zloty. One may expect the maximum appreciation of the current exchange rate (by approx. 25% as related to the first quarter of 2009) in mid-2013. Overvaluation of zloty would amount then to 9%. In subsequent periods, the current exchange rate should move forwards the long-run equilibrium exchange rate depreciating by approx. 10%.

Figure 13



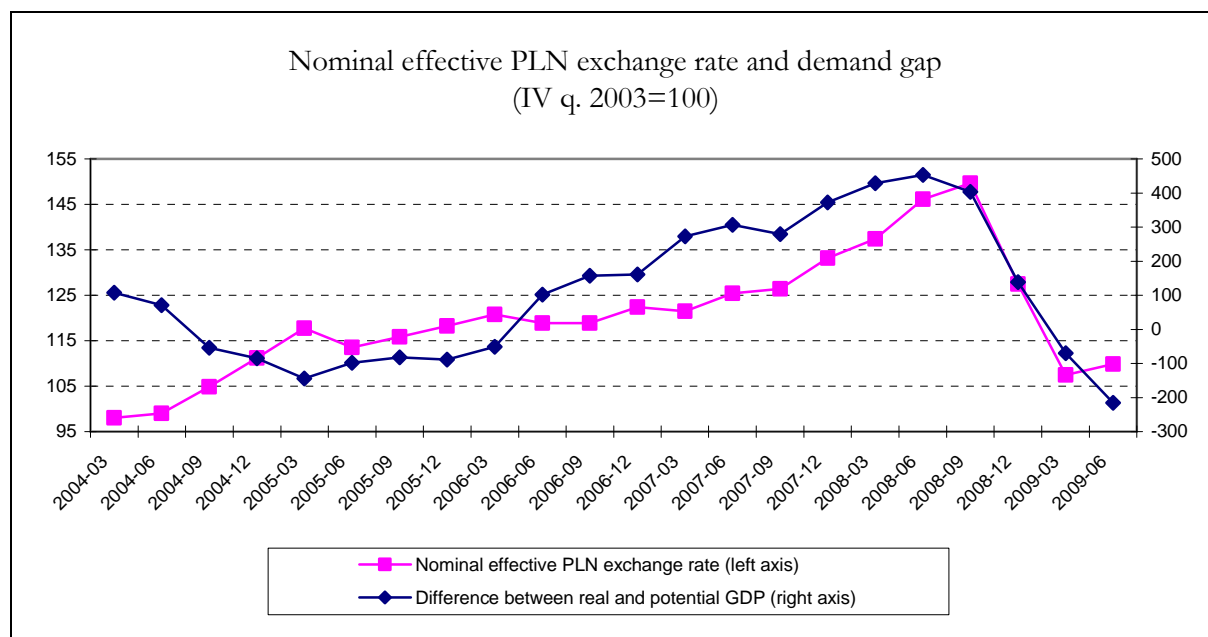
Source: Author's own calculations.

The presented study concerns the real equilibrium exchange rate, which is a composition of the nominal exchange rate and prices. Changes in the real exchange rate may be, therefore, caused either by changes of the nominal exchange rate or by changes in prices (or by change of the both these factors at the same time). Assuming that:

- the Monetary Policy Council will not allow permanent surpassing the inflation target;
- there is a positive correlation between changes in consumer prices in Poland and abroad,

one may expect that changes in the real exchange rate of zloty will be not only correlated but also proportional to the changes in the nominal exchange rate. In such a situation, with all reservations made earlier, one may assume that in the forecast of exchange rate the dynamics of the real and nominal exchange rate will be similar. The fact that the nominal exchange rate (like the real exchange rate) is being explained, among other things, by the expected demand gap, makes this thesis more realistic (Figure 14).

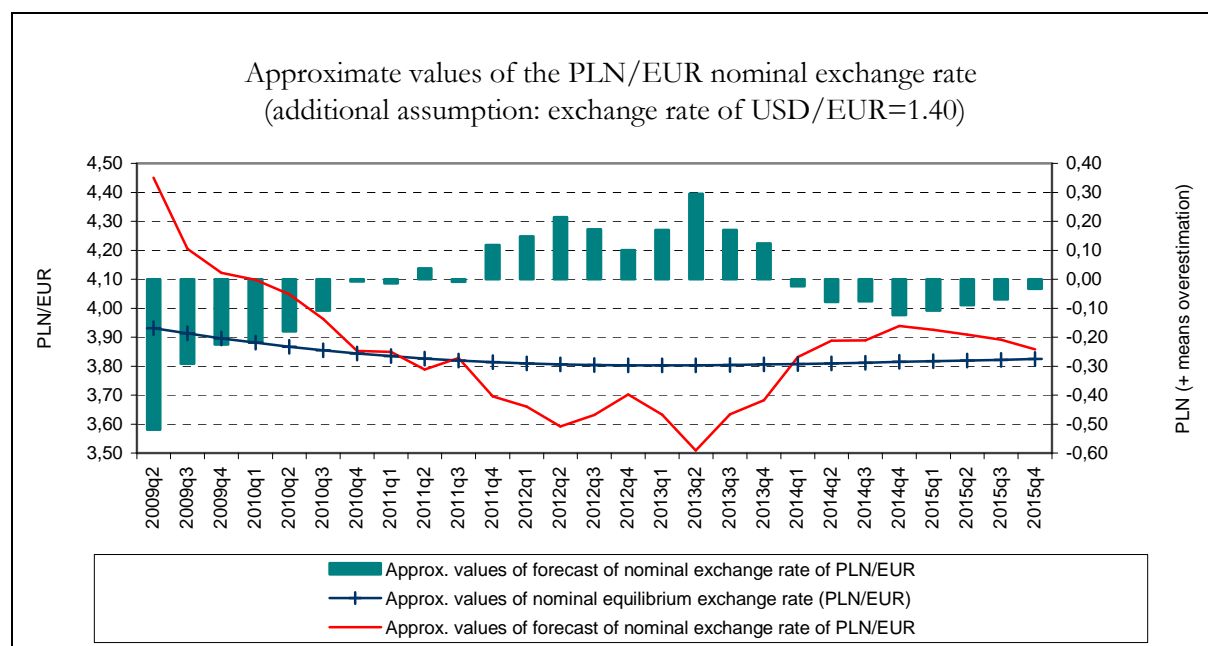
Figure 14



Source: Author's own calculations based on the CSO and NBP data.

In Figure 15, there are presented estimations of the PLN/EUR nominal exchange rate, the nominal equilibrium exchange rate and deviations of the forecasted exchange rate from the equilibrium exchange rate. An additional assumption adopted when calculating exchange rates is constant USD/EUR exchange rate over the entire forecast horizon. **The USD/EUR rate is assumed to be at the level of 1.40; this is a technical assumption.**

Figure 15



Source: Author's own calculations.

Remembering the reservations made earlier as to the possibility of merely approximate estimation of nominal exchange rates and as to uncertainty in forecasting exchange rates, one may state that the long-

term equilibrium in the economy is assured with the exchange rate of 3.80-3.90 zlotys for 1 euro. The current exchange rate will probably approach the equilibrium exchange rate at the turn of 2010 and 2011, and it will remain nearby over 5-6 quarters. This means that, in that period, the cost of retention of the PLN exchange rate within a narrow band of fluctuations is relatively the lowest. The next period where the current exchange rate should approach the optimal exchange rate is 2014 – then, in the medium run, the PLN exchange rate should be comprised within the interval of 3.80-3.90.

If, in the forecast horizon, the USD/EUR exchange rate differs from the level of 1.4 adopted as a technical assumption, then, for example, in case of dollar depreciation to the level of 1.8 per 1 euro (depreciation by 30%) the long-term equilibrium in the economy will be assured with the exchange rate 4.00-4.10 of zloty for 1 euro.

From the point of view of Poland's participation in the ERM-2 system and later on, in the euro zone, it is important that the accession exchange rate does not deviate too radically from the exchange rates considered as the equilibrium exchange rates. Well, an excessive depreciation will increase competitiveness of Polish goods abroad but, first of all, in the group of low-processed goods (approx. 20% of the entire export). In the other groups, for which specific is a high import intensity, there will grow costs of production, affecting directly the selling price. Another problem related to an excessive depreciation is drop of the purchasing power of personal domestic income expressed in a foreign currency. In the situation of direct recalculation of income into euro, this will reduce domestic consumption. In turn, an excessive appreciation will decrease competitiveness of Polish goods abroad, with a simultaneous increase in attractiveness of foreign goods in the domestic market. This will reduce domestic supply with all consequences for wages and employment and, in result, both for domestic consumption and for investment.

Making decisions on the grounds of forecast of long-term tendencies occurring in the economy, one should also pay attention to frequency of the data adopted for estimating those tendencies. In this case, those are quarterly data. The forecast, or in fact, simulation of long-term relations, for few dozens of periods ahead, serves to show the most probable behaviours, provided that there are met the adopted assumptions. This is, therefore, exclusively a coherent approximation of the course of individual economic processes in various periods; however, dating of periods is also approximate. Nevertheless, rigorous fulfilment of the conditions required by the selected concept (economic theory) allows for better results (also in case of exchange rate) than relying on subjective opinions or on the assumption of random nature of events.

## Resumption – conclusions

1. In the study, we analyse the real exchange rate of zloty deflated by the index of prices of consumer goods and services {composition of the nominal effective exchange rate of zloty (NEER) and the difference of consumer prices in Poland (consumer price index, CPI) and abroad (harmonised index of consumer prices, HICP)}.
2. For estimation of the long-run equilibrium exchange rate, there was taken the natural exchange rate (NATREX) hypothesis comprising less subjective elements than the frequently used fundamental equilibrium exchange rate (FEER) and its modifications. Moreover, the solutions provided by NATREX are stable over time, dependent, first of all, on relations of labour and capital in the Cobb-Douglas production function. Results are similar to the estimations being received from the general equilibrium models.
3. Estimated by the NATREX method the long-run real equilibrium exchange rate ensures the internal equilibrium (matching the potential GDP growth rate with the GDP real growth) with annual growth rates of GDP amounting to 4.1%, comprised of growth of consumption by 4% p.a., investment by 8.7%, volume of exports by 8.5% and volume of imports by 8.1% p.a. The specified growth rates ensure the external equilibrium understood as stabilisation of foreign indebtedness at the level of the end of 2008.
4. Related to the long-run equilibrium exchange rate, overvaluation of the current real exchange rate of zloty in the first quarter of 2009 amounted to 15%.
5. The current exchange rate and equilibrium exchange rate will probably match each other at the turn of 2010 and 2011. Related to the first quarter of 2009, the current exchange rate will then be appreciated by approx. 17%, and the equilibrium exchange rate by 3.5%.
6. The maximum appreciation of the current exchange rate (by approx. 25% against the first quarter of 2009) can be expected in the mid-2013. Overvaluation of zloty would amount then to 9%. In subsequent periods, the current exchange rate should move towards the long-run equilibrium exchange rate depreciating by approx. 10%.
7. Estimating on the ground of real exchange rates an approximate value of nominal exchange rates, one can state that the long-term equilibrium in the economy is ensured with the exchange rate of 3.80-3.90 zlotys for 1 euro. The current exchange rate will probably approach the equilibrium exchange rate at the turn of 2010 and 2011, and it will remain near that level over 5-6 quarters. This means that in that period cost of retention of the PLN exchange rate within a narrow band of fluctuations is relatively the least. The next period where the current exchange rate should approach the optimal exchange rate is 2014. Then, also in the medium term, the exchange rate of zloty should be comprised within the interval of 3.80-3.90 (assuming the stable exchange rate of USD/EUR=1.40).
8. If, in the forecast horizon, the USD/EUR exchange rate differs from the level of 1.4 adopted as a technical assumption, then, for example, in case of dollar depreciation to the level of 1.8 per 1 euro (depreciation by 30%), the long-term equilibrium in the economy will be ensured with the exchange rate of 4.00-4.10 zloty for 1 euro.

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