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Abstract

This paper investigates Bertrand competition of unionized mixed duopoly when the public firm is less efficient than the private firm, including endogenous imposition of the budget constraint on the public firm. Thus, we show that if the public firm's inefficiency is sufficiently small, no imposition of budget constraint is more likely to improve welfare and vice versa. Moreover, we suggest that in either budget constraint or non-budget constraint, the wages of the public firm can be smaller or larger than those of private firm depending upon the degree of inefficiency, which draws contrast to the finding of the previous literature. These results means that both the public firm and its union may or may not have different preferences with regard to the imposition of budget constraint depending upon both the degree of inefficiency and imperfect substitutability.
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1 Introduction

There are several studies of mixed oligopolies where a public firm traditionally maximizes social welfare, while the private firms compete with the public firm for maximizing their own profits[1]. Recently, the economic implications of mixed-oligopoly markets have been an issue for both market-structure efficiency and privatization with respect to the change in competition. Some pioneering studies conducted by De Fraja and Delbono (1989) and Beato and Mas-Colell (1984) on mixed oligopolies employed game-theoretic analysis of public and private firms. In particular, De Fraja (1993) and Haskel and Sanchis (1995) investigated the wage bargaining process for labor unions in a mixed duopoly in order to examine the effects of privatization. Although some theoretical studies have been successful in explaining mixed oligopolies where a public firm maximizes social welfare in a Cournot fashion of unionized mixed oligopoly, while by adopting the union's bargaining game under Bertrand competition, this paper addresses the issue of endogenous budget-constraints of the public firm. More specifically, we illustrate how, when the public firm is assumed to be less efficient than the private firms in a simple Bertrand competition of a mixed duopoly model, improved options for imposing budget constraints on the public firm may impact social welfare.

While the existing theoretical literature shows the relationship between the unionization structure and the union's utility for foreign direct investment, the interaction between labor markets and mixed oligopolies under Bertrand competition has received little attention in the theoretical literature[2]. On the other hand, there have been some attempts to introduce the union's utility into a model of Cournot competition of a mixed duopoly market; these included De Fraja (1993), Haskel and Sanchis (1995), Barcena-Ruiz and Campo (2009), and Haskel and Szymanski (1993). Furthermore, focusing on wage regulation that is imposed on the public firm and the union, Ishida and Matsushima (2006, 2009) analyzed the optimal framework. The papers

[1]Many of the issues concerning mixed oligopolies have been analyzed: partial privatization, capacity choice, and endogenous timing and so on. For the recent literature on the mixed oligopoly models, see Barcena-Ruiz (2007), Matsumura (1998), Matsumura and Matsushima (2004), Lu and Poddar (2006) and references therein. See also De Fraja and Delbono (1990), Bös (1991) and Nett (1993) for the general reviews of the mixed oligopoly model.

[2]Motivated by institutional diversity in unionization structures and the growth of foreign direct investment (FDI), the current bargaining process between firms and unions has developed independently. As identified by Naylor (1998, 1999), Zhao (1995), Skaksen and Sørensen (2001), Haucap and Wey (2004) and Leahy and Montagna (2000), the amount of domestic production is decided through union bargaining when a firm undertakes FDI. In another related paper, the relationship between the amount of production and the union has been explored among domestic private firms (Naylor, 2002). There are many studies that consider unionized international oligopolies; see for instance, Straume (2003), Ishida and Matsushima (2008), Mukherjee and Suetrong (2007), and the references therein.

that are closest to the present model of budget constraints for the public firm are Ishida and Matsushima (2006, 2009)[3]. However, the model of Ishida and Matsushima (2006, 2009) under *Cournot competition* becomes complicated with budget constraints which the authors consider to be: (1) the budget constraint that the public firm must earn non-negative profits (i.e., if π_0 denotes the public firm's profit, then $\pi_0 \geq 0$) and (2) a tighter budget constraint that the public firm pursues more profit (i.e., $\pi_0 \geq \overline{\pi}_0$ for some $\overline{\pi}_0 > 0$). By using numerical examples where the wage regulation was not imposed on the public firm, the authors showed that the tighter budget constraint is generally welfare-improving[4]. This is because a tighter budget constraint forces the public firm's union to be less aggressive in wage bargaining and to lower its wages, including the private firm's wage; thus, the market price is lowered.

While many of the previous analyses that have been conducted on union's utility focused on the power of collective bargaining, most analyses have not investigated the relative efficiency of private firms over public firm under Bertrand competition. Any discussion of the issue of unionization structure in Bertrand competition of mixed oligopolies ignores the inherent inefficiency of public firm. In fact, on the basis of the present model of a unionized mixed oligopoly under Bertrand competition, we propose that budget constraint or non-budget constraint motives are endogenously determined rather than exogenously imposed[5]. Consequently, our paper differs from prior studies that have been conducted on unionized mixed oligopolies, which have tended to focus on privatization without giving any attention to the endogenous budget constraints of the public firm under Cournot competition. In contrast, our paper investigates how wage-bargaining process functions in a mixed duopoly in order to explicitly examine the budget constraints of public firm. Concerning Bertrand competition, this paper compares the case where a public firm must make enough profit to at least break even (i.e., where profitability is constrained) with the case where no budget constraint is imposed whatsoever (i.e., where π_0 can be anything).

Consequently, we find that if a public firm is sufficiently inefficient in terms of the cost function in a certain condition of imperfect substitutability, it is desirable from the viewpoint of

[3]Willner (1999) considered that the public firm's payoff in the bargaining is its profits with restriction that the price should not be allowed fall below an exogenous parameter.

[4]Moreover, in the work of Ishida and Matsushima (2006, 2009), budget constraints imposed on the public firm were treated as exogenous rather than endogenous.

[5]Although we do not consider worker effort as did Schmidt (1996), De Fraja (1993), Haskel and Sanchis (1995), and Haskel and Szymanski (1993), we assume that each firm in the mixed oligopoly bargains with workers over the degree of wages. Direct empirical evidence in the UK suggests that employment is not a subject of the bargain–see Oswald and Turnbull (1985) and Haskel and Sanchis (1995) as well as references therein.

social welfare to force the public firm to try to break even through the imposition of a budget constraint–and vice versa if the degree of inefficiency is sufficiently small. This means that even though the profit of the public firm is becomes a negative (respectively, zero) value without (respectively, with) the imposition of budget constraint on the public firm, there is a possibility that improves more social welfare if the degree of inefficiency is sufficiently small (respectively, large) in a certain condition of imperfect substitutability. This is because regardless of whether a budget constraint is imposed, the public firm's union will be more (conversely, less) aggressive in wage bargaining, and pushing for higher (conversely, lower) wages if the public firm will be sufficiently efficient (conversely, inefficient) in terms of the cost function. The comparison of social welfare hinges critically on the fact that the wage levels in the public firm are determined according to the efficiency of the firm, since each level of social welfare is determined by a representative consumer's utility and the type of budget constraint effects. That is, even though a representative consumer's utility and prices are higher when a budget constraint is imposed on the public firm than when none is imposed on the public firm, social welfare changes with the type of budget constraint. Furthermore, both the public firm (or government) and its union may or may not have different preferences with regard to the imposition of the budget constraint depending on the various ranges of imperfect substitutability and the degree of inefficiency of the public firm. Thus, present paper suggests a new optimal governmental policy with regard to the imposition of the budget constraint into the trade-off between the public firm and its union. This trade-off depends on both the degree of inefficiency and imperfect substitutability, since our model suggests that the wages of the public firm are lower than those of the private firm if the former's degree of inefficiency is sufficiently large, and vice versa.

On the other hand, Ishida and Matsushima (2009) show that under Cournot competition, it is often welfare-improving to impose a tighter budget constraint rather than the imposition of budget constraint on the public firm. Thus, they show that when the public firm the potential to be as efficient as private firm, a tight budget constraint allows the public firm pursue a positive profit. However, in contrast to Ishida and Matsushima's (2006, 2009) findings, this paper shows that when the public firm is less efficient than the private firms under Bertrand competition, it is welfare-improving or welfare-harming to impose a budget constraint and force the public firm to break even from negative or positive profit to zero profit. Since the public firm studied in this paper ended in negative or positive profit when a budget constraint was not imposed, it seems clear that the implementation of budget constraints effectively forces public firm to more or less actively pursue profit depending on both the degree of inefficiency and imperfect substitutability. Due to the strategic complementarity that exists between total output and wage levels, wage level in private firm remains stable, while public firm is less efficient because it is presumed that the wage rate will fluctuate. The level of the wage in the public firm when the public firm does not face budget constraints works to further reduce social welfare by increasing inefficiency, and vice versa. For this result, we obtain that differences in the implementation of budget constraint depend on both the degree of inefficiency and imperfect substitutability between the public firm and its union; these effects were not found in Ishida and Matsushima's (2009) study. This finding, the main one that emerged in our study, directly contradicts the findings of Ishida and Matsushima (2006, 2009), who found that tighter budget constraints in a mixed duopoly enhanced the overall social welfare level in cases where a public firm was as efficient as a comparable private firm.

In a companion paper, the formal structure of the model is closely related to Choi (2009), which introduces only Cournot-type of product market competition into the endogenous budget-constraint motives under a unionized mixed oligopoly. However, present paper differs from Choi (2009), which demonstrates that regardless of the degree of inefficiency, only the imposition of budget constraints on the public firm is more desirable in terms of improving welfare than the lack of such imposition when public firm is assumed to be less efficient than the private firm. Instead, by introducing Bertrand competition into unionized mixed duopoly, we investigate how the properties of social welfare are altered when there are endogenous budget-constraint motives in a unionized mixed duopoly depending upon both the degree of inefficiency of the public firm and imperfect substitutability.

The organization of the paper is as follows. In Section 2, we describe the model. Section 3 presents the market equilibrium outputs under unionized mixed duopoly. Section 4 compares each level of social welfare, the union's utiliy and the wage of the public firm with each budget constraint's strategy. Section 4 closes the paper.

2 The Basic Model

The basic structure is a differentiated duopoly model. Consider two single-product firms that produce differentiated products that are supplied by a public firm (firm 0) and a private firm

(firm 1). We assume that the representative consumer's utility is a quadratic function given by

$$U = x_i + x_j - \frac{1}{2}\left(x_i^2 + 2cx_ix_j + x_j^2\right), \quad i \neq j;\ i,j = 0,1,$$

where x_i denotes the output of firm i ($i = 0,1$). The parameter $c \in (0,1)$ is a measure of the degree of substitutability among goods, while a negative $c \in (-1,0)$ implies that goods are complements. In the main body of analysis we focus on the imperfect substitutability case of $c \in (0,1)$. Thus, the inverse demand is characterized by

$$p_i = 1 - cx_j - x_i, \quad i \neq j;\ i,j = 0,1, \tag{1}$$

where p_i is firm i's market price. Hence, we can obtain the direct demands as

$$x_i = \frac{1 - c + cp_j - p_i}{1 - c^2} \tag{2}$$

provided the quantities are positive.

All firms are homogeneous with respect to the productivity. Each firm adopts a constant returns-to-scale technology, where one unit of labor is turned into one unit of the final good. The price of labor (i.e., wage) that firm i has to pay is denoted by $w_i (i = 0,1)$. However, the public firm is assumed to be less efficient than the private firm. Let $a > 1$ be the inefficiency of the public firm. That is, we also assume that $a > 1$. Therefore, the profit of private firm 1 and the profit of the public firm (firm 0) are given by

$$\pi_1 = (p_1 - w_1)x_1 \quad \text{and} \quad \pi_0 = (p_0 - aw_0)x_0,$$

respectively. The public firm maximizes a utilitarian measure of welfare that takes into account the inefficiency of the internal public firm, viz., $a > 1$. This parameter is a measure of inefficiency due to X-inefficiency[6].

[6] As described in Choi (2009), "the weight, a, which is attached to the public firm's cost function has been assumed to be an exogenously given parameter. This assumption is *necessary* to obtain a well-defined equilibrium because if $a = 1$, the public firm's wage diverges to infinity. This assumption reflects that the public firm is less efficient than the private firms, which is well known when the marginal cost of each firm is constant. It is often plausible for political or economic reasons (e.g., X-inefficiency in the public firm) to assume the inefficiency of the public firm. This is why the public firm's inefficiency is proportional to the wage level, which we will describe later with regard to the union's utility. A similar framework is represented by the introduction of the shadow cost of the public firm in the public firm's objective function. This approach is frequently adopted in contract theory (Laffont and Tirole, 1993). We apply the same analysis to a public firm competing in a unionized mixed oligopoly given getting money for reducing public debt or distortionary taxes because of the public firm's inefficiency. Capuano and De Fei (2008) introduced this cost in a mixed oligopoly. See also Matshumura and Tomaru (2009) for the model of subsidization with excess taxation burden under a mixed duopoly. Additionally, Grönblom and Willner (2008) also adopt that marginal costs are constant with respect to output, which are denoted $c = c_0 + lw$ where l stands for labor input per unit of output and c_0 for non-wage components under the setup of a mixed oligopoly."

As is customarily assumed in Bertrand competition of a mixed oligopoly, the public firm chooses p_0 to maximize the social welfare,

$$W = U - \sum_{i=0}^{1} p_i x_i + \sum_{i=0}^{1} (\pi_i + u_i), i = 0, 1,$$

which consists of the consumer surplus, $U - \sum_{i=0}^{1} p_i x_i$, the various unions' utilities and the producer surpluses of the public and private firms.

We assume that the public and private firms are unionized, and that wages are determined as a consequence of bargaining between firms and unions. Let $\overline{w}$ denote the reservation wage. Thus, we assume that the union sets the wage while public and private firms unilaterally decide the level of employment. Taking $\overline{w}$ as given, the union's optimal wage-setting strategy w_i regarding firm i is defined as

$$\max_{w_i} \; u_i = (w_i - \overline{w})^{\theta} x_i, i = 0, 1, \tag{3}$$

where θ is the weight that the union attaches to the wage level. As Haucap and Wey (2004), Leahy and Montagna (2000) and Lommerud *et al.* (2003) suggested, we assume that $\theta = 1$ and $\overline{w} = 0$ to show our results in a simple way (see also Booth, 1995)[7].

Finally, a three-stage game is conducted. The timing of the game is as follows. At the first stage, the government decides whether or not to impose a budget constraint on the public firm's profit[8]. At the second stage, each union chooses its wage w_i after being made aware of the budget-constraint strategy of the public firm. At the third stage, each firm chooses its price p_i simultaneously to maximize its respective objective, knowing the budget-constraint motive of the public firm and the wage levels.

3 Results

Before presenting the equilibrium outcome of a comparison of the social welfare, the two cases of fixed-budget constraint are explained when the public firm is assumed to be less efficient than the private firm: (1) the budget constraint whereby the public firm must earn non-negative profits and (2) the case where no budget constraint is imposed whatsoever (i.e., the public

[7] As Naylor (1998, 1999), Haucap and Wey (2004), Leahy and Montagna (2000) and Lommerud *et al.* (2003) suggested, this is because wage claims are decided by the elasticity of labor demand rather than the firm's profit. See also Oswald and Turnbull (1985).

[8] Note that welfare maximizing behavior of the public firm assumed to be based on the objective of benevolent government. Thus, such behavior implicitly assumes that (i) the government can control over the public firm, and (ii) the objective of government is social welfare.

firm's profit can be anything). Since we focus on a symmetric Nash equilibrium in this paper, we assume that all firms choose the same type of bargaining type. We adopt a subgame perfect Nash equilibrium (SPNE); thus, the game is solved backward.

3.1 Market Equilibrium

The public firm's objective is to maximize welfare, which is defined as the sum of the consumer surplus, the firms' profits, and the unions' utilities:

$$\begin{aligned} W &= U - \sum_{i=0}^{1} p_i x_i + \sum_{i=0}^{1} \pi_i + \sum_{i=0}^{1} u_i \\ &= U + (1-a) w_0 x_0. \end{aligned}$$

We first present the market equilibrium without a budget constraint, when private firm is assumed to be more efficient than the public firm.

[**No Budget Constraint**]: Given w_i for each firm, the public firm's maximization problem is as follows:

$$\begin{aligned} \max_{p_0} W = & \frac{(1-c)+cp_1-p_0}{1-c^2} + \frac{(1-c)+cp_0-p_1}{1-c^2} + \frac{w_0(1-a)[(1-c)+cp_1-p_0]}{1-c^2} \\ & -\frac{1}{2}\left\{\frac{[(1-c)+cp_1-p_0]^2+[(1-c)+cp_0-p_1]^2+2c[(1-c)+cp_0-p_1][(1-c)+cp_1-p_0]}{(1-c^2)^2}\right\}. \end{aligned}$$

Taking w_i as given, the best reply function of the public firm is given by

$$\frac{\partial W}{\partial p_0} = 0 \Leftrightarrow p_0 = cp_1 - (1-a)w_0. \tag{4}$$

On the other hand, the first-order condition for the private firm is given by

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow p_1 = \frac{1-c+cp_0+w_1}{2}. \tag{5}$$

By using x_i and solving the these two equations (4) and (5) problems yields

$$x_0 = \frac{1-c+(1-a)w_0}{1-c^2}, \tag{6}$$

$$x_1 = \frac{1-c-(1-c^2)w_1-(1-a)cw_0}{(2-c^2)(1-c^2)}. \tag{7}$$

which each x_i is a positive value because we have already assumed that $a > 1$.

In the second stage of this case, each wage is set to maximize the utility of the corresponding firm's union. To do this, the two independent maximization problems should be considered simultaneously. Using (6) and (7), the problem for union i is defined as

$$\max_{w_0}\ u_0 = w_0 x_0 = \frac{w_0[1-c+(1-a)w_0]}{1-c^2}, \tag{8}$$

$$\max_{w_1}\ u_1 = w_1 x_1 = \frac{w_1[1-c-(1-c^2)w_1-(1-a)cw_0]}{(2-c^2)(1-c^2)}. \tag{9}$$

Each first-order condition then leads to

$$w_0 = \frac{1-c}{2(a-1)}, \quad w_1 = \frac{1-c-c(1-a)w_0}{2(1-c^2)}.$$

Therefore, straightforward computation yields the following Lemma 1.

Lemma 1: *Suppose that under Bertrand competition, the budget constraint is not imposed on the public firm and that each union is allowed to bargain collectively. Then, the equilibrium values, denoted as w_i^N, x_i^N, p_i^N and u_i^N, are given by*

$$\begin{aligned}
w_0^N &= \frac{1-c}{2(a-1)}, \quad w_1^N = \frac{2+c}{4(1+c)};\\
x_0^N &= \frac{1}{2(1+c)}, \quad x_1^N = \frac{2+c}{4(2-c^2)(1+c)};\\
p_0^N &= \frac{4+6c-3c^2-4c^3}{4(1+c)(2-c^2)}, \quad p_1^N = \frac{(2+c)(3-2c^2)}{4(1+c)(2-c^2)};\\
u_0^N &= \frac{1-c}{4(a-1)(1+c)}, \quad u_1^N = \frac{(2+c)^2}{16(2-c^2)(1+c)^2}.
\end{aligned}$$

Lemma 1 suggests that given that the equilibrium wage in the public firm is determined by public firm's output in the third stage, the inefficiency of the public firm in the calculation of each output level and price level is canceled out. This is why each output level and price level is independent of a.

Noting that $W^N = U^N + (1-a)w_0^N x_0^N$ without the budget constraint, we can compute the social welfare W^N,

$$W^N = \frac{44+92c+7c^2-80c^3-32c^4+16c^5+8c^6}{32(1+c)^2(2-c^2)^2} > 0, \tag{10}$$

even though the public firm's profit depends on its degree of inefficiency;

$$\pi_0^N = \frac{a(6c+3c^2-4c^3-2c^4)-4-6c+3c^2+4c^3}{8(a-1)(1+c)^2(2-c^2)}. \tag{11}$$

Therefore, if $a > a^N \equiv \frac{4+6c-3c^2-4c^3}{6c+3c^2-4c^3-2c^4} > 1$, then $\pi_0^N > 0$. Otherwise $\pi_0^N < 0$ if $a^N > a > 1$. That is, regardless of the public firm's profit, the lack of imposition of a budget constraint on the public firm makes the social welfare to be a positive value because the public firm's goal is to maximize social welfare.

At sum, even though the outputs and prices of all firms and social welfare without budget constraint are independent of a, this does occur in the equilibrium path.

[**Imposition of Budget Constraint**]: Similar to "No Budget Constraint," at the third stage the public firm's objective is to maximize welfare which is defined as

$$\begin{aligned}\max_{p_0}\ W =& \frac{(1-c)+cp_1-p_0}{1-c^2} + \frac{(1-c)+cp_0-p_1}{1-c^2} + \frac{w_0(1-a)[(1-c)+cp_1-p_0]}{1-c^2} \\ &- \frac{1}{2}\left\{\frac{[(1-c)+cp_1-p_0]^2+[(1-c)+cp_0-p_1]^2+2c[(1-c)+cp_0-p_1][(1-c)+cp_1-p_0]}{(1-c^2)^2}\right\}, \\ &\text{s.t.} \quad \frac{(p_0-aw_0)[(1-c)+cp_1-p_0]}{1-c^2} \geq 0.\end{aligned}$$

The constraint implies that there is some lower-bound restriction on the public firm's profit, i.e., the public firm faces a budget constraint. Therefore, since we assume that each firm's output is a positive value as in (2), we can rewrite the budget constraint as follows: $(p_0 - aw_0)x_0 \geq 0 \Leftrightarrow p_0 - aw_0 \geq 0$.

Denoting the multiplier of the budget constraint λ and given wage level w_i in the third stage, the first-order conditions are given by

$$\frac{\partial L}{\partial p_0} = 0 \Leftrightarrow \frac{-(cp_1-p_0)+(1-a)w_0}{1-c^2} = \lambda, \tag{12}$$

$$\frac{\partial L}{\partial \lambda} = p_0 - aw_0 = 0. \tag{13}$$

On the other hand, the optimal price for the private firm is equal to Eq. (5): $p_1 = \frac{1-c+cp_0+w_1}{2}$. Given these results, we now obtain the output level for each firm. By using x_i and solving the these two equations (5) and (13) problems yields

$$x_0 = \frac{(1-c)(2+c)+cw_1-a(2-c^2)w_0}{2(1-c^2)}, \tag{14}$$

$$x_1 = \frac{1-c-w_1+acw_0}{2(1-c^2)}. \tag{15}$$

To solve for Lagrangian equation, suppose that the budget constraint is momentarily binding. We check ex-post that this constraint is binding.

In the second stage of this case, each wage is set to maximize the its own firm's union: $u_i = x_i w_i$. Using (14) and (15), the problem for union i is defined as

$$\max_{w_0} \ u_0 = w_0 x_0 = \frac{w_0[(1-c)(2+c) + cw_1 - a(2-c^2)w_0]}{2(1-c^2)},$$
$$\max_{w_1} \ u_1 = w_1 x_1 = \frac{w_1(1-c-w_1+acw_0)}{2(1-c^2)}.$$

Straightforward computation yields each firm's reaction function as follows:

$$w_0 = \frac{(1-c)(2+c)+cw_1}{2a(2-c^2)}, \quad w_1 = \frac{1-c+acw_0}{2}.$$

Therefore, straightforward computation yields the following Lemma 2.

Lemma 2: *Suppose that under Bertrand competition, the budget constraint is imposed on the public firm and that each union is allowed to bargain collectively. Then, the equilibrium values, denoted as w_i^B, x_i^B, p_i^B and u_i^B, are given by*

$$w_0^B = \frac{4-c-3c^2}{a(8-5c^2)}, \quad w_1^B = \frac{4-2c-3c^2+c^3}{8-5c^2};$$
$$x_0^B = \frac{8-2c-10c^2+c^3+3c^4}{2(1-c^2)(8-5c^2)}, \quad x_1^B = \frac{4-2c-3c^2+c^3}{(1-c^2)(8-5c^2)};$$
$$p_0^B = \frac{4-c-3c^2}{8-5c^2}, \quad p_1^B = \frac{12-6c-9c^2+3c^3}{2(8-5c^2)};$$
$$u_0^B = \frac{(4-c-3c^2)(8-2c-10c^2+c^3+3c^4)}{2a(1-c^2)(8-5c^2)^2}, \quad u_1^B = \frac{(4-2c-3c^2+c^3)^2}{(1-c^2)(8-5c^2)^2}.$$

As in Lemma 1, Lemma 2 suggests that given that the equilibrium wage in the public firm is determined by public firm's output in the third stage, the inefficiency of the public firm is canceled out. That is, even though the outputs and prices of all firms with budget constraint are independent of a, this does occur in the equilibrium path.

Finally, noting that $W^B = U^B + (1-a)w_0^B x_0^B$ with imposition of a budget constraint, we can compute the social welfare as follows;

$$\begin{aligned} W^B =& \frac{a(304-144c-816c^2+316c^3+787c^4-222c^5-320c^6+50c^7+45c^8)}{8a(1-c^2)^2(8-5c^2)^2} \\ &+ \frac{4(1-a)(1-c^2)(4-c-3c^2)(8-2c-10c^2+c^3+3c^4)}{8a(1-c^2)^2(8-5c^2)^2}. \end{aligned} \tag{16}$$

Substituting Lemma 2 into (12) then we have

$$\lambda = \frac{-a(12c - 14c^2 - 7c^3 + 9c^4) + 8 - 2c - 14c^2 + 2c^3 + 6c^4}{2a(8 - 5c^2)(1 - c^2)} > 0 \text{ if } \; c \in [0.90, 1), \tag{17}$$

which the budget constraint is binding because we have assumed that $a > 1$ (i.e., $a > 1 > a^B \equiv \frac{-8+2c+14c^2-2c^3-6c^4}{-12c+14c^2+7c^3-9c^4}$ when $c \in [0.90, 1)$). At the same time, if $a > a^B > 1$ when $c \in (0, 0.89]$, then the budget constraint is also binding. Otherwise, if $a^B > a > 1$ when $c \in (0, 0.89]$, the Lagrange multiplier is negative, which means that relaxing the constraint makes social welfare even lower since the Lagrange multiplier is marginal social welfare. Thus, if $a = a^B > 1$ when $c \in (0, 0.89]$ which means that $\lambda = 0$, then $p_0 = cp_1 - (1 - a)w_0$ (see also Eq. (4)) and plugging into the private firm's response function yields the same equilibrium values such as w_i^N and x_i^N[9].

4 Comparative Statics

By applying equilibrium values to each of the cases outlined in the previous subsection, we examine the budget constraint strategy by comparing each level of social welfare in each case. Hence, we can state the following results:

Proposition 1: *Suppose that under Bertrand competition, each firm's union is allowed to bargain collectively and that the public firm is less efficient than the private firm. Then, there can exist a critical value of* a^* *such that for all* $a > a^* > 1$ *when* $c \in (0, 0.89]$*, we obtain that* $W^B > W^N$*. Otherwise,* $W^B \leq W^N$ *when* $c \in (0, 0.89]$ *if* $a^* \geq a > 1$*. On the other hand, when* $c \in [0.90, 1)$*, we obtain that* $W^B > W^N$*.*

Proof: Comparing W^N with W^B yields

$$W^B \gtreqless W^N \Leftrightarrow a \gtreqless \frac{A}{B} \equiv a^* \quad \text{where } A = -2048 - 3072c + 8064c^2 - 10240c^3 - 11712c^4 - 1280c^5$$
$$+ 6624c^6 + 24192c^7 + 624c^8 - 12672c^9 - 2448c^{10} + 3456c^{11} + 1040c^{12} - 384c^{13} - 144c^{14}$$
$$\text{and} \quad B = -1536c - 896c^2 + 7296c^3 + 4500c^4 - 14012c^5 - 13447c^6 + 4680c^7$$
$$+ 8610c^8 + 548c^9 - 1963c^{10} + 448c^{11} + 660c^{12} - 224c^{13} - 164c^{14}.$$

Since comparing A with B becomes complicated, so we need to use numerical examples of critical values to illustrate the impact of the degree of c. Thus, the numerical analysis of Table 1 given by as follows:

[9] Solving from reaction functions, we find that any pair (w_i^N, x_i^N) is obtained such as is given by Lemma 1.

Table 1: Critical Values a^* under Bertrand Competition

	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = \cdots$	$c = 0.87$	$c = 0.88$	$c = 0.89$	$c = 0.90$	$c = 0.99$
a^*	14.7513	7.4469	8.8857	4.4584	$\cdots$	1.2795	1.1621	1.0493	0.9407	0.0876

Given above Table 1, if $a > a^* \equiv \frac{A}{B}$ and $c \in (0, 0.89]$, then $W^B > W^N$ because $a^* > 1$ when $c \in (0, 0.89]$. Otherwise, if $a \leq a^* \equiv \frac{A}{B}$ when $c \in (0, 0.89]$, then $W^B \leq W^N$. On the other hand, we obtain that $W^B > W^N$ because we have assumed that $a > 1 > a^*$ when $c \in [0.90, 1)$. ■

Proposition 1 suggests that even though the profit of the public firm is becomes a negative (respectively, zero) value without (respectively, with) the imposition of budget constraint on the public firm, there is a possibility that improves more social welfare if the degree of inefficiency is sufficiently small (respectively, large) in a certain condition of imperfect substitutability. That is, if the degree of inefficiency is sufficiently large in the range of $c \in (0, 0.89]$, it is desirable in terms of improving welfare to force the public firm to try to break even through the imposition of a budget constraint, and vice versa if a is sufficiently small. This is because regardless of whether a budget constraint is imposed or not, the public firm's union will be more (conversely, less) aggressive in wage bargaining, and pushing for higher (conversely, lower) wages if the public firm is sufficiently efficient (conversely, inefficient) regarding the cost function, that is, $w_0^B > w_0^N$ (respectively, $w_0^B < w_0^N$) if $a^w = \frac{8-2c-6c^2}{c(6-c-5c^2)} > a > 1$ (conversely, $a > a^w = \frac{8-2c-6c^2}{c(6-c-5c^2)} > 1$). In the case of $c \in (0, 0.89]$, since relaxing the constraint makes social welfare even lower, due the fact that the Lagrange multiplier is negative when $a^B = a^* > a$, this effect forces social welfare to be higher when no budget constraint is imposed than when the budget constraint is imposed on the public firm. In particular, if c is sufficiently large (i.e., in the range of $c \in [0.90, 1)$), then we obtain that regardless of the degree of inefficiency, $W^B > W^N$.

Moreover, with regard to the type of budget constraint, $(1-a)w_0^N x_0^N$ and $(1-a)w_0^B x_0^B$ in W^N and W^B have the effect of lowering social welfare. Therefore, each level of social welfare is determined by a representative consumer's utility U^B and U^N, and the type of budget constraint affects $(1-a)w_0^N x_0^N$ and $(1-a)w_0^B x_0^B$. As a result, even though a representative consumer's utility and prices are higher when a budget constraint is imposed than when no budget constraint is imposed on the public firm because of $x_0^B + x_1^B > x_0^N + x_1^N$, the social welfare changes with the type of budget constraint[10]. We call the former, the *"consumer effect,"* and the latter, the *"budget effect."* Thus, if the degree of inefficiency is sufficiently large (conversely, small), welfare

[10]We can easily calculate that $U^B > U^N$ and $p_i^N > p_i^B$.

improvement in W^B (conversely, W^N) is possible because the negative budget-effect is lower in W^B (conversely, W^N) than in W^N (conversely, W^B), whereas the consumer effect in W^B is larger than in W^N.

However, Proposition 1 states that the finding of Ishida and Matsushima (2006, 2009) does not hold true when the wage regulation is not imposed on the public firm. According to Ishida and Matsushima (2006, 2009), when there is no cost difference between public and private firm under Cournot-type of product market competition, their numerical examples illustrate the impact of budget constraints. That is, a tighter budget constraint under Cournot competition forces the public firm's union to be less aggressive in wage bargaining and to lower its wage, which also lowers the private firms' wages as well. A decrease in wages serves to improve welfare by increasing the total output and lowering the market price (Ishida and Matsushima, 2006, p. 18). On the other hand, introducing Cournot competition into the unionized mixed oligopoly, Choi (2009) demonstrates that regardless of the degree of inefficiency, only the imposition of budget constraints on the public firm is more desirable in terms of improving welfare than the lack of such imposition when public firm is assumed to be less efficient than the private firm[11]. However, in contrast to Ishida and Matsushima (2006, 2009) and Choi (2009), we find that if the degree of inefficiency is sufficiently large in the range of $c \in (0, 0.89]$ under Bertrand competition, the imposition of a budget constraint on the public firm is more desirable in terms of welfare improvement than the lack of such imposition and vice versa.

We now examine under what conditions each union made better off when either the budget constraint is imposed or no budget constraint is imposed on the public firm.

Proposition 2: *Suppose that under Bertrand competition, each firm's union is allowed to bargain collectively and that the public firm is less efficient than the private firm. Then, there can exist a critical value of a^{**} such that for all $a > a^{**} > 1$ when $c \in (0, 0.73]$, we obtain that $u_0^N > u_0^B$. Otherwise, $u_0^N \leq u_0^B$ when $c \in (0, 0.73]$ if $a^{**} \geq a > 1$. On the other hand, when $c \in [0.74, 1)$, we obtain that $u_0^N > u_0^B$.*

[11] The detailed computations of Cournot competition are available from author upon request. See the Appendix 1 of Choi's (2009) detailed-version, which will not be included in the main paper.

Proof: Comparing u_0^N with u_0^B yields

$$u_0^N \gtreqless u_0^B \Leftrightarrow a \gtreqless a^{**} \equiv \frac{-64+32c+124c^2-40c^3-82c^4+12c^5+18c^6}{-32c-20c^2+104c^3+33c^4-127c^5-7c^6+25c^7}.$$

Similar to the proof of Proposition 1, since comparing u_0^N with u_0^B becomes complicated, so we need to use numerical examples to illustrate the impact of the degree of c. Using this computation, the numerical analysis of Table 2 given by as follows:

Table 2: Critical Values a^{**} under Bertrand Competition

	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = \cdots$	$c = 0.73$	$c = 0.74$	$c = 0.75$	$c = 0.98$	$c = 0.99$
a^{**}	18.0962	5.2135	2.75	1.2913	$\cdots$	1.0517	0.9716	0.8921	0.0018	0.0004

As in the Table 2, if $a > a^{**}$ and $c \in (0, 0.73]$, then $u_0^N > u_0^B$ because $a^{**} > 1$ when $c \in (0, 0.73]$. Otherwise, if $a \leq a^{**}$ when $c \in (0, 0.73]$, then $u_0^N \leq u_0^B$. On the other hand, we obtain that $u_0^N > u_0^B$ because we have assumed that $a > 1 > a^{**}$ when $c \in [0.74, 1)$. ■

Contrary to Proposition 1, Proposition 2 suggests that the differences in the implementation of a budget constraint depend on both the degrees of inefficiency and imperfect substitutability between the public firm and its union. That is, when imperfect substitutability falls into the range of $c \in (0, 0.73]$, the public firm has an incentive to impose the budget constraint, whereas the union of the public firm does no have an incentive to accept the budget constraint if the degree of inefficiency is sufficiently small–and vice versa if imperfect substitutability falls into the range of $c \in (0, 0.73]$. Moreover, if c is sufficiently large (i.e., in the range of $c \in [0.74, 1)$), then we obtain that regardless of the degree of inefficiency, $u_0^B < u_0^N$. Hence, as compared to Proposition 1, both the public firm and its union have incentives to impose the budget constraint if the degree of inefficiency is in the range of $c \in [0.74, 0.89]$. Consequently, the difference in the union's utility of the public firm between the imposition of budget constraint and no imposition of budget constraint can be interpreted as follows. Proposition 2 provides new insight into the trade-off between the public firm and its union that depends on both the degree of inefficiency and imperfect substitutability in the government's optimal policy with regard to the imposition of the budget constraint.

On the other hand, having derived the equilibria for two instances of fixed-wage bargaining, budget-constraint motives and social-welfare levels, we can state the following results.

Corollary 1: *Suppose that under Bertrand competition, each firm's union is allowed to bargain collectively and that the public firm is less efficient than the private firms. Then, if the degree of inefficiency is sufficiently large in the range of* $c \in (0, 0.89]$*, the government should impose the budget constraint on the public firm in the first stage, and vice versa.*

Corollary 1 suggests that one significant result can be derived: either the imposition of the budget constraint or no imposition of the budget constraint on the public firm can be used as a commitment device to improve the social welfare if the public firm is assumed to be less efficient than the private firm.

Although the union's utility under a budget constraint is smaller than its utility without a budget constraint when the degree of inefficiency is sufficiently large, the government has the incentive to choose to impose a budget constraint on the public firm because of the welfare improvement. However, this result greatly depends on the strategic commitment that is made by both the degree of inefficiency and imperfect substitutability in the first stage. As a result, whether or not a budget constraint is imposed does impact upon the public and private firms' wages, unions' utilities and social welfare. Thus, Corollary 1 suggests that the condition under which is efficient to impose a budget constraint depends on whether that choice is endogenous or not. Hence, from the point of view of the government (or public firm), what this paper does is to compare the case where there is no budget constraint on the public firm with the case where there is a budget constraint on the public firm.

Finally, from Lemma 1 and 2 and Proposition 1, we can state the following results.

Proposition 3: *Suppose that under Bertrand competition, each firm's union is allowed to bargain collectively and that the public firm is less efficient than the private firms. Then, when the budget constraint is imposed (respectively, not imposed) on the public firm, the public firm pays a lower wage than the private firms if the degree of inefficiency is sufficient large, and vice versa.*

Proof: Comparing w_0^k with w_1^k where $k = N, B$ yields

$$w_1^N \gtreqless w_0^N \Leftrightarrow a \gtreqless a^{\dagger} \equiv \frac{4 + c - 2c^2}{2 + c},$$
$$w_1^B \gtreqless w_0^B \Leftrightarrow a \gtreqless a^{\ddagger} \equiv \frac{4 - c - 3c^2}{4 - 2c - 3c^2 + c^3}$$

because $a^{\ddagger}$ and $a^{\ddagger}$ are always larger than unity. ■

Proposition 3 states that the higher wages often observed in public firms are not necessarily due to the different objective functions. According to De Fraja (1993, p. 468), the "reasons (the effect of higher wages in the public firm) are essentially the different objective function and the oligopolistic interdependence between private and public producers." In contrast, De Fraja (1993) ignored the case where the public firm is less efficient than the private firm[12]. Thus, the reason for the higher wages of the public firm in our paper is essentially a sufficiently small degree of inefficiency in the public firm. Although we do not consider worker effort as Schmidt (1996), Haskel and Sanchis (1995), and Haskel and Szymanski (1993) did, we obtain the result that a lower wage (or same wage level with the private firm) can be observed in the public firm due to the inefficiency of the public firm. Furthermore, Choi (2009) demonstrates that only when the budget constraint is imposed on the public firm under Cournot competition, the level of wages between the public firm and the private firms is compared since the imposition of budget constraint can improve social welfare, while Proposition 3 under Bertrand competition compares each wage level between the public and private firm where a public firm must make enough profit to at least break even with each wage level where no budget constraint is imposed whatsoever.

5 Concluding Remarks

Concerning Bertrand competition, this study investigated changes in social welfare based on the motive of a budget constraint in a mixed-duopoly market when the public firm is assumed to be less efficient than the private firm. The decision of whether or not to impose a budget constraint was proposed endogenously as the first stage depending upon both the degree of inefficiency of the public firm and imperfect substitutability. We found that even though the profit of the public firm is becomes a negative (respectively, zero) value without (respectively, with) the imposition of budget constraint on the public firm, there is a possibility that improves more social welfare if the degree of inefficiency is sufficiently small (respectively, large) in a certain condition of imperfect substitutability. This result differs from the findings of Ishida and Matsushima (2006, 2009) that in Cournot competition of a unionized mixed-duopoly, a tight budget-constraint can

[12]The imposition of a budget constraint and the discounting of the public firm's profit, as done by De Fraja (1993), are two ways of avoiding this problem.

enhance social welfare. Our model also suggests that in either budget constraint or non-budget constraint, the wage of the public firm is smaller than those of private firms if the degree of inefficiency of the public firm is sufficiently large; the converse is also true. Thus, we provide new insight into the trade-off between the public firm and its union that depends on both the degree of inefficiency and the imperfect substitutability in the government's optimal policy with regard to the imposition of the budget constraint.

However, we have used simplifying assumption that each firm's union is allowed to engage in decentralized bargaining. Furthermore, a limitation of this study is that there is different bargaining structure among the firms, which can be considered the preferred bargaining structure. Therefore, this paper does not investigate the existence of degree of centralization of union bargaining matters for all firms to choose different bargaining regime for different firms. There could be important economic implications if the analysis was expanded to relax the assumption that the unions and the firms have different motives regarding bargaining structure in the framework of the motive of a budget constraint in a mixed-duopoly market. Finally, we did not extend the model to consider a situation where the public firm competes with both domestic and foreign private firms. Also, in this paper, we do not analyze privatization policy; in addition, a broader range of policies – such as tariffs and subsidies – is worth considering. The extension of our model in this regard remains a direction for future research.

References

[1] Barcena-Ruiz, J.C., 2007,"Endogenous Timing in a Mixed Duopoly: Price Competition," *Journal of Economics*, 91, 263-272.

[2] Barcena-Ruiz, J.C. and M. L. Campo, 2009, "Relocation and Public Ownership of Firms," *Journal of the Japanese and International Economies*, 23, 71-85.

[3] Beato, P. and A. Mas-Colell, 1984, "Marginal Cost Pricing as a Regulation Mechanism in Mixed Markets," in M. Marchand, P. Pestieu and H. Tulkens, (eds.), *The Performance of Public Enterprises,* pp. 81-101, North-Holland, Amsterdam.

[4] Booth, A., 1995, *The Economics of the Trade Union*, Cambridge, Cambridge University Press.

[5] Bös, D., 1991, *Privatization: A Theoretical Treatment*, Oxford University Press.

[6] Capuano, C. and G. De Feo, 2008, "Mixed Duopoly, Privatization and the Shadow Cost of Public Funds," CORE Discussion Paper 2008-15.

[7] Choi, K., 2009, "Strategic Budget Constraints in a Unionized Mixed Oligopoly," mimeo, Pusan National University.

[8] De Fraja, G., 1993, "Union and Wages in Public and Private Firms: A Game Theoretic Analysis," *Oxford Economic Papers*, 45, 457-469.

[9] De Fraja, G. and F. Delbono, 1989, "Alternative Strategies of a Public Enterprise in Oligopoly," *Oxford Economic Papers*, 41, 302-311.

[10] De Fraja, G. and F. Delbono, 1990, "Game Theoretic Models of Mixed Oligopoly," *Journal of Economic Surveys*, 4, 1-17.

[11] Grönblom, S. and J. Willner, 2008, "Privatization and Liberalization: Costs and Benefits in the Presence of Wage-Bargaining," *Annals of Public and Cooperative Economics,* 79, 133-160.

[12] Haskel, J. and A. Sanchis, 1995, "Privatisation and X-Inefficiency: A Bargaining Approach," *Journal of Industrial Economics,* 43, 301-321.

[13] Haskel, J. and S. Szymanski, 1993, "Privatization, Liberalization, Wages and Employment: Theory and Evidence for the UK,"*Economica,* 60, 161-182.

[14] Haucap, J. and C. Wey, 2004,"Unionisation Structures and Innovation Incentives," *Economic Journal*, 114, C149-C465.

[15] Ishida, J. and N. Matsushima, 2006, "Should Civil Servants be Restricted in Wage Bargaining? A Mixed-Duopoly Approach," Kobe University, GSBA Discussion Paper Series 2006-7.

[16] Ishida, J. and N. Matsushima, 2008, "Outward FDI in Unionized Oligopoly: Welfare and Policy Implications," Osaka University, OSIPP Discussion Paper DP-2008-E-005.

[17] Ishida, J. and N. Matsushima, 2009, "Should Civil Servants be Restricted in Wage Bargaining? A Mixed-Duopoly Approach,"*Journal of Public Economics,* 93, 634-646.

[18] Laffont, J-J, and J. Tirole, 1993, *A Theory of Incentives in Procurement and Regulation,* Cambridge, MA: MIT Press.

[19] Leahy, D. and C. Montagna, 2000, "Unionisation and Foreign Direct Investment: Challenging Conventional Wisdom?" *Economic Journal*, 110, C80-C92.

[20] Lommerud, K. E., F. Meland, and L. Sørgard, 2003,"Unionised Oligopoly, Trade Liberalisation and Location Choice," *Economic Journal*, 113, 782-800.

[21] Lu, Y. and S. Poddar, 2006, "The Choice of Capacity in Mixed Duopoly under Demand Uncertainty," *Manchester School*, 74, 266-272.

[22] Matsumura. T., 1998, "Partial Privatization in Mixed Duopoly," *Journal of Public Economics,* 70, 473-483.

[23] Matsumura, T. and O. Kanda, 2005,"Mixed Oligopoly at Free Entry Markets," *Journal of Economics,* 84, 27-48.

[24] Matsumura, T. and N. Matsushima, 2004, "Endogenous Cost Differentials between Public and Private Enterprises: A Mixed Duopoly Approach," *Economica*, 71, 671-688.

[25] Matshumura. T. and Y. Tomaru, 2009, "Mixed Duopoly, Privatization and Subsidization with Excess Taxation Burden," G-COE GLOBE 2 Working Paper Series, No. 19, Waseda University.

[26] Mukherjee, A. and K. Suetrong, 2007,"Unionisation Structure and Strategic Foreign Direct Investment," GEP Research Paper Series 2007/22, University of Nottingham.

[27] Naylor, R. A., 1998,"International Trade and Economic Integration when Labour Markets are Generally Unionised," *European Economic Review,* 42, 1251-1267.

[28] Naylor, R. A., 1999,"Wage Union Strategies and International Trade," *Economic Journal,* 109, 102-125.

[29] Naylor, R. A., 2002,"Industry Profits and Competition under Bilateral Oligopoly," *Economics Letters,* 77, 169-175.

[30] Nett, L., 1993, "Mixed Oligopoly with Homogeneous Goods," *Annals of Public and Cooperative Economics,* 64, 367-393.

[31] Oswald, A. J. and P. Turnbull, 1985, "Pay and Employment Determination in Britain: What are Labour Contracts Really Like," *Oxford Review of Economic Policy*, 1, 80-97.

[32] Schmidt, K. M., 1996, "The Costs and Benefits of Privatization: An Incomplete Contracts Approach," *Journal of Law, Economics and Organization,* 12, 1-24.

[33] Skaksen, M. Y. and J. R. Sørensen, 2001, "Should Trade Union Appreciate Foreign Direct Investment?," *Journal of International Economics,* 55, 379-390.

[34] Straume, O. R., 2003, "International Merger and Trade Liberalization: Implications for Unionized Labour," *International Journal of Industrial Organization,* 21, 717-735.

[35] Willner, J., 1999, "Policy Objectives and Performance in a Mixed Market with Bargaining," *International Journal of Industrial Organization,* 17, 137-145.

[36] Zhao, L., 1995, "Cross-Hauling Direct Foreign Investment and Unionized Oligopoly," *European Economic Review,* 39, 379-390.

It is available from author upon request

Appendix 1: Case of Cournot Competition

For the reviewers and editor, this appendix will not be included in the main paper. However, this is only available for the reviewers and editor: the case of Cournot competition. This appendix is more detailed-version of Choi (2009) with differentiated demand function. Note that our result basically remains the same with no differentiated demand function as in Choi (2009). In this case where we have been abbreviated, we present on separate page.

6 The Basic Model under Cournot Competition

The basic structure is a differentiated duopoly model under Cournot competition. Consider two single-product firms that produce differentiated products that are supplied by a public firm (firm 0) and a private firm (firm 1). Assume that the inverse demand is characterized by

$$p_i = 1 - cx_j - x_i,$$

where p_i is the market price, x_i is the level of output of each firm.

On the demand side of the market, the representative consumer's utility is a quadratic function given by

$$U = x_0 + x_1 - \frac{1}{2}\left(x_0^2 + x_1^2 + 2cx_1x_0\right).$$

Therefore, the profit of private firm 1 and the profit of the public firm (firm 0) are given by

$$\pi_1 = (p_1 - w_1)x_1 \quad \text{and} \quad \pi_0 = (p_0 - aw_0)x_0,$$

respectively. The public firm maximizes a utilitarian measure of welfare that takes into account the inefficiency of the internal public firm, viz., $a > 1$. We assume that $a > 1$. This parameter is a measure of inefficiency due to X-inefficiency.

As is customarily assumed in a mixed oligopoly, the public firm chooses x_0 to maximize the social welfare,

$$W = U - \sum_{i=0}^{1} p_i x_i + \sum_{i=0}^{1}(\pi_i + u_i),\ i = 0, 1,$$

which consists of the consumer surplus, $U - \sum_{i=0}^{1} p_i x_i$, the various unions' utilities and the producer surpluses of the public and private firms.

We assume that the public and private firms are unionized, and that wages are determined as a consequence of bargaining between firms and unions. Let $\overline{w}$ denote the reservation wage. Thus, we assume that the union sets the wage while public and private firms unilaterally decide the level of employment. Taking $\overline{w}$ as given, the union's optimal wage-setting strategy w_i regarding firm i is defined as

$$\max_{w_i} \; u_i = (w_i - \overline{w})^{\theta} x_i, \; i = 0, 1, \tag{18}$$

where θ is the weight that the union attaches to the wage level. As Haucap and Wey (2004), Leahy and Montagna (2000) and Lommerud *et al.* (2003) suggested, we assume that $\theta = 1$ and $\overline{w} = 0$ to show our results in a simple way (see also Booth, 1995).

Finally, a three-stage game is conducted. The timing of the game is as follows. At the first stage, the public firm decides whether or not to impose a budget constraint on its profit. At the second stage, each union chooses its wage w_i after being made aware of the budget-constraint strategy of the public firm. At the third stage, each firm chooses its quantity x_i simultaneously to maximize its respective objective, knowing the budget-constraint motive of the public firm and the wage levels.

7 Results under Cournot Competition

7.1 Market Equilibrium

We first present the market equilibrium without a budget constraint, when the public firm is assumed to be less efficient than the private firm.

[**No Budget Constraint**]: Given w_i for each firm, the public firm's maximization problem is as follows:

$$\max_{x_0} \quad W = U + (1 - a) w_0 x_0.$$

Taking w_i as given, the best reply function of the public firm is given by

$$\frac{\partial W}{\partial x_0} = 0 \Leftrightarrow x_0 = 1 - c x_1 - (a - 1) w_0. \tag{19}$$

On the other hand, the best reply function of the private firm is given by

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{1}{2}(1 - cx_0 - w_1). \tag{20}$$

Given these results, we now obtain the output level for each firm. Solving the first-order conditions (19) and (20), we obtain,

$$x_0 = \frac{2 - c + cw_1 - 2(a-1)w_0}{2 - c^2}, \tag{21}$$

$$x_1 = \frac{1 - c - w_1 - c(1-a)w_0}{2 - c^2}. \tag{22}$$

In the second stage of this case, each wage is set to maximize the utility of the corresponding firm's union. To do this, the two independent maximization problems should be considered simultaneously. Using (21) and (22), the problem for union i is defined as

$$\max_{w_0}\ u_0 = w_0 x_0 = \frac{w_0[2 - c + cw_1 - 2(a-1)w_0]}{2 - c^2}, \tag{23}$$

$$\max_{w_1}\ u_1 = w_1 x_1 = \frac{w_1[1 - c - w_1 - c(1-a)w_0]}{2 - c^2}. \tag{24}$$

Each first-order condition then leads to

$$w_0 = \frac{2 - c + cw_1}{4(a-1)}, \quad w_1 = \frac{1 - c - c(1-a)w_0}{2}.$$

Therefore, straightforward computation yields the following Lemma A-1.

Lemma A-1: *Suppose that under Cournot competition, the budget constraint is not imposed on the public firm and that each union is allowed to bargain collectively. Then, the equilibrium values, denoted as w_i^N, x_i^N and u_i^N, are given by*

$$w_0^N = \frac{4 - c - c^2}{(a-1)(8 - c^2)}, \quad w_1^N = \frac{4 - 2c - c^2}{8 - c^2};$$

$$x_0^N = \frac{2(4 - c - c^2)}{(2 - c^2)(8 - c^2)}, \quad x_1^N = \frac{4 - 2c - c^2}{(2 - c^2)(8 - c^2)};$$

$$u_0^N = \frac{2(4 - c - c^2)^2}{(a-1)(2 - c^2)(8 - c^2)^2}, \quad u_1^N = \frac{(4 - 2c - c^2)^2}{(2 - c^2)(8 - c^2)^2}.$$

Lemma A-1 suggests that the public firm pays lower wages than the private firms if a is sufficiently large. On the other hand, if the public firm becomes more efficient (i.e., a approaches unity), the public firm pays higher wages than the private firms[13]. Furthermore, given that the equilibrium wage in the public firm is determined by public firm's output in the third stage, the inefficiency of the public firm is canceled out. This is why each output level is independent of a.

Noting that $W^N = U^N + (1-a)w_0^N x_0^N$ without the budget constraint, we can compute the social welfare W^N,

$$W^N = \frac{176 - 80c - 136c^2 + 44c^3 + 31c^4 - 4c^5 - 5c^6}{2(2-c^2)^2(8-c^2)^2} > 0, \tag{25}$$

even though the public firm's profit is a negative value;

$$\pi_0^N = -\frac{2(4-c-c^2)(8-2c-6c^2+c^3+c^4)}{(a-1)(2-c^2)^2(8-c^2)^2} < 0. \tag{26}$$

Therefore, even though the lack of imposition of a budget constraint on the public firm is not equivalent to letting the public firm pursue more profit, the social welfare is positive because the public firm's goal is to maximize social welfare.

[**Imposition of Budget Constraint**]: Similar to "No Budget Constraint," at the third stage the public firm's objective is to maximize welfare which is defined as

$$\max_{x_0} \quad W = U + (1-a)w_0 x_0,$$
$$\text{s.t.} \quad (p_0 - aw_0)x_0 \geq 0.$$

The constraint implies that there is some lowerbound restriction on the public firm's profit, i.e., the public firm faces a budget constraint.

Denoting the multiplier of the budget constraint λ, the Lagrangian equation can be written as

$$L = x_1 + x_0 - \frac{(x_1^2 + x_0^2 + 2cx_0x_1)}{2} + (1-a)w_0x_0 + \lambda(x_0 - x_0^2 - cx_0x_1 - ax_0w_0).$$

Taking w_i as given and assuming symmetry of private firms, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = 1 + (1-a)w_0 - cx_1 - x_0 + \lambda(1 - 2x_0 - cx_1 - aw_0) = 0, \tag{27}$$

$$\frac{\partial L}{\partial \lambda} = 1 - cx_1 - x_0 - aw_0 = 0. \tag{28}$$

[13]Since we assume that $a > 1$, we can exclude the case where there is a discontinuity between the results at point $a = 1$ and also avoid the case that the public firm becomes a monopoly (which is the situation when $a = 1$).

On the other hand, symmetry across private firms implies that the optimal output for the private firm is equal to Eq. (20): $x_1 = \frac{1}{2}(1 - cx_0 - w_1)$. Given these results, we now obtain the output level for each firm. Solving the first-order conditions (20) and (28), we obtain,

$$x_0 = \frac{2 - c + cw_1 - 2aw_0}{2 - c^2}, \tag{29}$$

$$x_1 = \frac{1 - c - w_1 + acw_0}{2 - c^2}. \tag{30}$$

To solve for Lagrangian equation, suppose that the budget constraint is momentarily binding. We check ex-post that this constraint is binding.

In the second stage of this case, each wage is set to maximize the its own firm's union: $u_i = x_i w_i$. Using (29) and (30), the problem for union i is defined as

$$\max_{w_0}\ u_0 = w_0 x_0 = \frac{w_0(2 - c + cw_1 - 2aw_0)}{2 - c^2},$$
$$\max_{w_1}\ u_1 = w_1 x_1 = \frac{w_1(1 - c - w_1 + acw_0)}{2 - c^2}.$$

Straightforward computation yields each firm's reaction function as follows:

$$w_0 = \frac{2 - c + cw_1}{4a}, \quad w_1 = \frac{1 - c + acw_0}{2}.$$

Therefore, straightforward computation yields the following Lemma A-2.

Lemma A-2: *Suppose that under Cournot competition, the budget constraint is imposed on the public firm and that each union is allowed to bargain collectively. Then, the equilibrium values, denoted as w_i^B, x_i^B and u_i^B, are given by*

$$w_0^B = \frac{4 - c - c^2}{a(8 - c^2)}, \quad w_1^B = \frac{4 - 2c - c^2}{8 - c^2};$$
$$x_0^B = \frac{2(4 - c - c^2)}{(2 - c^2)(8 - c^2)}, \quad x_1^B = \frac{4 - 2c - c^2}{(2 - c^2)(8 - c^2)};$$
$$u_0^B = \frac{2(4 - c - c^2)^2}{a(2 - c^2)(8 - c^2)^2}, \quad u_1^B = \frac{(4 - 2c - c^2)^2}{(2 - c^2)(8 - c^2)^2}.$$

The wages of public firm in Lemma A-2 are smaller when the budget constraint is imposed on the public firm than those when the budget constraint is not imposed on the public firm.

Finally, noting that $W^B = U^B + (1-a)w_0^B x_0^B$ with imposition of a budget constraint, we can compute the social welfare as follows;

$$\begin{aligned} W^B =& \frac{a(304 - 144c - 256c^2 + 92c^3 + 67c^4 - 12c^5 - 6c^6)}{2a(2-c^2)^2(8-c^2)^2} \\ &+ \frac{(1-a)(126 - 64c - 120c^2 + 48c^3 + 36c^4 - 8c^5 - c^6)}{2a(2-c^2)^2(8-c^2)^2}. \end{aligned} \tag{31}$$

Substituting Lemma A-2 into (27) then we have

$$\lambda = \frac{-2(4-c-3c^2) - c^3 - c^4}{-2a(4-c-3c^2)} > 0, \tag{32}$$

which the budget constraint is binding.

7.2 Comparative Statics under Cournot Competition

By applying equilibrium values to each of the cases outlined in the previous subsection, we examine the budget constraint strategy by comparing each level of social welfare in each case. Hence, we can state the following results:

Proposition A-1: *Suppose that under Cournot competition, each firm's union is allowed to bargain collectively. Then, each level of social welfare is determined by* $W^B > W^N$.

Proof: Comparing W^B with W^N yields

$$W^B > W^N \Leftrightarrow a > \hat{a} \equiv \frac{50 - 16c - 16c^2 - 4c^3 - 5c^4 + 4c^5 + 4c^6}{176 - 80c - 136c^2 + 44c^3 + 31c^4 - 4c^5 - 5c^6} \in (0,1). \qquad \blacksquare$$

Thus, we obtain the Proposition A-2 as follows:

Proposition A-2: *Suppose that under Cournot competition, each firm's union is allowed to bargain collectively and that the public firm is less efficient than the private firms. Then, the government should impose the budget constraint on the public firm in the first stage.*