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## **Relative risk aversion: increasing or decreasing?**

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RELATIVE RISK AVERSION:  
INCREASING OR DECREASING?

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I. Introduction

The existence of risk aversion in portfolio theory can be explained by positing a concave utility function of wealth. In some cases it is useful to construct some measure of risk aversion rather than merely accept its existence.

Arrow [2,3] defines a measure of relative risk aversion which is invariant to positive linear transformations and further involves only the first two derivatives of the utility function:<sup>1</sup>

$$(1) \quad R(W) = -WU''(W)/U'(W) = \text{Relative Risk Aversion.}$$

In (1) it is assumed that  $U'(W)$  is positive (more wealth always being desirable). With  $U''(W)$  negative (for those who are risk averse), the size of  $R(W)$  will measure the degree of risk aversion of the individual under consideration, larger  $R(W)$  implying more risk aversion.  $R(W)$  could, of course, be a constant or any more general function of wealth,  $W$ .

Arrow hypothesizes that  $R(W)$  is an increasing function of  $W$  and demonstrates that, if a safe (zero variance) asset exists, then a larger  $R(W)$  implies a greater proportion of the portfolio will be put in the safe form as wealth is increased. For differentiable utility functions, this is equivalent to the hypothesis that if both wealth and the size of a bet are proportionally increased, an individual would be less willing to accept the bet and would require more favorable odds (see Stiglitz [17]).

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<sup>1</sup>This is actually  $R_R(W)$  in Arrow's article where the subscript is used to differentiate this measure from the related  $R_A(W)$ , absolute risk aversion. The absolute risk aversion measure and its related hypothesis are intuitively plausible and will not be discussed here--hence the subscript simplification.

The theoretical justification for advancing this hypothesis involves the boundedness of the utility function which does not preclude any relative risk aversion behavior over relevant (finite) wealth ranges. Empirically, the Arrow hypothesis explains the time series results of Friedman [6], Selden [16], Latane [10], and Meltzer [12]. These findings, reviewed recently in Goldfeld [7], suggest wealth elasticities of demand for cash balances in excess of unity--a result predicted by the increasing relative risk aversion hypothesis. As income (presumed to be a proxy for wealth) increased at a certain percent over time, average cash balances were found to increase at a greater percent.

Section II argues that the Strict Safety-First Principle, which has portfolio implications consistent with decreasing, rather than increasing, relative risk aversion, is more plausible than the Safety Principle. Section III brings together cross-sectional and (reexamined) time series evidence which point empirically toward acceptance of the decreasing relative risk aversion hypothesis, while Section IV contains concluding remarks.

## II. Relative Risk Aversion and Safety-First

Pyle and Turnovsky [15] have shown, for the simple case of one riskless and one risky asset, that a parallel exists between the implicit risk attitudes, as wealth changes, in the various chance-constrained portfolio models and the relative risk aversion of expected utility maximizers. In particular, the follower of what has come to be called the Safety Principle will exhibit increasing relative risk aversion behavior. The Safety Principle is being adhered to when one attempts to minimize the probability of falling below some fixed ("disaster") level of wealth. Similarly, a follower of the Strict Safety-First Principle exhibits wealth-induced portfolio changes which would be akin to those of an expected utility maximizer with decreasing relative risk aversion. In this safety model, the decision maker is presumed to be interested in maximizing the expected value of his portfolio subject to some probability constraint on going below the disastrous level of wealth. The watershed case of behavior analogous to constant relative risk aversion results from following the Safety-First Principle--the decision maker picks a probability of "disaster" and then maximizes the "disaster" level obtainable.

So far nothing substantive has been added to the discussion of whether Arrow's hypothesis is correct or incorrect. Another dimension has, however, been added: many people, who lack an intuitive feel for any variant of risk aversion, might have definite preferences among the safety models. It is quite likely that few people would possess intuition about the sign of the derivative with respect to wealth of Arrow's risk measure (itself a wealth-weighted ratio of first and second derivatives of the utility function of wealth). Yet a strong preference for a particular safety-first model might, through its portfolio implications, point to the preferred relative risk aversion hypothesis. It is argued in this section that most people, upon considering the safety models, find the Strict Safety-First Principle to be the most appealing of these models.

Indeed, the Safety Principle, which implies behavior that is consistent with the hypothesis of increasing relative risk aversion, has behavioral implications which most people would find quite at odds with "normal" behavior.

What evidence can be presented to support the notion that the Strict Safety-First Principle (maximizing the expected value given the constraints on the probability of falling below the fixed disaster level) is to be preferred to the other safety models? Agnew *et al* [1] have shown that a portfolio selected by the Strict Safety-First Principle has the advantage of being "Baumol efficient," as well as just "Markowitz efficient" (see Baumol [4], Markowitz [11]). Baumol argues convincingly that the investor is not interested in the standard deviation of the portfolio *per se*. Most people would "prefer the danger of coming out ten dollars short on a 100 dollar expected return as against an eight dollar shortfall below a fifty dollar expected return." [4, p. 177]. Hence, although for a given expected return greater variance is always undesirable, this need not be the case when the expected value is allowed to vary.

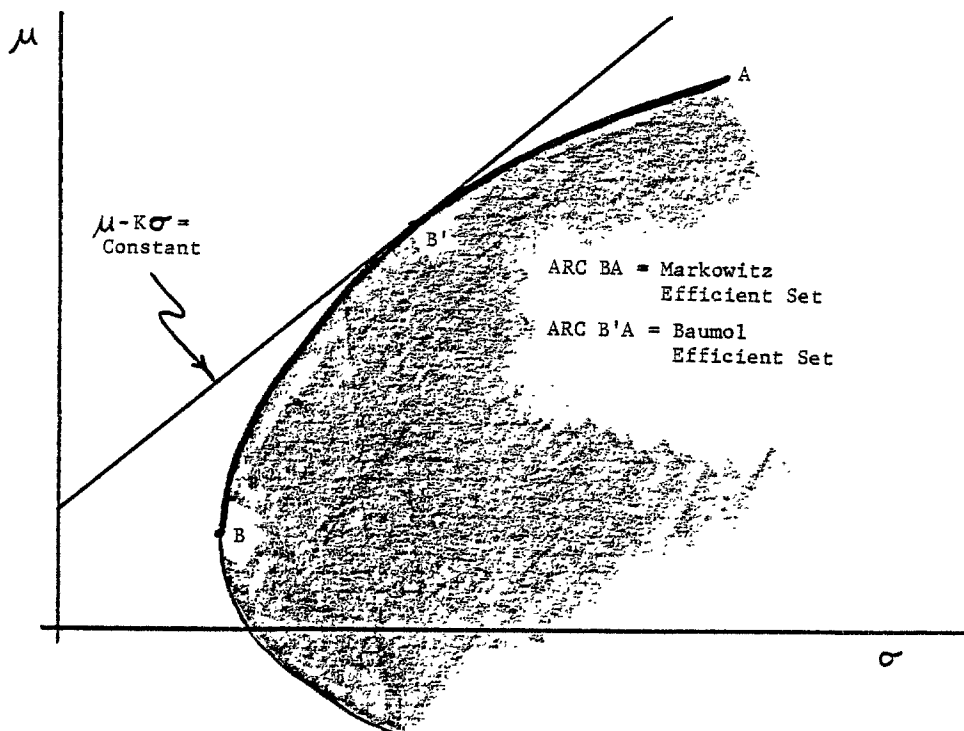
Baumol then proposes an alternative measure of investor risk [4, pp. 176-77]:

One possibility is suggested by standard probability theory which tells us the following: If our basic random variable (the return on our investment) is normally distributed, then there is only about a 16 percent probability that the realized return will ever fall below  $E - \sigma$ , there is only a 2 percent probability that it will ever be lower than  $E - 2\sigma$ , no more than a 0.1 percent probability that it will fall below  $E - 3\sigma$ , etc. Thus, we may say that the risk involved in a given portfolio is represented by  $E - K\sigma$  for some appropriately chosen value of  $K$ . Here  $E - K\sigma$  may be considered the lower confidence limit for the investor's return.

This alternative efficiency set is shown to be a subset of the Markowitz efficient set of portfolios in Figure 1. The follower of the Safety Principle (minimizing the probability of disaster) will, as his wealth increases, move out of any given Baumol efficient range (moving into arc  $BB'$  from arc  $B'A$  in Figure 1). The owner of a business firm would, under the Safety Principle, exchange his equity for safety as his wealth increases--eventually selling his business entirely. As one gets wealthier he would commit an ever-larger percentage of his assets to the safe asset (cash, savings accounts, savings bonds, etc.) and an ever-smaller percentage to risky assets (corporate stock, etc.). These behavioral implications appear to be quite opposed to what most people would plan on doing as they get wealthier--expanding their business, buying relatively more corporate stock, and so on.

It would appear that most peoples' thought processes are more likely to be illustrated in the following manner: Consider an individual faced with a coin-toss type of situation. He realizes that the odds are, objectively 50-50, and he might wish to be indifferent toward acceptance of the bet. Indeed, if the bet were sufficiently small as to affect negligibly the probability of his going below his disaster level, as e.g., an approximately fair lottery ticket, the individual may be near indifference. However, as the bet size gets larger, relative to his wealth and its disaster level, he is increasingly wary of it--the probability of going below his disaster level becomes more appreciable as the bet gets larger (e.g., house or car insurance is paid to avoid the bet). But, if

FIGURE I



his wealth and the bet size are both to double, he would be more likely to take the bet than prior to the doubling since he would have more left if he were to lose the bet--the probability of going below any given disaster level will be lower after both the bet and wealth have been doubled. This more plausible behavior pattern results from following the Strict Safety-First Principle (maximizing expected value under constraints), while under the Safety Principle our decision maker would be ever less likely to take a bet if its size and wealth were to rise in proportion.

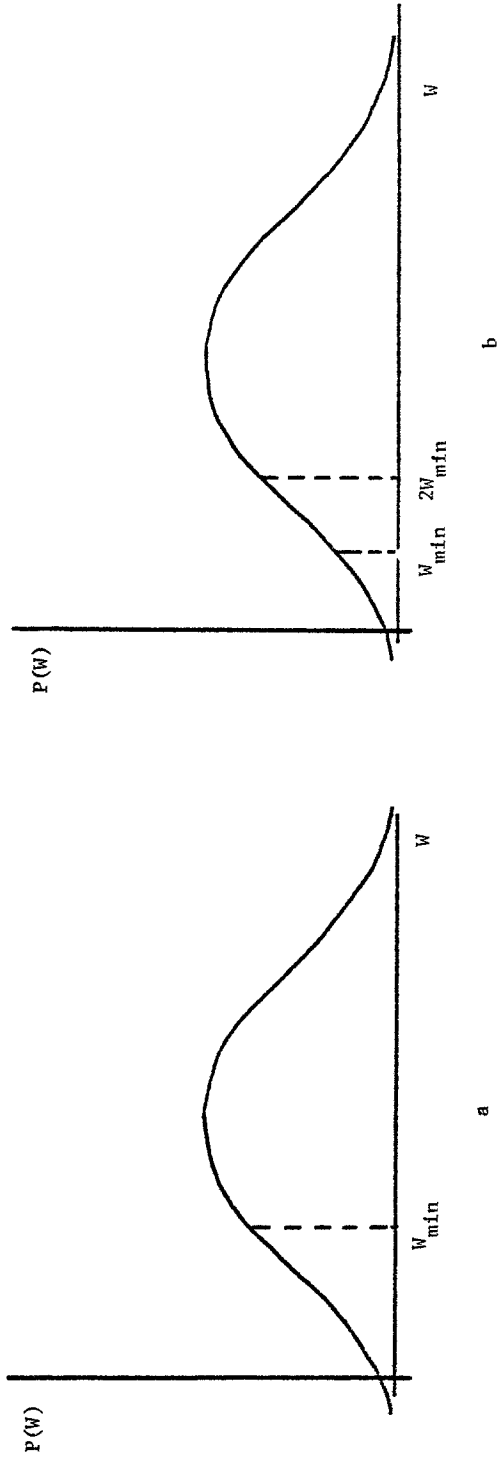
It is quite easy to see graphically how the wealth-doubling experiment will affect the portfolio behavior of the follower of the Strict Safety-First Principle. In Figure 2a, the (hypothetical) random return of the optimal portfolio (the one with maximum expected value given no more than the chosen probability of going below  $W_{\min}$ ) is depicted. Figure 2b is drawn under the assumption that our individual's entire portfolio has been doubled. Note that this is equivalent to just relabeling the returns axis, doubling all previous values. Is this new portfolio optimal under our behavioral assumptions? No, it certainly is not--the probability of going below  $W_{\min}$  is now much lower than before. There will be some other portfolio which will yield a higher expected return and which will have a variance such that the probability of going below  $W_{\min}$  will again be that which our individual is willing to accept. Specifically, the probability of going below  $2W_{\min}$  after doubling wealth is the same as the probability of going below  $W_{\min}$  prior to the wealth increase. Unless the level of  $W_{\min}$  rises proportionally to the wealth increase as wealth is increased, the doubled portfolio will be nonoptimal.  $W_{\min}$  might indeed rise (some part of the disaster level of wealth may be wealth-related), but to get an Arrow-type implication from this model,  $W_{\min}$  would have to more than double when wealth is doubled. This is unlikely in view of the fact that at least some of the precautionary desires reflected in the disaster level are not related to wealth, as for example college costs, bankruptcy expenses, etc. The intuitive plausibility of the Strict Safety-First Principle seems to lend sufficient a priori justification for expecting portfolio choices comparable to those obtained under decreasing relative risk aversion in the portfolio over most of the relevant range.

From the foregoing discussion, it is not at all intuitively clear, based on purely theoretical considerations, that the hypothesis of increasing relative risk aversion is sufficient to describe individual portfolio behavior.

### III. Empirical Evidence

Time series data on average cash balances appear to support the increasing relative risk aversion hypothesis. In the studies cited in the introduction, the wealth elasticity of demand for cash (presumably a low-risk asset) was found to be greater than unity. Hence the cash proportion increased over time based on these findings. Further, Perlman [13,14] reports additional support for the time series findings using cross-country comparisons of money holdings and income. However, Graves [8] has recently shown that both of these apparent findings are reversed when theoretically important socioeconomic variables are included (percent urban, median age, household

FIGURE II



size, and the like). Upon inclusion of these omitted variables the time series and cross-country results are consistent with decreasing, not increasing, relative risk aversion.<sup>2</sup>

Table 1 displays regression results similar to those in Graves [8] except that they are updated to include an additional ten-year period and for convenience the dependent variable is presented in terms of the cash proportion of income (a wealth proxy) rather than as velocity. These results indicate that the Friedman findings are not robust to alternative, plausible specification. Any of a number of variables (percent urban, PURB or median age, AGE shown here)<sup>3</sup> will upon inclusion reverse the Friedman result which forms the empirical basis of the Arrow hypothesis. In fact, the Durbin-Watson statistics suggest that the model is much better specified with these additional economic variables included. It would appear that the assumption of a stable utility function over a 90-year period is not a valid assumption. Rather the true downward relation between income and average cash balances has been shifting upward over time as the population has become progressively more urban and older.

Moreover, it is not even clear that the Arrow hypothesis is appropriately tested with aggregated data. It would be impossible to say how an individual would react to an increase in his wealth--since an individual increase might, in large part, be relative to other individuals in the economy. There are strong reasons for suspecting that the utility derived from wealth is not independent of the amount of wealth possessed by others. This might be one explanation of the militancy of poverty groups now when they are better off from an absolute wealth standpoint than ever before in our history. To take a less serious example, consider the behavior of some children on Christmas morning: the jealousy often observed can easily be explained by a "relative-versus-absolute" wealth hypothesis.<sup>4</sup>

These informal observations have, in fact, been corroborated (see for example Becker and McClintock [5] by game theory experiments which demonstrated that "whether a prize of 5 dollars is valued more than a prize of 4 dollars may be a function of what the other player is receiving." This relative wealth phenomenon is the clear counterpart to consumption emulation (keeping up with the Jones).

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<sup>2</sup>The implied relative risk aversion in Graves' regression results is not significantly less than unity in all specifications. However, the results are seen not to be inconsistent with the cross-sectional findings which are discussed at a later point in the text.

<sup>3</sup>Other variables considered were education and number of people per household as well as the rate of inflation and the interest rate. While these had the expected sign, the high multicollinearity among the independent variables for such a small number of observations (11 census years for which all data were available) resulted in very imprecise coefficients. That increased income resulted in smaller cash proportions, counter to both Arrow's hypothesis and Friedman's findings, continued to hold regardless of the inclusion or exclusion of these additional variables.

<sup>4</sup>The presumption here is that any differences in gifts received are random--that there is no "permanent component" representing overt preferences among the offspring. If such discrimination does, in fact, exist any observed sibling jealousy becomes much more rational and may be explained without resorting to relative wealth hypotheses.



Since the increasing relative risk aversion hypothesis is designed to explain the behavior of an individual in the face of a (presumably relative) wealth change, it is not justifiable to use data of the type employed by Arrow in testing his hypothesis. Cross-sectional data, in which high wealth implies high relative wealth, would seem to be the appropriate way to test the Arrow hypothesis since this hypothesis was designed to explain individual, as opposed to aggregate, behavior. This oversight by Arrow in empirically evaluating his hypothesis is quite important, particularly in view of the attention given the relative-versus-absolute distinction in consumption theory.<sup>5</sup>

When one considers the cross-sectional evidence as would appear appropriate from the foregoing discussion, the hypothesis of decreasing relative risk aversion is strongly supported. In many countries, using a variety of empirical techniques and specifications on many data sources, the proportion of assets devoted to nonrisky forms has declined with wealth rather than increased as would be expected under the increasing relative risk aversion hypothesis. The result that cash balances declined with wealth for any reason definition of either variable is extremely robust in all of the cross-sectional studies (see Graves [9] for greater detail on these findings and their relation to the time series results). As already indicated, the time series and cross-country results cited as support for the increasing relative risk aversion hypothesis exhibit sensitivity to alternative specification; indeed these results support the cross-sectional findings upon inclusion of variables which are theoretically expected to matter in the portfolio decision.

#### IV. Conclusions

The increasing relative risk aversion hypothesis was seen to lack theoretical plausibility when one considers the implications of the various safety models. In view of the recent work casting doubt on the veracity of the time series and cross-country findings and in light of the strong opposing results for more appropriate cross-sectional data, it appears that in portfolio situations where the degree of relative risk aversion is important one should prefer the hypothesis of decreasing relative risk aversion over relevant ranges of wealth.

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<sup>5</sup>This point, in portfolio theory, is what makes Markowitz's modification of the well-known Friedman and Savage utility function compelling. He suggests that the origin of an individual's utility curve for money be taken as his customary financial status, and that on either side of the origin the curve will be first concave, then convex. Then, if a person's wealth changes and everyone else's does too, he might continue to behave in risky situations as before. But if his wealth changes relative to others, his utility curvature reflects changing behavior patterns in risky situations. The point is that the time series data omit too much information, a common aggregation problem.

TABLE 1

CASH PROPORTION REGRESSIONS  
(t-VALUES IN PARENTHESIS)

Equation	Constant	Income <sup>a</sup>	PURB <sup>b</sup>	AGE <sup>c</sup>	R <sup>2</sup>	S.E.	D-W
(1)	.2550 (3.81)	.00789 (2.15)			.340	.107	.567
(2)	-.1051 (1.32)	-.01698 (3.20)	.0153 (5.04)		.84	.056	2.06
(3)	-.6673 (4.09)	-.00678 (2.21)		.0457 (5.76)	.87	.050	1.77
(4)	-.7566 (4.64)	-.02179 (4.99)	.0072 (2.15)	.0441 (3.96)	.96	.033	1.95

<sup>a</sup> Per capita income in 1958 dollars.

<sup>b</sup> Percent of the population that is urban.

<sup>c</sup> Median age of the population.

Source for the variables: Historical Statistics of the United States, Colonial Times to 1970. (U. S. Bureau of the Census, Washington, D.C., 1975). The observations are at ten-year intervals from 1870 to 1970 inclusive, 11 observations in total.

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