# Modelling and forecasting volatility of East Asian Newly Industrialized Countries and Japan stock markets with non-linear models

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**Abstract**

This paper explore the forecasting performance of several non-linear models, namely GARCH, EGARCH, APARCH used with three distributions, namely the Gaussian normal, the Student-t and Generalized Error Distribution (GED). In order to evaluate the performance of the competing models we used the standard loss functions that is the Root Mean Squared Error, Mean Absolute Error, Mean Absolute Percentage Error and the Theil Inequality Coefficient. Our result show that the asymmetric GARCH family models are generally the best for forecasting NICs indices. We also find that both Root Mean Squared Error and Mean Absolute Error forecast statistic measures tend to choose models that were estimated assuming the normal distribution, while the other two remaining forecast measures privilege models with t-student and GED distribution.

*Keywords*: GARCH; Volatility forecasting; forecast evaluation.
*JEL Classification*: C22, G15; G17.

## 1. Introduction

As pointed out by Poon and Granger (2003), financial market volatility is an important aspect in setting up strategies related to portfolio management, option pricing and market regulation in both developed and emerging stock markets.

Returns on financial markets are not independently distributed over time due to the presence of volatility clustering which occurs when large changes in these returns tend to be followed by large changes and small changes by small changes (Mandelbrot, 1963). Taking into account the time-varying behaviour of volatility, ARCH (Auto Regressive Conditional Heteroscedasticity) models developed by Engle (1982), which have been further developed into the GARCH (Generalized Auto Regressive Conditional Heteroscedasticity) models by Bollerslev (1986). A number of extensions of the basic GARCH model that are especially suited to estimating the conditional volatility of financial time series have been developed. An interesting feature of asset prices is that "bad" news seems to have a more pronounced effect on volatility than "goods" news. For many stocks, there is a strong negative correlation between the current return and the future volatility (De Gooijer and Hyndman, 2006). The tendency for volatility to decline when returns rise and to rise when return fall is often called the leverage effect. Among models that takes into account asymmetric and leverage effects, we have Exponential-GARCH (EGARCH) introduced by Nelson (1991), Threshold-GARCH (TARCH) introduced by Glosten et al. (1993) and Asymmetric-Power-ARCH (PARCH) introduced by Ding et al. (1993). It must point out that GARCH family models are not able to capture the tails property of high frequency time series. This problem can be solved by using non-normal distributions such as Student-t distribution and generalized error distribution (GED hereafter). This paper aims to compare modelling and forecasting the performance of

GARCH, EGARCH, GJR-GARCH and APARCH models and we also introduce different densities (Normal, Student-t and GED). Non-linear models been broadly used relatively to advanced stock markets. For instance Franses and van Dijk (1996) evaluated the out-of-sample forecasting performances of non-linear GARCH class models for five European stock market indices (i.e. Germany, Holland, Spain, Italy and Sweden). Kanas and Yannopoulos (2001) used a an artificial neural network (ANN) model to estimate the out-of-sample forecasts for two US stock indices (namely the Dow Jones and the Financial Times indices). Franses and Ghijsels (1996) used a modified version of GARCH models in order to avoid the effect of outliers in an out-of-sample forecasting exercise relatively to the Netherlands stock index.

Stock market volatility of East Asian Newly Industrialized Countries (NICs hereafter) is particularly interesting to model in these export oriented economies. International investor from developed countries who are not willing to invest in riskiest emerging countries may prefer to invest in NICs stock markets. So far stock market volatility in NICs markets has not been extensively explored so far.

Fong and Koh (2002) examined the existence of the asymmetric effects in the Hong Kong index by a Markow Switching EGARCH model. Their results show the presence of significant asymmetric effects in the periods during high volatility levels. Chan and Fung (2007) analyzed the predictability of the Hong Kong stock market volatility from 1999 to 2004. Their results show that GARCH models provide a satisfactory forecast. Liu and Morley (2009) explore the forecasting performance of several GARCH family models of the Hong Kong stock index from 2002 to 2007. By using different assumptions to the distribution of the conditional variance they found that GARCH models offers a better performance than historical averaging models.

Kim et al. (2005) examined the volatility of the Korean stock index before and after the 1997 Financial crisis. They found evidence that the Korean stock market volatility increased in the period after the crisis and that this effect seem to become a normal feature. Selcuk (2005) examined stock market volatility in a sample of emerging stock markets by using asymmetric stochastic volatility models. Using a dataset spanning from 1973 to 2000, they found that Korea and Taiwan stock markets have a very low leverage effect, while the same effect seem to be higher for Hong Kong and Singapore. One explanation of this different behaviour seem to be that Korea and Taiwan stock market are more domestic-oriented than Hong Kong and Singapore.[1]

Because of volatility can be used as a measurement of risk and stock market stability receives a great deal of concern from both investors and financial authorities (Yu, 2002), it is interesting to shed some light on these issues with a special focus with NICs stock markets given that there is no recent extensive research on these markets.

The paper is organized as follows. In section 2 we outline the methodologies employed in estimating models and for forecasting exercise. Section 3 describes the data. In section , we discuss the results finally Section 5 concludes.

## 2. Methodology

GARCH class model are one way to model and forecast the volatility of financial time series. Simple symmetric GARCH model provide a simple approximation of both modelling and forecasting. The GARCH model was introduced by Bollerslev (1986). In its simple form a GARCH (*q,p*) model involves the joint estimation of both conditional mean and a conditional variance equations. That is

$$r_t = c + \alpha r_{t-1} + \varepsilon_t \tag{1}$$

[1] Xing (2004) points out that stock market volatility differs across countries for several reasons such as market industry concentration, the relative size of the stock market and the number of firms listed in stock index.

$$\sigma_t^2 = c + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i}^{p} \beta_i \, \sigma_{t-i}^2 \tag{2}$$

where $r_t$ are returns at time $t$, while $c$ is a constant term, $\varepsilon_{t-i}^2$ are news about volatility from the previous period and $\sigma_{t-i}^2$ are last period forecast variance. The basic GARCH is symmetric and does not capture the asymmetry that characterize most of the financial time series and that it is known as the "leverage effect". This effect refers to the characteristic of time series on asset prices that "bad news" tends to increase volatility more than "good news". In order to capture the asymmetry shock to the conditional variance, Nelson (1991) proposed the exponential GARCH (EGARCH) model. In the EGARCH model the natural logarithm of the conditional variance is allowed to vary over time as a function of the lagged error terms rather that lagged squared errors. The EGARCH ($q,p$) model can be written as follows:

$$log(\sigma_t^2) = \omega + \sum_{j=1}^{q} \beta_j log\left(\sigma_{t-j}^2\right) + \sum_{i=1}^{p} \alpha_i \left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \tag{3}$$

The exponential nature of the EGARCH ensures that the conditional variance can never be negative even if it is permissible for the coefficients to be negative. Note that the presence of the leverage effects can be tested by the hypothesis that $\gamma_i < 0$, whereas the impact is asymmetric if $\gamma_i \neq 0$.

Ding et al (1993) proposed the Asymmetric Power ARCH (APARCH) model. That is:

$$\sigma_t^\delta = c + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^\delta + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^\delta \tag{4}$$

The asymmetric effects are present if $\gamma_i \neq 0$. Finally TARCH or Threshold GARCH were introduced by Glosten et al. (1993). The generalized specification for the conditional variance is given by:

$$\sigma_t^2 = \omega + \Sigma_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \Sigma_{i=1}^{p} \alpha_i \epsilon_{t-j}^2 + \Sigma_{k=1}^{r} \gamma_k \epsilon_{t-k}^2 D_{t-k}^- \tag{5}$$

Another common finding in the GARCH literature is the leptokurtosis of the empirical distribution of financial returns. From a theoretical point of view, very often in applying

GARCH model is assumed that the return series is conditionally normally distributed. With this assumption, GARCH model cannot explain fully the volatility clustering phenomenon that characterizes financial data (Thavaneswaran et al. 2005). In order to overcome this drawback and modelling such fat-tailed distributions researchers have adopted the Student's *t* or the Generalized Error Distribution (GED). Therefore, in addition to the classical gaussian assumption, we suppose that errors $\varepsilon_t$ are assumed to be distributed according to a Student's *t* or a GED distribution. In the first case, the probability density function (pdf) of $\varepsilon_t$ is:

$$f(\varepsilon_t) = \frac{\Gamma\left((v+1)/2\right)}{\Gamma(v/2)\sqrt{(v-2)\pi}}\left(1+\frac{\varepsilon_t^2}{v-2}\right)^{-(v+1)/2} \qquad (6)$$

where $v > 2$, $\Gamma(.)$ is the Gamma function and $v$ is the degree-of-freedom parameter. On the other side if $\varepsilon_t$ assumes a GED distribution, the pdf is as follows:

$$f(\varepsilon_t) = \frac{v\, exp\left(-\frac{1}{2}\left|\varepsilon_t/\lambda^v\right|\right)}{\lambda 2^{(1+1/v)}\Gamma(1/v)} \qquad (7)$$

with $-\infty < x < \infty$ , $0 < v \leq \infty$ and $\lambda = \left[2^{(-2/v)}\Gamma(1/v)/\Gamma(3/v)\right]^{1/2}$.

We also divided our data into two subsamples. The first subsample is used to build a nonlinear model and the second subsample is used to evaluate the forecasting performance of the model. We refer to the two subsamples of data as estimation and forecasting subsamples.

The forecasting performance of the different models is evaluated using a number of different measures. They are the Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and the Theil Inequality Coefficient (TIC). Suppose the forecast sample is j = T+1,T+2,…,T+h, and denote the actual and forecasted value in period *t* as $y_t$ and $\hat{y}_t$, respectively. For *h*-step-ahead forecasts, these measures are defined as:

$$RMSE(h) = \sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h} \qquad (8)$$

$$MAE(h) = \sum_{t=T+1}^{T+h} |\hat{y}_t - y_t| - h \qquad (9)$$

$$MAPE(h) = 100 \sum_{t=T+1}^{T+h} \left| \frac{\hat{y}_t - y_t}{y_t} \right| / h \qquad (10)$$

$$TIC(h) = \frac{\sqrt{\sum_{t=T+1}^{T+h} (\hat{y}_t - y_t)^2 / h}}{\sqrt{\sum_{t=T+1}^{T+h} \hat{y}_t^2 / h} + \sqrt{\sum_{t=T+1}^{T+h} y_t^2 / h}} \qquad (11)$$

The smaller is the error in the first three forecast error statistics, the better the forecasting ability of that model according to that criterion. The TIC is a scale invariant measure that always lies between zero and one, where zero indicates a perfect fit. Because of the best model is the one with the smallest forecast value of that measure, we rank the forecasting ability of the estimated GARCH family models by ranking the magnitudes of the forecast errors.

**3. Data**

The data used in this paper are closing daily prices for the stock markets in Hong Kong (Hang Seng), Japan (Nikkei 225), South Korea (Kospi), Singapore (STI), and Taiwan (TSE). All data come from *Thomson Financial Datastream* and span 10 years from 4 January 1999 to 17 June 2009. Figure 1 presents the patterns of price series for the period under review. From 2003 to 2007 all market experienced an upward trend which turn to a downward trend in correspondence with the credit crunch. Signals of recovering are evident from the late 2008.[2] We convert the index data to returns as follows $r_t = ln(p_t/p_{t-1})$, where $r_t$ is the daily return whereas $p_t$ denotes the value of the index in local currency on day *t*. In Figure 2 we present graph for Asian markets return series: visual

[2] As pointed out by Barkoulas and Travlos (2008) the dynamics of prices of less developed stock markets is associated with the developments occurred in developed economies but also expectations about political situation in less developed economies may play a role in explaining stock market movements in these less developed economies.

inspection show that volatility changes over time and it tends to cluster with periods with low volatility and periods with high volatility, after august 2007 volatility seems to increase in all markets. Table 1 provides some descriptive statistics. Mean equity market returns are positive for all the sample indices with the exception of Japan for which mean returns are negative. The highest daily return of 0.134 was reported for the Japanese stock market while the lowest daily return of -0.135 was earned by the Singapore stock market. The kurtosis values of all market indices are much higher than three indicating that the return distributions in all the markets are fat-tailed. The skewness values are negative in all markets indicating that the asymmetric tail extends more towards negative values than positive ones. The Barque-Bera statistics clearly rejects the null hypothesis of a normal distribution for all markets series returns. This last characteristics can be seen in figure 3: quantiles of stock market returns do not lie along the straight line that represents normal distribution quantiles.

In order to cayy on the forecasting exercise, we divided the sample in two parts. The first 2850 observations (from January 1, 1999 to July 17, 2009) are used as the in-sample for estimation purposes, while the remaining 65 observations (from July 18, 2009 to October 16, 2009) are taken as the out-of-sample for forecast evaluation purposes.

## 4. Empirical results

In this section we present some within-sample estimation results to give an idea of possible usefulness of (non-linear) GARCH models. We estimate the models using 2750 (smpl 01/04/1999 to 07/17/2009) observations and saving the last 65 observations, respectively for out-of-sample forecasting comparisons between models. To evaluate the performance of the non-linear models in describing the stock indices volatility, we compare the out-of-sample forecasts. The post-sample forecast comparisons are carried out as follows. First, we reserve the last 65 observations for forecast comparison. Secondly, all the models used in forecasting are estimated using the first 2750 observations. Such a scheme provides 65 one-step ahead forecasts. We summarize the forecast performance by considering the error statistics defined previously.

In table 2, 3, 4, 5 and 6, we report some estimation results for the GARCH family models. The possible usefulness of non-linear modifications to the linear GARCH models seems to be confirmed by the Log likelihood values, although both the AIC and SC do not suggest a clear favourite. At the same time use of asymmetric GARCH models is further justified by the consideration that all asymmetric coefficients are significant at standard levels. As is typical of GARCH model estimates for financial asset returns data, the sum of the coefficients on the lagged squared error and the lagged conditional variance is closed to unity, this implies that shocks to the conditional variance will be highly persistent indicating that large changes and small changes tend to be followed by small changes, this means that volatility clustering is observed in all financial returns. Another reason relatively to the high volatility persistence may be due to the presence of structural break in the variance equation as pointed out by Lamoureux and Lastrapes (1990).

For all the models, the dynamics of the first two moments of the series are tested with the Box-Pierce statistics at lag 36, (i.e. Q 36) of the standardized residuals which do not reject the null hypothesis that there is no correlation up to 36 lags. Also Lagrange multiplier test (LM hereafter) show that generally standardized residuals do not exhibit additional ARCH effects. Both Box-Pierce and LM indicate that we have successfully removed the conditional heteroskedasticity.

The forecasting ability of GARCH class models was evaluated with four different measures and is reported by ranking the models with respect to these measures. As pointed out by Brailsford and Faff (1996), the various model rankings are sensitive to the error statistic used to assess the accuracy of the forecasts.

For the Nikkei 225 stock index (table 7), the RMSE and MAE measures indicate that GARCH models gives the better forecast while the asymmetric models give the poorest forecasts. On the other side we found that MAPE and TIC measures indicate the asymmetric GARCH as better forecast models. With these results we were unable to draw a general conclusion about the best model to forecast Nikkei 225 index returns.

For the Hang Seng stock market index (table 8), the GED distribution is the most successful in forecasting the Hang Seng conditional variance. MAE, MAPE and TIC indicate that asymmetric models outperform the GARCH models which provide less satisfactory results with poorest forecasts. From these results we may infer that asymmetric GARCH models are massively preferred for forecasting aims compared to GARCH models

For the STI stock index (table 9), the normal distribution is the best one in forecasting STI conditional variance given that 3 out of 4 error statistic measures indicate that model using this distribution forecast volatility better than model using the other two distribution. It must also be point out that RMSE, MAE and TIC statistics indicate that symmetric GARCH models are better in forecasting volatility while MAPE statistic indicate an asymmetric model as the best one. The main conclusion coming from STI index forecasting techniques is that STI returns can be usefully forecast with symmetric GARCH models.

For the Kospi stock index (table 10), the RMSE statistic indicates that the symmetric GARCH model provides the most accurate forecasts with all distributions, while the GJR-GARCH model with the GED distribution is the worst performing model. MAE, MAPE and TIC statistics indicate that asymmetric GARCH models give better forecasts than the GARCH models.

For the TWSE stock index (table 11), symmetric GARCH models generate lower RMSEs and MAEs than asymmetric models. Further results indicate that asymmetric models are superior to GARCH models in forecasting Taiwan stock market volatility with model selections based on the MAPEs and TICs error statistics. These results imply that between GARCH and asymmetric GARCH models there is no clear favourite for forecasting aims.

Why did we find different results in our forecasting exercises? In other words what are the reason that lead to different indication about the best models for forecasting aims. As pointed out by Engle and Patton (2001) as well as Andersen and Bollerslev (1998), when using GARCH class model empirical results may depend on the sampling frequency. These authors suggest using intraday data but one of the main obstacle in doing so is that additional cost and complications in model constructing seem to be the main barrier to use intraday returns for forecasting.

Chen (1997) argues that forecast index volatility also depends on time periods volatility, in other word stock index characterized by more volatile time periods can be best forecasted by a kind of model rather than another. This may explain while some indices can be forecasted with symmetric GARCH models while asymmetric models are more suitable for other indices. It could be interesting to evaluate whether changing the sampling frequency, results are somewhat the same or changes massively for NICs stock indices. For all stock index we also found that symmetric models often perform better than asymmetric model in forecasting volatility. These result are not surprising given that Ballie and Bollerslev (1989), Hsieh (1989) and McCurdy and Morgan (1988) find that symmetric GARCH model often provide a good approximation in modelling financial time series.

To summarize, our results indicate that several different models can be used in forecasting stock market volatility of NICs stock markets although simple symmetric GARCH models can be usefully used in this task.

## 5. Conclusions

This study examine the ability of non linear models in an out-of-sample forecasting for daily return volatility of NICs stock markets. This work cover the period from 1999 to 2009. The forecasting models that were considered in this study ranged from the relative simple GARCH models to relatively complex GARCH models (including EGARCH, APARCH and GJR-GARCH models). Our result show that GARCH models with the normal distribution may be still useful used for forecasting purposes rather than more sophisticated asymmetric models.

**References**

Andersen T.G., Bollerslev T. (1998), Answering the Skeptics: Yes, Standard Volatility Models do Provide Accurate Forecasts, *International Economic Review*, 39(4), 885-905.

Baillie R.T., Bollerslev T. (1989). The message in Daily Exchange Rates: A Conditional Variance Tale, Journal of Business and Economic Statistics, 7, 297-305.

Barkoulas J., Travlos N. (1998). Chaos in an emerging capital market? The case of the Athens Stock Exchange, *Applied Financial Economics*, 8, 231-243

Bollerslev, Tim, (1986), Generalized Autoregressive Conditional Heteroscedasticity, *Journal of Econometrics*, 31, 307-327.

Brailsford T.J., Faff R.W. (1996), An evaluation of volatility forecasting techniques, *Journal of Banking and Finance*, 20, 419-438.

Chan H., Fung D. (2007). Forecasting volatility of Hang Seng Index and its application on reserving for investment guarantees. Working paper- The Actuarial Society of Hong Kong.

Chen A.S., (1997), Forecasting the S&P 500 Index Volatility, *International Review of Economics and Finance*, 6(4), 391-404.

De Gooijer J-G-, Hyndman R.J., (2006), 25 years of time series forecasting, *International Journal of Forecasting*, 22, 443-473.

Ding, Z., Granger, C. and Engle, R. (1993). A long memory property of stock returns and a new model, *Journal of Empirical Finance*, 1, 83-106.

Engle, R.F., (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica*, 50, 987-1007.

Engle R.F., Patton A.J. (2001). What good is a volatility model? *Quantitative Finance*, 1(2), 237-245

Fong W.M., Koh S.K. (2002). The political economy of volatility dynamics in the Hong kong stock market. *Asia-Pacific Financial Markets*, 9, 259-282.

Franses P.H., Van Dijk D. (1996). Forecasting Stock Market Volatility Using (Non-Linear) Garch Models, *Journal of Forecasting*, 15, 229-235.

Franses P.H., Ghijsels H. (1999). Additive outliers, GARCH and forecasting volatility, International Journal of Forecasting, 15, 1-9.

Glosten, L.R., Jaganathan, R., Runkle D. (1993). On the relation between the Expected Value and the Volatility of the Normal Excess Return on Stocks, *Journal of Finance*, 48, 1779-1801.

Hsieh D.A. (1989), Modelling Heteroskedsaticity in Daily Foreign Exchange Rates, Journal of Business and Economic Statistics, 7, 307-317.

Kanas A., Yannopoulos A. (2001), Comparing linear and nonlinear forecasts for stock returns, *International Review of Economics and Finance*, 10, 383-398.

Kim J., Kartsaklas A., Karanasos M. (2005). The volume-volatility relationship and the opening of the Korean stock market to foreign investors after the financial turmoil in 1997, *Asia-Pacific Financial Markets*, 12, 245-271.

Lamoureux C., Lastrapes W. (1990). Persistence in Varaince Structural Cahnge, and the GARCH Model, *Journal of Business and Economic Statistics*, 8, 225-234.

Liu W., Morley B. (2009), Volatility Forecasting in the Hang Seng Index using the GARCH Approach, *Asia-Pacific Financial Markets*, 16, 51-63.

Mandelbrot, B. (1963). The variation of Certain Speculative Prices, *Journal of Business*, 36, 394-419.

McCurdy T., Morgan I. (1988), testing the Martingale Hypothesis in Deutschemark Futures with Models Specifying the form of Heteroskedasticity, *Journal of Applied Econometrics*, 3, 187-202.

Nelson, D. (1991), Conditional heteroskedasticity in asset returns: a new approach, *Econometrica*, 59, 349-370.

Poon S.H., Granger C. (2003), Forecasting volatility in financial markets: a review, *Journal of Economic Literature*, XLI, 478-539

Selcuk F. (2005). Asymmetric Stochastic Volatility in Emerging Stock Markets, forthcoming in *Applied Financial Economics*.

Thavaneswaran A., Appadoo S.S., Peiris S. (2005). Forecasting volatility, *Statistics & Probability Letters*, 75, 1-10.

Xing X. (2004). Why does stock market volatility differ across countries? Evidence from thirty seven international markets, *International Journal of business*, 9(1), 84-102.

Yu J. (2002), Forecasting volatility in the New Zealand stock market, *Applied Financial Economics*, 13, 193-202.

# Appendix

Table 1 – Summary statistics of data on returns

| | Hang Seng | Nikkei 225 | STI | Kospi | TSEW |
|---|---|---|---|---|---|
| Sample size | 2814 | 2814 | 2643 | 2814 | 2814 |
| Mean | 0.000286 | -9.54e-05 | 8.11e-05 | 0.000365 | 6.54e-05 |
| Minimum | -0.135820 | -0.121110 | -0.086960 | -0.128047 | -0.099360 |
| Maximum | 0.134068 | 0.132346 | 0.075305 | 0.112844 | 0.085198 |
| St. Dev. | 0.016709 | 0.015668 | 0.013438 | 0.018945 | 0.016163 |
| Skewness | -0.014823 | -0.306255 | -0.233510 | -0.429812 | -0.117786 |
| Kurtosis | 10.46626 | 9.773552 | 7.284275 | 7.004575 | 5.586180 |
| Jarque-Bera test | 6536.213<br>(0.00) | 5423.537<br>(0.00) | 2045.365<br>(0.00) | 1966.936<br>(0.00) | 790.713<br>(0.00) |

Notes. The numbers in parentheses are p-values.

Figure 1 – Price series for Asian stock markets

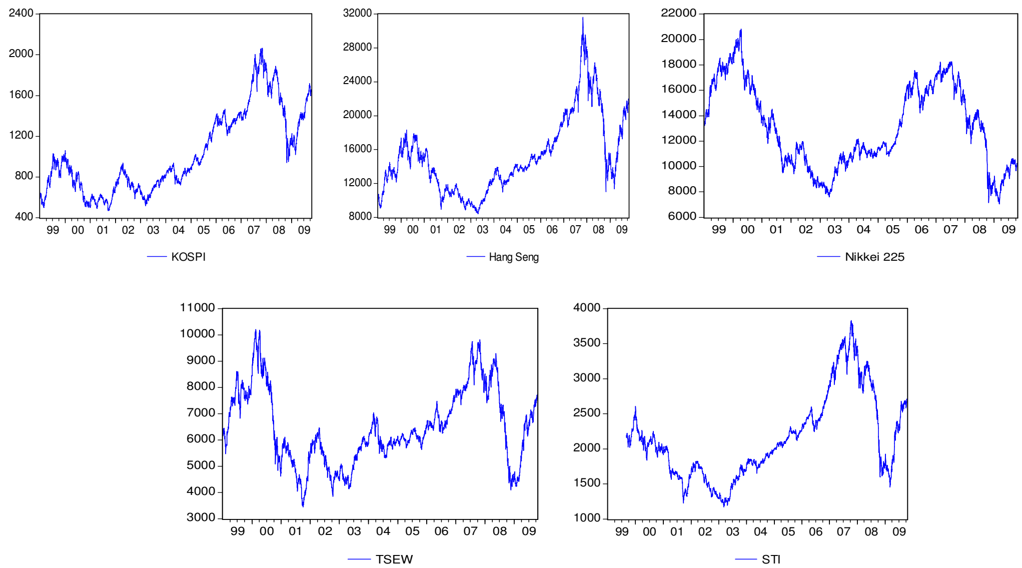


Figure 2 – Returns series for Asian stock markets

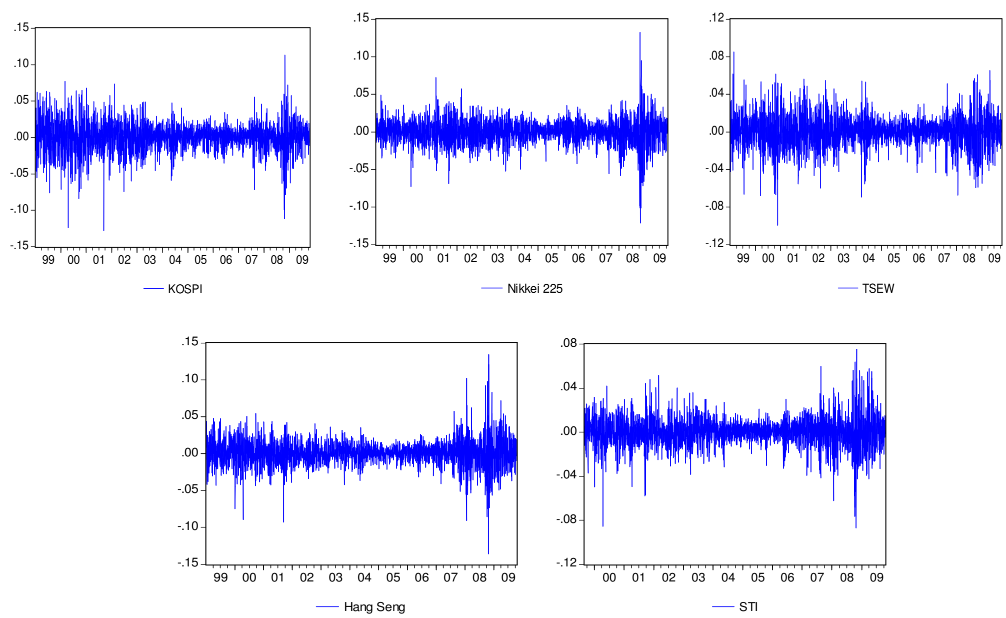

Figure 3: Q-Q normal plot about return normal distribution of each index

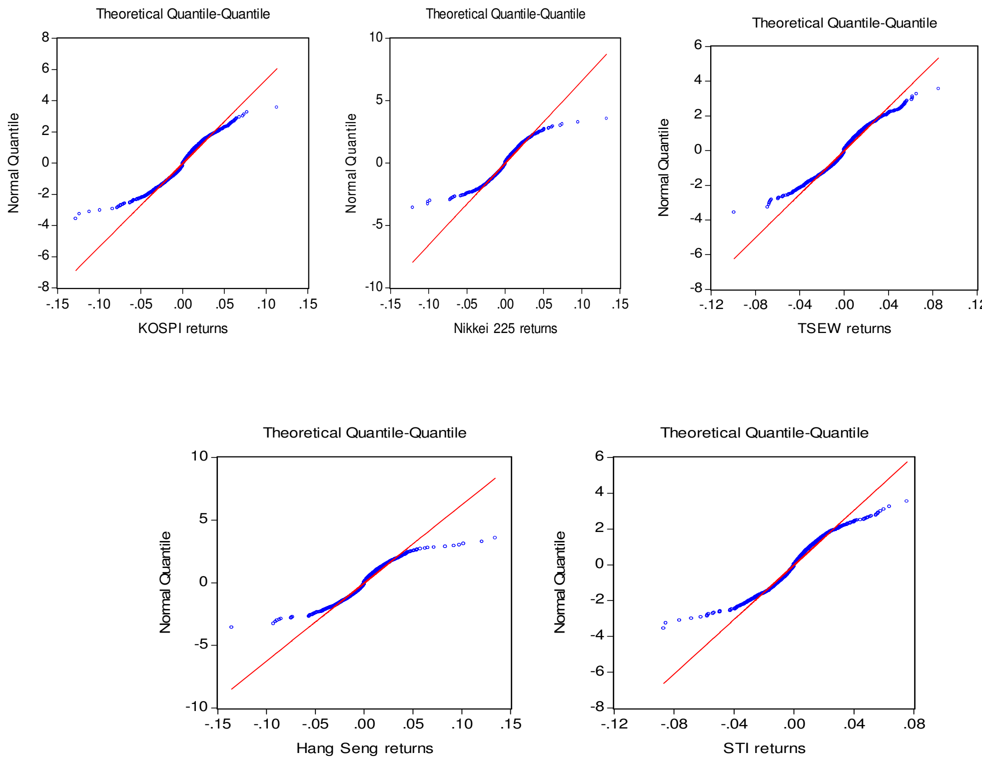

Table 2 – Non linear models estimates and out-of-sample comparisons for the volatility of the Nikkei 225 stock market returns.

| | GARCH(1,1) Normal distribution | EGARCH(2,1) Normal distribution | APARCH(2,1) Normal distribution | GJR-GARCH Normal distribution |
|---|---|---|---|---|
| c | 0.0004* | -0.0002 | 4.18e-05 | 9.70E-05 |
| | (0.0002) | (0.0002) | (0.0002) | (0.0002) |
| α | -0.045*** | -0.004 | -0.04** | -0.05*** |
| | (0.017) | (0.0117) | (0.017) | (0.017) |
| ω | 2.89E-06*** | -0.399*** | 4.66e-05 | 3.92E-06*** |
| | (6.34E-07) | (0.038) | (3.16E-05) | (6.41E-07) |
| $\alpha_1$ | 0.0867*** | -0.071** | 0.035 | 0.034*** |
| | (0.008) | (0.0310) | (2.97) | (0.007) |
| $\alpha_2$ | - | 0.244*** | 0.053** | - |
| | | (0.030) | (0.02) | |
| β | 0.903*** | 0.968*** | 0.891*** | 0.899*** |
| | (0.0087) | (0.003) | (0.01) | (0.009) |
| γ | - | -0.087*** | 0.999 | 0.098*** |
| | | (0.008) | (113.23) | (0.013) |
| Q(36) | 24.821 | 20.904 | 24.500 | 27.641 |
| | [0.920] | [0.979] | (0.927) | (0840) |
| LM(36) | 39.847 | 30.454 | 33.001 | 36.190 |
| | [0.302] | [0.729] | (0.611) | (0.459) |
| AIC | -5.745 | -5.769 | -5.767 | -5.761 |
| SC | -5.734 | -5.754 | -5.750 | -5.748 |
| Log likelihood | 7902.135 | 7937.843 | 7935.417 | 7925.168 |
| RMSE | 0.013237 | 0.013328 | 0.013271 | 0.013261 |
| MAE | 0.010150 | 0.010347 | 0.010230 | 0.010205 |
| MAPE | 92.61459 | 92.39440 | 89.698 | 89.31126 |
| TIC | 0.945300 | 0.982260 | 0.957701 | 0.945389 |
| | GARCH(1,1) t distributions | EGARCH(2,1) t distributions | APARCH(1,1) t distributions* | GJR-GARCH t distributions |
| c | 0.0004 | 8.69E-05 | 0.00017 | 0.0002 |
| | (0.0002) | (0.0002) | (0.0002) | (0.0002) |
| α | -0.0128 | -0.0024 | -0.021 | -0.02 |
| | (0.018) | (0.0171) | (0.017) | (0.018) |
| ω | 1.92E-06*** | -0.347*** | 4.34e-05 | 2.94E-06*** |
| | (6.68E-07) | (0.0466) | (4.33E-05) | (7.39E-07) |
| $\alpha_1$ | 0.0718*** | -0.122*** | 0.0713*** | 0.027*** |
| | (0.0094) | (0.0443) | (0.011) | (0.009) |
| $\alpha_2$ | - | 0.284*** | - | - |
| | | (0.044) | | |
| β | 0.922*** | 0.973*** | 0.920*** | 0.914*** |
| | (0.009) | (0.0048) | (0.009) | (0.010) |
| γ | - | -0.0871*** | 0.486*** | 0.088*** |
| | | (0.0117) | (0.09) | (0.016) |
| Q(36) | 20.347 | 20.801 | 22.03 | 21.969 |
| | (0.983) | (0.980) | (0.967) | (0.968) |
| LM(36) | 46.901 | 32.510 | 45.295 | 38.417 |
| | (0.105) | (0.635) | (0.137) | (0.360) |
| AIC | -5.722 | -5.793 | -5.783 | -5.782 |
| SC | -5.759 | -5.776 | -5.766 | -5.767 |
| Loglikelihood | 7940.043 | 7970.905 | 7958.021 | 7955.635 |
| RMSE | 0.013257 | 0.013295 | 0.013270 | 0.013266 |
| MAE | 0.010192 | 0.010261 | 0.010220 | 0.010212 |
| MAPE | 95.10989 | 92.78961 | 92.22754 | 92.59386 |
| TIC | 0.964149 | 0.992481 | 0.973610 | 0.971509 |
| | GARCH(1,1) GED distributions | EGARCH(1,1) GED distributions | APARCH(2,1) - GED distributions | GJR-GARCH GED distribution |
| c | 0.00016 | 6.20E-06 | 6.96e-05 | 9.08e-05 |
| | (0.0002) | (0.0002) | (0.0002) | (0.0002) |
| α | -0.0095 | -0.0136 | -0.01 | -0.013 |
| | (0.0166) | (0.0166) | (0.016) | (0.0168) |
| ω | 2.29E-06*** | -0.290*** | 6.19E-05 | 3.30e-06*** |
| | (8.45E-07) | (0.0478) | (6.52E-05) | (9.07e-07) |
| $\alpha_1$ | 0.0762*** | 0.145*** | 0.036 | 0.028** |
| | (0.0112) | (0.0198) | (0.077) | (0.011) |
| $\alpha_2$ | - | - | 0.051* | - |
| | | | (0.031) | |
| β | 0.917*** | 0.978*** | 0.9*** | 0.911*** |
| | (0.012) | (0.004) | (0.014) | (0.012) |
| γ | - | -0.0788*** | 0.978 | 0.093*** |
| | | (0.0129) | (2.628) | (0.0187) |
| Q(36) | 20.209 | 21.146 | 20.623 | 21.282 |
| | (0.984) | (0.977) | (0.981) | (0.976) |
| LM(36) | 45.859 | 50.534 | 35.586 | 39.111 |
| | (0.125) | (0.054) | (0.488) | (0.331) |
| AIC | -5.787 | -5.798 | -5.802 | -5.797 |
| SC | -5.774 | -5.783 | -5.782 | -5.782 |
| Loglikelihood | 7961.162 | 7976.617 | 7984.235 | 7975.687 |
| RMSE | 0.013281 | 0.013287 | 0.013277 | 0.013285 |
| MAE | 0.010236 | 0.010251 | 0.010235 | 0.010246 |
| MAPE | 93.02791 | 91.80493 | 91.70377 | 92.09659 |
| TIC | 0.983332 | 0.984891 | 0.976684 | 0.984515 |

*Convergence not achieved after 500 iterations.

Table 3 – Non linear models estimates and out-of-sample comparisons for the volatility of the Hang Seng stock market returns.

| | GARCH(1,1) | EGARCH(1,1) | APARCH(1,1) | GJR-GARCH(1,1) |
|---|---|---|---|---|
| c | 0.0005** | 0.0003 | 0.00035 | 0.0003 |
| | (0.0002) | (0.0002) | (0.0002) | (0.0002) |
| α | 0.015 | 0.0231 | 0.0189 | 0.0113 |
| | (0.017) | (0.016) | (0.0171) | (0.017) |
| ω | 1.05e-06*** | -0.180*** | 5.37e-05 | 1.46E-06*** |
| | (2.94e-07) | (0.0217) | (4.53e-05) | (3.04e-07) |
| $\alpha_1$ | 0.06*** | 0.122*** | 0.064*** | 0.029*** |
| | (0.005) | (0.011) | (0.006) | (0.006) |
| $\alpha_2$ | - | - | - | - |
| β | 0.936*** | 0.989*** | 0.940*** | 0.935*** |
| | (0.005) | (0.002) | (0.005) | (0.006) |
| γ | - | -0.045*** | 0.354*** | 0.056*** |
| | | (0.006) | (0.059) | (0.012) |
| Q(36) | 44.905 | 41.725 | 41.844 | 43.285 |
| | [0.147] | (0.236) | (0.232) | (0.188) |
| LM(36) | 32.639 | 43.544 | 44.430 | 42.604 |
| | [0.629] | (0.181) | (0.158) | (0.208) |
| AIC | -5.735 | -5.749 | -5.748 | -5.745 |
| SC | -5.724 | -5.736 | -5.733 | -5.732 |
| Loglikelihood | 7887.975 | 7908.172 | 7908.249 | 7903.374 |
| RMSE | 0.016499 | 0.016545 | 0.016533 | 0.016516 |
| MAE | 0.013632 | 0.013677 | 0.013666 | 0.013650 |
| MAPE | 115.0845 | 99.67824 | 102.6582 | 106.7653 |
| TIC | 0.960983 | 0.969425 | 0.971054 | 0.974722 |
| | GARCH(1,1) t distributions | EGARCH(1,1) t distributions | APARCH(1,1) t distributions* | GJR-GARCH(1,1) t distributions |
| c | 0.0005*** | 0.0004** | 0.0004** | 0.0004** |
| | (0.0002) | (0.0002) | (0.0002) | (0.0002) |
| α | 0.024 | 0.028 | 0.026 | 0.022 |
| | (0.0181) | (0.0181) | (0.018) | (0.018) |
| ω | 6.96E-07** | -0.150*** | 4.74e-05 | 1.01e-06 |
| | (3.27E-07) | (0.0249) | (5.45E-05) | (3.49E-07) |
| $\alpha_1$ | 0.0512*** | 0.113*** | 0.0589*** | 0.023*** |
| | (0.007) | (0.0151) | (0.008) | (0.008) |
| $\alpha_2$ | - | - | - | - |
| β | 0.948 | 0.992*** | 0.947*** | 0.945*** |
| | (0.007) | (0.002) | (0.007) | (0.007) |
| γ | - | -0.049*** | 0.422*** | 0.065*** |
| | | (0.009) | (0.098) | (0.013) |
| Q(36) | 45.120 | 41.335 | 41.247 | 46.377 |
| | (0.142) | (0.249) | (0.252) | (0.115) |
| LM(36) | 34.614 | 45.830 | 46.406 | 42.718 |
| | (0.534) | (0.126) | (0.114) | (0.204) |
| AIC | -5.771 | -5.780 | -5.779 | -5.778 |
| SC | -5.758 | -5.765 | -5.762 | -5.763 |
| loglikelihood | 7938.338 | 7952.542 | 7952.243 | 7948.975 |
| RMSE | 0.016522 | 0.016543 | 0.016539 | 0.016449 |
| MAE | 0.013646 | 0.013669 | 0.013666 | 0.013410 |
| MAPE | 108.8903 | 102.0694 | 102.8276 | 102.3247 |
| TIC | 0.957240 | 0.960284 | 0.961439 | 0.961726 |
| | GARCH(1,1) GED distributions | EGARCH(2,1) GED distributions | APARCH(2,1) GED distributions | GJR-GARCH(1,1) GED distributions |
| c | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| | (0.00018) | (0.00019) | (0.00019) | (0.0001) |
| α | 0.0121 | 0.015 | 0.014 | 0.009 |
| | (0.0157) | (0.0165) | (0.016) | (0.016) |
| ω | 7.09e-07* | -0.183*** | 9.54e-05 | 1.10E-06 |
| | (3.94e-07) | (0.0323) | (0.0001) | (4.21E-07)*** |
| $\alpha_1$ | 0.053*** | -0.0874* | 0.025 | 0.025** |
| | (0.0086) | (0.045) | (0.065) | (0.011) |
| $\alpha_2$ | - | 0.219*** | 0.045 | - |
| | | (0.0482) | (0.029) | |
| β | 0.946 | 0.989*** | 0.937*** | 0.943*** |
| | (0.008) | (0.002) | (0.01) | (0.009) |
| γ | - | -0.054*** | 0.999 | 0.057*** |
| | | (0.01) | (3.857) | (0.013) |
| Q(36) | 45.344 | 42.598 | 42.284 | 43.555 |
| | (0.137) | (0.208) | (0.218) | (0.181) |
| LM(36) | 34.217 | 39.086 | 36.954 | 43.072 |
| | (0.553) | (0.332) | (0.424) | (0.194) |
| AIC | -5.795 | -5.809 | -5.806 | -5.801 |
| SC | -5.782 | -5.792 | -5.786 | -5.786 |
| Loglikelihood | 7971.981 | 7993.645 | 7989.58 | 7981.52 |
| RMSE | 0.016526 | 0.016540 | 0.016536 | 0.016530 |
| MAE | 0.013662 | 0.013676 | 0.013672 | 0.013667 |
| MAPE | 103.3463 | 99.50209 | 100.3890 | 101.8057 |
| TIC | 0.978328 | 0.979082 | 0.978811 | 0.983149 |

*Convergence not achieved after 500 iterations.

Table 4 – Non linear models estimates and out-of-sample comparisons for the volatility of the STI stock market returns.

| | GARCH(1,1) | EGARCH(2,1) | APARCH(2,1) | GJR-GARCH(1,1) |
|---|---|---|---|---|
| c | 0.0005** | 0.0003 | 0.0002 | 0.0002 |
| | (0.0001) | (0.0002) | (0.0002) | (0.0002) |
| $\alpha$ | 0.00142 | 0.0315** | 0.011 | -0.003 |
| | (0.0165) | (0.016) | (0.017) | (0.0168) |
| $\omega$ | 1.42E-06*** | -0.3*** | 1.06E-05 | 1.65e-06*** |
| | (3.50E-07) | (0.028) | (1.09e-05) | (3.39e-07) |
| $\alpha_1$ | 0.0994*** | 0.06** | 0.036 | 0.051*** |
| | (0.0079) | (0.028) | (0.0216) | (0.0098) |
| $\alpha_2$ | - | 0.125*** | 0.058*** | - |
| | | (0.029) | (0.017) | |
| $\beta$ | 0.898*** | 0.982*** | 0.899*** | 0.902*** |
| | (0.007) | (0.002) | (0.008) | (0.007) |
| $\gamma$ | - | -0.065*** | 0.703 | 0.0806*** |
| | | (0.009) | (0.439) | (0.0131) |
| Q(36) | 49.445 | 48.151 | 49.343 | 49.651 |
| | (0.067) | (0.08) | (0.068) | (0.065) |
| LM(20) | 39.915 | 40.357 | 38.418 | 39.505 |
| | (0.301) | (0.283) | (0.360) | (0.316) |
| AIC | -6.065 | -6.076 | -6.076 | -6.075 |
| SC | -6.054 | -6.060 | -6.058 | -6.061 |
| Loglikelihood | 7826.45 | 7842.45 | 7843.69 | 7840.01 |
| RMSE | 0.011671 | 0.011708 | 0.0117 | 0.011703 |
| MAE | 0.009362 | 0.009447 | 0.009387 | 0.009416 |
| MAPE | 104.3299 | 98.53686 | 97.31510 | 99.25508 |
| TIC | 0.953288 | 0.955710 | 0.970431 | 0.974058 |
| | GARCH(2,1) t distributions | EGARCH(1,1) t distributions | APARCH(1,1) t distributions | GJR-GARCH(1,1) t distributions |
| C | 0.00052*** | 0.000379** | 0.0003* | 0.00036** |
| | (0.00018) | (0.00019) | (0.00018) | (0.00019) |
| $\alpha$ | 0.0152 | 0.026 | 0.014 | 0.0098 |
| | (0.0176) | (0.017682) | (0.0178) | (0.017) |
| $\omega$ | 1.89e-06*** | -0.2705*** | 1.46e-05 | 1.85e-06*** |
| | (6.04e-07) | (0.04273) | (2.03e-05) | (5.24e-07) |
| $\alpha_1$ | 0.04144* | 0.164*** | 0.087*** | 0.049*** |
| | (0.0231) | (0.0203) | (0.013) | (0.0133) |
| $\alpha_2$ | 0.0622** | - | - | - |
| | (0.0256) | | | |
| $\beta$ | 0.891*** | 0.983*** | 0.911*** | 0.904*** |
| | (0.0130) | (0.004) | (0.0122) | (0.0115) |
| $\gamma$ | - | -0.06511*** | 0.297*** | 0.0757*** |
| | | (0.0124) | (0.078) | (0.0182) |
| Q(36) | 49.852 | 46.720 | 47.293 | 47.906 |
| | (0.062) | (0.109) | (0.099) | (0.089) |
| LM(36) | 36.463 | 44.662 | 42.478 | 40.224 |
| | (0.447) | (0.152) | (0.211) | (0.288) |
| AIC | -6.103 | -6.108 | -6.108 | -6.108 |
| SC | -6.087 | -6.092 | -6.090 | -6.092 |
| Loglikelihood | 7876.88 | 7883.52 | 7884.72 | 7883.67 |
| RMSE | 0.011674 | 0.011695 | 0.01169 | 0.011692 |
| MAE | 0.009408 | 0.009432 | 0.009382 | 0.009416 |
| MAPE | 101.3552 | 98.83255 | 98.51928 | 98.53398 |
| TIC | 0.947301 | 0.953829 | 0.961599 | 0.962756 |
| | GARCH(1,1) GED distributions | EGARCH(1,1) GED distributions | APARCH(1,1) GED distributions | GJR-GARCH(1,1) GED distributions |
| C | 0.0004** | 0.0003* | 0.0002* | 0.00029 |
| | (0.0001) | (0.0001) | (0.00018) | (0.0001) |
| $\alpha$ | 0.012 | 0.0212 | 0.009 | 0.0065 |
| | (0.017) | (0.0169) | (0.017) | (0.0172) |
| $\omega$ | 1.81e-06*** | -0.273*** | 0.00018*** | 1.82e-06*** |
| | (6.24e-07) | (0.043) | (4.74e-05) | (5.38e-07) |
| $\alpha_1$ | 0.0442* | 0.165*** | 0.087*** | 0.0489*** |
| | (0.024) | (0.021) | (0.014) | (0.014) |
| $\alpha_2$ | 0.0604** | - | - | - |
| | (0.0268) | | | |
| $\beta$ | 0.890*** | 0.983*** | 0.909*** | 0.904*** |
| | (0.0132) | (0.0041) | (0.011) | (0.0116) |
| $\gamma$ | - | -0.0649*** | 0.280*** | 0.078*** |
| | | (0.0130) | (0.078) | (0.019) |
| Q(36) | 50.163 | 47.170 | 48.025 | 48.381 |
| | (0.059) | (0.101) | (0.087) | (0.081) |
| LM(36) | 36.067 | 44.047 | 41.37 | 39.881 |
| | (0.465) | (0.167) | (0.247) | (0.30154) |
| AIC | -6.107 | -6.112 | -6.113 | -6.113 |
| SC | -6.093 | -6.096 | -6.095 | -6.097 |
| Loglikelihood | 7881.24 | 7889.31 | 7891.1 | 7890.53 |
| RMSE | 0.011682 | 0.0117 | 0.011697 | 0.011696 |
| MAE | 0.009372 | 0.009396 | 0.009382 | 0.009379 |
| MAPE | 99.47394 | 97.63679 | 97.59994 | 97.63206 |
| TIC | 0.956756 | 0.962817 | 0.969918 | 0.970759 |

Table 5 – Non linear models estimates and out-of-sample comparisons for the volatility of the KOSPI stock market returns.

| | GARCH(1,1) | EGARCH(1,1) | APARCH(2,1) | GJR-GARCH(1,1) |
|---|---|---|---|---|
| C | 0.0009*** | 0.00076*** | 0.00065** | 0.0006** |
| | (0.0002) | (0.00029) | (0.00029) | (0.0002) |
| $\alpha$ | -0.0084 | -0.022* | -0.009 | -0.021* |
| | (0.0133) | (0.0136) | (0.014) | (0.012) |
| $\omega$ | 1.191e-06*** | -0.225*** | 1.87E-05 | 2.66e-06*** |
| | (4.89e-07) | (0.022) | (1.67E-05) | (5.12e-07) |
| $\alpha_1$ | 0.0663*** | 0.143*** | 0.024 | 0.032*** |
| | (0.0061) | (0.0119) | (0.144) | (0.007) |
| $\alpha_2$ | - | - | 0.044*** | - |
| | | | (0.0157) | |
| $\beta$ | 0.930*** | 0.985*** | 0.921*** | 0.926*** |
| | (0.0058) | (0.0023) | (0.006) | (0.0063) |
| $\gamma$ | - | -0.054*** | 0.992 | 0.067*** |
| | | (0.0062) | (7.239) | (0.009) |
| Q(36) | 34.238 | 34.369 | 32.031 | 36.562 |
| | [0.553] | [0.546] | (0.658) | (0.443) |
| LM(36) | 19.603 | 25.388 | 17.758 | 20.691 |
| | [0.988] | [0.906] | (0.995) | (0.980) |
| AIC | -5.314 | -5.324 | -5.328 | -5.325 |
| SC | -5.303 | -5.311 | -5.311 | -5.312 |
| Loglikelihood | 7309.87 | 7324.71 | 7332.18 | 7326.27 |
| RMSE | 0.01064 | 0.010658 | 0.010683 | 0.010670 |
| MAE | 0.008152 | 0.008150 | 0.008177 | 0.008159 |
| MAPE | 131.6475 | 119.1879 | 117.2050 | 115.8114 |
| TIC | 0.903759 | 0.921032 | 0.933199 | 0.928770 |
| | GARCH(1,1) t distributions | EGARCH(1,1) t distributions | APARCH(1,1) [a] t distributions | GJR-GARCH(1,1) t distributions |
| c | 0.0011*** | 0.001*** | 0.001*** | 0.001*** |
| | (0.00026) | (0.0002) | (0.00026) | (0.00026) |
| $\alpha$ | 0.0352** | 0.0334** | 0.029* | 0.029* |
| | (0.0172) | (0.0169) | (0.0169) | (0.017) |
| $\omega$ | 1.73e-06** | -0.212*** | 1.16E-05 | 2.43e-06*** |
| | (6.84e-07) | (0.0337) | (1.45E-05) | (7.16e-07) |
| $\alpha_1$ | 0.0602*** | 0.137*** | 0.061*** | 0.024** |
| | (0.009) | (0.018) | (0.0126) | (0.01) |
| $\alpha_2$ | - | - | - | - |
| $\beta$ | 0.938*** | 0.986*** | 0.934*** | 0.934*** |
| | (0.0084) | (0.003) | (0.008) | (0.008) |
| $\gamma$ | - | -0.0632*** | 0.369*** | 0.067*** |
| | | (0.011) | (0.098) | (0.014) |
| Q(36) | 28.066 | 24.179 | 25.161 | 25.952 |
| | (0.825) | (0.934) | (0.912) | (0.864) |
| LM(36) | 19.241 | 25.703 | 21.645 | 20.111 |
| | (0.989) | (0.898) | (0.974) | (0.984) |
| AIC | -5.370 | -5.376 | -5.376 | -5.377 |
| SC | -5.357 | -5.361 | -5.359 | -5.362 |
| Loglikelihood | 7387.69 | 7396.53 | 7398.58 | 7397.97 |
| RMSE | 0.010662 | 0.010676 | 0.010672 | 0.010670 |
| MAE | 0.008189 | 0.008197 | 0.008193 | 0.008159 |
| MAPE | 149.9777 | 142.5020 | 141.2126 | 115.8114 |
| TIC | 0.881678 | 0.894535 | 0.896282 | 0.928770 |
| | GARCH(1,1) GED distributions | EGARCH(1,1) GED distributions | APARCH(1,1) GED distributions | GJR-GARCH(1,1) GED distributions |
| C | 0.001*** | 0.00081*** | 0.0007*** | 0.0007*** |
| | (0.024) | (0.00023) | (0.0002) | (0.0002) |
| $\alpha$ | 0.0241 | 0.0223 | 0.019 | 0.019 |
| | (0.0153) | (0.0151) | (0.015) | (0.015) |
| $\omega$ | 1.78E-06** | -0.217*** | 1.02e-05 | 2.49e-06*** |
| | (7.79e-07) | (0.0384) | (1.50E-05) | (8.06e-07) |
| $\alpha_1$ | 0.06*** | 0.1379*** | 0.061*** | 0.025** |
| | (0.00982) | (0.0199) | (0.014) | (0.011) |
| $\alpha_2$ | - | - | - | - |
| $\beta$ | 0.937*** | 0.986*** | 0.933*** | 0.933*** |
| | (0.01) | (0.0038) | (0.009) | (0.01) |
| $\gamma$ | - | -0.06*** | 0.345*** | 0.066*** |
| | | (0.0113) | (0.0995) | (0.014) |
| Q(36) | 28.805 | 25.029 | 26.328 | 27.068 |
| | (0.797) | (0.915) | (0.881) | (0.859) |
| LM(36) | 19.563 | 25.687 | 21.240 | 20.310 |
| | (0.988) | (0.898) | (0.975) | (0.983) |
| AIC | -5.389 | -5.394 | -5.395 | -5.396 |
| SC | -5.376 | -5.379 | -5.378 | -5.381 |
| Loglikelihood | 7414.26 | 7422.29 | 7424.4 | 7423.94 |
| RMSE | 0.010676 | 0.010687 | 0.010687 | 0.010688 |
| MAE | 0.008194 | 0.008202 | 0.008201 | 0.008201 |
| MAPE | 136.3604 | 131.4645 | 129.8050 | 129.5985 |
| TIC | 0.904720 | 0.914296 | 0.917048 | 0.917438 |

Notes. [a]Convergence not achieved after 500 iterations.

Table 6 – Non linear models estimates and out-of-sample comparisons for the volatility of the TWSE stock market returns.

| | GARCH(1,1) | EGARCH(1,1) | APARCH(1,1) | GJR-GARCH(1,1) |
|---|---|---|---|---|
| c | 0.0005** | 0.0003 | 0.0003*** | 0.0003 |
| | (0.0002) | (0.0002) | (0.0002) | (0.0002) |
| α | 0.028 | 0.036** | 0.0271 | 0.022 |
| | (0.017) | (0.016) | (0.017) | (0.017) |
| ω | 1.89e-06*** | -0.240*** | 4.07e-05 | 2.75e-06*** |
| | (3.99e-07) | (0.02) | (2.76E-05) | (4.39e-07) |
| $\alpha_1$ | 0.068*** | 0.130*** | 0.0675*** | 0.029*** |
| | (0.005) | (0.008) | (0.005) | (0.006) |
| $\alpha_2$ | - | - | - | - |
| β | 0.926*** | 0.983*** | 0.929*** | 0.923*** |
| | (0.005) | (0.002) | (0.005) | (0.064) |
| γ | - | -0.056*** | 0.378*** | 0.07*** |
| | | (0.007) | (0.067) | (0.01) |
| Q(36) | 41.591 | 43.125 | 43.456 | 44.490 |
| | [0.240] | [0.182] | (0.184) | (0.157) |
| LM(36) | 55.572 | 56.410 | 52.283 | 48.191 |
| | [0.019] | [0.016] | (0.038) | (0.084) |
| AIC | -5.605 | -5.619 | -5.620 | -5.617 |
| SC | -5.594 | -5.606 | -5.605 | -5.605 |
| Loglikelihood | 7709.93 | 7730.50 | 7731.79 | 7727.83 |
| RMSE | 0.010828 | 0.010851 | 0.010860 | 0.010860 |
| MAE | 0.008683 | 0.008713 | 0.008722 | 0.008721 |
| MAPE | 103.5447 | 101.3630 | 101.2525 | 101.4866 |
| TIC | 0.932027 | 0.943878 | 0.952472 | 0.953741 |
| | GARCH(1,1) t distributions | EGARCH(3,1) t distributions | APARCH(3,1) t distributions | GJR-GARCH(1,1) t distributions* |
| c | 0.0006*** | 0.0005** | 0.0005** | 0.0005** |
| | (0.0002) | (0.0002) | (0.0002) | (0.0002) |
| α | 0.035** | 0.0344* | 0.0335* | 0.03* |
| | (0.0175) | (0.0172) | (0.017) | (0.017) |
| ω | 1.03E-06** | -0.201*** | 8.61e-05 | 1.68e-06*** |
| | (4.61E-07) | (0.0313) | (8.13e-05) | (5.20e-07) |
| $\alpha_1$ | -0.006 | -0.098** | 0.0246 | 0.0243** |
| | (0.014) | (0.0456) | (0.018) | (0.0096) |
| $\alpha_2$ | 0.126*** | 0.296*** | 0.999 | - |
| | (0.03) | (0.0640) | (12.280) | |
| $\alpha_3$ | -0.012 | -0.07 | -0.032 | - |
| | (0.0378) | (0.0467) | (0.028) | |
| $\alpha_4$ | -0.053* | - | - | - |
| | (0.027) | | | |
| β | 0.945*** | 0.987*** | 0.936*** | 0.942*** |
| | (0.009) | (0.003) | (0.008) | (0.008) |
| γ | - | -0.053*** | 0.999 | 0.059*** |
| | | (0.009) | (1.105) | (0.01246) |
| Q(36) | 44.448 | 44.526 | 43.148 | 42.949 |
| | (0.213) | (0.135) | (0.192) | (0.198) |
| LM(36) | 44.182 | 53.088 | 54.918 | 54.116 |
| | (0.164) | (0.033) | (0.0225) | (0.026) |
| RMSE | 0.010814 | 0.010830 | 0.010829 | 0.010835 |
| AIC | -5.656 | -5.669 | -5.664 | -5.658 |
| SC | -5.637 | -5.649 | -5.643 | -5.643 |
| Loglikelihood | 7783.69 | 7801.18 | 7795.98 | 7785.092 |
| MAE | 0.00866 | 0.008686 | 0.008686 | 0.008692 |
| MAPE | 104.3766 | 103.0671 | 103.1592 | 102.9106 |
| TIC | 0.920571 | 0.931946 | 0.931797 | 0.935976 |
| | GARCH(3,1) GED distributions | EGARCH(3,1) GED distributions | APARCH(2,1) GED distributions | GJR-GARCH(1,1) GED distributions |
| c | 0.00013 | 9.58E-05 | 6.36e-05 | 4.51e-05 |
| | (0.0002) | (0.0002) | (0.0002) | (0.0002) |
| α | 0.0124 | 0.015 | 0.009 | 0.006 |
| | (0.0151) | (0.015) | (0.015) | (0.015) |
| ω | 1.39e-06** | -0.228*** | 9.90e-05 | 2.18e-06*** |
| | (6.22e-07) | (0.04) | (0.0001) | (6.93e-07) |
| $\alpha_1$ | 0.001 | -0.082 | 0.0296 | 0.024** |
| | (0.018) | (0.0516) | (0.072) | (0.011) |
| $\alpha_2$ | 0.125*** | 0.290*** | 0.048 | - |
| | (0.035) | (0.069) | (0.032) | |
| $\alpha_3$ | -0.066 | -0.075 | - | - |
| | (0.033) | (0.0510) | | |
| β | 0.938 | 0.984*** | 0.924*** | 0.936*** |
| | (0.011) | (0.004) | (0.011) | (0.01) |
| γ | - | -0.0573*** | 0.998 | 0.067*** |
| | | (0.0118) | (3.388) | (0.015) |
| Q(36) | 46.853 | 49.245 | 47.430 | 48.382 |
| | (0.106) | (0.070) | (0.096) | (0.081) |
| LM(36) | 47.956 | 48.709 | 47.180 | 49.402 |
| | (0.0877) | (0.0767) | (0.01) | (0.067) |
| AIC | -5.681 | -5.693 | -5.689 | -5.685 |
| SC | -5.664 | -5.673 | -5.670 | -5.669 |
| Loglikelihood | 7817.16 | 7834.29 | 7829.68 | 7821.12 |
| RMSE | 0.010898 | 0.010902 | 0.010912 | 0.010918 |
| MAE | 0.008760 | 0.008765 | 0.008773 | 0.008778 |

| | | | | |
|---|---|---|---|---|
| MAPE | 99.60854 | 99.20624 | 98.95432 | 98.80546 |
| TIC | 0.978502 | 0.979062 | 0.986624 | 0.999701 |

Table 7 – Forecast error statistics for the Nikkei 225 stock market returns.

| | RMSE | Rank | MAE | Rank | MAPE | Rank | TIC | Rank |
|---|---|---|---|---|---|---|---|---|
| GARCH( normal) | 0.013237 | 1 | 0.010150 | 1 | 92.61459 | 9 | 0.945300 | 12 |
| EGARCH(normal) | 0.013328 | 12 | 0.010347 | 12 | 92.39440 | 7 | 0.982260 | 5 |
| APARCH(normal) | 0.013271 | 6 | 0.010230 | 6 | 89.698 | 2 | 0.957701 | 10 |
| GJR-GARCH(normal) | 0.013261 | 3 | 0.010205 | 3 | 89.31126 | 1 | 0.945389 | 11 |
| GARCH( t-student) | 0.013257 | 2 | 0.010192 | 2 | 95.10989 | 12 | 0.964149 | 9 |
| EGARCH(t-student) | 0.013295 | 11 | 0.010261 | 11 | 92.78961 | 10 | 0.992481 | 1 |
| APARCH(t-student) | 0.013270 | 5 | 0.010220 | 5 | 92.22754 | 6 | 0.973610 | 7 |
| GJR-GARCH(t-student) | 0.013266 | 4 | 0.010212 | 4 | 92.59386 | 8 | 0.971509 | 8 |
| GARCH( GED) | 0.013281 | 8 | 0.010236 | 8 | 93.02791 | 11 | 0.983332 | 4 |
| EGARCH(GED) | 0.013287 | 10 | 0.010251 | 10 | 91.80493 | 4 | 0.984891 | 2 |
| APARCH(GED) | 0.013277 | 7 | 0.010235 | 7 | 91.70377 | 3 | 0.976684 | 6 |
| GJR-GARCH(GED) | 0.013285 | 9 | 0.010246 | 9 | 92.09659 | 5 | 0.984515 | 3 |

Table 8 – Forecast error statistics for the Hang Seng 225 stock market returns.

| | RMSE | Rank | MAE | Rank | MAPE | Rank | TIC | Rank |
|---|---|---|---|---|---|---|---|---|
| GARCH( normal) | 0.016499 | 1 | 0.013632 | 2 | 115.0845 | 12 | 0.960983 | 10 |
| EGARCH(normal) | 0.016545 | 11 | 0.013677 | 12 | 99.67824 | 2 | 0.969425 | 7 |
| APARCH(normal) | 0.016533 | 6 | 0.013666 | 6 | 102.6582 | 7 | 0.971054 | 6 |
| GJR-GARCH(normal) | 0.016516 | 2 | 0.013650 | 4 | 106.7653 | 10 | 0.974722 | 5 |
| GARCH( t-student) | 0.016522 | 3 | 0.013646 | 3 | 108.8903 | 11 | 0.957240 | 12 |
| EGARCH(t-student) | 0.016543 | 10 | 0.013669 | 9 | 102.0694 | 5 | 0.960284 | 11 |
| APARCH(t-student) | 0.016539 | 8 | 0.01366 | 8 | 102.8276 | 8 | 0.961439 | 9 |
| GJR-GARCH(t-student) | 0.016449 | 12 | 0.013410 | 1 | 102.3247 | 6 | 0.961726 | 8 |
| GARCH( GED) | 0.016526 | 4 | 0.013662 | 5 | 103.3463 | 9 | 0.978328 | 3 |
| EGARCH(GED) | 0.016540 | 9 | 0.013676 | 11 | 99.50209 | 1 | 0.979082 | 2 |
| APARCH(GED) | 0.016536 | 7 | 0.013672 | 10 | 100.3890 | 3 | 0.978811 | 4 |
| GJR-GARCH(GED) | 0.016530 | 5 | 0.013667 | 7 | 101.8057 | 4 | 0.983149 | 1 |

Table 9 – Forecast error statistics for the STI stock market returns.

| | RMSE | Rank | MAE | Rank | MAPE | Rank | TIC | Rank |
|---|---|---|---|---|---|---|---|---|
| GARCH( normal) | 0.011671 | 1 | 0.009362 | 1 | 104.3299 | 12 | 0.953288 | 2 |
| EGARCH(normal) | 0.011708 | 12 | 0.009447 | 9 | 98.53686 | 8 | 0.9555710 | 5 |
| APARCH(normal) | 0.0117 | 9 | 0.009387 | 5 | 97.31510 | 1 | 0.970431 | 10 |
| GJR-GARCH(normal) | 0.011703 | 11 | 0.009416 | 8 | 99.25508 | 10 | 0.974058 | 12 |
| GARCH( t-student) | 0.011674 | 2 | 0.009408 | 7 | 101.3552 | 11 | 0.947301 | 1 |
| EGARCH(t-student) | 0.011695 | 6 | 0.009432 | 10 | 98.83255 | 9 | 0.953829 | 3 |
| APARCH(t-student) | 0.01169 | 4 | 0.009382 | 4 | 98.51928 | 6 | 0.961599 | 4 |
| GJR-GARCH(t-student) | 0.011692 | 5 | 0.009416 | 8 | 98.53398 | 7 | 0.962759 | 7 |
| GARCH( GED) | 0.011682 | 3 | 0.009372 | 2 | 99.47394 | 5 | 0.956756 | 6 |
| EGARCH(GED) | 0.0117 | 9 | 0.009396 | 6 | 97.63679 | 4 | 0.962817 | 8 |
| APARCH(GED) | 0.011697 | 8 | 0.009382 | 4 | 97.59994 | 2 | 0.969918 | 9 |
| GJR-GARCH(GED) | 0.011696 | 7 | 0.009379 | 3 | 97.63206 | 3 | 0.970759 | 11 |

Table 10 – Forecast error statistics for the KOSPI stock market returns.

| | RMSE | Rank | MAE | Rank | MAPE | Rank | TIC | Rank |
|---|---|---|---|---|---|---|---|---|
| GARCH( normal) | 0.01064 | 1 | 0.008152 | 2 | 131.6475 | 6 | 0.903759 | 8 |
| EGARCH(normal) | 0.010658 | 2 | 0.008150 | 1 | 119.1879 | 3 | 0.921032 | 3 |
| APARCH(normal) | 0.010683 | 8 | 0.008177 | 4 | 117.2050 | 2 | 0.933199 | 1 |
| GJR-GARCH(normal) | 0.010670 | 4 | 0.008159 | 3 | 115.8114 | 1 | 0.928770 | 2 |
| GARCH( t-student) | 0.010662 | 3 | 0.008189 | 5 | 149.9777 | 9 | 0.881678 | 10 |
| EGARCH(t-student) | 0.010676 | 6 | 0.008197 | 8 | 142.5020 | 8 | 0.894535 | 9 |
| APARCH(t-student) | 0.010672 | 5 | 0.008193 | 6 | 141.2126 | | 0.896282 | 8 |
| GJR-GARCH(t-student) | 0.010670 | 4 | 0.008159 | 3 | 115.8114 | 1 | 0.928770 | 2 |
| GARCH( GED) | 0.010676 | 6 | 0.008194 | 7 | 136.3604 | 7 | 0.904720 | 7 |
| EGARCH(GED) | 0.010687 | 9 | 0.008202 | 10 | 131.4645 | 5 | 0.914296 | 6 |
| APARCH(GED) | 0.010687 | 9 | 0.008201 | 9 | 129.8050 | 4 | 0.917048 | 5 |
| GJR-GARCH(GED) | 0.010688 | 10 | 0.008201 | 9 | 129.5985 | 4 | 0.917438 | 4 |

Table 10 – Forecast error statistics for the TWSE stock market returns.

| | RMSE | Rank | MAE | Rank | MAPE | Rank | TIC | Rank |
|---|---|---|---|---|---|---|---|---|
| GARCH( normal) | 0.010828 | 2 | 0.008683 | 2 | 103.5447 | 11 | 0.932027 | 4 |
| EGARCH(normal) | 0.010851 | 6 | 0.008713 | 5 | 101.3630 | 6 | 0.943878 | 6 |
| APARCH(normal) | 0.010860 | 7 | 0.008722 | 7 | 101.2525 | 5 | 0.952472 | 7 |
| GJR-GARCH(normal) | 0.010860 | 7 | 0.008721 | 6 | 101.4866 | 7 | 0.953741 | 8 |
| GARCH( t-student) | 0.010814 | 1 | 0.008660 | 1 | 104.3766 | 12 | 0.920571 | 1 |
| EGARCH(t-student) | 0.010830 | 4 | 0.008686 | 3 | 103.0671 | 9 | 0.931946 | 3 |
| APARCH(t-student) | 0.010829 | 3 | 0.008686 | 3 | 103.1592 | 10 | 0.931797 | 2 |
| GJR-GARCH(t-student) | 0.010835 | 5 | 0.008692 | 4 | 102.9106 | 8 | 0.935976 | 5 |
| GARCH( GED) | 0.010898 | 8 | 0.008760 | 8 | 99.60854 | 4 | 0.978502 | 9 |
| EGARCH(GED) | 0.010902 | 9 | 0.008765 | 9 | 99.20624 | 3 | 0.979062 | 10 |
| APARCH(GED) | 0.010912 | 10 | 0.008773 | 10 | 98.95432 | 1 | 0.986624 | 11 |
| GJR-GARCH(GED) | 0.010915 | 11 | 0.008775 | 11 | 98.986624 | 2 | 0.999701 | 12 |