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Threshold Effect and Financial Intermediation in
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ABSTRACT

This paper analyzes the theoretical finance-growth nexus. Using the Neoclassical growth framework, we raise a new issue where our finance-growth nexus has multiple stationary states with threshold effect. Threshold effect prevents the economy to reach long-run steady state equilibrium of capital and hence financial economists in developing countries should be aware of such an impediment. We show that the development of banking sector should be more supported than financial market, since banking sector is better than financial market in order to reduce threshold effect and ensure the existence and uniqueness of a higher long-run steady state equilibrium of capital stock.

Keywords: Threshold Effect, Financial Intermediation, Economic Growth, Developing Countries

JEL Classification: C61, C62, O16

1. Introduction

During the last two decades, the literatures on the nexus between financial development and economic growth emerge, but the findings are still subject to relevant debate until nowadays2. In developing countries study, particularly, financial development is associated with banking sector development, since financial market is underdeveloped. However, the more recent literature suggests that financial market should be also taken into account to spur economic growth, even in developing countries. Using a very large cross-country sample incorporating both developed and developing countries, Levine and Servos (1998) show that stock market liquidity leads to faster rate

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2 In empirical study, see King and Levine (1993a, 1993b), Levine (1998); Rajan and Zingales (1998) at the country level study, and Fisman and Love (2002) at the industry level; or recently Demirgüç-Kunt and Maksimovic (2002) at the firm level. In theoretical study, see Bencivenga and Smith (1991), or recently Hung and Cothren (2002). Levine (2005) provide a comprehensive literature review.
of growth, productivity improvement, and capital accumulation. Their paper supports Levine (1991) and Bencivenga et al (1995), where stock market liquidity facilitates long-term investment, since investors can easily sell their stake in the project if they need liquidity before their project matures. Enhanced liquidity and long-term investment, therefore, increase higher-return projects that boost productivity growth.

However, it is also well accepted that financial market tends to be more prone to asymmetric information problems and thus, financial liberalization fostering stock market liquidity is often blamed for macroeconomic downturn, as well as banking vulnerability and crisis (Bihde, 1993; Demirgüç-Kunt and Detagriache, 1999). Thus, the adverse effect of financial market appears. This is why according to Diamond (1984) the presence of bank as financial intermediation is necessary, since banks have technology to gain information from investors which enhance investor’s rational decision based on their consumption profile.

Building on the previous literatures on the importance of financial intermediation on economic growth, Bencivenga and Smith (1991) show that financial intermediation is better than financial market (financial autarky) in order to spur economic growth. In their contribution, there are basic lists of bank activities such as loans funded deposits, holding liquid reserves against predictable withdrawal demands, issuing liabilities that are more liquid than their primary asset, and reducing the need of self-investment. The main result of their model is that financial intermediation promotes the productive long-term (illiquid) investment rather than short-term (liquid) ventures.

Surprisingly, the optimal proportion long-term investment is decreasing in the income of long-term investment itself, although it is increasing in the fraction of entrepreneurs. It is also surprising that the optimal proportion of long-term investment is increasing in the income of short-term ventures and the fraction of non-entrepreneurs. Hence, although the income of long-term investment is higher than the income of short-term ventures, it does not always incite agents to be entrepreneur. This implies that entrepreneurship is not always a growth-enhancing factor, since the opportunity cost of being entrepreneur are very high.

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3 Stock market liquidity refers to the less expensive cost of equities trading.
Recently, both theoretical and empirical studies have questioned the positive link between financial intermediation and economic growth. In the theoretical study, Deidda and Fattouh (2002) show a non-linear relationship between financial intermediation and endogenous growth. In their model, the effect of financial intermediation on economic growth remains ambiguous at low initial levels of the development of banking sector and the existence of risk-averse agents. This is because risk-averse agents always prefer to incur financial transaction costs even though the expected return on their savings is lower than under financial autarky. Such a situation occurs because financial intermediation can fully perform in risk diversification process. As a consequence, economic growth rate under banking sector is lower than under financial autarky. Conversely, the relationship between banking sector development and economic growth will be always positive, and the level of banking sector development depends on the initial level of real per capita income.

Moreover, in the empirical examination, Deidda and Fattouh (2002) also find that there is no significant effect of financial development on economic growth in low-income countries, whereas in high-income countries, there is a positive link between financial development and economic growth. In the cross-country study, Mihci (2006) find that the relationship between finance and growth does not necessarily positive when substantial variations across different periods and country groups are taken into account. In the single country study, Crouzille et al (2007) indicate the presence of threshold effect on the link between rural bank development and regional growth in the Philippines.

The aim of this paper is therefore to reevaluate the theoretical finance-growth nexus with the existence of threshold effect. We modify several hypothesis used by Bencivenga and Smith (1991). First, since our motivation is to model the most suitable condition for developing countries, we consider that externalities changes due to technological innovation may be less important, so that they may not much play a pivotal role in boosting economic growth. Hence, we use the Neo-classical growth without externalities in an overlapping generation (OLG) model with three periods instead of drawing endogenous growth model as developed by Bencivenga and Smith (1991), and Deidda and Fattouh (2002). Using the Neo-classical growth framework allows us to obtain more realistic growth rate in developing countries, where the growth rate in
consecutive years is not necessarily positive. Second, we distinguish the behaviour *vis-à-vis* of risk between non-entrepreneur and entrepreneur. More precisely, the entrepreneurs are supposed to be risk neutral\(^4\). This hypothesis allows us to consider that entrepreneurs’ risk-taking behaviour may be the source of costly overinvestment which reduces long-term economic growth\(^5\).

Using these features, our contributions are threefold. First, we show that entrepreneurship is always growth-enhancing factor in both bank-based and market-based financial system, since the optimal proportion of long-term investment is increasing in the fraction of entrepreneurs, the income of long-term investment and short-term ventures, as well as the agent’s savings rate. Second, we show that agent’s saving is a main determinant of the optimal proportion of long-term investment, where in Bencivenga and Smith (1991), financial intermediation is not incitated to raise agents’ savings as input. Therefore, we characterize the traditional role of bank as financial intermediation (deposits and investments). Third, our model is characterized by the existence of multiple steady states with the threshold effect of capital stock as development trap problem which impedes the economy to reach the higher long-run steady state equilibrium. In this case, financial intermediation is better than financial autarky, since threshold level under financial intermediation is lower than under financial autarky, and financial intermediation yields a higher transition of capital stock than financial autarky.

Our results differ from that of Deidda and Fattouh (2002) for several reasons. First, we use the Neoclassical growth framework, while they use endogenous growth. Second, we emphasize that banking sector development is *always* better than financial autarky to decrease threshold level and increase long-run capital stock, while in Deidda and Fattouh (2002) the opposite is true at low levels of the financial development. Third, our threshold effect is due to the initial level of capital stock, while in their model, threshold effect is due to the initial level of real per capita income. Since the real per

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\(^4\) Azariadis and Smith (1998) also use this hypothesis for the different framework of model.

\(^5\) Baumol (1990) analyzes the riskiness of entrepreneurship activity which may be unproductive or even destructive. This fact should not be neglected by financial sectors whose role is to provide financial supports for entrepreneurship activity.
capita income depends on the initial level of capital stock for production, their model may suffer from major reverse causality problems on the finance-growth nexus.

The rest of this paper is organized as follows. Section 2 describes the model set-up. Section 3 models the financial market through financial market. Section 4 models the bank-based financial system. Section 5 builds the study of capital stock dynamic and threshold effect. Section 6 concludes.

2. The Set-Up

The framework we use is one of overlapping generations (OLG) model with three periods and a unique good. We suppose that there is no population growth in the economy and each generation consists of a continuum of agents with size $N_t = N = 1$. Each agent may live for two or three periods. Let $t$ be the time index, where the young and middle-age generations are endowed with an initial per firm capital stock of $k_0$ units at $t = 0$ and $k_1$ units at $t = 1$, respectively. Moreover, each young agent supplies inelastically one unit of labour in the first period.

At the first period, all agents of a generation are identical. At the beginning of the second period, the agents learn whether they will be either non-entrepreneurs (two-period-lived agents) or entrepreneurs (three-period-lived agents) with probability $(1 − \pi)$ and $\pi$, respectively. Thus, there are $(1 − \pi)N$ agents who will be non-entrepreneur at the second period and $\pi N$ agents who will be entrepreneur at the third period. All young agents save entirely their labour income in the first period. Meanwhile, if agents are non-entrepreneur, they consume their second period incomes, $c_u$. If the agent is entrepreneur, he consumes the profit of production in the third period, $c_{2u}$. Thus, agents have different liquidity needs, where the non-entrepreneurs have higher liquidity need than entrepreneurs, since because non-entrepreneurs only live for two periods. Meanwhile, the young agents have incentive to be entrepreneur because the profit of long-term investment is relatively higher than the return of non-entrepreneur’s saving. We assume that entrepreneurs are risk-neutral. Finally, whatever the type of agents, we can define the agent’s preferences by the following expected utility function.
\[ U(c_{it}, c_{2t}) = \frac{(1 - \pi)}{1 - \gamma} (c_{it})^{1 - \gamma} + \pi \phi c_{2t}, \quad \text{where } c_{0t} = 0 \]  

We define \( c_{it} \) as the period \( i \) consumption of an agent who is born at \( t \). The constant relative risk aversion is denoted by \( \gamma > -1 \). The variable \( \phi \) stands for the individual specific random variable realized at the beginning of period 2. Thus, the value of \( \phi \) is equal to 0 with probability \( 1 - \pi \), or 1 with probability \( \pi \).

In order to complete this model, we characterize the production function and the entrepreneur’s behaviour. The entrepreneur’s production \( y_t \) is realized by physical capital \( k_t \) and units of labour \( L_t \). We follow the Cobb-Douglas production function as follows

\[ y_t = A k_t^\theta L_t^{1 - \theta} \]  

where \( \theta \in [0, 1] \) is the part of production that uses \( k_t \) and \( A \) is an arbitrary coefficient. For simplification, we assume that capital depreciates completely at the end of period. Furthermore, there is no endowment of capital at period \( t > 0 \) except for the initial old generation and middle-age generation. In order to complete the entrepreneur program, the profit function must be established. The entrepreneur’s profit \( \Pi_t \) is the difference between the production and the cost of quantity units of labour defined

\[ \Pi_t(k_t, L_t) = A k_t^\theta L_t^{1 - \theta} - w_t L_t. \]  

At the equilibrium of labour market, labour demand \( L_t \) is equal to labour supply, \( N_t = N \), which is obtained by maximizing the entrepreneur’s profit subject to \( L_t \). Thus, we have \( w_t = A(1 - \theta)k_t^\theta \pi^\theta \) and the maximized profit function at each period \( t \) as much as

\[ \Pi_t = A \theta \psi k_t^\theta, \text{ with } \psi = L_t^{1 - \theta} = \pi^{\theta - 1} \]  

3. **Financial Market**

This system refers to an economy without the presence of bank as financial intermediation. In the first period, the agents divide their savings \( s_t \) between liquid and illiquid assets. Liquid assets are considered as inventory of consumption goods. One unit invested in liquid asset at \( t \) directly yields \( n > 0 \) units of consumption goods at both \( t + 1 \) and \( t + 2 \). On the other hand, one unit invested in the illiquid asset yields \( R \) units of
capital goods at $t+2$. If illiquid asset is liquidated at $t+1$, then the agents receive the “scrap value” of $x$ units of consumption goods, where $0 < x < n$.

In order to establish the agents’ budget constraint, we define $z^m_t$ and $q^m_t$ as the proportion of liquid asset and illiquid asset invested at $t$, respectively. The superscript $m$ stands for the financial market. Hence, we have

$$z^m_t + q^m_t = 1, \quad \text{where} \quad z^m_t \geq 0, \quad q^m_t \geq 0$$  

(4)

At the first period, the agents’ saving is equal to labour income, $s_t = w_t$, and is divided into $z^m_t s_t$ units of liquid asset and $q^m_t s_t$ units of illiquid asset. Let $i_L, i_R, i_S$ be the interest rate of the liquid asset, illiquid asset, and “scrap” value, respectively. At the second period, let $\omega_t$ be the income of non-entrepreneur after one period, then

$$\omega_t = (nz^m_t + xq^m_t) w_t, \quad \text{where} \quad n = 1 + i_L \quad \text{and} \quad x = (1 + i_s)$$  

(5)

By the hypothesis, if the agents are entrepreneur, then their consumption at the second period is equal to zero. At the beginning of the third period, the entrepreneur sells his illiquid assets and reinvests them in the physical capital, so that $(1 + i_R) q^m_t s_t = k^m_{t+2}$. This situation corresponds to the financial autarky case. At the third period, let $\omega_{2t}$ be the income received by entrepreneur before the production, then

$$\omega_{2t} = nz^m_t w_t + Rq^m_t w_t, \quad \text{where} \quad R = 1 + i_R, \quad \text{and} \quad Rq^m_t w_t = k^m_{t+2}$$  

(6.a)

and $0 < x < n < R$  

(6.b)

Using the profit function (3) and the budget constraints in the equation (4), (5) and (6.a), we now define the agent’s expected utility function when investment is self-financed.

$$U(q^m_t) = \left( -\frac{1-\pi}{\gamma} (xq^m_t w_t + n(1-q^m_t)w_t) \right)^{-\gamma} + \pi \left( A \theta \psi (Rq^m_t w_t) \right)^{\theta} + (1-q^m_t) nw_t \right)$$  

(7)

Meanwhile, the agents’ optimization program is defined as $\arg \max_{0 \leq q^m_t \leq 0} \{U(q^m_t)\}$.

From the first order condition, we obtain the optimal proportion of illiquid asset ($\bar{q}^m_t$) as follows.

$$\bar{q}^m_t = \bar{q}^m_t(w_t) = \frac{n}{(n-x)} \left( \frac{B(w_t)^{\frac{1-\gamma}{\gamma}}}{w_t(n-x)} \right)$$  

(8)
where \( B(w) = \frac{\pi}{\pi - 1} \left( \frac{nw - AR^b w^b \theta \psi}{w(n - x)} \right) \)

The optimal proportion of illiquid investment \( q_t^m \) depends on the labour income \( w_t \). Moreover, the existence of \( q_t^m \) in which \( 0 \leq q_t^m \leq 1 \) can be examined by the limit value of \( q_t^m \) when \( w_t \to 0^+ \) and \( w_t \to \infty^+ \). From (8), it is straightforward to obtain

\[
\lim_{w_t \to 0^+} q_t^m = -\infty \quad \text{and} \quad \lim_{w_t \to \infty^+} q_t^m = 1, \quad \text{if} \quad AR^b w_t^b \theta \psi > nw_t .
\]

Hence, there is a value of \( w_t \) which implies that \( q_t^m = 0 \).

4. Financial Intermediation

We assume that agent’s financial decisions are intermediated through the banking system. Therefore, we can directly define the program of financial intermediaries realized by an institution called as “bank”. We assume that bank is a coalition of young agents who can be either non-entrepreneur or entrepreneur. Let \( z_t \) and \( q_t^b \) be the proportion of liquid and illiquid investment realized by banks, respectively. Thus, we have

\[ z_t^b + q_t^b = 1 \] (9)

Banks ensure non-entrepreneur to receive \( R_t^b \) units of consumption goods at \( t+1 \) from each unit invested at \( t \) as following\(^7\)

\[ (1 - \pi)R_t^b = \alpha_{1t} z_t^b n + \alpha_{2t} q_t^b x \] (10)

where \( \alpha_{1t} \) and \( \alpha_{2t} \) are the part of liquid and illiquid asset liquidated at the second period, respectively. The bank chooses the values of \( \alpha_{1t} \) and \( \alpha_{2t} \). Moreover, banks also ensure entrepreneurs to receive \( R_{2t}^b \) units of capital goods at \( t+2 \) from each unit of time \( t \) illiquid investment and \( \tilde{R}_{2t}^b \) units of time \( t+1 \) consumption goods from each unit liquid asset invested at \( t \). For the withdrawal after two periods, there are \( \pi \) entrepreneurs who must receive \( R_{2t}^b \) units of capital goods from each unit of illiquid investment. Thus, \( \pi R_{2t}^b \) factor

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\(^{6}\) In Bencivenga and Smith (1991), this optimal proportion of illiquid investment is constant.

\(^{7}\) The index \( b \) refers the banking interest factor \( R^b \), where \(-1 \leq R^b \leq \infty \)
must be equal to the rest of illiquid asset \((1-\alpha_{2t})\) multiplied by the income of investment \(R q^b_t\). Thus, the bank must provide capital goods for entrepreneurs as much as

\[
\pi R^b_{2t} = (1 - \alpha_{2t}) R q^b_t \tag{11}
\]

In addition, entrepreneurs must also receive \(\tilde{R}^b_{2t}\) units of consumption goods for each unit of liquid investment at \(t\). The constraint \(\pi \tilde{R}^b_{2t}\) must be equal to the rest of consumption goods \((1-\alpha_{1u})\) multiplied by \(z^b_t n\). Thus, banks must provide consumption goods for entrepreneurs as much as

\[
\pi \tilde{R}^b_{2t} = (1 - \alpha_{1u}) z^b_t n \tag{12}
\]

In the next step, we define the program of financial intermediation for two types of agent. Firstly, there are \((1-\pi)\) non-entrepreneurs who will liquidate their investment at \(t+1\). Thus, the bank must ensure the non-entrepreneur by holding \(R^b_{1t} w_t\) units of consumption goods to be distributed at \(t+1\). Secondly, there are also \(\pi\) entrepreneurs who will liquidate their investment at the beginning of \(t+2\). Thus, the bank must ensure entrepreneurs by holding \(R^b_{2t} w_t\) units of capital goods and \(\tilde{R}^b_{2t} w_t\) units of consumption goods to be distributed at \(t+2\). Using budget constraints in the equation (10), (11), and (12) we define the program of financial intermediation in the following relation

\[
U(c_{1t}, c_{2t}) = \frac{-((1-\pi))}{\gamma} (R^b_{1t} w_t)^\gamma + \pi (A \theta \psi (R^b_{2t} w_t)^\delta + \tilde{R}^b_{2t} w_t) \tag{13}
\]

Note that in the third period \((t+2)\), entrepreneurs will use their income of investment to finance physical capital and use it in the production. Hence, we have \(R^b_{2t} w_t = k^b_{t+2}\). In order to simplify condition in the equation (13), we assume that the bank should provide the liquidity at \(t+1\), since none of the capital assets is liquidated “prematurely”. Thus, the bank should fulfil the following liquidity constraint

\[
A \theta \psi R > n \tag{14}
\]

By this assumption, we can reduce some variables as follows. In the third period \((t+2)\), the bank will only consider the existence of \(\pi\) entrepreneur. From (11), we have \(k^b_{t+2} = \frac{R q^b_t w_t}{\pi}\) as individual capital. Since the entrepreneur runs the production to get
the profit, then their profit should be superior to all income of liquid investment. Such condition provides incentive for agents to become entrepreneur. In other words, 
\[ A\theta\psi R > n, \] and
\[ A\theta\psi \left( (1 - \alpha_{2}) \left( \frac{R}{\pi} q_{i}^{b} w_{i} \right) \right) > \left( \frac{n}{\pi} q_{i}^{b} w_{i} \right) \] (15.a)
Equation (15.a) is fulfilled if and only if the bank set
\[ \alpha_{2i} = 0 \] (15.b)
Meanwhile, the bank also maximizes the expected utility of non-entrepreneur. It means that the bank will reallocate the non-entrepreneur’s illiquid assets into liquid assets at the beginning of \( t+1 \). For realizing this strategy, the bank will therefore set
\[ \alpha_{t} = 1 \] (15.c)
Using (15.b) and (15.c), we simplify (10), (11) and (12) respectively become
\[ R_{ti}^{b} = \frac{z_{t}^{b}}{1 - \pi} n \] (16)
\[ R_{2ti}^{b} = \frac{R}{\pi} q_{t}^{b} \] (17)
\[ \tilde{R}_{2t}^{b} = 0 \] (18)
Using (16), (17), and (18), and the budget constraint (9) we establish the program of financial intermediaries as follows
\[ U(q_{i}^{b}) = - \frac{(1 - \pi) \left( 1 - q_{i}^{b} - n w_{i} \right) \gamma}{\gamma} + \pi \left( A\theta\psi \left( \frac{R q_{i}^{b} w_{i}}{\pi} \right)^{\theta} \right) \] (19)
Hence, banks will choose \( q_{i}^{b} \) to maximize \( U(q_{i}^{b}) \). From the first-order condition, we obtain the optimal proportion of illiquid asset \( (\tilde{q}_{i}^{b}) \) as follows
\[ \tilde{q}_{i}^{b} = \tilde{q}_{i}^{b} (w_{i}) = 1 - \frac{(1 - \pi)(B_{i})^{\frac{1}{1 - \gamma}}}{n w_{i}} \] (20)
where \( B_{i} = \frac{\pi^{\theta} w_{i}^{\theta} \psi}{n w_{i}} \).

5. Capital Stock Accumulation and Threshold Effect
In comparing the level of steady state equilibrium of capital stock under financial market and banking, we establish Proposition 1 and 2 as follows.

**Proposition 1**

*From (8) and (20), we denote that the optimal value of illiquid investment under financial intermediation is higher than the optimal value of illiquid investment under financial market. In other words, we prove that \( \overline{q}_{t}^{b} > \overline{q}_{t}^{m} \)*

**Proof:**

For \( x = 0 \), we then show that \( (1-\pi)(B_{t})^{\frac{1}{1-\gamma}}/nw_{t} < (B)^{\frac{1}{1-\gamma}}/nw_{t} \). Thus, we examine whether \( B_{t} < B \). From \( B_{t} \) and \( B \), we only examine if

\[
(1-\pi) \left( A\pi \left( \frac{R}{\pi} \right)^{\theta} w^{\theta} \theta^{\psi} \right)^{\frac{1}{1-\gamma}} < \left( \frac{\pi}{1-\pi} (AR^{\theta} w^{\theta} \theta^{\psi} - nw_{t}) \right)^{\frac{1}{1-\gamma}}
\]

Let \( D_{1} = \left( A\pi \left( \frac{R}{\pi} \right)^{\theta} w^{\theta} \theta^{\psi} \right)^{\frac{1}{1-\gamma}} \) and \( D_{2} = \left( \frac{\pi}{1-\pi} (AR^{\theta} w^{\theta} \theta^{\psi} - nw_{t}) \right)^{\frac{1}{1-\gamma}} \), then we simplify \( (1-\pi)D_{1} < D_{2} \). Since \( q_{t}^{b}, q_{t}^{m} \in [0,1] \), then \( \max\{D_{1}\} = \max\{D_{2}\} = 1 \). Thus, the inequality \( (1-\pi)D_{1} < D_{2} \) is proved because \( 0 < (1-\pi) < 1 \). Finally, Proposition 1 is proved.

**Proposition 2**

*The existence of banks in an economy enhances economic growth more significantly than the absence of banks.*

**Proof:**

In the case of financial intermediation based on banking sector, economic growth is determined by the value of \( k_{t+2}^{b} = \frac{R \overline{q}_{t}^{b}w_{t}}{\pi} \). Meanwhile, in the case of financial market, economic growth is determined by the value of \( k_{t+2}^{m} = R \overline{q}_{t}^{m}w_{t} \). From Proposition 1, it is straightforward to find \( \mu_{t}^{b} > \mu_{t}^{m} \), where \( \mu_{t}^{b} = \frac{k_{t+2}^{b}}{k_{t}} \) and \( \mu_{t}^{m} = \frac{k_{t+2}^{m}}{k_{t}} \) are the change of
capital stock in the economy based on banking sector and financial market, respectively. Proposition 2 is thus proved.

In order to illustrate the dynamics of capital accumulation, we run a numerical example and the graphic is shown as follows\(^8\).

In Figure 1, we denote that there are threshold effects at \(k_{t+2}^b\) and \(k_{t+2}^m\) for the bank-based economy and the financial market-based economy, respectively. The following proposition shows that the existence of financial intermediaries is important to reduce the threshold effect and increase long-run steady state equilibrium of capital accumulation.

**Proposition 3**

*There is always a threshold effect in the finance-growth nexus, so that financial development cannot support economic growth, unless its initial level exceeds the threshold effect. However, threshold effect in the economy based on banking sector is lower than the one based on financial market. It suggests that the bank-based financial system is less costly than financial market based-system to ensure the existence and uniqueness of long-run steady state equilibrium of capital accumulation and hence, boost economic growth.*

\(^8\) Numerical examples are available on request.
Proof:

To prove Proposition 3, we firstly show that threshold effect exists in both the bank-based and financial market-based economy. Then, we compare both of them.

(i) The economy based on banking sector

From (20) and the stylized fact that \( k_{t+2}^{b} = \frac{R \bar{q}_{t}^{b} w_{t}}{\pi} \), we denote that the functional form of \( k_{t+2}^{b} = f(k_{t}) \) is difficult to be identified. And we cannot solve \( k_{t} \) algebraically from the equation \( k_{t+2}^{b} = f(k_{t}) = k_{t} \) to find the roots \( k_{t}^{b*} \) and \( k_{t}^{m*} \). Hence, we should derive \( k_{t+2}^{b} = \frac{R \bar{q}_{t}^{b} w_{t}}{\pi} \) in order to obtain its first-order condition as follows

\[
\frac{d k_{t+2}^{b}}{d k_{t}} = \frac{R(-1 + \theta) \theta \left( \frac{\Omega}{n} \right)^{-1}}{k_{t} n \pi (1 + \gamma)} \left( -1 + \pi + Ak_{t}^{0} \pi \theta (1 + \gamma) \left( \frac{\Omega}{n} \right)^{1/\gamma} \right) \tag{21}
\]

where \( \Omega = \left( \frac{A}{n} \left( \frac{R}{\pi} \right)^{0} \pi (-Ak_{t}^{0} \pi \theta (-1 + \theta)^{-1 + \theta} \theta \psi) \right)^{-1/\gamma} \)

The threshold effect \( k_{t}^{b*} \) exists, if and only if there is \( k_{t} \) that implies \( \frac{d k_{t+2}^{b}}{d k_{t}} > 1 \). In other words, \( \frac{d k_{t+2}^{b}}{d k_{t}} - 1 > 0 \) and at this point, the curve \( k_{t+2}^{b} = \frac{R \bar{q}_{t}^{b} w_{t}}{\pi} \) cuts \( k_{t+2}^{b} = k_{t} \) as shown at Figure 1. In order to simplify the functional form of the first-order condition, we examine the special case of model. Suppose that \( \pi \to 1 \) and as a consequence, \( \psi \to 1 \).

Under this condition, we simply obtain

\[
\lim_{\pi \to 1} \frac{d k_{t+2}^{b}}{d k_{t}} - 1 = - \frac{k_{t} + Ak_{t}^{0} R(\theta - 1) \theta}{k_{t}} \tag{22}
\]

Note that in this proof, despite assuming that \( \pi \to 1 \), we do not change the purpose of the model. As long as our purpose is to model the role of financial intermediation in boosting
long-term investment and entrepreneurship, the absence of non-entrepreneurs does not affect the change of capital stock. It is because economic growth should not be depended on non-entrepreneurs but entrepreneurs. Through (22), we should find \( k_i \) in order to make its right-hand side becomes positive. In other words,

\[
-k_i + A k_i^\theta R(\theta - 1)\theta \frac{1}{k_i} > 0
\]

\[
\Leftrightarrow \frac{1}{k_i} > 0 \text{ and } (-k_i - A k_i^\theta R(\theta - 1)\theta) > 0
\]

\[
\Leftrightarrow k_i < \infty \text{ and } k_i > \left(\frac{1}{A R (1-\theta)\theta}\right)^{\frac{1}{\theta-1}} = k_i^{**}
\]  \( (23) \)

Since \( A, R > 0 \) and \( 0 < \theta < 1 \), then \( \left(\frac{1}{A R (1-\theta)\theta}\right)^{\frac{1}{\theta-1}} > 0 \). Hence, we obtain

\[
k_i^{**} = \left(\frac{1}{A R (1-\theta)\theta}\right)^{\frac{1}{\theta-1}} \]

Equation (24) is simply defined as the threshold level of bank-based financial system, since for every \( k_i < \infty \) and \( k_i > k_i^{**} \), we always find the function

\[
\frac{d k_i^m}{d k_i} > 1
\]

The existence of threshold effect in the bank-based financial system is therefore confirmed.

(ii) The economy based on financial market

To prove the existence of threshold effect under financial market, we use the same characterization of the bank-based economy. Suppose that \( \pi \to 1 \) and consequently, \( \psi \to 1 \). It means that financial market exists only for responding the entrepreneur’s needs. Solving the first-order condition for \( k_i^m = f(k_i) = R \tilde{q}_i w_i \) and its limit for \( \pi \to 1 \), we obtain

\[
\lim_{\pi \to 1} \frac{d k_i^m}{d k_i} - 1 = \frac{A k_i^{\theta-1} n R (1-\theta)\theta}{n-x} - 1
\]  \( (25) \)
The threshold effect \( k_t^{m^*} \) exists, if and only if there is \( k_t > 0 \) that implies \( \frac{d k_t^{m^*}}{d k_t} > 1 \) or \( \frac{d k_t^{m^*}}{d k_t} - 1 > 0 \). By solving \( k_t \) in (25), we obtain

\[
k_t^{m^*} = \left( \frac{AnR\theta}{n-x} - \frac{AnR\theta^2}{n-x} \right)^{\frac{1}{\theta-1}}
\]

(26)

By the fact that \( 0 < \theta < 1 \), then it is straightforward to denote that \( k_t^{m^*} > 0 \). Hence, the existence of threshold effect in the economy based on financial market is proved.

(iii). Bank vs. Financial Market

To prove that threshold effect in the bank-based economy is lower than in the economy based on financial market, we should show that

\[
\frac{AnR\theta}{n-x} (1-\theta) < \frac{1}{AR(1-\theta)\theta}
\]

(27)

By substituting \( \theta \to 1 \) into (27), it simply shows that the left-hand side tends to 0, but the right-hand side tends to infinity. Instead, if we substitute \( \theta \to 0 \), then we denote the left-hand side tends to 0, and the right-hand side tends to 1. By these results, Proposition 3 is finally proved. Differently phrased, threshold effect in the bank-based economy system is lower than in the economy based on financial market.

Why threshold effect is so important in the finance-growth nexus? Suppose that \( k_0 \) is the initial capital of an economy that lies below the threshold level of market-financed system \( k_t^{m^*} \) (see Figure 1). In order to reach the long-run steady state capital, \( k_0 \) should be iterated by financial intermediation (see bank curve at Figure 1). Such situation may drive the economy to converge to \( k_t^{bss} \). Conversely, if \( k_0 \) is only iterated by the financial market curve (see market curve at Figure 1), the economy will disappear because the steady state capital stock tends to zero. Hence, we find that the bank-based economy is better than the economy based on financial market in order to ensure the existence and uniqueness of long-run steady state capital stock, and to reduce threshold effect. In turn, long-run economic growth can be boosted due to an increase in long-term productive investments and a decline in short-term ventures. By extension, the potential
source of speculations from short-term ventures is therefore reduced. However, since $k_0 < k^*_{t'}$ as shown in Figure 2, the capital accumulation will also converge to zero even if there is financial intermediation in the economy. In such a case, there is no positive link between financial development and economic growth.

6. Conclusion

In providing further issue on the finance-growth nexus, we have reevaluated the model of financial intermediation à la Bencivenga and Smith (1991). Our originality is twofold. First, in modelling the finance-growth nexus, we use the Neo-classical growth framework instead of drawing endogenous growth as developed by Bencivenga and Smith (1991). Second, we distinguish the behaviour vis-à-vis of risk between non-entrepreneur and entrepreneur.

Using these features, we find that the bank-based economy is better than the economy based on financial market in order to ensure the existence and uniqueness of long-run steady state equilibrium of capital accumulation, which is a necessary condition to achieve long-run economic growth. Moreover, we found that any level of financial development (either through banking or financial market) has a threshold effect. But the presence of banks as financial intermediaries clearly reduces the threshold level. The presence of threshold effect is a new finding in the finance-growth nexus, since it captures the difficulty of raising initial capital stock and reaching long-run economic growth. Thus, threshold effect should be acknowledged in future empirical research on the finance-growth nexus, notably in developing countries, where externalities due to human capital and technological innovations are not yet well-developed.

References


