Modelling the Demand for Money in Pakistan

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The study estimates the dynamic demand for money (M2) function in Pakistan by employing cointegration analysis and error correction mechanism. The parameters of preferred model are found to be super-exogenous for the relevant class of interventions. It is found that the rate of inflation is an important determinant of money demand in Pakistan. The analysis reveals that the rates of interest, market rate, and bond yield are important for the long-run money demand behaviour. Since the preferred model is super-exogenous, it can be used for policy analysis in Pakistan.

I. INTRODUCTION

Money demand holds a key position in macroeconomics in general and monetary economics in particular. The knowledge of the factors affecting the demand for money is crucial in the conduct of monetary policy, and for the choice of instruments and intermediate targets of monetary policy. A well-specified and empirically stable money demand function is crucial for statistical inference, forecasting, and policy analysis. A stable demand function for money means that the quantity of money is predictably related to a small set of key variables linking money to the real sector of the economy [Judd and Scadding (1982) and Friedman (1987)].

In Pakistan, like other countries, considerable effort has been made in estimating money demand functions. For example, Akhtar (1974), Abe, *et al*. (1975), Mangla (1979), Khan (1980, 1982, 1982a), Nisar and Aslam (1983), Ahmed and Khan (1990), Hossain (1994), Khan and Ali (1997), Qayyum (1998, 2001), etc., have estimated money demand functions by using alternative specifications. Some of these studies such as Ahmed and Khan (1990) and Qayyum (2001) have also examined the stability of their estimated money demand functions. Generally, the M2 function is found to be stable. However, with the exception of Hossain (1994), Khan and Ali (1997), and Qayyum (1998, 2001), these studies have ignored the time series properties of the relevant variables and therefore may be prone to spurious regression. Furthermore, according to our knowledge these studies did not rigoursly

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test the suitability of their estimated models to be used for forecasting and policy analysis. The decisive answer to these important questions is given by the theory of exogeneity, particularly testing of super-exogeneity of the parameters of interest [Ericsson (1992)].

Non-stationarity of time series data, an important characteristics of time series, has been taken care of by the theory of cointegration. Whereas the question as to whether the estimated model is valid for statistical inference, forecasting and policy analysis or not is addressed by the theory of exogeneity [Engle, *et al.* (1983)].

Moreover, most recent studies, such as Hossain (1994) and Qayyum (2001) among others, cover period of a few years of early 1990s. The full period of the decade of 1990s is not generally covered in these papers. During this decade Pakistan has gone through a wide range of financial sector reforms. These reforms and opening up of the economy provided the economic agents a wider choice of portfolio diversification. It is interesting to use extended data set covering reforms period and to obtain fresh estimates of the real money demand function to be used in the conduct and analysis of monetary policy.

It is strongly argued that the analysis of exogeneity of parameters of interest is required to derive policy implications from the cointegration analysis. The exogeneity of variables depends upon the parameters of interest and the purpose of the model. If the model is to be used only for statistical inference/analysis then we require the analysis of weak exogeneity. If the purpose of modelling is forecasting the future observations then we need to conduct the analysis of strong exogeneity. Finally the concept of super-exogeneity is relevant if the objective of the study is that the money demand model to be used for policy analysis.

Considering the importance of money demand in the macroeconomic analysis and exogeneity in statistical analysis, forecasting and policy simulation, this paper attempts to provide congruent money (M2)[1] demand function by employing cointegration analysis, estimating dynamic error correction model and testing the super-exogeneity of the parameters of interest.

The rest of the paper is organised as follows. The next section explains the methodology relating to the examination of time series properties of individual variables, estimation of long-run demand functions for real money, obtaining dynamic money demand function and testing of super-exogeneity of the parameters of interest. Section III presents the findings regarding the properties of data, cointegration analysis, and preferred model of the demand for real money. It also presents results from stability analysis of the estimated function as well as individual

[1]The main reason for considering M2 definition of money for analysis is that the State Bank of Pakistan has been using M2 as an important target variable in the conduct of monetary policy in Pakistan.

parameters of conditional model. The next section presents the outcome of the analysis of super-exogeneity of the parameters of interest. The final section contains the summary and conclusions.

II. METHODOLOGY

According to Friedman (1987) theory has done its job through highlighting the important determinants of real money demand. Therefore, we specify the money demand function that relates the demand for real balances ($RM_t = M_t/P_t$) to real income ($RY_t = Y_t/P_t$) and the set of variables representing the opportunity cost of holding money. Specifically, the real money demand function can be written as,

$$M_t/P_t = f(Y_t/P_t, R_t, INF_t, u_t) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where

M_t = Money Stock (M2)
P_t = Price Level (GDP Implicit Price Deflator, 1980-81=100)
Y_t = Nominal Income (GNP at current prices)
R_t = Vector of Interest rates including Call Money Rate (CMR), Government Bond Yield (BY)
INF_t = Rate of Inflation
u_t = Random term obeying classical assumptions of IID(0, σ^2).

Granger representation theorem [Granger (1986)] asserts that if two variables are non-stationary that is I(1) and these variables have cointegrating relationship among them then the dynamic function can be represented as an Error Correction Mechanism [Engle and Granger (1987)]. The concept of error correction mechanism (ECM) in economic literature has been introduced by Phillips (1954, 1957), and it is successfully applied in economics by Sargan (1964), Davidson, *et al.* (1978), among others. This error correction mechanism, however, is popularised by Devid Hendry through a number of studies [Hendry, *et al.* (1984), Hendry and Ericsson (1991), Hendry and Mizon (1993)]. In the literature the ECM has different formulations. One of the processes of formulation of the error correction model is the application of Vector Autoregressive (VAR) process [Johansen (1988)]. For this study we adopt the p-dimension vector autoregressive (VAR) process, which can be written as

$$x_t = \sum_{i=1}^{k} \Pi_i x_{t-i} + \mu_t + \varepsilon_t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Where x_t is a vector of variables included in the model, μ_t is constant term and ε_t is IN(0, Ω) disturbance term. With some manipulation this model can be written as reduced form vector error correction model, which is presented below;

$$\Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \mu_t + \varepsilon_t \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

In statistical terms, Equation 3 can be said as the dynamic joint density function, F_x (x ; θ), of real money demand. If we decompose the vector x_t into m_t (the real money demand) and z_t (= RY_t, INF_t, R_t)′ represents determinants of money demand, then the joint density function (Equation 3) can be factorised into the conditional density function of m_t given z_t (i.e., $F_{m/zt}$ (m/z_t ; λ_1) and the marginal density function of z_t. (i.e., F_{zt} (z_t ; λ_2). The dynamic conditional density function of real money demand can be written as;

$$\Delta m_t = \omega \Delta z_t + \sum_{i=1}^{k} \Gamma \Delta m_{t-i} + \Pi x_{t-1} + \mu_t + \varepsilon_t \qquad \ldots \quad \ldots \quad \ldots \quad (4a)$$

and the marginal density function of z_t is written as;

$$\Delta z_t = \alpha_z \beta' x_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta x_{t-i} + \mu_t + \varepsilon_{2t} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4b)$$

The explanation of the conditional model (Equation 4a) is that it considers the immediate impact that change in z_t has on the change in m_t, the term Πx_{t-1} (with condition $\Pi < 0$ that is required for dynamic stability) indicates the impact on change in m_t of having m_{t-1} out of equilibrium with βz_{t-1}. The long-run static solution to the error correction model requires that $m_t = \beta z_t$. On close inspection the model indicates that the cointegrating vector (β) enters into conditional as well as into the marginal function. In this type of situation the parameters of conditional and marginal models are interrelated, which implies that full system analysis is required to get inference about the parameters efficiently. However, if z_t is weakly exogenous for α and β, then the information about the cointegrating relations and the adjustment coefficients contained in the conditional model are equal to the information obtained from the full system analysis [Engle, *et al*. (1983)].

On of the objectives of the study is to test the exogeneity of preferred money demand function. In other words we are interested in to investigate whether the estimated model can be used for forecasting and policy analysis or not. The answer lies in the exogeneity analysis of the parameters of interest of the conditional model. Engle, *et al.* (1983) introduced three important concepts of exogeneity that is weak exogeneity, strong exogeneity and super-exogeneity.

Theory indicates that there are two important conditions for the existence of weak form of exogeneity. These conditions are that (i) the parameters of interest be a function of conditional models parameters only, and (ii) the parameters of conditional and marginal models be variation-free [Ericsson (1992)]. If the parameters of cointegrating vector (β) are the parameters of interest, then the restriction that $\alpha_z = 0$ on the marginal function (Equation 4b) ensures the weak exogeneity of z_t. However, in this situation other loading coefficients enter the conditional model. This situation leads to the conclusion that the parameters of conditional and marginal models are variation-free [Johansen (1991)].

In the case of the dynamic model, which we are considering here, the concept of strong exogeneity is most relevant. The strong exogeneity is the combination of two important features that is weak exogeneity and the Granger non causality [Ericsson (1992)]. The existence of strong exogeneity permits multi-step-ahead predictivity of money from the model, conditional on the predicting z_t generated from marginal model with conditions that $\alpha_z = 0$, when prediction of z_t depends on their own lags.

In case of policy analysis the concept of super-exogeneity is the most relevant. Super-exogeneity is also a combination of two important conditions that is weak exogeneity and invariance [Ericsson (1992)]. The validity of policy simulation is ensured through the existence of super-exogeneity. In case of policy analysis we have to introduce changes in the marginal process. The validity of the conditional model requires that the parameters of conditional model remain invariant to the changes in the marginal models. The answer to this important question whether the estimated model can be valid for policy analysis or not lies in the analysis of super-exogeneity of a variable with respect to specified class of interventions.

The reduced form equation may be non constant due to some external shocks such as oil shock, change in policy rule or financial innovation. In this case the factorisation of joint density function into conditional function and marginal model helps to isolate effects of these shocks. This is to say that the shocks only affect the parameters of marginal process whereas the parameters of conditional model remain constant. This implies that the parameters of conditional model are invariant to the interventions to the marginal model. In other word the parameters of conditional model do not depend on the parameters of marginal process.

Before estimating the final model we first investigate data generating process of individual time series to be used. For the purpose of the testing for the stationarity of the data and determination of the order of integration of each variable, we used Augmented Dickey Fuller (ADF) test of unit roots [Dickey and Fuller (1979, 1981)] following Hall's (1994) sequential rule. The variables having the same order of integration are then tested for cointegration, the long-run relationship, in the second step.

The hypothesis of cointegration is formulated as restriction on the Π matrix of Equation (3) leaving other parameters unrestricted. The hypothesis, that there are at most 'r' cointegrating relationships, is defined as reduced rank condition

$$H_0 : \text{Rank } (\Pi = \alpha \beta') = r, \text{ where } 0 < r < p \text{ and } p \text{ is full rank}$$

For this purpose, we used the Likelihood Ratio (LR) test based on both Trace and Maximum eigenvalue of stochastic matrix as proposed by Johansen (1988). Then the Maximum Likelihood Method of Johansen is used to estimate the long run real money demand function. In the third step, we estimate the dynamic error correction real money demand function. The parsimony of the model is achieved by applying general-to-specific methodology. During this process of model selection we use a battery of diagnostic tests (LM version) relating to the problems of serial correlation, functional form misspecification, non-normality, heteroscedasticity and Auto-regressive Conditional Heteroscedasticity (ARCH). Finally, to test the stability of the preferred model (conditional model) and the marginal model the test statistics such as CUSUM and CUSUMQ proposed by Brown, *et al.* (1975) and dummy variables by Gugarati (2003) are applied.

The study is based on annual data from 1960 to 1999. The principal data source is *50 Years of Pakistan in Statistics* supplemented by issues of *Economic Surveys* and State Bank's *Annual Report* and *Bulletin*.

III. EMPIRICAL RESULTS

1. Stationarity of Data

The data are transformed into the logarithmic form on the basis of preliminary analysis. This transformation reduces the variability of variance of the data. At the first step, the ADF unit root test is applied to all the variables to test for the stationarity of these variables. The test is applied to both the original series (in log form) and to the first differences. The results, reported in Table 1, indicate that all the series are non-stationary at their level, that is they are random walk series. They, except prices, become stationary after employing difference operator of degree one. That is, these series are integrated of order one, I(1), whereas, prices are I(2) implying that rate of inflation is I(1)[2].

Table 1

Augmented Dickey-Fuller (ADF) Test of Stationarity of Time Series Data

(*ADF Equation;* $\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{i=1}^{n} \lambda_i \Delta y_{t-i} + \varepsilon_t$)

Description	Variables	Lags	t-statistics	Variables	Lags	t-statistics

[2] The results are confirmed with application of the Phillips-Perron (PP) test.

1.	Real Money	$LRM2_t$	2	–2.191	$\Delta LRM2_t$	1	–5.62
2.	Real Income	LRY_t	1	–1.49	ΔLRY_t	0	–6.53
3.	Call Money Rate	LCR_t	1	–1.763	ΔLDR_t	1	–4.94
4.	Government Bond Yield	LBY_t	1	–2.25	ΔLBY_t	0	–5.29
5.	GDP Implicit Price Deflator (1980-81=100)	$LCPI_t$	1	–1.685	$\Delta LCPI_t$	2	–2.74
6.	Rate of Inflation	INF_t	0	–2.74	ΔINF_t	0	–8.01

Note: ADF τ<–3.44 at the 5 percent level of significance, *L* is for log and Δ shows first difference.

2. Cointegrating Analysis of Real Money Demand

This section presents the outcome of second step of the methodology. This step consists of two parts, that is, the test of cointegration and estimation of long run money demand function. To test for cointegration, the Johansen Likelihood Ratio Tests based on trace and maximum eigenvalue statistics are applied. We use different combinations of variables and different order of VAR. Final selection is made on the basis of error term that became white noise and the cointegrating vector gives theoretically plausible results. The lag selection criteria such as AIC and SBC are also used for this purpose. Similarly, the final versions of the ECM of demand for M2 are reported here which is selected on the basis of diagnostic tests. For all the test statistics we use 5 percent level of significance unless otherwise stated.

Another important issue in the cointegration analysis is appropriate treatment of deterministic components such as constant and trend term. Different treatment of constant and trend terms in the analysis lead towards different critical values [see for example, Johansen (1991) and Johansen and Juselius (1990), among others]. Johansen (1995) discussed five different choices about the usage of constant and trend term. It is suggested that if the variables included in the system show growth then constant term should not be placed into cointegrating space. It should rather be used unrestrictedly in the analysis. For detailed discussion see for example Banerjee, *et al.* (1993), Enders (2004), Johansen (1991, 1995), Johansen and Juselius (1990), Harris and Sollis (2003), Hamilton (1994), Hendry (1995), Pesaran and Smith (1998), and Pesaran, *et al.* (2000) among others.

Furthermore, considering the growth pattern of money and income variables a number of authors have used the intercept term unrestrictedly in the cointegration analysis of money demand function. These include, Johansen and Juselius (1990) for Finland, Petursson (2000) for Iceland, Brissimi, *et al.* (2003) for Greece, Artis and Beyer (2004) for Europe, Doornik, *et al.* (1998) for UK, Bruggeman (2000) for Europe and Boswijk and Doornik (2004), etc.

Plot of the series, not presented here, show that the real money and real income have linear trend and both variables grow over time. As suggested in the literature that in the presence of the variables showing growth, the intercept term cannot be used by restricting it into cointegrating space, we left intercept term unrestricted while doing cointegration analysis.

Table 2 reports the results of Johansen's likelihood ratio test based on maximal eigenvalue and trace statistics for testing of cointegrating relationship between the variables. The results indicate that there exist two cointegrating relations between real M2, real income, the rate of inflation, call money rate and long term government bond yield.

Table 2

Johansen Maximum Likelihood Procedure
(Variables = $LRM2_t$, LRY_t, INF_t, $LCMR_t$ and LBY_t, Order of VAR=2)

Null	Maximal Eigenvalue		Trace	
	Alternative	Chi-square	Alternative	Chi-square
r = 0	R = 1	46.23*	r ≥ 1	95.35*
r ≤ 1	R = 2	32.13*	r ≥ 2	49.11*
r ≤ 2	R = 3	13.27	r ≥ 3	16.98
r ≤ 3	R = 4	02.48	r ≥ 4	03.71
r ≤ 4	R = 5	01.23	r ≥ 5	01.23

Note: *Indicates significant at the 5 percent level.

In case of multiple cointegrating vectors, it is difficult to explain the results. However, traditionally, in such a situation the first vector (normalised on LRM2) is used as a long-run money demand function. Following this tradition, we present the estimated long-run money demand function by normalising the first cointegrating vector on LRM2 in the following. (Chi-square values are in parentheses.)

$$LRM2_t = 1.09\ LRY_t - 13.38\ INF_t + 1.08\ LCMR_t - 0.70\ LBY_t \quad \ldots \quad (5)$$
$$(6.59) \qquad (32.94) \qquad (15.69) \qquad (7.45)$$

It is revealed that in the long run demand for money is determined by income, rate of inflation, call money rate and government bond yield. The result is inline with the results of Qayyum (2001) regarding the significance and importance of the determinants of money demand in Pakistan. The most striking result is the high coefficient of inflation rate. The dominant role of inflation rate in affecting real M2 demand implies that economic agent prefer to invest in the real sector rather than to hold money. Furthermore, the high coefficient of the rate of inflation suggests that a one percentage point increase in inflation results in a 13 percent decrease in M2.

Therefore, it implies that the variability of inflation make the M2 targeting in Pakistan very difficult.[3] The difficulty in achieving in the targeted value of M2 is witnessed during previous decade in Pakistan (i.e., from 1994-2003). During the last decade on average the actual level of M2 remains 10.97 percent higher than the targeted level, with very high coefficient of variation. Whereas the rate of inflation is concerned, during the previous decade the average change in the rate of inflation remains less than one percentage point.

3. Dynamic Error Correction Model

From the long run estimated function (Equation 5) we obtained an error correction term. The error correction term is used as one of the determinants of the dynamic error correction money demand function. The function is thus estimated by OLS method by considering general to specific approach. The estimation process started with having lag length of two then tested down to the specific model. The preferred model is presented here (*t*-ratios are in parentheses);

$$\Delta LRM2_t = -0.10 + 1.02\,\Delta LRY_t - 0.94\,\Delta INF_t - 0.10\,ECM_{t-1} \quad \ldots \quad (6)$$
$$\quad (-3.04)\;(2.79) \qquad (-4.15) \qquad (3.73)$$

$R^2 = 0.54$ $F_{(3, 31)} = 12.31$ Auto $\chi^2_{(1)} = 1.34$ Norm $\chi^2_{(2)} = 0.30$ Hetro$\chi^2_{(1)} = 0.06$

The estimated model has many desirable statistical properties. The residual term is white noise and homoscedastic. Moreover, residual is normally distributed. The estimated parameters of the preferred Equation (6) satisfy the theoretical sign restrictions of the short run dynamics to be interpretable as the money demand function. The estimated model has important policy implications. The estimated coefficients imply that the immediate response of money demand for change in the rate of inflation and change in the real income. The current variability of rate of inflation enters significantly into the equation with estimated parameter near to unity. In this situation the preferred money demand Equation (6) can be written as $\Delta M2_t$ as a dependent variable, which implies that the current variability in the rate of inflation does not matter for nominal money in the short run.

The estimated constant term shows negative value. Burggeman (2000) and Choudhry (1999) have found negative constant terms for their dynamic money demand functions. Burggeman (2000) concluded that the estimated constant term has no direct implication since it indicates both the long run and the short run constant term.

However, it could be interpreted that there is decline in the unconditional growth in money (M2) demand during the study period. It implies that changing pattern of velocity of money (M2) in Pakistan.

[3]This point is suggested by the referee.

The estimated coefficient of error correction term has negative sign which is consistent with the theory. However the estimated value of parameters is low which indicates slow speed of adjustment towards equilibrium state. There can be several reasons for slow speed of adjustment. One reason that is often given is the minimal cost of being out of equilibrium. Another reason may be that the adjustment of actual money holding to equilibrium level is costly. The speed at which portfolio adjustment takes place depends on two types of costs that is the cost of moving to the new equilibrium and the cost of being out of equilibrium. Higher the ratio of the cost of moving to the new equilibrium relative to the cost of being out of equilibrium lowers the speed of adjustment [Thornton (1983)]. Therefore the economic agents may move slowly over time to adjust. It is also argued that the real side shocks such as natural disasters, oil prices etc., are responsible for long time persistence of disequilibrium [Thornton (1983)].

Further reason of low speed of adjustment may be the saving behaviour of the household sector. In Pakistan savings are held as part of M2. If precautionary savings depends on the long run consideration of future income and interest rates then we expect slow adjustment [Cuthbertson and Taylor (1990)]. Further, the saving behaviour in Pakistan is determined by the budget deficit, rate of inflation [Hook (1997)]. These variables remained on high side throughout the study period, so exerting pressure on the saving behaviour.

The parameter constancy of the conditional money demand function is important for the choice of the appropriate instrument of monetary policy. This property also plays a crucial role regarding the issue of exogeneity of the parameters of interest. For the purpose of checking the parameter constancy we used recursive least square estimation method. From this method we can easily construct CUSUM and CUSUMSQ [Brown, *et al.* (1975)] statistics. Figures 1 and 2 presented the graph of CUSUM and CUSUMSQ statistics with relevant standard errors. Furthermore, Figures 3–6 presented recursive coefficients of the constant, ΔINF_t, ΔLRY_t, and the error correction term (ECM_{t-1}) with sequential errors indicating 95 percent of confidence interval at each time. It shows that all coefficients are highly significant for all the samples.

Fig. 1. Plot of Cumulative Sum of Recursive Residual and 5 Percent Significance Level Critical Bands.

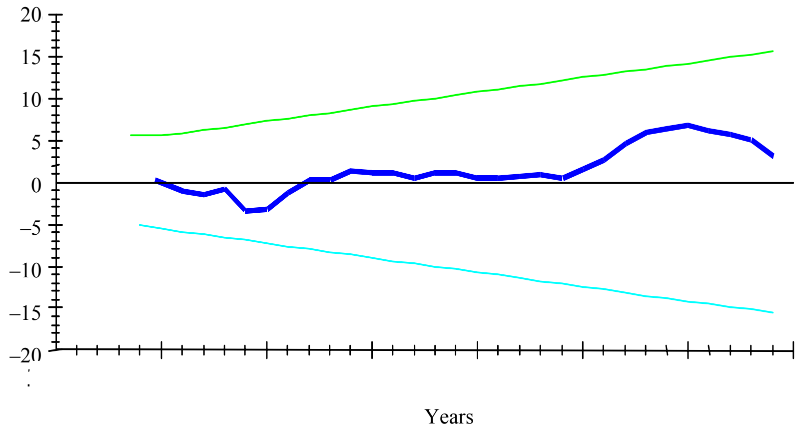


Fig. 2. Plot of Cumulative Sum of Squares of Recursive Residual and 5 Percent Significance Level Critical Bands.

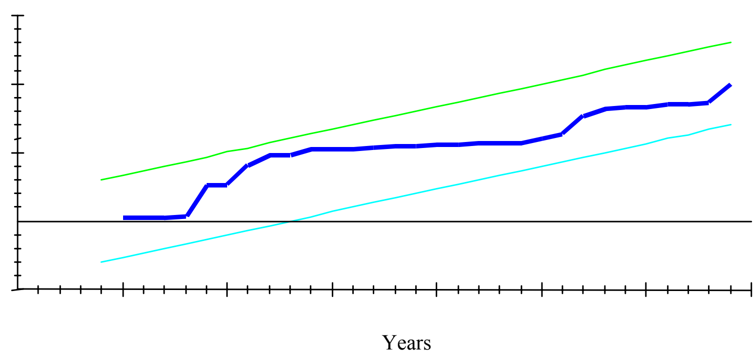


Fig. 3. Coefficient of Constant and Its 2 Standard Error Bands.

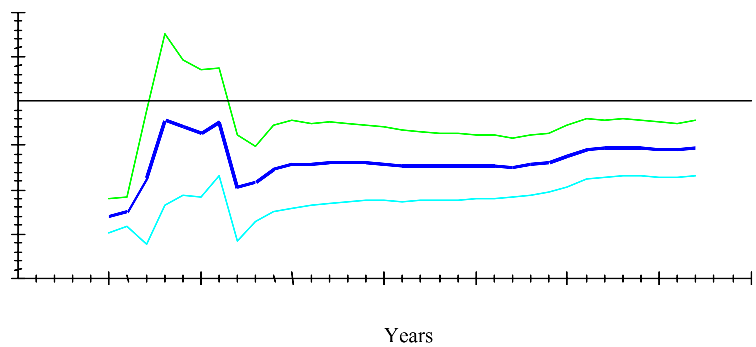


Fig. 4. Coefficient of ΔY_t and Its 2 Standard Error Bands.

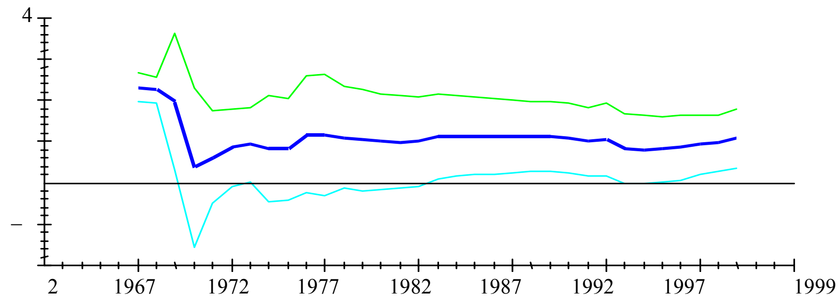


Fig. 5. Coefficient of ΔINF_t and Its 2 Standard Error Bands.

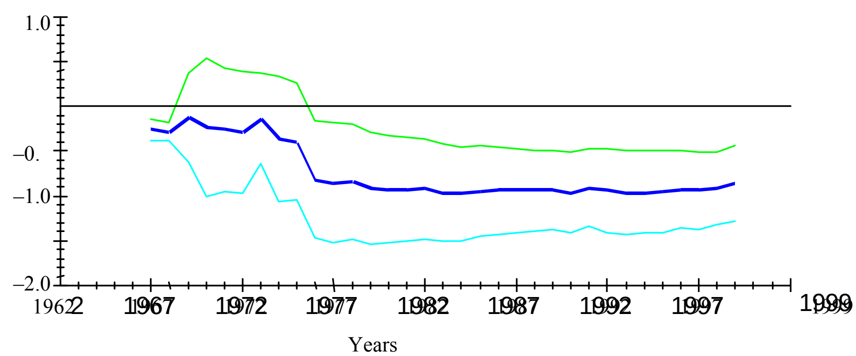
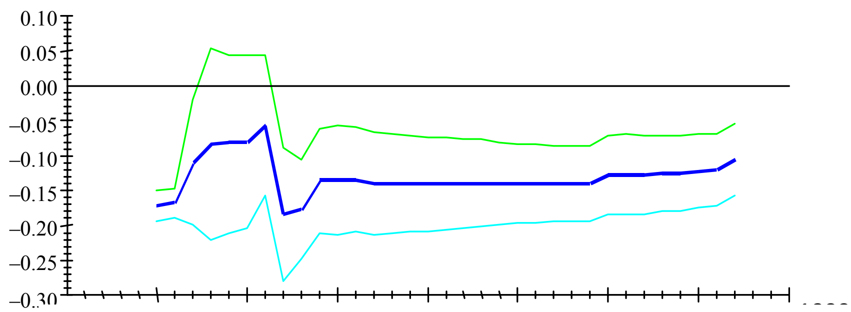


Fig. 6. Coefficient of ECM_{t-1} and Its 2 Standard Error Bands.



IV. TEST OF SUPER-EXOGENEITY

Sometime due to some external shocks such as changes in policy rule, financial innovation, oil shock, etc., the parameters of the reduced form model, like Equation 3, do not remain constant. Therefore the model having non-constant parameters could not be used for policy analysis [Lucas (1976)]. In this case the concept of super-exogeneity is crucial, because it has empirical consequences for Lucas's critique, instability of money demand function, and invariance of parameters of interest.

In order to test the super-exogeneity, we factorised the joint density function (Equation 3) into the conditional density function (Equation 4a) and the marginal density function (Equation 4b). The main objective of factorisation (Equation 4) is to isolate non-constancy of parameters into vector parameter of marginal process (i.e., λ_2) whereas the parameters of conditional model (i.e., λ_1) remain invariant to changes that have occurred. It implies that the external shocks such as policy change, financial innovation, etc., affected the parameters of marginal processes (i.e., λ_2), the inflation and income functions; and the parameters of the conditional model, the money demand function, are invariant to the change that have affected the marginal process. Therefore, tests for constancy of parameters play a pivotal role in order to test the super-exogeneity.

There are two tests that are used for testing for super-exogeneity. First test is to establish the constancy of the parameters of conditional model and the non-constancy of the parameters of marginal model. For this purpose the marginal model is obtained by inverting the conditional model. The super-exogeneity requires constancy of the conditional model and non-constancy of the marginal process. Therefore, under super-exogeneity the constant conditional money demand model is not interpretable as a re-parameterisation expectations model in which the re-parameterisation involves functions of the underlying structural parameters and the time dependent parameters of the marginal process. This implies that constant marginal model cannot be obtained by inverting conditional model. The non invertibility of the conditional model into marginal model can be used as evidence of super-exogeneity because the invertibility of conditional model into marginal model is precluded if the variables are super exogenous for the parameters of the conditional model [Hendry and Ericsson (1991)]. Therefore, finding the instability of marginal process and stability of conditional process is sufficient to conclude super-exogeneity [Parez (2000)]. The existence of super-exogeneity also implies weak exogeneity of current dated regressors.

Another test of super-exogeneity of parameters of interest against the external shocks that has changed the parameters of marginal density function is to develop the marginal model by adding dummies variables in the marginal process. Then add those dummy variables that are significant in the marginal model to the conditional model and test their significance by *F*-statistics. The *F*-statistics is calculated like a conventional test of joint significance of interventional variables in the conditional models [Engle and Hendry (1993)]. Therefore, the insignificance of dummy variables in the conditional model leads super-exogeneity of conditional model. Moreover, the introduction of dummy variable into the marginal processes and conditional process and their relevance in achieving the constancy of the marginal process and irrelevance in effecting the constancy the conditional process validates results of single equation.[4]

The preferred model (Equation 6) indicates that the real income and the rate of inflation enter into the model at their current rates. Therefore, to test the super-exogeneity we have to test, (i) the constancy of marginal processes of income and the rate of inflation and show that the marginal processes are non-constant, and (ii) the constancy of money demand model (Equation 6).

In order to test super-exogeneity of the parameter of the conditional money demand function against the known shocks, such as famous 1973 oil shock and exchange rate policy change from fixed to floating exchange in 1980, which could have affected the stability of the marginal process, we used dummy variable method proposed by Gujarati (1970). This method of dummy variables is used by Hendry and Ericsson (1991) while testing the super-exogeneity of the parameters of interest. The significance of dummy variable is tested by *t*-statistics for individual case, and *F*-test proposed by Engle and Hendry (1993) is used to test the joint significance of intervention dummies.

First we estimated univariate model of inflation and income to test the super-

[4]This point is also suggested by the referee.

exogeneity. Hendry and Ericsson (1991) and Cuthbertson (1988) have estimated autoregressive model in their analysis. We started with fifth-order autoregressive process of ΔINF_t and ΔRY_t. Final models are achieved by employing general to specific methodology. So by testing down the following specific models are obtained.

$$\underset{(2.27)}{\Delta INF_t = 0.03} + \underset{(5.31)}{0.66\,\Delta INF_{t-1}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (7)$$

$R^2 = 0.43$ $F = 28.2$ Auto $\chi^2(1) = 0.25$ Norm $\chi^2(2) = 1.70$ Hetro $\chi^2(1) = 1.07$

$$\Delta RY = \underset{(14.1)}{.053387} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

$R^2 = 0.00$ Auto $\chi^2(1) = 1.38$ Norm $\chi^2(2) = 0.08$

The stability of the estimated model is tested for the known shock. For this purpose we introduced identified dummy variable. The results are presented in the following two equations. We only reported the results containing significant dummies.

$$\Delta INF_t = \underset{(2.63)}{0.03} + \underset{(5.79)}{0.61\, INF_{t-1}} + \underset{(4.08)}{0.09D73} \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

$R^2 = 0.61$ $F = 28.3$ Auto $\chi^2(1) = 2..53$ Norm $\chi^2(2) = 0.71$ Hetro $\chi^2(1) = 0.27$

$$\Delta RY = \underset{(12.9)}{0.06} - \underset{(-2.5)}{0.02\, DEX} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (10)$$

$R^2 = 0.15$ $F = 6.31$ Auto $\chi^2(1) = 0.003$ Norm $\chi^2(2) = 0.2$ Hetro $\chi^2(1) = 0.25$

As may be seen the dummy variable D73 that is representing the oil shock significantly enters the marginal process of inflation (Equation 9) and the dummy variable indication exchange rate regime shift (DEX) is significant in the marginal equation representing income (Equation 10). The Oil shock shifted inflation function upwards whereas the exchange rate policy change pushed income growth downward. The significance of dummies implies non-constancy of marginal process against the known shocks.

Super-exogeneity has another implication that the constant conditional money demand model cannot be inverted to obtain the constant model of prices given money [Hendry and Ericsson (1991)]. Non-invertibility of money demand function also implies that only constant money demand function is not sufficient for policy implications. For the full policy implications we require additional information about the well-specified inflation equation. To test the super-exogeneity of inflation, we inverted the estimated money demand function (Equation 6) into inflation and income equations and tested the constancy of the marginal processes. The estimated model of inflation (inverted from the conditional model) along with the *t*-ratios (in the parentheses) is presented in the following equation.

$$\Delta INF_t = -0.05 + 0.31\ \Delta LRY_t - 0.38\ \Delta LRM2_t - 0.06\ ECM_{t-1} \quad \ldots \quad (11)$$
$$(-2.33)\ (1.21)\qquad (-4.15)\qquad (-3.10)$$

$R^2 = 0.39$ $F_{(3, 31)} = 6.72$ DW statistic = 1.69 Norm $\chi^2_{(2)} = 2.07$ Hetro $\chi^2_{(1)} = 0.38$

Income variable is not significant. Diagnostic test statistics show that residual term is serially correlated. Theoretically, one of the reasons of presence of serial correlation in the error term is mis-specification of the model. The mis-specification of the above Equation (11) has clear policy implications. It is that by simple inverting constant money demand function we cannot get well specified constant inflation equation. Another implication of super-exogeneity of inflation implies that the Lucas critique does not apply for relevant class of interventions [Hendry and Ericsson (1991)]. We have also estimated the equation by introducing shift dummies and presented below;

$$\Delta INF_t = -0.06 + 0.20\ \Delta LRY_t - 0.26\ \Delta LRM2_t - 0.06\ ECM_{t-1} + 0.07\ D73 \quad \ldots(12)$$
$$(-2.22)\ (0.71)\qquad (-2.59)\qquad (-3.20)\qquad (2.68)$$

$R^2 = 0.46$ $F_{(6, 32)} = 4.72$ DW statistic = 1.63 Norm $\chi^2_{(2)} = 1.15$ Hetro $\chi^2_{(1)} = 0.25$

As may be seen, the dummy for 1973 oil crisis (*D*73) has significant positive effect on the inflation process (Equation 12), indicating a shift in the estimated model.

Moreover, in order to establish the super-exogeneity of the parameters money demand function, we also estimated marginal function of income (i.e,. ΔLRY_t) by simply inverting the estimated money demand function. The ΔLRY_t model is presented below (*t*-ratios are in the parentheses);

$$\Delta LRY_t = 0.06 + 0.15\ \Delta INF_t + 0.19\ \Delta LRM2_t + 0.02\ ECM_{t-1} \quad \ldots \quad (13)$$
$$(+4.47)\ (1.21)\qquad (2.79)\qquad (1.12)$$

$R^2 = 0.21$ $F_{(3, 31)} = 2.71$ DW statistic = 1.60 Norm $\chi^2_{(2)} = 0.21$ Hetro $\chi^2_{(1)} = 0.004$

From the estimated model it is revealed that at the conventional level the estimated parameters of ΔINF_t and the error correction term (ECM_{t-1}) are insignificant. Moreover, the residual term is not white noise indicating mis-specification of the model. It could be due to inclusion of irrelevant variables or exclusion of important variables from the model. Though the model does not seem to be well specified to complete the process we have tested the stability of the model by including dummy variables for the events of 1973 oil crises (D73) and exchange rate regime shift in 1980 (*DEX*). The final form of the estimated model is presented model is;

$$\Delta LRY_t = 0.06 + 0.08\ \Delta INF_t + 0.19\ \Delta LRM2_t + 0.009\ ECM_{t-1} - 0.017\ DEX \quad (14)$$
$$(+4.47)\ (0.71)\qquad (3.31)\qquad (0.67)\qquad (-1.91)$$

$R^2 = 0.39$ $F_{(6, 32)} = 3.39$ DW statistic = 2.05 Norm $\chi^2_{(2)} = 0.06$ Hetro $\chi^2_{(1)} = 0.69$

The results show that the dummy variable indicating change in the exchange rate policy in 1980 is significant in ΔLRY_t Equation (14). This shows that this event of exchange rate regime change affected the model and introduced instability in the parameter of marginal process of income.

Finally, in order to the check the constancy of the preferred model against the identified events, we introduced dummy variables, that is *D*73 and *DEX*, in the preferred model (Equation 6). It is already proved that two of these dummy variables are already significantly entered into the marginal equations. The estimated model with these dummies is presented below (*t*-ratios are in the parentheses):

$$\begin{aligned} \Delta LRM2_t = & -0.10 + 1.03\ \Delta LRY_t - 0.68\ \Delta INF_t - 0.08\ ECM_{t-1} \\ & (-2.30)\quad (3.30)\qquad (-2.60)\qquad (-2.72) \\ & -0.03\ D73 + 0.01 DEX \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (15) \\ & (-0.80)\qquad (0.57) \end{aligned}$$

$R^2 = 0.54$ $F_{(6, 32)} = 6.35$ Auto $\chi^2_{(1)} = 0.69$ Norm $\chi^2_{(2)} = 0.16$ Hetro $\chi^2_{(1)} = 0.009$

The results show that the dummy variables that are significant in the marginal equations of income (Equation 14) and inflation (Equation 12) are individually and jointly insignificant when included in the preferred model of real money demand. The joint significance *F*-statistics is $F(2, 32)$ being 0.69. It is not surprising that the intervention variables are not significant in the preferred model because their influence is captured in the inflation and growth variables, as demonstrated by the marginal process (i.e., Equations 12 and 14).[5] This exercise also proves that the estimated parameters of the conditional mean equation remained stable against the identified external shocks. These identified shocks have caused instability in the parameters of marginal equations. Thus it can safely be said that the estimated model of money demand can be used for policy simulation.

V. SUMMARY AND CONCLUSIONS

The purpose of the paper is to estimate the dynamic demand for money function in Pakistan that could be used for policy analysis. The model is estimated by using long data set (i.e., 1960–1999) and taking care of time series properties of variables, ignored in earlier studies. The cointegration method is applied and the error correction specification is used in the analysis. Moreover, the CUSUM, the CUSUMQ, and the dummy variable method are carried for testing the stability of the model.

The analysis indicates that the measure of money (M2) seems to have a long-run stable relationship with variables like real income, rate of inflation, call money

[5]This point is suggested by the referee.

rate, and the government bond yield. In the long-run, the money demand appears to be dominantly affected by inflation rate. The intensity of inflation rates affecting M2 is even greater than that of real income which indicates a problem with targeting M2 in Pakistan. The preferred money demand function indicates that in the short-run, income and rate of inflation are important determinants. It is demonstrated that the variability of the rate of inflation is not important for growth in M2. An important part of the analysis deals with testing super-exogeneity of the parameters of the preferred model. The preferred model is found to be stable, whereas the marginal models could not survive against the stability test. The 1973 oil crisis shifted the inflation upwards, and the exchange rate regime shifted income growth downwards. This implies that the preferred model can be used for policy analysis in Pakistan.

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