The Optimum Quantity of Money Revisited: Distortionary Taxation in a Search Model of Money

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Abstract

This paper incorporates a distortionary tax into the microfoundations of money framework and revisits the optimum quantity of money. An optimal policy may consist of both a positive tax rate and a positive nominal interest rate: if the buyer's surplus share is inefficiently small, the intensive margin is distorted and the constrained optimal policy combines a sales tax with a money growth rate above that prescribed by the Friedman rule. Monetary, but not fiscal, policy alters the agent's bargaining position, leaving a special role for a deviation from the Friedman rule. Under similar conditions, this conclusion carries over to competitive pricing.

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1. Introduction

The Friedman rule has been one of the key doctrines of monetary theory for which traditional monetary economics has provided extensive support over the past decades. These traditional models rely on short-cuts, such as money-in-the-utility-function and cash-in-advance, to introduce money into their environments. Conversely, the microfoundations of money literature that does not resort to these shortcuts has pointed to a potential trade-off between the number of transactions (the extensive margin) and the quantity of goods exchanged in each trade (the intensive margin), which may render the Friedman rule a suboptimal monetary policy. However, these monetary search models have thus far mostly ignored fiscal policy, whose absence could potentially drive the shortfall of the Friedman rule – inflation may serve as a substitute for fiscal policy.

This paper incorporates fiscal policy considerations by introducing a sales tax into the monetary search environment, and revisits the optimum quantity of money. As such, it links the microfoundations of money to the traditional money and public finance literature. The key finding is that deviating from the Friedman rule, i.e. setting a positive nominal interest rate, may result in welfare gains – inflation is not a mere substitute for omitted fiscal policy. In fact, the suboptimality of the Friedman rule arrises even if no government revenue is needed; the deviation may help to improve efficiency in the number of matches, a feature nonexistent in the traditional literature, which does not model the extensive margin of trade.

The traditional money and public finance literature originates from Phelps' (1973) critique of the Friedman rule. Phelps argues that a government, which needs to resort to distortionary taxation to raise revenue should also tax money holdings. However, subsequent research has not supported this supposition. Those studies focus on the Ramsey problem, i.e. the optimal policy mix between a (sales) tax and a tax on money holdings, and tend to support the optimality of the Friedman rule: a sales tax dominates a tax on money holdings as an instrument to raise revenue (e.g. Kimbrough, 1986, Correia and Teles, 1996 and Chari et al., 1996). Faig (1988) demonstrates that the optimality of a zero inflation tax crucially depends on the assumption imposed on the utility and shopping-time functions. However, one shortfall of this literature is that it introduces money through short-cuts instead of deriving it from primitives.

Conversely, the environment in the microfoundations of money literature gives rise to the importance of money as a medium of exchange – agents are anonymous and engage in random, bilateral meetings, where all trades have to be made *quid pro quo*. Due to the spatial and time separation of trades, agents acquire money for consumption at a later point in time, which results in a positive opportunity cost of holding money. Agents therefore choose inefficiently low money balances unless the nominal interest rate is zero (Friedman, 1969). If the quantity exchanged is the only margin that might be distorted, following the Friedman rule is the welfare maximizing monetary policy because it ensures that agents are fully compensated for their impatience.[1]

[1] Berentsen and Rocheteau (2003) provide a detailed discussion of the Friedman rule in search models of money.

There is, however, a group of models in microfoundations literature that points out that not only the intensive, but also the extensive margin can be distorted. Therefore, deviating from the Friedman rule may be welfare improving under certain conditions: in the large household framework (Shi, 1997), the Friedman rule assures that the intensive margin is undistorted. Nevertheless, the number of trades might be inefficient because agents are not fully compensated for the externalities their search decisions generate; for this, the agent's surplus share has to equal her contribution to the creation of the match, i.e. the Hosios rule (Hosios, 1990) also needs to be satisfied. In this case, money growth may increase the number of matches and consequently welfare.[2]

Berentsen et al. (2006) further study this link between money growth and the extensive margin and show that the buyer's match surplus becomes endogenous if she is constrained by her money balances. This gives rise to a second-best result: if the buyer's exogenous bargaining weight is small relative to her contribution to the creation of the match, i.e. if the Hosios rule is violated, deviating from the Friedman rule will be welfare improving. However, these models do not study policy instruments other than monetary ones. If a social planner had lump-sum taxes and transfers available, the Friedman rule would indeed be optimal. In the presence of distortionary taxation, however, the outcome is less clear, as the optimal taxation literature has illustrated.

As this paper shows, introducing a sales tax into the money search framework (from Shi, 1997)

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[2] In a series of papers, Shi further demonstrates the robustness of the extensive margin effect to a number of extensions of the basic search framework: Shi (1998) introduces labour market frictions and Shi (1999) includes capital accumulation into the monetary search environment. Head and Kumar (2005) show that the suboptimality is also robust to price posting.

helps to correct an inefficiency in the extensive margin. A sales tax creates a tax burden that buyers and sellers split according to their bargaining weights. Consequently, if the seller pays a larger fraction of the tax, he will reduce his search effort more than the buyer will. This improves efficiency if the seller's search intensity exceeds the optimal level, i.e. if the seller's bargaining weight is larger than his contribution to the creation of the match.

However, if monetary policy obeys the Friedman rule, the two surplus shares are unaffected by fiscal policy. As long as the money constraint is not binding, the buyer cannot credibly limit her offer and will only receive the fraction of the surplus equal to her Nash bargaining weight. This limits the effectiveness of the sales tax – in order to improve along the lines of the Hosios rule, the buyer's surplus share needs to approach her contribution to the creation of the match. Thus, deviating from the Friedman rule is welfare improving. By making the money constraint bind, a positive nominal interest rate allows the buyer to extract a larger fraction of the total surplus. This *surplus share effect* is exclusive to monetary policy – monetary and fiscal policy are not substitutes in this environment.

This key finding carries over to an environment in which buyers and sellers do not bargain, but rather take prices as given (competitive pricing). With price taking, inflation will reduce the present value of the amount received by sellers, making it less attractive to be a seller and hence reducing the seller's search effort. At the same time, the buyer has an incentive to find a trading partner more quickly, inducing her to search more. This reduces the market congestion and increases efficiency. Inflation again implicitly changes the surplus split between agents; fiscal policy cannot achieve the same – taxation simply creates a burden on both buyers and sellers, and reduces their search efforts.

Monetary policy, on the other hand, does induce the buyer to search more, because it reduces the future value of money and thus creates a *surplus share effect*.

2. Environment

2.1. Households

The environment follows Berentsen et al. (2006).[3] The economy is comprised of a large number of households of size measure 1; each household is of a certain type, denoted h, with $H > 3$ different types in the economy. A household of type h produces good h and consumes good $(h + 1)$. Lower case letters denote household level variables, and capital letters denote the corresponding aggregates. Households are made up of two types of agents: buyers and sellers. The fraction of buyers is $n \in (0, 1)$. In each period, buyers and sellers enter a decentralized market to search for a trading partner.

All members of the household share the utility generated by the household's consumption. Thus their common objective is to maximize the household's utility, given as follows

$$U = \sum_{t=0}^{\infty} \beta^t \left[u' q_t^b - c(q_t^s) - n\phi(\sigma_{bt}) - (1-n)\phi(\sigma_{st}) \right], \qquad \beta \in (0, 1). \tag{2.1}$$

Here β is the discount factor, and $c(q)$ the cost of producing q in utility terms. The cost function

[3] This is a version of the large household framework by Shi (1997). The other framework commonly used in the microfoundations of money literature is the so-called LW model (Lagos and Wright, 2005). A brief discussion about possible differences in implications can be found in section 5.3.

$c(q)$ satisfies the usual properties $c(0) = 0$, $c'(q) > 0$, $c''(q) > 0$ and $c'(0) = 0$. The assumption of constant marginal utility is made for simplicity.[4] The number of buyers and sellers in the household is denoted by n and $(1-n)$ respectively. The function $\phi(\sigma)$ represents the disutility associated with a search intensity σ.[5] For simplicity, assume $\phi(\sigma) = \phi_0 (\sigma^{\alpha} - 1)$, with $\phi_0 > 0$ and $\alpha > 1$.

In each period, buyers and sellers go to the market where they engage in random bilateral matching. Since double coincidence of wants is ruled out by assumption and all members of a certain type of household are indistinguishable, a medium of exchange is needed. In each period t, the total stock of money is given by $M_t H$, where M_t is the average holding of money per household. As sellers have no use for money, all money is carried by the buyers, and each buyer carries m_t/n units of money when entering the market. Let ω denote the value of next period's money to the household and Ω the value of next period's money to other households. Finally, at the beginning of each period, the government gives a lump-sum transfer L_t to each household, so that the money stock grows at a rate of γ.

2.2. Fiscal Policy

Following the traditional money and public finance literature, the fiscal policy instrument is a sales tax. Whenever a buyer and a seller trade, the government imposes a tax at rate τ; i.e. if the

[4] The assumption of constant marginal utility has a bearing in the case of price taking, but does not alter the main result.

[5] The expressions "search intensity" and "search effort" are used synonymously thoughout this paper.

buyer pays x, the seller receives an after tax total of $x/(1+\tau)$.[6] The tax revenue raised is returned to the agents at the beginning of the next period as a lump-sum transfer to the household. This government transfer has to satisfy

$$L_t = (\gamma - 1)M_t + \frac{\Psi_{t-1}}{H}\tau x, \quad \gamma \geq \beta \tag{2.2}$$

where Ψ denotes the total number of matches. In this environment, this taxation scheme may also be regarded as an income tax on the seller.

An underlying assumption is that the government is able to observe matches and the amount of money exchanged although there is no sufficient record keeping technology (which makes money essential as a medium of exchange). This assumption is not as restrictive as it may seem – the monitoring technology necessary to implement this policy does not provide the type of memory that makes money dispensable (Kocherlakota, 1998). In fact, forms of sales or consumption based taxes have been used for centuries (e.g. salt taxes in Europe throughout the middle ages), with a general sales tax ("Generalkonsumakzise") first introduced in Saxony in 1754, at a time when record keeping was neither as easy nor as advanced as today.

[6]It can be shown that in this economy this tax scheme is equivalent to the buyer paying $x(1+\tau)$ and the seller receiving x.

2.3. Matching Function

In the decentralized market, buyers and sellers meet at random, and the total number of trade matches is determined by a matching function $\Psi(B\Sigma_b, S\Sigma_s)$, where B denotes the total number of buyers and S the total number of sellers in the market; Σ_i is their average search intensity. B and S are exogenously given as $B = HN$ and $S = H(1-N)$. In contrast, the search intensities are optimally chosen by the households. The function $\Psi(.)$ satisfies standard assumptions, such as homogeneity of degree 1 and concavity in both arguments (following e.g. Mortensen and Pissarides, 1994 and Berentsen et al., 2006).

It is useful to define the market thickness (for buyers) as the ratio of effective sellers to buyers

$$T \equiv \frac{S\Sigma_s}{B\Sigma_b} = \frac{(1-N)\Sigma_s}{N\Sigma_b} \tag{2.3}$$

Denote the marginal contribution of either side to the number of matches as $K_i(T)$

$$K_i(T) = \frac{\partial\Psi(B\Sigma_b, S\Sigma_s)}{\partial(i\,\Sigma_i)}, \qquad i = B, S$$

Then, one can rewrite the number of matches as $\Psi(B\Sigma_b, S\Sigma_s) = K_b(T)B\Sigma_b + K_s(T)S\Sigma_s$. Define the share of buyer's contribution to the total number of matches respectively as

$$\eta(T) = \frac{K_b(T)B\Sigma_b}{\Psi(B\Sigma_b, S\Sigma_s)}, \tag{2.4}$$

and analogously for the seller. Finally, it will prove convenient to define the average matching rate for the buyer and seller as

$$A_b(T) = \frac{\Psi(B\Sigma_b, S\Sigma_s)}{B\Sigma_b} = \Psi(1,T)$$

$$A_s(T) = \frac{\Psi(B\Sigma_b, S\Sigma_s)}{S\Sigma_s} = \frac{\Psi(1,T)}{T}$$

2.4. The Bargaining Process in the Decentralized Market

The bargaining process is the key mechanism through which fiscal and monetary policy change the agents' search behaviour: in the decentralized market, the buyer is potentially constrained by her money holdings. As a result, the bargaining outcome will depend on the buyer's money constraint whenever it is binding, i.e. in this framework the bargaining shares of buyer and seller are endogenous. Moreover, as shown below, they depend on both monetary and fiscal policy. This channel for policy is a crucial feature of this framework and is missing in the previous literature on money and public finance.

After a buyer and a seller meet in the market, they bargain over q, the quantity of goods, and x, the amount of money to be exchanged in the trade. The bargaining process is modeled as a sequential game with an exogenous risk of breakdown. In each round, one agent proposes a pair (q, x) and the respondent accepts or rejects. If the proposal is accepted, the trade takes place immediately on the agreed upon terms; if not, time Δ elapses and the respondent may make a counteroffer. During this "waiting time", the game might break down; the probability of breakdown depends on the rejecting agent's type. If a seller rejects the buyer's offer, the probability of breakdown is $\theta\Delta$; if the buyer rejects the seller's proposal the probability is $(1-\theta)\Delta$, where $\theta \in (0;1)$. This paper focuses on the limit case when Δ approaches 0, and there is no first-mover advantage.[7]

Assume all agents follow a stationary bargaining strategy, i.e. a buyer always proposes (q^b, x^b) and a seller always proposes (q^s, x^s). First, consider the buyer's problem: When making her proposal, the buyer faces two constraints; she is restricted by her own money holdings, and she may not leave the seller less surplus than his reservation surplus. Upon accepting, the seller obtains x^b units of money with a present value of Ωx^b and incurs a disutility of $c(q^b)$. Hence, his surplus from accepting is $\left[\Omega x^b - c(q^b)\right]$. If he decides to reject, he will make a counteroffer (Q^s, X^s) with probability $(1-\theta\Delta)$ that gives him a surplus of $[\Omega X^s - c(Q^s)]$. So, the buyer's proposal must satisfy

$$\frac{m}{n} \geq x^b \tag{2.5}$$

$$\Omega x^b/(1+\tau) - c(q^b) \geq (1-\theta\Delta)\ [\Omega X^s/(1+\tau) - c(Q^s)] \tag{2.6}$$

Similarly, the seller's proposal (q^s, x^s) needs to satisfy

$$\frac{M}{N} \geq x^s \tag{2.7}$$

$$u'q^s - \Omega x^s \geq (1-(1-\theta)\Delta)\ \left[u'Q^b - \Omega X^b(1+\tau)\right] \tag{2.8}$$

[7] For a more in-depth discussion of sequential bargaining games see Muthoo (1999)

In equilibrium, (2.6) and (2.8) will be satisfied with equality. To see why, suppose to the contrary that (2.6) holds as a strict inequality, then the buyer could increase her utility by rising q^b without increasing x^s until the constraint is satisfied with equality. Likewise, the seller could decrease q^s and hence his disutility of production if (2.8) were not satisfied with equality.

The solution to the bargaining game is summarized in Lemma 1.

Lemma 1. *In a symmetric equilibrium with $x^i = X^i$ and $q^i = Q^i$, when $\Delta \to 0$, $x^b = x^s = x$ and $q^b = q^s = q$ and the buyer's surplus is given by*

$$\Theta(q,\tau)\left[u'q - c(q)\right] - \Theta(q,\tau)\tau c(q). \tag{2.9}$$

The seller's surplus is given by

$$(1-\Theta(q,\tau))\left[u'q - c(q)\right] - (1-\Theta(q,\tau))\frac{\tau}{(1+\tau)}u'q. \tag{2.10}$$

Where $\Theta(q,\tau)$ is defined as

$$\Theta(q,\tau) = \frac{\theta u'}{\theta u' + (1+\tau)(1-\theta)c'(q)}. \tag{2.11}$$

Proof. *See appendix* ■

As noted above, the buyer's surplus share $\Theta(q,\tau)$ is endogenous as a result of the money constraint; only if $\theta = 0$ (a take-it or leave-it offer by the seller) or if $\theta = 1$ (a take-it or leave-it offer by the buyer) does it coincide with θ. The buyer's surplus depends directly on the tax rate and indirectly through the quantity exchanged in the market, on the rate of money growth. This effect of fiscal and monetary policy is critical for the results.

Fiscal policy has three effects on the agents' surplus. The first is the effect on total surplus $[u'q - c(q)]$, the second the tax burden levied on buyers, $\Theta(q,\tau)\tau c(q)$, and sellers, $(1-\Theta(q,\tau))\frac{\tau}{(1+\tau)}u'q$, and the third the effect on the surplus share the buyer receives, $\Theta(q,\tau)$. The first two effects reduce both the buyer's and seller's surplus because a distorting tax will reduce total surplus and impose a tax burden on both agents. The last effect goes in opposite directions for buyers and sellers, and is discussed in detail in section 3.2.

3. The Monetary Equilibrium with Fiscal Policy

3.1. The Household's Problem

In each period the household chooses its buyers' and sellers' bargaining proposals, their search intensity and next period's money stock, taking other households' choices as given. The problem can be written as a dynamic programming problem

$$v(m) = \max_{\{q^b,x^b,q^s,x^s\sigma_b,\sigma_s,m_{+1}\}} \left\{ \begin{array}{c} n\sigma_b A_b(T)u(q^b) - (1-n)\sigma_s A_s(T)c(q^s) \\ -n\phi(\sigma_b) - (1-n)\phi(\sigma_s) + \beta v(m_{+1}) \end{array} \right\} \tag{3.1}$$

subject to (2.5) - (2.8) and the law of motion for money

$$m_{+1} = m + (1-n)\sigma_s A_s(T)\frac{x^s}{(1+\tau)} - n\sigma_b A_b(T)x^b + L.$$

The first order conditions are given by

$$\begin{aligned}
u' &= \frac{\omega+\lambda}{\Omega}(1+\tau)c'(q^b) && (3.2)\\
c'(q^s) &= \frac{\omega - \pi(1+\tau)}{\Omega(1+\tau)}u' && (3.3)\\
\phi'(\sigma_b) &= A_b(T)\left(u(q^b) - \omega x^b\right) && (3.4)\\
\phi'(\sigma_s) &= A_s(T)\left(\frac{\omega x^s}{(1+\tau)} - c(q^s)\right) && (3.5)\\
\frac{\omega_{-1}}{\beta} &= \omega + \sigma_b A_b(T)\lambda && (3.6)
\end{aligned}$$

where $\omega = \beta v_{+1}(m_{+1})$, the discounted expected value of money next period. λ and π are the lagrange multipliers associated with the money constraints (2.5) and (2.7) respectively.

Equation (3.2) describes the trade-off a proposing buyer faces; the monetary cost of an extra marginal unit of the consumption good is given by $(1+\tau)c'(q^b)/\Omega$. This amount of money is valued at $(\omega+\lambda)$ by the buyer, where ω is next period's value of money and λ represents the tighter cash or resource constraint. Thus, the right hand side of (3.2) represents the marginal cost (in utility terms) of an extra unit of the consumption good for the buyer. The optimal proposal equalizes this marginal cost with the marginal utility. Similarly, equation (3.3) requires that the marginal gain (in utility terms) is equal to the marginal cost of producing.

Equations (3.4) and (3.5) describe the optimal choices for the search intensities. The right hand side of each of these equations is the marginal gain from increasing the search intensity. To see this, note that $A_i(T)$ is the average matching rate per "unit of search effort", while $\left(u(q^b) - (1+\tau)\omega x^b\right)$ and $(\omega x^s - c(q^s))$ are the gains from a trade match for a buyer and a seller respectively. A marginal increase in the search intensity increases the probability of a match by $A_i(T)$, and increases the expected utility by $A_i(T)\left(u(q^b) - (1+\tau)\omega x^b\right)$. In equilibrium, this marginal gain needs to equal the marginal cost of searching.

Using the result of Lemma 1, (3.4) and (3.5) can be rewritten as

$$\phi'(\sigma_b) = A_b(T)\,\Theta(q,\tau)\,[u(q) - (1+\tau)\,c(q)] \tag{3.7}$$

$$\phi'(\sigma_s) = A_s(T)\,\frac{(1-\Theta(q,\tau))}{(1+\tau)}\,[u(q) - (1+\tau)c(q)] \tag{3.8}$$

(3.6) is the envelope condition for money.

3.1.1. Stationary and Symmetric Monetary Equilibrium

Definition 1. *A stationary and symmetric monetary equilibrium consists of a sequence of individual household's choices* $\{d_t\}_{t=0}^{\infty}$, *where* $d = (q^b, x^b, q^s, x^s, m_{+1}, \sigma_b, \sigma_s)$, *other households' choices* $\{D_t\}_{t=0}^{\infty}$, *and the shadow prices* $(\omega, \Omega, \lambda, \Lambda, \pi, \Pi)$. *The sequence satisfies the following requirements for all t:*

(i) optimality: d_t *solves the households problem given* D_t,

(ii) symmetry: $d_t = D_t$,

(iii) stationarity: $d_t = d$,

(iv) $0 < \omega_{t-1} M_t < \infty$ *and* $\omega_{t-1} M_t$ *constant.*

Conditions (i) - (iii) are standard. The first part of condition (iv) requires that money has a positive and finite value, where ω_{t-1}/β is the period t value of one unit of money.[8]

The stationary and symmetric monetary equilibrium allocation $(q, \omega x, \sigma_b, \sigma_s)$ can be obtained from (3.7), (3.8), (2.3) and the following equations:

$$\frac{u'}{c'(q^s)} = \left[1 + \frac{1}{\sigma_b A_b(T)} \left(\frac{\gamma}{\beta} - 1\right)\right] (1+\tau) \tag{3.9}$$

$$u(q) - \omega x = \Theta(q,\tau) \left[u(q) - (1+\tau)\, c(q)\right] \tag{3.10}$$

Equation (3.9) comes from combining (3.2) and (3.6) and imposing stationarity. Equation (3.10) follows from Lemma 1.

From (3.9), we can see that if the economy follows the Friedman rule, $u' = (1+\tau)c'(q^s)$. In this case, we can solve for the equilibrium quantity without having to specify the matching function. It is obvious that only if $\tau = 0$, the intensive margin will be undistorted. If $\tau \neq 0$, the quantity q deviates from the social optimum even if $\gamma = \beta$.[9] From (3.9) it also follows that deviating from the Friedman rule will result in an inefficiency in the intensive margin. This follows directly from the fact that the buyer is constrained by his real money balances. Whenever $\gamma > \beta$, there is an

[8] The existence of an equilibrium was established in Berentsen et al. (2006); it is necessary that $\gamma \geq \beta$ and $\lambda > 0$ if and only if $\gamma > \beta$.

[9] See section 4 for the characterization of the social optimum.

opportunity cost of holding money and hence the buyer will choose to hold inefficiently low real money holdings; q will thus also be inefficiently low.

3.2. The Buyer's Surplus Share Revisited

Recall from Lemma 1 that the buyer's fraction of the surplus is given by $\Theta(q, \tau)$. Using the steady state condition (3.9), $\Theta(q, \tau)$ can be rewritten as

$$\Theta(\gamma, \tau, T) = \frac{\theta}{\theta + \frac{(1-\theta)}{\left[1+\frac{1}{\sigma_b A_b(T)}\left(\frac{\gamma}{\beta}-1\right)\right]}}. \tag{3.11}$$

From (3.11), it can be seen that the buyer's surplus share critically depends on the rate of money growth. If monetary policy follows the Friedman rule, $\Theta(\gamma, \tau, T)$ reduces to θ, the exogenous bargaining parameter. This implies that the surplus split is independent of the fiscal policy at the Friedman rule; the buyer will receive a fixed fraction of the total surplus regardless of τ.

The buyer's surplus share becomes endogenous only if monetary policy deviates from the Friedman rule; from (3.11), it follows that $\Theta(\gamma, \tau, T)$ is increasing in the rate of money growth. Equivalently, examining (3.3) shows that the higher the rate of money growth, the tighter the money constraint and the larger the lagrange multipliers λ/ω and π/ω. Substituting (3.3) into $\Theta(q, \tau)$ shows that the buyer's surplus share is increasing in π/ω, i.e. the higher the rate of money growth, the larger the buyer's surplus share.

The intuition for this result follows from the latter argument. The money constraint serves as a credible upper bound to the buyer's offer and becomes tighter as the rate of money growth increases. This improves the buyer's threat point in the bargaining game and allows her to extract a larger fraction of the total surplus, creating the *surplus share effect* of monetary policy.

The *surplus share effect* of monetary policy is also essential for the effectiveness of fiscal policy. At the Friedman rule, fiscal policy does not have an impact on the surplus split: $\Theta(\gamma = \beta, \tau) = \theta$. If, however, the rate of money growth exceeds the rate of time preference $(\gamma > \beta)$, the sales tax will further improve the buyer's bargaining position. To see this, note that $\sigma_b A_b(T)$ decreases as τ increases.[10] The intuition is the same as for monetary policy: if the buyer is constrained by her money holdings, increasing the tax rate makes this constraint even more binding, allowing the buyer to extract a larger fraction of the total surplus. However, this channel only works if the buyer's money constraint is binding, giving a special role to monetary policy. This link between fiscal and monetary policy is a novel feature of the model and is crucial for the optimal policy as discussed below.

4. Social Optimum

Now consider the problem of a benevolent planner who seeks to maximize social welfare – the total trade surplus generated by all matches, less the cost of searching incurred to create these matches.

[10] The buyer's search intensity falls because the reduction in total surplus and the tax burden outweigh the increase in the buyer's surplus share. The reduction in the average matching probability for the buyer results from the reduction in the market tightness T.

Hence, the social welfare function can be written as

$$W = \Psi(N\Sigma_b, (1-N)\Sigma_s)\left[u'q - c(q)\right] - N\phi(\Sigma_b) - (1-N)\phi(\Sigma_s) \tag{4.1}$$

The planner chooses the quantity produced in each match, q, and agents' search intensities (Σ_b, Σ_s). The first order conditions are given by

$$u' = c'(q) \tag{4.2}$$

$$\phi'(\Sigma_b) = K_b(T)\left(u'q - c(q)\right) = \eta(T)A_b(T)\left(u'q - c(q)\right) \tag{4.3}$$

$$\phi'(\Sigma_s) = K_s(T)\left(u'q - c(q)\right) = (1-\eta(T))A_s(T)\left(u'q - c(q)\right). \tag{4.4}$$

The social optimum $(q^*, \Sigma_b^*, \Sigma_s^*)$ is characterized by (4.2) - (4.4) and (2.3).

In comparing the social optimum characterized by (4.2) - (4.4) and the monetary equilibrium characterized above, several differences are apparent: the quantity, and the search intensities, may differ. From (4.2) and (3.9), it follows that in order for the quantity traded to be efficient, the tax rate must be zero and monetary policy needs to follow the Friedman rule.

Moreover, comparing (3.7) to (4.3) and (3.8) to (4.4), it follows that efficiency in the number of trades requires $\Theta(q,\tau) = \eta(T)$: the buyer's surplus share needs to equal her contribution to the creation of the match. Since at the Friedman rule $\Theta(q,\tau)$ reduces to θ, the Hosios rule calls for $\theta = \eta(T^*)$. However, there is no apparent reason to believe that this requirement is satisfied, since this condition links a property of the matching function to the buyer's bargaining power. If $\theta < \eta(T^*)$, the buyer's bargaining share is too small and her search intensity too low.[11] In order to improve efficiency, σ_b needs to increase relative to σ_s, so T needs to decrease. Conversely, if $\theta > \eta(T^*)$, the equilibrium market thickness is too low and an increase in T will improve efficiency.

5. The Welfare Effects of Fiscal and Monetary Policy

From the previous sections, it is evident that the monetary equilibrium is unlikely to coincide with the first best outcome if there is no fiscal policy and monetary policy simply follows the Friedman rule. This section studies how fiscal and monetary policy can improve efficiency in this environment and demonstrates that monetary, but not fiscal, policy alters the agents' bargaining position, rendering a deviation from the Friedman rule optimal whenever the buyer's bargaining weight is small relative to her contribution to the match. This result is summarized in the following proposition.

Proposition 1. *If $\theta < \eta(T^*)$, $\tau^* > 0$ and $\gamma^* > 0$, the optimal policy calls for a positive sales tax and a deviation from the Friedman rule.*

The proof is laid out in three parts. First, section 5.1 derives the optimal tax rate at the Friedman rule. Section 5.2 then establishes that without fiscal policy, it is optimal to deviate from the Friedman rule if $\theta < \eta(T^*)$. Lastly, section 5.3 combines the previous two results and proves that an optimal policy mix consists of using the two instruments jointly.

[11] To see this, compare (3.7) to (4.3) with $\tau = 0$ and note that $\phi(\sigma)$ is a convex function.

5.1. Fiscal Policy

If the buyer's bargaining weight is too low relative to her contribution to the match $(\theta < \eta(T^*))$, the buyer is not sufficiently rewarded for her search effort and will hence choose an inefficiently low search effort. This renders the market tight for sellers and thick for buyers, i.e. T is inefficiently high. In order to improve efficiency, any policy needs to decrease the market tightness. To see that the market tightness decreases as the tax rate increases, divide (3.7) by (3.8) and impose the Friedman rule:

$$T_{|\gamma=\beta} = \left[\left(\frac{(1-N)}{N} \right)^{\alpha-1} \left(\frac{(1-\theta)}{\theta} \right) \frac{1}{(1+\tau)} \right]^{\frac{1}{\alpha}} \tag{5.1}$$

From (5.1), it follows that the market tightness at the Friedman rule is decreasing in the tax rate: $\frac{\partial T}{\partial \tau}_{|\gamma=\beta} < 0$.

As discussed above, the sales tax has three effects on the agents' surplus. The sales tax reduces both agents' surpluses because it distorts the quantity of goods exchanged and levies a tax burden on them. The third effect, a change in the surplus split between buyer and seller, is not present here. At the Friedman rule, the buyer's surplus share is equal to θ, and is independent of the tax rate τ. As a result, both the buyer's and seller's search intensity fall. However, the seller's search effort falls relatively more than the buyer's. This is because an increase in the tax reduces the present value of the amount of money exchanged in the match, $\omega x/(1+\tau)$, making it relatively less attractive to be a seller and consequently lowering T.

Now, taking the derivative of the welfare function (4.1) with respect to the tax rate and evaluating at the Friedman rule gives

$$\begin{aligned}
\frac{\partial W}{\partial \tau}_{|\gamma=\beta} &= \Psi\tau\left[c'\frac{\partial q}{\partial \tau}+\left(\theta\frac{1}{\Sigma_b}\frac{\partial \Sigma_b}{\partial \tau}c(q)+\frac{(1-\theta)}{(1+\tau)}\frac{1}{\Sigma_s}\frac{\partial \Sigma_s}{\partial \tau}u(q)\right)\right] \\
&\quad +\Psi(\eta(T)-\theta)\frac{\Sigma_s}{\Sigma_b}\frac{\partial\left(\frac{\Sigma_b}{\Sigma_s}\right)}{\partial \tau}(u-c) \\
&= \Psi\frac{1}{T}\frac{\partial T}{\partial \tau}\left[-(\eta(T)-\theta)(u-c)+\tau\begin{pmatrix} \alpha\frac{(c')^2}{c''}+\frac{\theta}{\alpha-1}c(q)\left((1-\eta)+\alpha\frac{(1+\tau)c}{u-(1+\tau)c}\right) \\ +\left(\frac{1-\theta}{1+\tau}\right)\frac{1}{\alpha-1}u(q)\left(-\eta+\alpha\frac{(1+\tau)c}{u-(1+\tau)c}\right)\end{pmatrix}\right]
\end{aligned} \tag{5.2}$$

Since $\frac{\partial T}{\partial \tau}<0$, (5.2) indicates that $\tau^*>0$ if and only if $\theta<\eta(T)$.

The optimal tax rate at the Friedman rule, τ^*, solves

$$(\eta(T)-\theta)(u-c)=\tau\begin{pmatrix} \alpha\frac{(c')^2}{c''}+\frac{\theta}{\alpha-1}c(q)\left((1-\eta)+\alpha\frac{(1+\tau)c}{u-(1+\tau)c}\right) \\ +\left(\frac{1-\theta}{1+\tau}\right)\frac{1}{\alpha-1}u(q)\left(-\eta+\alpha\frac{(1+\tau)c}{u-(1+\tau)c}\right)\end{pmatrix}. \tag{5.3}$$

If $\theta<\eta(T)$, the exogenous bargaining share of the buyer is too small and as a result her search intensity is too low and the seller's effort too high. However, contrary to what may seem intuitive, paying a subsidy does not improve welfare in this case. As argued above, the buyer and seller split any subsidy or tax burden through the bargaining process. If the seller has a high bargaining power, he can extract most of the subsidy, further increasing his inefficiently high search effort and hence decreasing welfare.

A positive tax rate, on the other hand, reduces the seller's search intensity relative to the buyer's, making the market tighter for buyers and increasing efficiency. Both the buyer's and seller's search efforts fall, closing the wedge between social marginal benefit and social marginal cost of searching for the seller and widening the wedge for the buyer. The positive welfare effect is a result of the first effect dominating the second because the seller's search effort falls relatively more.

5.2. Monetary Policy

To see how T responds to an increase in the rate of money growth, combine the equilibrium conditions (4.3) and (3.9) to obtain

$$\left(\frac{\gamma}{\beta}-1\right)^{\alpha-1} = \left(\frac{u'}{c'(q)(1+\tau)}-1\right)^{\alpha-1} \frac{[A_b(T)]^{\alpha}\left(u(q)-(1+\tau)c(q)\right)}{\alpha\phi_0\left(1+T^{\alpha}(1+\tau)\left(\frac{N}{1-N}\right)^{\alpha-1}\right)} \tag{5.4}$$

After solving for $q = q(T)$ by dividing (3.7) by (3.8) and substituting into (5.4), the resulting expression gives $\frac{\partial T}{\partial \gamma} < 0$. This is a result of the *surplus share effect* described above. That is, if the rate of money growth exceeds the Friedman rule, the buyer becomes constrained by her money holdings, which allows her to credibly limit her offer to the seller, thus increasing her share of the total surplus. As a result, the buyer's search effort increases relative to the seller's effort and the market tightness T decreases.

To analyze the welfare effect of increasing the rate of money growth above the rate prescribed by the Friedman rule, take the derivative of the welfare function (4.1) with respect to γ and evaluate at $\gamma = \beta$.

$$\begin{aligned}
\frac{\partial W}{\partial \gamma}_{|\gamma=\beta} &= \Psi\tau\left[c'\frac{\partial q}{\partial \gamma} + \left(\theta\frac{1}{\Sigma_b}\frac{\partial \Sigma_b}{\partial \gamma}c(q) + \frac{(1-\theta)}{(1+\tau)}\frac{1}{\Sigma_s}\frac{\partial \Sigma_s}{\partial \gamma}u(q)\right)\right] \\
&\quad +\Psi(\eta(T)-\theta)\frac{\Sigma_s}{\Sigma_b}\frac{\partial\left(\frac{\Sigma_b}{\Sigma_s}\right)}{\partial \gamma}(u-c) \\
&= \Psi\frac{1}{T}\frac{\partial T}{\partial \gamma}\left[-(\eta(T)-\theta)(u-c) + \tau\left(\begin{array}{c} \alpha\frac{(c')^2}{c''} + \frac{\theta}{\alpha-1}c(q)\left((1-\eta)-\alpha\left(1-\theta\right)\right) \\ +\left(\frac{1-\theta}{1+\tau}\right)\frac{1}{\alpha-1}u(q)\left(-\eta+\alpha\theta\right) \end{array}\right)\right]
\end{aligned}$$

(5.5)

With $\tau = 0$, there is a positive effect of increasing the rate of money growth at the Friedman rule if $\theta < \eta(T)$. As argued, deviating from the Friedman rule raises the buyer's share of the surplus by tightening the money constraint, causing her to exert a higher search intensity and increasing social welfare. If, however, $\theta > \eta(T)$, the Friedman rule is still constrained optimal. In this case, the Hosios rule demands a negative nominal interest rate which is not a feasible policy option.

The welfare improvement in deviating from the Friedman rule results from trading off the efficiency of the quantity traded against reducing the inefficiency of the search intensity. This is a typical second best/ optimal taxation result – the second order loss in the intensive margin is smaller than the first order gain in the extensive margin.

5.3. Optimal Policy Mix

It is apparent that both fiscal and monetary policy in isolation will be welfare improving, at least if $\theta < \eta(T)$. However, the key question remains: is the Friedman rule optimal in the presence of fiscal policy, and how does the optimal policy mix (γ^*, τ^*) compare to the optimal rate of money growth in an environment without a sales tax?

Substituting the implicit solution for the optimal tax (5.3) into the first order condition for the rate of money growth (5.5) gives

$$W_\gamma(\tau^*, \beta) = \frac{1}{T}\frac{\partial T}{\partial \gamma}\tau^* \left(\begin{array}{c} \theta \frac{\alpha}{\alpha-1} c(q) \left(-(1-\theta) - \frac{(1+\tau^*)c}{u-(1+\tau^*)c} \right) \\ + \left(\frac{1-\theta}{1+\tau^*} \right) \frac{\alpha}{\alpha-1} u(q) \left(\theta - \frac{(1+\tau^*)u}{u-(1+\tau^*)c} \right) \end{array} \right) \begin{array}{l} > 0 \text{ if } \theta < \eta \\ < 0 \text{ if } \theta > \eta \end{array} .$$

With distortionary taxation, increasing the rate of money growth above the rate of time preference is still welfare improving if the buyer's bargaining power is lower than her contribution to the match creation. Thus, the conditions for the suboptimality of the Friedman rule are the same as without fiscal policy.

To understand this optimal policy, it is important to remember the *surplus share effect* of monetary policy and how it affects the effectiveness of fiscal policy. At the Friedman rule, fiscal policy does not have an impact on the bargaining weights: the buyer receives a θ fraction of the surplus, independent of the tax rate. If, however, the rate of money growth exceeds the rate of time preference, the sales tax will further improve the buyer's bargaining position and the buyer's surplus share increases in τ. This channel of fiscal policy only works if the buyer's money constraint is binding, leaving a special role for monetary policy.[12]

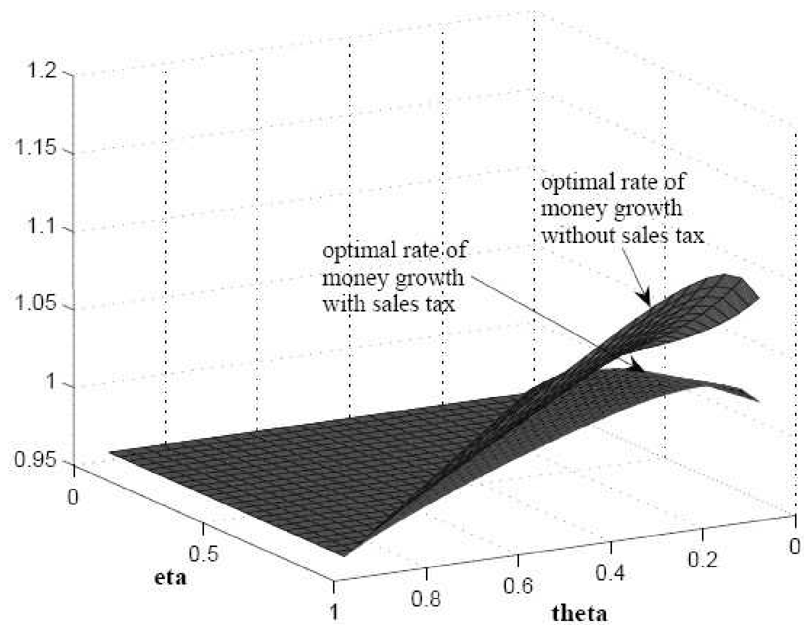

Figure 1: The optimal rate of money growth γ^*

While this establishes the key result, it is interesting to see how the optimal rate of money growth changes after the introduction of a sales tax. Unfortunately, it is necessary to resort to

[12]The other framework commonly used in the microfoundations literature is the so-called LW model (Lagos and Wright, 2005). A key difference from the model above is that buyers are constrained by their money balances even at the Friedman rule. This is because of a hold-up problem: buyers do not carry sufficient amounts of money to the decentralized market because they will not get fully compensated for their investment unless they have all the bargaining power. Consequently, the buyer's surplus share is endogenous even at the Friedman rule. Furthermore, because the intensive margin is distorted at the Friedman rule, both sales tax and monetary policy cannot improve welfare by alleviating an inefficiency in the extensive margin.

computational methods to answer this question. As is common in the literature, a Cobb-Douglas type matching function is used, i.e. $\Psi(B\Sigma_b, S\Sigma_s) = \Psi(B\Sigma_b)^\eta(S\Sigma_s)^{1-\eta}$, which implies $A_b(T) = T^{1-\eta}$ and $A_s(T) = T^{-\eta}$. Also, the contributions of the buyer and seller to the match are constant and independent of T. The cost of production is given by $c(q) = \frac{1}{\chi}q^\chi$, $\chi > 1$.[13]

Introducing fiscal policy reduces the optimal rate of money growth unambiguously. Figure 1 shows the optimal rate of money growth as a function of the buyer's contribution to the match and her bargaining weight. The rate of money growth is increasing in $(\eta - \theta)$ almost throughout the range; only if θ is very small does γ^* increase in θ. The smaller the buyer's exogenous bargaining parameter relative to her contribution to the match, the larger the rate of money growth.

Examining (5.5), it is apparent that the positive welfare effect of money growth is falling as the tax rate increases. Fiscal policy reduces the marginal gain of increasing the rate of money growth, as it serves the same objective, while at the same time increasing the cost. With a positive tax rate, deviating from the Friedman rule does not result in a second order, but rather in a first order loss because of the distortion in the number of goods. In fact, there is a critical tax rate $\widetilde{\tau}$ such that if $\tau^* > \widetilde{\tau}$, the Friedman rule is optimal. However, as shown above, $\tau^* < \widetilde{\tau}$ and a deviation from the Friedman rule is still welfare improving. Nevertheless, the existence of a critical tax rate may become important if the government uses taxes not only to improve efficiency, but also to raise tax revenue.[14]

[13] For the numerical exercise, let $u' = 1$, which gives $q^* = 1$. All figures are based on $N = 0.5$, $\chi = 3$ and $\alpha = 3$.

[14] Aruoba and Chugh (2006) study the Ramsey problem in a monetary search environment and find that deviating from the Friedman rule is optimal.

As Figure 2 shows, the optimal tax rate displays a similar pattern. If monetary policy follows the Friedman rule, the optimal tax rate is higher than with optimal monetary policy.

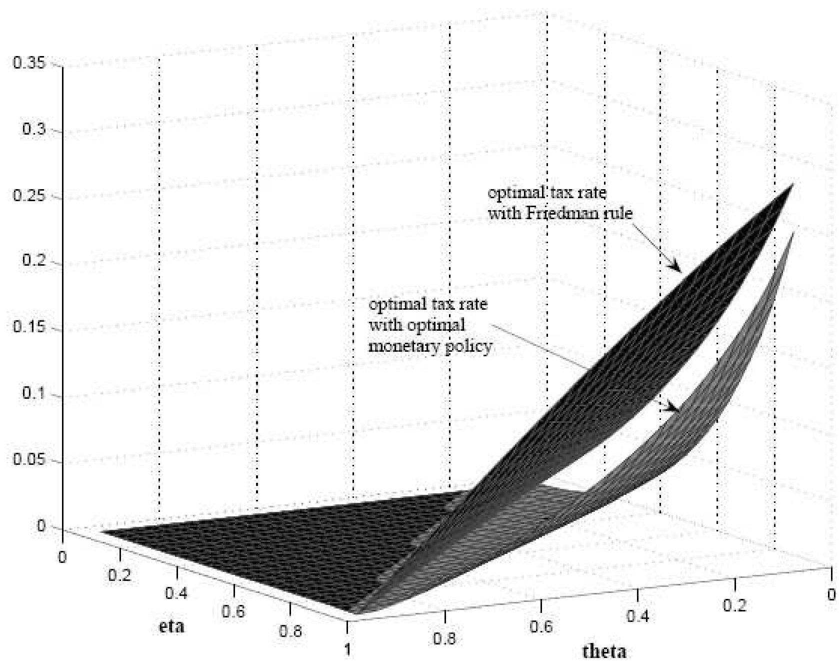

Figure 2: The optimal tax rate τ^*

6. Competitive Pricing

While the analysis in section 5 demonstrates that monetary policy can play a special role in achieving efficiency and cannot be replaced by distortionary taxation, it is important to note that this conclusion carries over to different pricing mechanisms. As this section demonstrates, a deviation from the Friedman rule is optimal with competitive pricing.[15]

[15] In monetary search models, a variety of pricing mechanisms other than bargaining have been used. Head and Kumar (2005) introduce price-posting with random search into the large household framework. Rocheteau and

With price taking, the search friction is modelled as entry into a specific market for good h. Once in the market, buyers and sellers observe the price and the buyer demands q^b while the seller offers q^s. To make the two models comparable, let the entry function be the matching function described earlier; in this case, the number of buyers equals the number of sellers in the market. Thus, this setup can also be interpreted as bilateral matching with the buyers and sellers taking prices as given. So, the probability of a buyer entering the market is the same as her finding a trading partner in the environment with bilateral matching and bargaining, $A_b(T)\sigma_b$; similarly, the probability of trading for the seller is $A_s(T)\sigma_s$.

The household's problem is now given by

$$v(m) = \max_{\{q^b, q^s, \sigma_b, \sigma_s, m_{+1}\}} \left\{ \begin{array}{c} n\sigma_b A_b(T)u(q^b) - (1-n)\sigma_s A_s(T)c(q^s) \\ -n\phi(\sigma_b) - (1-n)\phi(\sigma_s) + \beta v(m_{+1}) \end{array} \right\} \tag{6.1}$$

subject to

$$pq^b \leq \frac{m}{n}$$

Wright (2005) study several different pricing mechanisms (bargaining, competitive equilibrium, competitive search) in the LW framework (Lagos and Wright, 2005). They show that competitive search leads to an efficient monetary equilibrium – fiscal policy is unnecessary and deviating from the Friedman rule is not welfare improving. However, in the case of price taking, the Friedman rule may not be optimal in their environment.

and the law of motion for money

$$m_{+1} = m + (1-n)\sigma_s A_s(T)\frac{pq^s}{(1+\tau)} - n\sigma_b A_b(T)pq^b + L.$$

Imposing stationarity and symmetry, the equilibrium condition is given by

$$\frac{u'}{c'(q)} = \left[1 + \frac{1}{\sigma_b A_b(T)}\left(\frac{\gamma}{\beta} - 1\right)\right](1+\tau). \tag{6.2}$$

which is the same as the one with bargaining in (3.9). Solving for the buyer's surplus gives

$$(u(q) - \omega pq) = u(q) - u'q + \frac{1}{\sigma_b A_b(T)}\left(\frac{\gamma}{\beta} - 1\right)(1+\tau)c'(q). \tag{6.3}$$

Evaluating (6.3) at the Friedman rule and imposing the Hosios rule gives

$$\frac{(u(q) - u'q)}{(u(q) - c(q))}_{|\gamma=\beta} = \eta. \tag{6.4}$$

This condition in general need not to be satisfied. With linear utility, it furthermore reduces to $\eta = 0$, which is not admissible. However, it is obvious that this condition need not be satisfied with a concave utility function either.

Imposing a sales tax at the Friedman rule reduces both the buyer's and seller's surpluses, with the seller's surplus falling more than the buyer's. Hence, imposing a sales tax will improve welfare if the market thickness at the Friedman rule was too high, i.e. if the market was congested by sellers.[16] However, just as with bargaining, increasing the rate of money growth is the more efficient policy: from (6.3), it is apparent that the buyer's surplus increases as the rate of money growth increases (in the neighborhood of the Friedman rule), which grows the buyer's search intensity. Similarly, the seller's surplus and search intensity fall, reducing the congestion and increasing efficiency.

The rationale behind this result is similar as with bargaining. Deviating from the Friedman rule makes it less attractive to be a seller. The discounted value of the amount of money paid for one unit of goods, ωp, falls; the seller's market power is not sufficient to increase the price such that ωp stays constant, which is essentially the *surplus share effect* from above. On the other hand, the buyer will try to spend the money more quickly because it looses value over time, while the present value of her expenditure falls.

7. Conclusion

This paper introduces a sales tax into the monetary search framework. A departure from the Friedman rule is optimal whenever there is a thick market on the buyer's side, i.e., if there are too many sellers relative to buyers in the market. If the buyer's bargaining weight is too small, the buyer's search effort will be inefficiently low, causing a congestion in the market. By restricting the buyer's money balance, monetary policy changes the agents' bargaining position, allowing the

[16] This is a result of the constant marginal utility. It makes the demand very elastic, hence the seller has to bear most of the burden. With decreasing marginal utility, the buyer bears a greater share of the burden, making a sales tax even less effective.

buyers to extract a larger fraction of the trade surplus. A sales tax cannot reproduce this *surplus share effect*, leaving a role for a deviation from the Friedman rule as an optimal monetary policy.

Moreover, this finding does not depend on bargaining as the pricing mechanism; the *surplus share effect* of monetary policy carries over to an environment with competitive pricing. Key to this effect is the change in the future value of money due to inflation, which makes it less attractive to be a seller and induces buyers to search more. This reduces the market tightness, which cannot be achieved by a sales tax.

This paper focuses only on the scenario in which the buyer's contribution to the creation of the match exceeds the exogenous Nash bargaining weight. In that case, deviating from the Friedman rule can improve efficiency because it increases the buyer's surplus share above her exogenous bargaining parameter. In the opposite situation, the exogenous bargaining parameter is the lower bound for the buyer's surplus share; a negative nominal interest rate is not a feasible policy option, so the Friedman rule is the constrained optimal policy. However, fiscal policy can achieve some welfare improvement by setting a negative tax rate – paying a subsidy to the agents.

8. Appendix

Proof. Lemma 1

The constraints are given by

$$\begin{aligned}
\Omega x^b/(1+\tau) - c(q^b) &= (1-\theta\Delta)\left[\Omega X^s/(1+\tau) - c(Q^s)\right] \\
u'Q^s - \omega X^s &= (1-(1-\theta)\Delta)\left[u'q^b - \omega x^b\right] \\
x^b &= X^s = \frac{m}{n}
\end{aligned}$$

rearranging gives

$$\begin{aligned}
Q^s &= Q^s(q^b,\Delta) = \frac{1}{u'}\left[\omega\frac{m}{n} + (1-(1-\theta)\Delta)\left(u'q^b - \omega x^b\right)\right] \\
\frac{\partial}{\partial\Delta}Q^s(q^b,\Delta) &= -\frac{1-\theta}{u'}\left[u'q^b - \omega\frac{m}{n}\right]
\end{aligned}$$

in equilibrium $Q^s(q^b,0) = q^b$, so

$$\Omega\frac{m/n}{1+\tau} - c(q^b) = (1-\theta\Delta)\left[\Omega\frac{m/n}{1+\tau} - c\left(Q^s(q^b,\Delta)\right)\right]$$

rearranging gives

$$\theta\Omega\frac{m/n}{1+\tau} = \frac{1}{\Delta}\left[c(q^b) - (1-\theta\Delta)c\left(Q^s(q^b,\Delta)\right)\right]$$

take limit $\Delta \to 0$

$$\theta\Omega\frac{m/n}{1+\tau} = \theta c(q^b) - c'\frac{-(1-\theta)}{u'}\left[u'q^b - \omega\frac{m}{n}\right]$$

rearranging gives

$$\omega \frac{m}{n} = \frac{(1+\tau)u'}{\theta u' + (1-\theta)(1+\tau)c'} \big(\theta c(q) + (1-\theta)c'(q)q\big)$$

so, the seller's surplus is given by

$$\frac{\omega \frac{m}{n}}{1+\tau} - c = \frac{(1-\theta)c'}{\theta u' + (1-\theta)(1+\tau)c'} \big(u'q - (1+\tau)c(q)\big)$$

and the buyer's surplus is given by

$$u'q - \omega \frac{m}{n} = \frac{\theta u'}{\theta u' + (1-\theta)(1+\tau)c'} \big(u'q - (1+\tau)c(q)\big)$$

■

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