Foreign Direct Investment, Child Labour and Unemployment of Unskilled Labour in a Dual Economy

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Abstract: Using a three-sector specific-factor Harris-Todaro type general equilibrium model the paper demonstrates how an inflow of foreign capital might produce favourable effect on the incidence of child labour in a small open dual economy. The welfare of the working families is likely to improve due to the policy even though the urban unemployment situation of unskilled labour may not get better.

Keywords: Child labour, general equilibrium, Harris-Todaro model, foreign capital, return to education, wage inequality.

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1. Introduction

Poverty has been attributed as the single largest factor behind the incidence of child labour in a developing economy. It compels people to have large families and children to go out in the job market and earn their own means of livelihood. However, empirical evidences that incomes of the poorer section of the population have not changed significantly in absolute terms\(^1\) and that the incidence of child labour has decreased satisfactorily in most of the developing economies\(^2\) suggest that that there are factors other than poverty that have contributed to the decline of the child labour problem in the liberalized era.

Empirical studies e.g. Cigno et al. (2002) and Neumayer and Soysa (2005) have reported that trade and investment reforms have produced a favorable impact on child labour. However, whatever little impact the liberalized policies have so far made on child labour must have come through channels other than the income effect. Unfortunately, the theoretical literature on how economic reforms can impinge on the incidence of child labour is yet to evolve\(^3\). The emergence of this literature is urgently needed that should identify the different channels through which

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1 See Wade and Wolf (2002), Khan (1998) and Tendulkar et al. (1996) in this context.

2 ILO (2006) has reported that the number of economically active children in the 5-14 age group declined by 11 per cent in 2004 from the 2000 figure. The decline is the sharpest for Latin America and Caribbean, whereas Asia and Pacific and Sub-Saharan Africa registered small decline in activity rates. For Asia and Pacific and Sub-Saharan Africa, child labour participation rate (5-14 age group) declined from 19.4 and 28.8 to 18.8 and 26.4, respectively (during 2000 to 2004). For Latin America and Caribbean, activity rate declined from 16.1 % in 2000 to 5.1 % in 2004.

3 Theoretical papers on child labour like Basu and Van (1998), Basu (1999), Ranjan (1999, 2001), Baland and Robinson (2000) and Jafarey and Lahiri (2002) have not discussed the relationship between economic reforms and the child labour incidence. Dwibedi and Chaudhuri (2010) is, however, a notable exception. But, they have considered a full-employment general equilibrium structure that does not take into consideration some of the essential features of a developing economy like imperfections and persistence of unemployment in the market for unskilled labour. Furthermore, Dwibedi and Chaudhuri (2010) paper does not examine the consequences of economic reforms on the welfare of the child labour supplying families.
economic reforms like an inflow of foreign capital can affect the child labour problem in a developing economy. The need for identifying the diverse routes has also been recognized by Neumayer and Soysa (2005).

In the circumstances, the present paper is designed to ascertain the different ways through which economic reforms can have a bearing on the incidence of child labour in a developing economy using a three-sector specific-factor Harris-Todaro type general equilibrium model. Although the model only analyzes the consequence of liberalized investment policies, it may be useful in studying the effects of trade reforms as well. The workers are not the owners of capital and the capitalist class does not supply child labour. The supply function of child labour of each working family is derived from its intertemporal utility-maximizing behaviour. Sector 1 in the general equilibrium model is the rural sector where child labour is used along with adult unskilled labour and capital to produce an agricultural commodity. Sector 2 is an urban sector where a low-skill manufacturing good is produced by means of adult unskilled labour and capital. Finally, sector 3, another urban sector, employs skilled labour and capital to produce a high-skill commodity. There are imperfections in the market for unskilled labour in sector 2 where the unskilled workers receive a high unionized wage while their counterparts in the rural sector earn a competitive wage. There is unemployment of unskilled labour in the urban sector. Using this setup it is found that inflows of foreign capital can indeed lower the problem of child labour by raising the return to education and the non-child income of the working households and by lowering the earning opportunities of children. Besides the paper shows that inflows of foreign capital are likely to improve the welfare of the child labour supplying families although the urban unemployment problem of the unskilled labour may not improve. The paper, thus, demonstrates that reduction in poverty is not a necessary condition for the problem of child labour to improve. There are factors other than reduction of poverty that not only mitigate the child labour problem but also improve the welfare of the families that supply child labour.

2. Derivation of family supply function of child labour

The supply function of child labour by each working family is determined from its intertemporal utility maximizing behaviour. Let us consider a two period optimizing problem of the representative working family consisting of one adult member (the guardian) and \( n \) number of children with \( n \geq 1 \). Staying in line with the traditional model of the household (Becker 1964),
we consider each household as a single decision-making unit. On behalf of the family the guardian unilaterally takes decision regarding allocation of consumption in the two periods and the labour supply of his children. The guardian in the first period works in the adult labour market and earns a wage $W_0$. In this period, he takes decision about his children’s work effort and schooling. $lC$ number of children are sent out to work at the wage rate $W_C$. The non-existence of a market for loans against future earnings compels the parent to use income from child work to smooth out the family consumption. The remaining children who are not sent out to work are sent to school. Hence $(n-lC)$ numbers of children are sent to school. So, $lC$ number of child workers earns the child wage $(W_C)$ in the first period and the unskilled adult wage $(W)$ in the second period while the remaining $(n-lC)$ children earn nothing in the first period and the skilled wage $(W_S)$ in the second period. In the presence of positive return on education, $W_S$ is greater than $W$. In the second period, the guardian earns nothing and lives on the income he receives from his children who have become adult workers by this time.

We assume that parent cares only about the lifetime family consumption and does not attach any value to the child’s leisure. The utility is therefore a function of consumption levels in the two periods (1 and 2). For algebraic simplicity we assume a logarithmic utility function with unitary intertemporal elasticity of substitution.

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4 $W_0$ can take two values, $W$ (unskilled wage) and $W_S$ (skilled wage), depending on the type of the representative working household.

5 There are informal credit markets in developing countries as a substitute to missing formal credit market, but they mainly deal with short-term loans. Poor households need long-term credit to be able to substitute for the foregone earnings of their children, which is missing in the developing countries. See for example, Baland and Robinson (2000), Jafery and Lahiri (2002), Ranjan (1999, 2001) in this context.

6 This is a simplifying assumption that ignores the existence of non-labour non-school goers.

7 Introduction of uncertainty in securing a skilled job in the second period would be an interesting theoretical exercise. However, the major findings of the model remain unaffected if the probability in finding a high-skill job is given exogenously.

8 This is a marked departure from the Basu and Van (1998) paper that considers an altruistic parent who cares about the well-being of his children and derives disutility from the labour supplied by his offspring.
\[ V = \log C_1 + \beta \log C_2 \]  
(1)

where \( \beta \) is the time discount factor.

The first period’s consumption \( (C_1) \) consists of wage income of the guardian and child wage income from the working children. So we have

\[ C_1 = (W_0 + l_C W_C) \]  
(2)

The second period’s consumption \( (C_2) \) can be thought of as the sum of skilled wage income of educated adult (schooled in the first period) workers and unskilled wage income of uneducated adult labourers (worked in the first period). Therefore, \( C_2 \) is given as follows:

\[ C_2 = (l_C W + (n - l_C) W_S) \]  
(3)

We assume that the only cost of education is the opportunity cost in terms of forgone earnings of children. \(^9\)

The guardian maximizes the lifetime utility (Equation (1)) with respect to \( l_C \) and subject to (2) and (3). Maximization gives the following first-order condition.

\[ \frac{(l_C W + (n - l_C) W_S)}{(W_0 + l_C W_C)} = \frac{\beta (W_S - W)}{W_C} \]  
(4)

Solving equation (4) the following child labour function by each working family is obtained.

\[ l_C = \frac{n W_S}{(1 + \beta)(W_S - W)} - \frac{\beta W_0}{(1 + \beta) W_C} \]  
(5)

The properties of the child labour supply function, given by (5), are as follows. An increase in current income, \( W_0 \), (income from non-child source) raises both \( C_1 \) and \( C_2 \) and hence lowers \( l_C \) through a positive income effect. An increase in the child wage rate implies an increase in the opportunity cost of education and hence leads to more child labour supply (i.e. less schooling). Any changes in skilled and/or unskilled wage affect the return to education and therefore influence the guardian’s decision regarding allocation of his children between education and

\(^9\) One can, of course, incorporate direct schooling cost without affecting the qualitative results of the model.
work. For example, an increase in skilled wage ($W_S$) or a decrease in unskilled wage ($W$) will make education more attractive and raises the number of school-going children from each family thereby lowering the supply of child labour by the household.

3. **The General Equilibrium Analysis**

We consider a small open dual economy with two broad sectors: rural and urban. The urban sector is further subdivided into two subsectors so that in all we have three sectors. Sector 1 produces an agricultural commodity, $X_1$, using adult unskilled labour ($L$), child labour ($L_C$) and capital ($K$). The capital-output ratio in sector 1, $a_{K1}$, is assumed to be technologically given.\footnote{Although this is a simplifying assumption it is not completely without any basis. Agriculture requires inputs like fertilizers, pesticides, weedicides etc. which are to be used in recommended doses. Now if capital is used to purchase those inputs, the capital-output ratio becomes constant technologically. However, even if the capital-output ratio is not given technologically the results of the paper still hold under an additional sufficient condition relating to partial elasticities of substitution between capital and other inputs in sector 1.}

Sector 2 is an urban sector that produces a low-skill manufacturing good, $X_2$, by means of capital and unskilled labour\footnote{Even if sector 2 is allowed to use child labour the results of model hold under different sufficient conditions containing terms of relative intensities in which child labour and other two inputs are used in the first two sectors.}. The presupposition that child labour is used only in the agricultural sector is simplifying. However, it is partly justified on the ground that more than 70 per cent of economically active children in the developing countries are engaged in agriculture and allied sectors and less than 9 per cent are involved in manufacturing (ILO (2002) report). Finally, sector 3, another urban sector, uses capital and skilled labour ($S$) to produce a high-skill commodity, $X_3$. Skilled labour is a specific input in sector 3 while child labour is specific to sector 1. Unskilled labour is imperfectly mobile between sectors 1 and 2 while capital is completely mobile among all the three sectors of the economy.

Sector 2 faces a unionized labour market where unskilled workers receive a contractual wage, $W^*$, while the unskilled wage rate in the rural sector, $W$, is market determined with $W^* > W$. The two wage rates are related by the Harris-Todaro (1970) condition of migration
equilibrium where the expected urban wage equals the rural wage rate. Hence, there is urban unemployment of unskilled labour. The capital endowment of the economy includes both domestic capital, $K_D$, and foreign capital, $K_F$. Incomes from foreign capital are completely repatriated. Sector 2 uses capital more intensively with respect to unskilled labour vis-à-vis sector 1. Production functions exhibit constant returns to scale with positive and diminishing marginal productivity to each factor. Markets, except the urban unskilled labour market, are perfectly competitive. All the three commodities are traded internationally. Hence their prices are given internationally. Finally, commodity 3 is chosen as the numeraire.

A general equilibrium of the system is represented by the following set of equations:

\begin{align*}
Wa_{L1} + W_C a_{C1} + Ra_{K1} &= P_1 \\
W^* a_{L2} + Ra_{K2} &= P_2 \\
W_s a_{S3} + Ra_{K3} &= 1
\end{align*}

where $a_{ji}$ are input-output ratios; and, $R$ is the return to capital.

\begin{align*}
a_{C1}X_1 &= L_C \\
a_{K1}X_1 + a_{K2}X_2 + a_{K3}X_3 &= K_D + K_F = K \\
a_{S3}X_3 &= S \\
a_{L1}X_1 + a_{L3}X_3 + L_U &= L
\end{align*}

Equations (6) – (8) are the three competitive industry equilibrium conditions in the three sectors. On the other hand, equations (9) – (11) are the full-employment conditions for child labour, capital\(^{12}\) and skilled labour, respectively. The unskilled labour endowment is given by (12).

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\nonumber
\text{\ref{footnote12} Even though the capital-output ratio in sector 1 is technologically given, adult labour and child labour are substitutes and the production function displays the constant returns to scale property in these two inputs.}

\nonumber
\text{\ref{footnote13} It is assumed that the capital stock of the economy consists of both domestic capital and foreign capital which are perfect substitutes. It may be mentioned that this assumption has been widely used in the theoretical literature on trade and development.}
Since the probability of finding a job in the low-skill urban manufacturing sector is \( a_{l,2}X_2 \left/ \left( a_{l,2}X_2 + L_U \right) \right. \), the expected unskilled wage in the urban area is \( (W^*a_{l,2}X_2) \left/ \left( a_{l,2}X_2 + L_U \right) \right. \). Therefore, the allocation mechanism of adult unskilled labour between rural and urban areas is expressed as

\[
(W^*a_{l,2}X_2) \left/ \left( a_{l,2}X_2 + L_U \right) \right. = W,
\]

or equivalently,

\[
(W^*/W)a_{l,2}X_2 + a_{i,1}X_1 = L \tag{13}
\]

The firms in the low-skill urban sector have well-organized trade unions. One of the most important roles of the labour unions is to bargain with their respective employers in respect of the betterment of the working conditions. Through offer of negotiation, threat of strike, actual strike etc. the trade unions exert pressure on the employers (firms) in order to secure higher wages, reduced hours of work, share in profits and other benefits. Organized workers in large firms leave no stones unturned so as to reap wages higher than their reservation wage i.e. the rural sector unskilled wage \(^{14}\). The relationship for the unionized wage rate is specified as \(^{15}\):

\[
W^* = W^*(W,U) \tag{14}
\]

This function satisfies the following properties.

\[
W^* = W \text{ for } U = 0; W^* > W \text{ for } U > 0; \text{ and, } (\partial W^* / \partial W), (\partial W^* / \partial U) > 0 .
\]

Equation (14) states that in the absence of any bargaining power of the trade unions i.e. when \( U = 0 \), the rural and the urban unskilled wage rates are equal. However, the urban sector wage rate, \( W^* \), exceeds the competitive rural sector wage rate, \( W \), when there is at least some power to the trade unions. The unionized wage is scaled upward as the rural sector wage rate rises. Also with an increase in the bargaining power, the unions bargain for a higher wage.

Using (14) equation (7) can be rewritten as

\(^{14}\) See Bhalotra (2002) in this context.

\(^{15}\) Assuming that each formal sector firm has a separate trade union, the unionized wage function may be derived as a solution to the Nash bargaining game between the representative firm and the representative union in the low-skill manufacturing sector. For detailed derivation see Chaudhuri and Mukhopadhyay (2009).
Both unskilled and skilled working families are potential suppliers of child labour and their current wage incomes \((W_0)\) are \(W\) and \(W_s\), respectively. Besides, there are \(L\) and \(S\) numbers of unskilled and skilled working families in the economy, respectively. Using equation (5) the aggregate child labour supply in the economy is obtained as follows.

\[
L_C = (\frac{1}{1+\beta})[L \left\{ \frac{nW_s}{(W_s - W)} - \frac{\beta W}{W_C} \right\} + S \left\{ \frac{nW_s}{(W_s - W)} - \frac{\beta W_s}{W_C} \right\}] \quad (15)
\]

Using (15) equation (9) can be rewritten as follows.

\[
a_{c1}X_1 = (\frac{1}{1+\beta})[L \left\{ \frac{nW_s}{(W_s - W)} - \frac{\beta W}{W_C} \right\} + S \left\{ \frac{nW_s}{(W_s - W)} - \frac{\beta W_s}{W_C} \right\}] \quad (9.1)
\]

4. **Comparative Statics**

The general equilibrium structure consists of nine equations, ((6), (7.1), (8), (9.1), (10) – (13)) and (15), and the same number of variables namely; \(W, W_C, W_s, R, X_1, X_2, X_3, L_C\) and \(L_U\). This is an indecomposable system. The factor prices depend on both commodity prices and factor endowments. Given the child wage rate, sectors 1 and 2 together effectively form a miniature Heckscher-Ohlin system as they use both adult unskilled labour and capital. It is sensible to assume that sector 1 is more adult labour-intensive than sector 2 with respect to capital. Totally differentiating equations (6), (7.1), (8), (9.1), (10) – (11) and (13) and solving by Cramer’s rule the following proposition can be established\(^{16}\).

**Proposition 1:** An inflow of foreign capital leads to (i) increases in both adult unskilled wage and skilled wage; (ii) a decrease in child wage rate; and, (iii) an expansion of the low-skill urban manufacturing sector. The skilled-unskilled wage inequality\(^{17}\) worsens if the high-skill sector is capital-intensive (in a special sense) relative to the low-skill sector.

\(^{16}\) These results have been derived in appendix I.

\(^{17}\) There are three groups of unskilled workers in this system earning different wages. Unskilled workers employed in the rural and the low-skill urban sectors receive a competitive wage, \(W\), and the unionized wage, \(W^*\), respectively while the unemployed urban workers earn nothing.
Proposition 1 can be intuitively explained in the following fashion. An inflow of foreign capital lowers the return on capital, \( R \), as the supply rises given the demand. A Rybczynski effect takes place in the miniature HOS system comprising of sectors 1 and 2 leading to a contraction of sector 1 and an expansion of sector 2 as the latter is more capital-intensive relative to the former sector with respect to adult unskilled labour. Sector 3 also expands as it uses capital but a different type of labour (skilled labour). The demand for child labour falls in sector 1 while that of skilled labour rises in sector 3 as these are the two sector-specific inputs. Consequently, the child wage falls while the skilled wage rises. Owing to reduction in capital cost, the unionized unskilled wage, \( W^* \), has to rise so as to satisfy the zero profit condition in sector 2 (see equation (7.1)). But, \( W^* \) can increase only if the competitive unskilled wage, \( W \), rises. Why \( W \) and \( W^* \) increase is easily understandable. Sector 2 expands both in terms of output and employment following an inflow of foreign capital. The expected urban wage for a prospective rural unskilled migrant rises unambiguously that paves the way for a fresh migration into the urban sector. The availability of unskilled labour in the rural sector falls, which in turn causes the rural unskilled wage, \( W \), to rise. \( W^* \) also rises as \( W \) rises. What happens to the skilled-unskilled wage inequality depends on the rates of increase in \( W \) and \( W^* \). If \( \theta_{K2} / \theta_{L2} < (\theta_{K3} / \theta_{L3}) \) the saving on capital cost in the low-skill manufacturing sector (sector 2) is less than that in the high-skill sector, which in turn, implies that the rate of increase of the unionized unskilled wage, \( W^* \) (and hence that of \( W \) as \( 1 > E_W \geq 0 \)), is smaller than that of the skilled wage, \( W \). Thus, the wage inequality worsens if the low-skill manufacturing sector is less capital-intensive vis-à-vis the high-skill sector in a special sense.  

The average wage for unskilled labour is given by \( \bar{W}_A = (\lambda_{L1} W + \lambda_{L2} W^*) \) where \( \lambda_{L1} \) and \( \lambda_{L2} \) denote the proportion of unskilled labour employed in sectors 1 and 2, respectively. Using (13), we can write \( \bar{W}_A = W \). It may be mentioned that the average wage of the workers (unskilled workers in this case) in a Harris-Todaro economy is equal to the rural sector wage. This is known as the ‘envelope property’. 

\( ^{18} \) Here sectors 2 and 3 use two different types of labour. However, there is one intersectorally mobile input which is capital. So, these two sectors cannot be classified in terms of factor intensities which is usually done in the Heckscher-Ohlin-Samuelson model. Despite this, a special type of factor intensity classification in terms of the relative distributive shares of the mobile factor i.e. capital can be made for analytical purposes. The sector in which this share is higher relative to the other may be considered as capital-intensive in a special sense. See Jones and Neary (1984) for details.
To analyze the outcome of foreign capital inflows on the supply of child labour in the economy after totally differentiating equation (1.5) the following proposition can be proved.\footnote{The proof is available in appendix II.}

**Proposition 2:** An inflow of foreign capital lowers the incidence of child labour in the economy if the high-skill sector is capital-intensive relative to the low-skill sector.

We intuitively explain proposition 2 as follows. In proposition 1 we have stated how different factor prices and the relative wage inequality respond to inflows of foreign capital. A fall in the child wage rate, $W_C$, means a decrease in the opportunity cost of education. On the other hand, the return to education rises as the wage inequality rises. Finally, the initial incomes from non-child source of both the unskilled and skilled working families have increased which lower the supply of child labour by each family via the positive income effect. Hence, under the sufficient condition that the high-skill sector is capital-intensive all these three effects work in the same direction and lower the problem of child labour in the society.

Let us now turn to analyze the outcome of an inflow of foreign capital on the unemployment of unskilled labour in the urban sector. Subtraction of (12) from (13) yields

$$L_U = a_{t_2}X_2\left(\frac{W^*}{W} - 1\right)$$

(16)

Differentiating (16) the following proposition can be established.\footnote{See appendix III for detailed derivations.}

**Proposition 3:** An inflow of foreign capital produces an ambiguous effect on the unemployment of unskilled labour in the urban sector.

We explain proposition 3 in the following manner. In the migration equilibrium the expected urban wage for a prospective unskilled rural migrant equals the actual unskilled rural wage. An inflow of foreign capital affects the migration equilibrium in two ways. First, the low-skill urban manufacturing sector expands following a Rybczynski effect. This leads to an increase in the number of jobs available in this sector. The expected urban wage for a prospective rural migrant,$[W^* / \{1 + (L_U / a_{t_2}X_2)\}]$, increases as the probability of getting a job in this sector rises for every unskilled worker. This is the centrifugal force which paves the way for fresh migration
from the rural to the urban sector. Second, an inflow of foreign capital raises the rural unskilled wage (see proposition 1). This is the centripetal force that prevents rural workers from migrating into the urban sector. Thus, there are clearly two opposite effects working on determination of the size of the unemployed urban unskilled workforce. The net effect on unemployment is, therefore, ambiguous.

Finally, we would like to examine the consequence of foreign capital inflows on the welfare of the child labour supplying families. Differentiating equations (1) – (3) and using (5) the final proposition of the model can be established as follows.  

**Proposition 4:** An inflow of foreign capital improves the welfare of the child labour supplying families if $\beta \geq nW_5(W_5 - W)$.

As the two wage rates and the relative wage inequality increase and the incidence of child labour declines the consumption of the household in period 2, $C_2$, rises unequivocally owing to inflows of foreign capital. However, the effect on the consumption level in period 1, $C_1$, is not so obvious. This is because $C_1$ rises as $W_0$ rises while it falls as the income from child labour, $W_cl_c$, declines. However, the expansionary effects on $C_2$ outweigh the negative effects on $C_1$ under the sufficient condition as stated in the proposition. Consequently, the welfare of the working families improves.

**5. Concluding remarks**

The paper has identified the different channels through which liberalized economic policies can influence the incidence of child labour in a developing economy in terms of a three-sector specific-factor Harris-Todaro type general equilibrium model. The existence of imperfections in the market for unskilled labour and the persistence of unemployment in the urban sector have been taken into consideration. The interesting result is that inflows of foreign capital might exert a downward pressure on the child labour incidence by raising the return to education (premium on skill) and the initial non-child incomes of the working families and by lowering the child wage i.e. the opportunity cost of schooling. Hence the child labour incidence may improve even if non-child incomes of the families do not increase. There are enough other forces brought about by

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21 This result has been proved in appendix IV.
economic reforms that can overcompensate for decreased parental incomes. Hence, reduction of poverty is not a necessary condition for the problem of child labour to improve in the developing economies following economic reforms. These results are consistent with empirical findings that the incidence of child labour has decreased at least in relative terms although the problem of poverty has increased in many developing countries following economic reforms. Besides, the analysis shows that inflows of foreign capital are likely to improve the welfare of the families that supply child labour although the urban unemployment problem of unskilled labour may not improve.

References:


Appendices

Appendix I:

Totally differentiating equations (6), (7.1) and (8) and using envelope conditions the following expressions are obtained.

\[ \theta_{El1}\hat{W} + \theta_{c1}\hat{W}_C + \theta_{K1}\hat{R} = 0 \quad (A.1) \]

\[ \theta_{E2}\hat{W} + \theta_{K2}\hat{R} = 0 \quad (A.2) \]

\[ \theta_{S3}\hat{W}_S + \theta_{K3}\hat{R} = 0 \quad (A.3) \]

where: \( \theta_{ji} \) = distributive share of the \( j \) th input in the \( i \) th sector; and, 'ˆ' = proportional change.

Totally differentiating equations (9.1), (10), (11) and (13), collecting terms and simplifying we get the following expressions.

\[ \bar{S}_{LL}\hat{W} + \lambda_{L1}\bar{S}_{LC}\hat{W}_C + \bar{S}_{LK}\hat{R} + \lambda_{L1}\hat{X}_1 + \lambda_{L2}\hat{X}_2 = 0 \quad (A.4) \]

\[ \bar{S}_{KL}\hat{W} + A_2\hat{R} + A_4\hat{W}_S + \lambda_{K1}\hat{X}_1 + \lambda_{K2}\hat{X}_2 = \hat{K} \quad (A.5) \]

\[ (S^1_{CL} + E)\hat{W} + (S^1_{CC} - F)\hat{W}_C + G\hat{W}_S + \hat{X}_1 = 0 \quad (A.6) \]

[Note that we have used \( \hat{X}_3 = -S^3_{SS}\hat{W}_S - S^3_{SR}\hat{R} \) from (11)]

where:

\[ \bar{S}_{LL} = [\lambda_{L1}^* (E_{W1} - 1) + S^2_{LL}] + \lambda_{L1}\bar{S}_{LC} ] < 0; \quad \bar{S}_{LK} = \lambda_{L2}^* S^2_{LK} > 0; \]

\[ \bar{S}_{KK} = (\lambda_{K2}^2 S^2_{KK} + \lambda_{K2}^3 S^3_{KK}) < 0; \quad \bar{S}_{KL} = \lambda_{K2}^2 S^2_{KL} > 0; \quad A_1 = \lambda_{K3}(S^3_{SK} + S^3_{SS}) > 0; \]

\[ A_2 = (\bar{S}_{KK} - \lambda_{K3}^3 S^3_{SK}) < 0; \quad \lambda^*_{L2} = \frac{W^*}{W} - \lambda_{L2} > 0; \quad (A.7) \]

\[ A = \frac{W^*}{W} > 0; \quad B = \frac{\beta}{(1 + \beta)W} > 0. \]

\[ E = (-nA(L + S) + BLW); \quad F = B(LW + SW^*_S) > 0; \quad G = [nA(L + S) + BSW^*_S] > 0; \]

\( S^k_{ji} \) = the degree of substitution between factors \( j \) and \( i \) in the \( k \) th sector, \( j,i = L,S,L_C,K \); and, \( k = 1,2,3 \). \( S^k_{ji} > 0 \) for \( j \neq i \); and, \( S^k_{ji} < 0 \); and, \( \lambda_{ji} \) = proportion of the \( j \) th input employed in the \( i \) th sector.

Arranging (A.1) – (A.6) in the matrix notation we get the following.
Solving (A.8) by Cramer’s Rule the following expressions are obtained.

\[
\hat{W} = -\frac{\theta_{s3}\theta_{c1}\theta_{k2}\lambda^*_{l2}}{\Delta}\hat{K}
\]

(A.9)

\[
\hat{W}_c = \frac{\theta_{s3}\theta_{l2}\lambda^*_{l1}}{\Delta}\hat{K}
\]

(A.10)

\[
\hat{R} = \frac{\theta_{s3}\theta_{c1}E_w\theta_{l2}\lambda^*_{l2}}{\Delta}\hat{K}
\]

(A.11)

\[
\hat{W}_s = -\frac{\theta_{k1}\theta_{c1}E_w\theta_{l2}\lambda^*_{l2}}{\Delta}\hat{K}
\]

(A.12)

\[
(\hat{W}_s - \hat{W}) = -\frac{\theta_{c1}\lambda^*_{l2}(\theta_{l2}E_w\theta_{k3} - \theta_{s3}\theta_{k2})}{\Delta}\hat{K}
\]

(A.13)

\[
\hat{X}_2 = \frac{\hat{K}}{\Delta}[-\theta_{s3}\lambda_{l1}\theta_{l2}\lambda_{l1}^*|S_{lc}^i - (S_{cc}^i - F) - \theta_{l2}\theta_{c1}E_w(\theta_{k3}\lambda_{l1} + \theta_{s3}\bar{S}_{lk})

+ \theta_{c1}\theta_{k2}\theta_{s3}(\bar{S}_{ll} - \lambda_{l1}(S_{cl}^i + E))]
\]

where,

\[
\Delta = -\theta_{k3}\theta_{c1}E_w\theta_{l2}\{BSW_S|\lambda| + A_{l2}\lambda_{l2}^*\} - \theta_{s3}\theta_{c1}\theta_{l2}E_w(\bar{S}_{lk}\lambda_{k2} + A_{l2}\lambda_{l2}^*)

+ \theta_{s3}\theta_{c1}\theta_{k2}\{S_{cc}^i - F|\lambda| - \lambda_{k2}\lambda_{l1}S_{lc}^i\}

- \theta_{s3}\theta_{c1}\theta_{k2}\{(S_{cl}^i + BLW)|\lambda| - (\lambda_{k2}\bar{S}_{ll} - \lambda_{l2}\bar{S}_{kl})\}

+ nA(L+S)\theta_{c1}\lambda_{l1}|\lambda|(|\theta_{k2} - \theta_{k3})
\]

\[|\lambda|_{lk} = \lambda_{l1}\lambda_{k2} - \lambda_{k1}\lambda_{l2}^* > 0; \text{ and,} \]

\[|\theta|_{lk} = (\theta_{l1}\theta_{k2} - \theta_{k1}\theta_{l2}E_w) > 0
\]

(A.16)
(Note that $|\hat{\theta}| > 0$ as sector 2 is more capital-intensive than sector 1 with respect to adult unskilled labour).

Using (A.7) and (A.16) from (A.15) it follows that
\[ \Delta < 0 \text{ if } \theta_{K3}E_w\theta_{L2} > \theta_{K2}\theta_{S3}. \] (A.17)

However, $\theta_{K3}E_w\theta_{L2} > \theta_{K2}\theta_{S3}$ is only a sufficient condition for $\Delta$ to be negative.

Using (A.7), (A.15) and (A.16) from (A.9) – (A.14) we can obtain the following results.
\[
\begin{align*}
(i) \quad & \hat{W} > 0 \text{ when } \hat{K} > 0; \\
(ii) \quad & \hat{W}_C < 0 \text{ when } \hat{K} > 0; \\
(iii) \quad & \hat{R} < 0 \text{ when } \hat{K} > 0; \\
(iv) \quad & \hat{W}_S > 0 \text{ when } \hat{K} > 0; \\
(v) \quad & (\hat{W}_S - \hat{W}) > 0 \text{ when } \hat{K} > 0 \text{ iff } \theta_{K3}E_w\theta_{L2} > \theta_{K2}\theta_{S3}; \\
(vi) \quad & \hat{X}_2 > 0 \text{ when } \hat{K} > 0.
\end{align*}
\] (A.18)

Appendix II:

We use equation (15) to examine the impact of foreign capital inflows on the incidence of child labour in the economy. Totally differentiating equation (15) we get
\[
\dot{\hat{L}}_c = -nA(L+S)(\hat{W}_S - \hat{W}) - LBW\hat{W} - SBW_S\hat{W}_S + BW_C(W_S + LW) \tag{A.19}
\]

Using (A.9) – (A.13), the expression (A.19) may be rewritten as follows.
\[
\begin{align*}
\dot{\hat{L}}_c &= \left(\frac{1}{\Delta}\right)[nA(L+S)\theta_{c1}\lambda^*_L (\theta_{K3}E_w\theta_{L2} - \theta_{S3}\theta_{K2}) + LBW\theta_{S3}\theta_{c1}\theta_{K2}\lambda^*_L + SBW_S\theta_{K3}\theta_{c1}\theta_{L2}\lambda^*_L \\
&\quad + B(SW_S + LW)\theta_{S3}|\theta|\lambda^*_L]\hat{K} \tag{A.20}
\end{align*}
\]

From (A.20) we find that.
\[ \hat{L}_c < 0 \text{ when } \hat{K} > 0 \text{ if } \theta_{K3}E_w\theta_{L2} > \theta_{S3}\theta_{K2}. \]
So, the incidence of child labour decreases following inflows of foreign capital under the sufficient condition: \( \theta_{K,1}E_w\theta_{L,2} > \theta_{S,3}\theta_{K,2} \). This implies that sector 3 is capital-intensive relative to sector 2. However, this result may hold under other sufficient conditions as well.

**Appendix III:**

Differentiating (16) one gets

\[
\frac{\dot{L}_U}{K} = \left(\frac{\dot{X}_2}{K}\right) - [S_{LK}^2 E_w + \left(\frac{\lambda_{L,2} + \lambda_{LU}}{\lambda_{LU}}\right)(1 - E_w)]\left(\frac{\dot{W}}{K}\right) + S_{LK}^2 \left(\frac{\dot{R}}{K}\right) \tag{A.21}
\]

Using (A.9), (A.11) and (A.14) and simplifying from the above equation we obtain

\[
\frac{\dot{L}_U}{K} = \left(\frac{1}{\Delta}\right)\left[\theta_{S,3}\{\theta_{C,1}\theta_{K,2}(S_{LL} - \lambda_{L,1}S_{CL}^1) - \lambda_{L,1}\theta_{C}[S_{LC}^1 + S_{CL} + F]\}
- nA(L + S)\theta_{C,1}\lambda_{L,1}(\theta_{K,3}E_w\theta_{L,2} - \theta_{S,3}\theta_{K,2})\right]
- \left(\frac{\theta_{C,1}\theta_{K,2}\theta_{S,3}}{\Delta}\right)\left[\lambda_{L,1}B\left(\frac{\theta_{L,2}E_w\theta_{K,3}S_{W,5}}{\theta_{K,2}\theta_{S,3}} + WL\right) - \lambda_{L,2}^*E_w S_{LK}^2 + \left(\frac{\lambda_{L,2} + \lambda_{LU}}{\lambda_{LU}}\right)(1 - E_w)\right] \tag{A.22}
\]

\[
\frac{\dot{L}_U}{K} > 0 \text{ if } \left[\frac{\lambda_{L,1}^*B\left(\frac{\theta_{L,2}E_w\theta_{K,3}S_{W,5}}{\theta_{K,2}\theta_{S,3}} + WL\right)}{\lambda_{L,2}^*} \geq \frac{E_w S_{LK}^2 + \left(\frac{\lambda_{L,2} + \lambda_{LU}}{\lambda_{LU}}\right)(1 - E_w)}{}\right] \tag{A.23}
\]

**Appendix IV:**

Differentiation of equation (1) yields

\[
dV = \dot{C}_1 + \beta\dot{C}_2 \tag{A.24}
\]

Substituting the expression for \( l_c \) from (5) into (2) and (3) and simplifying one gets

\[
C_1 = \left[\frac{W(W_{S,5} - W) + nW_{c}W_{S,5}}{(1 + \beta)(W_{S,5} - W)}\right]; \text{ and,}
C_2 = \beta\left[\frac{W(W_{S,5} - W) + nW_{c}W_{S,5}}{(1 + \beta)W_{c}}\right] \tag{A.25}
\]

Differentiating (A.25) and (A.26) we respectively, find
\[
\hat{C}_1 = \left[ \frac{(W_s - W)[W(W_s - W)^2 \hat{W} + nW_c W_s \{(W_s - W)\hat{W}_c - W(\hat{W}_s - \hat{W})\}]}{[W(W_s - W) + nW_c W_s]} \right]; \text{and,} \quad (A.27)
\]

\[
\hat{C}_2 = \left[ \frac{W_c [nW_s \hat{W}_s + (\frac{W}{W_c}) \{W_s \hat{W}_s + \hat{W}_s - W\} - (W_s - W)\hat{W}_c \}}{[W(W_s - W) + nW_c W_s]} \right]
\]

Substitution of the expressions for \( \hat{C}_1 \) and \( \hat{C}_2 \) into (A.24) and simplification produce

\[
dV = \left[ \frac{1}{[W(W_s - W) + nW_c W_s]} \right] [W(W_s - W)\hat{W}_s^2 + nW_c W_s \{(W_s - W)^2 + nW_c W_s + \beta \}]
\]

\[
+ \beta W(W_s \hat{W}_s - W\hat{W})
\]

\[
+(W_s - W)\hat{W}_c \{nW_c W_s (W_s - W) - \beta W\} + nW_c W_s \hat{W}_s \{\beta - W(W_s - W)\}
\]

or,

\[
\frac{dV}{dK} = \left[ \frac{1}{[W(W_s - W) + nW_c W_s]} \right] [(W_s - W) \left( \frac{dW}{dK} \right)(\{(W_s - W)^2 + nW_c W_s + \beta \})
\]

\[
+(+) \quad (+) \quad (+)
\]

\[
+ \beta W \left( \frac{dW_c}{dK} - \frac{dW}{dK} \right)
\]

\[
+(+) \quad (+)
\]

\[
+(W_s - W) \left( \frac{dW_c}{dK} \right) \{nW_s (W_s - W) - W\} + nW_c \left( \frac{dW_s}{dK} \right) \{\beta - W(W_s - W)\} \]
\]

(A.29)

From (A.29) it follows that

\[
\left( \frac{dV}{dK} \right) > 0 \text{ if } \beta W \geq nW_c W_s (W_s - W)
\]

(A.30)

As \( n \geq 1; W_s > W \); and, \( W_c < W \) from (A.30) it follows that

\[
\left( \frac{dV}{dK} \right) > 0 \text{ if } \beta \geq nW_s (W_s - W)
\]

(A.31)

However, from (A.29) it is easily seen that \( \beta \geq nW_s (W_s - W) \) is only a sufficient condition for \( \left( \frac{dV}{dK} \right) > 0 \). One can find out several other sufficient conditions under which \( \left( \frac{dV}{dK} \right) > 0 \).