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Estimation of Technical and Allocative Inefficiencies in a Cost System: An Exact Maximum Likelihood Approach

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Abstract

Estimation and decomposition of overall (economic) efficiency into technical and allocative components goes back to Farrell (1957). However, in a cross-sectional framework joint econometric estimation of efficiency components has been mostly confined to restrictive production function models (such as the Cobb-Douglas). In this paper we implement a maximum likelihood (ML) procedure to estimate technical and allocative inefficiency using the dual cost system (cost function and the derivative conditions) in the presence of cross-sectional data. Specifically, the ML procedure is used to estimate simultaneously the translog cost system and cost increase due to both technical and allocative inefficiency. This solves the so-called 'Greene problem' in the efficiency literature. The proposed technique is applied to the Christensen and Greene (1976) data on U.S. electric utilities, and a cross-section of the Brynjolfsson and Hitt (2003) data on large U.S. firms.

JEL classification: C 31

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1. Introduction

The standard neoclassical production theory assumes that producers are always efficient. This assumption, however, is not consistent with reality. Consequently, the idea of measuring efficiency of firms is becoming popular both in the government and private sectors. In measuring efficiency of producers, the focus is mostly on technical efficiency, which is often associated with managerial efficiency and estimated from the production/distance functions or the dual cost, revenue and profit functions. Although achieving technical efficiency is perhaps the utmost concern, producers can be inefficient in using their input mixes as well. This is labeled as allocative inefficiency, which increases cost. Thus a producer is economically efficient when it is both technically and allocatively efficient. Estimation of both technical and allocative efficiency requires the use of a system approach. However, the system to be used depends on type of data available and the behavioral assumption on the part of the producers. For example, the cost system is appropriate if the assumption of cost minimization holds (as is the case with regulated utilities).

In the stochastic frontier literature there are two approaches to estimate a system and obtain measures of increased cost due to technical and allocative efficiencies under the behavioral assumption of cost minimization. First, the primal system approach was originally proposed by Schmidt and Lovell (1979). Their system, constructed from a Cobb-Douglas (CD) production function and the first-order conditions of cost minimization, is used to estimate the parameters of the model as well as technical and allocative inefficiencies. Since inefficiency increases cost, measurement of increased cost due to each type of inefficiency is needed. This requires knowledge of the cost function, which for a CD production function can be easily derived analytically. Once the cost function is derived, computation of increased cost associated with technical and allocative inefficiencies (using the estimated parameters and estimates of technical and allocative inefficiencies from the primal system) is trivial.¹ The second approach is to estimate the dual cost system and was first suggested by Greene (1980). There are two problems in this approach, especially when a flexible functional form is used. The first problem is related to specification of the cost function that accommodates both technical and allocative inefficiency and the second problem is related to joint estimation of the cost system and computation of cost increase due to each inefficiency component. Note that these problems are not trivial even for production functions for which the cost function can be derived analytically², unless one uses the

¹ Although the Schmidt-Lovell procedure can be extended to accommodate flexible functional form such as the translog (Kumbhakar and Wang (2006)), it is not possible to derive the cost function analytically. That is, for flexible production functions computation of cost associated with technical and allocative inefficiency is not trivial.

² There are not many production functions for which cost function can be derived analytically.

Schmidt-Lovell (1979) primal system to estimate the model and then uses the analytical cost function to compute cost of each inefficiency components. In other words, although the dual cost system helps one to specify the technology and identify costs of each inefficiency component, estimation of such a system using cross-sectional data is yet to be done.³

Kumbhakar (1997) solved the specification problem in a dual cost function framework that incorporates both technical and allocative inefficiency in any cost system. This formulation uses an input-oriented technical inefficiency specification and introduces allocative inefficiency following Schmidt and Lovell (1979). It solves the specification problem theoretically; however, no estimation technique is offered. The difficulty is that the cost function and the deviations of optimal shares from their observed counterparts are complicated functions of allocative inefficiency. Although the link between allocative inefficiency and increase in cost therefrom is well established for any cost function in Kumbhakar (1997), the stochastic structure for the cost system consisting of the cost function and the cost shares is, unfortunately, very complicated for the standard ML method. This estimation problem has been a major stumbling block for more than two decades and is labeled as the Greene problem by Bauer (1990).

It is often argued that the Greene problem can be circumvented if decomposition of cost efficiency into technical and allocative components is not desired. The logic behind this argument is that since both inefficiencies increase cost inclusion of a one-sided error term in the cost function will capture the cost of overall (technical plus allocative) inefficiency, which can easily be estimated by the standard ML method using a single equation approach. However, it is shown by Kumbhakar and Wang (2005), in terms of a detailed Monte Carlo simulation, that failure to include allocative inefficiency explicitly in the cost function biases estimates of (i) the cost function parameters, (ii) returns to scale (RTS), (iii) input price elasticities, and (iv) cost-inefficiency. They also found that the true and estimated distributions of RTS, elasticities, and cost-inefficiency are quite different. Based on these results they concluded that the widely held view that ignoring costs due to allocative inefficiency (or aggregating it with the cost of technical inefficiency) is not likely to affect the technological parameters and measures of RTS, elasticities, etc., is not correct. These findings send a strong message indicating the importance of solving the Greene problem, i.e., estimating costs of technical and allocative inefficiency jointly from a system.

We are not aware of any application where the Greene problem is solved using a flexible cost function and cross-sectional data with the exception of the Bayesian approach in Kumbhakar and Tsionas (2005). Empirical

³ The exception is Kumbhakar and Tsionas (2005) who uses a Bayesian approach to estimate a translog cost system.

application of the Kumbhakar (1997) model in a sampling theory framework is difficult because of the complex error structure of the model, especially when allocative inefficiency is represented by random variables a la Schmidt and Lovell (1979).⁴ Because of this difficulty no one has applied the method proposed in Kumbhakar. In this paper we propose a classical solution and use an exact ML approach to estimate the translog cost system in a cross-sectional setup where both technical and allocative inefficiency are random variables. The proposed method solves the long lasting Greene problem.

The rest of the paper is organized as follows. The cost system of Kumbhakar (1997) is briefly presented in Section 2. The ML estimation procedure is discussed in Section 3, first with only allocative inefficient and then both technical and allocative inefficiencies. Data and results are discussed in Section 4, followed by some concluding remarks in Section 5.

2. The model

Since our focus is to estimate the cost system, in this section we lay out the model proposed in Kumbhakar (1997). Let the production technology be specified as $q_i = f(x_i e^{-u_i})$ where q_i is output and x_i is a vector of J inputs for firm i ($i = 1, \dots, n$), $f(\cdot)$ is the production function, and $u_i \geq 0$ measures input-oriented (IO) technical inefficiency (Farrell (1957)). This specification implies that a technically inefficient producer over-uses all the inputs by $u \cdot 100$ percent compared to an efficient producer producing the same output. Consequently, the IO measure of technical inefficiency is useful when the objective of the producers is to allocate inputs in such a way that cost is minimized for an exogenously given level of output. In allocating inputs producers may make mistakes. These mistakes are labeled as allocative inefficiency. Here we follow Schmidt and Lovell (1979) and Kumbhakar (1997) in modeling allocative inefficiency, viz., $f_j(x_i e^{-u_i}) / f_1(x_i e^{-u_i}) = w_{j,i} e^{\xi_{j,i}} / w_{1,i}$, $j = 2, \dots, J$. Here a non-zero value of $\xi_{j,i}$ indicates the presence of allocative inefficiency for the input pair $(j,1)$ for firm i . Note that unless the production function is homogeneous the IO technical inefficiency term (u) will not drop out from the first-order conditions.

⁴ With panel data these problems can be avoided by modeling allocative inefficiency parametrically. This involves the assumption that either price distortions are time-invariant or they can be related functionally to certain explanatory variables. Furthermore, additional error terms are to be introduced in the cost share equations without including them in the cost function (integrability condition, see McElroy (1987)). If panel data are not available or one does not want to make the assumption that allocative inefficiency can be completely explained in terms of some covariates, the only alternative is to model technical and allocative inefficiency as random variables.

Schmidt and Lovell (1979) used the above framework to estimate a Cobb-Douglas production function for which the cost function can be derived analytically. Since there are not many production functions for which the cost function can be derived analytically and the interest is mostly on costs of inefficiency, the natural alternative is to consider a dual cost function, which is widely used in empirical studies. The cost function specification with input-oriented technical inefficiency is strongly separable in costs of technical and allocative inefficiency. The complicating factor is that the cost share equations are affected by the presence of allocative inefficiency. Thus, the model has to take into account the link between allocative inefficiency appearing in the cost share equations and increase in cost therefrom that appears in the cost function. The modeling approach proposed by Greene (1980) did not make use of this link in a theoretically consistent manner. Kumbhakar (1997) derived the exact relationship between allocative inefficiency and cost therefrom using a translog cost function, thereby solving the Greene problem theoretically. This relationship is crucial from estimation point of view because it connects the error terms in the cost share equations and the cost function.

Since $\xi_{j,i}$ represents allocative inefficiency for the input pair $(j, 1)$ the relevant input prices to the firm i ($i = 1, \dots, n$) are $w_i^* \equiv (w_{1,i}, w_{2,i} \exp(\xi_{2,i}), \dots, w_{J,i} \exp(\xi_{J,i}))'$ where $\xi_{2,i}, \dots, \xi_{J,i}$ are random variables that capture allocative inefficiency. Kumbhakar (1997) showed that actual cost could be expressed as

$$\ln C_i^a = \ln C^*(w_i^*, q_i) + \ln G(w_i, q_i, \xi_i) + u_i \quad (1)$$

where $C_i^a = \sum_j w_{j,i} x_{j,i}$ and $C^*(w_i^*, q_i)$ is the minimum cost function obtained from solving the following problem:

$\min_{x_i e^{-u}} w_i^* x_i e^{-u}$ subject to $q_i = f(x_i e^{-u})$. The $G(w_i, q_i, \xi_i)$ function in (1) is defined as $G(\cdot) = \sum_j S_{j,i}^* e^{-\xi_{j,i}}$ where

$S_{j,i}^* = \partial \ln C^*(\cdot) / \partial \ln w_{j,i}^*$. Since (1) is strongly separable in u_i , cost of technical inefficiency (percentage increase in

cost due to technical inefficiency) is represented by $u_i \geq 0$. The allocative inefficiency terms (ξ_j) appear both in the

$C^*(\cdot)$ and the $G(\cdot)$ functions. Thus, to separate the cost of allocative inefficiency, we need to define $C^0(w_i, q_i)$, the

cost frontier (also labeled as the neoclassical cost function). For this we rewrite the cost function in (1) as

$\ln C_i^a = \ln C^0(w_i, q_i) + \ln C^{AL}(w_i, q_i, \xi_i) + u_i$ where $C^0(w_i, q_i)$ is the cost frontier (the neo-classical cost function),

which can be obtained from the cost function in (1) by imposing restrictions that firms are efficient both technically

and allocatively. That is, $\ln C^0(\cdot) = \ln C^a(\cdot | \xi_{j,i} = 0 \forall j, u_i = 0) = \ln C^*(\cdot) | \xi_{j,i=0}$ (since $\ln G(\cdot) | \xi_{j,i=0} = 0$) and

$\ln C^{AL}(w_i, q_i, \xi_i) = \ln C^a |_{u_i=0} - \ln C^0(\cdot) = \ln C^*(w_i^*, q_i) + \ln G(w_i, q_i, \xi_i) - \ln C^0(\cdot)$. The $\ln C_i^{AL}$ term can be interpreted as the percentage increase in cost due to allocative inefficiency.

To get a better understanding of the above decomposition we start with a parametric functional form on $C^*(\cdot)$. Once a functional form for $C^*(\cdot)$ is assumed, the $G(\cdot)$ and $C^0(\cdot)$ functions can be easily derived. Using these functions we can easily obtain $\ln C^{AL}$. Since the cost function in (1) is naturally expressed in logarithms, it is convenient to discuss both modeling and estimation issues in terms of the Cobb-Douglas and translog cost functions. It is worth pointing out, however, that the methodology is equally applicable for any other flexible parametric form for $C^*(\cdot)$. For example, if we assume a parametric functional form (e.g., translog) for $C^*(\cdot)$, i.e.,

$$\ln C^*(w_i^*, q_i) = \alpha_0 + \sum_j \alpha_j \ln w_{j,i}^* + \gamma_q \ln q_i + \frac{1}{2} \gamma_{qq} (\ln q_i)^2 + \frac{1}{2} \sum_j \sum_k \beta_{jk} \ln w_{j,i}^* \ln w_{k,i}^* + \sum_j \gamma_{jq} \ln w_{j,i}^* \ln q_i$$

then $S_{j,i}^* = \alpha_j + \sum_k \beta_{jk} \ln w_{k,i}^* + \gamma_{jq} \ln q_i$. Imposing the linear homogeneity restrictions on $C^*(\cdot)$, and noting that

$w_{1,i}^* = w_{1,i}$ by definition, we can rewrite (1) as $\ln(C_i^a / w_{1,i}) = \ln \tilde{C}^*(\tilde{w}_i^*, q_i) + \sum_j S_{j,i}^* \exp(-\xi_{j,i})$, where

$\tilde{C}^*(\tilde{w}_i^*, q_i) = C^*(w_i^*, q_i) / w_{1,i}$ and $\tilde{w}_i^* = (w_{2,i}^* / w_{1,i}, \dots, w_{J,i}^* / w_{1,i})$. Now we write the cost function and the associated cost share equations in terms of $C^0(\cdot)$. These are

$$\ln(C_i^a / w_{1,i}) = \ln C_i^0(\tilde{w}_i, q_i) + \ln C_i^{AL}(\tilde{w}_i, q_i, \xi_i) + u_i, \quad (2)$$

$$S_{j,i}^a = S_{j,i}^0(\tilde{w}_i, q_i) + \eta_{j,i}(\tilde{w}_i, q_i, \xi_i), \quad i = 1, \dots, n; j = 2, \dots, J \quad (3)$$

where $\tilde{w}_i = (w_{2,i} / w_{1,i}, \dots, w_{J,i} / w_{1,i})$, $S_{j,i}^a = w_{j,i} x_{j,i} / C_i^a$ is the actual (observed) cost share of input j ($j = 2, \dots, J$),

$C_i^0(\tilde{w}_i, q_i)$ is the normalized (by $w_{1,i}$) cost frontier and $S_{j,i}^0 = \partial \ln C_i^0(\cdot) / \partial \ln w_{j,i}$ ($j = 2, \dots, J$). For the above

translog cost function $\ln C_i^0(\tilde{w}_i, q_i)$ is

$$\ln C_i^0(\cdot) = \alpha_0 + \sum_{j=2}^J \alpha_j \ln \tilde{w}_{j,i} + \gamma_q \ln q_i + \frac{1}{2} \gamma_{qq} (\ln q_i)^2 + \frac{1}{2} \sum_{j=2}^J \sum_{k=2}^J \beta_{jk} \ln \tilde{w}_{j,i} \ln \tilde{w}_{k,i} + \sum_{j=2}^J \gamma_{jq} \ln \tilde{w}_{j,i} \ln q_i, \quad (4)$$

$$S_{j,i}^0 = \alpha_j + \sum_{k=2}^J \beta_{jk} \ln \tilde{w}_{k,i} + \gamma_{jq} \ln q_i, \quad j = 2, \dots, J, \quad (5)$$

$$\ln C_i^{AL} = \ln G_i + \sum_{j=2}^J \alpha_j \xi_{j,i} + \sum_{j=2}^J \sum_{k=2}^J \beta_{jk} \xi_{j,i} \ln \tilde{w}_{k,i} + \frac{1}{2} \sum_{j=2}^J \sum_{k=2}^J \beta_{jk} \xi_{j,i} \xi_{k,i} + \sum_{j=2}^J \gamma_{jq} \xi_{j,i} \ln q_i, \quad (6)$$

$$\eta_{j,i} = \frac{S_{j,i}^0 \{1 - G_i \exp(\xi_{j,i})\} + a_{j,i}}{G_i \exp(\xi_{j,i})}, \quad j = 2, \dots, J \quad (7)$$

where

$$G_i = \sum_{j=2}^J (S_{j,i}^0 + a_{j,i}) \exp(-\xi_{j,i}), \quad (8)$$

and

$$a_{j,i} = \sum_{k=2}^J \beta_{jk} \xi_{k,i}. \quad (9)$$

The cost system defined in (2) and (3) serves two purposes. First, technical and allocative inefficiencies are modeled in a coherent manner. Second, the exact link between allocative inefficiency (ξ_j) and its cost is given in (6).

The cost function decomposes the overall increase in cost due to inefficiency into two components, viz., the percentage increase in cost due to allocative inefficiency, $\ln C_i^{AL}$, and the percentage increase in cost due to technical inefficiency, u_i . The decomposition formula also establishes an exact link between the error terms in the cost share equations (which are functions of allocative inefficiency) and cost of allocative inefficiency, which is very important from estimation point of view. In general, the link is provided by the relationship $\ln C^{AL}(w_i, q_i, \xi_i) = \ln C^*(w_i^*, q_i) + \ln G(w_i, q_i, \xi_i) - \ln C^0(\cdot)$. For the Cobb-Douglas case, this link is established in Schmidt and Lovell

$$(1979), \text{ viz., } \ln C^{AL} = \sum_{j=2}^J \alpha_j \xi_j + \ln \left[\alpha_1 + \sum_{j=2}^J \alpha_j e^{-\xi_j} \right] - \ln \left[\sum_{j=1}^J \alpha_j \right]. \quad \text{Since Schmidt and Lovell used the system}$$

consisting of the production function and the first-order conditions of cost minimization, it was not necessary to use the above link in estimation. It was, however, used to compute the cost of allocative inefficiency. Thus, Schmidt and Lovell avoided the Greene problem by not estimating the cost system. Kumbhakar and Wang (2006) did the same when they extended the CD model of Schmidt and Lovell to accommodate flexible production functions (such as the translog). Although they avoided the Greene problem by following the Schmidt and Lovell procedure, they had to solve a system of nonlinear equations to compute the costs of technical and allocative inefficiencies.

The Greene problem is associated with estimating a cost system in (2) and (3) using the link between cost of allocative inefficiency and errors in the cost share equations (which are functions of allocative inefficiency), given in equations (6), (8) and (9). Thus to address the Greene problem, we focus on estimating the cost system. It can be seen that the error structure based on u and ξ_j in (2) and (3) is quite complicated. Because of this the model has not been

estimated using cross-sectional data.⁵ In the following section we discuss an estimation method, first with only allocative inefficiency and then with both technical and allocative inefficiency.

3. Estimation

3.1. Only allocative inefficiency

For simplicity we consider first the case with only allocative inefficiency, and write the cost system in (2) and (3) as

$$\begin{aligned}\ln C_i^a &= \ln C_i^0(\beta) + \ln C_i^{AL}(\beta, \xi_i) + v_i, \\ S_{j,i}^a &= S_{j,i}^0(\beta) + \eta_{j,i}(\beta, \xi_i), \quad j = 2, \dots, J, i = 1, \dots, n,\end{aligned}\tag{10}$$

where β is the parameter vector associated with the translog cost function, and v is the stochastic noise component added to the cost function. To avoid singularity problem, one share equation is dropped and we normalize the corresponding element of ξ_i to zero. We impose the standard distributional assumptions on the error components, v_i and ξ_i , in the above system.. More specifically, we assume $v_i \sim i.i.d.N(0, \sigma^2)$ and $\xi_i \sim i.i.d.N_{J-1}(\mu, \Omega)$, distributed independently of v_i . Here, the $(J-1) \times 1$ vector μ represents systematic allocative inefficiency, and the $(J-1) \times (J-1)$ matrix Ω is the covariance matrix of allocative inefficiency components.

We write the i th observation for the above system compactly as

$$y_i = X_i \beta + \begin{bmatrix} \ln C^{AL}(\xi_i, \beta) + v_i \\ \eta_i(\xi_i, \beta) \end{bmatrix} \equiv X_i \beta + \begin{bmatrix} \varepsilon_i \\ \eta_i \end{bmatrix},\tag{11}$$

where $y_i = [\ln C_i^a, S_{2,i}^a, \dots, S_{J,i}^a]'$, $X_i = [\ln C_i^0(\beta), S_{2,i}^0(\beta), \dots, S_{J,i}^0(\beta)]'$, $\eta_i(\xi_i, \beta) = (\eta_i(\xi_{2,i}, \beta), \dots, \eta_i(\xi_{J,i}, \beta))'$. Since the error vector in (11) is $(\varepsilon_i, \eta_i)'$, for the ML method one has to derive the joint pdf of $(\varepsilon_i, \eta_i)'$ starting from the distributions on v_i and ξ_i . The joint pdf of $(\varepsilon_i, \eta_i)'$ is $p(\varepsilon_i, \eta_i) = p_{\varepsilon|\eta}(\varepsilon_i | \eta_i) p_{\eta}(\eta_i)$ and

⁵ The system described in (2) and (3) is somewhat similar to the Kumbhakar and Tsionas (2005) model which assumed the presence of additional error terms in the share equations. Integrability condition requires that if there are errors in the share equations, these errors should also appear in the cost function (McElroy (1987)). No such allowance was made in the Kumbhakar and Tsionas (2005) model. Furthermore, they used a Bayesian approach to estimate the system. Here we propose a classical ML method without any extra error terms in the share equations.

$\varepsilon_i | \eta_i \sim N(\ln C^{AL}(\xi_i(\eta_i), \beta), \sigma_v^2)$, where $\xi_i(\eta_i)$ is the solution of ξ_i in terms of η_i from $\eta_i = \eta_i(\xi_i, \beta)$. Furthermore, the pdf of $p_\eta(\eta_i)$ can be expressed as

$$p_\eta(\eta_i) = p_\xi(\xi_i(\eta_i)) \cdot |\det D\xi_i(\eta_i)|, \quad (12)$$

where $D\xi_i(\eta_i)$ is the Jacobian matrix (derivatives of ξ_i with respect to η_i). Therefore, the joint pdf of the error vector in (11) is

$$p(\varepsilon_i, \eta_i) = p_{\varepsilon|\eta}(\varepsilon_i | \eta_i) p_\eta(\eta_i) = (2\pi)^{-J/2} (\sigma_v^2)^{-1/2} \det(\Omega)^{-1/2} \times \exp \left\{ -\frac{[\xi_i - (\ln C^{AL}(\xi_i(\eta_i)))]^2}{2\sigma_v^2} - \frac{1}{2} e_i(\eta_i, \beta)' \Omega^{-1} e_i(\eta_i, \beta) \right\} |\det D\xi_i(\eta_i)| \quad (13)$$

where $e_i(\eta_i, \beta) = \xi_i(\eta_i, \beta) - \mu$.

In practice, to implement the likelihood function based on (13) we have to show that (i) ξ can be solved in terms of η , and (ii) the Jacobian matrix can be derived analytically. We show these next.

For notational simplicity now we drop the observation index i . The first task is to solve for ξ in terms of η . Note that

$$\begin{aligned} \eta_j &= S_j^a - S_j^0 = \frac{S_j^0 [1 - G \exp(\xi_j)] + \sum_{k=1}^J \beta_{jk} \xi_k}{G \exp(\xi_j)} \\ \Rightarrow (\eta_j + S_j^0) G \exp(\xi_j) &= S_j^0 + \sum_{k=1}^J \beta_{jk} \xi_k \\ \Rightarrow S_j^a G \exp(\xi_j) &= S_j^0 + \sum_{k=1}^J \beta_{jk} \xi_k = S_j^0 + \sum_{k=2}^J \beta_{jk} \xi_k, \quad j = 1, \dots, J. \end{aligned} \quad (14)$$

For the last equality we used the normalization $\xi_1 = 0$. The equations in (14) can be expressed in ratio form to generate the following system of nonlinear equations,

$$\lambda_j \exp(\xi_j) = \frac{S_j^0 + \sum_{k=2}^J \beta_{jk} \xi_k}{S_j^0 + \sum_{k=2}^J \beta_{jk} \xi_k}, \quad j = 2, \dots, J, \quad (15)$$

where $\lambda_j = S_j^a / S_1^a$. In the Appendix we use fixed point arguments to show that a solution of ξ_j exists and is unique.

Once the ξ_j s are obtained, the value of G can be obtained as $G = \frac{S_1^0 + \sum_{k=2}^J \beta_{1k} \xi_k}{S_1^a} = \frac{S_1^*}{S_1^a}$. Note that we need G to

compute $\ln C^{AL}(\xi_i, \beta)$.

The second task is to derive the Jacobian of the transformation from ξ to η . To compute it we start again from the definition of η_j , i.e.,

$$\eta_j = \frac{S_j^0(1-h_j) + \sum_{k=2}^J \beta_{jk} \xi_k}{h_j}, \quad j = 2, \dots, J, \quad \text{where } h_j = G \exp(\xi_j). \quad \text{Differentiating it with respect to } \eta_l \text{ gives}$$

$$h_j \delta_{jl} + S_j^a \sum_k \frac{\partial h_j}{\partial \xi_k} \frac{\partial \xi_k}{\partial \eta_l} = \sum_k \beta_{jk} \frac{\partial \xi_k}{\partial \eta_l}, \quad (16)$$

where δ_{jl} is the Kronecker delta. The system in (16) can be written as

$$\Theta D = M, \quad (17)$$

where $\Theta_{jk} = S_j^a \frac{\partial h_j}{\partial \xi_k} - \beta_{jk}$, $D_{kj} = \frac{\partial \xi_k}{\partial \eta_j}$, $M = -\text{diag}(h_2, \dots, h_J)$. Here, D (the short form of $D\xi_j(\eta_j)$) is the Jacobian

of the transformation. The solution of D from (17) is $D = \Theta^{-1}M$, and $\det(D) = \det(\Theta)^{-1} \prod_{j=2}^J h_j$. To evaluate the

components of Θ we obtain

$$\frac{\partial h_j}{\partial \xi_k} = \left(\frac{\partial G}{\partial \xi_k} + G \delta_{jk} \right) \exp(\xi_j), \quad j, k = 2, \dots, J,$$

$$\frac{\partial G}{\partial \xi_k} = \sum_m \beta_{km} \exp(-\xi_m) - \exp(-\xi_k) S_k^*, \quad k = 2, \dots, J,$$

with the understanding that all the previous expressions are evaluated at the solution of the system which is $\xi = \xi(\eta)$.

Thus, the solution for the i th observation can be written as $\xi_i = \xi_i(\eta_i, \beta)$, and the likelihood function is

$$L(\beta, \sigma_v, \Omega; y, X) = (2\pi)^{-Jn/2} (\sigma_v^2)^{-n/2} \det(\Omega)^{-n/2} \times$$

$$\exp \left\{ -\frac{1}{2\sigma_v^2} \sum_{i=1}^n [\varepsilon_i - \ln C^{AL}(\beta, \xi_i(\eta_i, \beta))]^2 - \frac{1}{2} \sum_{i=1}^n e_i(\eta_i, \beta)' \Omega^{-1} e_i(\eta_i, \beta) \right\} \prod_{i=1}^n |\det D\xi_i(\eta_i, \beta)| \quad (18)$$

where $e_i(\eta_i, \beta) = \xi_i(\eta_i, \beta) - \mu$.

The ML estimators of σ_v^2 , μ and Ω are

$$\tilde{\sigma}_v^2(\beta) = n^{-1} \sum_{i=1}^n [\varepsilon_i - (\ln C^{AL}(\xi_i(\eta_i, \beta)))]^2, \quad \tilde{\mu}(\beta) = n^{-1} \sum_{i=1}^n \tilde{\xi}_i(\beta) \equiv \bar{\xi}(\beta),$$

$$\tilde{\Omega}(\beta) = n^{-1} \sum_{i=1}^n [\tilde{\xi}_i(\beta) - \bar{\xi}(\beta)][\tilde{\xi}_i(\beta) - \bar{\xi}(\beta)]',$$

where $\tilde{\xi}_i(\beta) \equiv \xi_i(\tilde{\eta}_i, \beta)$, and $\tilde{\eta}_i$ is the cost share residual vector. The concentrated log-likelihood function is:

$$\ln L^C(\beta; y, X) = \text{const.} - (n/2) \ln \tilde{\sigma}_v^2(\beta) - (n/2) \ln(\det(\tilde{\Omega}(\beta))) + \sum_{i=1}^n \ln |\det D\xi_i(\tilde{\eta}_i, \beta)|. \quad (19)$$

Note that the concentrated log-likelihood function in the absence of systematic allocative inefficiency can be obtained by setting $\mu = 0_{J-1}$ and $\tilde{\Omega} = n^{-1} \sum_{i=1}^n \tilde{\xi}_i \tilde{\xi}_i'$ in (19).

3.2. Both technical and allocative inefficiency

With both technical and allocative inefficiency the system is

$$y_i = X_i \beta + \begin{bmatrix} \ln C^{AL}(\xi_i, \beta) + v_i + u_i \\ \eta(\xi_i, \beta) \end{bmatrix} \equiv X_i \beta + \begin{bmatrix} \zeta_i \\ \eta_i \end{bmatrix} \quad (20)$$

where $u_i \sim i.i.d.N(0, \sigma_u^2)$ ($u_i \geq 0$) distributed independently of v_i and ξ_i . The convolution $\omega_i \equiv v_i + u_i$ has a familiar distribution, namely, $f(\omega_i) = \frac{2}{\sigma} \phi\left(\frac{\omega_i}{\sigma}\right) \Phi\left(\frac{\lambda \omega_i}{\sigma}\right)$, where $\sigma^2 = \sigma_v^2 + \sigma_u^2$, $\lambda = \sigma_u / \sigma_v$, and ϕ , Φ denote, respectively the pdf and cdf of the standard normal variable (see Kumbhakar and Lovell (2000), p. 140). Consequently, $p(\zeta_i | \xi_i) = p_\omega(\omega_i - \ln C^{AL}(\xi_i, \beta))$.

Assuming $\xi_i \sim i.i.d.N_{J-1}(0, \Omega)$ as before, we obtain the following joint probability density function

$$\begin{aligned} p(\zeta_i, \eta_i) &= p(\zeta_i | \eta_i) \cdot p_\eta(\eta_i) = p(\zeta_i | \xi_i(\eta_i, \beta)) \cdot p_\xi(\xi_i(\eta_i, \beta)) \cdot |\det D\xi_i(\eta_i, \beta)| \\ &= \frac{2}{\sigma} \Phi\left(\frac{\lambda}{\sigma} [\zeta_i - \ln C^{AL}(\xi_i, \beta)]\right) \cdot |\det D\xi_i(\eta_i, \beta)| \times \\ &= (2\pi)^{-J/2} \det(\Omega)^{-1/2} \exp\left\{-\frac{[\zeta_i - \ln C^{AL}(\xi_i, \beta)]^2}{2\sigma^2} - \frac{1}{2} e(\eta_i, \beta)' \Omega^{-1} e(\eta_i, \beta)\right\}. \end{aligned}$$

Using

$$\zeta_i = \ln C_i^a - \ln C_i^0(\beta) - \ln C_i^{AL}(\beta, \xi_i(\eta_i, \beta))$$

$$\eta_{j,i} = S_{j,i}^a - S_{j,i}^0(\beta), \quad j = 1, \dots, J-1,$$

the likelihood function becomes

$$L(\beta, \sigma_v, \sigma_u, \Omega; y, X) \propto$$

$$\sigma^{-n} \prod_{i=1}^n \Phi\left(-\frac{\lambda}{\sigma} [\zeta_i - \ln C^{AL}(\xi_i, \beta)]\right) \cdot \prod_{i=1}^n |\det D\xi_i(\eta_i, \beta)| \times \\ \det(\Omega)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2} \sum_{i=1}^n [\zeta_i - \ln C^{AL}(\xi_i, \beta)]^2 - \frac{1}{2} \sum_{i=1}^n e_i(\eta_i, \beta)' \Omega^{-1} e_i(\eta_i, \beta)\right\}, \quad (21)$$

where $e_i(\eta_i, \beta) = \xi_i(\eta_i, \beta) - \bar{\xi}(\eta_i, \beta)$, and $\bar{\xi}(\eta_i, \beta) = n^{-1} \sum_{i=1}^n \xi_i(\eta_i, \beta)$. The above likelihood function can be concentrated with respect to Ω , the ML estimator of which is

$$\hat{\Omega}(\beta) = n^{-1} \sum_{i=1}^n e_i(\eta_i, \beta) e_i(\eta_i, \beta)'.$$

Thus, the concentrated log-likelihood function is proportional to

$$\ln L^C(\beta, \sigma_v, \sigma_u; y, X) = -\frac{n}{2} \ln(\sigma^2) + \sum_{i=1}^n \ln \Phi\left(-\frac{\lambda}{\sigma} [\zeta_i - \ln C^{AL}(\xi_i, \beta)]\right) + \sum_{i=1}^n \ln |\det D\xi_i(\eta_i, \beta)| \\ - \frac{n}{2} \det(\hat{\Omega}(\beta)) - \frac{1}{2\sigma^2} \sum_{i=1}^n [\zeta_i - \ln C^{AL}(\xi_i, \beta)]^2. \quad (22)$$

Here, σ and λ are functions of the original parameters σ_v and σ_u . To maximize the log-likelihood functions shown in (22) we use the Nelder-Mead simplex maximization technique that does not require numerical derivatives. To compute standard errors for the parameters we have used the BHHH formula which is based on first-order derivatives of the log-density with respect to the parameters.

4. Results

4.1 Electric utility data

Data for the first application is from Christensen and Greene (1976) who used this cross-sectional data on steam electricity generating firms. The inputs are capital, labor and fuel. Output is electricity (million kwh) generated

by steam power. The total cost is the sum of labor, fuel and capital cost. The total number of firms in the data set is 123. Details on the construction of output and input quantities, input prices and output can be found in Christensen and Greene (1976).

First, we consider the model with only allocative inefficiency, which is estimated with and without systematic components. In both cases fuel price is used as a numeraire to impose homogeneity (of degree one) in input prices. Thus, our estimates of allocative inefficiency in labor and capital (ξ_L and ξ_K) are relative to fuel. Since both the mean and median values of ξ_L and ξ_K are negative (without systematic allocative inefficiency), we can argue that, on average, both labor and capital are overused (relative to fuel). Overuse (underuse) of capital relative to labor can be computed by comparing $w_K e^{\xi_K} / w_L e^{\xi_L}$ with w_K / w_L (i.e., from $\xi_K - \xi_L$) which shows that capital is overused relative to labor since $\xi_K - \xi_L < 0$ (at the mean and median). When we allowed systematic allocative inefficiency, we find that capital (but not labor) is overused relative to fuel. We also find that, on average, capital is overused. Since $\xi_K < 0$ and $\xi_L > 0$, the degree of over-use (indicated by $\xi_K - \xi_L$) is stronger in the model with persistent allocative inefficiency. Note that these results hold at the mean/median. To get an idea about relative over-use of capital and labor for each firm, we plot the distribution of ξ_K and ξ_L in Figures 1(a) and 1(b) for the model without systematic/persistent allocative inefficiency. Figures 1(c) and 1(d) plot these for the model that allows systematic/persistent allocative inefficiency. It is clear from the graphs that the mean values of ξ_K and ξ_L are close to zero (an artifact of the distributional assumption that their means are zero). This is, however, not the case when non-zero means (systematic allocative inefficiency) are allowed on ξ_K and ξ_L . For these models the mean values of ξ_K and ξ_L are found to be negative, which means that capital and labor are overused relative to fuel.⁶ The percentage of firms that are not overusing capital (relative to fuel and/or labor) is quite small, especially when technical inefficiency is allowed.

Since we use input-oriented technical inefficiency, the percentage increase in cost is the same as percentage over-use of inputs due to technical inefficiency, u . We find that, on average, cost is increased about 9% due to technical inefficiency. This is true irrespective of whether allocative inefficiency is systematic or not (see Table 2). Technical inefficiency of the top (bottom) 5% of the firms is 3% or less (20% or more). The distribution u (cost of

⁶ These results are in line with those of Kumbhakar and Wang (2006) who used a more recent data (1986-1996) on 72 steam electric power generation plants, although they used a different modeling approach.

technical inefficiency) is given in Figure 2. It can be seen from this figure that technical inefficiency for about 90% of the firms is less than 15%. That is, technical efficiency of about 90% of the firms is at least 85%. Using a more recent data (1986-1996) on 72 steam electric power generation plants, Kumbhakar and Wang (2006) found similar results although they used a different modeling approach.

Next we report (in Table 1) the increase in cost due to allocative inefficiency, $\ln C^{AL}$, which is found to be quite small for both models. The mean (median) values of $\ln C^{AL}$ is 0.0444 (0.0296), which shows that cost is increased, on average, by 4.44%. For the top (bottom) 50% of the firms the cost is increased by 2.96% due to input misallocation. For the bottom 5% of firms, misallocation of inputs increased cost by 14.44%. These figures are much higher (mean 10.13% and median 9.16%) when firms are allowed to make systematic allocative errors in input-use decisions. Increase in cost due allocative inefficiency for about half of the firms is more than 9.16%. These figures for the bottom 5% of the firms are more than 22.39%. Distributions of $\ln C^{AL}$ (with and without systematic allocative inefficiency) are plotted in Figure 2. It can be seen that the distribution of $\ln C^{AL}$ without systematic allocative inefficiency is very tight compared to the one that allows systematic allocative inefficiency. Since these firms were regulated, then from a static optimization point of view it is expected that inputs, especially capital, won't be used optimally (the Averch-Johnson hypothesis). Thus the results from the model that ignores systematic allocative inefficiency are likely to be biased. This finding accords with the Monte Carlo results in Kumbhakar and Wang (2005).

Results from the models with both technical and allocative inefficiency are reported in Table 2. Increase in cost due to technical inefficiency, on average, is 9.07% to 9.94% depending on whether or not systematic allocative inefficiency is allowed. For about half of the firms, the increase in cost due to technical inefficiency is around 8%, irrespective of whether allocative inefficiency is systematic or not. However, the increase in cost due to allocative inefficiency, $\ln C^{AL}$, vary widely depending on whether or not allocative inefficiency is assumed to be systematic. For example, compared to the model without systematic allocative inefficiency it can be seen that the estimated value of $\ln C^{AL}$ in the model with systematic allocative inefficiency is more than twice (the median values are 8.46% and 3.8%, respectively). In both models we find overuse of capital and labor (relative to fuel), although the degree of such overuse is more pronounced in the model that allows systematic allocative inefficiency. This is one of the reasons why $\ln C^{AL}$ is more than double in the model that allows systematic allocative inefficiency. Further insight can be

obtained from the plots of $\ln C^{AL}$ in Figure 3. It is clear that distributions of $\ln C^{AL}$ are much tighter when systematic allocative inefficiency is not allowed.

Now we examine the returns to scale results, reported in Table 3 and Figure 4. In fact, we report cost elasticity with respect to output, $E_{CQ} = \partial \ln C^0 / \partial \ln q$, (which is the reciprocal of returns to scale). Scale economies (*SCE*) can then be defined as $SCE = 1 - E_{CQ}$, which shows economies (diseconomies) of scale if *SCE* is positive (negative). We find evidence of scale economies (at the mean/median) from all models. The models with only allocative inefficiency show smaller scale economies when compared to those with both technical and allocative inefficiency (evaluated at the mean/median). Figure 4 shows plots of E_{CQ} from all four models. Percentage of firms with scale economies ($E_{CQ} < 1$) varies depending on the model. For example, about 94% of the firms in the models with both technical and with or without allocative inefficiency were operating with economies of scale. However, if one assumes that firms were operating fully efficiently (technically), then these percentages drop dramatically. For example, only 62.6% of firms showed economies of scale in the model that allows systematic allocative inefficiency. On the other hand, the percentage of firms with economies of scale increased to 82.9% if one makes the assumption that firms were operating without systematic allocative inefficiency. Systematic allocative inefficiency matters more in the model with no technical inefficiency so far as the estimates of scale economies are concerned. However, all the models predict the presence of substantial unexploited scale economies, which is also found by Christensen and Greene (1976). However, these results are in contrast to those reported in Kumbhakar and Wang (2006) who used the more recent data and found no evidence of substantial scale economies. This is not surprising because the average size of firms has increased over time to exploit scale economies.

Since the Christensen and Greene formulation comes closer to our model with no technical and systematic allocative inefficiency (Model 1), we compare scale economies derived from these two models. Note that the error terms in the cost share equations in the Christensen and Greene model have no economic interpretation while in our model these error terms (η) are functions of allocative inefficiency (ξ). The summary statistics of cost elasticity from the Christensen and Greene model (Model 5) are reported in the last column of Table 3. A comparison of these numbers with those from Model 1 shows that estimates E_{CQ} are underestimated (*SCE* are biased upward) in Model 5. For example, the mean (median) *SCE* in Model 5 is 0.11 (0.090) whereas in Model 1 it is 0.084 (0.062).

It is often argued that since the data is from the 1970s, the empirical results are not relevant to the electric utility industry of today. Comparing our results to those that used the recent data (for example, Kumbhakar and Wang (2006)), we find that efficiency results (both technical and allocative) are in fact qualitatively similar. However, the estimates of returns to scale are found to be different. While the industry was mostly operating under increasing returns to scale, no scale economies are found in the recent years. Our finding that most of the firms were operating under increasing returns to scale is consistent with the findings of Christensen and Greene (1976). That is, the inclusion of technical and allocative inefficiencies in the Christensen and Greene model did not change their main result that most of the firms experienced scale economies (although the scale economies results are overestimated). In general, other results of our models such as own and cross-elasticities of substitution are found to be in agreement with those reported in Christensen and Greene⁷.

4.2 Firm-level IT productivity data

We also apply the proposed technique to another very different type of data set that is used by Brynjolfsson and Hitt (2003) to measure computer (IT) productivity. Since our objective is to apply the cost system approach, we focused on a single cross-section of 518 firms for the year 1994. The output variable is value added (Q) and the three variable inputs are: ordinary capital stock (K), computer (IT) capital stock (C), and labor (L). The total cost is the sum of ordinary capital, computer capital, and labor cost. The details on the definition and construction of these variables can be found in Brynjolfsson and Hitt (2003). To conserve space we discuss the main results from this data, viz., from the models with and without systematic allocative inefficiency.⁸ In both models price of ordinary capital is used as a numeraire to impose homogeneity (of degree one) restrictions. Thus, our estimates of allocative inefficiency in labor and IT capital (ξ_L and ξ_C) are relative to ordinary capital. The mean values of ξ_L and ξ_C are found to be positive, which means that labor and IT capital are underused relative to ordinary capital. Alternatively, ordinary capital is overused relative to both IT capital and labor. It can be seen from the distributions of ξ_C and ξ_L (Figures

⁷ To conserve space, we do not report these results. Details can be obtained from the authors upon request.

⁸ We do not compare our results with those of Brynjolfsson and Hitt (2003) mainly because their focus was on measuring productivity and in doing so they estimated the TFP change regression without estimating the technology directly from the production/cost function. On the other hand, our interest is to estimate the underlying technology and in doing so we used the flexible translog cost system.

5a and 5b) that IT capital under-use (relative to capital) is much less than that of labor. The spread of ξ_L is quite large in both models with and without systematic allocative inefficiency.

Since misallocation of inputs increase costs which is what matters most to the producers, it is natural to examine increase in cost due to allocative inefficiency. The distributions of percentage increase in cost due to allocative inefficiency for both models (with and without systematic allocative inefficiency) are reported in Figure 8. On average, cost due to allocative inefficiency, $\ln C^{AL}$, is increased by 6.2% (9.8%) in the model without (with) systematic allocative inefficiency. The median values of $\ln C^{AL}$ in these models are 6.8% and 9.7%, respectively. Thus, distributions of $\ln C^{AL}$ in both models are quite symmetric. However, the variation (spread) in $\ln C^{AL}$ in the model with systematic allocative inefficiency is more pronounced.

Next we examine costs of technical inefficiency (which is the same as input-oriented technical inefficiency), u . The distribution of u is plotted in Figure 7 for both models. On average, technical inefficiency increased cost by 7.12% (5.81%) in the models without (with) systematic allocative inefficiency. The high mean value of technical inefficiency in the model without systematic allocative inefficiency is due to the fact that the distribution is highly skewed to the right and technical inefficiency of several firms is found to be quite high. The median values of technical inefficiency in both models are quite low in both models (1.54% vs. 1.64%).

Finally, we report cost elasticities, E_{CQ} , in Figure 6. From the estimates of E_{CQ} we can obtain the measure of scale economies (SCE) which is $SCE = 1 - E_{CQ}$, and returns to scale which is the reciprocal of E_{CQ} . It can be seen from Figure 6 that almost all these firms enjoyed economies of scale (operated under increasing returns to scale) in the model without systematic allocative inefficiency. The mean value of SCE is 0.0736 and is significantly different from zero. On the other hand, in the model with systematic allocative inefficiency, the mean value of SCE is 0.046. Finally, the mean value of SCE is reduced to 0.036 when one assumes the firms to be both technically and allocatively efficient. A closer look at the distribution of E_{CQ} shows that although the magnitude of scale economies is smaller in the model with systematic allocative inefficiency, RTS is found to be greater than unity for about 90% of the firms. In summary, we find evidence of economies of scale for most of the firms regardless of which model is used. The distributions of E_{CQ} for both models are almost symmetric, and their spreads are quite similar.

5. Conclusions

In this paper we build on the work of Kumbhakar (1997) and developed an exact maximum likelihood method for his postulated cost system which models technical and allocative inefficiencies in a theoretically consistent way. In essence, this methodology permitted us to estimate jointly the cost function parameters as well as increased cost due to each inefficiency component using cross-sectional data. Our approach solves the long standing Greene problem, which is associated with estimation of a flexible cost system in the presence of both technical and allocative inefficiency. We illustrated the working of our model first with only allocative and then both technical and allocative inefficiencies. The Christensen and Greene (1976) electric utility data are used to estimate the parameters of our models. We found that costs of technical and allocative inefficiency are sensitive to whether one assumes only allocative or both technical and allocative inefficiency. The same conclusion is reached for the estimates of returns to scale. Comparing our results with those that use a much more recent data, we find that efficiency results (both technical and allocative) are qualitatively similar, but the returns to scale are not.

Our second application used cross-sectional data on 518 large U.S. firms taken from the Brynjolfsson and Hitt (2003) data. In this data we found that costs of allocative inefficiency are somewhat sensitive to whether or not allocative inefficiency is assumed to be symmetric. Finally, we found evidence of scale economies in both models, although it is found to be more pronounced in the model without systematic allocative inefficiency.

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Table 1: Only allocative inefficiency

	Without systematic allocative inefficiency			With systematic allocative inefficiency		
	$\ln C^{AL}$	ξ_L	ξ_K	$\ln C^{AL}$	ξ_L	ξ_K
Mean	0.0444	-0.0892	-0.4553	0.1013	0.0486	-0.6030
Median	0.0296	-0.1183	-0.4334	0.0916	0.0489	-0.5844
Std deviation	0.0472	0.4037	0.5598	0.0697	0.3154	0.4154
5%	0.0001	-0.6388	-1.364	0.0039	-0.4070	-1.230
95%	0.1444	0.5448	0.6033	0.2239	0.4194	-0.1526

Table 2: Both technical and allocative inefficiency

	Without systematic allocative inefficiency				With systematic allocative inefficiency			
	u	$\ln C^{AL}$	ξ_L	ξ_K	u	$\ln C^{AL}$	ξ_L	ξ_K
Mean	0.0994	0.0569	-0.0884	-0.5324	0.0907	0.1052	-0.1465	-0.8078
Median	0.0882	0.0380	-0.1184	-0.5359	0.0784	0.0846	-0.2029	-0.7929
Std. dev.	0.0533	0.0582	0.4009	0.5731	0.0504	0.0874	0.4145	0.6331
5%	0.0305	0.0005	-0.7264	-1.472	0.0321	0.0052	-0.7074	-1.8190
95%	0.2057	0.0005	0.5890	0.2178	0.2046	0.2854	0.4915	0.0713

Table 3: Output cost elasticity

	Only allocative inefficiency		Both technical and allocative inefficiency		
	without systematic	with systematic	without systematic	with systematic	Model 5
	Model 1	Model 2	Model 3	Model 4	
Mean	0.9155	0.9595	0.8870	0.8898	0.8866
Median	0.9378	0.9799	0.9078	0.9099	0.9066
Std deviation	0.1003	0.0970	0.0967	0.0954	0.0929
5%	0.6217	0.6699	0.6051	0.6115	0.6146
95%	1.052	1.093	1.018	1.019	1.013

Appendix: Existence and uniqueness of solution to the system of equations

Theorem: If

(A1) the actual share $S_i^a > 0$ for all $i \in \mathbb{Z} = \{1, \dots, J\}$,

(A2) for every $i \in \mathbb{Z}$, we have ($\beta_{ij} \neq 0$ for some $j \in \mathbb{Z}$), where β_{ij} represents the second-order translog coefficients with respect to prices,

(A3) $\sum_{j \in \mathbb{Z}} \beta_{jk} = 0$, and

(A4) $\sum_{j \in \mathbb{Z}} S_j^0 = 1$,

then (i) there exists a solution of ξ in the system of equations in (14), and (ii) the solution is unique.

Proof: Before proving the existence and uniqueness, we note that the condition in (A1) follows from the definition of cost shares, while that in (A2) is necessary for flexibility of the translog cost function. Finally, the conditions (A3) and (A4) follow from homogeneity (of degree one in input prices) of the cost function.

(i) *Existence.*

The system in (14) is of the form $S_j^a \exp(\xi_j) G = S_j^0 + \sum_{k \in \mathbb{Z}} \beta_{jk} \xi_k$, $j \in \mathbb{Z}$. Note that here we are considering the system in which the homogeneity restrictions are not directly imposed by expressing all prices and cost relative to one input price. Let $\kappa_j = S_j^a \exp(\xi_j) G$, $j \in \mathbb{Z}$, so that $\ln \kappa_j = \ln S_j^a + \xi_j + \ln G$, $j \in \mathbb{Z}$. Then the system in (14) can be written in the form

$$\kappa_j = S_j^0 - \sum_{k \in \mathbb{Z}} \beta_{jk} \ln S_k^a + \sum_{k \in \mathbb{Z}} \beta_{jk} \ln \kappa_k - \ln G \sum_{k \in \mathbb{Z}} \beta_{jk} = S_j^0 - \sum_{k \in \mathbb{Z}} \beta_{jk} \ln S_k^a + \sum_{k \in \mathbb{Z}} \beta_{jk} \ln \kappa_k, \quad j \in \mathbb{Z}. \quad (\text{A.1})$$

such that $\sum_{j \in \mathbb{Z}} \kappa_j = 1$. Then $\kappa \in \mathbb{S} \equiv \{\kappa \in \mathbb{R}_+^n \mid \sum_{j \in \mathbb{Z}} \kappa_j = 1\}$, the unit simplex in \mathbb{R}^n . Write the residual from the cost share system (3) as

$$f_j(\kappa) = \kappa_j - S_j^0 + \sum_{k \in \mathbb{Z}} \beta_{jk} \ln S_k^a - \sum_{k \in \mathbb{Z}} \beta_{jk} \ln \kappa_k, \quad j \in \mathbb{Z}. \quad (\text{A.2})$$

Define the mapping

$$g_j(\kappa) = \frac{\kappa_j + f_j(\kappa)^2}{1 + \sum_{k \in \mathbb{Z}} f_k(\kappa)^2}, \quad j \in \mathbb{Z}.$$

Clearly, $g: \mathbb{S} \rightarrow \mathbb{S}$, i.e., it maps \mathbb{S} into itself and is continuous. By Brouwer's fixed point theorem, there exists a $\kappa^* \in \mathbb{S}$ such that $g(\kappa^*) = \kappa^*$, which implies that

$$f_j(\kappa^*)^2 = \kappa_j^* \sum_{k \in \mathbb{Z}} f_k(\kappa^*)^2, \quad j \in \mathbb{Z}.$$

Multiplying both sides by $f_j(\kappa^*)$ and summing over j we obtain

$$\sum_{j \in \mathbb{Z}} f_j(\kappa^*)^3 = \sum_{j \in \mathbb{Z}} \kappa_j^* f_j(\kappa^*) \sum_{k \in \mathbb{Z}} f_k(\kappa^*)^2 .$$

Suppose $f_j(\kappa^*) = 0$ for all $j \neq l$ but $f_l(\kappa^*) \neq 0$. Then the above equation implies $f_l(\kappa^*)^3 = \kappa_l^* f_l(\kappa^*)^3$, which gives $\kappa_l^* = 0$. Write $\kappa_l^* = S_l^a \exp(\xi_l^*)$. By assumption (A1) since $\kappa_l^* = 0$ we have $\xi_l^* = -\infty$. For $\kappa_l^* = 0$ by (A.2) we obtain $f_j(\kappa^*) = \pm\infty$ for some $j \in \mathbb{Z}$ provided assumption (A2) holds. Now, $g_l(\kappa^*) = \kappa_l^* = \frac{\kappa_l^* + f_l(\kappa^*)^2}{1 + f_l(\kappa^*)^2}$. Although $\kappa_l^* = 0$, the limit of the right hand side expression as $\kappa_l^* \rightarrow 0$ (and therefore as $f_l(\kappa^*)^2 \rightarrow +\infty$) is equal to one, a contradiction since $g_l(\kappa)$ is continuous. Therefore, we conclude that at the fixed point κ^* we must have $f_j(\kappa^*) = 0$ for all $j \in \mathbb{Z}$ which means that κ^* represents a solution.

(ii) *Uniqueness.*

Suppose $\{\xi_j\}$ and $\{\varphi_j\}$ are distinct solutions. Therefore they must satisfy

$$S_j^a \exp(\xi_j) G = S_j^0 + \sum_{k \in \mathbb{Z}} \beta_{jk} \xi_k$$

$$S_j^a \exp(\varphi_j) G = S_j^0 + \sum_{k \in \mathbb{Z}} \beta_{jk} \varphi_k , \quad (\text{A.3})$$

for all $j \in \mathbb{Z}$, and they also satisfy the following equality

$$\sum_{j \in \mathbb{Z}} S_j^a \exp(\xi_j) = \sum_{j \in \mathbb{Z}} S_j^a \exp(\varphi_j) = 1. \quad (\text{A.4})$$

Define $\varepsilon_k = \xi_k - \varphi_k$ so that we have $S_j^a \exp(\varphi_j) [\exp(\varepsilon_j - 1)] = \sum_{k \in \mathbb{Z}} \beta_{jk} \varepsilon_k$. Let $\Lambda_j = S_j^a \exp(\varphi_j)$ and notice that

$\exp(\varepsilon_j - 1) \geq \varepsilon_j$ to obtain $\sum_{k \in \mathbb{Z}} \beta_{jk} \varepsilon_k \geq \Lambda_j \varepsilon_j$. This system can be written in the form $[B - \text{diag}(\Lambda)]\varepsilon \geq 0$. Now choose

a vector c such that $c' \varepsilon < 0$, which is always possible provided not all the ε_j s are zero (a fact that we have to accept

since we have assumed the existence of two different solutions). By applying the Farkas' lemma we obtain that since

the above system has a solution, the system $[B - \text{diag}(\Lambda)]\varepsilon = c$, $\varepsilon \geq 0$ must have no solution. Therefore, there exists

no nonnegative vector ε to satisfy $\sum_{k \in \mathbb{Z}} \beta_{jk} \varepsilon_k = \Lambda_j \varepsilon_j + c_j$. We set $\varepsilon_j = w$ for all j so we know that there exists no

nonnegative w to satisfy $c_j = -w \Lambda_j$. We will obtain an obvious contradiction provided we can show that the

inequality $c' \varepsilon < 0$ is satisfied. But $c' \varepsilon = \sum_{j \in \mathbb{Z}} c_j \varepsilon_j = - \sum_{j \in \mathbb{Z}} w^2 \Lambda_j < 0$ since the Λ_j s are positive. The contradiction shows

that the solution must be unique.

Farkas' lemma: The system $Ax = c, x \geq 0$ has no solution if and only if the system $A'y \geq 0, c'y < 0$ has a solution.

Figure 1a. Allocative inefficiency in capital (no technical inefficiency)

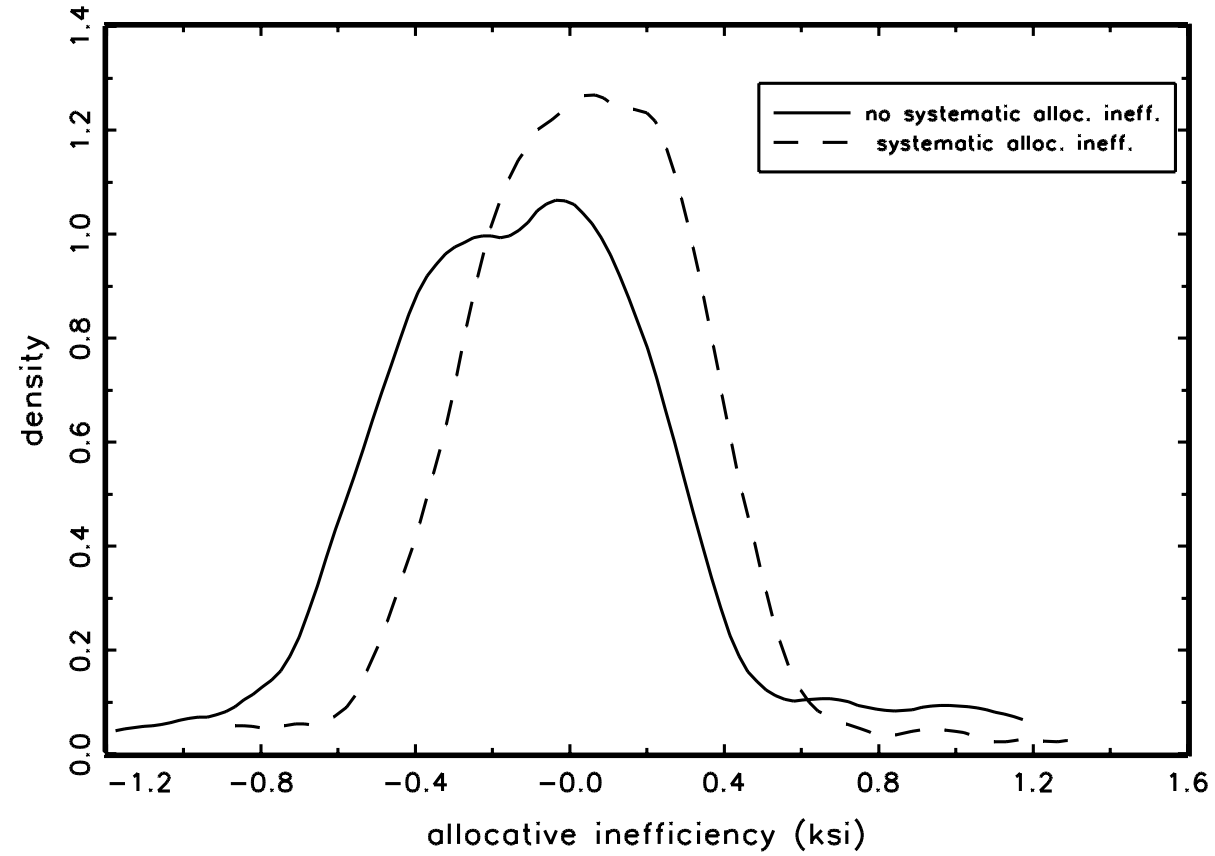


Figure 1b. Allocative inefficiency in labor (without technical inefficiency)

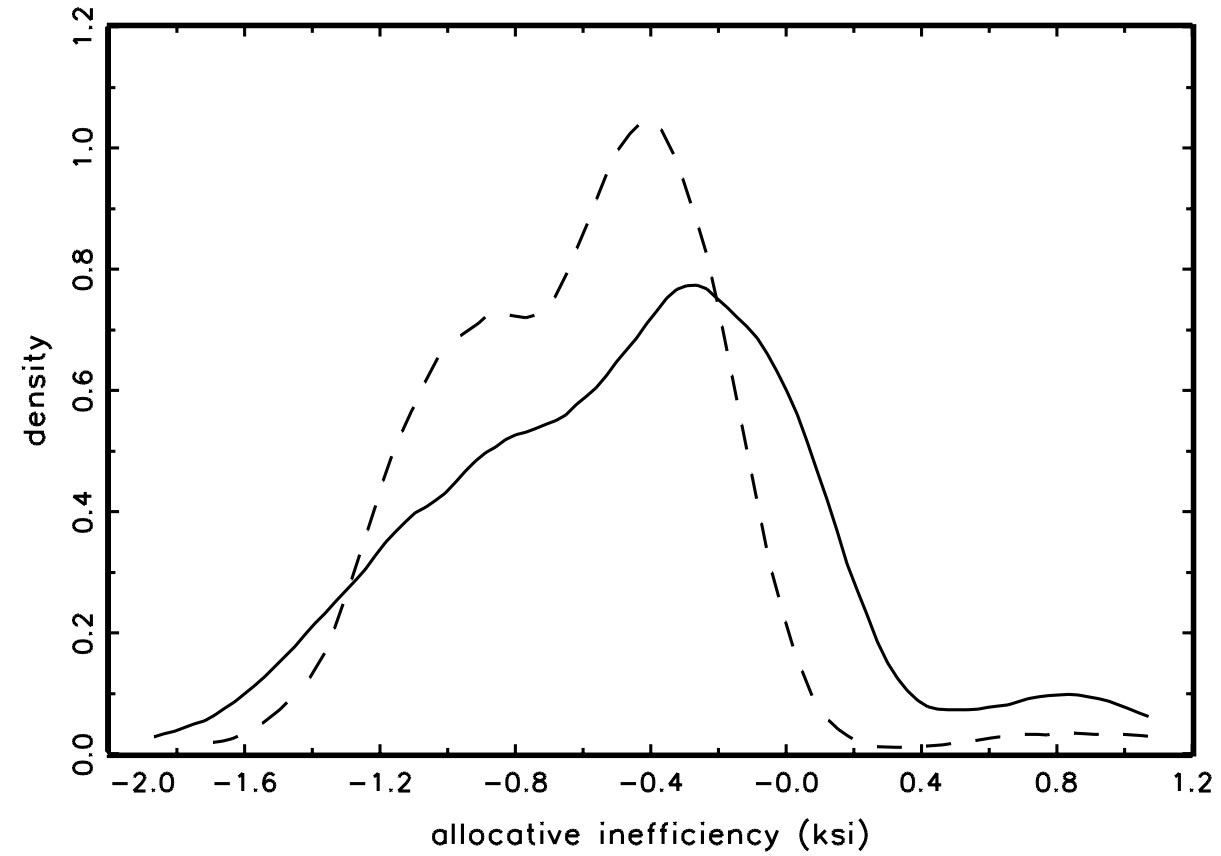


Figure 1c. Allocative inefficiency in capital (with technical inefficiency)

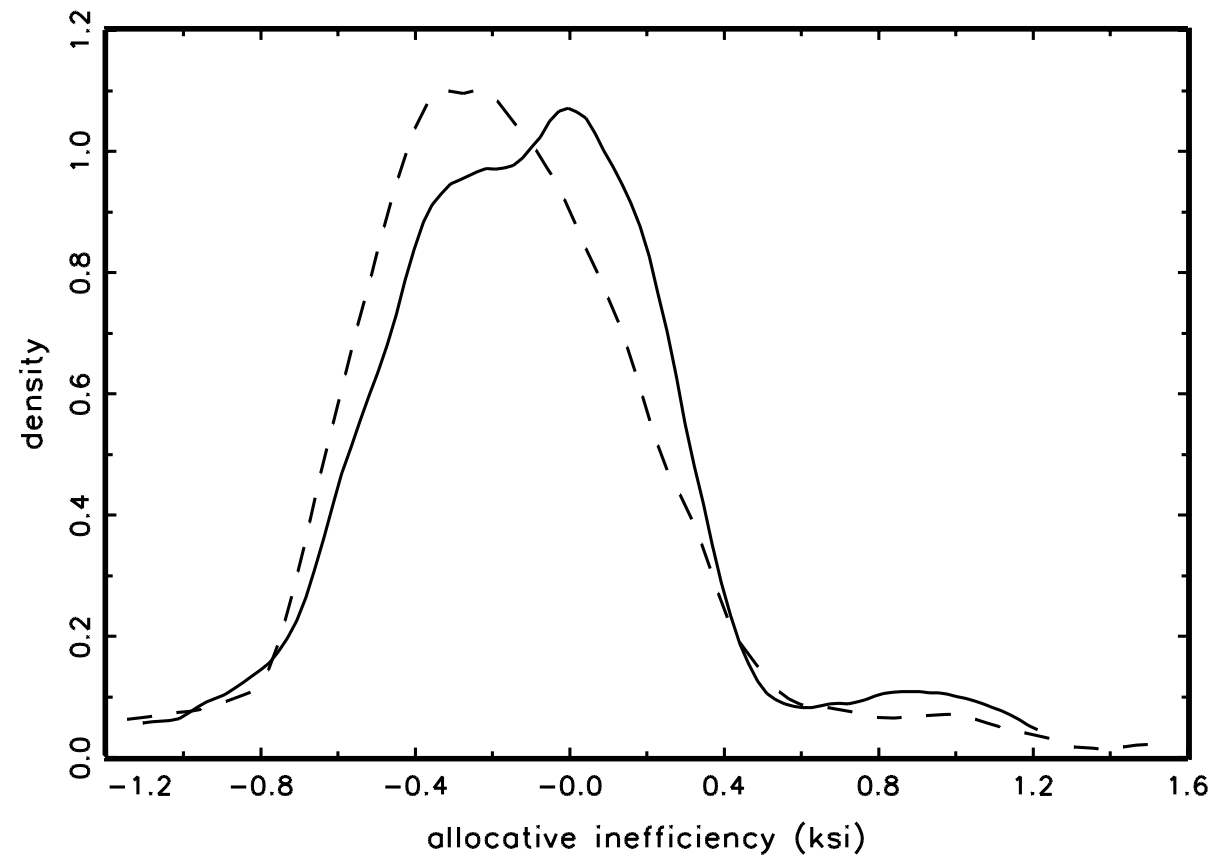


Figure 1d. Allocative inefficiency in labor (with technical inefficiency)

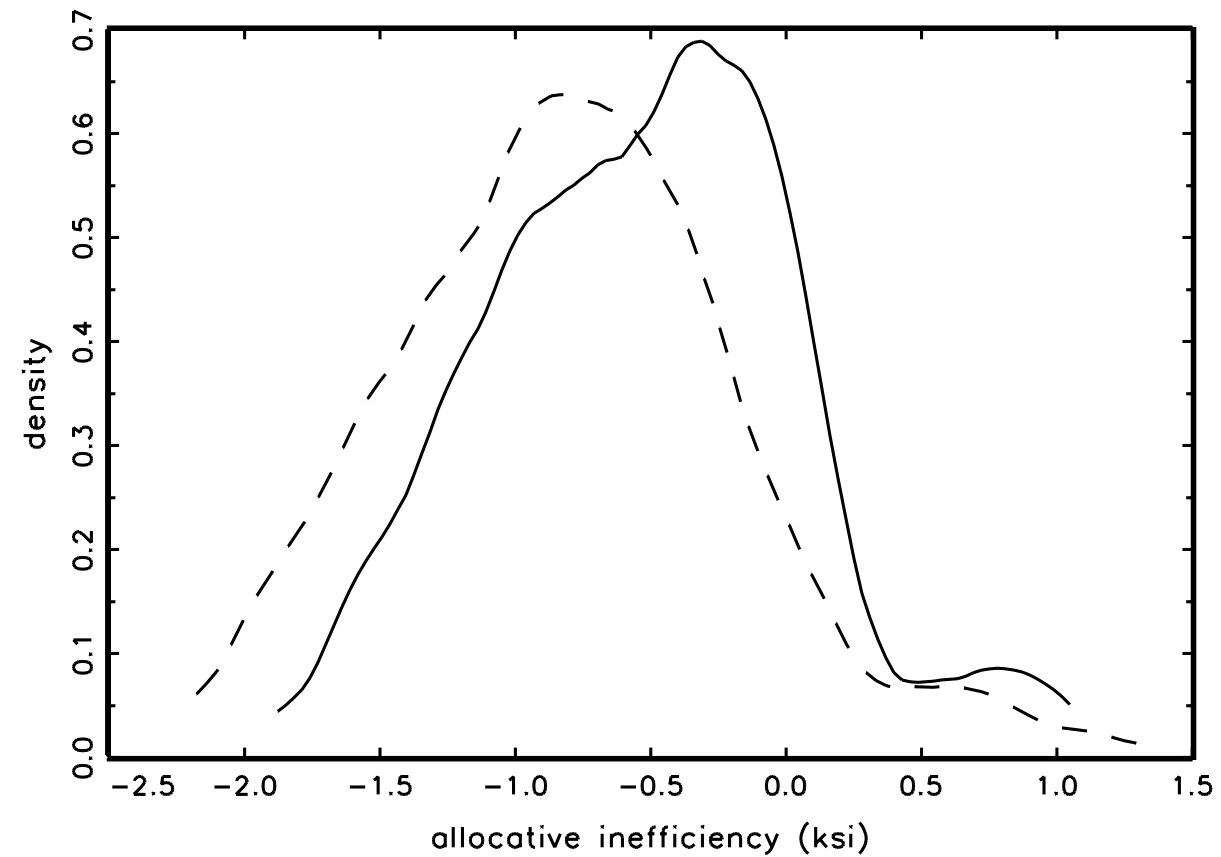


Figure 2. Technical inefficiency

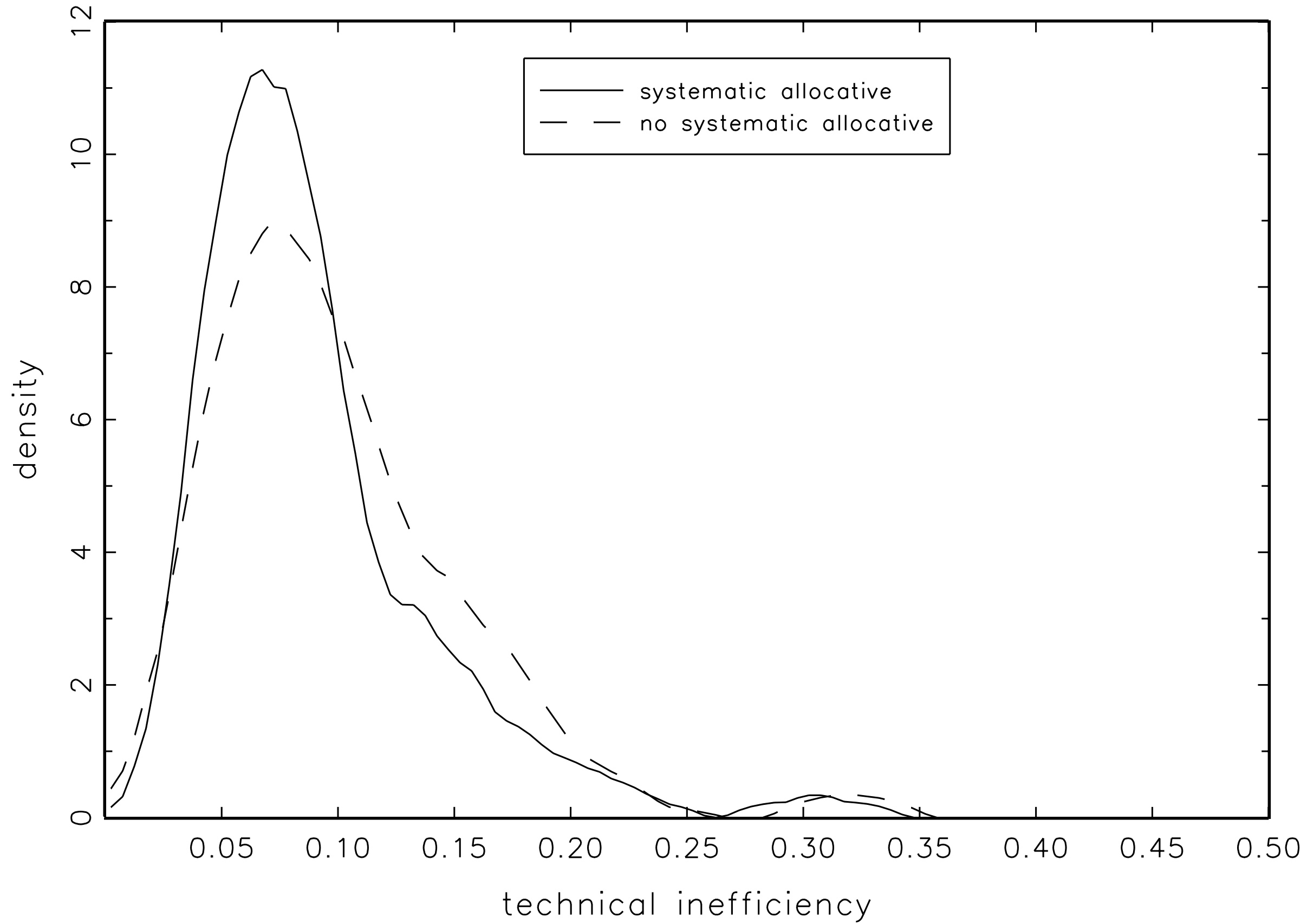


Figure 3. Costs of allocative inefficiency

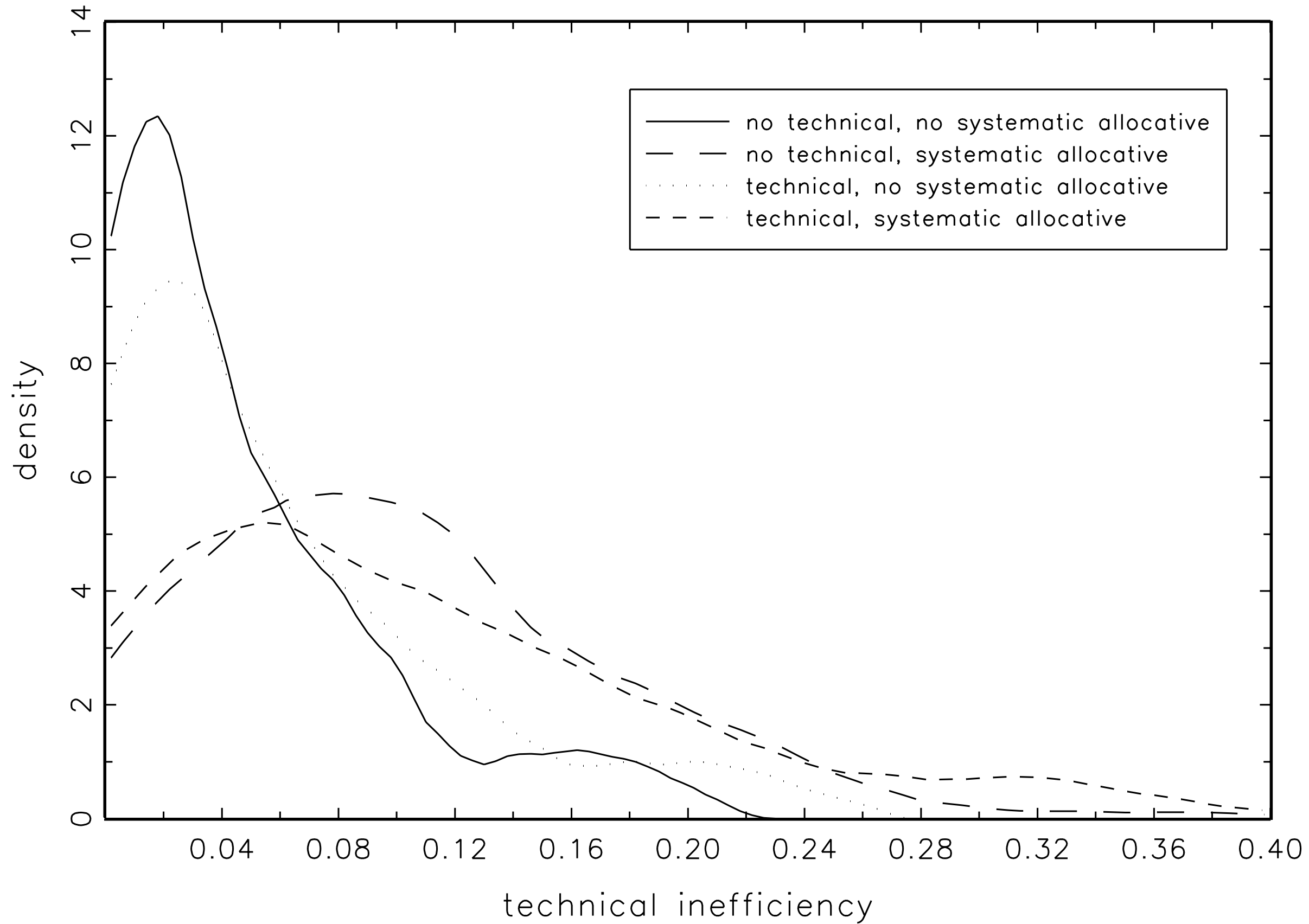


Figure 4. Output cost elasticity

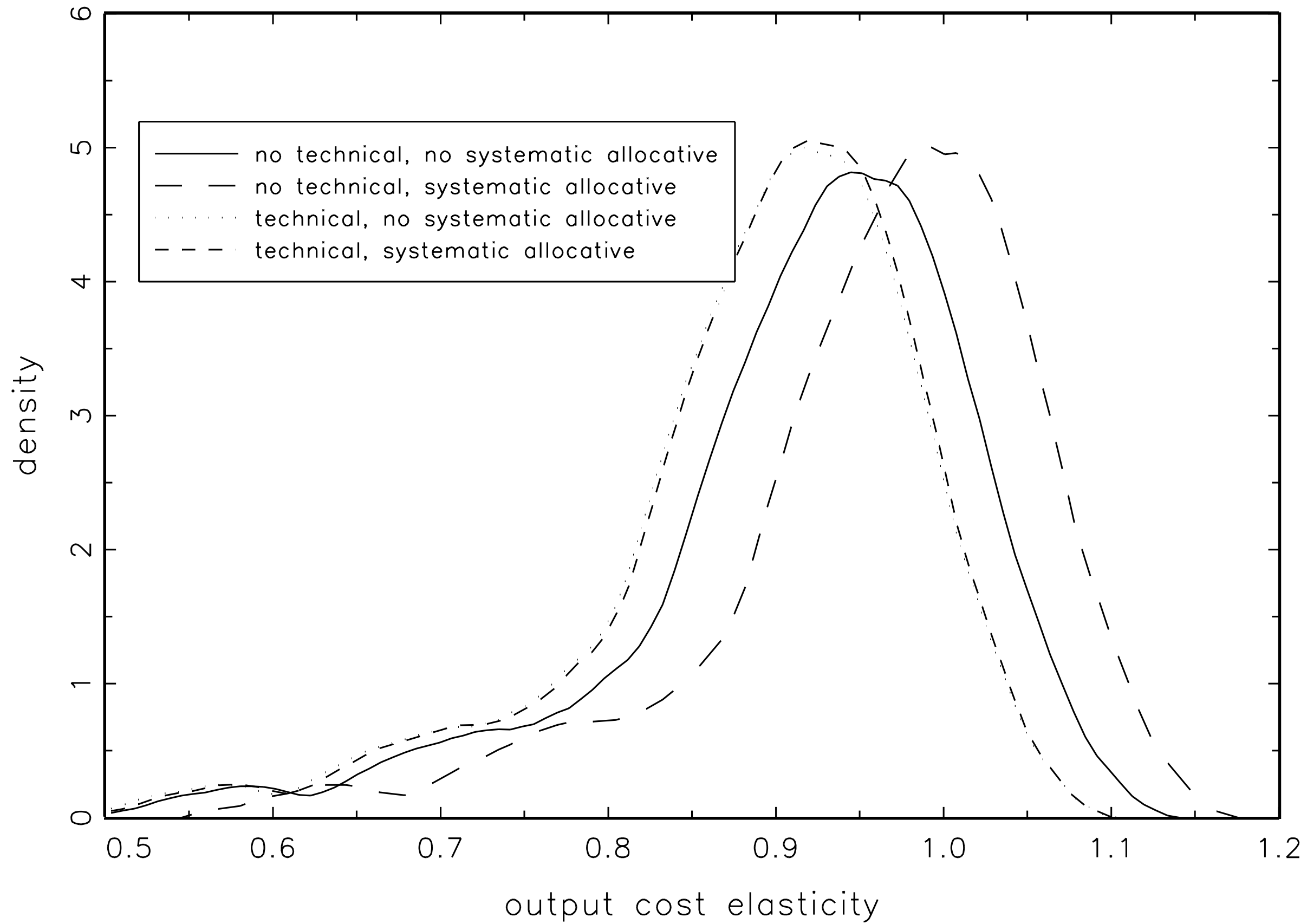


Figure 5a. Allocative inefficiency in information capital

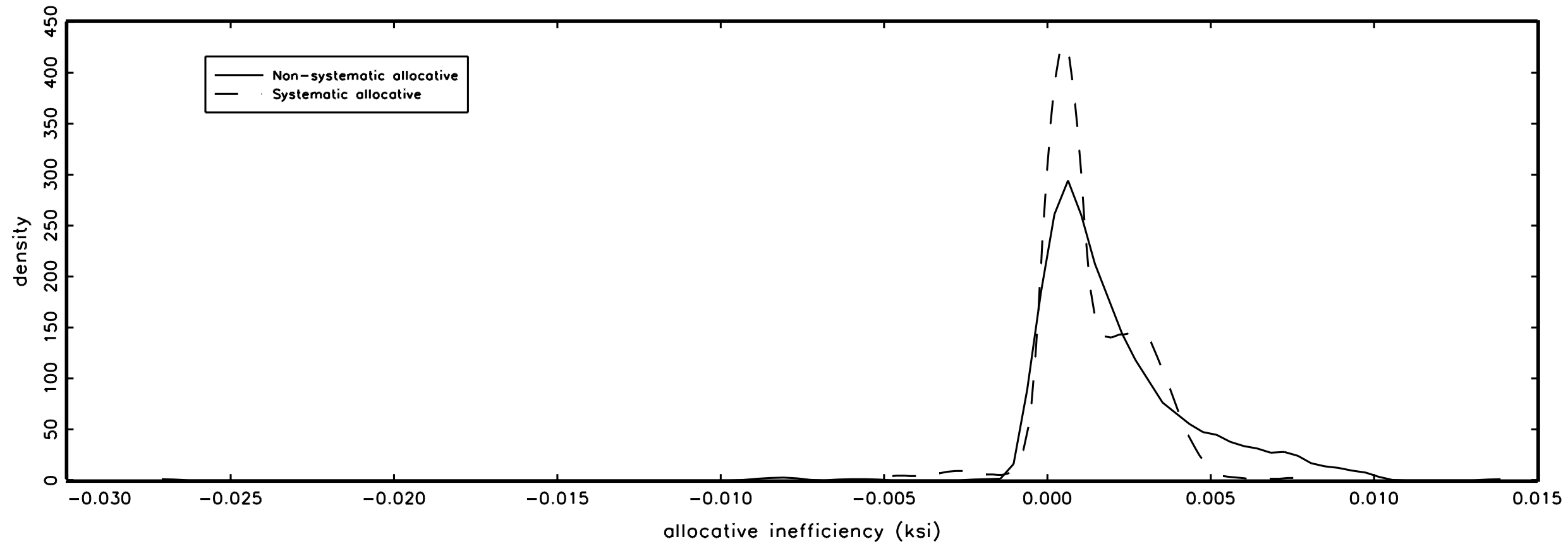


Figure 5b. Allocative inefficiency in labor

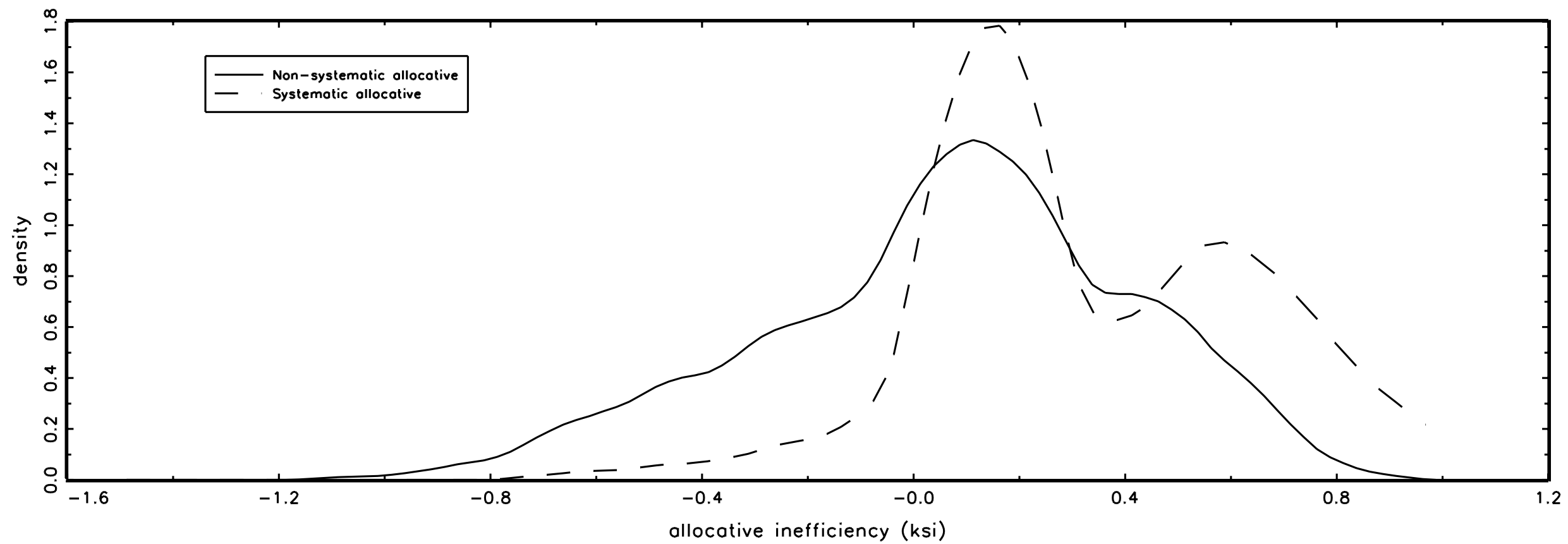


Figure 6. Technical inefficiency

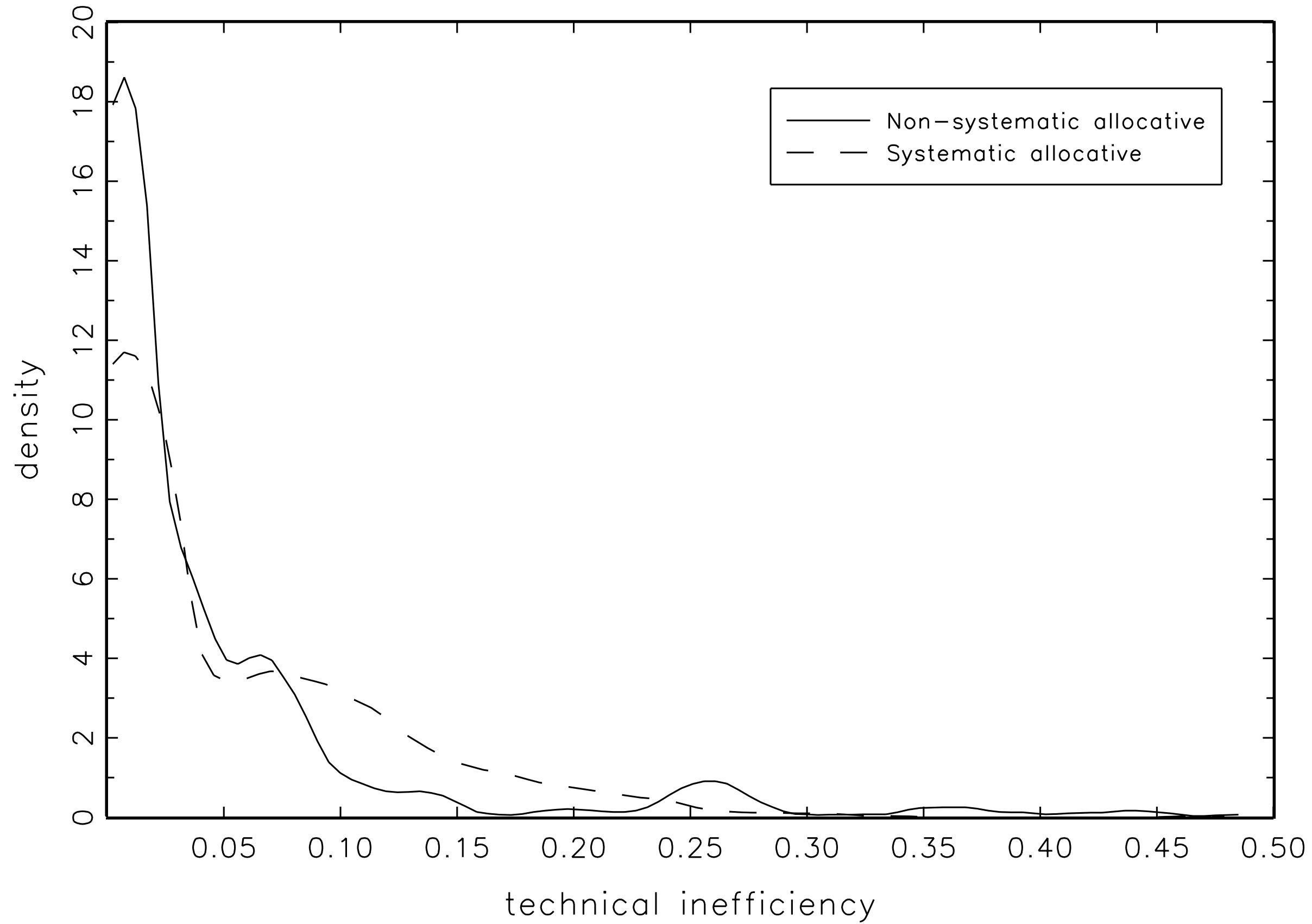


Figure 7. Output cost elasticity

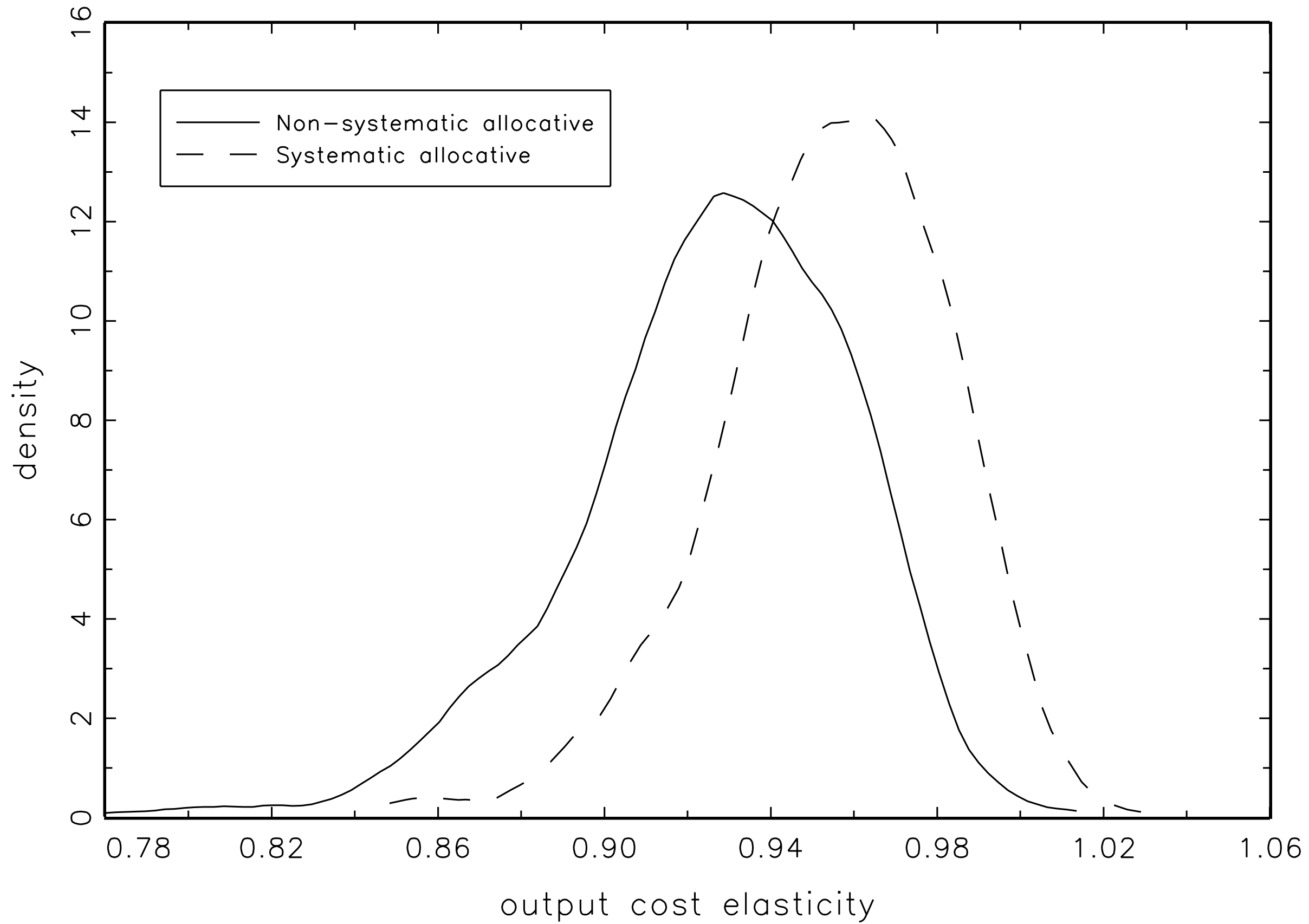


Figure 8. Costs of allocative inefficiency

