Munich Personal RePEc Archive

Monopoly Supply

Kripalani, G.K. and Tolley, G.S. and Graves, P.E. and Sexton, R.L.

Western Michigan University, University of Chicago, University of Colorado, Pepperdine University

1990

Online at https://mpra.ub.uni-muenchen.de/19897/
MPRA Paper No. 19897, posted 11 Jan 2010 07:35 UTC

Monopoly Supply

G. K. KRIPALANI, G. S. TOLLEY, P. GRAVES, AND R. SEXTON*

I. Introduction

Conventional economic theory takes the position that the supply curve construct is only applicable to the purely competitive firm, being neither meaningful nor relevant in a monopoly context. A sampling of representative statements over the last two decades supporting this conclusion are: [Lloyd 1967, p. 168] "for a monopolistic firm, the supply curve is not easy to come by;" [Nicholson 1985, p. 423] "However, to connect the resulting series of equilibrium points on the market demand curves would have little meaning. This locus might have a very strange shape depending on how the market demand curve's elasticity (and hence its associated MR curve) changes as the curve is shifted. In this sense the monopoly firm has no well defined supply curve" and; [Lancaster 1974 pp. 192-3] "It is meaningless to ask what output the monopolist will supply at each of several prices . . . A market with a monopoly supplier has no supply curve of the ordinary kind." Some authors mention a *point* supply curve for the monopoly case.

Gould and Ferguson [1980, p. 261] correctly note "If demand shifts while cost conditions remain stationary, a monopoly supply curve can be constructed. But the curve depends upon the precise set of demand shifts." However, they also state [p. 261], "in neither case does the monopoly supply curve have the clear and exact meaning that competitive supply has; and the concept of supply price is entirely meaningless in monopoly."

The apparent lack of generality of the supply curve notion will be shown to result from an inappropriately narrow definition of the supply curve. That is, the supply curve has been defined as the locus of quantities a firm will be willing to supply at various prices, holding constant the conditions of supply. A broader definition, subsuming this as a special case, is to define a firm's supply curve, *ceteris paribus*, as the locus of profit maximizing supply price-quantity equilibrium positions.

Supply generally depends on demand parameters, except in the case of perfect competition. If single parameter shifts are observed (e.g., parallel shifts outward due to rising income), then the expected supply response can be estimated. This latter approach allows the supply response to depend on more general demand parameters, since in all market structures other than perfect competition firms are not, in fact, price takers.

Section II presents more formally the generalized definition of supply, while Section III considers in greater detail some empirically interesting special cases of monopoly supply prior to the summary and conclusion of Section IV.

II. A Generalized Definition of Supply

A formal definition of a supply curve, from which the supply curve of a perfectly competitive firm emerges as a special case, is as follows: Given the marginal cost curve and given the specific family of demand curves (numbering infinity of order one) describing the possible implicitly stipulated shifts in demand conditions facing a profit-maximizing firm, the supply curve shows the quantities the firm will supply at various prices.

In general, the demand relation is an n-parameter family of curves where n, the number of demand shifters, is a positive integer. Changes in only one parameter, holding the remaining $(n-1)$ parameters constant, generates a family of demand curves infinite in number. If all the n parameters are allowed to undergo simultaneous and independent change, the number of demand curves thus generated will be of the *nth* order of infinity. As can be readily shown, demand curves of order of infinity greater than 1 will not generate a supply curve but rather a two-dimensional supply region in the (P, Q) plane lying above the marginal cost curve. A region may be defined as a set

*Western Michigan University, University of Chicago, University of Colorado, and Pepperdine University, respectively. The authors wish to acknowledge the helpful comments of any anonymous referee on an earlier version of this paper.

of points in the (P, Q) plane such that to each P of an infinite set of P's there corresponds an infinite set of Q's, and conversely. Unless otherwise stipulated, the discussion presumes changes in only one parameter.

Hence, $P = a$ represents a one-parameter possible family of infinite demand curves — that faced by a perfectly competitive firm. A two-parameter family of demand curves, say $P = a - bQ$, has associated with it two supply curves, one obtained by varying a holding b constant (parallel demand shifts) and another which results from varying b holding a constant (rotational changes).

While Section III presents a more detailed treatment of such specific cases, for present purposes, the fundamental similarity of all such shifts requires emphasis. That is, it is always changes in the demand curve facing the firm which determine the whole set of a firm's equilibrium supply price-quantity positions. In the special case of the perfectly competitive firm, the price it faces is equivalent to the demand curve at that price. Hence, to apply the concept of supply to firms under imperfect competition, what is needed is not a change in supply concept but rather a more explicit statement of the concept as suggested here. The particular family of demand curves must be explicitly identified in order to derive the set of supply price-quantity equilibrium positions.

In the general case, the demand curve facing the firm is

$$P = D(Q, a, b, d, \ldots), \qquad (1)$$

where $a, b, d, \ldots$ are parameters of demand. The equation of the marginal revenue curve will then involve the same parameters:

$$r' = r'(Q, a, b, d, \ldots). \qquad (2)$$

The total cost function takes the following general form, where $\bar{e}, \bar{f}, \bar{g}, \ldots$ are the fixed parameters of the cost function:

$$C = C(Q, \bar{e}, \bar{f}, \bar{g}, \ldots). \qquad (3)$$

Since throughout the discussion, a single marginal cost curve is assumed, the equation of the marginal cost curve may be simplified as:

$$C' = C'(Q). \qquad (4)$$

Setting marginal revenue equal to marginal cost, the profit-maximizing quantity is determined as:

$$Q^* = Q^*(a, b, d, \ldots). \qquad (5)$$

Substituting this quantity value into the demand relation, equation (1), the profit-maximizing price as a function of the demand curve parameters is obtained:

$$P^* = P^*(a, b, d, \ldots). \qquad (6)$$

The locus of equilibrium supply points is defined by

$$P = D(Q, a, b, d, \ldots), \qquad (1)^1$$

subject to,

$$r'(Q, a, b, d, \ldots) = C'(Q) \qquad (7)$$

and is obtained by eliminating one parameter of the system between (1) and (7). Choosing parameter a for elimination, the locus can be solved as

$$P = F(Q, b, d, \ldots). \qquad (8)$$

This equation represents the supply set, the profit-maximizing equilibrium supply price-quantity values, for the general case. The perfect competition case is seen as a special case in which eliminating the a parameter results in a supply set not involving any demand parameters.

III. Special Cases of Monopoly Supply

To illustrate the analysis of the preceding section, consider the family of parallel linear monopoly demand curves having constant (non zero) slope:

$$P = a - \bar{b}Q. \qquad (9)$$

Marginal revenue in this case is:

$$r' = a - 2\bar{b}Q. \qquad (10)$$

Assuming for simplicity a linear marginal cost curve:

$$C' = \bar{e} + \bar{f}Q, \tag{11}$$

the set of supply price-quantity equilibrium points is given by the demand relation subject to the optimizing condition that marginal revenue equals marginal cost:

$$a - 2\bar{b}Q = \bar{e} + \bar{f}Q. \tag{12}$$

Solving for a in equation (12) and substituting into equation (9) yields the monopoly supply curve for parallel linear demand shifts:

$$P = \bar{e} + (\bar{f} + \bar{b})Q. \tag{13}$$

That supply in perfect competition is a special case, which may be seen by letting $\bar{b}$ equal 0 in equation (13). In this case, the supply relation degenerates into a form identical to the marginal cost curve.

Tables 1 & 2 give derivations of the monopoly supply curves for specified linear and non-linear demand curve cases.

To add empirical content to the preceding

TABLE 1

Summary Results Showing the Supply Curve When the Demand Curve Family and the Marginal Cost Curves are Straight Lines

	A system of concurrent st. lines with different slopes	**A system of parallel st. lines with changing intercept**
A. Linear Demand Curve	$P = a - bQ$	$P = a - bQ$
B. Specified System of Demand Curves	$P = \bar{a} - bQ$ $\bar{a}$ constant b changing	$P = a - \bar{b}Q$ $\bar{b}$ constant a changing
C. Given Marginal Cost Curve (straight line)	$C' = \bar{e} + \bar{f}Q$	$C' = \bar{e} + \bar{f}Q$
D. Supply Curve	$P = (\bar{a} + \bar{e})/2 + \bar{f}Q/2$ straight line supply curve	$P = \bar{e} + (\bar{f} + \bar{b})Q$ straight line supply curve
E. Elasticity of Supply $\eta\,(s)$ Compared with Elasticity of Marginal Cost $\eta(C')$	$\eta(s) \geq \eta\,(C')$	$\eta(s) \leq \eta\,(C')$
F. Constant Marginal Cost Case	$\bar{f} = 0$	$\bar{f} = 0$
(i) supply curve	$P = (\bar{a} + \bar{e})/2$	$P = \bar{e} + \bar{b}Q$
(ii) relations between supply price and marginal cost	a constant supply price	a supply price increasing with quantity

TABLE 2

Summary Results Showing the Supply Curve When the Family of Demand Curves and the Marginal Cost Curves are Non-Linear Constant Elasticity Curves.

	Elasticity Parameter λ Held Constant	Efficiency Parameter a Held Constant
A. Non-linear Demand Curve	$P = aQ^{\lambda}$	$P = aQ^{\lambda}$
B. Specified System of Demand Curves	$P = aQ^{\bar{\lambda}}$ $\bar{\lambda}$ constant, a changing	$P = \bar{a}\,Q\lambda$ $\bar{a}$ constant, λ changing
C. Given Marginal Cost Curve (non-linear constant elasticity curve)	$\bar{C}\,' = eQ^{\bar{\mu}}$	$C' = e\bar{Q}^{\,\bar{\mu}}$
D. Supply Curve	$P = \bar{e}\,Q^{\bar{\mu}}/(1 + \bar{\lambda})$	$P = \bar{a}\,Q\ \exp.\ [\bar{e}\,Q^{\bar{\mu}}/P\text{-}1]$
E. Elasticity of Supply $\eta(s)$ Compared with Elasticity of Marginal Cost $\eta(C') = 1/\bar{\mu}$	same constant elasticity as the marginal cost $\eta(s) = \eta(C') = 1/\bar{\mu}$	same constant elasticity as the marginal cost $\eta(s) = \eta(C') = 1/\bar{\mu}$
F. Constant Marginal Cost Curve	$\bar{\mu} = 0$	$\bar{\mu} = 0$
(i) Supply curve	$P = \bar{e}/(1 + \bar{\lambda})$	$P = \bar{a}\,Q\ \exp.\ [\bar{e}/P\text{-}1)$
(ii) relation between supply price and marginal cost	constant supply price, a multiple of marginal cost $\bar{e}$	"

discussion, consider the case in which income is the demand shifter and the wage rate is the shifter of marginal cost. Let, as before, the demand curve and marginal cost curve be represented by:

$$P = a - bQ \tag{14}$$

and

$$C' = e + fQ. \tag{15}$$

Taking the parameter a to be a linear function of income, Y, and the parameter e to be a linear function of the wage rate, W, equations (14) and (15) may be written as:

$$P = a_0 + a_1Y - bQ \tag{16}$$

$$C' = e_0 + e_1W + fQ. \tag{17}$$

The marginal cost equals marginal revenue condition yields:

$$e_0 + e_1 W + fQ = a_0 + a_1 Y - 2bQ.$$

Hence, the reduced form expressing Q in terms of shifters Y and W is:

$$Q = [a_0 - e_0 + a_1 Y - e_1 W]/(f + 2b). \quad (18)$$

Substituting the value of Q in the demand relation and simplifying the reduced form for P in terms of shifters Y and W results in:

$$P = [a_0(f + b) + be_0 + a_1(f + b)Y - be_1 W] / (f + 2b) \quad (19)$$

One may write:

$$Q = \gamma_{qo} + \gamma_{qy} Y + \gamma_{qw} W \quad (20)$$

and

$$P = \gamma_{po} + \gamma_{py} Y + \gamma_{pw} W, \quad (21)$$

where γ's are functions of parameters of demand and marginal cost. Given a set of corresponding observations on P, Q, Y, and W, one may regress each P and Q, Y, and W, and estimate the six γ's, noting that:

$$\gamma_{qo} = (a_0 - e_0)/(f + 2b)$$

$$\gamma_{qy} = a_1/(f + 2b)$$

$$\gamma_{qw} = -e_1/(f + 2b)$$

$$\gamma_{po} = [a_0(f + b) + be_0]/(f + 2b) = \{[a_0 - e_0)(f + b)]/(f + 2b)\} + e_0$$

$$\gamma_{py} = a_1(f + b)/(f + 2b)$$

$$\gamma_{pw} = -be_1/(f + 2b).$$

Hence, estimates of parameters b, e, and f may be obtained as:

$$\hat{b} = \text{estimate of } b = \gamma_{pw}/\gamma_{qw} \quad (22)$$

$$\widehat{(b + f)} = \text{estimate of } (f + b) = \gamma_{py}/\gamma_{qy}. \quad (23)$$

Hence,

$$\hat{f} = \text{estimate of } f = \widehat{(f + b)} - \hat{b} = [\gamma_{py}/\gamma_{qy}] - [\gamma_{pw}/\gamma_{qw}] \quad (24)$$

$$\hat{e}_o = \text{estimate of } e_o = \gamma_{po} - \widehat{(f + b)}\, \gamma_{qo} = \gamma_{po} - [\gamma_{py}/\gamma_{qy}] \cdot \gamma_{qo}. \quad (25)$$

Thus, if the marginal cost curve remains fixed and the family of demand curves is generated by money income changes where money income changes are assumed to affect parameter a of the demand relation, then the supply curve, given by $P = \bar{e} + (\bar{f} + \bar{b})Q$, can be estimated. Note that when the marginal cost curve remains fixed and unchanged one obtains $e = e_0$, since $e_1 = 0$. Hence, when the marginal cost curve is unchanged and demand shifts are generated by changes in parameter a due to income changes, the supply curve can be estimated. The estimates of the remaining parameters a_0, a_1, and e_1 can also be obtained from the six γ values.

IV. Summary and Conclusions

In the preceding sections, it is shown that a supply curve in the ordinary sense does exist and can be derived, which corresponds to a specified family of demand curves described by changes in a single parameter of demand. If the demand curve has n parameters, an equal number of supply curves can be derived, one corresponding to each family of demand curves generated by variation in one of the parameters, holding the remaining n-1 constant. Since the demand curve family facing the perfectly competitive firm has the form $P = a$, there is only one supply curve in this case. Further, that supply curve does not involve demand parameters since the sole demand parameter, a, is eliminated in its derivation. In general, this is not the case and the marginal cost schedule will be insufficient to describe the supply relation for imperfectly competitive firms.

In the teaching of graduate and undergraduate microeconomics, students often become confused by conflicting discussion suggesting that monopoly supply is meaningless, irrelevant, or a point. If there is only one demand curve, the equilibrium price-quantity supply will be one point regardless of whether monopoly or perfectly competitive firms are being considered. The broader definition of supply advocated here offers the pedagogic advantage of stressing the fundamental uniformity of the supply concept in the theory of the firm. With shifting demand, there is a meaningful supply relation in both the competitive and monopoly case.

As indicated in Section III, this generalized definition of supply has strong implications for empirical work, showing that it is tractable to estimate monopoly supply relations as well as marginal cost curves in the monopoly case and thus opening the door to useful new empirical work.

REFERENCES

Milton Friedman, *Price Theory*, Aldine, 1976, pp. 126-7.

Ragner Frisch, "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in a Model with Many Sectors," *Econometrica*, 27, 1959, pp. 327-41.

J. P. Gould and C. E. Ferguson, *Microeconomic Theory*, Irwin, Inc., 1980.

Kelvin Lancaster, *Introduction to Modern Microeconomics*, Rand-McNally, 1974, pp. 192-3.

Cliff Lloyd, *Microeconomic Analysis*, Irwin, Inc., 1967, p. 186.

Walter Nicholson, *Microeconomic Theory*, The Dryden Press, 1985, p. 423.

Joan Robinson, *The Economics of Imperfect Competition*, Macmillan Co., 1948, Chapter 4.

Myron Ross, *Supply Curve in Imperfect Condition*, (Unpublished paper, privately circulated), 1976.

Henri Theil and R. B. Brooks, "How Does the Marginal Utility of Income Change When Real Income Changes?," University of Chicago, Center for Mathematical Studies in Business and Economics, Center Report #7012, March 1970.