# Policy irreversibility and interest rate smoothing

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**Abstract**

Many empirical studies argue that the inertial behavior of the policy rates in industrialized countries can be well explained by a linear partial adjustment version of the Taylor rule. However, the explanatory power of the lagged interest rate has been questioned from various points of view. This paper formally examines a situation in which a central bank has an aversion for frequent policy reversals. Imposing an irreversibility constraint on the control space makes the lagged interest rate a state variable, but the policy function cannot then be expressed as a partial adjustment form even if the original Taylor rule is the correct policy function in the absence of the constraint. The simulation results reveal that the conventional regression tends to falsely support the functionally misspecified partial adjustment model. This implies that the significant role of the lagged interest may simply reflect the central banks' reversal aversion.

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*Keywords*: gradualism, interest rate smoothing, irreversibility, Taylor rule.

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# 1 Introduction

It is widely recognized that the credibility of central banks is a key to the effectiveness of monetary policy. Major central banks thus have an aversion for losing credibility, and that this aversion would make them behave conservatively, especially when the future economic condition is highly uncertain.

The gradual adjustment of the policy rates has long been deemed as good evidence of the central banks' preference for conservatism. The most popular way to measure the extent of gradualism is to estimate the coefficient on the lagged interest rate in the following Taylor rule:

$$i_t = \psi_i i_{t-1} + (1 - \psi_i)(c_0 + c_1\pi_t + c_2 y_t),$$

where $i_t$ is the policy rate at time $t$, and $\pi_t$ and $y_t$ are inflation and an output gap, respectively. This partial adjustment version of the Taylor rule, which is occasionally called the "generalized Taylor rule", has been heavily used not only in empirical studies, but also in theoretical studies especially in the new-Keynesian DSGE models (e.g., Clarida, Galí and Gertler, 1999, Woodford, 2003a,b).[1]

Despite its popularity, the partial adjustment version of the Taylor rule has been criticized. Rudebusch (2002) argues that if the central bank conducts interest rate smoothing, then future short-term interest rates must be predicted quite accurately. He shows that the predictability of future short-term rates is not high enough to support interest rate smoothing, insisting that the explanatory power of the lagged interest rate is due to the presence of serially correlated omitted variables. Sack (2000) states that the presence of parameter uncertainty would lead the Fed to act less aggressively compared to the optimal policy under certainty. Trehan and Wu (2007) show that failing to include the time-varying equilibrium real interest rate in the estimation equation can exaggerate the degree of interest rate smoothing. More recently, Consolo and Favero (2009) reestimate the Taylor rule using GMM, taking into account the possibility of weak instruments. They show that GMM estimation that takes care of the weak instruments problem makes the estimated coefficient on the lagged interest rate significantly smaller than suggested by the previous studies.

In this paper, I present an alternative explanation about why the partial adjustment model can be incorrect. I consider a situation in which a central bank has an aversion for frequent policy reversals. In practice, the central banks apparently try to avoid policy reversals, since reversals send a signal to the market that the policy shifts may be modified or even neutralized in the near future. Such a possibility of altering policy directions undermines the central bank's credibility and thereby the effectiveness of monetary policy. As of December 2009, the Fed has changed the Federal funds target 95 times since 1990, and there are only two cases in which the Fed changed the direction of the policy shift

[1] See Sack and Wieland (2000) for a survey of the literature on interest rate smoothing.

within two quarters. In fact, the shortest interval between opposite target shifts is about 5 months (Figure 1). This aspect of infrequent policy reversals is inconsistent with the quarterly partial adjustment model that allows for immediate reversals.

The possibility that an aversion for policy reversals would lead the central banks to be conservative is pointed out by Lowe and Ellis (1997), while I introduce the central banks' reversal aversion in a more formal way. I consider a situation in which the central bank faces an irreversibility constraint that prohibits the policy rate to move in opposite directions over the two consecutive periods. This can be interpreted as a special case of the situation in which the central bank bears the cost of policy-rate reversals without any restriction on the control space. Thus, as noted by Lowe and Ellis (1997), this discussion is similar to the irreversibility of investment (e.g., Dixit and Pindyck, 1994). The introduction of irreversibility makes the optimal policy less aggressive than would be attained under no constraint. This is simply because a larger policy shift would increase the probability that the policy rate must be reversed in the next period. In other words, there arises an option value to wait as is the case with investment irreversibility.

The simulation results reveal that the conventional regression tends to yield a wrong conclusion that the linear partial adjustment model is correct. Obviously, the linear partial adjustment equation is functionally misspecified and omits an important state variable as long as the central bank faces the irreversibility constraint. I also show that the application of English, Nelson and Sack's (2003) specification test, which simultaneously allows for both the partial adjustment term and serial correlations, would not correctly detect the misspecification. This suggests that the widely used partial adjustment model may be falsely accepted since the policy rates under irreversibility are observationally indistinguishable from those under the partial adjustment model. If this is the case, the use of the partial adjustment version of the Taylor rule in a theoretical study will be subject to the Lucas critique.

## 2 A simple model of gradualism

This section presents a very simple model in which there is no interaction between the central bank's policy action and the desired level of interest rate for the purpose of clarifying the pure effect of introducing an irreversibility constraint. Assume that the central bank knows the process of the desired interest rate, which is given as

$$i_t^* = \rho i_{t-1}^* + \varepsilon_t, \quad \rho \in [0, 1), \tag{1}$$

where $i_t^*$ is the current desired interest rate and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. $i_t^*$ is called the "desired" interest rate in the sense that the central bank always sets its policy rate, $i_t$, at $i_t^*$ if there is no restriction on the behavior of the policy rate. However, if the central bank has an aversion for frequent policy reversals, then $i_t^*$ is not necessarily the *optimal* interest rate since the previous policy rate becomes an upper (a lower) bound for the current policy rate when the policy rate was previously lowered (increased).

A central bank that has an aversion for policy reversals not only avoids current policy reversal, but also takes into account the influence of its current policy action on the likelihood of future policy reversals. For example, suppose that the desired level of the interest rate is currently quite high, but is likely to decline in the next period. In this situation, an upward shift in the policy rate will increase the probability of policy reversal (i.e., a negative policy shift) in the next period. The central bank may decide to keep the level of the policy rate unchanged if such a future policy reversal is highly likely. Thus, the decision-making of a central bank that tries to avoid frequent policy reversals is intrinsically a dynamic problem and the optimal policy will be forward-looking.

Assume that the central bank has a quadratic return function that takes the maximum value zero when the policy rate is equal to $i_t^*$. One way to incorporate the central bank's reversal aversion is to directly introduce an irreversibility constraint in the Bellman equation:

$$V(i_{t-1}, \delta_{t-1}, i_t^*) = \max_{i_t \in \Omega_t} \{-(i_t - i_t^*)^2 + \beta E_t V(i_t, \delta_t, i_{t+1}^*)\}, \tag{2}$$

where $V(\cdot)$ is the value function of the central bank, $\delta_t = \text{sign}(i_t - i_{t-1})$ and $\beta$ is the discount factor. The control space for $i_t$ is constrained by $\Omega_t$, where

$$\begin{aligned} \Omega_t &= \{i_t |\ i_t \leq i_{t-1}, i_t \in \Omega\} \quad \text{if} \ \ \delta_{t-1} = -1, \\ &= \Omega \quad \text{if} \ \ \delta_{t-1} = 0, \\ &= \{i_t |\ i_t \geq i_{t-1}, i_t \in \Omega\} \quad \text{if} \ \ \delta_{t-1} = 1. \end{aligned} \tag{3}$$

$\Omega$ denotes the overall control space. There are three state variables in the value function. The most obvious one is the desired level of the policy rate.[2] In addition, $i_{t-1}$ and $\delta_{t-1}$ become state variables since they determine the current control space. It should be noted that the current policy not only affects the one-period return, $-(i_t - i_t^*)^2$, but also determines the next period's state variables $i_t$ and $\delta_t$, which constrain the future control space. It is this forward-looking aspect that creates the possibility that the central bank favors gradualism.

Another possible way to examine the central bank's aversion for policy reversals is to introduce penalties or costs of reversals. In this case, the control space is not constrained by the direction of previous policy shifts, but policy reversals impose penalties or costs on the central bank through the one-period return function. This type of problem setting is called the penalty method or the barrier method, in which case the solution could be obtained by a perturbation method once the constraint is rewritten as $\Delta i_t \Delta i_{t-1} \geq 0$.[3] However, a shortcoming of the penalty method is that the problem is not exactly the

[2] It is assumed that the $i_t^*$ is observable at the time the central bank sets $i_t$.

[3] See, for example, Luenberger (2003) for an explanation of the penalty and barrier methods.

same as the corresponding constrained problem unless the size of the penalty is infinite outside the boundary. den Haan and Wind (2009) also argue that for the perturbation methods to be accurate, the penalty function must not have a singularity, say $x^*$, since the radius of convergence of the Taylor series is bounded above by $||\bar{x} - x^*||$, where $\bar{x}$ is the point at which the approximation is based.[4] This requires the penalty function or the barrier function to be differentiable at the boundary, but this is done at the expense of solution accuracy around the boundary. These properties will cause a serious problem since the irreversibility constraint demands arbitrary accuracy around the boundary due to the fact that the next period's control space depends largely on $\text{sign}(\Delta i_t)$. For this reason, introducing a penalty function or a barrier function will be inappropriate for a problem with the irreversibility constraint.

## 2.1 Stochastic simulation

The introduction of an irreversibility constraint makes the central bank's policy function highly nonlinear, and an analytical solution is thus no longer available even in this otherwise simple linear-quadratic framework. In the following, I solve the dynamic programming numerically using value function iteration.

The baseline parameters and grid sizes are as follows. $\beta = .9$, $\sigma_\varepsilon = .005$ and I consider two alternative cases regarding the persistence of the desired interest rate: $\rho = .8$ and 0. $\Omega = [-.03, .03]$, and the grid size for state $i_{t-1}$ is set at .0025. As for $i_t^*$, the AR(1) process is discretized by the method proposed by Tauchen (1986). The number of nodes for $i_t^*$ is 11.[5] The optimal policy in each iteration is obtained by using a modified golden search algorithm, where the optimal value attained by the standard golden search method is compared with the value under no policy shift (i.e., $i_t = i_{t-1}$). Such a modification is necessary because the standard golden search method often fails to calculate corner solutions.[6] The next period's expected value is approximated using piece-wise linear spline at points not on the grid.[7] The approximated value function is given by

$$V(\hat{i}_j, \hat{\delta}_k, \hat{i}_l^*) = \max_{\bar{i} \in \Omega_{j,k}} \{-(\bar{i} - \hat{i}_l^*)^2 + \beta \sum_{m=1}^{11} P(l,m) V(\bar{i}, \text{sign}(\bar{i} - \hat{i}_j), \hat{i}_m^*)\}, \tag{4}$$

where the current control space depends on indices $j$ and $k$ since the space is determined by the previous interest, $\hat{i}_j$, and the previous increment, $\hat{\delta}_k$. $P(l,m)$ denotes the transition probability from state $\hat{i}_l^*$ to $\hat{i}_m^*$.

Figure 2 illustrates sample paths of the policy rate and the desired rate. The figure shows that there is a large discrepancy between the two interest rates. The point is that

[4]See also Judd (1998, Theorem 6.1.2).

[5]The maximum and minimum values of $i_t^*$ are .015 and -.015, respectively.

[6]Recall that $i_{t-1}$ becomes an upper or a lower bound of $i_t$ when $\delta_{t-1}$ is nonzero. See Miranda and Fackler (2002) for an explanation of the standard golden search algorithm.

[7]The criterion of convergence is $||\frac{V^k(\cdot) - V^{k-1}(\cdot)}{V^{k-1}(\cdot)}||_\infty < 10^{-6}$, where $V^k(\cdot)$ is the value after $k$th iterations.

the discrepancy is not observed only when the irreversibility constraint is binding. The policy rate often deviates from the desired rate when the constraint is not binding. The latter phenomenon corresponds to the central bank's gradualism, where the central bank moves the policy rate to a lesser extent than the change in the desired rate. As noted above, this is because a larger shift in the policy rate will increase the probability of the next period's policy reversal.

Another interesting feature is that the size of the discrepancy between the policy rate and the desired rate depends on the degree of persistence, $\rho$. It turns out that, on average, the discrepancy is wider under $\rho = 0$ than under $\rho = .8$. The intuition is that the central bank follows changes in the desired rate more frequently as the degree of persistence increases since the probability of reversal in the desired rate is decreasing in $\rho$. This implies that the extent of gradualism is endogenously determined, depending on the behavior of the desired rate.

Figure 3 shows the policy function when $i_{t-1} = 0$. The two figures reconfirm the two properties shown in the simulated paths: i) the central bank that faces the irreversibility constraint conducts gradual monetary policy even when the constraint is not binding, and ii) the degree of gradualism depends on the degree of persistence in $i_t^*$. In particular, Figure 3a reveals that it is optimal to do nothing when the change in the desired rate is sufficiently small and the persistence of the desired rate is sufficiently small. The relationship between the extent of gradualism and the persistence in $i_t^*$ is far from linear.

### 2.2 Conventional regression: A partial adjustment model

Since the influential work by Taylor (1993), a large number of empirical studies have attempted to estimate a simple policy rule, and many of them supports a linear partial adjustment model of the form:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) c_{i^*} i_t^* + \xi_t, \tag{5}$$

where $\xi_t$ is a residual term and $i_t^*$ typically represents the original Taylor rule, or the "Taylor rate", that depends only on inflation and an output gap.

Eq.(5) is obviously not a correct policy function in the presence of the irreversibility constraint, while the gradual adjustment of the simulated optimal policy shown in Figure 2 does not seem inconsistent with the partial adjustment model. Even in theoretical studies, it is often *assumed* that the linear partial adjustment model can well capture the actual central banks' conservative behavior, which is called "interest rate smoothing". However, if the central banks' conservativeness stems from the aversion for policy reversals, then such a conventional assumption will not hold true.

One might argue that the partial adjustment model may still describe the central banks' conservatism as first-order approximation to the possibly nonlinear policy function. Unfortunately, the policy function under the irreversibility constraint is not only nonlinear, but also depends on additional state variable $\delta_{t-1}$. Therefore, any linear partial adjustment model is functionally misspecified even around the steady state.

In the following I estimate eq.(5), using simulated data obtained by the model presented in the previous subsection. If the estimated values of $\rho_i$ and $c_{i^*}$ are statistically significant, then the possibility arises that the conventional regression of the partial adjustment model may be spurious.

### 2.3 Estimation results

Since the data used in the previous studies are mostly quarterly and start from the late 80's, I set the length of simulated data used in the estimation at 100. In doing so, I generated 5000 sets of 200-period-long data, and the initial 99 periods were discarded.[8] The distributions of the OLS estimates are shown in Figures 4 and 5.

First, it turns out that the coefficient on the lagged interest rate, $\rho_i$, is significantly different from 0 in both cases and its size tends to be larger under $\rho = 0$ than under $\rho = .8$. This difference in the estimated values of $\rho_i$ can be thought of as reflecting the extent of discrepancy between the policy rate and desired rate. Second, the coefficient on $i_t^*$, $c_{i^*}$, is less than 1 in most cases, while $c_{i^*}$ should be unity under the assumption that the linear partial adjustment model is correct.

Figures 4d and 5d also show that the Breusch-Godfrey's LM test often fails to reject the null hypothesis that the OLS residuals are serially uncorrelated.[9] Although, in general, the presence of omitted variables often makes the OLS residuals serially correlated, the result that serial correlations are not detected does not necessarily mean that the estimated equation correctly contains all the true explanatory variables. Obviously, the estimated policy rule is functionally misspecified in terms of nonlinearity and a key state variable $\delta_{t-1}$ is incorrectly omitted from the model. The Monte Carlo experiments reveal that this type of error is more likely to occur as the desired rate becomes more persistent.

This simple simulation exercise clearly states that the OLS estimation of the functionally misspecified equation can easily lead to an incorrect conclusion that the actual policy rate can be expressed by a linear partial adjustment model. Recall that the above results are obtained under the assumption that the current desired rate, $i_t^*$, is known with certainty. Many empirical studies focus on correctly estimating the Taylor rate $i_t^*$, but the conventional regression would be inappropriate even if we have accurate information about the Taylor rate.

[8] I discarded 99 periods, not 100 periods, since lagged data are needed.

[9] The Durbin-Watson test is inappropriate when the regression contains a lagged dependent variable, for the test statistic will be biased toward a finding of no serial correlation (e.g., Nerlove and Wallis, 1966).

# 3 The Ball-Svensson backward-looking model

## 3.1 The basic framework

Thus far, I examined the situation in which the central bank's policy action does not affect the level of the desired interest rate. In practice, however, explanatory variables in the policy rule, typically inflation and an output gap, are inevitably affected by the policy rule itself. This section employs a simple structural model in order to take into account the interaction between the policy rule and its explanatory variables.

The economic structure is expressed by the following Ball-Svensson's model:[10]

$$y_{t+1} = \rho_y y_t - \delta_y (i_t - E_t \pi_{t+1}) + \nu_{t+1}, \tag{6}$$
$$\pi_{t+1} = \gamma_\pi \pi_t + \alpha_\pi y_t + \eta_{t+1}, \tag{7}$$

where $y_t$ is an output gap and $\pi_t$ is the rate of inflation. $\nu_{t+1}$ and $\eta_{t+1}$ are white noises whose variances are $\sigma_\nu^2$ and $\sigma_\eta^2$, respectively. Since the expected inflation is expressed as $E_t\pi_{t+1} = \gamma_\pi \pi_t + \alpha_\pi y_t$, the output gap in period $t+1$ leads to

$$y_{t+1} = (\rho_y + \delta_y \alpha_\pi) y_t - \delta_y i_t + \gamma_\pi \delta_y \pi_t + \nu_{t+1}. \tag{8}$$

A virtue of this simple backward model is that the standard Taylor rule is obtained as the optimal policy function. To see this, assume that the central bank has a quadratic one-period return function. The Bellman equation is written as[11]

$$V(y_t, \pi_t) = \max_{i_t} \{-y_t^2 - \lambda \pi_t^2 + \beta E_t V(y_{t+1}, \pi_{t+1})\}. \tag{9}$$

It can be easily shown that the policy function leads to

$$\tilde{i}_t = \gamma_\pi \pi_t + \frac{\rho_y + \delta_y \alpha_\pi}{\delta_y} y_t + \frac{\gamma_\pi \beta \alpha_\pi a}{\delta_y (1 + \beta \alpha_\pi^2 a)} (\gamma_\pi \pi_t + \alpha_y y_t), \tag{10}$$

where $a = (-(1 - \lambda\beta\alpha_\pi^2 - \beta\gamma_\pi^2) + \sqrt{(1 - \lambda\beta\alpha_\pi^2 - \beta\gamma_\pi^2)^2 + 4\lambda\beta\alpha_\pi^2})/(2\beta\alpha_\pi^2)$. Thus, in the absence of any restriction on the policy space, the optimal policy is to follow the Taylor rule without the lagged interest term.

Now, let us define the problem under the irreversibility constraint. The Bellman equation is written as

$$V(i_{t-1}, \delta_{t-1}, y_t, \pi_t) = \max_{i_t \in \Omega_t} \{-y_t^2 - \lambda \pi_t^2 + \beta E_t V(i_t, \delta_t, y_{t+1}, \pi_{t+1})\}, \tag{11}$$

[10] See Ball (1999) and Svensson (1997). Kato and Nishiyama (2005) investigate the effect of the zero-lower bound for the policy rates within this model. The only difference from their model is that I allow the non-unitary coefficient on $\pi_t$, $\gamma_\pi$. The reason for this modification is described below.

[11] Since $i_t$ does not affect either $\pi_t$ or $y_t$, the Bellman equation can be reformulated by employing $E_t y_{t+1}$ and $E_t \pi_{t+1}$ as a control variable and a state, respectively. However, I use eq.(9) in the following since i) the values of $\pi_t$ and $y_t$ are needed in estimation and ii) the irreversibility constraint should be defined with respect to the policy rate.

where control space $\Omega_t$ is defined in the same way as (3).

Since the Taylor rate can be defined as $\tilde{i}_t$, the partial adjustment equation to be estimated should be written as

$$\begin{aligned} i_t &= \phi_i i_{t-1} + (1-\phi_i)\tilde{i}_t + \xi_t \\ &= \phi_i i_{t-1} + (1-\phi_i)(\phi_\pi \pi_t + \phi_y y_t) + \xi_t. \end{aligned} \tag{12}$$

The coefficients $\phi_i$, $\phi_\pi$ and $\phi_y$ will be estimated simultaneously.

### 3.2 Simulation

The parameter values are as follows: $\lambda = 1$, $\rho_y = \delta_y = \gamma_\pi = .3$ and $\alpha_\pi = .05$. The parameter values for structural equations are all set at levels smaller than those used in Kato and Nishiyama (2007) for two reasons. First, the aim of the simulation is not to replicate the actual moment size of economic variables. Second, employing larger coefficients increases the possibility of extrapolation when calculating the expected value. Since extrapolation often undermines the accuracy of value function approximation, I set parameters sufficiently small so that extrapolation is not needed. In doing so, I searched for the largest parameter combination by gradually lowering each parameter value.

Determination of the overall control space $\Omega$ should also be done with great care. Since our focus is neither on the upper nor the lower bound of interest rate, but on the effect of the irreversibility constraint, any influence of restrictions other than the irreversibility constraint must be eliminated. To do this, I chose a sufficiently large control space so that the Taylor rate would be covered for all nodes of $(y_t, \pi_t)$. Given that the optimal policy under the irreversibility constraint is more conservative compared to the policy under no constraint, this choice of control space will cover all the nodes for the optimal interest rate under irreversibility.

Specifically, the maximum and minimum nodes for $i_{t-1}$ are .1 and -.1, respectively. The maximum and minimum nodes for $y_t$ are .08 and -.08, respectively, while the maximum and minimum nodes for $\pi_t$ are .04 and -.04, respectively. The grid size is .005 for all states other than $\delta_{t-1}$. The distributions for shocks, $\nu$ and $\eta$, are obtained in the same way as above, where the variances are both set at $.01^2$. The number of nodes for each shock is 9. With these parameter values, the theoretical values of $\phi_y$ and $\phi_\pi$ are 1.05 and 0.31, respectively.[12] The numerical value function is written as

$$\begin{aligned} V(\hat{i}_j, \hat{\delta}_k, \hat{y}_l, \hat{\pi}_n) &= \max_{\bar{i} \in \Omega_{j,k}} \{-\hat{y}_l^2 - \lambda \hat{\pi}_n^2 \\ &+ \beta \sum_{m=1}^{9} \sum_{s=1}^{9} q_{\nu,m} q_{\eta,s} V(\bar{i}, \mathrm{sign}(\bar{i} - \hat{i}_j), \hat{y}_+(\hat{y}_l, \hat{\pi}_n, \bar{i}, \hat{\nu}_m), \hat{\pi}_+(\hat{y}_l, \hat{\pi}_n, \hat{\eta}_s))\}, \end{aligned} \tag{13}$$

[12] In this model, the well known Taylor principle does not need to be satisfied to ensure a stable process for inflation and output. It can be shown that the solution is stable under a wide range of reasonable parameter values.

where $\hat{y}_+(\hat{y}_l, \hat{\pi}_n, \bar{i}, \hat{\nu}_m) = (\rho_y + \delta_y \alpha_\pi)\hat{y}_l - \delta_y \bar{i} + \gamma_\pi \delta_y \hat{\pi}_n + \hat{\nu}_m$ and $\hat{\pi}_+(\hat{y}_l, \hat{\pi}_n, \hat{\eta}_s) = \gamma_\pi \hat{\pi}_n + \alpha_\pi \hat{y}_l + \hat{\eta}_s$. $q_{\nu,m}$ and $q_{\eta,s}$ are probabilities that $\nu_{t+1}$ and $\eta_{t+1}$ are in states $\nu_m$ and $\eta_s$, respectively.

The paths of the optimal interest rate and the Taylor rate, $\tilde{i}_t$, are shown in Figure 6. The fundamental properties are the same as those observed in Figure 2: the policy rate often deviates from the Taylor rate in periods when the irreversibility constraint is not binding. This can also be confirmed by Figure 7, which illustrates policy responses to output and inflation.

Figure 8 shows the distribution of the OLS estimates of the coefficients in (12). The coefficient on the lagged interest rate, $\phi_i$, is estimated to be around .4 and is statistically significant. Interestingly, as is the case with the simple model above, the estimated coefficients on inflation and output are both biased downward. This has an important implication that the central banks' conservatism could not be captured only by the lagged interest rate. In the literature, it is often presumed that the value of $\phi_i$ represents the degree of policy inertia, but the influence of conservatism would also affect the coefficients in the Taylor rate. This suggests that the central banks in industrialized countries may have had more hawkish Taylor rates over the past two decades than suggested by the previous empirical studies.

As argued in the previous sections, the optimal policy under the irreversibility constraint may lead to a misleading conclusion that the inertial policy rule can be expressed as a partial adjustment model. The analysis of this section shows that such a misperception could also occur in an environment in which the policy rate and its explanatory variables interact with each other. The coefficient on the lagged interest rate can be viewed as reflecting the central bank's gradualism stemming from the irreversibility constraint, which should be distinguished from the conventional interest rate smoothing expressed by the linear partial adjustment model.

# 4 The English-Nelson-Sack test

Rudebusch (2002, 2006) argue that the explanatory power of the lagged interest rate in the Taylor rule arises if a serially correlated variable is incorrectly omitted from the estimated equation. If this is the case, the standard estimation of the coefficient on the lagged interest will be biased due to the omitted variables, and the lagged interest rate cannot be able to correctly capture the extent of gradualism.

Based on this argument, English, Nelson and Sack (2003) proposed a hypothetical function that allows for both serially correlated residuals and the lagged interest rate. The equations to be simultaneously taken into account are:

$$i_t = \phi_i i_{t-1} + (1 - \phi_i)\tilde{i}_t + u_t, \quad (14)$$

$$u_t = \theta u_{t-1} + \zeta_t, \quad (15)$$

where, as above, $\tilde{i}_t$ is the Taylor rate, $u_t$ is the possibly serially correlated residual, and $\zeta_t$ is a white noise. Combining the two equations yields

$$\begin{aligned} \Delta i_t &= (1-\phi_i)\Delta\tilde{i}_t + (1-\phi_i)(1-\theta)(\tilde{i}_{t-1} - i_{t-1}) + \phi_i\theta\Delta i_{t-1} + \zeta_t, \\ &= b_1\Delta\tilde{i}_t + b_2(\tilde{i}_{t-1} - i_{t-1}) + b_3\Delta i_{t-1} + \zeta_t. \end{aligned} \tag{16}$$

As long as $\theta$ is nonnegative, the estimation result will be one of the following four cases

1. $b_1 = b_2 = 1, b_3 = 0$: No inertia, no serial correlation,
2. $b_1 = 1, b_2 \in (0,1), b_3 = 0$: No inertia, serial correlation,
3. $b_1, b_2 \in (0,1), b_3 = 0$: Inertia, no serial correlation,
4. $b_1, b_2, b_3 \in (0,1)$: Inertia, serial correlation.

English, Nelson and Sack (2003), Gerlach-Kristen (2004) and Castelnuovo (2007) show that case 4 is most likely. Here, I estimate $b_1$, $b_2$ and $b_3$ without imposing parameter restrictions.

Let us first consider the simplest model of section 2. Figures 9 and 10 illustrate the distributions of the OLS estimates. It turns out that the coefficients $b_1$, $b_2$ and $b_3$ are all greater than zero and less than one in most cases. This is consistent with the results obtained by English, Nelson and Sack (2003), Gerlach-Kristen (2004) and Castelnuovo (2007), who insist that the joint formalization of the two hypotheses will be the best approximation to policy rules. Figure 11 shows the estimation results under the backward-looking model. The figure states that the essential results are the same as those of the simple model.

Obviously, the theoretically correct policy function under irreversibility cannot be estimated by (16) even if one allows for the possibility of serially correlated errors. The "statistically significant" serially correlated errors ($u_t$) arose as a result of failing to incorporate nonlinearity and state variable $\delta_{t-1}$, which would make a standard OLS estimation inconsistent. Therefore, the English, Nelson and Sack's modification to the traditional regression will not work when the central banks' gradualism stems from irreversibility. The results illustrated in Figures 9, 10 and 11 reveal that it would be hard to distinguish between the irreversible monetary policy and the inertial policy under the partial adjustment model just by looking at the estimates of the nested equation eq.(16).

# 5 Term structure implications

While the above results suggest observational equivalence between the policies under the irreversibility constraint and the policies based on the partial adjustment form, there remains another question: which type of policy is more plausible as a description of the Fed's behavior? To address this issue, it is useful to recall Rudebusch's (2002, 2006) argument that the actual policy rate contains less information about future policies than suggested by the partial adjustment model.

Unfortunately, however, the simulated statistics obtained by using the simple models above cannot be directly compared with the actual statistics shown by Rudebusch since the models are too simple to capture quantitative details. Therefore, I conduct some exercises to show the relation between the policy rate and future interest rates. It could be said that for the irreversible policies to be plausible, the policy rate under irreversibility should not have much information about future policies.

### 5.1 The term structure regression

Firstly, let us consider the standard term structure regression employed by Rudebusch (2002):

$$i_{t+j} - i_{t+j-1} = c_0 + c_1(E_t i_{t+j} - E_t i_{t+j-1}) + \xi_{t+j}, \quad j = 1, 2, 3. \tag{17}$$

Under rational expectations, $i_{t+j} = E_t i_{t+j} + e_{t+j}$, where $e_{t+j}$ denotes the expectational error which has no correlation with time-$t$ information. Therefore, it is expected that $c_0 = 0$ and $c_1 = 1$ if the future changes in the short-term rates are forecast in an unbiased manner. Moreover, if the current policy rate has sufficient information to forecast future policy rates, then the obtained $R^2$ would take a value close to one.

In order to obtain simulated data on expected interest rates, an "expectations function" needs to be obtained by conducting simulation since it is impossible to obtain interest expectations analytically. Here, I adopt the following procedure:

1. For a given combination of state variables, 500 sets of $(n+1)$-period-long interest rate data are generated.
2. The mean of period $(j+1)$'s simulated interest rates is calculated for $j = 1, 2, \ldots, n$.
3. Repeat steps 1-2 for all the combinations of state variables.

I obtained expectations functions of various horizons up to 3 periods ahead (i.e., $n = 3$).

Table 1 reports the 95% confidence intervals for $c_1$ and $R^2$ under alternative models.[13] It shows that the predictive power of the current policy rate, which is measured by $R^2$, deteriorates dramatically as the forecast horizon increases, although the predictive accuracy is not very different between $j = 1$ and 2 in the backward-looking model. Judging from Rudebusch's (2002) discussion that the forecastability of 2-period-ahead policy shift is very low, $R^2$ in the case of $j = 2$ in the backward model seems too high.[14] This would be because there are too many cases where the constraint is binding. For example, if it

[13]The case of $\rho = 0$ under the simple model is not considered since the current desired rate has no predictive power regarding future interest rates.

[14]Rudebusch (2002) reports that $R^2$ is 0.11 when $j = 2$.

is highly likely that the constraint is binding in the next period, then the forecast of the next policy rate will necesarily be accurate. In other words, the simulated Taylor rate may unrealistically exhibit many reversals. In fact, the confidence interval of $R^2$ regarding the forecast of the 2-period-ahead Taylor rate's shift takes very low values because of the frequent reversals in the Taylor rate. In the simple model, where reversals in the desired rate are less frequent, $R^2$ under the irreversible policy is not significantly different from that under the no-constraint policy. This would be indirect evidence that the actual desired rate is persistent enough to prohibit the irreversibility constraint from binding frequently. In the case of $j = 3$, on the other hand, $R^2$ is very close to zero in both models.

### 5.2 The policy rate's correlation with long-term rates

Many authors, such as Goodfriend (1991), Amato and Laubach (1999) and Woodford (2003b), argue that policy rate smoothing is useful for controlling long-term interest rates since market participants are more likely to expect current policy shift to continue as the persistence of the policy rate increases. On the other hand, the actual policy rate would not have a close correlation with long-term rates if Rudebusch's argument is correct.

Figure 12 illustrates the correlation between the policy rate and long-term interest rates under the expectations theory. The figure shows that if persistence in the policy rate comes from the central bank's reversal aversion, then the conventional argument is not necessarily correct. While the introduction of policy irreversibility can increase the correlation between the policy rate and interest rates of relatively short maturities, it weakens the correlation of the policy rate with longer-term rates. This is because forecasting the value of the additional state $\delta_{t+k}$ becomes more difficult as the time horizon $k$ increases, and the presence of this additional state variable turns into a disturbing factor when $k$ is sufficiently large. This kind of non-monotonic impact of gradualism on the relation between long- and short-term interest rates would not arise under the linear partial adjustment version of the Taylor rule. Arguably, the result that the policy rate may have less correlation with long-term rates than the desired rate would is also consistent with Rudebusch's finding that the current policy rate contains little information about future policies.

## 6 Conclusion

This paper shows that the significant role of the lagged coefficient in the estimated Taylor rule may reflect the central bank's aversion for frequent policy reversals since the lagged interest rate becomes a state variable in the presence of an irreversibility constraint. If this is the case, the widely used partial adjustment model is functionally misspecified in terms of nonlinearity and the presence of an omitted variable. This suggests that the degree of policy inertia should be endogenously determined in accordance with the degree of persistence in the desired rate. The use of a partial adjustment policy rule in a theoretical study thus lacks a microfoundation.

An important extension is to introduce forward-looking expectations in AS and IS equations. While it is well understood within the linear quadratic framework that the virtue of making the current policy rate dependent on the lagged policy rates comes from the current policy rate's increased influence on inflation expectations, it might not be the case once policy irreversibility is taken into account. As discussed in section 5, "policy inertia" stemming from the irreversibility constraint may make it even more difficult to controll long-term rates. One difficulty of introducing expectations in structural equations is to maintain the accuracy of the solution. Considering nonlinear policies under irreversibility will be hard to justify as long as the forward-looking AS and IS equations are obtained by a linearization technique. Ideally, therefore, the model should be solved by a non-local approximation method.

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| | $j=1$ | $j=2$ | $j=3$ |
|---|---|---|---|
| **Simple model ($\rho = .8$)** | | | |
| $c_1$ (irreversibility) | (.748, 1.30) | (.659, 1.62) | (.588, 1.45) |
| $c_1$ (no constraint) | (.700, 1.81) | (.641, 1.78) | (.525, 1.80) |
| $R^2$ (irreversibility) | (.214, .506) | (.086, .284) | (.047, .198) |
| $R^2$ (no constraint) | (.069, .184) | (.028, .139) | (.010, .109) |
| **Backward model** | | | |
| $c_1$ (irreversibility) | (.758, 1.10) | (.857, 1.25) | (.530, 1.47) |
| $c_1$ (no constraint) | (.846, 1.18) | (-2.37, 4.24) | (-9.99, 12.16) |
| $R^2$ (irreversibility) | (.475, .698) | (.341, .586) | (.040, .247) |
| $R^2$ (no constraint) | (.410, .573) | (0, .036) | (0, .029) |

Table 1: 95% confidence intervals for the estimates of eq.(17)

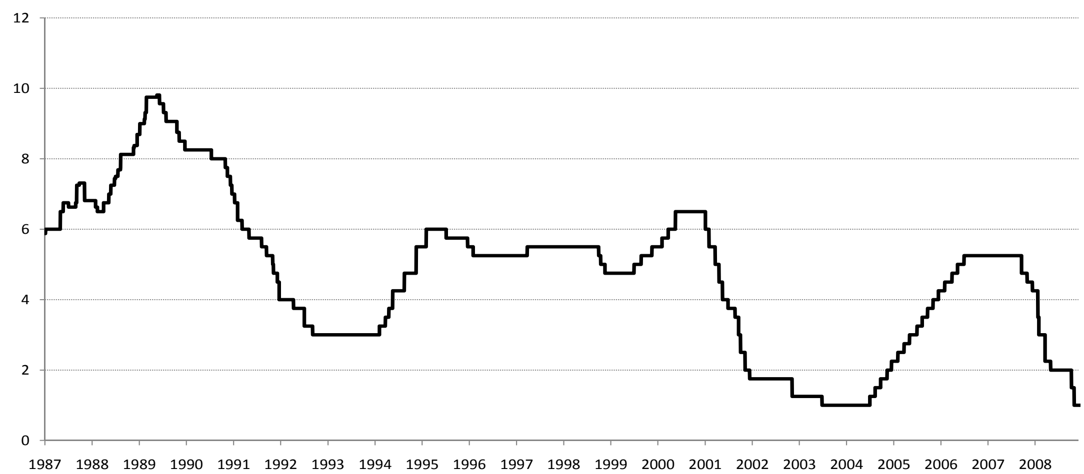


Figure 1: The Federal funds target

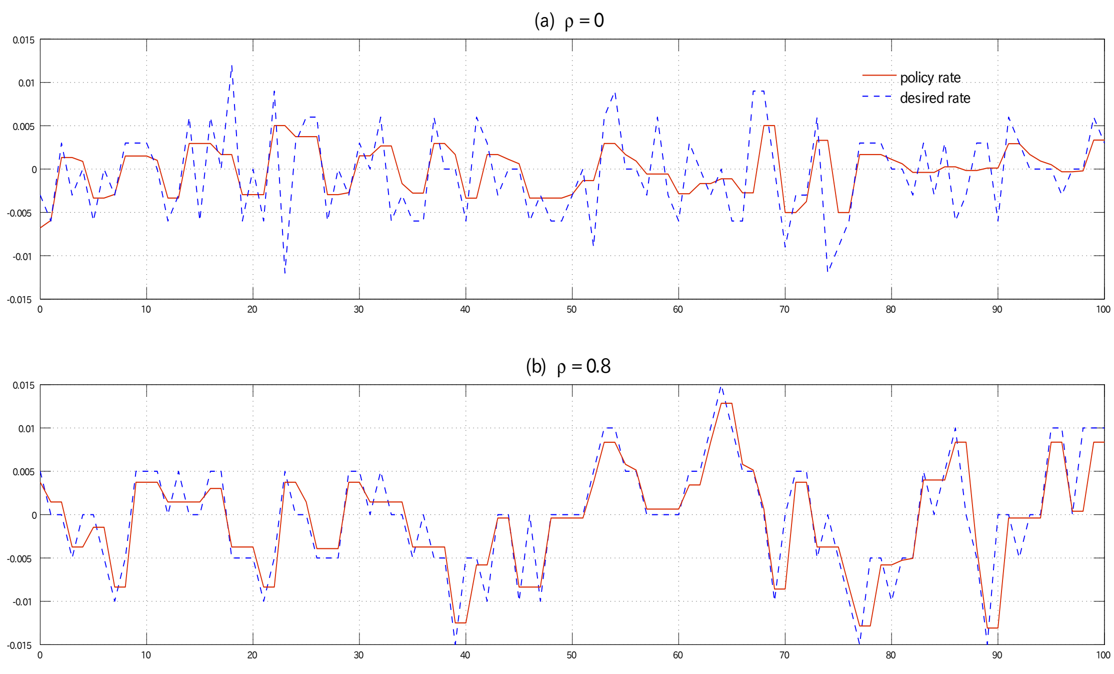


Figure 2: Simulated paths: The simple model

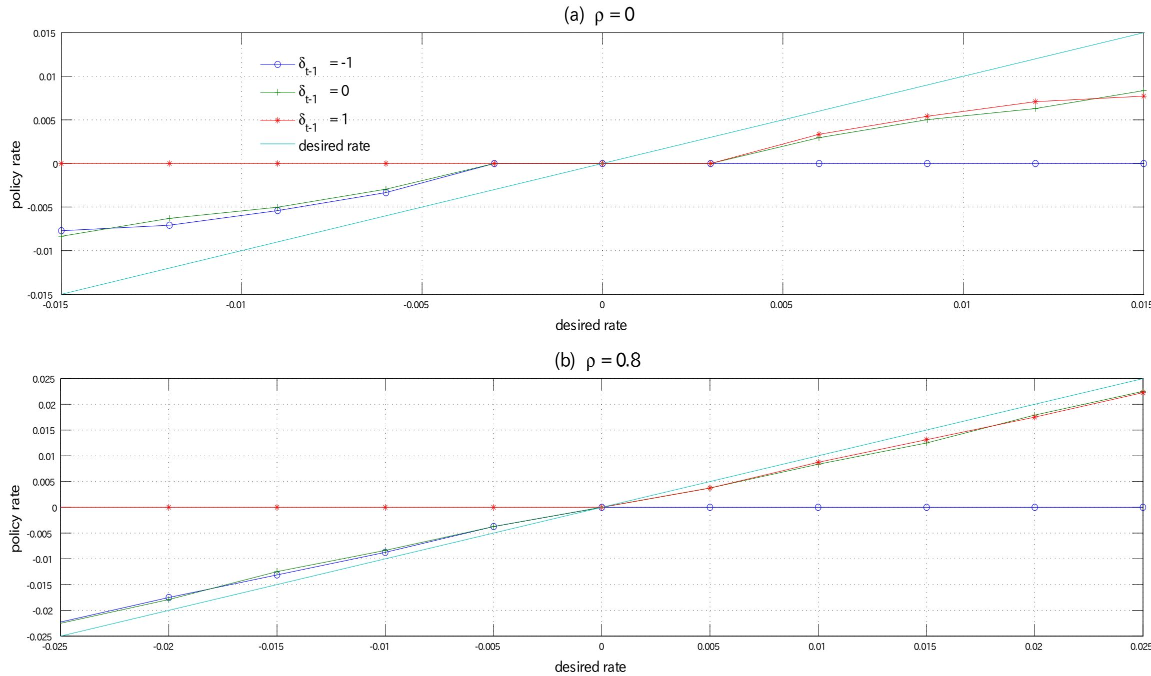


Figure 3: Policy function: The simple model

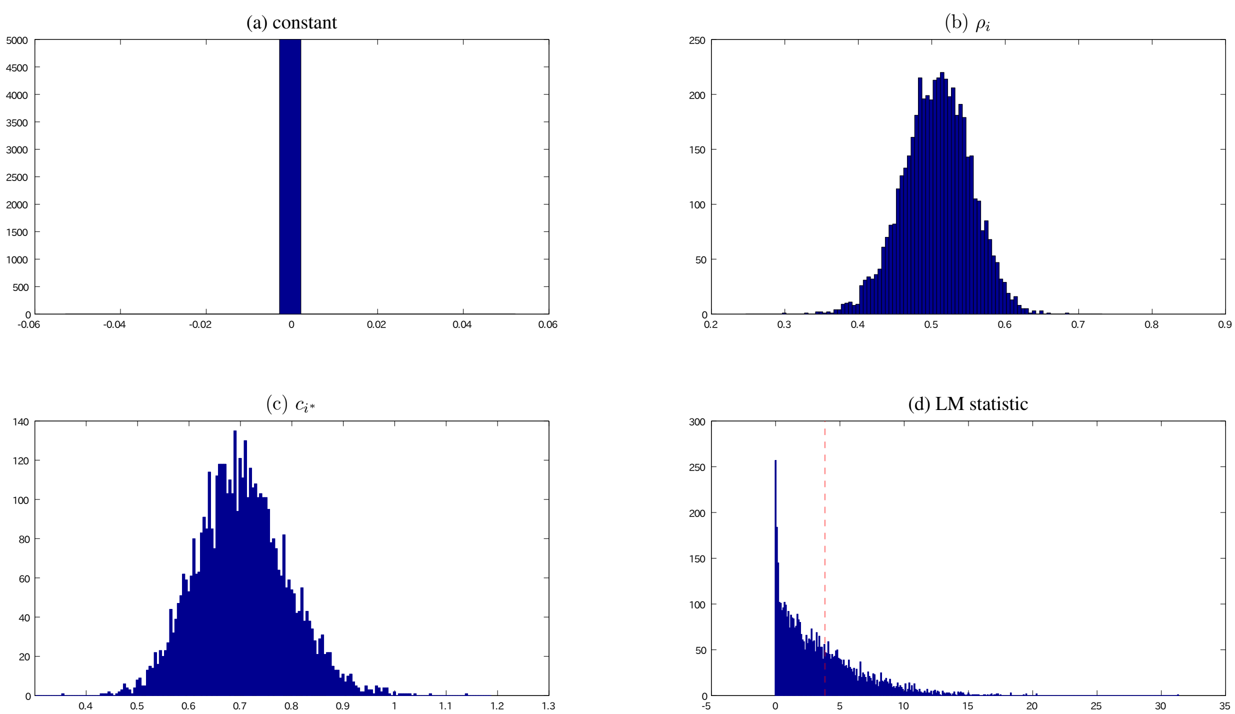


Figure 4: Distribution of the OLS estimates of eq.(5): $\rho = 0$. (Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.)

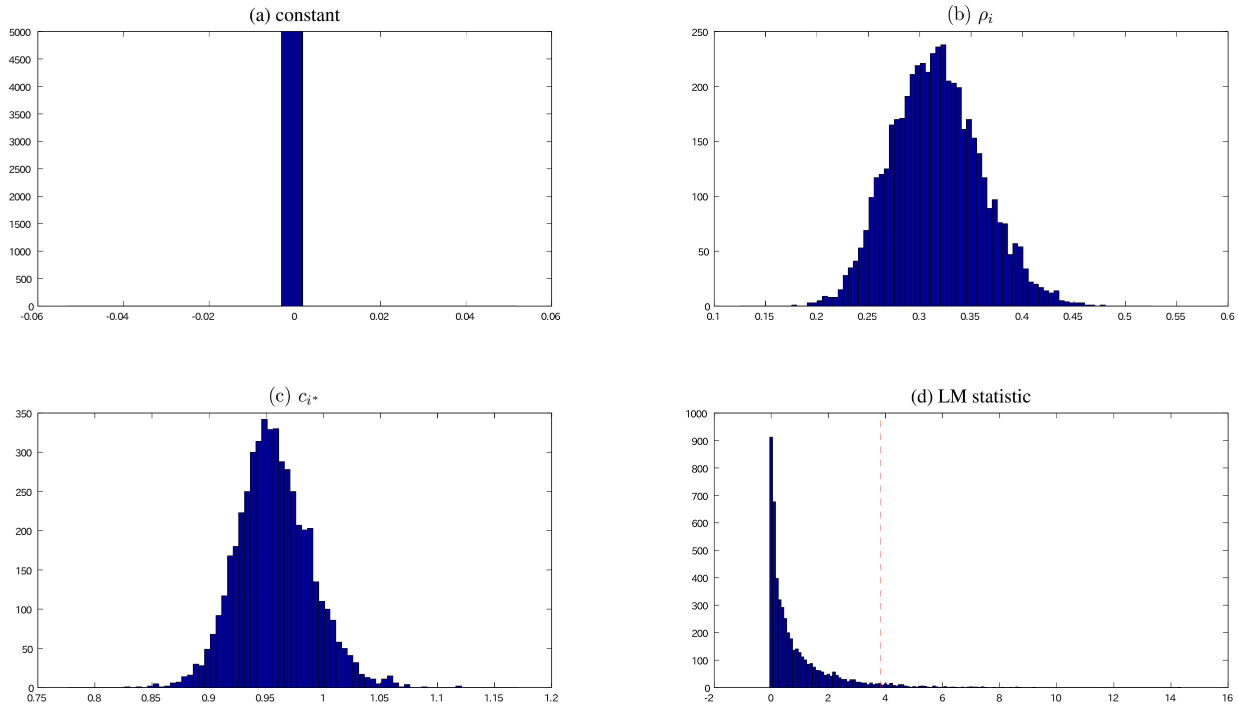


Figure 5: Distribution of the OLS estimates of eq.(5): $\rho = .8$. (Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.)

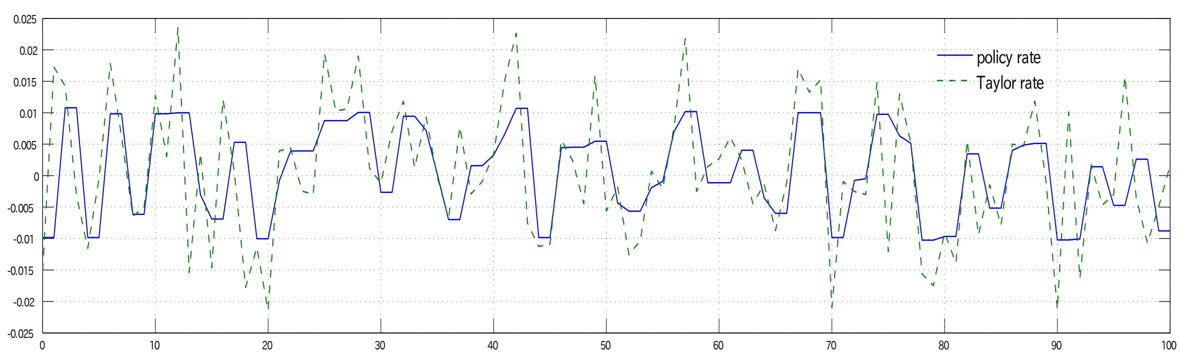


Figure 6: Simulated paths: The backward-looking model

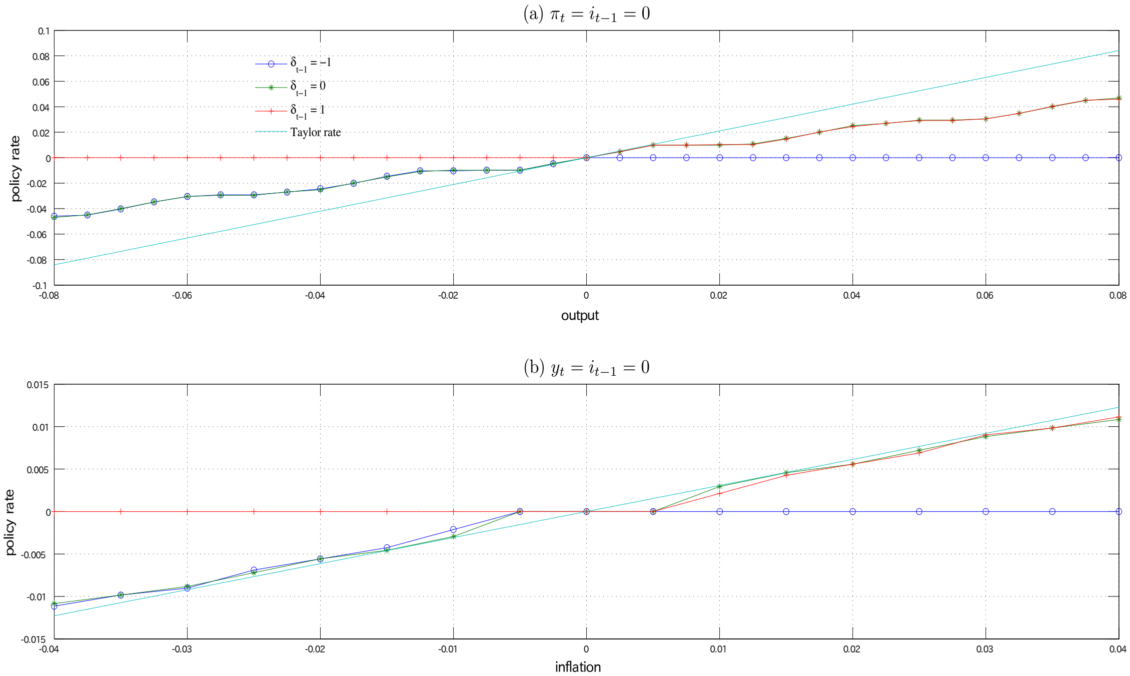


Figure 7: Policy function: The backward-looking model

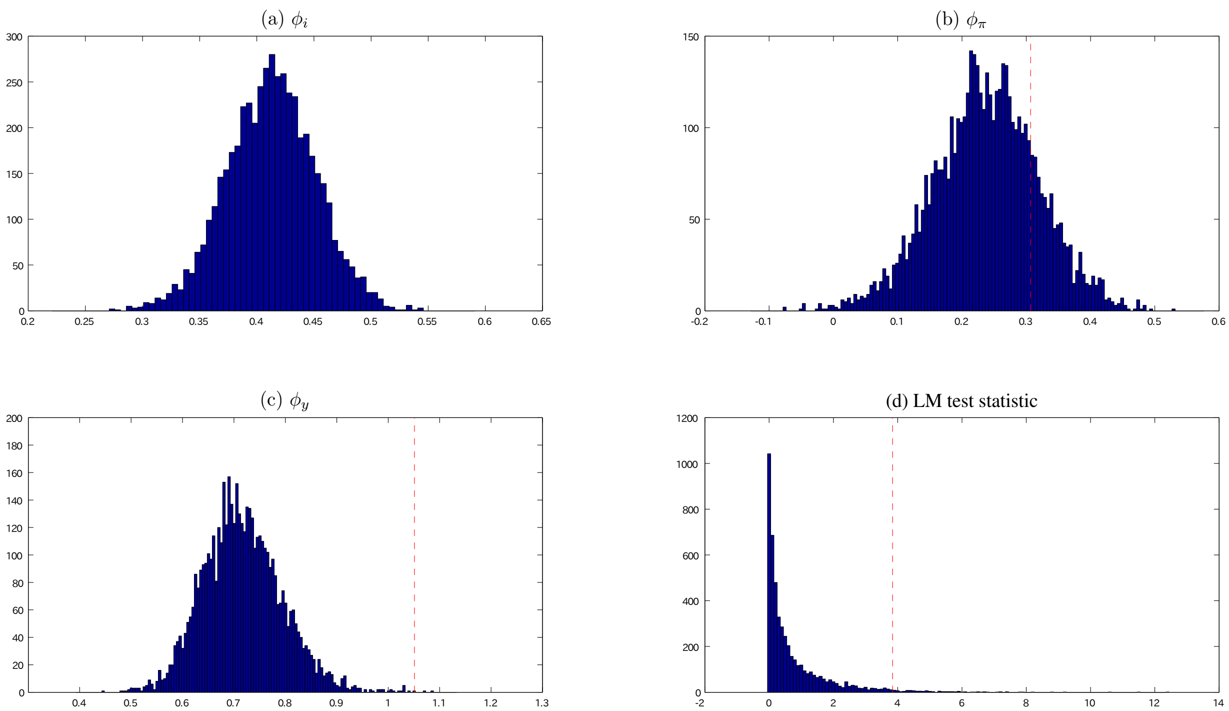


Figure 8: Distribution of the OLS estimates of eq.(12): The backward-looking model (Dashed vertical lines: theoretical values (b and c), the 95% critical value for the Breusch-Godfrey LM test statistic (d) ).

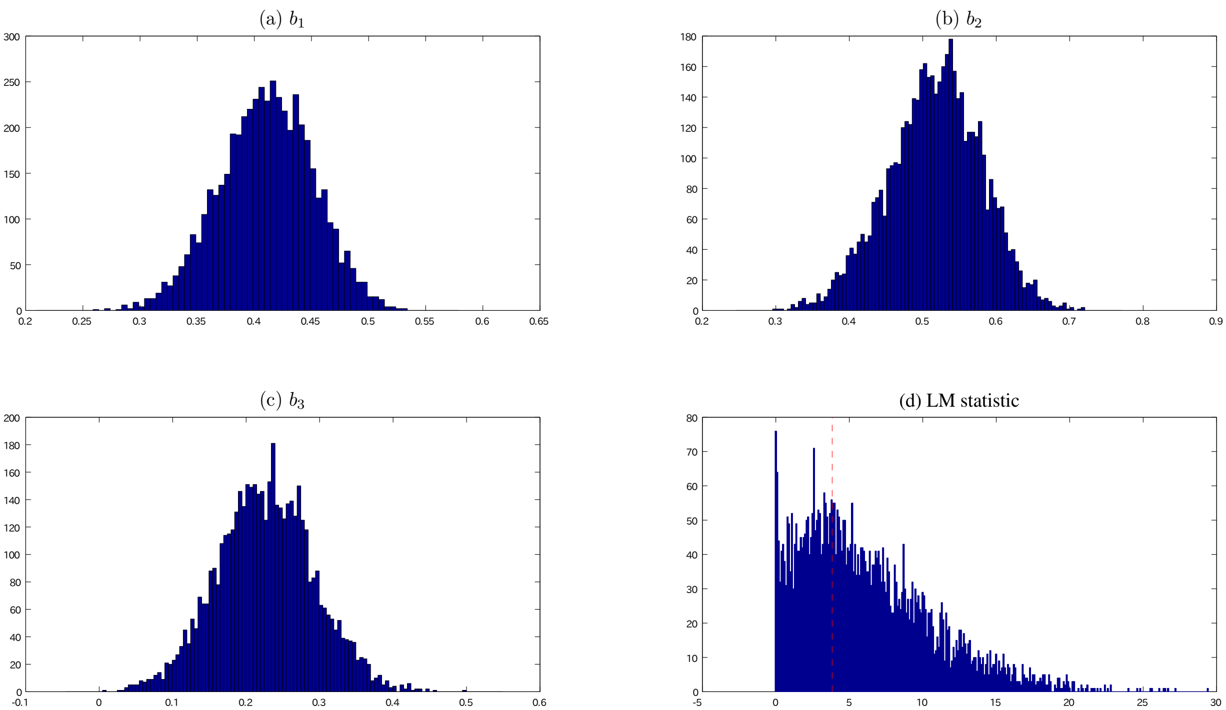


Figure 9: The OLS estimates of English-Nelson-Sack equation (16): $\rho = 0$. (Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.)

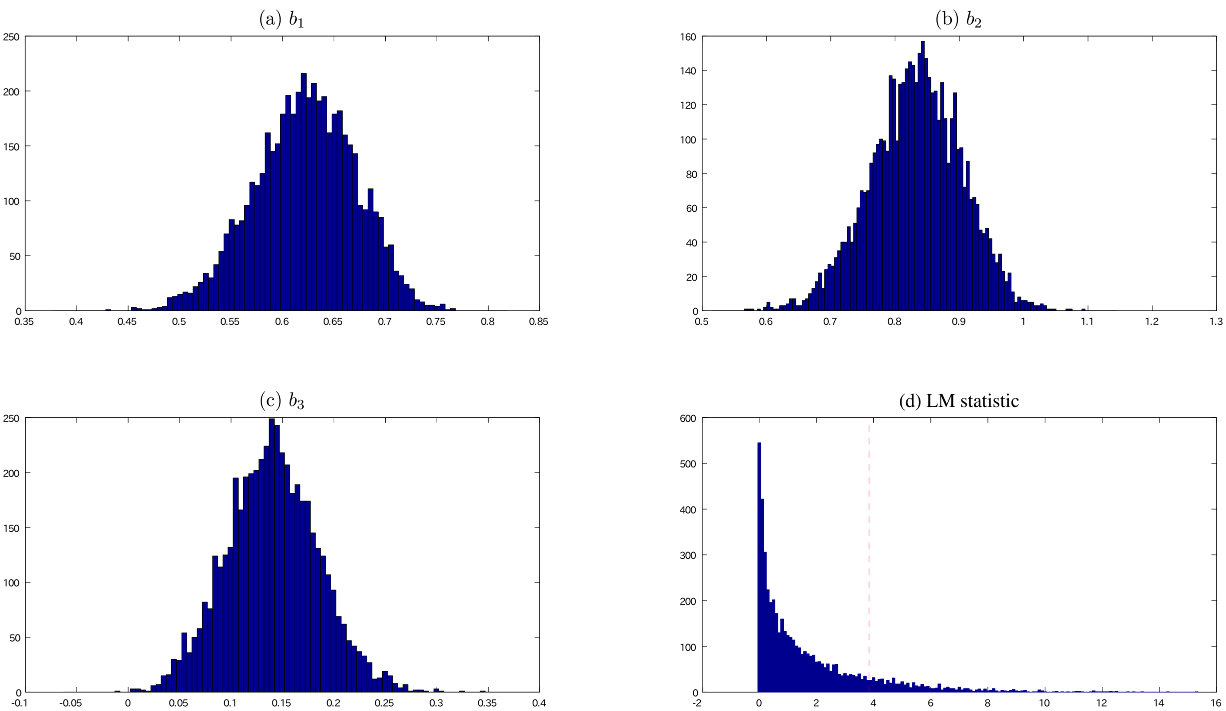


Figure 10: The OLS estimates of English-Nelson-Sack equation (16): $\rho = .8$. (Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.)

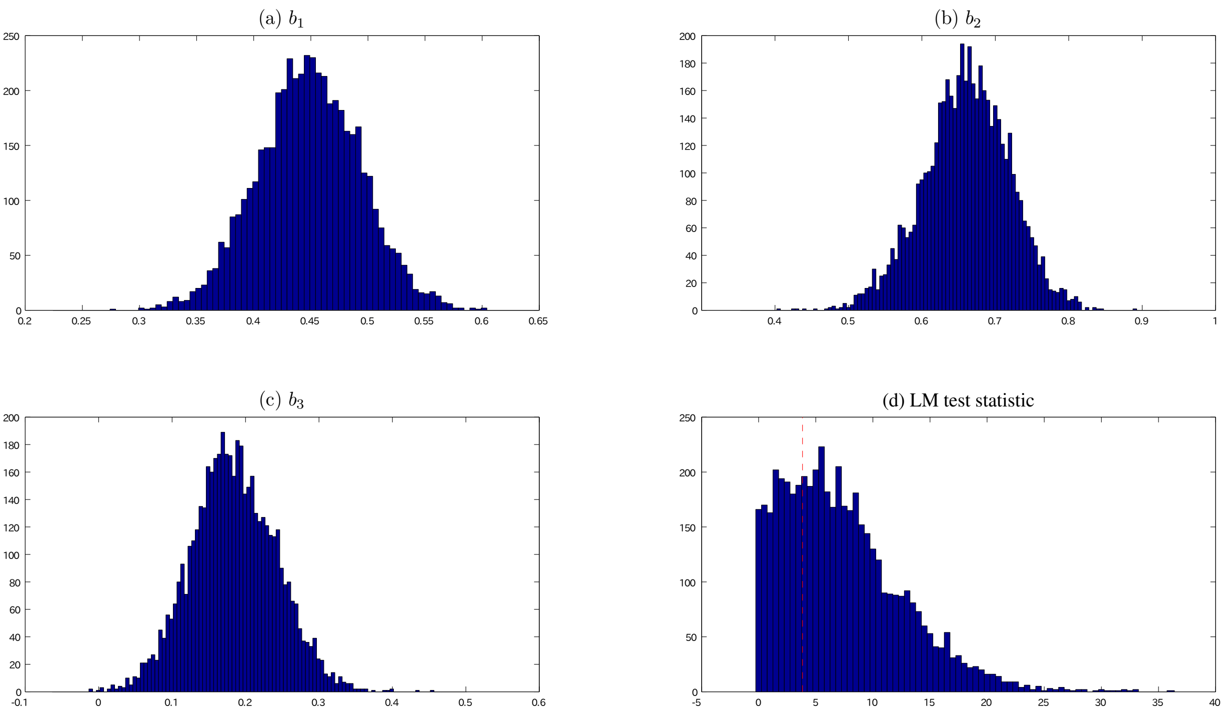


Figure 11: The OLS estimates of English-Nelson-Sack equation (16): The backward-looking model (Dashed vertical line: the 95% critical value for the Breusch-Godfrey LM test statistic.)

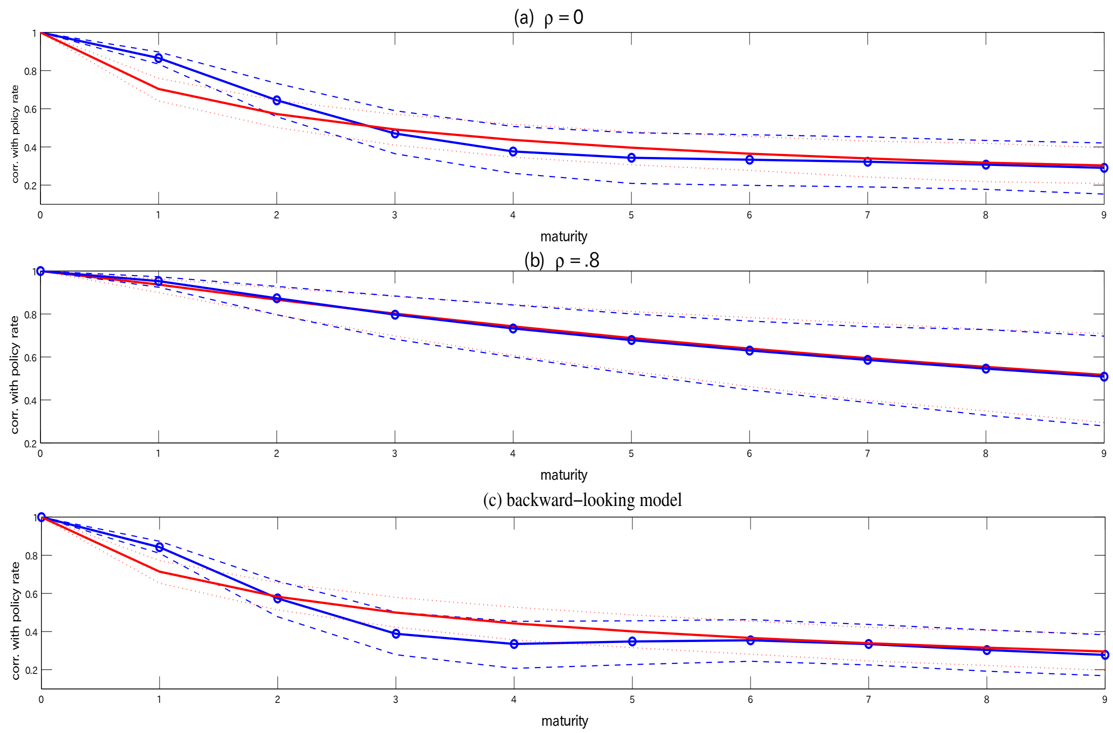


Figure 12: The correlation between the policy rate and long-term rates (solid line with circles: correlations under irreversible policies. Solid line: correlations under no constraint policies. 95% confidence intervals for irreversible and no-constraint policies are illustrated by dashed and dotted lines, respectively.)