Hierarchical Reasoning versus Iterated Reasoning in p-Beauty Contest Guessing Games

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Abstract

This paper analyzes strategic choice in p-beauty contests. We first show that it is not generally a best reply to guess the expected target value (accounting for the own weight) even in games with $n > 2$ players and that iterated best response sequences are not unique even after perfect/cautious refinement. This implies that standard formulations of "level-k" models are neither exactly nor uniquely rationalizable by belief systems based on iterated best response. Second, exact modeling of iterated reasoning weakens the fit considerably and reveals that equilibrium types dominate the populations. We also show that "levels of reasoning" cannot be measured regardless of the underlying model. Third, we consider a "nested logit" model where players *choose* their level. It dispenses with belief systems between players and is rationalized by a random utility model. Besides being internally consistent, nested logit equilibrium fits better than three variants of the level-k model in standard data sets.

JEL–Codes: C19, C44, C72

Keywords: logit equilibrium, hierarchical response, level-k, beauty contest

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1 Introduction

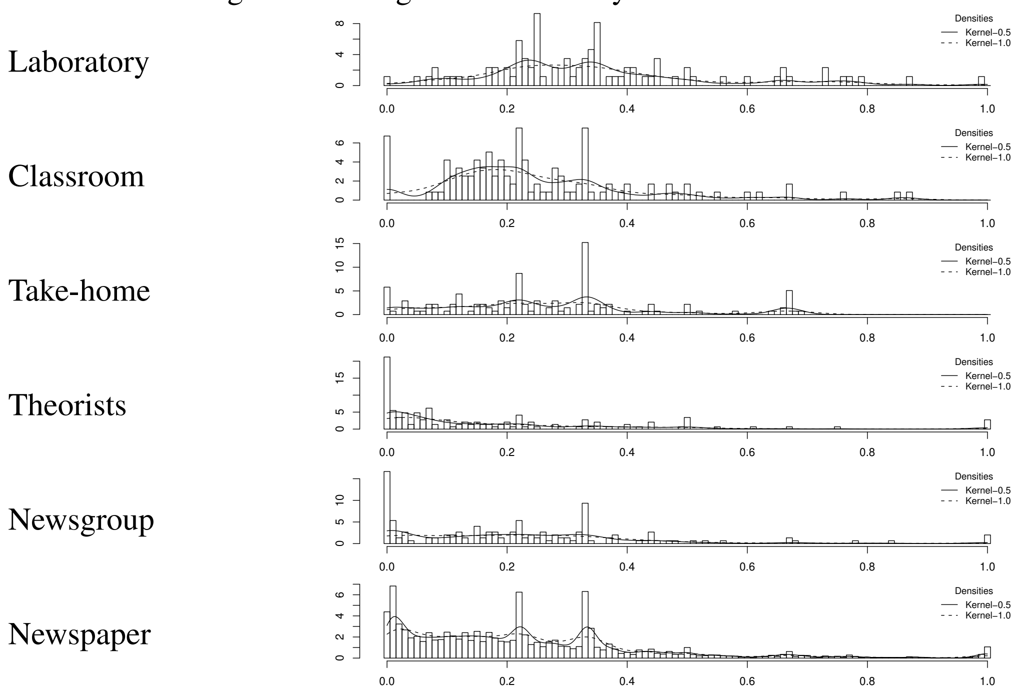
The "p-beauty contest" is a game where n players guess numbers $x_i \in [0,1]$ and the player whose guess is closest to $p \in (0,1)$ times the mean of all guesses wins a fixed prize. Nagel (1995) reported the first experimental investigation of such guessing games and advanced an interpretation of "iterated best response" to explain her observations, leaning on the level-k model of Stahl and Wilson (1995). Accordingly, the population consists of level-0 types who randomize uniformly, of level-1 types who assume all their opponents are level 0, of level-2 types who assume all their opponents are level 1, and so on. This model seems to explain two focal observations made in many beauty contest experiments, increased densities around .333 and .222 in case $p = 2/3$, and meanwhile, models of iterated reasoning have been adopted by many authors.[1] Currently, level-k models are considered the leading theory of strategic choice not only in p-beauty contests, but in novel situations in general.

The present study challenges the level-k interpretation of strategic choice in guessing games and presents evidence in favor of an alternative concept called nested logit equilibrium (NLE). The NLE concept generalizes logit equilibrium, which is the conventional formalization of quantal response equilibrium as defined by McKelvey and Palfrey (1995), by employing the more general hierarchical concept "nested logit response" instead of "logit response" at the level of individual choice (see e.g. McFadden, 1981, 1984). We will discuss various level-k, logit, and nested logit models based on the data set compiled by Bosch-Domenech et al. (2002), which combines observations from six different sets of p-beauty contest experiments.

Figure 1 presents histograms and density estimates of the observations. As indicated, the level-k approach essentially seeks to explain the spikes at .333 and .222, and combines with a notion of noise to approximate the overall distribution. The spikes at .333 and .222 rarely have the highest density in these samples, however. In most cases the Nash equilibrium guess has higher density, and the "Laboratory" data have the highest density in the neighborhood of .333 and .222 rather than exactly there.

[1] See e.g. Ho et al. (1998), Costa-Gomes et al. (2001), Crawford and Iriberri (2007), and Stahl and Haruvy (2008) for level-k analyses and Camerer et al. (2004), Kübler and Weizsäcker (2004), and Rogers et al. (2009) for related analyses.

Figure 1: Histograms and density estimates of the data



Moreover, these spikes are hardly significant in the density estimates, and thus there is no necessity that an explanation of these spikes has to be the basis of models explaining the distribution of guesses.[2] In addition to this lack of necessity, the present paper shows the following. Increased densities at .333 and .222 (in case $p = 2/3$) are generally not rationalizable by iterated best response, and hence level-k models as discussed in the literature do not exactly comply with iterated reasoning (Section 2). Exact modeling of iterated reasoning suggests that equilibrium types dominate the population but leads to significantly worse goodness-of-fit (Section 3). Nested logit equilibrium rationalizes subjects' levels using a random utility model, which is internally consistent, and it significantly improves the goodness-of-fit (Section 4).

To be precise, Section 2 shows that the guess $p \cdot 0.5$, or $\frac{n-1}{n-p} \cdot p \cdot 0.5$ as in Ho et al. (1998), is in general not the best response to uniformly randomizing players. The best response may differ strikingly if the number of players is low or if the players behave as if their opponents' guesses would be highly correlated (such perceived cor-

[2]This applies in particular for standard risk-minimizing kernel bandwidths ("Kernel-1.0" in Figure 1), but to a lesser degree also if we use half these bandwidths ("Kernel-0.5").

relation has been reported by Ho et al., 1998, and it is found below as well). Section 2 further discusses the non-uniqueness of best responses to deterministically acting opponents. Assuming all opponents pick .333 (i.e. level 1 as understood conventionally), all actions x_i satisfying $.111 \leq x_i < .333$ are best responses in case $p = 2/3$ (the lower bound .111 is not tight if the number of players is $n < \infty$). While this issue has been noted in the existing literature (see e.g. Footnote 1 in De Giorgi and Reimann, 2008), an argument based on refinement concepts clarifying why .222 is the most reasonable best response has not been put forward. We consider various refinements of best responses, e.g. perfection (Selten, 1975) and cautiousness (Pearce, 1984), and find that .222 itself is weakly dominated and that the focus on a response approximating .222 cannot be justified by refinement concepts. It follows that level-k responses approximating $p^k \cdot 0.5$ for all k cannot be derived from an assumption of iterated best responses, and hence the conventionally assumed beliefs and actions of level-k players are neither necessary nor sufficient for satisfying iterated rationality.

Section 3 investigates to which degree the standard assumptions underlying level-k actions can be relaxed without obstructing the goodness-of-fit of level-k models. We consider three increasingly self-sustaining incarnations of level-k models: (i) a non-strategic model following Stahl (1996) where level-k players randomize normally with mean $p^k \cdot 0.5$ and variance σ_k^2, (ii) is similar to (i), but now the mean μ_k equates with the actual best response to level $k-1$, and (iii) a model of iterated logit response (with precision λ_k for level k) that explains both mean and distribution of level-k actions in a way that is compatible with the assumed beliefs of level-k players and the "choice axiom" (Luce, 1959, 1977). Models relying on the actual expected payoffs, i.e. (ii) and (iii), have to my knowledge not been estimated yet.[3] We find that if the level-k means in variant (i) are treated as free parameters, then the hypothesis $\mu_k = p^k \cdot 0.5$ is rejected in three out of six data sets. In addition, models of variant (i) fit significantly better than variant (ii), and models of variant (ii) fit better than

[3]A reason seems to be that computing expected payoffs is comparably tedious in p-beauty contests (see Section 2). The existing literature focuses on variations of (i), e.g. Ho et al. (1998) and De Giorgi and Reimann (2008), and rests on the assumption that the unique best response of level-k players equates with p or $\frac{n-1}{n-p} \cdot p$ times the mean of level $k-1$. This assumption is not satisfied, but it circumvents the computation of expected payoffs.

variant (iii). That is, the goodness-of-fit of level-k models decreases significantly if we require consistency with iterated reasoning and the choice axiom. We therefore consider the level-k theory of strategic choice in guessing games incomplete.

Another observation made in Section 3 can be summarized as follows. Let $\Pr(L = k|X = x)$ denote the ex-post probability that a subject is of level k conditional on the guess being x (computed using Bayes' Rule and the model estimates). Intuitively, one may expect that a subject guessing $x \in [0,1]$ believes that the opponents guess about $x \cdot p^{-1}$ on average. Hence, if a subject guesses $x = 0.4$ for example, then one would expect that this subject is most likely of level $k = 1$. The existing literature universally assumes that "levels of reasoning" can be measured in this way. We show that the conditional probabilities $\Pr(L = k|X = x)$ derived from level-k model estimates strikingly contradict this intuition—in all data sets and for all level-k model variants (i), (ii), and (iii). In most cases, the most probable type k is not even weakly decreasing in the guess x. Hence, under the assumptions of level-k models, it is not possible to infer a subject's belief from its guess as it is done intuitively. In contrast, subjects' descriptions of their own strategies generally allow for such inference (see for example Bosch-Domenech et al., 2002).

Section 3 finally shows that most subjects are classified as equilibrium types (i.e. as level $k = \infty$) in the model variants (ii) and (iii), which shows that subjects are more aptly modeled as logit equilibrium types than as low-level logit responders. As indicated already, the goodness-of-fit of variant (iii), i.e. of iterated logit response and logit equilibrium, is rather poor, however. This may come as a surprise, as logit equilibria proved powerful in explaining strategic choice,[4] but as Haile et al. (2008) discuss, their result that QRE can fit everything has typically no bite if we restrict attention to logit models in games with many strategies. Clearly, a guessing game has "many" strategies, even if we do not equate the strategy set with the continuum, and for this reason, the logit specification does not imply a good fit automatically. Aside

[4] To name just a few studies following McKelvey and Palfrey (1995), McKelvey and Palfrey (1998) apply it to extensive form games, Anderson et al. (1998a,b) apply it to public-goods and rent-seeking games, Weizsäcker (2003) extends it to heterogenous precision parameters, Turocy (2005) relates the principal branch of logit equilibria to the Harsanyi-Selten tracing procedure, and Breitmoser et al. (2009) apply it to dynamic games with infinite time horizon.

from this, closer inspection of the data reveals a possible explanation for its failure in guessing games: the densities are not monotonic in the expected payoffs,[5] which suggests that the choices are not independent from irrelevant alternatives (IIA).

Models relaxing IIA have a long tradition in choice theory and economics (see e.g. Train, 2003, and Tversky, 2004) and if we reinterpret the level-k model as a model where players "choose their level" themselves, rather than living it without individual reflection, then we actually obtain a model violating IIA: the nested logit model of McFadden (1978, 1984). Its simplest variant, the two-level nested logit model, can be described as follows: if $(X^1, X^2, \ldots, X^n)$ denotes a partition of $[0,1]$, then players first choose a nest X^k and secondly choose an element $x_i \in X^k$. Besides being intuitive in cases with large choice sets, nested logit response is compatible with random utility maximization (Daly and Zachary, 1978; McFadden, 1978) for all partitions $(X^k)_{k \leq n}$. That is, internal consistency of the model is generally given if we allow that subjects choose their "level" themselves. Note also that the nested logit model avoids two unintuitive implications of level-k models even if we do not stress their inconsistencies. By definition, level-k players are perfectly sure to make a winning guess, and they believe to be exactly one step ahead of all opponents. Both of these assumptions contradict the uncertainty experimental subjects express to have (see e.g. Bosch-Domenech et al., 2002). Finally, analyses of nested logit models are well established in other branches of economics.[6] The analysis of nested logit equilibria, i.e. of mutual nested logit responses, appears novel, however.

Section 4 defines nested logit equilibrium formally, derives the model estimates for the six data sets, and shows that it fits the observations better than the non-strategic level-k model variant (i) following Stahl (1996) in the majority of data sets, and it fits the data better than the strategic level-k variants (ii) and (iii) in all data sets. Section 5 concludes. The supplementary material contains relegated technical material and the parameter estimates of the models discussed in the text.

[5]The most obvious violation is that guesses $x_i \approx 0$ are more profitable than $x_i = 0$ if players hold non-degenerate beliefs, yet $x_i = 0$ has higher density than any $x_i \approx 0$.

[6]For example in analyses of multiproduct firms (Anderson and De Palma, 1992), voting behavior (Whitten and Palmer, 1996), phone calling patters (Train et al., 1987; Lee, 1999), and recreation trip choices (Shaw and Ozog, 1999).

2 Iterated best response sequences

This section derives several results on iterated best responses and relates them to assumptions made in the level-k literature. The best-known assumption is that if one faces opponents randomizing on $[0,1]$ according to a probability distribution with mean $\bar{x}$, then one's best response is $\frac{n-1}{n-p} \cdot p\bar{x}$, accounting for the weight of the own guess (or $p \cdot \bar{x}$ for simplicity). In addition, there seems to be consensus that x_i is a best response if it minimizes the distance to the expected target value, and if the best response is not unique, then it is most plausible to focus on this minimizer.[7] Guessing the expected target value is known to be sub-optimal in two-player games (under full support, assuming $p < 1$, $x_i = 0$ is optimal there), and as the following shows, it is not generally optimal if $n = 3$ (next) and as n becomes large (below). We discuss refined best responses to deterministically acting opponents and show that there is no particular reason to focus on said minimizer. Hence, the conventionally considered level-k beliefs and actions cannot be derived from iterated best response.

2.1 Optimal level-1 strategies in three-player games

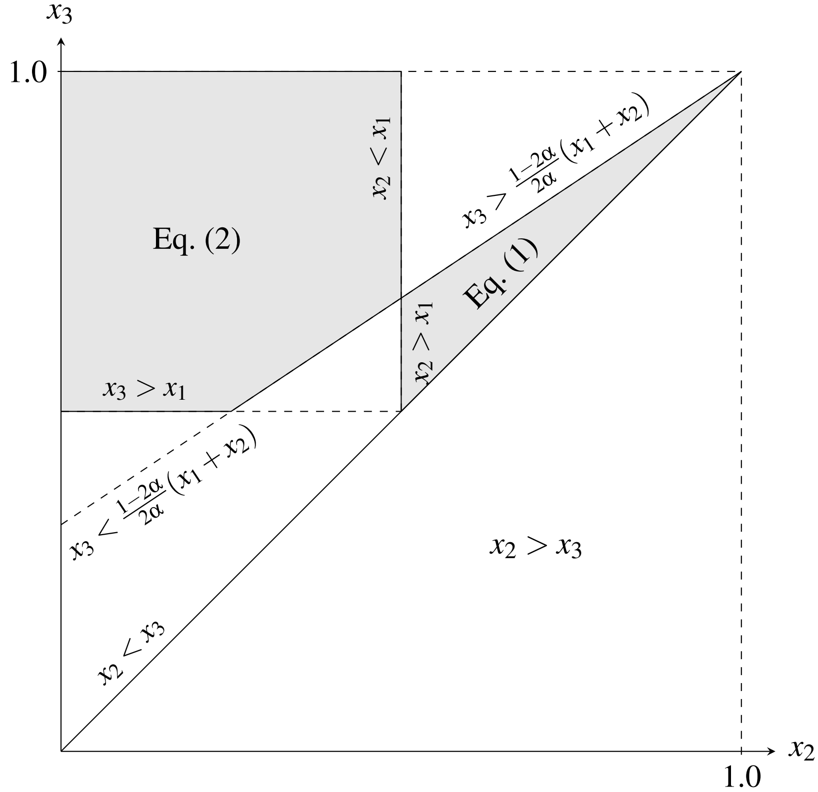
The notation is standard. The set of players is denoted as $N = \{1, \ldots, n\}$, typical players are denoted as $i, j \in N$, and actions are denoted as $x_i \in [0,1]$ for all $i \in N$. All players move simultaneously, and the payoff of $i \in N$ is denoted as $\pi_i(x)$ for all $x = (x_i)_{i \in N}$. The strategy of $i \in N$ is a probability distribution on $[0,1]$.

Assume $n = 3$ and consider player $i = 1$ in response to two opponents randomizing uniformly on $[0,1]$. The set of $(x_2, x_3) \in [0,1]^2$ in response to which a given x_1 is closest to the target value is called *winning region* of x_1. Without loss of generality, assume $x_2 < x_3$. Two cases have to be distinguished. On the one hand, player 1 wins (a share of) the prize in case $x_1 \leq x_2 < x_3$ if, using $\alpha = p/3$,

$$\alpha(x_1 + x_2 + x_3) \leq \frac{1}{2}(x_1 + x_2) \qquad \Leftrightarrow \qquad x_3 \leq \frac{1-2\alpha}{2\alpha} x_1 + \frac{1-2\alpha}{2\alpha} x_2. \qquad (1)$$

[7] A particularly recent and explicit reference in this direction can be found in De Giorgi and Reimann (2008, Fn. 1 and Ass. 1).

Figure 2: Winning regions of player 1 in case $x_2 < x_3$ (for $x_1 = 0.5$ and $p = 0.9$)



On the other hand, player 1 wins in case $x_2 < x_1 \leq x_3$ if

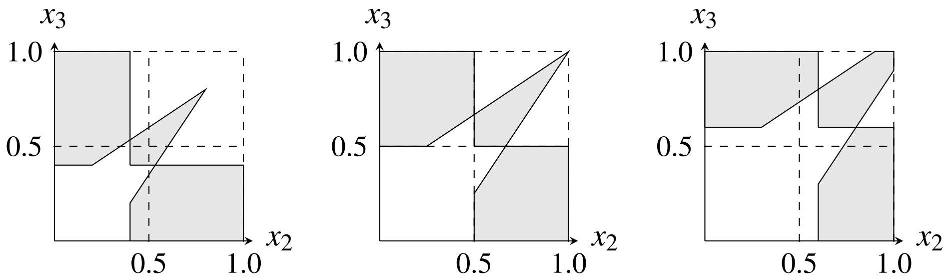
$$\alpha(x_1+x_2+x_3) \leq \frac{1}{2}(x_1+x_2) \qquad \Leftrightarrow \qquad x_3 \leq \frac{1-2\alpha}{2\alpha}x_1 + \frac{1-2\alpha}{2\alpha}x_2. \tag{2}$$

These two cases correspond with two disjoint winning regions in $[0,1]^2$ and are illustrated in Figure 2. A second set of restrictions (and winning regions) applies when $x_3 > x_2$. Figure 3 depicts the aggregate winning regions for three values of x_1.

In the case of uniformly randomizing opponents, the expected payoff of guessing x_1 simply equates with the aggregate area size of the winning regions associated with x_1. The expected payoff can be computed for all x_1 in closed form, but due to the case distinctions involved, it is relegated to Appendix A. The following results.

Proposition 2.1. *Assume $n = 3$ and $p \in (0,1)$. The payoff-maximizing choice in*

Figure 3: Winning regions for $x_1 \in \{0.4, 0.5, 0.6\}$ and $p = 0.9$



(a) $x_1 = 0.4$ (b) $x_1 = 0.5$ (c) $x_1 = 0.6$

response to two uniformly randomizing opponents is, using $\alpha := p/3$,

$$x_1^* = \frac{2\alpha(1-2\alpha)}{(4-7\alpha)(1-3\alpha)} \qquad \text{if } p \leq 0.908, \tag{3}$$

$$x_1^* = 2/\left(4 - \frac{(2-6\alpha)^3}{\alpha(1-2\alpha)(4\alpha-1)}\right) \qquad \text{if } p > 0.908. \tag{4}$$

That is, best response functions are considerably more complex than $x_i^* = \frac{n-1}{n-p} \cdot p\bar{x}$. As an example, consider $p = 0.9$, which has been chosen in three-player treatments by Ho et al. (1998). In this case, the payoff-maximizing choice in response to uniformly randomizing players is $x_1^* = 0.5217$ (note that Ho et al. actually assume truncated normal randomization at level 0, but a similar argument applies in this case). This solution is greater than both mean and median of the opponent's strategies, and in relation to the expected target value, which is $\alpha * (.5 + .5 + x_1^*) = .457$, the optimal x_1^* is on the "wrong side" of the opponent's means. That is, the best response of level-1 players is *greater* than the average guess of level-0 players in this case, and since the best response in case $p = 1$ is to pick the median of (i.i.d.) opponents (if $n = 3$), the optimal choice is *not monotonically increasing* in p. Both of these observations contradict the assumptions conventionally made in level-k analyses.

The best response to uniformly randomizing opponents converges to $\frac{n-1}{n-p} \cdot p\bar{x}$, and hence to $p\bar{x}$, as n approaches infinity. This convergence suggests that strategic effects becomes negligible when n is greater than say 10. In this context, let us recollect a result of Ho et al. (1998) stating that subjects behave as if their opponents' guesses would be correlated with $\rho = 1$. We will obtain a similar conclusion, although our formalization of the stochastic dependence differs slightly. These observations relate to the bias known as "belief in small numbers" and suggests that subjects do not account for the actual distributional properties of the opponents' guesses if n is large. Hence, the strategic effects inducing deviations from $p\bar{x}$ do not become negligible as n grows.

2.2 Iterated elimination and iterated best response

Guessing games are special in that they are solvable by iterated elimination of weakly dominated strategies although almost all strategies are rationalizable (all but the upper bound in case $p < 1$). The discrepancy between iterated weak dominance and common knowledge of rationality could not be larger in any game. One's intuition may suggest that refined notions of rationalizability, e.g. perfect rationalizability (Bernheim, 1984) or cautious rationalizability (Pearce, 1984), will allow us to bridge this gap. Since a closely related issue exists in refining iterated best response sequences, and hence in justifying beliefs in level-k models, such refinement concepts are investigated in some detail now.

We begin with specifying the solution path according to weak dominance. Assume one acts in response to opponents whose guesses are restricted to $[0,y]$ with $0 < y \leq 1$. Lemma A.1 shows that all $x_i \geq \mathrm{DR}(y)$ are weakly dominated where

$$\mathrm{DR}(y) = \frac{n-2}{n-2p} \cdot py \equiv \frac{n-2}{1/\alpha - 2} \cdot y \qquad \text{for } \alpha = p/n. \tag{5}$$

The largest non-dominated action $\mathrm{DR}(y)$ will be called "dominant response" in the following. It equates with the smallest guess x_i that is optimal in response to all x_{-i} where $x_j \in (x_i, y]$ for all $j \in i$ (i.e. it is optimal as long as all opponents' guesses x_j are greater than x_i). If any of the opponents picks some $x_j \leq \mathrm{DR}(y)$, in turn, then the resulting target value is less than or equal to $\mathrm{DR}(y)$, and combined these observations imply that $\mathrm{DR}(y)$ weakly dominates all $x_i > \mathrm{DR}(y)$. Note also that $\mathrm{DR}(y)$ is the unique action that is optimal whenever $n-2$ opponents stick with y and one opponent "deviates" to any $x_j \in [0,y]$. Finally, this argument of weak dominance implies that one should not minimize the distance to the expected target value when all opponents pick some $y \in (0,1]$. Consider for example $y \approx 1$. The distance is minimized if one picks $x_i = p \cdot \frac{n-1}{n-p} \cdot 1$, which is weakly dominated by $\mathrm{DR}(1)$.

We now turn to the solution according to refined notions of rationalizability, namely perfect and cautious rationalizability. In short, a strategy is rationalizable if it is a best response to rationalizable strategies of the opponents, a strategy is perfectly rationalizable if it continues to be a best response when the opponents tremble with positive probability to alternative strategies, and it is cautiously rationalizable if it continues to be a best response when the opponents tremble with positive probability to other rationalizable strategies. Note that the restriction on the support of trembles imposed by cautious rationalizability corresponds closely with the reasoning underlying iterated weak dominance, whereas perfect rationalizability relates more closely to the interpretation that trembles are made mistakingly. Formal definitions are provided in Appendix A (Defs. A.2 and A.4; see also Bernheim, 1984, and Pearce, 1984). We obtain the following result.

Proposition 2.2. *For all $i \in N$, all actions $x_i \in [0,1)$ are rationalizable, and in case $n > 2$, all actions $x_i \in [0, DR(1))$ are perfectly rationalizable, while only $x_i = 0$ is cautiously rationalizable.*

Since obeying weak dominance is in itself not an implication of rationality, refined notions of rationalizability are used to validate iterated eliminations of weakly dominated strategies. In our case, it turns out that the solution path depends on the refinement concept employed. If trembles are considered to be mistakes, as in perfect rationalizability, then many actions are rationalizable. If trembles are considered to express a specific requirement of robustness, as in cautious rationalizability, then the Nash equilibrium results. The proof, see appendix, also shows that the elimination sequence implied by cautious rationalizability coincides with the one derived above for iterated weak dominance. That is, after k iterations, all actions but $[0, \mathrm{DR}^k(1))$ are eliminated according to cautious rationalizability. It is therefore appropriate to say that players choosing actions $x_i \in \left[\mathrm{DR}^{k+1}(1), \mathrm{DR}^k(1)\right)$ are at most level-k cautiously rational. Since $\mathrm{DR}(y) \neq \frac{n-1}{n-p} \cdot py$, however, the standard intervals result only approximately.

Sequences of iterated best responses are related closely to the solution paths of rationalizability concepts. The only significant difference concerns the restrictions imposed on level-0 players. The level-0 strategies are unrestricted under rationalizability, whereas specific distributions (usually the uniform one) are assumed in iterated best response sequences. It follows that the limit points of iterated best response sequences are specific rationalizable strategies, but it is not obvious as to whether the restriction at level 0 induces a significant refinement effect at level ∞. In order to investigate this, assume uniform randomization at level 0. Aside from this, level-k players best respond to level $k-1$ for all $k \geq 1$. Following the literature, restrict attention to the case that all players at level k act symmetrically and deterministically. Potentially, this assumption induces a second refinement effect beyond restricting level-0 strategies, but in our case it does not. The level-k guess is denoted as $x_k \in [0,1]$, for $k \geq 1$.

A closed-form representation of the best response to $n-1$ opponents randomizing uniformly is not available. Let x_1 denote the best response to level 0. In general, x_2 is a best response to opponents choosing x_1 if and only if it is an element of the open set $(\mathrm{BR}_{\inf}(x_1), x_1)$ with the lower bound

$$\mathrm{BR}_{\inf}(x_k) = \max\left\{\tfrac{n*(2-1/p)-2}{n-2p} \cdot px_k, 0\right\}. \tag{6}$$

This implies that iterated best response sequences may converge either very quickly to 0, e.g. in case x_{k+1} is near the lower bound $\mathrm{BR}_{\inf}(x_k)$ for all k, or they converge to 0 rather slowly, or actually not at all. If all x_{k+1} are sufficiently close to the respective upper bounds, then the sequence would converge within any ε-neighborhood of x_1. Since x_1 may be greater than 0.5 even if $p < 1$ (e.g. if $p = 0.9$ and $n = 3$), this implies that iterated best response sequences may converge above 0.5. Our next result investigates the implications of perfection and cautiousness in this context.[8]

Proposition 2.3. *A best response sequence (x_k) converging to $r \in \mathbb{R}$ exists if and only if $r \in [0, x_1)$. Assuming $n > 2$, perfect response sequences (y_k) converging to r exist iff $r \in [0, \min\{DR(1), y_1\})$. All cautious response sequences (z_k) converge to 0.*

In other words, the (refined) best response dynamics may converge anywhere in the (correspondingly refined) set of rationalizable strategies capped at x_1. Restricting

[8]Perfect responses and cautious responses are defined formally in Def. A.2 and Def. A.4.

the level 0 strategies has no significant implications aside from capping the level-1 actions. Furthermore, neither perfect responses nor cautious responses are unique—see Lemmas A.3 and A.5—aside from the fact that guessing the expected target value is never a cautious response (under the assumption that opponents act symmetrically). Finally, note that while the notion of perfect response does not impose significant restrictions on iterated best responses, the more restrictive notion of iterated cautious response induces sequences with an upper bound that is strictly below the conventionally considered sequences, as $\left(\frac{n-2}{n-2p} \cdot p\right)^{k-1} * x_1 < \left(\frac{n-1}{n-p} \cdot p\right)^{k-1} * x_1 < p^{k-1} * x_1$. Hence, iterated best response sequences do not explain the increased densities at $p \cdot 0.5$ and $p^2 \cdot 0.5$, and they are neither necessary nor sufficient to rationalize the assumed actions of level-k players.

To further illustrate, consider a fourth kind of iterated best responses: level 1 best responds to level 0 as before, but for all $k > 1$, level-k players assume their opponents would be level $k-1$ with probability $1-\varepsilon$ and level 0 with probability ε (i.i.d.). In this case, level-k players are not unsure about the action that players at level $k-1$ pick, but they seek robustness with respect to the possibility that some of their opponents are not level $k-1$. Our result shows that as ε tends to 0, the resulting best response sequences converge to positive values in fairly general circumstances.

Proposition 2.4. *Assume $n \geq 3$. Consider a best response sequence (x_k) with level-0 perturbations as described above, as ε tends to zero. Such a sequence (x_k) exists if and only if $x_1 > \frac{p}{n-p(n-1)}$, and if it exists it converges to $\frac{p}{n-p(n-1)}$ as k tends to infinity.*

For example, if $p = 2/3$, which satisfies the restriction for all $n > 3$, the limit of the best response sequence is $2/(n+2)$, which equates with $1/4$ if $n = 4$ and with $1/10$ if $n = 18$. Hence, convergence above zero is remarkably robust even as n grows. To be sure, this effect is not an artifact of $\varepsilon \to 0$, as convergence to zero can be ruled out in general if ε is a positive constant. It follows that convergence to 0 as k tends to infinity is not a universal characteristic of best response dynamics—any deviation from cautious response potentially induces a deviation from this limit and thus a deviation from the conventionally assumed level-k actions.

3 Application of level-k models

We consider three different families of level-k models. The first one is the seminal model due to Stahl (1996) which we will refer to as "non-strategic" model.

Definition 3.1 ("Non-Strat")**.** The set of player types is $\mathcal{K} = \{0,\dots,K\}$; the type shares in the population are $(\rho_k)_{k\in\mathcal{K}}$. Players of type $k = 0$ randomize uniformly on $[0,1]$, and for all $k \geq 1$, type-k players randomize according to normal distributions $\mathcal{N}(p^k * 0.5, \sigma_k^2)$ truncated to $[0,1]$.

This model is non-strategic in the sense that its structure cannot be derived solely from assumptions of strategic choice. Assuming truncated normal distributions at levels $k \geq 1$ is not compatible with random utility maximization in response to level $k-1$. In addition, the modes of these distributions, $p^k * 0.5$, are not the best responses to the distributions at level $k-1$, in particular not when these distributions are truncated as they are here, nor are they the uniquely most plausible best responses if all opponents deterministically choose $x_j = p^{k-1} * 0.5$ (see above).[9]

The log-likelihood of the observations $\mathbf{o} = (o_m)_{m=1,\dots,M}$ (e.g. "Laboratory") under the parameters $(\sigma_k, \rho_k)_{k\in\mathcal{K}}$ is, using $f_k(x|\sigma_k)$ as the density of the level-k strategy,

$$LL(\mathbf{o}|\sigma,\rho) = \sum_{m=1}^{M} \ln \sum_{k\geq 0} \rho_k \cdot f_k(o_m|\sigma_k). \tag{7}$$

We estimate such finite-mixture models (Peel and MacLahlan, 2000) using the comparably robust procedure, albeit time consuming, of maximizing the full information likelihood jointly over all parameters using the Nelder-Mead algorithm (similar for example to Stahl, 1996; see also Arcidiacono and Jones, 2003). Standard errors are obtained from the information matrix. Model selection is discussed based on the Bayes Information Criterion (BIC, Schwarz, 1978). The ML estimates of the model parameters $(\sigma_k, \rho_k)_{k\in\mathcal{K}}$ for various specifications of $\mathcal{K}$, i.e. of $\max \mathcal{K}$, are reported in Table B.1, and the goodness-of-fit of these models is reported in Table 1. It is not

[9] Related models have been considered by Ho et al. (1998) and De Giorgi and Reimann (2008). They too assume that guessing the expected target value is the best response to level $k-1$, but they vary distributional assumptions (e.g. normal at level 0 or uniform at level $k > 0$) and introduce truncation toward subsets of $[0,1]$ at higher levels. Neither of these variations is implied by strategic choice.

necessary to memorize these estimates in detail. For the flow of our argument, it is sufficient to note that according to BIC, the populations contain types up to level 2 (in *Laboratory* and *Classroom*) or up to level 4 and 5 otherwise.

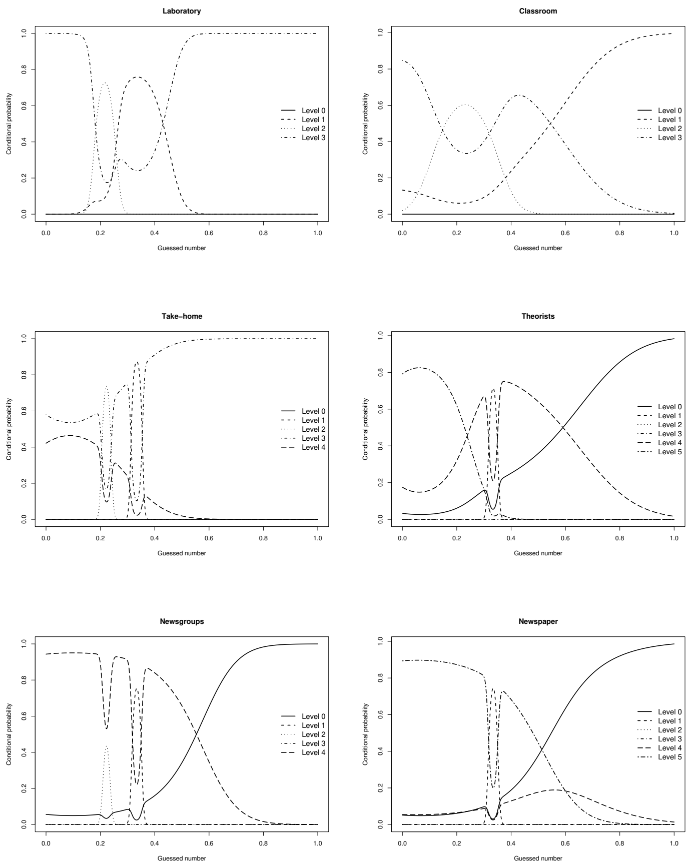
A first robustness check of the non-strategic model is obtained when we treat the level-k means (μ_k) as free parameters and estimate $(\mu_k, \sigma_k, \rho_k)$ jointly. To facilitate this test, Tables 1 and B.1 also report the ML estimates and the LL maxima of correspondingly generalized models. These models contain two additional parameters, $\tilde{x}_0$ and $\tilde{p}$, and the level-k mean is computed as $\mu_k = \tilde{x}_0 \cdot \tilde{p}^k$. According to likelihood-ratio tests, the hypothesis $\mu_k = p^k \cdot 0.5$ for all k is rejected in three of the six data sets (namely with respect to the non-student data sets *Theorists*, *Newsgroup*, and *Newspaper*). The explanation seems to be that quasi-equilibrium types (i.e. level $k = \infty$) are needed to explain a significant share of the observations in these data sets. In order to investigate this, consider the following extension of the non-strategic model.

Definition 3.2 ("Non-Strat$^\infty$")**.** The set of player types is $\mathcal{K} = \{0, \ldots, K, \infty\}$, otherwise equivalent to "Non-Strat" (Def. 3.1).

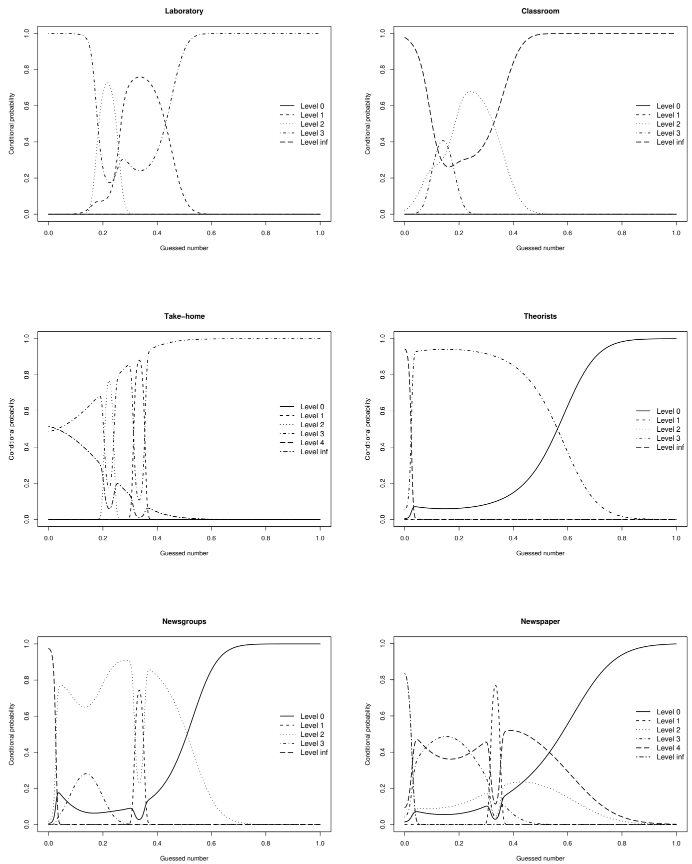
These models are considered solely as a second robustness check of the non-strategic models, since the "Non-Strat$^\infty$" models themselves are inconsistent. For, randomization according to $\mathcal{N}\left(0, \sigma_\infty^2\right)$ contradicts equilibration of mutual responses. In response to $\mathcal{N}\left(0, \sigma_\infty^2\right)$, one is best off choosing a strictly positive number, and at the very least, the mode of such equilibrium strategies should therefore be positive rather than zero. In relation to "Non-Strat", the results from "Non-Strat$^\infty$" (see Tables 1 and B.2) offer interesting insights, however. The BIC improves in four out of six data sets due to inclusion of $k = \infty$, namely in the three non-student samples and in *Take-home*, and the estimated shares of "quasi-equilibrium" players are above 50% in these four samples. In addition, the maximal level of non-equilibrium types drops to $K = 2$ or $K = 3$ in all samples. Finally, the level-k means $\mu_k = \tilde{x}_0 \cdot \tilde{p}^k$ are now robust to treating $(\tilde{x}_0, \tilde{p})$ as free parameters, i.e. the BICs do not improve significantly in any data set when $k = \infty$ is included (see Table 1). These results suggest that stochastic equilibrium may have explanatory power in guessing games, with the caveat that $\mathcal{N}\left(0, \sigma_\infty^2\right)$ is not a consistent notion of stochastic equilibrium.

Before we elaborate on this, let us turn to a second issue with level-k models.

Figure 4: A-posteriori classification based on non-strategic models (the model dimensions follow from $\max BIC$, at least $K = 3$)



(a) Without level ∞



(b) With level ∞

Given a data set $\mathbf{o}$ and model estimates (ρ_k, σ_k), the probability that subject m is of type k conditional on its guess o_m can be computed using Bayes' Rule.

$$\Pr(\text{type } = k | o_m) = \rho_k \cdot f_k(o_m) / \sum_{k' \in \mathcal{K}} \rho_{k'} \cdot f_{k'}(o_m | \sigma_{k'}). \tag{8}$$

Figure 4 depicts the ex-post classifications implied by our estimates for both "Non-Strat" and "Non-Strat$^\infty$." A standard assumption in level-k analyses is that a subject guessing o_m is classified as level k if and only if $o_m \in \left(p^{k+1/2} \cdot 0.5, p^{k-1/2} \cdot 0.5\right]$, or approximations thereof. This assumption violates Bayes' Rule in that it neglects the a-priori probabilities of the various types (and to a lesser degree also the type variances σ_k^2), and consequently this rule-of-thumb classification drastically contradicts the Bayes-consistent classification Eq. (8) in all cases. In "Non-Strat" for example, a guess of .4 is most likely to have been made by a level-1 player in only one out of these six data sets (*Laboratory*), and in all cases, a guess of .5 is more likely to have been made by a player of at least level three than by a player of level 0 or 1. This violation of "measurability" of levels of reasoning is a concern. For, if the assumptions underlying the non-strategic level-k model imply that a guess of .5 most likely has been made by a subject believing the opponents are level $k \geq 2$, universally across samples, then these assumptions may have to be reconsidered.

A solution may be to truncate the support of the level-k strategies, but we are hesitant to do so since no such restriction can be derived from rationalizability. An alternative solution may be to consider strategically consistent variants of level-k models. Two such variants are considered next. The first one is a model of iterated best response with strategy perturbations, the second one is a model of iterated logit response. Similar to Ho et al. (1998), we allow that subjects behave as if their opponents' draws are not independent (in order to model subjective overconfidence). Assume all opponents $j \neq i$ randomize according to the density f_j on $[0,1]$. We assume that i estimates the expected payoff in a thought experiment using $m \leq n-1$ draws from f_j, rather than exactly $n-1$ draws. Let $f^m(\cdot|f_j)$ denote the joint density of m i.i.d. random variables with density f_j, and define i's payoff from x_i as

$$\pi\left(x_i | m, f^m(\cdot|f_j)\right) = \int_{[0,1]^m} f^m(\mathbf{y}|f_j) p_i(x_i, \mathbf{y}) d\mathbf{y}, \tag{9}$$

where $p_i(x_i, \mathbf{y})$ indicates whether x_i wins in response to $\mathbf{y}$

$$p_i(x_i, \mathbf{y}) = \begin{cases} 1, & \text{if } |x_i - t(x_i, \mathbf{y})| \leq |y_j - t(x_i, \mathbf{y})| \ \forall j = 1, \ldots, m, \\ 0, & \text{otherwise,} \end{cases}$$

assuming the target value $t(x_i, \mathbf{y})$ is the weighted average of x_i and $\mathbf{y}$

$$t(x_i, \mathbf{y}) = \frac{1}{n} \left(x_i + \tfrac{n-1}{m} \sum_{j \leq m} y_j \right).$$

We assume $m \geq 2$ for computational reasons.[10] Unbiased behavior results if $m = n-1$, but the hypothesis $m = n-1$ is rejected in all cases. The estimated sample size m used by subjects is fairly consistent, between 2 and 6 in most cases, and guessing game subjects therefore seem to be overconfident or "believe in small numbers" during payoff estimation. The strategic level-k models are defined as follows.

Definition 3.3 ("INP")**.** The set of player types is $\mathcal{K} = \{0, 1, \ldots, K, \infty\}$. Players of type $k = 0$ randomize uniformly on $[0, 1]$, and for all $k \geq 1$, players of type k randomize according to $f_k = \mathcal{N}\left(\mu_k, \sigma_k^2\right)$ where $\mu_k \in \arg\max_{x_i} \pi(x_i | m, f^m(\cdot | f_{k-1}))$.

The acronym "INP" refers to Iterated best response with Normal Perturbations. Level $k = \infty$ induces equilibrium play if $m \geq 2$. Our second model is iterated logit response (ILR).

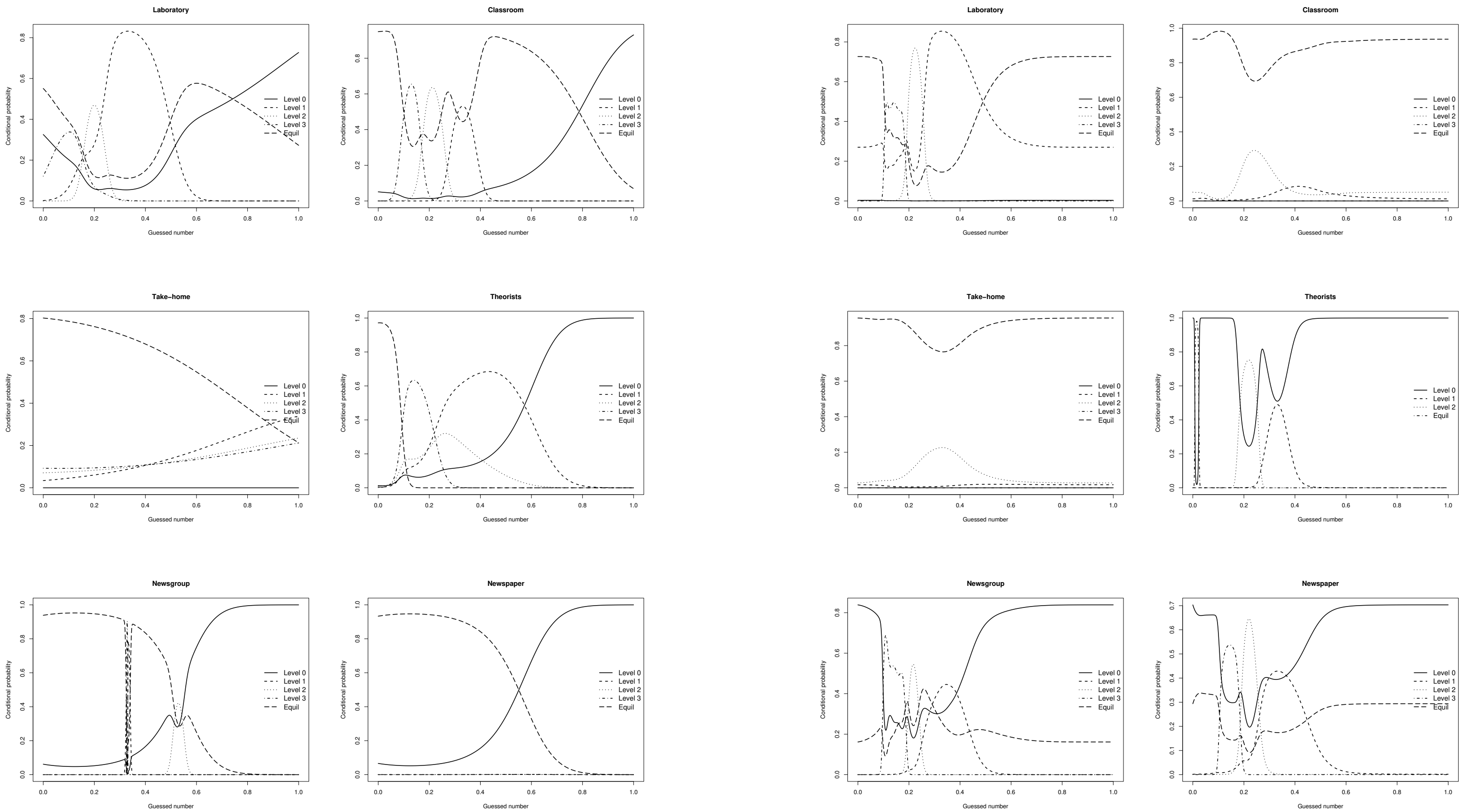
Definition 3.4 ("ILR")**.** Similar to INP, but now, for all $k \geq 1$, players of type k randomize on $[0, 1]$ according to the density (using $\lambda_k \geq 0$)

$$f_i(x_i) = \exp\{\lambda_k \cdot \pi(x_i | f^m(\cdot | f_{k-1}))\} \Big/ \int_0^1 \exp\{\lambda_k \cdot \pi(\tilde{x}_i | f^m(\cdot | f_{k-1}))\} \, d\tilde{x}_i. \quad (10)$$

Using standard procedure, we revert to maximum simulated likelihood to estimate these models (Train, 2003), and in particular we compute the expected payoffs $\pi(\cdot)$ by simulation. Otherwise, we proceed as before, maximizing jointly over all parameters using the Nelder-Mead algorithm. The model estimates are reported in Tables B.3 and B.4, the LL maxima and BICs are reported in Table 2, and the conditional type classifications can be found in Figure 5.

[10] This model of individual payoff computation generalizes to real $m \in \mathbb{R}$ as follows: the expected payoff is the weighted average of using $\lceil m \rceil$ and $\lfloor m \rfloor$, with weights $m - \lfloor m \rfloor$ and $\lceil m \rceil - m$, respectively.

Figure 5: A-posteriori classification based on strategic level-k models



(a) Iterated best response with normal perturbations

(b) Iterated logit response

Again, equilibrium types dominate the populations. In the INP models that maximize BIC, the equilibrium type $k = \infty$ makes up 60% of the population in all cases, and even 80% in five out six data sets. In the ILR models, the picture is similar aside from the fact that a significant share of level-0 subjects is estimated in the three non-student samples (around 50% in *Theorists*, *Newsgroup*, and *Newspaper*). Consequently, the conditional type classifications mainly fluctuate between $k = \infty$ and $k = 0$ according to these models, with very thin support for types $k \in \{1,2,3\}$, and in the majority of cases the estimated model dimension does not contain types $k \in \{1,2,3\}$ according to BIC (they do not in three out six cases for INP, and in five out of six cases for ILR).

These results show that iterated reasoning adds little explanatory power to models containing both $k = \infty$ and $k = 0$ if we model iterated reasoning in a strategically consistent way, but again, a caveat applies: overall the non-strategic model outperforms both INP and ILR. According to BIC, INP improves upon the non-strategic model in two non-student samples, *Theorists* and *Newsgroup*, but aside from this, the non-strategic model fits best. That is, the level-k mean μ_k is better described by $0.5 \cdot p^k$ than by the best response to level $k-1$, and the randomization employed by subjects is better described by the normal distribution than by logit response functions. Hence, these strategic models do not adequately reflect the way in which subjects reason in guessing games, but the following section shows that the nested logit model does.

4 Nested logit models of hierarchical reasoning

The hierarchical models considered in this section are two-level nested logit models, and as such, they are special cases of the *generalized extreme value* (GEV) model family (McFadden, 1981, 1984). In contrast to the choice-theoretic literature, we will of course consider mutual nested logit responses, i.e. nested logit equilibria. Many further generalizations of nested logit models exist, e.g. to multi-level hierarchies and to cross-nesting (see e.g. Wen and Koppelman, 2001, and Bierlaire, 2006), but to express our simple intuition, a two-level structure is sufficient: subjects first choose a "level" and second choose an action corresponding with this level. By endogenizing the level choice, we avoid the issues (e.g. non-uniqueness) that are involved with constructing belief systems that rationalize exogenous level assignments, while the basic intuition is kept intact. The choice procedure itself will be modeled in a way that is consistent with random utility maximization.

By "choosing a level" we mean that subjects choose a subset of $[0,1]$ from a partition of the strategy set as it results along the solution path according to cautious rationalizability, as $n \to \infty$, assuming the level-0 actions are uniform on $[0,1]$. We thereby follow the intuition expressed in the literature, and explicitly refrain from exploiting any degree of freedom in manipulating the underlying partition of $[0,1]$. The only difference to the literature is that we maintain level k as $\left(p^k \cdot 0.5, p^{k-1} \cdot 0.5\right]$, for consistency with cautious rationalizability, rather than centering it around $p^k \cdot 0.5$.

Definition 4.1 (Basic choice procedure)**.** Let $(\kappa_l)_{l \leq L}$ denote a partition of $[0,1]$, for some $L \in \mathbb{N}$. Players first choose $l \leq L$ and second choose $x_i \in \kappa_l$, using

$$\begin{aligned}&(\kappa_L, \kappa_{L-1}, \ldots, \kappa_1, \kappa_0)\\&= \left(\left[0, 0.5 \cdot p^{L-1}\right], \left(0.5 \cdot p^{L-1}, 0.5 \cdot p^{L-2}\right], \ldots, \left(0.5 \cdot p^1, 0.5 \cdot p^0\right], \left(0.5 \cdot p^0, 1\right]\right).\end{aligned}$$

The remainder defines the two-level nested logit model as it applies to guessing games. For all $x_i \in [0,1]$, define $l(x_i)$ such that $x_i \in \kappa_{l(x_i)}$. As before, the expected payoff of x_i in response to opponents picking f_j is denoted as $\pi\left(x_i | m, f^m(\cdot|f_j)\right)$. The nested logit response to opponents sticking to f_j has the density

$$f_i(x_i|f_j) = Q\big(l(x_i)\big) Q\big(x_i|l(x_i)\big) \tag{11}$$

where the density of choosing x_i conditional on having chosen category $l(x_i)$ is

$$\begin{aligned}Q\big(x_i|l(x_i)\big) &= \exp\left\{\lambda \cdot \pi\left(x_i|m, f^m(\cdot|f_j)\right) - \lambda \cdot \omega_{l(x_i)}\right\} / J(l(x_i))\\ &\quad\text{with } J(l) = \int_{\kappa_l} \exp\left\{\lambda \cdot \pi\left(\tilde{x}_i|m, f^m(\cdot|f_j)\right) - \lambda \cdot \omega_l\right\} d\tilde{x}_i\end{aligned} \tag{12}$$

and the probability of choosing category $l \leq L$ is

$$Q(l) = \exp\left\{\lambda\beta'\omega_l + \beta'' \ln J(l)\right\} / \sum_{l' \leq L} \exp\left\{\lambda\beta'\omega_{l'} + \beta'' \ln J(l')\right\}. \tag{13}$$

We say that the symmetric strategy profile $\sigma = (\sigma_i)_{i \in N}$ is a **nested logit equilibrium** if $\sigma_i = f_i(\cdot|\sigma_i)$. The extension to asymmetric strategy profiles is straightforward, but not relevant in guessing games.

The category weights $(\omega_l)_{l \leq L}$ remain to be defined, but regardless of their definition, this nested logit model is equivalent to the logit model if $\beta' = \beta'' = 1$. Choices of $\beta', \beta'' \neq 1$ induce interdependence of choices within categories. In the extreme case $\beta' = \beta'' = 0$, categories are chosen by a uniform draw, while the second-level choices of x_i conditional on category $l(x_i)$ are still made with positive precision if $\lambda > 0$. We consider two variants of GEV models. They differ with respect to the assumed category weights $(\omega_l)_{l \leq L}$: the weights are either zero or the average category payoff.

Definition 4.2 ("GEV-0")**.** The category weights are $\omega_l = 0$ for all $l \leq L$.

Definition 4.3 ("GEV-A")**.** The category weights are $\omega_l = \frac{1}{|\kappa_l|} \int_{\kappa_l} \pi(x_i|\cdot) dx_i$ for all l.

Besides GEV-equilibrium types, we control for the possibility that the population contains level-0 players (i.e. players that randomize uniformly), and in order to investigate the interaction between hierarchical and iterated reasoning, we will also allow for "level-1 types" who play a nested logit response to level 0. With respect to the number of categories, we assume $L \leq 20$. As before, we maximize the simulated likelihood jointly over all parameters to obtain efficient estimates (see Amemiya, 1978). The estimation results are reported in Table B.5 (ML estimates) and Table 2 (LL maxima and BICs).

Our discussion is kept brief, focusing on the main results. Controlling for level-0 players turned out unnecessary in the sense that their estimated shares in the population are zero in *all* GEV-models without a level-1 player type (for this reason, we did not count the level-0 type share as a parameter when computing the BIC in these cases). This suggests that the GEV-equilibrium comprehensively describes the distribution of guesses. Allowing for level 1, however, improves the goodness-of-fit in three out of six data sets (*Classroom*, *Take-home*, and *Newspaper*). Although the level-1 type shares are low (less than 20%), they are significant in these cases, which suggests that the subject pools are not optimally described by a homogenous GEV model in all cases. These level-1 actions can be modeled alternatively by introducing a second GEV-equilibrium type with appropriately adapted precision parameters, but to maintain the focus on the basic idea of hierarchical reasoning, an investigation of this is left as further research. The difference between GEV-0 and GEV-A is fairly small, i.e. the assumed category weights are only of secondary relevance. Similarly, the qualitative results do not depend on the assumed partition of $[0,1]$, but computations have shown that flexible parameterization of the partition would improve the goodness-of-fit in the three non-student samples.

In relation to the level-k models, the estimated GEV models fare rather well. The best GEV models are better than the best strategic level-k models (INP and ILR) in all cases, and they are better than the best non-strategic models in four cases, about as good in *Laboratory*, and it is worse than the best non-strategic model in *Newspaper*. The latter seems to be due to the fact that our GEV models focused on two-type populations (nested logit equilibrium plus level-1 players), which seems inadequate to model the heterogeneity of types in newspaper experiments. Finally, the GEV model fits better than the massively parameterized kernel density estimator with risk-minimizing bandwidth in all cases but *Laboratory*, and it is better than the kernel estimator using half this bandwidth in all cases but *Laboratory* and *Newspaper* (the exact numbers are reported in Table 2). We therefore conclude that the GEV models fit better overall than all alternative models considered in this study, and that they are close to what one may reasonably expect in terms of goodness-of-fit.

5 Concluding discussion

This paper discussed strategic choice in "p-beauty contest" guessing games by contrasting the level-k model of iterated reasoning with the nested logit model of hierarchical reasoning. Both of these approaches seek to capture the same intuition: subjects first pick categories and secondly pick numbers within categories. The level-k model does so indirectly, by assigning categories ("levels of reasoning") to players exogenously and assuming that players do not reflect on their category. Attempts to rationalize such category assignments based on beliefs derived from iterated best response sequences have been shown to be both insufficiently well-defined, since iterated best response sequences are generally not unique even after employing refinement concepts, and inconsistent, since best responses to randomizing opponents do generally not equate with guessing the expected target value. The latter applies in particular if players behave as if their opponents' guesses are correlated, which has been observed in our study, where subjects behave as if they had only $m \in [2,6]$ independent opponents, and previously by Ho et al. (1998). We have also seen that allowing for equilibrium types in the strategic level-k models (INP and ILR) implies that the majority of subjects are classified as equilibrium types. Finally, it was shown that ex-post classifications of subjects conditional on their guesses (using the model estimates and Bayes' Rule) are highly counter-intuitive for all data sets and all level-k models considered.

The nested logit model assumes that subjects choose their level themselves, based on perturbed payoff computations. By resorting to a hierarchical model of choice, we explain level choice in a way that is compatible with random utility maximization. More importantly, we avoid the assumption that subjects adhere to a specific belief system out of the continuum of belief systems that are compatible with iterated perfect/cautious response—while preserving a "level component" in the choice procedure and equilibration as it was observed in our level-k analysis. Nested logit equilibria are a currently neglected specification of quantal response equilibria (McKelvey and Palfrey, 1995), but the concept of nested logit response is well-established in choice theory and consumer theory.

We found that nested logit models generally fit better than strategic level-k models, and they fit better than non-strategic level-k models following Stahl (1996) in four out of six cases. These results are obtained despite our reluctance to exploit the freedom that exists in constructing hierarchical models. We focused on only two kinds of category weights (ω_l was either 0 or equal to the average payoff within category), we did not deviate from the (sub-optimal) partition implied by cautious rationalizability, and we restricted attention to populations of at most two GEV types (equilibrium and level 1). In light of this parsimony and the fact that two-level nested logit is an established and intuitive generalization of logit, let us briefly address the concern voiced by Haile et al. (2008). They show that if one fully exploits the distributional freedom in designing random utility models, including a relaxation of "i.i.d." in at least one dimension, then QRE fits are uninformative. To obtain perfect fits, one needs a random utility model with about one free parameter per strategy in generic games. Fitting two- and three-parametric GEV models in a game with a continuum of strategies, we seem to be save in this respect. A related concern is that equilibration is unlikely in one-shot guessing games. I tend to agree with this assessment, but let us recall that we did not assume equilibration from the outset. Instead, in a latent structure analysis allowing for non-equilibrium types (with corresponding logit/GEV response functions), we find that most subjects are equilibrium types in all data sets and according to all strategic models (i.e. INP, ILR, and GEV). It is a result rather than an assumption, albeit surprising.

To summarize, it seems reasonable to conclude that studying nested logit models appears to be a promising avenue for further research in analyses of strategic choice. As for studies of guessing games, let us sketch out how further research may extend the present work. Aside from investigating the role of category weights, category bounds, and heterogeneity of populations, it seems particularly interesting to study more deeply nested choice models. One may find, for example, that *Theorists* start out in the highest category (close to the Nash equilibrium) and stochastically work downwards to lower categories, that *Laboratory* subjects start out in the lowest category and stochastically work upwards, and that say *Newspaper* subjects start out in the middle and may deviate back and forth in either direction until they settle with a category. The exact paths that subjects follow in decision making may be highly individual ex post, but models of nested logit equilibrium allow us to study the regularities in the underlying stochastics, and this in turn, may lead to more precise predictions, and a better understanding of the limits of predictability, in future applications.

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Table 1: Goodness of fit of non-strategic models; $A \hat{=}$ standard $\mu_k = p^k \cdot 0.5$, $B \hat{=} \mu_k$ are free parameters

Levels	Laboratory		Classroom		Take-home		Theorists		Newsgroup		Newspaper	
	LL	BIC	LL	BIC	LL	BIC	LL	BIC	LL	BIC	LL	BIC
$A: 0-2$	**32.124**	**23.216**	**61.584**	**52.026**	58.434	48.579	46.432	36.465	57.106	47.085	3098.24	3080.29
$A: 0-3$	34.142	20.779	64.373	50.036	78.751	63.969	73.214	58.263	77.811	62.779	3709.37	3682.45
$A: 0-4$	34.656	16.839	66.516	47.400	**87.297**	**67.588**	86.683	66.749	**84.681**	**64.638**	4196.08	4160.19
$A: 0-5$	34.656	12.385	66.559	42.664	87.674	63.038	**99.467**	**74.549**	87.169	62.116	**4263.49**	**4218.62**
$B: 0-2$	**33.596**	**20.233**	65.273	50.935	60.048	45.266	**141.96**	**127.01**	**110.04**	**95.009**	4225.82	4198.90
$B: 0-3$	37.393	19.576	**71.568**	**52.451**	78.954	59.245	143.57	123.64	112.97	92.925	4309.96	4274.06
$B: 0-4$	37.393	15.122	71.697	47.801	**87.644**	**63.007**	148.32	123.40	118.91	93.855	**4336.34**	**4291.49**
$B: 0-5$	37.481	10.755	71.697	43.022	88.004	58.440	148.41	118.51	121.89	91.828	4344.63	4290.78
$A: 0-1, \infty$	29.419	20.510	56.676	47.118	58.767	48.912	119.89	109.92	81.448	71.427	4325.50	4307.55
$A: 0-2, \infty$	**34.015**	**20.651**	**64.856**	**50.519**	**87.718**	**72.936**	130.50	115.55	122.89	107.86	4496.14	4469.21
$A: 0-3, \infty$	34.142	16.324	66.624	47.507	88.010	68.301	**139.32**	**119.39**	**130.22**	**110.17**	**4698.16**	**4662.26**
$A: 0-4, \infty$	34.656	12.385	72.009	48.113	88.010	63.374	142.92	118.01	130.22	105.17	4703.00	4658.12
$B: 0-1, \infty$	31.605	18.242	65.240	50.902	65.035	50.253	143.59	128.63	**122.87**	**107.84**	4325.99	4299.07
$B: 0-2, \infty$	**36.567**	**18.750**	**71.772**	**52.655**	**88.053**	**68.344**	**148.59**	**128.66**	123.12	103.08	4496.81	4460.91
$B: 0-3, \infty$	37.393	15.122	72.795	48.899	88.326	63.690	149.62	124.70	132.55	107.50	**4701.14**	**4656.28**
$B: 0-4, \infty$	37.393	10.667	72.795	44.120	88.326	58.763	151.00	121.10	133.06	103.00	4706.31	4652.47

Table 2: Goodness-of-fit comparison over all models

	Laboratory		Classroom		Take-home		Theorists		Newsgroup		Newspaper	
	LL	BIC	LL	BIC	LL	BIC	LL	BIC	LL	BIC	LL	BIC
Kernel	31.464	n.a.	62.170	n.a.	68.263	n.a.	79.733	n.a.	53.498	n.a.	3685.85	n.a.
Kernel-0.5	38.12	n.a.	73.486	n.a.	88.304	n.a.	104.314	n.a.	77.106	n.a.	4368.10	n.a.
Non-Strat	**32.124**	**23.216**	**61.584**	**52.026**	**87.297**	**67.588**	**99.467**	**74.549**	**84.681**	**64.638**	**4263.49**	**4218.62**
Non-Strat$^{\infty}$	34.015	20.651	64.856	50.519	87.718	72.936	139.32	119.39	130.22	110.17	4698.16	4662.26
INP + L0	21.438	14.757	**59.606**	**52.437**	**59.208**	**51.818**	90.139	82.664	72.784	65.268	**3781.06**	**3767.60**
INP + L0-1	**30.072**	**18.936**	59.793	47.845	59.218	46.899	105.52	93.057	85.740	73.213	3780.39	3757.96
INP + L0-2	32.119	16.529	62.756	46.029	59.221	41.975	**120.35**	**102.91**	86.537	69.000	3780.67	3749.26
INP + L0-3	32.152	12.108	68.219	46.713	59.225	37.052	123.16	100.73	**98.549**	**76.001**	3780.40	3740.02
ILR + L0	**27.105**	**20.424**	**58.237**	**51.068**	**38.340**	**30.949**	**5.873**	**-1.603**	**21.207**	**13.691**	1171.19	1157.73
ILR + L0-1	30.481	19.345	58.245	46.297	38.370	26.052	8.554	-3.905	21.336	8.809	1278.92	1256.48
ILR + L0-2	32.283	16.693	58.899	42.172	38.837	21.591	14.926	-2.517	23.413	5.875	1453.61	1422.20
ILR + L0-3	33.420	13.375	58.913	37.407	38.920	16.747	19.863	-2.563	25.083	2.535	**1684.62**	**1644.24**
GEV-0	**31.000**	**22.091**	58.914	49.356	47.127	37.272	**163.52**	**153.56**	65.509	55.488	3704.89	3686.94
GEV-0 + L1	31.044	15.454	**80.878**	**64.151**	**142.64**	**125.39**	163.53	146.08	65.512	47.975	3815.38	3783.97
GEV-A	32.266	21.130	69.187	57.239	59.812	47.494	163.49	151.03	**110.01**	**97.486**	3835.75	3813.32
GEV-A + L1	32.386	12.342	80.878	64.151	142.64	125.39	163.49	141.07	110.02	87.472	**4151.58**	**4111.20**

Kernel and **Kernel-0.5** are kernel density estimates with standard bandwidth choice (using GNU R) and half this bandwidth, respectively. **Non-Strat** are the non-strategic models following Stahl (1996). **INP + L0-*K*** is iterated best response with normal strategy perturbations (*K* levels) and the corresponding equilibrium type. **ILR + L0-*K*** is the ILR model and *K* levels of iterated logit response. **GEV-0** and **GEV-A** are nested logit models.

Hierarchical Reasoning vs. Iterated Reasoning in Simple Guessing Games

Supplementary material

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A Relegated definitions and proofs

Proof of Proposition 2.1 Fix x_1 and define the auxiliary function $f(\cdot)$, to simplify notation relating to the conditions Eqs. (1) and (2), and also define g as follows.

$$f(x) = \frac{1-2\alpha}{2\alpha}(x_1 + x) \qquad\qquad g \in \{x|f(x) = x\} \Leftrightarrow g = \frac{1-2\alpha}{4\alpha-1}x_1 \tag{14}$$

The payoff maximizing x_1 can now be discussed by distinguishing three cases. First, it $x_1 \geq \frac{\alpha}{1-2\alpha}$, then the expected payoff is

$$\pi(x_1) = (1-x_1)^2 + \left[f^{-1}(x_1) + f^{-1}(1)\right] * (1-x_1). \tag{15}$$

This equates with

$$\pi(x_1) = \frac{1-(3-8\alpha)*x_1}{1-2\alpha} * (1-x_1) \qquad \text{if } \alpha > 1/4 \tag{16}$$

$$\pi(x_1) = (1-x_1)^2 + \left[\frac{2\alpha}{1-2\alpha} - x_1\right] * \left[1 - \frac{1-2\alpha}{2\alpha}x_1\right] \qquad \text{if } \alpha \leq 1/4 \tag{17}$$

and implies $\pi'(x_1) < 0$ in either case, i.e. for all $x_1 \geq \frac{\alpha}{1-2\alpha}$. Hence, the optimal guess satisfies $x_1 \leq \frac{\alpha}{1-2\alpha}$. Second, for all $x_1 \leq \frac{4\alpha-1}{1-2\alpha}$ (if any) the expected payoff is

$$\begin{aligned}\pi(x_1) &= 2*x_1*(1-x_1) - \left[x_1 - f^{-1}(x_1)\right] * [f(x_1) - x_1] + [f(x_1) - x_1] * [g - x_1] \\ &= 2*x_1*(1-x_1) - \frac{(2-6\alpha)^2}{(1-2\alpha)2\alpha}x_1^2 + \frac{(2-6\alpha)^2}{2\alpha(4\alpha-1)}x_1^2,\end{aligned} \tag{18}$$

and the first derivative of it is positive for all x_1 relevant to this case if and only if

$$2\alpha(1-2\alpha)^2+(2-6\alpha)^3-4\alpha\cdot(1-2\alpha)(4\alpha-1)>0, \tag{19}$$

i.e. iff $\alpha<0.3026$. If not, i.e. if $\alpha\geq 0.3026$, then the payoff maximizing choice is the zero of the first derivative of Eq. (18), which is

$$x_1^*=2/\left(4+\frac{(2-6\alpha)^2}{(1-2\alpha)\alpha}-\frac{(2-6\alpha)^2}{\alpha(4\alpha-1)}\right)=2/\left(4-\frac{(2-6\alpha)^3}{\alpha(1-2\alpha)(4\alpha-1)}\right). \tag{20}$$

If $\alpha<0.3026$, in turn, then the payoff maximizing guess x_1^* must satisfy $\frac{4\alpha-1}{1-2\alpha}\leq x_1^*\leq\frac{\alpha}{1-2\alpha}$, and in this case the expected payoff is

$$\begin{aligned}\pi(x_1)&=1-x_1^2-\left[x_1-f^{-1}(x_1)\right]*[f(x_1)-x_1]-\left[f^{-1}(1)-x_1\right]*[1-f(x_1)]\\&=1-x_1^2-\frac{(2-6\alpha)^2}{(1-2\alpha)2\alpha}x_1^2-\frac{[2\alpha-2x_1(1-2\alpha)]^2}{(1-2\alpha)2\alpha}.\end{aligned} \tag{21}$$

The zero of the first derivative is now

$$x_1^*=\frac{4\alpha(1-2\alpha)}{(2-6\alpha)^2+2(2-3\alpha)(1-2\alpha)}=\frac{2\alpha(1-2\alpha)}{(4-7\alpha)(1-3\alpha)}. \tag{22}$$

□

Lemma A.1 (Weak dominance)**.** *Let Y denote a convex subset of $[0,1]$, with $0=\min Y$ and $\bar{y}=\sup Y$, and assume all players $j\neq i$ are restricted to pick strategies from Y. Then, $x_i:=DR(\bar{y})$, see Eq.* (5)*, weakly dominates all $x_i'>x_i$.*

Proof. Define $X=\times_{i\in N}Y$, fix $i\in N$, define $\underline{x}_i$ such that

$$\underline{x}_i=p/n*\left(2\underline{x}_i+(n-2)\bar{y}\right)\qquad\Leftrightarrow\qquad\underline{x}_i=\frac{n-2}{n-2p}*p\bar{y}, \tag{23}$$

and define $X'_{-i}(x_i'):=\{x_{-i}\in X_{-i}\mid x_j>x_i'\ \forall j\neq i\}$. It is easy to verify that $x_i:=\underline{x}_i$ is a best response to all $x_{-i}\in X'_{-i}(\underline{x}_i)$ and that any $x_i'>x_i$ is not a best response to all $x_{-i}\in X'_{-i}(\underline{x}_i)$, since any such x_i' is not a best response if $\exists j\neq i$ such that $x_i<x_j<x_i'$. Next, for all $x_{-i}\notin X'_{-i}(x_i)$, i.e. for all $x_{-i}\in Y^{N\setminus\{i\}}\setminus X'_{-i}(x_i)$, there exists $j\neq i$ such that $x_j\leq x_i$. This implies that the target value satisfies $p/n*\left(x_i+\sum_{j\neq i}x_i\right)\leq x_i$ as well as $p/n*\left(x_i'+\sum_{j\neq i}x_i\right)<x_i'$ for all $x_i'>x_i$. It follows that $x_i'>x_i$ is a best response to any $x_{-i}\notin X'_{-i}(x_i)$ only if x_i is a best response to it. As a result, all $x_i'>\underline{x}_i$ are weakly dominated by $x_i=\underline{x}_i$. □

Definition A.2 (Perfect response)**.** Fix $x_i \in [0,1]$ and $x_{-i} \in [0,1]^{N\setminus\{i\}}$. We say that x_i is a perfect response to x_{-i} if there exist probability densities $f_{\varepsilon,j}(\cdot) : [0,1] \to \mathbb{R}_+$ for all $\varepsilon > 0$ and all $j \neq i$ such that x_i is the limit of best responses to the following mixed strategies (as ε approaches zero): for all $j \neq i$, j chooses x_j with probability $1-\varepsilon$ and randomizes according to $f_{\varepsilon,j}$ with probability ε.

Lemma A.3 (Perfect response)**.** *Fix $x' \in [0,1)$ and $x_i \in [0,1]$, and assume $n \geq 3$ as well as $BR_{\inf}(x') > 0$. x_i is a perfect response to $x_{-i} = \{x'\}^{N\setminus\{i\}}$ if and only if $x_i \in \left(BR_{\inf}(x'), \min\left\{DR(1), x'\right\}\right)$.*

Proof. "If"-part Fix any x' and assume first that $x_i \in \left[\mathrm{DR}(x'), \min\left\{\mathrm{DR}(1), x'\right\}\right)$. Define $y' := \mathrm{DR}^{-1}(x_i)$, which satisfies $y' \in [0,1]$ due to this restriction on x_i, as well as

$$\forall j \neq i\ \forall y \in [0,1]\ \forall \varepsilon : \qquad f_{\varepsilon,j}(y) := \begin{cases} c_\varepsilon, & \text{if } y' - \varepsilon \leq y \leq y' \\ 1, & \text{if } \mathrm{DR}(y') \leq y \leq \mathrm{DR}(y') + \varepsilon \\ \varepsilon^{1/\varepsilon}, & \text{otherwise} \end{cases}$$

with $c_\varepsilon \in \mathbb{R}$ such that $\int_0^1 f_{\varepsilon,j}(y)dy = 1$ (for all $j \neq i$). Given any $j \neq i$, let $Y_{\varepsilon,j}$ denote the random variable associated with density $f_{\varepsilon,j}$, for all ε. Also for all ε, let $\mathrm{Pr}_1(\varepsilon)$ denote the probability that $y' - \varepsilon \leq Y_{\varepsilon,j} \leq y'$, let $\mathrm{Pr}_2(\varepsilon)$ denote the probability that $\mathrm{DR}(y') + \varepsilon \leq Y_{\varepsilon,j} \leq \mathrm{DR}(y')$, and define $\mathrm{Pr}_3(\varepsilon) = 1 - \mathrm{Pr}_1(\varepsilon) - \mathrm{Pr}_2(\varepsilon)$. Note that $\mathrm{Pr}_3(\varepsilon) < \varepsilon^{1/\varepsilon}$, $\mathrm{Pr}_2(\varepsilon) = \varepsilon$, and $\mathrm{Pr}_1(\varepsilon) > 1 - \varepsilon - \varepsilon^{1/\varepsilon}$. It follows that, as ε approaches zero, $\mathrm{Pr}_2(\varepsilon)/\mathrm{Pr}_1(\varepsilon)^n \to 0$ and $\mathrm{Pr}_3(\varepsilon)/\mathrm{Pr}_2(\varepsilon)^n \to 0$, for all $j \neq i$. Guessing x_i in response to x_{-i} perturbed in this way yields an expected payoff of at least $1 - (n-1) * \varepsilon * \mathrm{Pr}_3(\varepsilon)$, while guessing any $x_i' < x_i$ yields $1 - (n-1) * \varepsilon^{n-1} * \mathrm{Pr}_1(\varepsilon)^{N-2} * \mathrm{Pr}_2(\varepsilon)$ at most, and guessing any $x_i' > x_i$ yields $1 - (n-1) * \varepsilon * \mathrm{Pr}_2(\varepsilon)$ at most (in both cases, this applies if $\varepsilon < |x_i' - x_i|$ and ε sufficiently close to zero). The former is generally greater than the payoff in either of the latter cases if ε is close to zero, and hence x_i is a perfect response to x_{-i}.

Second, assume $x_i \in (\mathrm{BR}_{\inf}(x'), \mathrm{DR}(x'))$. Again define $y' := \mathrm{DR}^{-1}(x_i)$, but the tremble density now becomes (noting that $x' < 1$ implies $y' < 1$)

$$\forall j \neq i \,\forall y \in [0,1]\, \forall \varepsilon: \qquad f_{\varepsilon,j}(y) := \begin{cases} c_\varepsilon, & \text{if } y' - \varepsilon \leq y \leq y' \\ 1, & \text{if } \mathrm{DR}(y'-\varepsilon) - \varepsilon \leq y \leq \mathrm{DR}(y'-\varepsilon) \\ \varepsilon^{1/\varepsilon}, & \text{otherwise} \end{cases}$$

with $c_\varepsilon \in \mathbb{R}$ such that $\int_0^1 f_{\varepsilon,j}(y)dy = 1$ (for all $j \neq i$). The argument is very similar to the one made above; the only difference is that $x_i' = \mathrm{DR}(y'-\varepsilon)$, for $y' := \mathrm{DR}^{-1}(x_i)$, is now the unique best response as ε approaches zero, and since it converges to x_i' as ε tends to zero, x_i is a perfect response to x_{-i}.

"Only-if"-part Since x_i is a perfect response only if it is a best response, $x_i > \mathrm{BR}_{\inf}(x')$ and $x_i < x'$ are necessary conditions. In addition, $x_i \leq \mathrm{DR}(1)$ is a necessary condition, since $x_i = \mathrm{DR}(1)$ weakly dominates all $x_i' > \mathrm{DR}(1)$ by Lemma A.1, and hence strictly dominates them under full support. □

Definition A.4 (Cautious response)**.** Same as perfect response with support of $f_{\varepsilon,j}$ restricted to $[0, x_j]$ for all ε and all $j \neq i$.

Lemma A.5 (Cautious response)**.** *Fix $x' \in [0,1)$ and $x_i \in [0,1]$, and assume $n \geq 3$ as well as $BR_{\inf}(x') > 0$. x_i is a cautious response to $x_{-i} = \{x'\}^{N\setminus\{i\}}$ if and only if $x_i \in (BR_{\inf}(x'), DR(x')]$.*

Proof. The proof is very similar to that of Lemma A.3, noting that $x_i \leq \mathrm{DR}(x')$ is a necessary condition in this case. For, by Lemma A.1, $x_i := \mathrm{DR}(x')$ weakly dominates all $x_i' > \mathrm{DR}(x')$ in response to $x_{-i} \in [0, x']^{N\setminus\{i\}}$, and hence, it strictly dominates all such x_i' if the support for trembles is restricted correspondingly. □

Proof of Proposition 2.2 First, since $x_i = 1$ is not a best response to any strategy profile in case $p < 1$, it is not rationalizable. Any $x_i < 1$, in turn, is a best response to any strategy profile x_{-i} satisfying $x_j \in (x_i, x_i/p)$ for all $j \neq i$. Such x_{-i} exist due to $p < 1$ for all $x_i < 1$, and hence all $x_i < 1$ are rationalizable. Second, Lemma A.1 implies that all $x_i > \mathrm{DR}(1)$ are not perfectly rationalizable, and thus $p < 1$ implies that $x_i = \mathrm{DR}(1)$ is not perfectly rationalizable either. Lemma A.3 shows that for all $x_i < \mathrm{DR}(1)$ there exists $x' < \mathrm{DR}(1)$ such that x_i is a perfect response to $x_{-i} =$

$\{x'\}^{N\setminus\{i\}}$, and hence all $x_i < \mathrm{DR}(1)$ are perfectly rationalizable. Third, note that for all $\bar{x} \in [0,1]$, Lemmas A.1 and A.5 imply that x_i is a cautious response to some $x_{-i} \in [0,\bar{x}]^{N\setminus\{i\}}$ if and only if $x_i \le \mathrm{DR}(\bar{x})$. Hence x_i is level-k cautiously rationalizable if and only if $x_i \le \mathrm{DR}^k(1)$, and thus x_i is cautiously rationalizable if and only if $x_i \le \lim_{k\to\infty} \mathrm{DR}^k(1) = 0$. □

Proof of Proposition 2.3 Since iterated response sequences must converge to rationalizable strategies as k tends to infinity, convergence toward r as restricted in the proposition for either of the three cases is a necessary condition (see Prop. 2.2). The following shows that the proposed restrictions are also sufficient. Fix $i \in N$, $y \in [0,1]$, and $x_{-i} = \{y\}^{N\setminus\{i\}}$. The guess $x_i \in [0,1]$ is a best response to x_{-i} iff $x_i < y$ and

$$\frac{1}{2}(x_i + y) > p/n\big[x_i + (n-1)y\big] \quad \Leftrightarrow \quad x_i > \frac{n*(2p-1)-2p}{n-2p} \cdot y =: \underline{x}_i, \tag{24}$$

i.e. iff $x_i \in (\underline{x}_i, y)$ in case $\underline{x}_i > 0$ and $x_i \in [0,y)$ otherwise. Now fix any $r \in [0,x_1)$ and consider the following sequence (x_k) for ε sufficiently close to zero.

$$\forall k > 1: \quad x_k = \begin{cases} r + \varepsilon^k, & \text{if } r \ge \frac{n*p-2p}{n-2p} \cdot x_{k-1}, \\ \frac{n*p-2p}{n-2p} * x_{k-1}, & \text{otherwise.} \end{cases} \tag{25}$$

Note that $p < 1$ implies $2p-1 < p$, and hence $\frac{n*p-2p}{n-2p} > \frac{n*(2p-1)-2p}{n-2p}$. It follows that for all $k > 1$, x_k is a best response when all opponents choose x_{k-1}, and since $\frac{n*p-2p}{n-2p} < 1$, the sequence converges to r. By Lemma A.3, (x_k) is also a perfect response sequence if $r \in [0, \mathrm{DR}(1))$, and by Lemma A.5 it is a cautious response sequence if $r = 0$ and $n > 2$. As for $n = 2$, Lemma A.5 implies that the unique cautious response sequence (x_k) satisfies $x_k = 0$ for all $k > 1$ and consequently converges to zero as well. □

Proof of Proposition 2.4 Fix $y \in [0,1)$, $\varepsilon > 0$, and let $\Pi_i(x_i|y,\varepsilon)$ denote the expected payoff of $i \in N$ in response to $x_{-i} = \{y\}^{N\setminus\{i\}}$ (under the assumed perturbations). Note that $\Pi_i(x_i|y,\varepsilon)$ can be expressed as

$$\Pi_i(x_i|y,\varepsilon) = \Pi_i^0(x_i|y) + \binom{n-1}{1} * \varepsilon * \Pi_i^1(x_i|y) + \binom{n-1}{2} * \varepsilon^2 * \Pi_i^2(x_i|y) + \dots \tag{26}$$

if $\Pi_i^m(x_i|y)$ denotes the expected payoff of $i \in N$ in case $m \leq n$ players randomize uniformly on $[0,1]$ and the remaining opponents pick y with probability 1. Note also that $d\Pi_i^0(x_i|y)/dx_i = 0$ for all $x_i < y$, and if we focus on $dx_i > 0$, as we do in the following, $d\Pi_i^m(x_i|y)/dx_i$ is well-defined and finite for all $m \geq 0$ and all $x_i < y$. Hence, as ε approaches zero, $d\Pi_i(x_i|y,\varepsilon)/dx_i \approx d\Pi_i^m(x_i|y)/dx_i$ in the sense that $\text{sign}\,(d\Pi_i(x_i|y,\varepsilon)/dx_i) = \text{sign}\,(d\Pi_i^m(x_i|y)/dx_i)$ if $d\Pi_i^m(x_i|y)/dx_i \neq 0$. We will see that $d\Pi_i^m(x_i|y)/dx_i \neq 0$ for all $x_i < y$, and hence the optimal x_i in response to y is $x_i^* = \sup\{x_i < y | d\Pi_i^m(x_i|y)/dx_i > 0\}$ if $x_i^* < y$, and an optimal x_i does not exist if $x_i^* = y$. For all $x_i < y$ and $dx_i > 0$, $d\Pi_i^1(x_i|y)/dx_i$ can be expressed as the sum of d^a and d^b, where (using $\alpha = p/n$)

$$d^a = \begin{cases} (1-\alpha)/\alpha, & \text{if } \alpha \cdot [1+(n-2)y+x_i] > (y+x_i)/2, \\ 0, & \text{otherwise,} \end{cases} \tag{27}$$

$$d^b = \begin{cases} -2, & \text{if } x_i < 2 * \text{DR}(y) \\ -1, & \text{otherwise.} \end{cases} \tag{28}$$

Since $n \geq 3$ is assumed, we know that $(1-\alpha)/\alpha > 2$, i.e. the sign of $d^a + d^b$ is independent of the actual value of d^b (which is -2 or -1). Now, two cases can be distinguished. First, if $y > \frac{p}{n-p(n-1)}$, the best response x_i is characterized by $\alpha \cdot [1 + (n-2)y + x_i] = (y+x_i)/2$ as ε approaches zero, i.e. it is

$$x_i = \frac{2\alpha + [2\alpha(n-2)-1]y}{1-2\alpha}. \tag{29}$$

Hence, if $x_1 > \frac{p}{n-p(n-1)}$, then the level-2 best response satisfies $x^2 > \frac{p}{n-p(n-1)}$, and as k approaches infinity, the best response sequence converges to $\frac{p}{n-p(n-1)}$. Second, if $y \leq \frac{p}{n-p(n-1)}$, then $d^a + d^b$ is positive for all $x_i < y$, due to $d^a = (1-\alpha)/\alpha > 2$ for all $x_i < y$, and thus a best response to y does not exist. As a result, under the assumed perturbation, a best response sequence starting in some $x_1 \leq \frac{p}{n-p(n-1)}$ does not exist. $\square$

B ML estimates

Table B.1: Parameter estimates of the non-strategic models

	Level 1		Level 2		Level 3		Level 4		Categ-Pars	
ρ_0	ρ_1	σ_1	ρ_2	σ_2	ρ_3	σ_3	ρ_4	σ_4	$\tilde{x}^0$	$\tilde{p}$
Laboratory										
0.39	0.4491 (0.1342)	0.0838 (0.0302)	0.1609 (0.0719)	0.0239 (0.0081)						
0.3296	0.5518 (0.1099)	0.1071 (0.0229)	0.1187 (0.0578)	0.0146 (0.0048)					0.401 (0.0637)	0.7676 (0.0662)
0	0.3241 (0.1258)	0.0618 (0.0262)	0.1701 (0.0729)	0.0234 (0.0071)	0.5058 (0.1199)	0.4918 (0.1222)				
0.0028	0.1412 (0.0485)	0.01 (0.0048)	0.1802 (0.0555)	0.0165 (0.0053)	0.6758 (0.3263)	0.3997 (0.1665)			0.4936 (0.0174)	0.6925 (0.019)
0	0.3788 (0.1316)	0.0695 (0.0223)	0.1723 (0.0705)	0.0241 (0.0072)	0.4154 (0.1376)	0.6014 (0.2804)	0.0336 (0.0372)	0.0209 (0.0146)		
0.0028	0.1412 (0.0447)	0.01 (0.0021)	0.1802 (0.0553)	0.0165 (0.0053)	0.6758 (0.3262)	0.3997 (0.1658)	0 (–)	– (–)	0.4936 (0.0162)	0.6925 (0.0185)
Classroom										
0	0.2112 (0.1026)	0.359 (0.1424)	0.7888 (0.1234)	0.1188 (0.0199)						
0.0833	0.6611 (0.1041)	0.1931 (0.0532)	0.2556 (0.1043)	0.0438 (0.0122)					0.2157 (0.115)	0.9102 (0.2663)
0	0.156 (0.2619)	0.3825 (0.3787)	0.3719 (0.1851)	0.0738 (0.0161)	0.472 (0.2216)	0.204 (0.1233)				
0	0.0783 (0.0302)	0.01 (0.0055)	0.393 (0.0945)	0.0516 (0.0096)	0.5287 (0.1024)	0.3267 (0.0418)			0.5953 (0.0385)	0.5555 (0.0343)
0	0 (–)	– (–)	0.3511 (0.1268)	0.0735 (0.0173)	0.1279 (0.0733)	0.0329 (0.0119)	0.5211 (0.1221)	0.3313 (0.0444)		
0	0.0789 (0.0301)	0.01 ($8e{-}04$)	0.3926 (0.0937)	0.0518 (0.0096)	0 (–)	– (–)	0.5285 (0.1014)	0.3467 (0.0431)	0.5917 (0.0384)	0.5589 (0.0348)
Take-home										
0	0 (–)	– (–)	1 (0.0925)	0.2156 (0.0162)						
0	0.6354 (0.2492)	0.2622 (0.0453)	0.3646 (0.2446)	0.1195 (0.0365)					0.2116 (0.0291)	0.99 (0.0913)
0	0.1638 (0.0392)	0.01 (0.001)	0.4445 (0.1303)	0.2964 (0.0505)	0.3917 (0.1281)	0.125 (0.0285)				
0	0.1643 (0.0385)	0.01 (0.0034)	0.4708 (0.1711)	0.2968 (0.0499)	0.3649 (0.1688)	0.1355 (0.0505)			0.5421 (0.1684)	0.6133 (0.1904)
0	0.1716 (0.0399)	0.01 (0.0035)	0.103 (0.0341)	0.01 (0.0028)	0.5136 (0.1495)	0.3175 (0.0503)	0.2119 (0.1407)	0.1353 (0.049)		
0	0.1714 (0.039)	0.01 (0.0755)	0.1047 (0.0337)	0.01 (0.0036)	0.5152 (0.1504)	0.3189 (0.0506)	0.2086 (0.1425)	0.1366 (0.0505)	0.5037 (0.0106)	0.6603 (0.0111)
Theorists										
0.0917	0 (–)	– (–)	0.9083 (0.0455)	0.2133 (0.0183)						
0.1137	0.627 (0.0587)	0.2237 (0.0243)	0.2593 (0.0396)	0.01 (0.002)					0.1 (0.055)	0.1 (0.0143)
0.0978	0 (–)	– (–)	0 (–)	– (–)	0.9022 (0.0415)	0.1795 (0.0159)				
0.1054	0.5333 (0.0815)	0.2492 (0.0314)	0.2368 (0.0431)	0.01 (0.0035)	0.1244 (0.0718)	0.0573 (0.0241)			0.1 (0.0233)	0.1 (0.0165)

	Level 1		Level 2		Level 3		Level 4		Categ-Pars	
ρ_0	ρ_1	σ_1	ρ_2	σ_2	ρ_3	σ_3	ρ_4	σ_4	$\tilde{x}^0$	$\tilde{p}$
0.1217	0 (−)	− (−)	0 (−)	− (−)	0 (−)	− (−)	0.8783 (0.0482)	0.1669 (0.0178)		
0.1063	0.0926 (0.0372)	0.0104 (0.0043)	0 (−)	− (−)	0.5327 (0.0666)	0.2535 (0.0291)	0.2684 (0.0408)	0.01 ($9e-04$)	0.5942 (1.947)	0.1016 (0.3299)
Newsgroup										
0.1016	0 (−)	− (−)	0.8984 (0.0464)	0.1852 (0.0173)						
0.0927	0.7072 (0.0572)	0.2778 (0.0259)	0.2 (0.0364)	0.01 (0.0025)					0.1 (0.0642)	0.1 (0.0157)
0.1131	0.0808 (0.0435)	0.01 (0.0212)	0 (−)	− (−)	0.8061 (0.0627)	0.1777 (0.0182)				
0.1048	0.6903 (0.0567)	0.2222 (0.0356)	0 (*NaN*)	0.1892 (0.1272)	0.2049 (0.0365)	0.01 ($9e-04$)			0.99 (0.0665)	0.1 (0.0393)
0.1196	0.0893 (0.0287)	0.01 (0.0768)	0.0378 (0.027)	0.01 (0.0768)	0 (−)	− (−)	0.7532 (0.0591)	0.1871 (0.0193)		
0.1292	0.6429 (0.0553)	0.1281 (0.02)	0 (−)	− (−)	0 (−)	− (−)	0.228 (0.0375)	0.01 (0.0012)	0.99 (0.0648)	0.2165 (0.0197)
Newspaper										
0.1101	0 (−)	− (−)	0.8899 (0.0068)	0.178 (0.0025)						
0.1111	0.8002 (0.0079)	0.2371 (0.0019)	0.0886 (0.0045)	0.01 ($7e-04$)					0.5161 (0.1135)	0.1 (0.0081)
0.1258	0 (−)	− (−)	0 (−)	− (−)	0.8742 (0.0067)	0.1787 (0.0026)				
0.1328	0.7563 (0.0089)	0.1713 (0.0047)	0 (−)	− (−)	0.1109 (0.0053)	0.0115 ($6e-04$)			0.99 (0.0108)	0.16 (0.0057)
0.1424	0.0768 (0.0038)	0.01 ($6e-04$)	0 (−)	− (−)	0 (−)	− (−)	0.7808 (0.0081)	0.1803 (0.0029)		
0.133	0.5659 (0.1004)	0.1308 (0.0114)	0.1184 (0.1025)	0.2778 (0.043)	0.0821 (0.0254)	0.0616 (0.0133)	0.1005 (0.0063)	0.0111 ($8e-04$)	0.99 (0.0107)	0.217 (0.0133)
0.1314	0.0799 (0.0039)	0.01 (0.001)	0 (−)	− (−)	0 (−)	− (−)	0.0702 (0.0706)	0.3049 (0.0476)		
0.1302	0.3335 (0.137)	0.1019 (0.0152)	0.2962 (0.1682)	0.2433 (0.0319)	0.1359 (0.0732)	0.1062 (0.0328)	0.017 (0.0206)	0.0212 (0.0082)	0.99 (0.0107)	0.2588 (0.0209)

Table B.2: Non-strategic models including "level" ∞

	Level 1		Level 2		Level 3		Level ∞		Categ-Pars	
ρ_0	ρ_1	σ_1	ρ_2	σ_2	ρ_3	σ_3	ρ_∞	σ_∞	$\tilde{x}^0$	$\tilde{p}$
Laboratory										
0	0.5192 (0.1355)	0.0903 (0.0171)					0.4808 (0.133)	0.575 (0.1469)		
0	0.567 (0.143)	0.0878 (0.0182)					0.433 (0.1354)	0.6789 (0.267)	0.4632 (0.9779)	0.6379 (1.3468)
0	0.3453 (0.1215)	0.0655 (0.0241)	0.1662 (0.0717)	0.0234 (0.0072)			0.4885 (0.1146)	0.606 (0.1529)		
0	0.1446 (0.0507)	0.01 (0.0044)	0.1847 (0.0593)	0.0168 (0.0055)			0.6707 (0.1059)	0.5157 (0.0819)	0.4941 (0.0171)	0.6918 (0.0188)
0	0.3241 (0.1258)	0.0618 (0.0262)	0.1701 (0.0729)	0.0234 (0.0071)	0.5058 (0.1199)	0.4919 (0.1222)	0 (−)	− (−)		

	Level 1		Level 2		Level 3		Level ∞		Categ-Pars	
ρ_0	ρ_1	σ_1	ρ_2	σ_2	ρ_3	σ_3	ρ_∞	σ_∞	$\tilde{x}^0$	$\tilde{p}$
0.0028	0.1412 (0.0478)	0.01 (0.0044)	0.1802 (0.0555)	0.0165 (0.0053)	0.6758 (0.3262)	0.3997 (0.1664)	0 (–)	– (–)	0.4936 (0.0172)	0.6925 (0.0189)
Classroom										
0.0748	0.204 (0.2097)	0.1275 (0.0535)					0.7212 (0.234)	0.2503 (0.0573)		
0	0.4658 (0.1278)	0.0744 (0.0155)					0.5342 (0.1298)	0.3709 (0.0468)	0.3758 (0.9125)	0.5541 (1.3456)
0	0.0127 (0.4637)	0.4882 (2.4749)	0.4637 (0.1335)	0.0773 (0.0142)			0.5236 (0.4355)	0.3582 (0.2392)		
0	0.0795 (0.0301)	0.01 (0.0069)	0.3937 (0.0928)	0.052 (0.0095)			0.5268 (0.1002)	0.3724 (0.0446)	0.5882 (0.0386)	0.5621 (0.0355)
0	0 (–)	– (–)	0.3711 (0.1225)	0.0749 (0.0168)	0.12 (0.073)	0.0323 (0.0124)	0.5089 (0.1178)	0.3786 (0.0481)		
0	0.0812 (0.0301)	0.01 (7e–04)	0.3646 (0.0886)	0.0474 (0.0098)	0.0414 (0.033)	0.01 (0.0829)	0.5127 (0.0953)	0.3787 (0.0458)	0.5809 (0.0219)	0.5692 (0.0193)
Take-home										
0	0.4264 (0.2675)	0.1906 (0.031)					0.5736 (0.2699)	0.2633 (0.0812)		
0	0.2667 (0.0887)	0.0602 (0.0152)					0.7333 (0.1071)	0.3344 (0.027)	0.4586 (0.9841)	0.6308 (1.3536)
0	0.1689 (0.0372)	0.01 (0.0052)	0.1057 (0.0333)	0.01 (0.0036)			0.7254 (0.0709)	0.3351 (0.0257)		
0	0.1683 (0.0392)	0.01 (0.0035)	0.1069 (0.0323)	0.01 (0.0074)			0.7248 (0.0715)	0.3354 (0.0257)	0.503 (0.0112)	0.661 (0.0113)
0	0.1736 (0.0396)	0.01 (0.0031)	0.1085 (0.0312)	0.01 (0.0068)	0.5345 (0.1468)	0.312 (0.0468)	0.1835 (0.1364)	0.153 (0.0722)		
0	0.1729 (0.043)	0.01 (0.0065)	0.1096 (0.0344)	0.01 (0.001)	0.5399 (0.154)	0.3123 (0.0476)	0.1776 (0.1407)	0.1517 (0.0769)	0.5031 (0.0127)	0.661 (0.012)
Theorists										
0.1025	0.3617 (0.0678)	0.1509 (0.0257)					0.5358 (0.0592)	0.0478 (0.011)		
0.1137	0.6275 (0.0588)	0.2236 (0.0286)					0.2588 (0.0396)	0.01 (0.002)	0.1 (0.1008)	0.1 (0.1008)
0.0972	0 (–)	– (–)	0.612 (0.0514)	0.1768 (0.0196)			0.2908 (0.0346)	0.01 (0.0024)		
0.1072	0.4851 (0.0896)	0.1967 (0.0696)	0.1296 (0.0629)	0.0149 (0.0055)			0.2781 (0.0417)	0.01 (4e–04)	0.3431 (0.6982)	0.4012 (0.4221)
0.1089	0 (–)	– (–)	0 (–)	– (–)	0.614 (0.0548)	0.176 (0.0216)	0.2771 (0.0366)	0.01 (0.0025)		
0.1068	0.4827 (0.0646)	0.2065 (0.0314)	0.0997 (0.0349)	0.01 (0.0012)	0.0385 (0.0283)	0.01 (0.0011)	0.2723 (0.0399)	0.01 (8e–04)	0.2342 (0.1178)	0.514 (0.1291)
Newsgroup										
0.1281	0.2947 (0.0945)	0.0816 (0.0152)					0.5773 (0.0902)	0.1467 (0.0333)		
0.128	0.6412 (0.0517)	0.1208 (0.0137)					0.2308 (0.0342)	0.01 (0.0021)	0.3945 (0.8772)	0.5925 (1.3175)
0.1295	0.0654 (0.1036)	0.0915 (0.0817)	0.5745 (0.1175)	0.118 (0.0217)			0.2306 (0.0371)	0.01 (0.0015)		
0.1289	0.6043 (0.0665)	0.1176 (0.0126)	0.0333 (0.0507)	0.0266 (0.0271)			0.2335 (0.0324)	0.01 (0.0024)	0.4595 (0.1762)	0.5292 (0.1989)
0.1315	0.0884 (0.0301)	0.01 (0.0042)	0.4851 (0.1121)	0.1294 (0.0198)	0.0658 (0.0892)	0.0469 (0.0479)	0.2292 (0.0323)	0.01 (0.0032)		

	Level 1		Level 2		Level 3		Level ∞		Categ-Pars	
ρ_0	ρ_1	σ_1	ρ_2	σ_2	ρ_3	σ_3	ρ_∞	σ_∞	$\tilde{x}^0$	$\tilde{p}$
0.1186	0.0953 (0.0329)	0.01 (0.0048)	0.4224 (0.2289)	0.1395 (0.0342)	0.1317 (0.1883)	0.0554 (0.0242)	0.232 (0.0517)	0.01 (0.0065)	0.4536 (0.0553)	0.7217 (0.0884)
Newspaper										
0.1354	0.0814 (0.0038)	0.01 ($6e{-}04$)					0.7832 (0.0082)	0.2224 (0.0031)		
0.1354	0.0815 (0.0038)	0.01 ($6e{-}04$)					0.7831 (0.0082)	0.2224 (0.0031)	0.5004 (0.0275)	0.667 (0.0366)
0.1445	0.0849 (0.0038)	0.01 ($7e{-}04$)	0.0586 (0.0039)	0.01 ($8e{-}04$)			0.7119 (0.0094)	0.2167 (0.0035)		
0.1446	0.085 (0.0038)	0.01 ($7e{-}04$)	0.0587 (0.0039)	0.01 ($8e{-}04$)			0.7118 (0.0094)	0.2166 (0.0035)	0.5007 (0.002)	0.6668 (0.0022)
0.124	0.0833 (0.004)	0.01 ($8e{-}04$)	0.2686 (0.0267)	0.2056 (0.0109)	0.3961 (0.0295)	0.1055 (0.0068)	0.1281 (0.0055)	0.0148 ($8e{-}04$)		
0.1262	0.0831 (0.0039)	0.01 ($6e{-}04$)	0.2966 (0.0311)	0.1987 (0.011)	0.3723 (0.0328)	0.1087 (0.0077)	0.1218 (0.0066)	0.0141 ($8e{-}04$)	0.522 (0.0128)	0.6395 (0.0156)
0.1268	0.0833 (0.0039)	0.01 ($7e{-}04$)	0.0988 (0.0793)	0.1931 (0.0307)	0.2535 (0.0479)	0.0966 (0.0071)	0.1169 (0.0064)	0.0136 ($8e{-}04$)		
0.1291	0.0832 (0.0039)	0.01 ($1e{-}04$)	0 (−)	− (−)	0.1809 (0.0449)	0.0809 (0.0088)	0.1186 (0.0057)	0.014 ($8e{-}04$)	0.476 (0.0086)	0.7014 (0.0126)

Table B.3: Iterated best response with normal perturbations

	Level 1		Level 2		Level 3		Equil		
ρ_0	ρ_1	λ_1	ρ_2	λ_2	ρ_3	λ_3	ρ_e	λ_e	m
Laboratory									
0.0941							0.9059 (0.0907)	0.2798 (0.0048)	3.0115 (0.016)
0.2602	0.4809 (0.1121)	0.0826 (0.002)					0.2589 (0.1589)	0.2361 (0.035)	3.2561 (0.005)
0.2093	0.4598 (0.0945)	0.081 ($8e{-}04$)	0.1295 (0.063)	0.03 (0.0012)			0.2015 (0.1995)	0.3521 (0.0246)	3.0716 (0.0019)
0.1525	0.4786 (0.1036)	0.0819 ($6e{-}04$)	0.0943 (0.06)	0.0303 (0.003)	0.0496 (0.072)	0.0737 (0.005)	0.225 (0.3156)	0.4022 (0.0491)	3.4717 (0.0035)
Classroom									
0.0891							0.9109 (0.0458)	0.1959 (0.0035)	3.2426 (0.0124)
0.0729	0.1077 (0.1477)	0.2165 (0.0201)					0.8194 (0.117)	0.1853 (0.0078)	3.2319 (0.0124)
0.0889	0.1062 (0.1965)	0.125 (0.0055)	0.1347 (0.0926)	0.0487 ($2e{-}04$)			0.6702 (0.2231)	0.1917 (0.0458)	3.217 (0.0182)
0.0476	0.1007 (0.0491)	0.0362 (0)	0.1647 (0.0306)	0.0309 ($4e{-}04$)	0.158 (0.0472)	0.0264 ($4e{-}04$)	0.5289 (0.1159)	0.2432 ($2e{-}04$)	3.1468 ($8e{-}04$)
Take-home									
0							1 (0.0098)	0.2416 (0.0081)	2.8732 (0.1521)
0	0.1088 (0.3337)	0.2355 (0.1505)					0.8912 (0.1832)	0.2284 (0.0082)	3.3483 (0.9736)
0	0.0854 (0.5542)	0.232 (0.0084)	0.1054 (0.5755)	0.2477 (0.0797)			0.8092 (0.0763)	0.2256 (0.0299)	3.7506 (0.0168)

	Level 1		Level 2		Level 3		Equil		
ρ_0	ρ_1	λ_1	ρ_2	λ_2	ρ_3	λ_3	ρ_e	λ_e	m
0	0.0822 (1.2689)	0.2294 (0.0404)	0.0941 (5.5198)	0.2457 (0.0782)	0.1023 (4.2155)	0.2522 (0.0481)	0.7214 (0.1306)	0.2223 (0.0114)	3.8701 (0.0299)
Theorists									
0.3792							0.6208 (0.0499)	0.0668 ($2e-04$)	3.832 (0.0024)
0.1044	0.2896 (0.0486)	0.131 (0.0019)					0.606 (0.0433)	0.0538 (0.003)	3.4584 (0.0022)
0.1044	0.2148 (0.0773)	0.1579 (0.0012)	0.1944 (0.0652)	0.0886 ($8e-04$)			0.4864 (0.0446)	0.0334 ($1e-04$)	3.0325 (0.0012)
0.1041	0.1795 (0.0298)	0.1365 (0.0095)	0.0997 (0.0669)	0.1261 ($5e-04$)	0.1293 (0.0547)	0.0496 (0.001)	0.4873 (0.0451)	0.0327 (0)	3.8984 ($1e-04$)
Newsgroup									
0.1212							0.8788 (0.0433)	0.1782 (0.006)	3.1094 (0.0279)
0.0938	0.0402 (0.0159)	0.0032 ($1e-04$)					0.866 (0.0359)	0.1674 (0.0039)	5.4127 (0.0021)
0.1321	0.0507 (0.0173)	0.0039 ($1e-04$)	0.0023 (0.0083)	0.0105 (0.0201)			0.8148 (0.0467)	0.1611 (0.0058)	5.6461 ($4e-04$)
0.1131	0.0469 (0.0176)	0.0044 ($1e-04$)	0.0069 (0.0136)	0.0163 ($2e-04$)	0.1041 (0.0242)	0.001 ($1e-04$)	0.729 (0.0485)	0.1665 (0.0053)	5.5655 ($7e-04$)
Newspaper									
0.1378							0.8622 (0.0063)	0.1782 ($2e-04$)	2.457 ($6e-04$)
0.1363	$2e-04$ (0.0034)	0.1915 (0.054)					0.8635 (0.0065)	0.1791 ($4e-04$)	2.6496 (0.0031)
0.1424	0 (−)	− (−)	0 (−)	− (−)			0.8576 (0.0064)	0.1768 ($2e-04$)	2.8562 ($7e-04$)
0.1406	$3e-04$ (0.0051)	0.2105 (0.0196)	$4e-04$ (0.0098)	0.2104 (0)	$4e-04$ (0.0084)	0.2104 (0.0012)	0.8583 (0.0072)	0.1767 ($5e-04$)	2.8579 (0.0025)

Table B.4: Iterated logit response

	Level 1		Level 2		Level 3		QRE		
ρ_0	ρ_1	λ_1	ρ_2	λ_2	ρ_3	λ_3	ρ_e	λ_e	m
Laboratory									
0							1 (0.1359)	4.8144 (0.0175)	3.2641 (0.0378)
0	0.2621 (0.1148)	8.2658 (0.0274)					0.7379 (0.1493)	5.1959 (0.0306)	3.7069 (0.0291)
0	0.4855 (0.1291)	5.8368 (0.0397)	0.168 (0.0612)	15.9037 (0.0165)			0.3465 (0.1178)	0.7929 (0.0321)	4.2835 (0.0179)
0.0012	0.4848 (0.1394)	6.2194 (0.033)	0.163 (0.0567)	19.888 (0.0183)	0.0367 (0.0321)	15.2417 (0.046)	0.3142 (0.2457)	0.8367 (0.1193)	5.098 (0.0159)
Classroom									
0							1 (0.0146)	6.249 (0.0318)	2 (0.0013)
0	0.0049 (0.0941)	6.7163 (0.3286)					0.9951 (0.0042)	6.2486 (0.0164)	2 (0.0057)

	Level 1		Level 2		Level 3		QRE		
ρ_0	ρ_1	λ_1	ρ_2	λ_2	ρ_3	λ_3	ρ_e	λ_e	m
0	0.0134 (0.0806)	5.8625 (0.035)	0.1119 (0.0311)	6.5013 (0.0146)			0.8747 (0.0916)	6.4678 (0.0204)	2 (0.0018)
0	0.0227 (0.0147)	5.8311 (0.0187)	0.1203 (0.0456)	6.5116 (0.018)	$3e-04$ (0.0243)	6.5222 (0.2945)	0.8566 (0.1178)	6.4689 (0.0308)	2 (0.0023)
Take-home									
0							1 (0.1142)	4.9867 (0.0227)	2.6908 (0.0152)
0	0 (−)	– (−)					1 (0.0044)	4.9056 (0.0234)	2.8806 (0.015)
0	0.0095 (0.2021)	1.8766 (0.0093)	0.1052 (0.1287)	7.3068 (0.0218)			0.8853 (0.2841)	5.0051 (0.0423)	2.8324 (0.0142)
0	0.0108 (0.2011)	1.8061 (0.0022)	0.1145 (0.1186)	8.406 (0.0224)	0 (−)	– (−)	0.8747 (0.279)	5.0499 (0.0139)	2.7833 (0.0041)
Theorists									
0.698							0.302 (0.0477)	15.832 (0.0458)	5.9584 (0.0128)
0.6709	0.0761 (0.007)	15.8849 (0.0411)					0.253 (0.0118)	17.5821 (0.0014)	7.0446 (0.0048)
0.6226	0.0494 (0.0256)	16.7127 (0.0099)	0.1052 (0.029)	16.711 (0.0332)			0.2228 (0.0101)	18.4968 ($9e-04$)	7.4117 ($4e-04$)
0.5639	0.0464 (0.0299)	17.2978 (0.0438)	0.0973 (0.0109)	17.5549 (0.0139)	0.1025 (0.0356)	17.0283 (0.0292)	0.19 (0.0421)	16.9117 (0.0334)	7.6555 ($4e-04$)
Newsgroup									
0.3928							0.6072 (0.0998)	6.4708 (0.0441)	2.7282 (0.0121)
0.4472	0.0684 (0.0666)	8.9952 (0.0534)					0.4843 (0.1043)	7.3124 (0.0298)	2.6742 (0.0062)
0.5064	0.1094 (0.048)	18.8729 (0.0286)	0.0319 (0.0333)	13.2986 (0.0925)			0.3522 (0.0923)	9.1548 (0)	2.2357 (0)
0.5064	0.1098 (0.077)	19.5951 (0.0261)	0.0592 (0.0395)	19.3964 (0.0285)	0.097 (0.0444)	11.6919 (0.0366)	0.2276 (0.2435)	5.3098 (0.0405)	2.6508 (0.0035)
Newspaper									
0.4263							0.5737 (0.0128)	7.7812 (0.0017)	2.7513 ($1e-04$)
0.4958	0.1091 (0.0082)	9.0895 (0.0044)					0.3951 (0.0106)	11.2319 (0.001)	2.5456 (0.001)
0.5608	0.1004 (0.0068)	10.7052 (0.0021)	0.0855 (0.0048)	9.6549 (0.0056)			0.2534 (0.0084)	13.7557 (0.0015)	2.8155 ($4e-04$)
0.5109	0.1113 (0.0064)	11.9588 (0.0107)	0.0898 (0.0041)	10.2531 ($4e-04$)	0.064 (0.0036)	9.1118 (0.0031)	0.2241 (0)	14.4376 (0)	2.8762 ($1e-04$)

Table B.5: Nested logit models

#1 ≙ GEV–Eq, #2 ≙ GEV–Eq+L1, #3 ≙ GEV-Aver–Eq, #4 ≙ GEV-Aver–Eq+L1

#	λ_1	β_1'	β_1''	ρ_1	λ_e	β_e'	β_e''	ρ_e	m	$L/10$
Laboratory										
1					6.0689 (0.0245)		0 (*NaN*)	1 (0.0081)	4.2902 (0.045)	0.4008 (0.0143)
2	6.7586 (14.5168)		0.0031 (0.0256)	0.0942 (0.0094)	6.3437 (0.0152)		0.0081 ($9e-04$)	0.9058 (0.0236)	4.3538 (0.0176)	0.4002 (0.0134)

#	λ_1	β'_1	β''_1	ρ_1	λ_e	β'_e	β''_e	ρ_e	m	$L/10$
3					6.1756 (0.0281)	0.8501 (0.003)	0 (*NaN*)	0.9983 (0.0072)	4.3623 (0.0152)	0.3 (0.1247)
4	6.317 (4.4954)	0.9247 (1*e*−04)	0.0049 (0.0292)	0.1 (0.1342)	5.7697 (0.009)	0.8883 (0.0082)	0.0018 (7*e*−04)	0.8984 (0.0529)	4.2652 (7*e*−04)	0.3165 (0.027)
Classroom										
1					7.4446 (0.004)		0.7738 (0.0055)	1 (0.0087)	2.1387 (0.0152)	0.7932 (0.261)
2	7.3423 (25.378)		0 (*NaN*)	0.0828 (0.0325)	6.6816 (0.0032)		1.3199 (2*e*−04)	0.9172 (0.0327)	2 (0.079)	2 (0.0174)
3					1.3644 (0.0119)	2.3021 (0.0101)	0.006 (5*e*−04)	1 (0.0087)	2.6799 (0.0335)	0.5299 (0.0161)
4	7.3423 (25.378)	0	0 (*NaN*)	0.0828 (0.0325)	6.6816 (0.0032)	0	1.3199 (2*e*−04)	0.9172 (0.0327)	2 (0.079)	2 (0.0174)
Take-home										
1					3.7128 (0.0121)		0.6282 (0.0019)	0.9981 (0.0038)	3.0181 (0.0097)	0.9009 (0.0166)
2	2.1751 (0.0084)		0 (*NaN*)	0.2004 (0.0772)	5.4347 (0.0157)		0.984 (0.0039)	0.634 (0.1407)	2 (0.1772)	2 (0.1682)
3					0.2593 (0.0145)	6.9833 (0.0134)	0.0161 (8*e*−04)	1 (0.009)	2.9418 (0.002)	0.5092 (0.0155)
4	2.1751 (0.0084)	0	0 (*NaN*)	0.2004 (0.0772)	5.4347 (0.0157)	0	0.984 (0.0039)	0.634 (0.1407)	2 (0.1772)	2 (0.1682)
Theorists										
1					0.2763 (0.0111)		0.2131 (0.0256)	1 (0.0113)	3.3036 (0.0477)	2 (0.0096)
2	0.2763 (0.0346)		0.2344 (0.1691)	0.1 (0.0512)	0.304 (0.0224)		0.2131 (0.0095)	0.9 (0.0501)	3.3036 (0.0372)	2 (0.0096)
3					0.1021 (0.0121)	0.1558 (0.0282)	0.2143 (0.0441)	1 (0.011)	3.1904 (0.0418)	2 (0.0096)
4	0.1021 (1.0389)	0.1558 (4.2761)	0.2143 (0.0234)	0.1 (1.3201)	0.1021 (0.0031)	0.1885 (0.0135)	0.2143 (0.0307)	0.9 (1.3201)	3.1904 (0.0752)	2 (0.0046)
Newsgroup										
1					0 (*NaN*)		0 (*NaN*)	0.9999 (0.0079)	6.7296 (*NaN*)	0.6181 (0.0299)
2	0.0117 (0.0027)		0 (*NaN*)	0.1015 (0.0279)	0 (*NaN*)		0 (*NaN*)	0.8985 (0.0288)	6.8449 (*NaN*)	0.6178 (0.0298)
3					0 (*NaN*)	0.2604 (*NaN*)	0.3228 (0.0384)	1 (0.0133)	4.2597 (*NaN*)	2 (0.0093)
4	0 (*NaN*)	0.2604 (*NaN*)	0.3906 (0.0042)	0.1 (3.29)	0 (*NaN*)	0.2604 (*NaN*)	0.3228 (0.2421)	0.9 (3.2899)	4.2597 (*NaN*)	2 (2*e*−04)
Newspaper										
1					3.0158 (0.0016)		0.3955 (0.002)	1 (8*e*−04)	3.0067 (0.0021)	1.2001 (0.0013)
2	3.7746 (4*e*−04)		0 (*NaN*)	0.1169 (0.0116)	4.7766 (0.0121)		0.5043 (8*e*−04)	0.8831 (0.0162)	3.24 (0.0012)	1.2214 (8*e*−04)
3					1.8878 (0.0023)	1.4323 (0.0049)	0.4334 (9*e*−04)	1 (0.0013)	4.7483 (0.004)	1.2399 (0.0089)
4	2.0381 (3*e*−04)	1.8469 (0)	0.276 (0)	0.0981 (0.1061)	1.9649 (0.003)	1.4532 (0.0047)	0.4459 (0.0016)	0.9019 (0.1062)	4.9269 (0.0058)	1.255 (1*e*−04)