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# On the sustainability of collusion in Bertrand supergames with discrete pricing and nonlinear demand\*

Paul R. Zimmerman<sup>†</sup>

## Abstract

In traditional industrial organization models of Bertrand supergames, the critical discount factor governing the sustainability of collusion is independent of key demand and supply parameters. Recent research has demonstrated that these counterintuitive results stem from the assumption that firms can change prices in infinitesimally small increments (*i.e.*, continuously). This note considers the effects of *demand curvature* in the context of a model of collusion where, as in Gallice (2008), Bertrand competitors can deviate only by lowering prices by some small, discrete amount. Two alternative demand specifications that capture the influence of demand curvature are considered. In either case, it is shown that with discrete price changes the critical discount factor is determined by the key demand parameters, including demand curvature. However, the direct effects of increased concavity (or convexity) in market demand on the sustainability of collusion runs in opposite directions across the two models. This discrepancy is shown to arise from the way in which the respective demand curves rotate in response to a change in the demand curvature parameter. The results support the conclusion of earlier research that determining the potential for collusion in homogenous goods industries likely requires careful case-by-case investigation.

*Keywords:* Bertrand supergames; cartels; collusion sustainability; discrete pricing; nonlinear demand

*JEL Classification:* L13, L41

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## 1 Introduction

Collusion is said to be *sustainable* in a discrete-time, infinitely repeated Bertrand-Nash oligopoly game ('supergame') if

$$\frac{1}{N} \Pi_M (1 + \delta + \delta^2 + \dots) \geq \pi_D + \pi_N (\delta + \delta^2 + \dots), \quad (1)$$

where  $N$  denotes the number of perfectly symmetric firms competing in homogenous products,  $\Pi_M$  the per-period collusive (monopoly) profits,  $\pi_D$  the one-period profits realized by a given firm deviating from ('cheating on') the collusive agreement,  $\pi_N$  the per-period Bertrand-Nash equilibrium profits, and  $\delta \in [0,1]$  the rate at which firms discount future profits (which is assumed to be identical across firms). The above expression can be solved for the critical value of the discount rate,  $\delta^*$ , for which collusion is sustainable:

$$\delta \geq \delta^* = 1 - \frac{\Pi_M}{N\pi_D}, \quad (2)$$

since  $\sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta}$  and, by the usual assumption,  $\pi_N = 0$  (*i.e.*, the firms revert to playing the one-period non-cooperative Bertrand-Nash game in each punishment period *ad infinitum*).<sup>1</sup>

The traditional industrial organization literature assumes that the cheating firm can capture all market sales by cutting its price by some infinitesimal amount, *i.e.*, firms can change their prices in a continuous fashion. A counterintuitive implication of this latter assumption is that  $\delta^*$  is unaffected by key demand (*e.g.*, intercept and slope) and supply (*i.e.*, marginal cost) parameters that are known to affect the *profitability* of collusion. However, an important and recent paper by Gallice (2008) assumes homogenous Bertrand competitors can change prices (*i.e.*, deviate from the optimal collusive price) only in *discrete* (or non-infinitesimal) increments.

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<sup>1</sup> This is the so-called 'trigger strategy' developed by Friedman (1971).

Under this arguably more plausible assumption, he shows that  $\delta^*$  is no longer independent of key demand or supply parameters.<sup>2</sup>

This note expands upon Gallice's framework and adds to the literature on collusion sustainability in Bertrand supergames by considering more general demand specifications that capture the effect of demand curvature (in addition to other key demand parameters) on the sustainability of collusion. Considering nonlinear demand curves is warranted since many (empirical) market demand curves are unlikely to be strictly linear, and in fact, some economists have opined that nonlinear demand curves are likely to be the norm rather than the exception from a theoretical perspective.<sup>3</sup> Furthermore, it has been shown that the derivation of demand curves that are linear in prices requires underlying utility functions (indifference maps) that are complex, *e.g.*, require quadratic orders of the consumption arguments (such as in Singh & Vives 1984), or that contain at least a single point of discontinuity (Alperovich & Weksler 1996).

It is shown that the curvature of market demand can impart sizeable effects on the critical discount factor both directly and indirectly (*i.e.*, through its relation to changes in other demand parameters), although the nature of the relationships depends on the particular specification of the market demand function—two of which are considered in the analysis. For example, in the first specification an increase in market demand (as affected by a positive change in the intercept parameter) can have no impact on the sustainability of collusion for 'sufficiently convex' demand. It is demonstrated (numerically) that collusion can become easier to sustain as the demand curve becomes more concave. However, the opposite result holds under the second

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<sup>2</sup> Gallice considers two demand specifications, linear and constant-elasticity. For the latter case he shows (numerically) that there is a range over which  $\delta^*$  is initially decreasing in the elasticity of demand but then jumps discontinuously to a point where collusion is never sustainable. Earlier work by Collie (2004), in the context of a Cournot model, also shows that collusion becomes easier to sustain the more elastic is market demand. The ensuing discussion relates to the results from Gallice's linear demand model.

<sup>3</sup> See, *e.g.*, Walters (1980), Formby *et al.* (1982), and Coughlin (1984). This body of literature focuses on the strong potential for and implications of upward-sloping marginal revenue functions that may arise out of convex demand curves. According to Formby *et al.* (1982, p.306): "the possibility of an upward sloping marginal revenue function can *never* safely be assumed away. Our theoretical analysis lends support to A.A. Walter's [1980] conjecture that the demand conditions [*i.e.*, convexity] leading to upward sloping marginal revenue may indeed be pervasive." (emphasis in original)

demand specification. Thus, exactly *how* nonlinearity ‘influences’ the demand specification can lead to disparate and, in some instances, somewhat counterintuitive implications for the sustainability of collusion.

The remainder of the paper proceeds as follows. Section 2 presents and discusses the two demand specifications considered in the analysis. It is shown that the concavity of demand can either (directly) increase or decrease the sustainability of collusion depending on the specific parameterization of the demand function. The differences in the results obtained from the two specifications are shown to stem from the effect that changes in the curvature parameters have on the rotation of the respective demand schedules. Section 3 summarizes and offers concluding remarks.

## 2 The model

### 2.1 Demand

Two specifications of a (potentially) nonlinear market demand curve are examined. The first is similar to that considered, *e.g.*, by Benson (1980) and Lambertini (1996):

$$Q(p) = \alpha - \beta p^\chi. \quad (3)$$

[Model I]

The second is similar in form to that used by Tyagi (1999) and is given by

$$Q(p) = (\alpha - \beta p)^\psi. \quad (4)$$

[Model II]

The parameters  $\alpha$  and  $\beta$  in either specification denote the strictly positive intercept and slope parameters, respectively, and  $Q(p) > 0$  where  $p > 0$  denotes price.

The parameters  $\chi > 0$  and  $\psi > 0$  are the respective demand curvature parameters.

Notice that in Model I only market price is raised to the power of the curvature parameter, while

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<sup>4</sup> Benson (1980), Lambertini (1996), and Tyagi (1999) exclusively or primarily consider ‘direct’ demand functions wherein price is a function of quantity, as opposed to the ‘inverse’ demand functions considered here. As such, the former papers model firms competing in *quantities* as opposed to *prices*.

in Model II the curvature parameter applies to *both* the market price and the key demand parameters.

The curvature parameters can be varied to consider a wide range of possible downward-sloping demand functions. Model I is linear for  $\chi = 1$ , while for  $\chi > 1$  ( $\chi < 1$ ) it is concave (convex). Model II is linear for  $\psi = 1$ , while for  $\psi > 1$  ( $\psi < 1$ ) it is convex (concave).

## 2.2 Supply

Unlike the cases Gallice (2008) considers, with the demand specification given in equations (3) and (4) it must be assumed that marginal costs are zero in order to derive closed-form solutions. At first blush the assumption of zero marginal costs may appear somewhat unappealing; however, it is a common one made in the literature even when not analytically required. *See, e.g.,* Deneckere (1983) and Rothschild (1992). Indeed, since marginal costs are assumed to be constant and symmetric across firms in the model, this treatment is analytically equivalent to simply normalizing marginal costs to zero. Alternatively, the intercept term can simply be conceptualized as being measured net of marginal costs (Deneckere 1983).

## 2.2 Model I

First consider the demand curve given by Eq. (3). Individual demand for firm  $i$  is

$$q_i(p) = \begin{cases} \frac{\alpha - \beta p_i^\chi}{1 + \sum_j (1_{\{p_j = p_i\}})} & \text{if } p_i \leq p_j \text{ for any } j \neq i \\ 0 & \text{otherwise} \end{cases}$$

Under the collusive equilibrium the firms cooperatively set the market price that maximizes total industry profits. Since the firms are symmetric this implies

$$p_1 = p_2 = \dots = p_N = p.$$

The joint (collusive) profit-maximization problem is

$$\begin{aligned} \max_p \Pi(p) &= pQ(p) \\ &= \alpha p - \beta p^{\chi+1}. \end{aligned}$$

The first-order condition (“FOC”) associated with the above problem is

$$\frac{\partial \Pi}{\partial p} = \alpha - \beta(\chi + 1)p^\chi = 0.$$

Solving the FOC for the collusive price yields

$$p_M = \left[ \frac{\alpha}{\beta(\chi + 1)} \right]^{1/\chi},$$

where  $M$  indexes collusive items or outcomes that are attained at the monopoly equilibrium levels. Each cartel member’s share of the per-period collusive profits equals

$$\begin{aligned} \frac{\Pi_M}{N} &= \frac{p_M Q(p_M)}{N} \\ &= \left( \frac{\chi}{1 + \chi} \right) \frac{\alpha}{N} p_M. \end{aligned}$$

Again, if one firm chooses to deviate from the collusive agreement, then the other non-deviating firms react by playing the Friedman (1971) non-cooperative (‘grim trigger’) strategy in all subsequent periods (so that each firm, including the deviator, earns  $\pi_N = 0$  in each period of the infinite punishment phase). Following Gallice, the cheating firm’s deviation from the collusive price (its price undercut) is given by the small and *positive* but *discrete* increment  $\Delta > 0$ ,<sup>5</sup> which in turn implies  $\pi_D < \Pi_M$ . The firm that deviates from the cartel therefore charges price  $p_D = p_M - \Delta$  and, for a single period, captures all market sales. The quantity set by the deviating firm at  $p_D$  is given by  $q_D = \alpha - \beta p_D^\chi$ , and the firm earns a one-period profit of  $\pi_D = \alpha p_D - \beta p_D^{\chi+1} > \Pi_M/N$ .

Substituting  $\Pi_M$  and  $\pi_D$  into the incentive constraint given by equation (1) and expressing it in terms of  $p_M$  and  $p_D$  indicates that collusion is sustainable if

$$\delta \geq \delta^* = 1 - \frac{1}{N} \frac{p_M(\beta p_M^\chi - \alpha)}{p_D(\beta p_D^\chi - \alpha)}. \quad (5)$$

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<sup>5</sup> As in Gallice (2008), it is assumed that the deviation price is in the range of 1-2 percent beneath the collusive price.

The above expression indicates that, as in Gallice's model, under the assumption of discrete price deviations  $\delta^*$  is a function of the intercept and slope parameters of the demand function.

Furthermore,  $\delta^*$  is also determined by the *curvature* of the market demand function since  $p_M$  and  $p_D$  are implicitly defined by  $\chi$ .<sup>6</sup>

Examination of equation (5) leads to the following intuitive results. *First*, if demand is linear ( $\chi = 1$ ) and prices change discretely ( $\Delta > 0$ ), then the critical discount factor pertaining to Gallice's model obtains (*i.e.*, where  $c = 0$  in the latter):

$$\delta^* \Big|_{\chi=1, \Delta>0} = 1 - \frac{\alpha^2}{N(\alpha^2 - 4\beta^2\Delta^2)}.$$

*Second*, if demand is linear and prices change continuously (regardless of the treatment of marginal costs), then the ordinary case in which the critical discount factor is independent of any demand factors is obtained:

$$\delta^* \Big|_{\chi=1, \Delta \approx 0} = 1 - \frac{\alpha^2}{N\alpha^2} = \frac{N-1}{N}.$$

That is, the critical discount factor is *only* a function of  $N$ , which indicates that as the number of firms in the market increases the sustainability of collusion decreases. The key parameters of the market demand function play no role in determining the sustainability of collusion in this case.

Now consider how  $\delta^*$ , as given by equation (5), changes with a perturbation in the intercept parameter of the market demand curve:

$$\frac{\partial \delta^*}{\partial \alpha} = \frac{p_M \Delta (\alpha - \beta p_M^\chi) (\alpha - \beta(1 + \chi) p_D^\chi)}{N \alpha \chi (-p_D)^2 (q_D)^2} \geq 0. \quad (6)$$

The sign of the marginal effect depends on the sign of  $z_D = \alpha - \beta(1 + \chi) p_D^\chi$ , which is not immediately apparent.<sup>7</sup> However, some simple numerical examples are sufficient to illustrate

<sup>6</sup> Notice that these prices in equation (5) are also raised to the power of the curvature parameter.

<sup>7</sup> Note that  $z_D$  is determined by  $\chi$  through its scaling of  $\beta$  as well as through its effect on  $p_D$ , which itself is raised to the power of  $\chi$ .



that it is non-negative. As such, an outward shift of the demand curve will tend to make collusion harder to sustain since all firms will have a greater incentive to deviate and capture the entirety of the larger market.

Consider  $\{\alpha = 100; \beta = 50; \Delta = 0.01; N = 10\}$  and  $\{\chi = 0.01\}$  (which corresponds to a highly convex market demand curve). Then  $z_D = 0$  and  $\partial\delta^*/\partial\alpha = 0$  — *i.e.*, an increase in the size of the market has no effect on the sustainability of collusion. Keeping all parameter values the same but setting  $\chi = 1000$  (which corresponds to a highly concave demand curve) results in  $z_D = 100$ ; clearly  $\partial\delta^*/\partial\alpha > 0$  in this case.<sup>8</sup> Other examples can be shown to demonstrate that, *ceteris paribus*, an outward shift of the demand curve will have a greater (smaller) effect on the critical discount factor the more concave (convex) the market demand curve.

Turning to the slope parameter  $\beta$ , it can be shown that

$$\frac{\partial\delta^*}{\partial\beta} = -\frac{\alpha}{\beta} \frac{\partial\delta^*}{\partial\alpha} \leq 0. \quad (7)$$

This result also mirrors Gallice (2008).<sup>9</sup> All else equal, an increase in the slope coefficient of the market demand function makes collusion easier to sustain. The implications of how the curvature of demand will affect the extent to which a change in  $\beta$  will affect the critical discount factor are straightforward given that the sign of equation (7) is simply the negative of equation (6) times  $\alpha/\beta > 0$ . As the demand curve becomes relatively more concave a marginal change in  $\beta$  results in a proportionally smaller (negative) change in  $\delta^*$ .

Gallice finds that the sustainability of collusion is increasing in  $\Delta$  — a lower deviation price effectively lowers the opportunity cost of cheating. This same result and the intuition

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<sup>8</sup> Recognize that  $\lim_{\chi \rightarrow 0} z_D = 0$  and  $\lim_{\chi \rightarrow \infty} z_D = \alpha$ , which are the fundamental results that underscore the relationship between the curvature of the market demand function and the impact of a change in demand on the critical discount factor.

<sup>9</sup> Gallice shows that changes in the slope parameter affect the sustainability of collusion through their impact on the intercept term of the *inverse* market demand curve.

behind it are also borne out in the present context. The effect of a marginal change in the price tick on the critical discount factor is given by

$$\frac{\partial \delta^*}{\partial \Delta} = -\frac{\alpha \chi}{\Delta} \frac{\partial \delta^*}{\partial \alpha} \leq 0. \quad (8)$$

Thus, the greater the spread between the collusive and deviation prices the easier it is to sustain collusion. Furthermore, the magnitude of the (negative) marginal effect rises as the market demand curve becomes more concave. This result holds since a higher  $\chi$  raises both the parenthetical expression given in equation (8) as well as  $\partial \delta^* / \partial \alpha$ , as discussed above.

It is also possible to consider the *direct* effect of a change in demand curvature in Model I on the critical discount factor under the assumption of discrete price changes, which is given by

$$\frac{\partial \delta^*}{\partial \chi} = \frac{\left( p_M \left\{ \beta p_M^\chi \pi_D \ln(p_M) + \frac{1}{\chi^2(1+\chi)} [-\alpha \chi (\alpha \Delta - \beta p_M^\chi) (\Delta(1+\chi) - \chi p_M)] \right. \right.}{N(-p_D)^2 (q_D)^2}, \quad (9)$$

$$\left. \left. + (1+\chi) \left\{ (-\alpha \beta \chi p_M) (p_M^\chi - p_D^\chi) + \Delta (-z_M) q_D \right\} \ln \left( \frac{\alpha}{\beta(1+\chi)} \right) \right. \right.$$

$$\left. \left. + \beta \chi p_D^\chi (\alpha \chi p_M^\chi + \Delta z_M - \chi(1+\chi) q_M p_D \ln(p_D)) \right\} \right)$$

where  $z_M = \alpha - \beta(1+\chi)p_M^\chi$  (which, like  $z_D$ , is strictly non-negative). The sign of equation (9) is not apparent due to the various (potentially) positive and negative terms that interrelate additively or multiplicatively in the numerator.

However, a numerical example is sufficient to demonstrate that the sign of equation (9) is negative, which implies that as demand becomes increasingly concave the sustainability of collusion increases (albeit at a decreasing rate). Again, consider the parameter values  $\{\alpha = 100; \beta = 50; \Delta = 0.01; N = 10\}$ . Figures 1.a and 1.b graph  $\delta^*$  and  $\partial \delta^* / \partial \chi$ , respectively, at these values while allowing  $\chi$  to vary over the range  $[0.0001, 10\,000]$ .

Figure 1.a clearly shows that the slope of  $\delta^*$  monotonically decreases over the relevant range of  $\chi$ . Figure 1.b shows that the rate of change in the partial derivative falls the higher the

value of the slope parameter, approaching a horizontal asymptote at  $\partial\delta^*/\partial\chi = 0$ . Therefore, while a marginal increase in  $\chi$  unambiguously make collusion *easier* to sustain, the magnitude of the effect is smaller the higher the (initial) concavity of the market demand curve.

Arguably, these results may appear somewhat counterintuitive. For instance, Hayrapetyan *et al.* (2007) note that: “A concave demand curve is applicable for modeling demand for a service with a comparable alternative (at a high enough price all users will switch to the alternate service).” One might therefore expect, *a priori*, that the critical discount factor would be *increasing* in the concavity of demand (*i.e.*, hinder the sustainability of collusion) since consumers could more readily substitute towards alternative goods/services at sufficiently high prices. Indeed, this is precisely the result obtained by Lambertini (1996, p. 332) with respect to a homogenous *Cournot* model.<sup>10</sup>

However, in the present context it is straightforward to demonstrate why the same intuition does not apply. Define  $\bar{\pi}(\chi) = \pi_D - (\pi_M / N) > 0$ , or the difference between the deviation profits and the collusive profits that the cheating firm would have otherwise obtained in each one-shot (or single-period) game over the range  $\chi \in [0.0001, 10\,000]$ . Figure 2 presents the graph of this function for the above parameter values. Notice that  $\partial\bar{\pi} / \partial\chi > 0$  (indicating that deviation profits are monotonically increasing in demand curvature) while  $\partial\bar{\pi} / \partial\chi \rightarrow 0$  as  $\chi \rightarrow \infty$ . Thus, the more concave the market demand curve of Model I the lower the *relative* gain from cheating, which implies that collusion will be easier to sustain as  $\chi$  increases.

### 2.3 Model II

The collusive profit-maximization problem for the demand specification given by equation (4) is

$$\max_p \Pi(p) = p(\alpha - \beta p)^\psi,$$

with the FOC:

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<sup>10</sup> Lambertini (1996) takes the deviation *quantity* set by the cheating firm as set under the assumption that each of the non-deviating firms continues to produce its joint profit-maximizing output level.

$$\frac{\partial \Pi(p)}{\partial p} = Q(p) - \beta p(\alpha - \beta p)^{-1+\psi} = 0.$$

The equilibrium collusive and deviation prices are given by

$$p_M = \frac{\alpha}{\beta(1 + \psi)}$$

$$p_D = \frac{\alpha - \beta\Delta(1 + \psi)}{\beta(1 + \psi)},$$

respectively, with the associated equilibrium profits

$$\Pi_M = \frac{\alpha \left( \frac{\alpha\psi}{1 + \psi} \right)^\psi}{\beta(1 + \psi)},$$

$$\Pi_D = \frac{\left( \beta\Delta + \frac{\alpha\psi}{1 + \psi} \right)^\psi (\alpha - \beta\Delta(1 + \psi))}{\beta(1 + \psi)}.$$

Plugging the latter two expressions into equation (2) gives the collusion sustainability condition:

$$\delta \geq \delta^* = 1 - \frac{\alpha \left( \frac{\alpha\psi}{1 + \psi} \right)^\psi \left( \beta\Delta + \frac{\alpha\psi}{1 + \psi} \right)^{-\psi}}{N(\alpha - \beta\Delta(1 + \psi))} \quad (10)$$

Equation (10) also indicates that under the assumption of discrete price changes the critical discount factor is a function of key demand parameters, including the curvature of demand.

Again, the sustainability of collusion is decreasing in the size of the market under this framework:

$$\frac{\partial \delta^*}{\partial \alpha} = \frac{\beta^2 \Delta^2 \left( \frac{\alpha\psi}{1 + \psi} \right) (1 + \psi)^2 (\beta\Delta + \frac{\alpha\psi}{1 + \psi})^{-(1+\psi)}}{N(\alpha - \beta\Delta(1 + \psi))^2} > 0.^{11} \quad (11)$$

The comparative statics results with respect to the slope and price-tick parameters are qualitatively similar to those found in Model I:

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<sup>11</sup> Note that for  $\Pi_D > \Pi_M / N$  it must be the case that  $\alpha - \beta\Delta(1 + \psi) > 0$ , and as such, equations (11)-(12) hold as a strict inequalities.

$$\begin{aligned}\frac{\partial \delta^*}{\partial \beta} &= -\frac{\alpha}{\beta} \frac{\partial \delta^*}{\partial \alpha} < 0 \\ \frac{\partial \delta^*}{\partial \Delta} &= -\alpha \Delta \frac{\partial \delta^*}{\partial \alpha} < 0.\end{aligned}\tag{12}$$

Now consider the direct effect of demand curvature, which is given by

$$\frac{\partial \delta^*}{\partial \chi} = -\frac{\left( \left( \frac{\alpha\psi}{1+\psi} \right) \left( \beta\Delta + \frac{\alpha\psi}{1+\psi} \right)^{-(1+\psi)} (\alpha\beta(1+\psi)) \right.}{N\psi(\alpha - \beta\Delta(1+\psi))^2} \left. + (\alpha - \beta\Delta(1+\psi)) \left( \ln\left( \frac{\alpha\psi}{1+\psi} \right) - \ln\left( \beta\Delta + \frac{\alpha\psi}{1+\psi} \right) \right) \right).\tag{13}$$

Again, numerical simulations must be used in order to gain insight into the sign of the numerator in equation (13). Figure 3 graphs equation (13) for the parameter values  $\{ \alpha = 100 ; \beta = 50 ; N = 10 ; \Delta = 0.01 \}$  where  $\psi$  varies over the interval  $[0.01, 100]$ . In this case, as the market demand curve becomes relatively more convex (concave), the critical discount factor diminishes (rises)—the exact *opposite* of the result obtained from Model I. Thus, Model II generates the seemingly more ‘intuitive’ result that—to the extent concavity reflects substitution options (*i.e.*, at least at the higher range of market price)—collusion is more difficult to sustain the more easily consumers can switch to alternatives besides the cartel’s output.

Given that both Models I and II appear to be reasonable specifications in which to capture the influence of demand curvature (and, again, both have been used in previous studies), the question remains why the corresponding results are so similar with respect to the (indirect) effects of perturbations in the demand parameters, but diametrically in opposition with respect to the (direct) effect of demand curvature. The economic intuition for the latter stems from the way in which the respective (direct) demand specifications rotate about their intercepts in  $(p, q)$  space in response to a perturbation in the relevant curvature parameter.

For Model I, a *ceteris paribus* increase in the curvature parameter  $\chi$  results in the market demand curve rotating inwards about its horizontal (quantity) intercept (*i.e.*,  $\alpha$ ). That is, as this demand curve becomes more concave it also rotates inwards, which (all else equal) results in a

lower deviation price relative to the collusive price at any *given* level of *output*, hence making collusion easier to sustain.<sup>12</sup> On the other hand, in Model II an increase in  $\psi$ , which makes the market demand curve relatively more convex, results an upward rotation of the curve about its vertical (price) intercept. Thus, all else equal, at any *given* level of *price*, greater convexity of demand is associated with a higher level of output and, correspondingly, a lower price that a firm could obtain from deviating. Therefore, increasing convexity in this case makes collusion easier to sustain; conversely, more concave demand makes collusion more difficult to sustain.<sup>13</sup>

### 3 Summary and concluding remarks

This paper considers the effect of demand curvature on the sustainability of collusion in Bertrand supergames in which firms can only deviate from the optimum cartel price by undercutting in small, discrete increments. Depending on how the market demand curve is parameterized, a change in demand curvature can have very different direct implications for the sustainability of collusion. The empirical literature on cartel duration generally finds that an increase in market demand tends to hinder collusion (Levenstein & Suslow 2006), although this need not be the case under the demand specification in Model I and sufficiently convex demand. Overall, the results support Tyagi's (1999, p. 298) conclusion (drawn in the context of a repeated quantity-setting game) that: "[W]ithout doing a case-by-case analysis, it is difficult to say whether firms in more homogenous product markets find it easier or harder to tacitly collude."

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<sup>12</sup> This inward rotation of the Model I market demand curve is what also drives the concavity of  $\tilde{\pi}$ .

<sup>13</sup> It is important to keep in mind that these results arise *because of the assumption of discrete price changes* in the pricing supergame, which it turn results in the critical discount factor being a function of the respective curvature parameters.

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FIGURE 1.A: THE EFFECT OF  $\chi$  ON  $\delta^*$  FOR  $\alpha = 100$ ,  $\beta = 50$ ,  $N = 10$ , AND  $\Delta = 0.01$

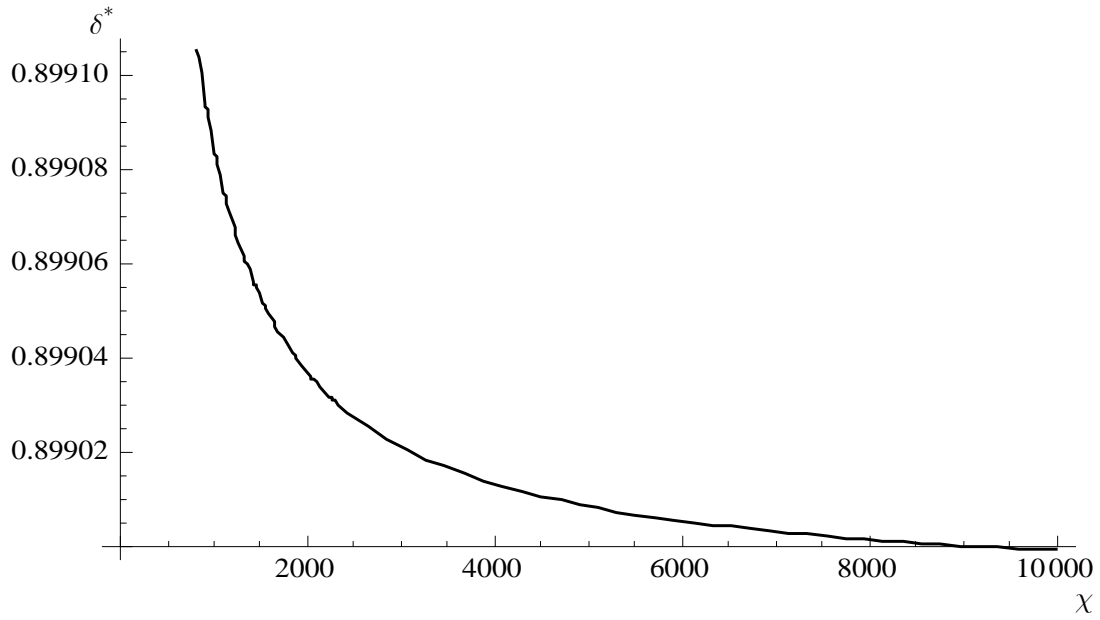


FIGURE 1.B: THE EFFECT OF  $\chi$  ON  $\partial\delta^*/\partial\chi$  FOR  $\alpha = 100$ ,  $\beta = 50$ ,  $N = 10$ , AND  $\Delta = 0.01$

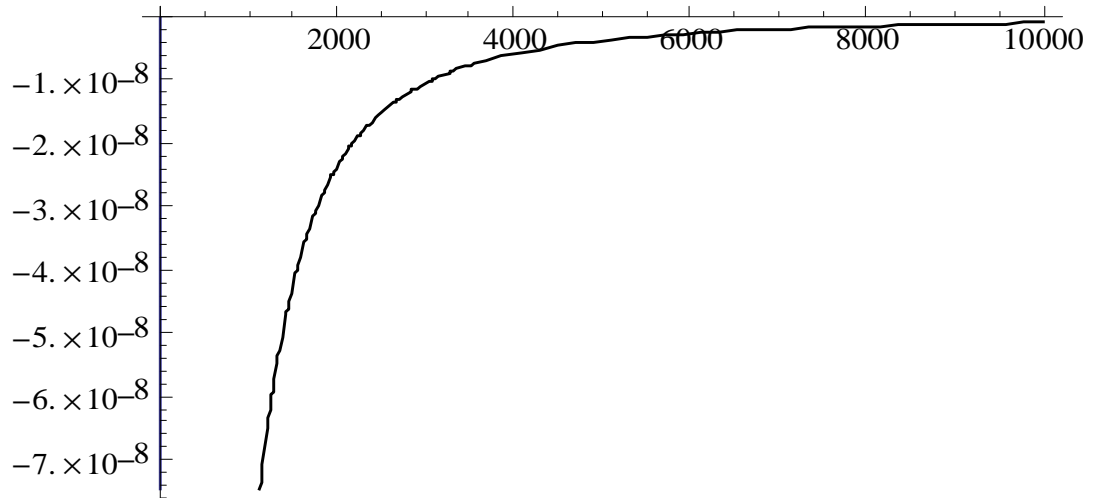




FIGURE 2: THE EFFECT OF  $\chi$  ON  $\tilde{\pi}$  FOR  $\alpha = 100$ ,  $\beta = 50$ ,  $N = 10$ , AND  $\Delta = 0.01$

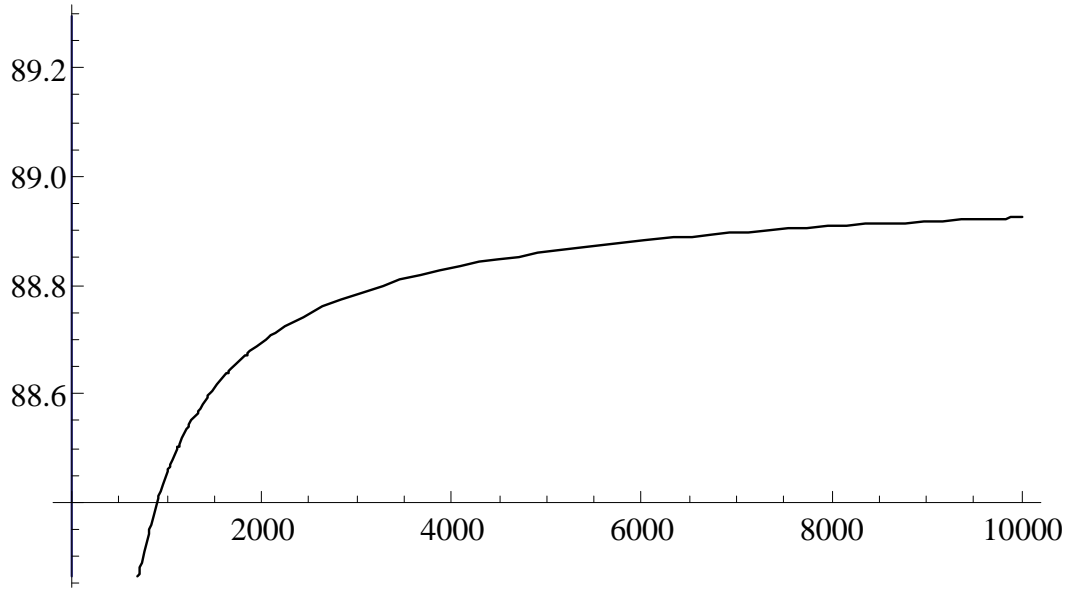


FIGURE 3: THE EFFECT OF  $\psi$  ON  $\delta^*$  FOR  $\alpha = 100$ ,  $\beta = 50$ ,  $N = 10$ , AND  $\Delta = 0.01$

