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Abstract

In this paper I consider the 1991 Grossman-Helpman model which analyses the role of innovation on growth. The model assumes constant returns to scale. I intend to show what happen in this model if I assume strong increasing returns. In particular, under the assumption of increasing returns of capital but leaving all other main features of the Grossman-Helpman model unchanged, I analyse the influence of the rate of innovation on three variables: the rate of growth of final output, the level of prices of final output and the rate of investment.

JEL classifications: D24, O31.

Key words: Grossman-Helpman model, growth, innovation, increasing returns to scale.

Innovation and growth in the Grossman-Helpman's model with increasing returns: a note.

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Introduction

In this paper I consider the 1991 Grossman-Helpman model[1] which analyses the role of innovation on growth. The model assumes constant returns to scale. However, since important studies[2] affirm that innovation is positively correlated with increasing returns, I intend to show what happen in this model if I assume strong increasing returns. In particular, under the assumption of strong increasing returns to scale but leaving all other main features of the Grossman-Helpman model unchanged, I analyse the influence of the rate of innovation on three variables: the rate of growth of final output, the level of prices of final output and the rate of investment. Firstly I describe the main aspects of the Grossman-Helpman (1991) model that I consider interesting for my analysis. Secondly I introduce increasing returns of capital in the production function and I discuss the new results.

1. *Basic Model*

There are three sectors. The first sector produces research with labour alone as an input and the rate of innovation is equal to

$\gamma > 0$.

The second sector produces intermediate goods with the following production function

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[1] Innovation and Growth in the Global Economy (1991, pp. 115-122).

[2] See for example Kaldor (1981), Arthur (1994), Buchanan and Yoon (1994), Toner (1999).

$$D = A_D X \quad (1),$$

where D is an index of intermediate goods output, X is the quantity of labour employed in this sector, A_D is the sector's productivity index.[3] The rate of growth of this index is

$$\frac{\dot{A}_D}{A_D} = \mu\gamma \quad (2),$$

where $\mu > 0$ is a parameter of specialization[4]. Because this parameter is positive, according to equation (2) research and development activity (R&D), indicated by the rate of innovation γ, improves the productivity of the production processes of intermediate goods. Finally, the third sector produces final goods Y with the following production function

$$Y = A_Y K^{\beta} D^{\eta} L_Y^{1-\beta-\eta} \quad (3),$$

with $0 < \beta, \eta, \beta + \eta < 1$. In equation (3), $A_Y > 0$ is a constant parameter of productivity, K is the aggregated capital stock, D is the index of intermediate goods input, and L_Y is the quantity of labour employed in this sector. In this sector there is perfect competition and constant returns to scale.

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3 The authors write: "The productivity measure [A_D] reflects either the available variety of components or the average quality of each component" (Grossman-Helpman, 1991 p.117).

4 This parameter can have two alternative meaning: the first with $\mu = (1-\alpha)/\alpha$ where $0 < \alpha < 1$ is the different quantity of intermediate goods (horizontal specialization) and in this case, D represents an index of intermediate goods $x(j)$ and is $D = A_D X = \left[\int_0^n x(j)^{\alpha} dj\right]^{1/\alpha}$ with $0 < \alpha < 1$; the second with $\mu = \log\lambda$ where $\lambda > 1$ is the different quality of intermediate goods (vertical specialization); in this case, the expression for *logD* is $\log D = \log A_D X = \int_0^1 \log\left[\sum_m \lambda^m x_m(j)dj\right]$ where "$x_m(j)$ denotes the input of the variety of component j whose quality is λ^m" (Grossman-Helpman, 1991 p.116).

The equation of equilibrium of steady state is obtained deriving, from equation (1), the following equation for the rate of final output

$$g_Y = \frac{\dot{A}_Y}{A_Y} + \beta \frac{\dot{K}}{K} + \eta \frac{\dot{A}_D}{A_D} + \eta \frac{\dot{X}}{X} + (1-\beta-\eta)\frac{\dot{L}_Y}{L_Y} \quad (4).$$

In equation (4), g_Y, $\frac{\dot{A}_Y}{A_Y}$, $\frac{\dot{K}}{K}$, $\frac{\dot{A}_D}{A_D}$, $\frac{\dot{X}}{X}$, $\frac{\dot{L}_Y}{L_Y}$ are the rates of growth respectively of final output, of A_Y, of the productivity index of the intermediate goods sector, of labour in the intermediate sector, of labour in the final sector. According to the model, in steady state I put: $\frac{\dot{A}_Y}{A_Y} = 0$ because A_Y is constant; $\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y}$, as in all endogenous growth model; $\frac{\dot{X}}{X} = \frac{\dot{L}_Y}{L_Y} = 0$ because the quantity of labour in the intermediate and final sectors is constant[5], and $\frac{\dot{A}_D}{A_D} = \mu\gamma$, as described by equation (2). The equation of steady state becomes

$$g_Y = \frac{\eta}{1-\beta}\mu\gamma > 0 \quad (5).$$

The rate of growth of final output is positively correlated with R&D activity through the rate of innovation γ and through the index of specialization of intermediate goods μ. The long run analysis shows that the level of price of the final output decreases, when the rate of innovation increases. In fact

[5] "In this calculation [growth –accounting, equation (6)] the allocation of labor to the production of intermediate [X] and final goods [L_Y] is taken as constant, as indeed it is in the steady state". (Grossman-Helpman, 1991 p.121)

from equation $\frac{p_Y}{w_K}=\frac{1}{(\rho+g_Y)}$ (Grossman-Helpman 1991, p.122) I can obtain the following relationship[6]

$$p_Y=\frac{w_K(1-\beta)}{(1-\beta)\rho+\eta\mu\gamma} \quad (6).$$

In equation (6), p_Y is the level of price of final output and w_K is "the rental rate on capital" (Grossman-Helpman 1991,p.115).

From equation (6) I have the following derivative

$$\frac{dp_Y}{d\gamma}=-\frac{w_K\eta\mu(1-\beta)}{[(1-\beta)\rho+\eta\mu\gamma]^2}<0 \quad (7).$$

Because of decreasing returns of capital, $0<\beta<1$, any increment in the rate of innovation implies in the long run a decrease of the price level.

I have also another result: the rate of investment is positively correlated with the rate of growth of final output as described by the following equation

$$\frac{\dot{K}}{Y}=\frac{\beta g_Y}{\rho+g_Y} \quad (8).$$

Equation (8) implies that the long run rate of investment is related positively with the rate of innovation. In fact I have[7]

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[6] In the model, from equation $\frac{p_Y}{w_K}=\frac{1}{(\rho+g_Y)}=\frac{1}{\rho+\frac{\eta\mu\gamma}{(1-\beta)}}=\frac{1}{\frac{(1-\beta)\rho+\eta\mu\gamma}{(1-\beta)}}$ I obtain $p_Y=\frac{w_K}{(1-\beta)\rho+\eta\mu\gamma(1-\beta)}$.

$$\frac{d(\dot{K}/Y)}{d\gamma} = \frac{\beta\eta\mu\rho(1-\beta)}{[(1-\beta)\rho+\eta\mu\gamma]^2} > 0 \quad (9).$$

Equation (9) is positive thanks to the assumption of constant returns to scale where I have $0 < \beta < 1$, then the rate of investment is positively related with the rate of innovation. According to the authors this theoretical result is confirmed by the empirical studies of positive correlation between the rate of investment and the increment of innovation

"In fact the authors write: "Our finding that innovation drives investment is at least consistent with another bit of cross-country evidence, namely, the high positive correlation between the growth rate of the capital stock (or the ratio of investment to GDP) and the realized gain in total factor productivity, which has been noted by Baumol et al. (1989). Moreover it is supported by evidence reported in Lach and Schankerman (1989) that, at the firm level, R&D Granger-causes investment, but investment does not Granger-cause R&D." (Grossman-Helpman, 1991, p.113).

2. ***Critique***

Result (9) shows the importance of innovation for growth and also the strong correlation between investment and innovation. In this respect, the authors write

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[7] From equation (8) I have $\frac{\dot{K}}{\mathrm{Y}} = \frac{\beta\frac{\eta\mu\gamma}{1-\beta}}{\rho+\frac{\eta\mu\gamma}{1-\beta}} = \frac{\beta\eta\mu\gamma}{(1-\beta)}\frac{(1-\beta)}{(1-\beta)\rho+\eta\mu\gamma} = \frac{\beta\eta\mu\gamma}{(1-\beta)\rho+\eta\mu\gamma}$. From this equation I can obtain the following derivative $\frac{d(\dot{K}/\mathrm{Y})}{d\gamma} = \frac{\beta\eta\mu[(1-\beta)\rho+\eta\mu\gamma]-\beta\eta\mu\gamma(\eta\mu)}{[(1-\beta)\rho+\eta\mu\gamma]^2} =$

$= \frac{\beta\eta\mu(1-\beta)\rho+\beta\eta^2\mu^2\gamma-\beta\eta^2\mu^2\gamma}{[(1-\beta)\rho+\eta\mu\gamma]^2} = \frac{\beta\eta\mu\rho(1-\beta)}{[(1-\beta)\rho+\eta\mu\gamma]^2}$.

"Capital accumulation might occur mostly in response to knowledge accumulation, as technological innovations raise the marginal productivity [and then the average productivity] of capital and so make investment in machinery and equipment more profitable. (Grossman-Helpman, 1991 p.113)

Indeed according to the model, innovation is the main instrument to attain the long term growth. However, as I shall show, all the results I have described depend on the assumption of constant returns to scale and in particular on decreasing returns to capital. In my opinion, this assumption is not adequate for analysing the growth processes generated by innovation. Empirical studies, like Verdoorn, (1949), Sylos Labini (1995), show that increasing returns to scale are widespread in the economy and theoretical studies argue that innovation is one of the main causes of increasing returns to scale, mainly in the industrial sector.

Thus in the Grossman-Helpman (1991) model there is a "theoretical question" about the relationship between the role of innovation and the assumption on returns: on the one side, the role of innovation is crucial for economic growth, as it is confirmed by theoretical and empirical studies; on the other side, in their model Grossman and Helpman assume constant returns scale, but this is in contradiction with their previous statement according to which innovation is crucial for economic growth. Indeed, beginning with Adam Smith, a lot of studies argue that increasing returns are generated by innovation processes.

Increasing returns to scale can be static, if they depend on the indivisibility of inputs or on the dimension of plants (Hufbauer 1966, Kaldor 1934) or dynamic, if they depend on different kinds of learning (Arrow 1962, Kaldor and Mirrlees 1962, Brian 1994). Moreover there is no reason for dismissing the possibility of "strong increasing returns", in the sense that each production factor can have increasing returns and in particular the capital. In fact, Sylos Labini (1995) shows that increasing returns to scale can be generated by increasing returns to capital[8].

Since a specific value for the parameters α and β cannot be established *a priori*, I will focus on this case by showing what happens in the Grossman-Helpman model if I substitute strong increasing returns to their assumption of constant returns. I will present three cases (*a*, *b*, *c*) where the capital has increasing returns $\beta > 1$.

Firstly, I consider the production function with increasing returns and I check whether the long-run rate of innovation remains positive. The production function becomes

$$Y = A_Y K^{\beta} D^{\eta} L_Y^{\theta} \quad (10).$$

With increasing returns, the exponent of labour in the final sector is not equal to $1-\beta-\eta$, but it is a generic parameter $\theta > 0$.

<u>*Case a*</u>

Let us express the relation between the rate of growth of final output and the rate of innovation. From equation (5), assuming increasing returns with $\beta > 1$, I obtain

$$\frac{dg_Y}{d\gamma} = \frac{\eta}{1-\beta}\mu < 0 \quad (11).$$

In this case innovation is an obstacle for the rate of growth of final output. This result is counterfactual because in theory I can argue that in the growth processes there is an important sequence: more

[8] See EEC (1961), Guarini and Tassinari (1990).

innovation, increasing returns to scale, positive rate of growth of final output. On the contrary, if equation (11) holds, along the balanced path the positive rate of growth of innovation corresponds to the negative rate of economic growth because in equation (5) g_Y becomes negative.

Case b

This case focus on the relation between innovation and prices of final output. In particular, I rewrite equation (7) considering the assumption $\beta > 1$.

$$\frac{dp_Y}{d\gamma} = -\frac{w_K \eta\mu(1-\beta)}{\left[(1-\beta)\rho + \eta\mu\gamma\right]^2} > 0 \quad (12).$$

According to equation (12), I can argue that an increase of the rate of innovation causes an increase of the output price level because the sign of the derivative is positive. Also this case is paradoxical, because innovation through increasing returns, that is decreasing costs, should reduce the level of price. In an open economy, as many authors in heterodox growth theory affirm (like Beckerman 1962; Boyer 1988, Boggio 2002), thanks to increasing returns to scale, the rate of labour productivity growth, that is an indicator of innovation processes, tends to reduce prices. This result, affecting international competitiveness, can increase the rate of growth of exports that, according to Smithian ideas, fostering market expansion, stimulates new innovations and thus new increases of productivity. In line with this view, it is possible to generate a virtuous growth circle between productivity and exports through increasing returns and decline of prices. Moreover, the size of the diminution of prices depends on the level of competition: this size grows with competition. Finally, with increasing returns the intensity of diffusion of innovation among firms depends on competition: with high competition, the decline of prices favours technological transfer processes.

Case c

In this last case I analyse how the rate of innovation influences the rate of investment with strong increasing returns to scale, rewriting derivative (9):

$$\frac{d(\dot{K}/Y)}{d\gamma} = \frac{\beta\eta\mu\rho(1-\beta)}{\left[(1-\beta)\rho + \eta\mu\gamma\right]^2} < 0 \quad (13).$$

The sign of derivative (17) is negative. Then there is a negative relation between rate of innovation and rate of investment because of assumption of increasing returns of capital, $\beta > 1$. This negative relationship is unrealistic because, as the Kaldorian approach affirms, in many cases the technological progress is "embodied" in new machinery; moreover, in the other cases the "disembodied" innovations turn out to be in a complementary relationship with machines. Thus in opposition to the result of equation (13), with strong increasing returns, the more realistic sign of the relationship between the rate of innovation and the rate of investment is the positive one.

Conclusion

In this paper, after describing the basic features of the Grossman-Helpman's model (1991), I have changed the technological condition of production, by assuming strong increasing returns and I have had counterfactual results: the rate of innovation is negatively correlated with the rate of growth of final output, with the level of prices of final output. Moreover it is possible to obtain a negative relation between the rate of innovation and the rate of investment. The main comment is that this theoretical exercise has shown that, with respect to the Grossman-Helpman model, innovation has "realistic" economic effects with the "unrealistic" assumption of constant returns, while innovation has "unrealistic" economic effects with the "realistic" assumption of increasing returns.

References

Arrow K.J. (1962): "The economic implications of learning by doing", *Review of Economic Studies*, vol.26, pp. 155-73.

Arthur B. (1994): *Increasing Returns and Path Dependence in the Economy*, University of Michigan Press.

Beckerman W. (1962): "Projecting Europe's growth", *Economic Journal*, 73 pp.912-925.

Boggio L. (2002): "A Model of Take – off and Fast Growth in Open Economies", *Metroeconomica* 54, pp. 301-325.

Boyer R. (1988): "Formalizing growth regimes", in Dosi, G. *Technical change and economic theory*, London, Pinter.

Buchanan J. M., Yoon J. Y. (1994): *The Return to Increasing Returns*, University of Michigan Press.

European Economic Community (1961): *Metodi di previsione dello sviluppo economico a lungo termine*, Informazioni statistiche, Paris.

Grossman G., Helpman E. (1991): *Innovation and Growth in the Global Economy*, MIT Press, Cambridge MA.

Guarini R., Tassinari F. (1990): *Statistica economica – Problemi e metodi di analisi*, il Mulino, Bologna.

Hufbauer G. C. (1966): *Synthetic Materials and the Theory of International Trade*, Cambridge: Harvard University Press.

Kaldor N. (1934): "A classificatory Note on the Determinateness of Equilibrium", *Review of Economic Studies,* 1, p.122-136.

Kaldor N. (1981): "The Role of Increasing Returns, Technical Progress and Cumulative Causation in the Theory of International Trade and Economic Growth", *Economie Appliquee*, 34, pp. 593-615.

Kaldor N., Mirrlees J.A. (1962): "A New Model of Economic Growth", *The Review of Economic Studies*, 29, pp. 174-92.

Sylos Labini P. (1995): "Why the interpretation of the Cobb Douglas production function must be radically changed", *Structural change and economic dynamics*, 3, pp.485-504.

Toner P. (1999): *Main Currents in Cumulative Causation: The Dynamics of Growth and Development*, Macmillan.

Verdoorn PJ. (1949): “Fattori che regolano lo sviluppo della produttività del lavoro”, *L'industria*, 1, pp. 3 -10.

Young A. A. (1928): “Increasing Returns and Economic Progress” *The Economic Journal*, 38, pp. 527-42.