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A Method to Evaluate Composite Performance Indices Based on the Variance-Covariance Matrix

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Abstract

In this paper we compute performance indices like those from Mereuţă et all. (2007) using the eigenvalues and the eigenvectors of the variance-covariance matrix of these indices. The eigenvalues are used in this paper to give natural weights to the performance indices in order to compute the weighted competitiveness indicators, and their corresponding eigenvectors are used to obtain the desired uncorrelated performance indices. In order to point out the mutual influence in the case of each pair of the considered correlated performance indices we compute also their correlation matrix.

After we order the composite performance indices (non-weighted or weighted) we classify them using either the maximum entropy principle, either the maximum separation (Chow breakpoint test). A comparison between the classifications using the weighted/non-weighted classifications using the maximum entropy principle and the maximum separation are also done in the paper.

As application we consider the GDP per capita, investment share in GDP, the unemployment rate, the Gini index of income inequality and the share of consumption of renewal energy resources (five performance indices) for the 27 countries of European Union. These performance indices are according to Indicators of Sustainable Development (www.un.org/esa/sustdev/publications/indisd-mg2001.pdf) approved by the Commission on Sustainable Development at its Third Session in 1995.

JEL Classification: O47, O57, C43.

Keywords: Sustainable development competitiveness indices, composite indices, weighted and non-weighted indices, Shannon entropy, Chow breakpoint test.

1 Introduction

In Mereuţă et all., 2007 there is evaluated the regional competiveness of the EU regions using five criteria: annual GDP growth rate (%), denoted by IC_1, annual average unemployment rate (%), denoted by IC_2, evolutions of the households disposable income, denoted by IC_3, the share of industry and services gross value-added in GDP (%), denoted by IC_4, and the share in total employment of the persons employed in competitiveness-enhancing sectors in industry and services, denoted by IC_5.

For a country having the value V_i for the criterion IC_i $\left(i=\overline{1,5}\right)$ the value IC_i is computed using the formula

$$IC_i = \frac{V_i - \min\limits_i V_i}{\max\limits_i V_i - \min\limits_i V_i}. \tag{1}$$

If the criterion IC_i is a non-performance index (the case of unemployment rate), the formula (1) becomes

$$IC_i = 1 - \frac{V_i - \min\limits_i V_i}{\max\limits_i V_i - \min\limits_i V_i} = \frac{\max\limits_i V_i - V_i}{\max\limits_i V_i - \min\limits_i V_i}. \tag{1'}$$

After computing the five competitives indices, in Mereuţă et all., 2007 there are computed composite indices using the formulae

$$ICFin = \frac{IC_1 + IC_2 + IC_3 + IC_4 + IC_5}{5}, \text{ and} \tag{2}$$

$$ICPond = \frac{IC_1 + IC_2 + IC_3}{3} \cdot 0.4 + (IC_4 + IC_5) \cdot 0.3. \tag{3}$$

Such composite indices are computed also in Albu, 2008, but none of these composite indices take into account the possible correlations between the performance indices.

In (Saporta, 1990) it is presented the PCR (Principal Components Regression), which differs from linear regression (Saporta, 1990, Jula, 2003) by the fact that the residues from the linear regression become Euclidean distances.

Definition 1 *The orthogonal linear variety of the dimension k for n points of $\mathbb{R}^p$ $X^{(1)}, ..., X^{(n)}$ with $X^{(i)} = \left(X_1^{(i)}, ..., X_p^{(i)}\right)$ is the linear variety of the dimension k for which the sum of squares of the Euclidean distances from these points to the linear variety is minimum.*

It is proved (Saporta, 1990) that the orthogonal linear variety of the dimension k is generated by the gravity center of the points and the first k eigenvectors of the sample variance-covariance matrix corresponding to the first maximum k eigenvalues. The above eigenvectors are also called principal components, and this is the reason for which the orthogonal regression is called principal components regression.

Remark 1 *The orthogonal linear variety of the dimension* 1 *is the orthogonal regression line. The orthogonal linear variety of the dimension* $p-1$ *is the orthogonal regression hyper-plane.*

Remark 2 *Because each linear variety of the dimension* k *is the intersection of* $p-k$ *hyper-planes, the orthogonal linear linear variety of the dimension* k *is* $A_{i0}+\sum\limits_{j=1}^{p}A_{ij}\cdot X_j^{(i)}=0$ *for any* i *with* $1\leq i\leq p-k$. *This is analogous to the simultaneous equation models (Jula, 2003).*

The principal components regression and an algorithm analogous to the $k-$means algorithm were used in (Ciuiu, 2007) to classify some banks. When we change the canonical basis of $\mathbb{R}^p$ with the eigenvectors of the variance-covariance matrix the new coordinates become uncorrelated. These new coordinates were used in (Ciuiu, 2008) together with a generalization of the Perceptron algorithm to classify the same banks, and for a consumer behaviour model (Jula, 2003).

For a discrete random variable X with $p_n=P\left(X=n\right)$ the Shannon entropy is defined by the formula (Onicescu and Ştefănescu, 1979, Petrică and Ştefănescu, 1982)

$$H=-\sum_{n=0}^{\infty}p_n\cdot\ln p_n. \tag{4}$$

Remark 3 *In fact in the original Shannon definition it is used* $\log_2$ *in the place of* $\ln$, *but this does not modify the properties of monotony and convexity, and we will use* $\ln$ *for the comodity of the computations, as other authors (Preda, 1992).*

In Chow, 1960 there is presented a test that verifies if the regressions (in vectorial writing)

$$Y_1=X_1\beta_1+\varepsilon_1 \text{ and} \tag{5}$$

$$Y_2=X_2\beta_2+\varepsilon_2 \tag{5'}$$

have the same coefficients, where X_1 is a $n\times p$ matrix, X_2 is a $m\times p$ matrix, Y_1 and ε_1 are vectors of the dimension n, Y_2 and ε_2 vectors of the dimension m, and β_1 and β_2 are vectors of the dimension p.

Considering the first regression given by the first n observations and the second one given by th last m observations there are estimated the coefficients β_1 by

$$b_1 = \left(X_1^T X_1\right)^{-1} X_1^T Y_1 \text{ and} \tag{6}$$

$$d = Y_2 - X_2 \cdot b_1. \tag{7}$$

In fact d is the difference between the values of Y for the next m observations and their estimations using the first n ones. Tacking into account that

$$d = X_2\beta_2 - X_2\beta_1 + \varepsilon_2 - X_2 \left(X_1^T X_1\right)^{-1} X_1^T \varepsilon_1 \tag{7'}$$

we obtain the expectation and the variance-covariance matrix of d (Chow, 1960)

$$E\left(d\right) = X_2\beta_2 - X_2\beta_1, \text{ and} \tag{8}$$

$$\begin{gathered} Cov\left(d\right) = Cov\left(\varepsilon_2\right) + Cov\left(X_2 \left(X_1^T X_1\right)^{-1} X_1^T \varepsilon_1\right) = \\ \sigma^2 \cdot I + X_2 \left(X_1^T X_1\right)^{-1} X_1^T \cdot Cov\left(\varepsilon_1\right) \cdot X_1 \left(X_1^T X_1\right)^{-1} X_2^T = . \\ \left(I + X_2 \left(X_1^T X_1\right)^{-1} X_2^T\right) \sigma^2 \end{gathered} \tag{8'}$$

The special case $m = 1$ is considered, d is a real number and we obtain (Chow, 1960)

$$Var\left(d\right) = \left(1 + X_2 \left(X_1^T X_1\right)^{-1} X_2^T\right) \sigma^2. \tag{9}$$

We estimate σ^2 by the sum of the squares of the first n residues divided by the number of degrees of freedom $n - p$ (p is the number of estimated parameters), and we denote this unbiased estimator by s_1^2. It is proved (Chow, 1960) that in the null hypothesis $\beta_2 = \beta_1 = \beta$ we have $E\left(d\right) = 0$ and the statistics

$$Ch = \frac{d^2}{\left(1 + X_2 \left(X_1^T X_1\right)^{-1} X_2^T\right) s_1^2} \tag{10}$$

has the distribution Snedecor—Fisher with 1 and $n - p$ degrees of freedom, $F_{1,n-p}$.

Similarly, for $m > 1$ new observations we compute first

$$\overline{d} = \frac{\sum\limits_{i=1}^{m} d_i}{m}, \tag{11}$$

and the statistics

$$Chmed = \frac{m^2 \cdot \overline{d}^2}{e^T \left(1 + X_2 \left(X_1^T X_1\right)^{-1} X_2^T\right) e \cdot s_1^2}, \tag{10'}$$

where the vector e has all the m components equal to 1 has also the distribution $F_{1,n-p}$ (Chow, 1960).

The above statistics $Chmed$ is used if we change the null hypothesis $\beta_2 = \beta_1 = \beta$ by $E\left(\overline{d}\right) = 0$. Instead of this we can consider the quadratic form $d^T \left(Cov\left(d\right)\right)^{-1} d$, and finally it results that (Chow, 1960)

$$Ch = \frac{d^T \left(1 + X_2 \left(X_1^T X_1\right)^{-1} X_2^T\right)^{-1} d}{m \cdot s_1^2} \tag{12}$$

has the distribution Snedecor—Fisher with m and $n-p$ degrees of freedom, $F_{m,n-p}$.

For all the above $F-$statistics we accept the null hypothesis if the considered statistics is less than the centil of the level $1-\varepsilon$ of the involved Snedecor—Fisher distribution.

2 The algorithm

The n points in $\mathbb{R}^p$ are n countries for which we consider p performance indices: in Mereuţă et all, 2007 we have $p = 5$ and $X_i = V_i$. These performance indices used in (1) and $(1')$ are correlated.

Suppose that all the values IC_i are performance indices (if it is non-performance, like IC_2-unemployment rate in Mereuţă et all., 2007 we replace first V_i by $-V_i$. After these eventual replacements we compute the variance-covariance matrix and its eigenvalues $\lambda_1, ..., \lambda_p$ and its eigenvectors $\mathcal{E}_1, ..., \mathcal{E}_p$.

Next we compute the new coordinates V_i' and these coordinates are used in (1) in the place of V_i to compute IC_i. The non-weighted compsite indices $ICFin$ are now computed with the same formula (2), but the (possible arbitrary in (3)) weights in weighted composite ones $ICPond$ are set to be

$$w_i = \frac{\sqrt{\lambda_i}}{\sum\limits_{j=1}^{p} \sqrt{\lambda_j}}. \tag{13}$$

Therefore the new formula of the composite indice $ICPond$ is

$$ICPond = \sum_{j=1}^{p} w_j \cdot IC_j, \tag{14}$$

and the performance indices weights are proportional to the square roots of the corresponding eigenvalues. Of course, the sum of the weights is also 1 as in Mereuţă et all., 2007.

After we compute the eigenvectors of the variance-covariance matrix we have to check if they are poitive oriented. If the sum of the components of the eigenvector corresponding to the maximum eigenvalue is negative, we change the sign of the principal component. If a determinant computed on the main diagonal is negative we change the sign of the corresponding row in the obtained matrix, and of course we change also the sign of the corresponding eigenvector.

After computing the composite competitiveness indices Mereuţă et all., 2007 consider two classifications. First classification consist in five classes as in the following table.

Table 1. Classification in five classes.

A^+	$m + S < IC_i$	Very high relative competitiveness
A	$m + \frac{S}{3} < IC_i \leq m + S$	High relative competitiveness
B	$m - \frac{S}{3} < IC_i \leq m + \frac{S}{3}$	Medium relative competitiveness
C	$m - S < IC_i \leq m - \frac{S}{3}$	Low relative competitiveness
C^-	$IC_i \leq m - S$	Very low relative competitiveness

Second classification consist in nine classes as in the following table.

Table 2. Classification in nine classes.

A^{++}	$m + 1.75 \cdot S < IC_i$	Very high relative competitiveness
A^+	$m + S < IC_i \leq m + 1.75 \cdot S$	High relative competitiveness
A	$m + 0.5 \cdot S < IC_i \leq m + S$	Heigh-medium relative competitiveness
B^+	$m < IC_i \leq m + 0.5 \cdot S$	Medium-heigh relative competitiveness
B	$m - 0.5 \cdot S < IC_i \leq m$	Medium relative competitiveness
B^-	$m - S < IC_i \leq m - 0.5 \cdot S$	Medium-low relative competitiveness
C	$m - 1.5 \cdot S < IC_i \leq m - S$	Low-medium relative competitiveness
C^-	$m - 1.75 \cdot S < IC_i \leq m - 1.5 \cdot S$	Low relative competitiveness
C^{--}	$IC_i \leq m - 1.75 \cdot S$	Very low relative competitiveness

In the above tables IC_i is the composite competitive indice for the country i, m is the average of the considered composite indice and S^2 is its variance (hence S is its standard deviation). This is the reason of using of the formulae (13) and (14) in this paper: the eigenvalues λ_i are the variances of the new coordinates.

For the above classifications Mereuţă et all., 2007 use the core method for obtaining the above tables. In this paper we will consider each classification in k classes ($k = 5$ or $k = 9$ as in the above tables) using the backtracking method. From all these classifications ($C_{26}^8 = 1562275$ for 9 classes and 27 countries) we choose that classification that optimizes the considered criterion.

First criterion is the maximum Shannon entropy. Tacking into account that the classes have countries with composite indices in inreasing order we compute first the separators of two successive classes as follows. Denote by $G_{i,i+1}$ the gravity center for the composite indices of the classes i and $i+1$, and by $\min_i$, $\max_i$, $\min_{i+1}$ and $\max_{i+1}$ the minimum and the maximum values of the considered composite indices for the class i, respectively for the class $i+1$. The separator of the classes i and $i+1$ is

$$sep_{i,i+1} = \begin{cases} G_{i,i+1} \ if \ \max_i \le G_{i,i+1} \le \min_{i+1} \\ \max_i \ if \ G_{i,i+1} < \max_i \\ \min_{i+1} \ if \ G_{i,i+1} > \min_{i+1} \\ \min_1 \ if \ i = 0 \\ \max_k \ if \ i = k \end{cases} . \tag{15}$$

Next we compute the probabilities of being in the class i

$$p_i = \frac{sep_{i,i+1} - sep_{i-1,i}}{sep_{k,k+1} - sep_{0,1}}, \tag{16}$$

and from here we compute the entropy of the classification using (4).

Another criterion to select the optimal classification is the maximum Chow breakpoint statistics. Denoting by S^2 the variance of all the classes for the considered composite indices and by S_i^2 the variance of the class i we compute the Chow statistics

$$Ch = \frac{(n-1) \cdot S^2 - \sum_{i=1}^{k} (n_i - 1) \cdot S_i^2}{\sum_{i=1}^{k} (n_i - 1) \cdot S_i^2} \cdot \frac{n-k}{k-1}, \tag{17}$$

which has a Snedecor—Fisher with $k-1$ and $n-k$ degrees of freedom. In the above formula we consider n countries classified in k classes such that the class i contains n_i countries.

3 Application

Consider the GDP per capita (IC_1), investment share in GDP (IC_2), unemployment rate (IC_3), Gini index of income inequality (IC_4) and share of consumption of renewal energy resources (IC_5) for the 27 countries of European Union.

The data source is from "European Comission. Eurostat. Your Key to European Statistics" (for GDP per capita, investment share in GDP, unemployment rate and share of consumption of renewal energy resources), and the Gini index of income inequality is from "2007/2008 Human Development Report. CIA World Factbook". The data are in the following table.

Table 3. The correlated data.

Country	AT	BE	BG	CY	CZ	DK	EE	FI	FR	DE
V_1	123.1	114.6	40.1	94.6	80.4	118.3	67.2	115	107.3	115.8
V_2	21.8	22.7	33.4	23.3	24	21	28.4	20.6	21.9	19.2
V_3	4.2	7.2	5.4	4.1	4.7	4.1	8.4	6.8	8.4	7.2
V_4	29.1	33	29.2	33.45	25.4	24.7	35.8	26.9	32.7	28.3
V_5	23.8	3.1	4.7	2.4	4.7	17.3	10	22.6	7	8.3

Country	GR	HU	IE	IT	LV	LT	LU	MT	NL	PL
V_1	95.3	62.9	139.5	100.5	55.7	61.3	252.8	76.4	134.6	57.5
V_2	19.3	20.1	21.1	20.9	30.2	24.8	20.1	15.8	20.5	22
V_3	7.9	8.4	8.7	7	11.3	9	5.5	6.1	2.8	7.1
V_4	34.3	26.9	34.3	36	37.7	36	34.6	26.15	30.9	34.5
V_5	5	5.3	2.9	6.9	29.7	8.9	2.5	7.8	3.6	5.1

Country	PT	RO	SK	SI	ES	SE	UK
V_1	75.3	43.56	71.9	89.8	103.9	121.4	117.5
V_2	21.7	33.3	25.9	28	29.4	19.5	16.9
V_3	8.2	5.9	9.3	4.2	14.7	7	6.5
V_4	38.5	31	25.8	28.4	34.7	25	36
V_5	17.6	11.9	5.5	10	7	30.9	2.1

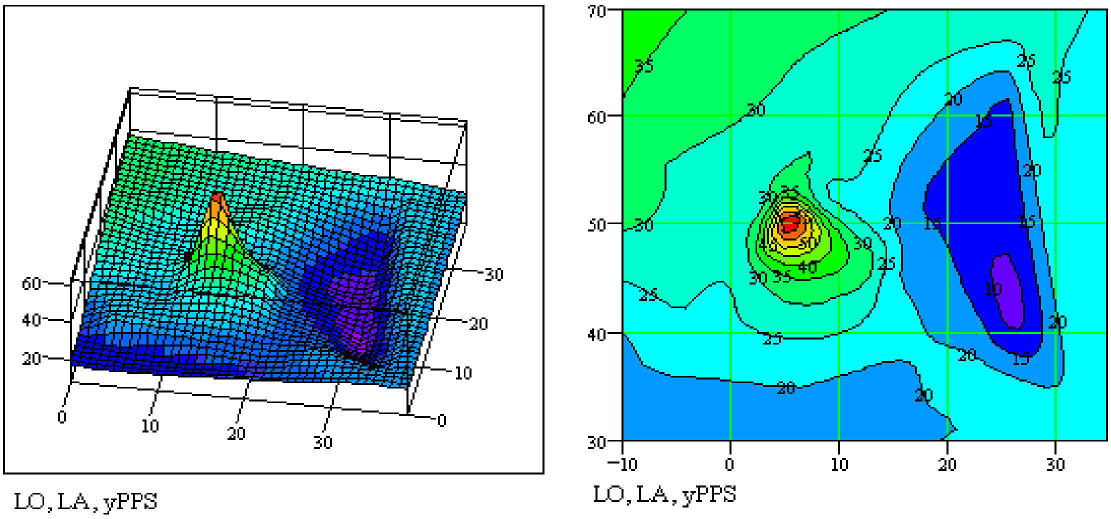
In the above table the GDP per capita and the share of investment in GDP are considred for the year 2008, the unemployment rate is considered for December 2008, the Gini index of incomes inequality is considered for 2007/2008 and the share of consumption of renewable energy resources are given for the year 2007.

For GDP per capita in the case of Romania the value of 45.8 is forecast for the year 2008, as the value 42.1 for 2007. But in Eurostat the value for Euro Area in 2008 is 108.9 and we know from "Banca Naţională a României. Raport Anual 2008" that the Romanian GDP per capita is 40% from Euro Area, hence the value is 43.56.

The values of GDP per capita are also forecasted for the year 2008 in the cases of Austria, Greece and United Kingdom, and estimated for Slovakia. The Gini indexes lack for Cyprus, Luxembourg and Malta, but their values were computed as the neighbours' average (the data in the source table are in order). Malta is also a lack in the table source of the share of consumption of renewable energy resources, but the value is replaced by the EU-27 average (7.8).

In Figure 1, the spatial distribution of GDP per capita in 2006 is shown as a stylised map of the EU-27, where LO is longitude (on the left side of the map in relation to the origin, 0 meridian, we changed West longitude, as it is marked usually on geographical maps, in negative values), LA is latitude, and yPPS is the level of GDP per capita in thousand

Euro PPS (Purchasing Power Standards). On the stylised map of the EU-27, we can see two distinct groups of regions delimited by 30 to 55 red contour lines and by 20 to 10 blue contour lines representing the highest and lowest GDP per capita levels respectively. In general, GDP per capita is increasing from the right side of EU-27 stylised map (eastern EU) to the left side (western EU) and from the bottom (southern EU) to the top (northern EU).



Source: own elaboration on EUROSTAT data.

Figure 1. 3-D map and contour plot of GDP per capital for EU-27 in 2006.

The variance-covariance matrix is

$$\begin{pmatrix} 1678.97444 & -91.78076 & 21.77701 & -3.94841 & -35.05565 \\ -91.78076 & 20.32617 & -2.67091 & -2.60698 & 3.38128 \\ 21.77701 & -2.67091 & 5.95130 & 4.19329 & -1.89291 \\ -3.94841 & -2.60698 & 4.19329 & 17.75154 & 5.70893 \\ -35.05565 & 3.38128 & -1.89291 & 5.70893 & 66.21525 \end{pmatrix}$$ with the eigenvalues 1685.09796, 66.14393, 20.5824, 13.24441 and 4.15001, and the eigenvectors on rows

$$\begin{pmatrix} 0.99816 & -0.05509 & 0.01305 & -0.00232 & -0.02175 \\ 0.02328 & 0.02116 & -0.01589 & 0.11273 & 0.993 \\ 0.03231 & 0.52199 & -0.28595 & -0.7995 & 0.0743 \\ 0.04475 & 0.84991 & 0.13084 & 0.50302 & -0.07417 \\ -0.00977 & 0.04122 & 0.94905 & -0.30832 & 0.04954 \end{pmatrix}.$$

If we want to check the correlations between the five criteria we have to compute the correlation matrix, which is

$$\begin{pmatrix} 1 & -0.49682 & 0.21786 & -0.02287 & -0.07515 \\ -0.49682 & 1 & -0.24284 & -0.13724 & 0.09217 \\ 0.21786 & -0.24284 & 1 & 0.40797 & -0.09536 \\ -0.02287 & -0.13724 & 0.40797 & 1 & 0.16652 \\ -0.07515 & 0.09217 & -0.09536 & 0.16652 & 1 \end{pmatrix}$$

The new performance values (uncorrelated) are in the following table.

Table 4. The uncorrelated data.

Country	AT	BE	BG	CY	CZ	DK	EE
V_1'	121.16702	113.05328	38.08104	93.11383	78.82491	116.55243	65.26739
V_2'	23.74665	2.62075	3.10154	1.37279	4.25796	17.6579	8.19313
V_3'	41.59159	44.22454	43.96878	43.31284	37.12594	36.98955	48.76278
V_4'	7.08391	6.64955	14.438	6.49566	10.25537	8.8977	7.29559
V_5'	5.86073	3.3107	5.09561	6.57697	3.80713	4.29096	4.07505

Country	FI	FR	DE	GR	HU	IE	IT
V_1'	113.13513	105.70962	114.31987	93.92872	61.51417	137.98331	99.00535
V_2'	22.63042	6.35948	8.26804	3.8508	4.25356	2.84521	5.68647
V_3'	39.59889	43.96415	39.06509	43.20686	36.82658	45.6473	45.45303
V_4'	6.55702	5.34757	5.70708	2.00974	4.87445	5.56863	2.72398
V_5'	2.68515	2.31091	1.96323	3.18976	0.79811	1.96878	4.6773

Country	LV	LT	LU	MT	NL	PL	PT
V_1'	53.22745	59.59316	251.18067	75.20007	133.17929	56.0584	73.56512
V_2'	27.35753	6.87428	4.9796	7.00731	3.70307	3.09207	15.47914
V_3'	53.14286	46.94272	48.0812	33.94667	40.82241	43.33346	48.19332
V_4'	5.51439	3.87438	10.08637	2.31674	7.26972	2.60958	0.06796
V_5'	3.07111	3.42206	3.9301	2.56439	6.57771	4.49625	5.11853

Country	RO	SK	SI	ES	SE	UK
V_1'	41.38119	70.1594	87.88541	101.82512	119.39636	116.30531
V_2'	10.1346	4.92272	9.4782	6.31372	31.21539	1.2232
V_3'	46.14543	39.53769	42.16695	51.16969	38.38636	43.41472
V_4'	13.00293	10.62741	12.23888	9.73939	6.22245	0.50655
V_5'	5.49482	-0.23417	5.54218	-2.70912	2.21285	4.58306

The performance indices are in the following table.

Table 5. The performance indices.

Country	AT	BE	BG	CY	CZ	DK	EE
IC_1	0.38989	0.35182	0	0.25825	0.1912	0.36824	0.12758
IC_2	0.75098	0.0466	0.06263	0.00499	0.10119	0.54797	0.23239
IC_3	0.39825	0.53541	0.52209	0.48792	0.16562	0.15852	0.77183
IC_4	0.48823	0.45801	1	0.4473	0.70893	0.61445	0.50297
IC_5	0.9228	0.64821	0.84041	0.99992	0.70167	0.75376	0.73051

Country	FI	FR	DE	GR	HU	IE	IT
IC_1	0.3522	0.31736	0.35776	0.26207	0.10996	0.46881	0.2859
IC_2	0.71376	0.17125	0.23489	0.08761	0.10104	0.05408	0.14881
IC_3	0.29445	0.52185	0.26664	0.4824	0.15003	0.60953	0.59941
IC_4	0.45157	0.3674	0.39242	0.13513	0.33448	0.38279	0.18483
IC_5	0.58085	0.54055	0.50312	0.63519	0.37766	0.50371	0.79537
Country	LV	LT	LU	MT	NL	PL	PT
IC_1	0.07108	0.10095	1	0.17419	0.44626	0.08436	0.16651
IC_2	0.87137	0.18842	0.12525	0.19285	0.08268	0.06231	0.47532
IC_3	1	0.67701	0.73632	0	0.35818	0.48899	0.74216
IC_4	0.37901	0.26489	0.69717	0.15649	0.50116	0.17687	0
IC_5	0.62241	0.6602	0.71491	0.56785	1	0.77587	0.84288
Country	RO	SK	SI	ES	SE	UK	
IC_1	0.01549	0.15053	0.23371	0.29913	0.38158	0.36708	
IC_2	0.29712	0.12335	0.27524	0.16973	1	0	
IC_3	0.63548	0.29126	0.42822	0.89721	0.23128	0.49323	
IC_4	0.90013	0.73482	0.84697	0.67303	0.42829	0.03052	
IC_5	0.8834	0.2665	0.88849	0	0.52999	0.78522	

For classification of the 27 countries we refer to "Class I" if we use the maximum entropy criterion, and to "Class II" for the Chow criterion. If we use the non-weighted indices we obtain the following table.

Table 6. Classification using non-weighted composite indices.

Country	AT	BE	BG	CY	CZ	DK	EE
Value	0.59003	0.40801	0.48503	0.43967	0.37372	0.48859	0.47305
Rank	2	14	8	13	20	7	11
Class I	A^+	B^-	B	B^-	C	B^+	B
Class II	A^+	C^-	B	C	C^{--}	B^+	B^-
Country	FI	FR	DE	GR	HU	IE	IT
Value	0.47857	0.38368	0.35097	0.32048	0.21463	0.40378	0.40286
Rank	9	18	21	23	27	16	17
Class I	B	B^-	C	C	C^{--}	B^-	B^-
Class II	B	C^{--}	C^{--}	C^{--}	C^{--}	C^{--}	C^{--}
Country	LV	LT	LU	MT	NL	PL	PT
Value	0.58877	0.37829	0.65473	0.21828	0.47766	0.31768	0.44537
Rank	3	19	1	26	10	24	12
Class I	A^+	C	A^{++}	C^{--}	B	C^-	B
Class II	A^+	C^{--}	A^{++}	C^{--}	B^-	C^{--}	C

Country	RO	SK	SI	ES	SE	UK
Value	0.54632	0.31329	0.53453	0.40782	0.51423	0.51423
Rank	4	25	5	15	6	22
Class I	A	C^{-}	A	B^{-}	B^{+}	C
Class II	A	C^{--}	A	C^{-}	B^{+}	C^{--}

In the above table we have the same first four classes even we use the maximum entropy or the maximum separation: Luxembourg is alone in the first class, the second contains Austria and Latvia, the third contains Romania and Slovenia and the fourth contains Sweden and Denmark. The other countries from the rank 8 (Bulgaria) to the rank 27 (Hungary) are have the division in classes $5+6+5+2+2$ in the case of maximum entropy, respectively 2+2+2+2+12 in the case of maximum separation. We notice in the last case the concentration of the last 12 countries in the last class.

The maximum entropy of the above classification (Class I) is 2.18454, very close to the maximum possible for 9 classes, namely $\ln 9 = 2.19722$. The Chow statistics (Class II) is 1.89817 and it has a Snedecor—Fisher distribution with 8 and 18 degrees of freedom. If we compute the c.d.f. we obtain $F(1.89817) = 0.87666$ using the Simpson method (Păltineanu et al., 1998), respectively $F(1.89817) = 0.874$ using the Monte Carlo method, simulating the normal random variables by the Box—Muler method (Văduva, 2004). Therefore in both cases the value of Chow statistics is grather than the centil of the order 0.13. For the above non-weighted indices we obtain the minimum 0.21463, the maximum 0.65473 and the variance 0.01158.

If we use the weighted indices we obtain the following table.

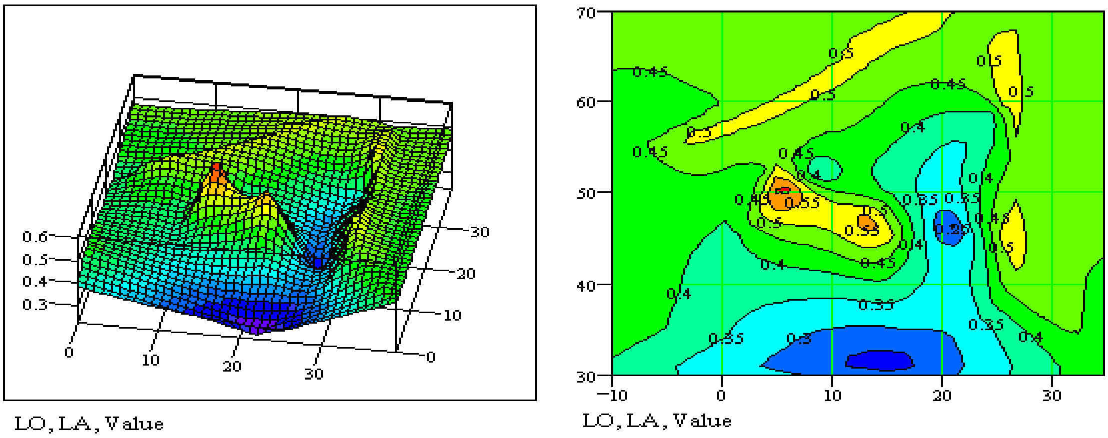
Table 7. Classification using weighted composite indices.

Country	AT	BE	BG	CY	CZ	DK	EE
Value	0.33521	0.34072	0.13855	0.27813	0.22615	0.40514	0.23482
Rank	10	8	26	16	20	6	19
Class I	B	B	C^{--}	B^{-}	C^{-}	B^{+}	C
Class II	B^{-}	B^{-}	C^{--}	C^{-}	C^{-}	B^{+}	C^{-}

Country	FI	FR	DE	GR	HU	IE	IT
Value	0.41123	0.32369	0.34109	0.26003	0.13474	0.41869	0.30235
Rank	5	11	7	18	27	3	14
Class I	B^{+}	B	B	C	C^{--}	A	B^{-}
Class II	B^{+}	C	B	C^{-}	C^{--}	A^{+}	C^{-}

Country	LV	LT	LU	MT	NL	PL	PT	RO	SK	SI	ES	SE	UK
Value	0.28939	0.18615	0.83175	0.17586	0.41211	0.14163	0.26576	0.18538	0.19734	0.31429	0.33974	0.46273	0.32017
Rank	15	22	1	24	4	25	17	23	21	13	9	2	12
Class I	B^-	C^-	A^{++}	C^{--}	B^+	C^{--}	C	C^-	C^-	B	B	A^+	B
Class II	C^-	C^-	A^{++}	C^{--}	A	C^{--}	C^-	C^{--}	C^-	C^-	B^-	A^+	C

In Figure 2 is presented the spatial distribution of Value (first row in the last table) in EU-27. In this case, we can see just a little different stylised map, signifying a more complex distribution within EU.



Source: own elaboration on EUROSTAT data.

Figure 2. 3-D map and contour plot of Value for EU-27.

When we use weighted composite indices instead of the non-weighted ones we can see that three countries remain on the same position: Luxembourg (first position), Czech Republic (position 20) and Hungary (last position, 27). We have also differences, the highest increasing being for Germany (14 positions from 21 to 7) and the highest decreasing being for Romania (19 positions from 4 to 23). The distribution of the 27 countries is 1+1+1+3+7+3+3+4+4 in the case of the maximum entropy, respectively $1 + 2 + 1 + 2 + 1 + 3 + 2 + 9 + 5$ in the case of maximum separation. We notice that we have also higher concentration for the last classes in the case of maximum separation, as in the case of non-weighted composite indices.

The obtained maximum entropy in this case is less than in the case of non-weighted indices: 1.91095. It means that the 27 countries are distributed less uniform than in the previous case. The obtained maximum Chow statistics is very high: 1401.82931, hence it is useless to compute the c.d.f. for this value. For the above weighted indices we obtain the minimum 0.13474, the maximum 0.83175 and the variance 0.01958.

4 Conclusions

We know (Onicescu and Ştefănescu, 1979, Petrică and Ştefănescu, 1982, Preda, 1992) that the maximum Shannon entropy for a simple random variable is reached for the uniform random variable. That's why if we use the maximum entropy principle as classification criterion the countries are distributed into the classes as uniform as possible (there are not huge differences between the numbers of classes members).

In applications we use more the Chow statistics for which the first number of degree of freedom is greather than 1 (as in formula (12), for instance). This is the case of Chow breakpoint test, where the denominator is the estimated variance of the groups tacking into account the break points, and the numberator is the difference between the sum of squares if we do not consider the breakpoints and the same sum if we do, divided by the number of degrees of freedom. This is the logical explanation of the formula (17).

The selected performance criteria are in agreement with sustainable development (Indicators of Sustainable Development) and were approved by the Commission on Sustainable Development at its Third Session in 1995. We notice that each obtained eigenvector has at least one negative component, even the principal component. Therefore it is wrong to try to reduce all the costs or increase all the benefits if the values are correlated: for instance we can increase the value of the principal component by decreasing the investment share in GDP or by increasing the Gini index.

The new coordinates have positive values, except the last one (corresponding to the eigenvalue 4.15001). In this case we have the values -0.23417 for Slovakia and -2.70912 for Spain.

If we want to apply the Chow breakpoint test in the considered application for non-weighted performance indices we accept the null hypothesis that we have no breakpoints even if the error level is 10% $(0.87666 < 0.9)$, but the test is not the goal of using the Chow statistics: we want only to obtain the maximum of this statistics (the maximum separation).

The weighted composite indices are more scattered than the non-weighted ones: in our example the minimum is less, and the maximum and the variance are greater in the first case. If we denote by σ_1^2 the variance in the case of non-weighted composite indices, and by σ_2^2 the variance in the case of weighted composite indices we obtain $\frac{\sigma_2^2}{\sigma_1^2} = \frac{0.01958}{0.01158} = 1.69117$. It is possible that this can explain also the lower Shannon entropy (hence the less uniform distribution) of the countries in the case of weighted indices.

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