Taxation without Commitment

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Abstract

This paper considers a Ramsey model of linear capital and labor income taxation in which a benevolent government cannot commit ex-ante to a sequence of taxes for the future. In this setup, if the government is allowed to borrow and lend to the consumers, the optimal capital income tax is zero in the long run. This result stands in marked contrast with the recent literature on optimal taxation without commitment, which imposes budget balance and typically finds that the optimal capital income tax does not converge to zero. Since it is efficient to backload incentives, breaking budget balance allows the government to generate surplus that reduces its debt or increases its assets over time until the lack of commitment is no longer binding and the economy is back in the full commitment solution. Therefore, while the lack of commitment does not change the optimal capital tax in the long run, it may impose an upper bound on the level of long run debt.

Keywords: Fiscal Policy, Optimal Taxation, Incidence, Debt
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1 Introduction

This paper explores the issue of optimal capital and labor income taxation when the government cannot commit to future taxes. By allowing the government to borrow and lend from households, the model generates results substantially different from the ones found by the previous literature on taxation without commitment. The reason for this is that governments with more assets need to use less distortionary taxation, which means that the incentive to default can be reduced by allowing asset accumulation.

A traditional question in the optimal taxation literature concerns the extent to which capital taxes should be used to finance public spending. While in the short run it is optimal to tax capital to collect costless revenue from a sunk investment, in the long run using this source of taxation will distort the accumulation of capital. Chamley (1986) and Judd (1985) show that in an economy of infinitely lived agents capital taxes lead to intertemporal distortions that compound over time, creating an infinite wedge between marginal utility in different periods. Therefore, in the long run, the capital income tax should asymptote to zero.

It has been believed the result of zero capital taxes in the long run critically hinges on the ability of the government to commit ex-ante to a sequence of future taxes. Namely, Judd (1985) says that his "results indicate that redistribution of income through capital income taxation is effective only if it is unanticipated and will persist only if policy-makers cannot commit themselves to low taxation in the long run." Later work by Benhabib and Rustichini (1997) and Phelan and Stachetti (2001) confirms this intuition by finding that when commitment binds, the long run capital tax will not be zero. Using numerical simulations, Fernadez-Villaverde and Tsyvinsky (2002) find that, in general, commitment will bind if the government is impatient enough, since the future reward of a better equilibrium will not be enough to prevent the government from deviating from the predefined plan. However, all these papers assume that the government has to keep budget balance in each period.

This paper shows that if instead the government is allowed to borrow and lend to consumers, the optimal capital tax still converges to zero in the long run, as in the full commitment case. The reason for this is that a government with a large amount of assets will not have an incentive to default since it does not need to use much distortionary taxation to finance its spending. Thus, governments can use asset accumulation (or debt reduction) as a commitment device for the future. As long as commitment binds, there will be an incentive to increase the government's assets. This is consistent with the result found independently by Dominguez (2006), who analyzes the model in Benhabib and Rustichini (1997) for the case where bonds are allowed but the value of default is exogenous and depends only on capital.

Although the economy without commitment converges to a steady state where commitment does not bind and capital income taxes are zero, some steady states that were feasible in the economy with commitment will never be reached without commitment. If the economy with commitment converged to a steady state with high government debt that is no longer sustainable without commitment, then in the economy without commitment the government will have to accumulate more assets in the short run and will converge to a new steady state with lower debt. Hence, while the lack of commitment does not change the optimal capital tax in the long run, it may impose an upper bound on the long run level of debt.

A rather unexpected consequence of the lack of commitment is that capital levels will tend to be higher in the long run when there is no commitment. This happens because the government has to accumulate assets to overcome its commitment problem and will therefore be richer in the long run. This allows labor taxes to be lower, which in turn increases labor supply. Higher labor will make capital more productive, which implies that capital will also be higher in steady state.

An interesting feature of the short run dynamics is that as long as commitment binds, capital may either be taxed or subsidized, depending on whether increasing capital makes the commitment constraint slacker or tighter. Numerical simulations will show an example where capital is being subsidized in the short run, so that the capital level is higher in the economy without commitment at all times.

On a more technical side, this paper provides a setup where the worst sustainable equilibrium can be determined in advance. Benhabib and Rustichini (1997) derive the best policy without commitment assuming that the worst punishment is known. Phelan and Stachetti (2001) argue that this is not always the case since the government's incentive constraint usually binds in the worst equilibrium, which means that the worst punishment has to be determined endogenously. This paper provides a sufficient condition for these two approaches to be equivalent. If the government is allowed to make lump sum transfers to consumers, which is a common assumption in most taxation models, then it is always credible to give the households the worst possible expectations regarding future capital taxes, since it is incentive compatible for the government to tax the initial sunk capital at maximal rates, given that any remaining revenue can be redistributed to consumers as a lump sum transfer. Thus, no incentives need to be given for the government to act according to consumers' expectations, which means that the continuation of a worst equilibrium is still a worst equilibrium in this model, which allows us to determine the worst sustainable equilibrium in advance, as was assumed in Benhabib and Rustichini (1997).

We can interpret the long run results in this paper as an example of backloading of incentives, which is also present in models of commitment in other settings, such as Kocherlakota (1996), Ray (2002), or Acemoglu, Golosov and Tsyvinsky (2005). The idea is that in order to make the government's choice incentive compatible at all points in time, it is optimal to provide rewards as far off in the future as possible, since this provides incentives in all periods until then. Here, in particular, the backloading of incentives is achieved by letting the government increase its assets until the lack of commitment stops binding. This mechanism was not allowed by previous models that imposed budget balance.

This paper is also related to the work of Klein and Rios-Rull (2002), Klein, Krusell and Rios-Rull (2004), Klein, Quadrini and Rios-Rull (2005), and Klein, Krusell and Rios-Rull (2006), who look at time consistent Markov equilibria in taxation models. Since Markov equilibria preclude the use of trigger strategies, the set of equilibria that can be implemented is significantly smaller and in general a steady state with zero capital taxes will not be optimal even if the government is allowed to break budget balance. An exception to this is provided by Azzimonti-Renzo, Sarte and Soares (2006), where zero capital and labor income taxes are reached in the long run by collecting enough capital taxes in the initial periods to finance all future government spending. Although this paper reaches somewhat similar conclusions to ours, the mechanism at work is not the same. In Azzimonti-Renzo, Sarte and Soares (2006), capital accumulation occurs because in the short run capital is sunk, and it is in the government's best interest to use non distortionary taxation to finance future spending. As a consequence, asset accumulation will not stop until the government has enough assets to finance all future spending. Here, on the other hand, asset accumulation is used to make the future without default better, so that the incentive to default is reduced. Thus, asset accumulation stops when the incentive constraint for the government stops binding, which happens before the government's asset limit is reached, which means there is still positive labor taxation in the long run. Furthermore, along the transition path the predictions of the two models are significantly different, since here capital may even be subsidized in the short run if higher capital levels loosen the government's incentive constraint.

The game played between households and the government builds on the stream of literature developed by Chari and Kehoe (1990) on sustainable equilibria, which allows a more parsimonious definition of subgame perfect equilibria when some agents are too small to behave strategically. Chari and Kehoe (1993a and 1993b) use a setup without capital to model debt default. They allow for government default, but they either assume that households can commit to their debt, or debt repayment cannot be enforced at all. This paper, on the other hand, allows households to default, but it also allows the government to punish them if they do so, which makes household default non trivial.

The paper proceeds as follows. Section 2 sets up the model with commitment and derives the optimal ex-ante plan for the government. Section 3 relaxes the assumption of commitment and characterizes the set of sustainable equilibria without commitment. Section 4 derives the best sustainable equilibrium under no commitment and analyses its long run properties. Section 5 presents a numerical example with short run dynamics and steady state results. Section 6 concludes with a brief summary of the main findings of the paper.

2 Taxation with Commitment

This section introduces the economy with commitment. It characterizes allocations which are attainable under commitment for an arbitrary policy, which will also be relevant when there is no commitment since, from the households' perspective, they will be best responding to the government's strategy, which they take as given. The benchmark Chamley (1986) and Judd (1985) result is also derived.

2.1 Model Setup

The economy has a continuum of measure one of infinitely lived identical consumers, an arbitrary number of firms who behave competitively and a benevolent government. Time is discrete.

2.1.1 Households

The households' derive utility from consumption c_t, labor n_t, and consumption of a public good g_t. They discount the future at rate β, with $0 < \beta < 1$, so that each consumer's lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t, n_t) + v(g_t)].$$

Assume u is increasing in consumption and decreasing in labor and globally concave. The usual Inada conditions hold $u_c(0, n) = \infty$, $u_c(\infty, n) = 0$, $u_n(c, 0) = 0$ and $u_n(c, \infty) = -\infty$. Assume also that the utility of the public good v is increasing and concave with $v'(0) = \infty$ and $v'(\infty) = 0$.

For each unit of work, households receive after tax wages of $w_t(1-\tau_t^n)$. The labor tax can take any real value. Households can transfer consumption between periods using capital k_t or government bonds b_t. At time $t-1$ households buy capital k_t. Each unit of capital costs one unit of consumption good. At time t households can rent this capital to firms for which they receive an after tax return of $R_t(1 - \tau_t^k)$.For simplicity, assume that capital is fully depreciated. If capital were depreciated at rate $\delta < 1$, which may be irreversible, all the results in the paper remain unchanged. Steady state simulations will illustrate the effect of introducing irreversible capital. Assume households can always choose not to use their capital, so that capital taxes cannot be higher than one $\tau_t^k \leq 1$. No lower bound on τ_t^k is imposed. A bond that pays one unit of consumption good in period t costs q_{t-1} units of consumption good in period $t-1$. Households may also receive lump sum transfers from the government T_t, which must always be positive.

The households' per period budget constraint is thus given by

$$c_t + k_{t+1} + q_t b_{t+1} \leq R_t(1-\tau_t^k)k_t + b_t + w_t(1-\tau_t^n)n_t + T_t.$$

They must also meet the following no Ponzi condition

$$\lim_{t\to\infty}\left[b_{t+1}\prod_0^t q_s\right] \geq 0.$$

2.1.2 Government

The government is benevolent, which means that it maximizes the utility of a representative consumer. It needs to collect revenue to finance expenditure in the public good g_t every period. It sets proportional taxes on labor τ_t^n and capital τ_t^k each period. It transfers revenues between periods using government bonds b_t. The government sets the bond price q_t and consumers decide how many bonds to purchase. The government can make positive lump sum transfers to the households $T_t \geq 0$. Given this, the government's per period budget constraint is given by

$$g_t + b_t + T_t = w_t\tau_t^n n_t + R_t\tau_t^k k_t + q_t b_{t+1}.$$

2.1.3 Firms

Each period firms maximize profits given the before taxes prices for labor w_t and capital R_t. They have access to the production function $F(k_t, n_t)$, which has constant returns to scale and decreasing marginal productivity of capital and labor. Assume $F_k(0,n) = \infty$, $F_n(k,0) = \infty$, and $F_k(\infty, n) < 1/\beta$.

2.1.4 Market equilibrium

Market must clear every period. For the goods market, this means that the resource constraint must be met every period

$$c_t + g_t + k_{t+1} = F(k_t, n_t).$$

Factor markets clear when factor prices equal the marginal productivity of each factor: $w_t = F_n(k_t, n_t)$ and $R_t = F_k(k_t, n_t)$.

2.2 Allocations Attainable under Commitment

Consider the commitment economy in which the government makes all its decisions for the future at the beginning of time. Households make their decisions after observing the policy plan decided by the government.

Let $\pi = (\pi_0, \pi_1, ...)$ denote the sequence of government policies $\pi_t = (\tau_t^k, \tau_t^n, T_t, q_t, g_t)$, let $x = (x_0, x_1, ...)$ denote the sequence of allocations $x_t = (c_t, n_t, k_{t+1}, b_{t+1})$, and let $p = (p_0, p_1, ...)$ denote the sequence of market clearing prices $p_t = (R_t, w_t)$.

An allocation x is *attainable under commitment* if there are policies π and prices p such that (i) households maximize utility subject to their budget constraints and no Ponzi condition, (ii) the government meets its budget constraint with $T_t \geq 0$ and, (iii) factor prices equal marginal productivity of factors and the resource constraint is met.

Following the approach developed by Lucas and Stokey (1983), we can plug the first order condition for the households' problem into the households' budget constraint and obtain the economy's implementability condition. Lemma 1 shows that an allocation is attainable under equilibrium if and only if it meets the implementability condition and the resource constraint, as well as a transversality condition. Lemma 1 is proven in the appendix.

Lemma 1 *An allocation x is attainable under commitment if and only if it meets the following conditions for $t \geq 0$*

$$\begin{aligned} m(c_t, n_t) + \beta a_{t+1} &\geq a_t \\ c_t + g_t + k_{t+1} &= F(k_t, n_t) \\ \lim_{t\to\infty} \beta^t a_{t+1} &= 0 \end{aligned}$$

given k_0 and $a_0 = u_c(c_0, n_0)[F_k(k_0, n_0)(1 - \tau_0^k)k_0 + b_0]$, with $m(c_t, n_t)$ and a_t given by

$$\begin{aligned} m(c_t, n_t) &\equiv u_c(c_t, n_t)c_t + u_n(c_t, n_t)n_t \\ a_t &\equiv u_c(c_t, n_t)\left[\frac{u_c(c_{t-1}, n_{t-1})}{\beta u_c(c_t, n_t)}k_t + b_t\right] \ \textit{for } t > 0. \end{aligned}$$

Using a change of variables, we have replaced the level of government bonds for the value of consumer assets a_t. This will allow us to write the problem recursively using as state variables the level of capital and the value of consumer assets. This approach builds on Werning (2003), who rewrites the problem in Aiyagari, Marcet, Sargent and Seppala (2002) using the value of debt and the state of an exogenous Markov process as state variables.

The capital and labor income taxes associated with a given allocation x are determined by

$$\begin{aligned}\tau_{t+1}^k &= 1-\frac{1}{\beta F_k(k_{t+1},n_{t+1})}\frac{u_c(c_t,n_t)}{u_c(c_{t+1},n_{t+1})}\\ \tau_t^n &= 1+\frac{1}{F_n(k_t,n_t)}\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)}.\end{aligned}$$

We can guarantee that the transversality condition $\lim \beta^t a_{t+1} = 0$ is met by constraining a to always be below the natural debt limit $\overline{a}(k_t)$ which is the maximum debt level that can be repaid by the government

$$\overline{a}(k_t) \equiv \max_{c,n,k} \sum_{s=t}^{\infty} \beta^{s-t} m(c_s, n_s) \text{ st } c_s + k_{s+1} \leq F(k_s, n_s).$$

From now on the implementability condition and the resource constraint will be used as necessary and sufficient conditions for an allocation to be attainable under commitment with the underlying condition that a_t must remain below this upper bound.

This characterization of allocations attainable under commitment will be useful to determine the optimal policies with or without commitment since in both cases the resulting allocations will have to be chosen by households who anticipate a given set of policies, which means that their outcomes must be attainable under commitment.

2.3 Optimal Taxes with Commitment

This section derives the optimal policy plan when the government can choose the policies for all future periods at time zero. It introduces a recursive formulation of the problem (that will also be used for the no commitment case) to derive the benchmark Chamley (1986) and Judd (1985) result of zero capital taxes in the long run.

The Ramsey problem chooses among all the allocations attainable under commitment, the one that maximizes the welfare of the representative consumer. The outcome of a *Ramsey equilibrium* is a sequence x that maximizes the present value of utility $\sum_{t=0}^{\infty} \beta^t [u(c_t, n_t) + v(g_t)]$ subject to x being attainable under commitment and given an initial stock of capital $k_0 > 0$ and an initial promise for the value of consumer assets $a_0 < \overline{a}(k_0)$. This formulation assumes that the initial planner has committed to a given a_0. If instead we wanted to assume that the government had an initial outstanding debt of b_0 we would have to add the following restriction for the initial period $a_0 = u_c(c_0, n_0)[F_k(k_0, n_0)(1 - \tau_0^k)k_0 + b_0]$. Given this, we can write the Ramsey problem using the following sequence formulation

$$
\begin{array}{rcl}
V(k_0, a_0) & \equiv & \max_{c,n,g,k,a} \sum_{t=0}^{\infty} \beta^t [u(c_t, n_t) + v(g_t)] \\
\text{subject to} & & m(c_t, n_t) + \beta a_{t+1} \geq a_t \\
& & c_t + g_t + k_{t+1} = F(k_t, n_t).
\end{array}
$$

Using a and k as state variables this problem can be written recursively in the following way

$$
\begin{array}{rcl}
V(k, a) & = & \max_{c,n,g,k,a} [u(c, n) + v(g) + \beta V(k', a')] \\
\text{subject to} & & m(c, n) + \beta a' \geq a \\
& & c + g + k' \leq F(k, n).
\end{array}
$$

If $m(c, n)$ is concave, then the constraint set is convex, which means that the value function $V(k, a)$ will be concave, and the first order conditions are necessary and sufficient for optimality. If this condition is not met, first order conditions are still necessary for an optimum, but no longer sufficient, since it is also necessary to verify that the second order conditions are met to make sure we are at a maximum.

Using μ as the multiplier on the implementability condition and ρ as the multiplier on the resource constraint, we can write the Lagrangean for this problem in the following way

$$
L = u(c, n) + v(g) + \beta V(k', a') + \mu[m(c, n) + \beta a' - a] - \rho[c + g + k' - F(k, n)].
$$

Combining the first order conditions for k' and a' with the envelope conditions for k and a, we get the following equations

$$
\begin{array}{ll}
(k) & V_k(k, a) = \beta F_k(k, n) V_k(k', a') \\
(a) & V_a(k', a') = V_a(k, a).
\end{array}
$$

A steady state for the Ramsey economy has constant c, n, g, k and a as well as constant multipliers μ and ρ. From the optimality condition for k, it is clear that in steady state $\beta F_k = 1$. Plugging into the expression for capital taxes, it is straightforward to see that capital taxes must be zero in steady state

$$
\tau^k = 1 - \frac{1}{\beta F_k} \frac{u_c}{u_c'} = 0.
$$

Labor taxes, on the other hand, will remain positive in steady state

$$
\tau_t^n = 1 + \frac{1}{F_n} \frac{u_n}{u_c}.
$$

This is the well known Chamley (1986) and Judd (1985) result that capital income taxes converge to zero in the long run.

3 Sustainable Equilibria without Commitment

This section introduces lack of commitment by modelling the taxation problem as a game where the government and the agents in the economy make sequential decisions every period. The equilibrium concept is defined and a simple characterization of equilibrium outcomes is formalized based on a maximum threat point of reversion to the worst equilibrium, in the spirit of Abreu (1988).

3.1 Game Setup

This section introduces a game where the lack of commitment is modelled explicitly and the value of default that sustains the initial plan is determined endogenously. Since households and firms behave competitively, whereas the government behaves strategically, I will use the notion of sustainable equilibria introduced by Chari and Kehoe (1990), where all strategies are conditional on the past history of the government's actions.

Chari and Kehoe (1993a and 1993b) model debt default in a setup without capital. Both papers allow the government to defult on its bonds. In Chari and Kehoe (1993a) it is assumed that households can always commit to repay their debt. Conversely, in Chari and Kehoe (1993b) it is assumed that households cannot commit to their debt, which means that debt repayment cannot be enforced at all, leading to no loans being made to households in equilibrium. By introducing an endogenous punishment for default, we now make the households' default decision non trivial since they will only default if they expect not to get punished harshly enough.

Assume that the government cannot commit to future taxes and transfers. Furthermore, both households and the government can default on their bonds. The government can punish consumers who defaulted. Namely, each period the government chooses $P_t \geq 0$, which is the utility loss that consumers who defaulted in the previous period experience. Let d_t^g be an indicator function for whether the government defaults and d_t^c be the percentage of consumers who defaulted in period t.

The timing of the game is as follows. At the beginning of the period, the government decides by how much to punish consumers who defaulted in the previous period, whether to default on its bonds, and which taxes and transfers to set for the current period. Note that the government does not need to know which households defaulted; it is sufficient that the government knows the percentage of households who defaulted. Since each household has mass zero, the action of a finite number of households does not affect the percentage of households defaulting, which means that they are still non strategic. To put the punishment into practice the government will then rely on an outside independent institution (maybe courts) which will be able to punish each defaulting household.

After the government has made its choices, allocations and prices are jointly determined by the households' and the firms' maximization problems at market clearing prices.

3.2 Strategies

The government's actions in period t now include the punishment and decision to default, so that the expanded vector of government's actions is now $\Pi_t = (\tau_t^k, \tau_t^n, T_t, q_t, g_t, d_t^g, P_t)$.

Each period, every household chooses how much to consume, work and invest in capital and bonds. It also decides whether to default on its debt. Let d_t^{ci} be an indicator function for whether household i chooses to default in period t. The vector of individual decisions in each period is $X_t^i = (c_t^i, n_t^i, k_{t+1}^i, b_{t+1}^i, d_t^{ci})$. The vector of aggregate choices that results from the households decisions is $X_t = (c_t, n_t, k_{t+1}, b_{t+1}, d_t^c)$, where the aggregate value of each aggregate variable is the integral over all the households in the economy of the individual variables. In equilibrium, since all households are identical and follow pure strategies, the aggregate action will be the same as each individual action.

The price vector is $p_t = (R_t, w_t)$ as before.

Let h_t be the history of government decisions until time t so that $h_t = (\Pi_0, ..., \Pi_t)$. Following Chari and Kehoe (1990), all the strategies in the game will be contingent only on this history, since households are infinitesimal and have no power to influence X_t, which means that they will not behave strategically. Thus, knowing the households' strategies and government's actions until time t is enough to characterize all the history until then.

The strategy for the government is given by σ. The strategy for each period t is a mapping from the history h_{t-1} into the government's decision space Π_t, so that $\Pi_t = \sigma_t(h_{t-1})$. When choosing a given strategy, the government anticipates that histories will evolve according to $h_t = (h_{t-1}, \sigma_t(h_{t-1}))$. Let σ^t denote the sequence of government strategies from time t onwards.

The strategy for a representative household is given by f. The strategy for each period t is a mapping from the history h_t into the households' decision space X_t, so that $X_t = X_t^i = f_t(h_t)$. Let f^t denote the sequence of household strategies from time t onwards.

Firms and markets jointly work as a third player that has strategy ϕ mapping the history h_t into the vector of factor prices p_t, so that $p_t = \phi(h_t)$. Let f^t denote the sequence of household strategies from time t onwards.

In the next section we will specify how each player chooses its strategy in a sustainable equilibrium.

3.3 Sustainable Equilibrium

At time t, the government and households choose an action for time t and a contingent plan for the future. This is equivalent to choosing an action for today while anticipating future behavior since both the government and households have time consistent preferences, which means that the plan they choose today will be optimal tomorrow. The problem solved by the government and households at time t is described below.

For every history h_{t-1}, given allocation rule f and pricing rule ϕ, the government chooses σ^t to maximize the present value of utility

$$\sum_{s=t}^{\infty} \beta^{s-t}[u(c_s(h_s), n_s(h_s)) + v(g_s(h_{s-1})) - d_t^c(h_{s-1})P_s(h_{s-1})]$$

subject to

$$\begin{aligned} g(h_{s-1}) + T_s(h_{s-1}) &= w_s(h_s)\tau_s^n(h_{s-1})n_s(h_s) + R_s(h_s)\tau_s^k(h_{s-1})k_s(h_{s-1}) + \\ &\quad q_t(h_{s-1})b_{s+1}(h_s) - b_s(h_{s-1})(1 - d_t^c(h_s))(1 - d_t^g(h_{s-1})) \\ T_s(h_{s-1}) &\geq 0 \end{aligned}$$

and realizing that future histories are induced by σ^t according to $h_s = (h_{s-1}, \sigma_s(h_{s-1}))$.

For every history h_t, given policy rule σ (and the histories it induces) and pricing rule ϕ, each household chooses f^t to maximize the present value of utility

$$\sum_{s=t}^{\infty} \beta^{s-t}[u(c_s^i(h_s), n_s^i(h_s)) - d_t^{ci}(h_{s-1})P_s(h_{s-1})]$$

subject to

$$\begin{aligned} c_s^i(h_s) &= w_s(h_s)(1 - \tau_s^n(h_{s-1}))n_s^i(h_s) + T_s(h_{s-1}) + \\ &\quad b_s^i(h_{s-1})(1 - d_t^{ci}(h_s))(1 - d_s^g(h_{s-1})) - q_s(h_{s-1})b_{s+1}^i(h_s) + \\ &\quad R_s(h_s)(1 - \tau_s^k(h_{s-1}))k_s^i(h_{s-1}) - k_{s+1}^i(h_s). \end{aligned}$$

Market clearing and firm optimality require that for every history h_t firm demand must equal household supply for every production factor, which happens when factor prices equal their marginal productivity, so that $\phi_t(h_t)$ is given by

$$\begin{aligned} w_t(h_t) &= F_n(k_t(h_{t-1}), n_t(h_t)) \\ R_t(h_t) &= F_k(k_t(h_{t-1}), n_t(h_t)). \end{aligned}$$

A *sustainable equilibrium* is a triplet (σ, f, ϕ) that satisfies the following conditions: (i) given f and ϕ, the continuation of contingent policy plan σ solves the government's problem for every history h_{t-1}; (ii) given σ and ϕ, the continuation of contingent allocation rule f solves the households' problem for every history h_t; (*iii*) given f and σ, the continuation of the contingent pricing rule ϕ is such that factor prices equal marginal productivity for every history h_t.

3.4 Worst Sustainable Equilibrium

Let $V(\sigma, f, \phi)$ denote the present value of utility that results from a sustainable equilibrium (σ, f, ϕ). Then the worst sustainable equilibrium is the sustainable equilibrium that leads to the lowest value $V(\sigma, f, \phi)$. It will be useful to find the worst sustainable equilibrium to then define which equilibria can be sustained without ex-ante commitment, since the worst sustainable equilibrium is the worst punishment that can be credibly inflicted on a government that deviates from a predefined plan.

Lemma 2 *The value of the worst sustainable equilibrium only depends on the current capital level:* $V(\sigma^w, f^w, \phi^w) = V^w(k)$.

A proof of this lemma can be found in the appendix. The idea is that the value of a worst equilibrium can only depend on the current payoff relevant variables. Furthermore, since the government can eliminate debt by defaulting, no additional punishment can be given to it, which means that the value of the worst sustainable equilibrium will not depend on the level of debt.

For our model the worst sustainable equilibrium is an equilibrium where all agents default on their debt and the government always expropriates capital.

The *default equilibrium* is a triplet (σ^d, f^d, ϕ^d) where agents have the following strategies:

(*i*) The government always defaults on b_t, never punishes consumers, always taxes capital at confiscatory rates, and sets $q_t = 0$. Transfers and labor taxes implement the solution to the following problem

$$\max_{c_t, n_t, g_t} \; [u(c_t, n_t) + v(g_t)]$$
$$\text{subject to } m(c_t, n_t) \geq 0$$
$$c_t + g_t = F(k_t, n_t)$$

(*ii*) Households default if $b_t < 0$, never invest in capital, and never lend or borrow from the government. Labor and consumption solve the following problem

$$\max_{c_t, n_t} u(c_t, n_t)$$
$$\text{subject to } c_t = T_t + w_t(1 - \tau_t^n)n_t$$

(*iii*) Factor prices equal marginal productivity of factors.

Lemma 3 *The default equilibrium is the worst sustainable equilibrium:* $V(\sigma^w, f^w, \phi^w) = V^w(k) = V^d(k)$.

The worst sustainable equilibrium punishes the government by giving households beliefs about the government's future behavior that lead to low future value. However, these beliefs have to be correct, so the government has to be given incentives to keep the plan. Phelan and Stachetti show that in general the government's incentive constraint will be binding, which means the continuation value of a worst sustainable equilibrium will not be a worst sustainable equilibrium. However, since we allow the government to make lump sum transfers to the households, even when the government gives households extremely pessimistic beliefs that capital will be fully expropriated in the following period, the government's incentive constraint will not bind since it is always willing to tax capital at maximal rates and then redistribute back to the households.

Since households always expect the government to fully expropriate capital, they will never invest even though capital is very productive. Furthermore, if any household actually invests, it will be in the government's best interest to expropriate it since capital is sunk ex-post. Thus, this lack of commitment will lead to an extremely inefficient investment decision. Since both households and the government are best responding to each other's strategy, the default equilibrium is sustainable. The appendix proves that the default equilibrium is the worst sustainable equilibrium.

Since the best the government can do when households are playing a default equilibrium is to maximize its per period utility, the flow utility reached in a period when the initial stock of capital is k is given by

$$U^d(k) = \max_{c,n,g}[u(c,n) + v(g)] \text{ st } m(c,n) \geq 0,\ c + g = F(k,n)$$

and the net present value of a default equilibrium is

$$V^d(k) = U^d(k) + \frac{\beta}{1-\beta}U^d(0),$$

which only depends on the initial level of k.

3.5 Characterization of Sustainable Outcomes

In the spirit of Abreu's optimal punishments, we will use reversion to the worst sustainable equilibrium as the maximum threat point that allows us to sustain equilibria. Thus, for an equilibrium to be sustainable, it must yield higher utility than the worst sustainable equilibrium in all future dates. We are using the terminology of Abreu (1988), who defines optimal punishments when firms deviate in a cartel. Here, there is not any kind of collusion *per se*, but we still need to enforce cooperation since the government's ex-ante and ex-post incentives are not aligned. Thus, the worst punishment is not inflicted by other firms, but rather by changing the consumer's expectations, which leads to a different equilibrium that is worse for everyone.

The next lemma characterizes the entire set of sustainable equilibrium outcomes, which are the allocations that are induced by a particular sustainable equilibrium.

Lemma 4 *An allocation x is the outcome of a sustainable equilibrium if and only if:*

(i) x is attainable under commitment

(ii) the continuation value of x is always better than the worst sustainable equilibrium

$$\sum_{t=i}^{\infty} \beta^{t-i}[u(c_t, n_t) + v(g_t)] \geq V^d(k_i).$$

The proof of this lemma can be found in the appendix. The idea is that for an allocation to be the outcome of a sustainable equilibrium, it is necessary that households and firms are optimizing given the government's strategy, which means that the resulting allocation must be attainable under commitment. Government optimality requires that it is never in the government's best interest to deviate. Since the worst punishment after a deviation is $V^d(k)$, this gives us a lower bound on the utility that can be reached in a sustainable equilibrium at any point in time.

4 Best Sustainable Equilibrium

Now that the set of sustainable equilibria has been characterized, we turn to finding among these, the one that maximizes the initial welfare for the society for given initial conditions for k_0 and a_0. As before, assuming commitment to a given a_0 in the initial period is a a simplifying assumption that can be relaxed. Assume that a_0 is within the necessary bounds for an equilibrium to exist.

4.1 Sequence Approach

The outcome of the best sustainable equilibrium solves

$$V(k_0, a_0) = \max_{c,n,g,k,a} \sum_{t=0}^{\infty} \beta^t [u(c_t, n_t) + v(g_t)]$$

$$
\begin{aligned}
\text{subject to } & m(c_t, n_t) + \beta a_{t+1} \geq a_t \\
& c_t + g_t + k_{t+1} = F(k_t, n_t) \\
& \sum_{t=i}^{\infty} \beta^{t-i}[u(c_t, n_t) + v(g_t)] \geq V^d(k_i).
\end{aligned}
$$

This follows directly from lemma 4. Since the three restrictions are necessary and sufficient for a sustainable equilibrium, then the allocation that maximizes welfare subject to them must be the outcome of the best sustainable equilibrium.

This formulation is equivalent to the Ramsey problem under commitment, with an additional incentive compatibility condition that ensures that, at each point in time, the government never wants to deviate from the predefined plan.

Notice that one of our restrictions now has an endogenous function $V^d(k_i)$, which is concave. Given this, the constraint set may not be convex, even if we assumed that $m(c_t, n_t)$ is concave. Thus, we cannot guarantee that $V(k, a)$ is a concave function. In the analysis that follows we proceed as if $V(k, a)$ were concave. The appendix shows that the same results follow through even if that is not the case.

Let $x^{fb}(k_0) \equiv \arg\max \sum_{t=0}^{\infty} \beta^t[u(c_t, n_t) + v(g_t)]$ st $c_t + g_t + k_{t+1} = F(k_t, n_t)$ be the first best allocation when the economy has initial capital k_0.

Then the implementability constraint will not be binding for $a_0 \leq \underline{a}(k_0)$, which is defined by $\underline{a}(k_0) \equiv \sum_{t=0}^{\infty} \beta^t m(c_t^{fb}(k_0), n_t^{fb}(k_0))$.

Notice that $V^{fb}(k) \equiv V(k, \underline{a}(k)) > V^d(k)$, which means that if the implementability condition is not binding, then the incentive compatibility condition will not bind either. The reason for this is straightforward: if the government does not have to use distortionary taxation to finance its spending, then it has no incentive to deviate from the optimal plan.

Since the implementability condition is not binding for $a_0 \leq \underline{a}(k_0)$, then having lower a_0 does not bring any additional benefit to the economy, which means that $V_a(k, a) = 0$ for $a \leq \underline{a}(k)$. On the other hand, if $a > \underline{a}(k)$, then starting off with a lower a_0 relaxes the implementability condition, which implies that $V_a(k, a) < 0$ for $a > \underline{a}(k)$.

4.2 Recursive Approach

The program to find the best sustainable equilibrium can also be written recursively as stated below. The appendix shows that the two formulations are equivalent, and from now on the recursive approach will be used.

$$
\begin{aligned}
V(k,a) = &\max_{c,n,g,k,a} [u(c,n) + v(g) + \beta V(k',a')] \\
&\text{subject to } m(c,n) + \beta a' \geq a \\
&\qquad c + g + k' = F(k,n) \\
&\qquad V(k',a') \geq V^d(k')
\end{aligned}
$$

The Lagrangean for this problem is

$$
\begin{aligned}
L &= u(c,n) + v(g) + \beta V(k',a') + \mu[m(c,n) + \beta a' - a] \\
&\quad -\rho[c + g + k' - F(k,n)] + \gamma\beta[V(k',a') - V^d(k')].
\end{aligned}
$$

Combining the first order conditions for $k\prime$ and $a\prime$ with the envelope conditions for k and a, we get the following equations

$$
\begin{aligned}
&(k')\ V_k(k',a') = \frac{V_k(k,a)}{\beta F_k(k,n)} + \gamma[V_k^d(k') - V_k(k',a')]. \\
&(a')\ V_a(k',a')(1+\gamma) = V_a(k,a).
\end{aligned}
$$

The optimality condition for k shows how the lack of commitment can distort the choice of capital in the short run. If commitment binds ($\gamma > 0$) and the value of default reacts more to changes in capital than the value of the optimal sustainable plan, then capital will be distorted downward, since this will help loosen the incentive compatibility constraint. If conversely, the value of the optimal sustainable plan varies more with capital, than capital will be distorted upward. Thus, if higher capital makes commitment less binding, it will be optimal to subsidize capital. This result is reminiscent of the findings of Benhabib and Rustichini (1997), who show that it could be optimal to either tax or subsidize capital. However, here this will only be true in the short run, since in the long run the economy will converge to a steady state where commitment does not bind.

The optimality condition for a says that it is optimal for the value of government assets to decrease over time as long as commitment is binding, which leads to an increase of government assets over time. The reason for this is that when the government accumulates assets, it gets a direct benefit of higher utility tomorrow, as well as an additional benefit from loosening the incentive compatibility condition in the future. Thus, to some extent, government assets work as a commitment mechanism that reduces the incentive to default by increasing the welfare of the equilibrium strategy.

The next section describes the long run properties of the economy without commitment.

4.3 Steady State

In this section we derive the paper's main result that capital taxes must converge to zero in the long run. We start by showing that any steady state must have zero capital taxes and then show that the economy will indeed converge to a steady state.

Proposition 1 ***(Zero capital taxes in steady state)*** *In steady state, the best sustainable equilibrium has zero capital taxes.*

Proof. Assume the economy is in a steady state with constant c, n, g, k, and a. The first order conditions for c and n ($u_c+\mu m_c = \rho$ and $u_n+\mu m_n = -\rho/F_n$) imply that ρ and μ must also be constant. We can now prove by contradiction that capital taxes cannot be different from zero in the long run.

If capital taxes are not zero, then $\beta F_k(k,n) \neq 1$. From the optimality condition for k derived above, this implies that $\gamma > 0$ (recall that $\gamma \geq 0$ since it is the multiplier on an inequality constraint). We can see in the optimality condition for a that when $\gamma > 0$, it must be true that $V_a(k,a) = 0$ is equal to zero in steady state. But then it must be true that $a \leq \underline{a}(k)$ and $V(a,k) = V^{fb}(k) > V^d(k)$, which means that the incentive compatibility condition cannot be binding and we must have $\gamma = 0$, which means we have reached a contradiction and capital taxes cannot be different from zero in steady state. ■

Proposition 2 ***(Convergence to steady state)*** *The best sustainable equilibrium converges to a steady state.*

Proof. The first order conditions describe the unique path for the economy for given initial conditions.

If γ converges to zero, then the long run dynamics of capital and a are like those of an economy without commitment, which are governed by

$$\begin{aligned} V_k(k',a') &= \frac{V_k(k,a)}{\beta F_k(k,n)} \\ V_a(k',a') &= V_a(k,a). \end{aligned}$$

This means that capital will increase as long as it is below its steady state level ($\beta F_k > 1$) and increase when it is above the steady state level ($\beta F_k < 1$), so that it converges to its steady state.

If γ does not converge to zero, then $V_a(k,a)$ must converge to zero since γ is weakly positive and $V_a(k',a')(1+\gamma) = V_a(k,a)$. But we have just seen that when $V_a(k,a)$ is close to zero, the implementability condition cannot be binding, which means that γ must converge to zero. ■

As long as commitment is binding, increasing government savings not only increases tomorrow's continuation value, but also loosens the incentive compatibility constraint. Thus, the government will keep saving until it has achieved enough assets for the incentive compatibility to stop binding. This will happen before the government reaches its asset limit, since $V(\underline{a}(k), k) > V^d(k)$.

We have seen so far that without commitment the economy will still converge to a steady state where commitment does not bind. Next we explore the long run implications that the lack of commitment may have.

With commitment, a steady state had to meet the following conditions

$$\begin{aligned} u_n + \mu m_n &= -\rho F_n & \qquad u_c + \mu m_c &= \rho \\ c + g + k &= F(k,n) & \qquad v_g &= \rho \\ \mu[m(c,n) - a(1-\beta)] &= 0 & \qquad \beta F_k &= 1 \end{aligned}$$

Without commitment, any steady state commitment still has to meet the previous conditions, but it also has to meet the incentive compatibility condition

$$V(k,a) = \frac{1}{1-\beta}[u(c,n) + v(g)] \geq V^d(k).$$

Thus, all steady states without commitment are also steady states under commitment. However, the converse need not be true. In particular, steady states with a very indebted government (which translates into a high level of a) need to use more distortionary taxation, which reduces the present value of utility, so that the incentive compatibility condition may not hold. As one would expect, more indebted governments have a higher incentive to default. Conversely, steady states where the government has a substantial amount of assets do not need to use much distortionary taxation, which reduces the incentive to default and expropriate capital.

5 Numerical Simulations

To illustrate the results of the model, consider an economy with preferences given by

$$u(c,n) + v(g) = \ln(c) - n^{1+\theta}/(1+\theta) + \ln(g)$$

and production function

$$F(k,n) = Ak^{\alpha}n^{1-\alpha} + Bn + Ck,$$

where the specific parameters are $\theta = 1$, $\beta = 0.4$, $A = 2$, $B = 0.5$, $C = 0.5$, and $\alpha = 0.5$.

We will start by looking at the short run dynamics of this economy for given initial conditions, in order to see how the lack of commitment changes the path towards the steady state and what welfare costs it entails.

Next, we turn too see how commitment changes the feasible steady states of this economy, in order to infer what the long run implications of lack of commitment are.

5.1 Short Run Dynamics

The recursive problem can be written as

$$\begin{aligned} V(k,a) &= \max_{k',a'} \left\{ U(k,a,k'a') + \beta V(k',a') \right\} \\ \text{subject to} & \quad V(k',a') \geq V^d(k') \end{aligned}$$

where $U(k,a,k'a')$ is given by

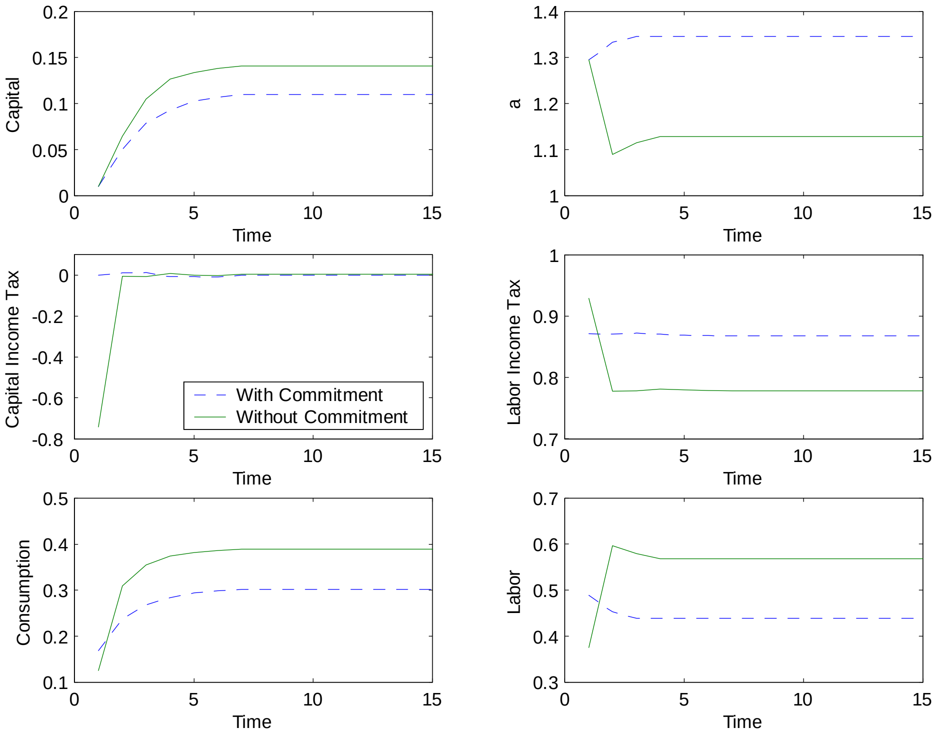
$$\begin{aligned} U(k,a,k'a') =& \max_{c,n,g} [u(c,n) + v(g)] \\ & \text{subject to } m(c,n) + \beta a' \geq a \\ & \qquad c + g + k' = F(k,n). \end{aligned}$$

The first step of the simulation is to construct $U(k,a,k'a')$. For computational convenience, it is useful to stack the state variables for each period into one single dimension. If we exclude initial conditions where the implementability condition is not binding, we can assume, without loss of generality, that the implementability condition holds with equality. Given this and the previous assumptions for the functional form, we can write the implementability condition as $1 - n^{1+\theta} + \beta a' \geq a$, which means we can solve for $n(k,a,k'a')$, which is a matrix that gives us the optimal choice of n for a given state today on one axis and a given state tomorrow on the other axis. Given the functional form for utility it is optimal to have $g = c$. Thus, we can use the resource constraint to find $g(k,a,k'a')$ and $c(k,a,k'a')$. Finally, we can use these matrices to compute $U(k,a,k'a') = u[c(k,a,k'a'), n(k,a,k'a')] + v[g(k,a,k'a')]$.

Given this, we can make an initial guess for $V(k,a)$ and iterate on it until we find a fixed point. For the case where there is no commitment, this iteration includes a large punishment when the incentive compatibility constraint is violates. This ensures that for the chosen solution the constraint is always met.

Figure I shows the optimal path for an economy with initial capital stock $k_0 = 0.1$ and an initial promise for the value of consumer assets $a_0 = 1.3$. Dashed lines represent the economy with commitment, whereas the solid lines represent the no-commitment economy.

Figure I - Transition Dynamics with and without Commitment



For the parameter values we have assumed, capital is being subsidized when the incentive compatibility is binding. This reflects the fact that the value of the best equilibrium is more sensitive to changes in the level of capital than the worst equilibrium. Thus, increasing capital relaxes the incentive compatibility condition.

In the economy without commitment, the value of a also decreases in initial periods (which is equivalent to reducing the government's debt) so that, in the future, labor taxes are lower and labor and capital levels are higher without commitment.

As a consequence, capital levels are higher in the economy without commitment both in the short run (due to the capital subsidy) and in the long run (due to lower labor taxes).

The initial value of welfare in the economy with commitment is -5.57, whereas, for the economy without commitment, initial welfare is -5.82. This difference represents the cost of lack of commitment.

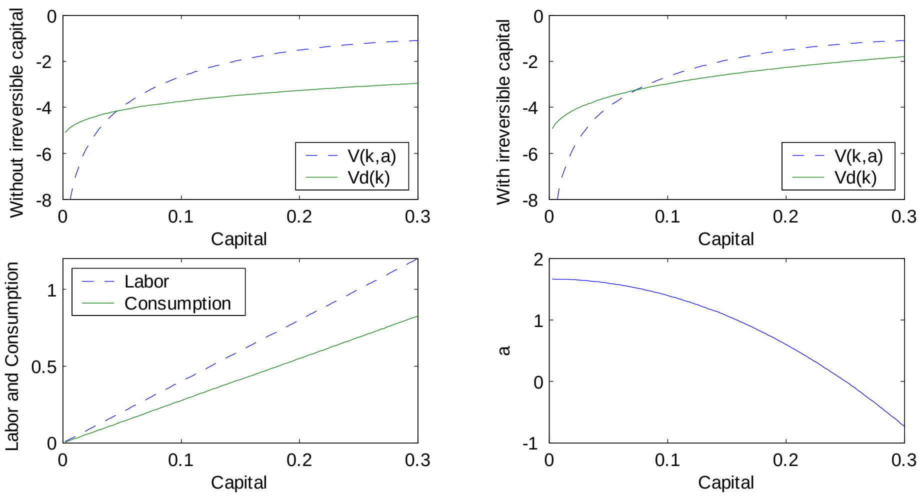
In the long run, however, the welfare in the economy without commitment (-3.41) is higher than in the economy with commitment (-4.08), since the economy without commitment reduced its debt in the initial periods, which means that it does not have to do as much distortionary taxation in steady state. Thus, while there is a short run cost to commitment, in the long run, an economy whose government cannot commit ex-ante may actually have higher welfare.

5.2 Long Run Implications

In the long run, commitment will not be binding even if the government cannot commit to policies ex ante. Section 4.3 shows that steady states without commitment are also steady states in the economy with commitment, albeit with different initial conditions. However, the converse need not be true since, without commitment, we cannot sustain steady state equilibria of the economy with commitment where the incentive compatibility condition is not met. Thus, introducing the inability of governments to commit can help us predict what kinds of equilibria we might expect to find.

Figure II plots the set of steady state equilibria under commitment, indexed by the steady state level of capital. Notice that steady states with higher capital also have lower a, which means that the government is less indebted. Thus, $V(k,a)$ increases with capital not only because capital is increasing but also because a is decreasing. Labor and consumption are increasing with capital, exactly because steady states with higher capital have lower labor taxes.

Figure II - Steady States achievable with and without Commitment



To find which steady states are still feasible when the government cannot commit to future policies, the top panels plot the value of default associated with the capital level that is chosen in each of the equilibria. The left panel shows the value of default in our baseline model, whereas the right panel plots the value of default when the depreciated capital cannot be reconverted into consumption goods. As would be expected, in the economy with irreversible capital, the value of default is higher, but the qualitative results do not change[1]. Without commitment, steady states must meet the incentive compatibility constraint $V(k,a) \geq V^d(k)$, which means that only steady states with a high enough level of capital are sustainable. In general, steady states where the government is very indebted will not be feasible without commitment. In particular, when governments approach their natural debt limit and need to tax labor income at very distortionary levels, it becomes more likely that they will default on their past promises.

6 Concluding Remarks

If the government is allowed to accumulate assets, then, even if it cannot commit to a stream of future taxes, in the long run capital taxes will converge to zero. The previous literature on taxation without commitment had found that this was not the case when the government was forced to keep budget balance in every period. The reason for this disparity is that, in the absence of commitment, government assets can work as a commitment device to discipline government behavior that was not available under budget balance.

Thus, in the short run, economies without commitment will tend to accumulate more assets. Furthermore, while commitment binds, capital may either be taxed or subsidized, depending on whether increasing capital loosens or tightens the government's incentive constraint.

In the long run, economies without commitment will tend to have a higher asset level, which leads to higher capital levels since an economy with a richer

[1] If capital depreciates at rate δ and the non-depreciated capital is irreversible then capital income taxes still converge to zero in the long run in the best sustainable equilibrium. The more substantial difference is that default is now more attractive, since in the default equilibrium the level capital remains positive and only decreases as it depreciates, which does not allow such as stark punishment as before. Let the value of a default equilibrium in this case be given by $V^D(k)$.

The outcome of the best sustainable equilibrium solves

$$
\begin{aligned}
V(k,a) = \max_{c,n,g,k,a} & [u(c,n) + v(g) + \beta V(k',a')] \\
\text{subject to } & m(c,n) + \beta a' \geq a \\
& c + g + k' = F(k,n) + (1-\delta)k_t \\
& k_{t+1} \geq (1-\delta)k_t \\
& V(k',a') \geq V^D(k')
\end{aligned}
$$

Since $V^D(k)$ is still lower than $V^{fb}(k)$ capital taxes are still zero in the long run.

The value for depreciation is $\delta = 1 - C$, so that the steady state equilibria with commmitment remain unchanged with or without irreversibility of capital.

government will have lower labor taxes. This in turn increases labor, leading to higher productivity of capital.

7 Appendix

7.1 Proof of Lemma 1

The intertemporal budget constraint can be written as

$$\begin{aligned} R_0(1-\tau_0^k)k_0+b_0 \quad &\leq \quad \sum_{t=0}^{\infty}\left[(c_t-w_t(1-\tau_t^n)n_t-T_t)\prod_0^t q_s\right] \\ &\quad + \lim_{t\to\infty}\left[(\frac{k_{t+1}}{q_t}+b_{t+1})\prod_0^t q_s\right]. \end{aligned}$$

Since the marginal utility of consumption is always positive, it will always be optimal for households to meet their budget constraint with equality and to choose the long run values of k an b such that the last term in the budget constraint is not positive, which would make it more binding.

Together with the no Ponzi condition and the non negativity constraint on capital, this implies that the following transversality condition must be met

$$\lim_{t\to\infty}\left[(\frac{k_{t+1}}{q_t}+b_{t+1})\prod_0^t q_s\right]=0.$$

Households thus solve the following problem

$$\max_{c,n,b,k}\sum_{t=0}^{\infty}\beta^t u(c_t,n_t)$$

$$\begin{aligned} c_t-w_t(1-\tau_t^n)n_t-T_t+k_{t+1}+q_tb_{t+1} \quad &= \quad R_t(1-\tau_t^k)k_t+b_t \\ \lim_{t\to\infty}\left[(k_{t+1}/q_t+b_{t+1})\prod_0^t q_s\right] \quad &= \quad 0 \\ c_t,n_t,k_{t+1} \quad &\geq \quad 0, \text{ given } k_0 \text{ and } b_0. \end{aligned}$$

Since we are maximizing a concave function on a convex set, the following first order conditions, together with the government's budget constraint and the transversality condition, are necessary and sufficient for household optimality

$$\begin{aligned} u_n(c_t,n_t)+w_t(1-\tau_t^n)u_c(c_t,n_t) \quad &= \quad 0 \\ k_{t+1}[u_c(c_t,n_t)-\beta R_{t+1}(1-\tau_{t+1}^k)u_c(c_{t+1},n_{t+1})] \quad &= \quad 0 \\ q_tu_c(c_t,n_t)-\beta u_c(c_{t+1},n_{t+1}) \quad &= \quad 0. \end{aligned}$$

Furthermore, the following conditions are necessary and sufficient for firm optimality and market clearing

$$\begin{aligned} w_t \quad &= \quad F_n(k_t,n_t) \\ R_t \quad &= \quad F_k(k_t,n_t) \\ c_t+g_t+k_{t+1} \quad &= \quad F(k_t,n_t). \end{aligned}$$

Finally, the government has to meet the following constraints

$$g_t + b_t + T_t = w_t \tau_t^n n_t + R_t \tau_t^k k_t + q_t b_{t+1} \text{ and } T_t \geq 0.$$

Thus an allocation is attainable under commitment if and only if there are prices and policies such that all the above conditions are met.

We can drop the government's budget constraint since the resource constraint and the households' budget constraint jointly imply that the government's budget constraint is always met.

Multiplying the households' budget constraint by $u_c(c_t, n_t)$ and plugging in the first order condition leads to the following implementability condition

$$\begin{aligned} c_t u_c(c_t, n_t) + n_t u_n(c_t, n_t) + \beta u_c(c_{t+1}, n_{t+1})[\frac{u_c(c_t, n_t)}{\beta u_c(c_{t+1}, n_{t+1})} k_{t+1} + b_{t+1}] \\ = u_c(c_t, n_t)[\frac{u_c(c_{t-1}, n_{t-1})}{\beta u_c(c_t, n_t)} k_t + b_t] + T_t u_c(c_t, n_t), \end{aligned}$$

which can replace the households' budget constraint.

The only conditions that constrain the allocations achievable under commitment are the implementability condition, the resource constraint, and the transversality condition. All the remaining conditions can be met by choosing prices and policies. Furthermore, all the initial conditions can be recovered using this characterization of the economy. Thus, an allocation is attainable under commitment if and only if it meets the following conditions

$$\begin{aligned} m(c_t, n_t) + \beta a_{t+1} &\geq a_t \\ c_t + g_t + k_{t+1} &= F(k_t, n_t) \\ \lim_{t \to \infty} \left[\beta^t a_{t+1}\right] &= 0 \end{aligned}$$

with $a_t \equiv u_c(c_t, n_t)\left[\frac{u_c(c_{t-1}, n_{t-1})}{\beta u_c(c_t, n_t)} k_t + b_t\right]$ for $t > 0$ and $a_0 \equiv u_c(c_0, n_0)[F_k(k_0, n_0)$ $(1 - \tau_0^k)k_0 + b_0]$.

The prices and policies that ensure that the remaining conditions are met are the following

$$\begin{aligned} T_t &= m(c_t, n_t) + \beta a_{t+1} - a_t \\ w_t &= F_n(k_t, n_t) \\ R_t &= F_k(k_t, n_t) \\ \tau_t^n &= 1 + \frac{1}{w_t} \frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} \\ \tau_{t+1}^k &= 1 - \frac{1}{\beta R_t} \frac{u_c(c_t, n_t)}{u_c(c_{t+1}, n_{t+1})}. \end{aligned}$$

7.2 Proof of Lemma 2

The value of the worst sustainable equilibrium depends only on the payoff relevant state variables, which are the current stock of capital and bonds, as well the percentage of households who defaulted in the previous period.

Suppose that this is not the case and that there are two histories A and B with the same k, b, and d^c, but with $V^w(A) > V^w(B)$. Then the value of A could be decreased if all agents followed the strategies from equilibrium B, which means it could not have been the worst equilibrium to begin with. Thus, it must be true that $V(\sigma^w, f^w, \phi^w) = V^w(k, b, d^c)$.

Since $V^w(k, b, 0)$ was the worst sustainable equilibrium when no consumers defaulted in the previous period, if $d^c > 0$ and the government decides not to punish consumers, the welfare cannot be lower than $V^w(k, b, 0)$. This implies the value of the worst sustainable equilibrium does not depend on whether consumers defaulted in the previous period.

The value of the worst sustainable equilibrium is also independent of b. Since $V^w(k, 0, d^c)$ is the worst equilibrium when the government has no debt, then if the government defaults on its debt no additional punishment can be given to him, which means that $V^w(k, b, d^c) \geq V^w(k, 0, d^c)$. If $b < 0$, then $V^w(k, b, d^c) \leq V^w(k, 0, d^c)$ since the politician can achieve $V^w(k, 0, d^c)$ by making transfers to consumers and equilibrium with $b < 0$ was already the worst possible, which means that no additional loss of welfare will be possible. Furthermore, it cannot be the case that $V^w(k, b, d^c) > V^w(k, 0, d^c)$ when $b > 0$ since then there would be a worst equilibrium where all households default.

Thus, the value of the worst sustainable equilibrium can only depend on the initial level of capital.

7.3 Proof of Lemma 3

In the worst sustainable equilibrium, households' beliefs about the following subgame are manipulated to yield the worst possible payoffs for the government. As we have seen, in each period households make their decisions according to

$$
\begin{aligned}
&u_c(c, n)F_n(k, n)(1 - \tau^n) + u_n(c, n) = 0 \\
&k'\{u_c(c, n) - \beta u_c(c', n')F_k(k', n')(1 - \tau^{k\prime})\} = 0 \\
&c_t + k_{t+1} = R_t(1 - \tau_t^k)k_t + w_t(1 - \tau_t^n)n_t + T_t
\end{aligned}
$$

Following Phelan and Stachetti (2001), define z' as the marginal value of capital for households tomorrow $z' \equiv u_c(c', n')F_k(k', n')(1 - \tau^{k\prime})$. The only way to affect households actions is by changing this value.

Let us now consider the government's problem when faced with a worst sustainable equilibrium. Let $V^w(k)$ be the expected payoff of the worst sustainable equilibrium when initial capital is k. Since the worst equilibrium the government can be given tomorrow is $V^w(k')$, then the following equilibrium is always available, which means that $V^w(k) \geq \underline{V}(k, z')$

$$\begin{aligned} \underline{V}(k, z') \quad &= \quad \max_{c,n,g,k'} \left\{u(c,n) + v(g) + \beta V^w(k')\right\} \\ \text{subject to} \quad & \quad m(c,n) + k' u_c(c,n) \geq 0 \\ & \quad c + g + k' = F(k,n) \\ & \quad k'[u_c(c,n) - \beta z'] = 0. \end{aligned}$$

Let $z' \in [\underline{z}, \overline{z}]$ be the values of z that can be sustained tomorrow. Then the worst sustainable equilibrium must be given by:

$$V^w(k) = \min_{z' \in [\underline{z}, \overline{z}]} \underline{V}(k, z').$$

From government's optimality we cannot have $V^w(k) < \min \underline{V}(k, z')$.

Suppose $V^w(k) > \min \underline{V}(k, z')$. Then this could not be the worst sustainable equilibrium since a lower payoff could be reached by giving consumers expectations $z' = \arg\min \underline{V}(k, z')$.

Since $\underline{V}(k, z')$ is increasing in z', it achieves its minima at the lower bound $\underline{z} = 0$. This means that consumers will expect capital to be taxed at confiscatory rates tomorrow, which implies that there will be no investment in capital. Furthermore, it will always be incentive compatible for the government to expropriate capital. Thus, the default equilibrium is the worst sustainable equilibrium.

7.4 Proof of Lemma 4

Suppose that the allocation x is the outcome of a sustainable equilibrium (σ, f, ϕ). Consumer optimality requires that x maximizes the households utility at time zero given the policies and prices along the equilibrium path. Government optimality implies that the government must satisfy its budget constraints from time zero on. Furthermore, in a sustainable equilibrium factor prices must equal marginal productivity of factors and the resource constraint must hold along the equilibrium path. Thus, the outcome of a sustainable equilibrium x must be attainable under commitment, which means that condition (i) must hold. At any time t, after history h_{t-1}, if the government deviates from the equilibrium path, it will get a payoff higher or equal to $V^d(k_t)$. Government optimality requires that not deviating must yield a higher payoff than deviating, which means that the present value of future profits must be at least as high as $V^d(k_t)$, which means that condition (ii) must hold at every time t. Thus, if an allocation x is the outcome of a sustainable equilibrium (σ, f, ϕ) it must meet conditions (i) and (ii).

Suppose now that an allocation x meets conditions (i) and (ii). Let π and ϕ be the policies and prices that implement this allocation under commitment. Consider the following strategy for households: as long as the government's action is according to π, choose allocation x; if the government deviates, follow the default equilibrium strategy. Likewise, consider the government's strategy where it acts according to π along the equilibrium path and plays the default strategies off equilibrium. Finally, consider factor prices that are equal to the marginal productivity of each factor of production for every possible history h_t. We will show that this is a sustainable equilibrium. First, consider histories where there have been no deviations until time t. Since x is attainable under equilibrium, and along the equilibrium path households will expect to face policies π and prices ϕ, this means that the continuation of x must be optimal for consumers. The government, on the other hand, can choose to deviate, in which case it would get $V^d(k_t)$, or it can follow the equilibrium path. Since condition (ii) ensures that the payoff along the equilibrium path is always higher than $V^d(k_t)$, then it is always incentive compatible for the government not to deviate. Now, consider histories where there has been a deviation before time t. Our strategy has specified that in this case both households and the government will play a default equilibrium, which we have shown to be sustainable. Thus, the specified set of strategies is a sustainable equilibrium that leads to outcome x.

7.5 Equivalence of Sequence and Recursive approaches

The Lagrangean for the sequence problem to find the best sustainable equilibrium can be written in the following way, where $\widehat{\mu}_t$ is the multiplier on the implementability condition, $\widehat{\rho}_t$ is the multiplier on the resource constraint, and $\widehat{\gamma}_t$ is the multiplier on incentive compatibility condition

$$\begin{aligned} n &= \sum_{t=o}^{\infty} \beta^t \left\{ \begin{array}{c} u(c_t, n_t) + \widehat{\mu}_t[m(c_t, n_t) + \beta a_{t+1} - a_t] \\ -\widehat{\rho}_t \left[c_t + g_t + k_{t+1} - F(k_t, n_t)\right] \end{array} \right\} \\ &\quad + \sum_{i=1}^{\infty} \beta^i \widehat{\gamma}_i \left[\sum_{t=i}^{\infty} \beta^{t-i} u(c_t, n_t) - V^d(k_i) \right]. \end{aligned}$$

The first order conditions for this problem are

$$\begin{aligned} u_{c_t}(1 + \sum_{i=1}^{t} \widehat{\gamma}_i) + \widehat{\mu}_t m_{c_t} &= \widehat{\rho}_t \\ u_{n_t}(1 + \sum_{i=1}^{t} \widehat{\gamma}_i) + \widehat{\mu}_t m_{n_t} &= -\widehat{\rho}_t F_{n_t} \\ \beta \widehat{\rho}_{t+1} F_{k_{t+1}} - \beta \widehat{\gamma}_{t+1} V_k^d(k_{t+1}) &= \widehat{\rho}_t \\ \widehat{\mu}_{t+1} &= \widehat{\mu}_t. \end{aligned}$$

The first order conditions for the recursive problem in section 4.2 are

$$
\begin{aligned}
&u_c + \mu m_c = \rho \\
&u_n + \mu m_n = -\rho F_n \\
&v_g = \rho \\
&V_a(a', k')(1+\gamma) = -\mu \\
&V_k(a', k') = \rho/\beta + \gamma[V_k^d(k') - V_k(a', k')]
\end{aligned}
$$

and the envelope conditions for a and k are

$$
\begin{aligned}
V_a(a, k) &= -\mu \\
V_k(a, k) &= \rho F_k(k, n).
\end{aligned}
$$

It is easy to see that the equilibrium conditions for the two problems lead to the same allocations for c, n, k, and a for as long as the multipliers for the constraints in the recursive formulation μ, ρ, and γ have the following relationship with the multipliers in the sequence approach:

$$
\begin{aligned}
\mu &= \frac{\widehat{\mu}_t}{1 + \sum^t \widehat{\gamma}_i} \\
\rho &= \frac{\widehat{\rho}_t}{1 + \sum^t \widehat{\gamma}_i} \\
1 + \gamma &= \frac{1 + \sum^{t+1} \widehat{\gamma}_i}{1 + \sum^t \widehat{\gamma}_i}.
\end{aligned}
$$

Furthermore, the following transversality condition must be met in both formulations

$$\lim_{t \to \infty} \beta^t V(k_t, a_t) = 0.$$

Since the best sustainable equilibrium converges to a steady state, which has positive and finite allocations, the transversality condition is met.

7.6 Impossibility of non zero capital taxes in steady state

We will now show that any steady state of the best sustainable equilibrium must have zero capital taxes without imposing concavity of $V(k_t, a_t)$.

The idea is that if commitment were binding in the long run, then the implementability condition would stop binding, and labor taxes would converge to zero. But this can not be an optimal steady state, since it is optimal to take a deviation where labor taxes are increased marginally (with zero first order cost) to increase government assets and make the incentive compatibility constraint less binding (which has a positive first order effect).

More formally, assume that there is a steady state where capital taxes are different from zero. Let c^{ss}, n^{ss}, g^{ss}, a^{ss} and k^{ss} be the allocations in this steady state. In order to meet first order conditions, these steady states must also have constant multipliers ρ^{ss}, μ^{ss}, γ^{ss}. In steady state, the optimality condition for capital becomes

$$\rho^{ss}F_k(k^{ss},n^{ss}) = \rho^{ss}/\beta + \gamma^{ss}[V_k^d(k^{ss}) - \rho^{ss}F_k(k^{ss},n^{ss}).$$

Given this we can only have capital taxes different from one and $\rho F_k(k^{ss},n^{ss}) \neq 1$ if $\gamma^{ss} > 0$.

The optimality condition for a implies that

$$\mu^{ss}(1+\gamma^{ss}) = \mu^{ss}.$$

Thus, when $\gamma^{ss} > 0$ we must have $\mu^{ss} = 0$.

But then the first order conditions for consumption, labor and public spending take the following form

$$\begin{aligned} u_c(c^{ss},n^{ss}) &= \rho^{ss} \\ u_n(c^{ss},n^{ss}) &= -\rho^{ss}F_n(k^{ss},n^{ss}) \\ v'(g^{ss}) &= \rho^{ss}. \end{aligned}$$

Furthermore, it must be true that

$$V(k^{ss},a^{ss}) = V^d(k^{ss}) < V(k^{ss},\underline{a}(k^{ss})) = V^{fb}(k^{ss}).$$

Now let us consider a departure from this steady state that will lead to higher welfare than our original candidate for a steady state, thus implying that it could not be an optimal solution to begin with. The departure is as follows. In an initial period (let us call it period 0), starting from our initial steady state, we will choose $a_1 = a^{ss} - \Delta a$ instead of a^{ss} and $k_1 = k^{ss}$. The new levels of consumption, labor and public spending are the solution to the following problem

$$\begin{aligned} [c_0,n_0,g_0] \quad &= \quad \arg\max_{c,n,g} \; [u(c,n)+v(g)] \\ \text{subject to} \quad & \quad w(c,n) + \beta(a^{ss}-\Delta a) \geq a^{ss} \\ & \quad c+g+k^{ss} = F(k^{ss},n). \end{aligned}$$

First, let us check that this deviation is feasible. Clearly the implementability condition and the resource constraint must be met, by construction of the previous problem. The government's incentive compatibility condition now becomes

$$V(k^{ss},a^{ss}-\Delta a) \geq V^d(k^{ss}).$$

Since $V(k,a)$ cannot be increasing in a this condition must be met since $V(k^{ss},a^{ss}) = V^d(k^{ss})$ in the original steady state.

Now we will show that taking this deviation has a zero cost up to a first order approximation. The new flow utility in period zero is given by

$$\begin{aligned} W(\Delta a \mid k^{ss}, a^{ss}) &= \max_{c,n,g} \; [u(c,n) + v(g)] \\ \text{subject to} \quad & w(c,n) + \beta(a^{ss} - \Delta a) \geq a^{ss} \\ & c + g + k^{ss} = F(k^{ss}, n). \end{aligned}$$

We can write a first order Taylor approximation of this expression in the following way

$$W(\Delta a) = W(0) + \Delta a W'(0).$$

Given that the implementability is not binding when $\Delta a = 0$, then $W(\Delta a) = W(0)$, which means that the cost of this deviation is zero to a first order approximation.

Let us now see if there is any benefit to it. The deviation proceeds as follows. From period 1 to period $T-1$ (which will be defined shortly), the chosen allocations will be

$$\begin{aligned} c_t &= c^{ss}, n_t = n^{ss}, g_t = g^{ss}, k_t = k^{ss} \text{ for } t = 1...T-1 \\ a_t &= a^{ss} - \beta^{1-t}\Delta a \text{ for } t = 1...T. \end{aligned}$$

Let us start by checking that the new plan is feasible from periods 1 to T. First, the resource constraint must be met since the allocations in this constraint are the same as in the initial steady state. The incentive compatibility must also hold since $V(k,a)$ cannot be increasing in a. Finally, the implementability condition will be met since $w(c^{ss},n^{ss}) \geq a^{ss}(1-\beta)$, which implies that $w(c,n) + \beta(a^{ss} - \beta^{-t}\Delta a) \geq a^{ss} - \beta^{1-t}\Delta a$.

Let T be defined by the following condition

$$\Delta a \frac{1}{\beta^T} = a^{ss} - \underline{a}(k^{ss}).$$

This means that in period T we will reach $a_t = \underline{a}(k^{ss})$ and will be able to increase $V(k^{ss}, \underline{a}(k^{ss})) > V(k^{ss}, a^{ss})$ by switching to the non constrained solution. Furthermore, this will lead to a first order positive welfare increase in the initial period

$$B(\Delta a) = \beta^T[V(k^{ss}, \underline{a}(k^{ss})) - V(k^{ss}, a^{ss})] = \frac{V(k^{ss}, \underline{a}(k^{ss})) - V(k^{ss}, a^{ss})}{a^{ss} - \underline{a}(k^{ss})}\Delta a.$$

Thus, the net benefit of our deviation is strictly positive for a small enough change in a, which means that the initial candidate for a steady state was not optimal.

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