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# Economic growth and stability with public PAYG pensions and private intra-family old-age insurance

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**Abstract** This paper investigates the steady state and dynamical effects of two historical alternatives as a means of old-age insurance – i.e., voluntary intra-family transfers from young to old members versus pay-as-you-go public pensions –, in a general equilibrium overlapping generations model with children as a desirable good. It is shown that the shift from a private system of old-age insurance to a public system of social security increases GDP per worker. Moreover, although in both cases the dynamics of capital, under myopic expectations, may be globally unstable depending on the size of the (private as well as public) inter-generational transfer, we show that such a shift significantly reduces, for plausible economies, the risk of cyclical instability which otherwise would be dramatically high, especially in countries with high degree of parsimony and low preference for children.

**Keywords** Endogenous fertility; Myopic foresight; OLG model; Private old-age support; Public PAYG pensions

**JEL Classification** C62; H55; J14; J18; J26

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## 1. Introduction

Most developed countries have introduced social security or public pensions programs since the first part of the past century (e.g., in the US social security was introduced the end of thirties). In practice, the social security system is mostly unfunded, or pay-as-you-go (PAYG). While all societies have been historically contributed to the livelihood of the elderly through voluntary intra-family transfers from children to parents (see, for instance, Ehrlich and Lui, 1991), at a certain stage of economic development, in order to secure old-age consumption, governments propagated public pension provision.[1]

Since economists agree with the reducing-fertility effect of the introduction of public pensions as a substitute of intra-family transfers, then for some developing economies public pensions might apparently be advised for the purpose of lowering fertility rates (e.g., the case of China). Otherwise, for many developed economies plagued by below-replacement fertility it has been pointed out that the public pension provision is responsible for such a fertility drop (see, e.g., Cigno and Werding, 2007).

In addition to the reduced fertility effect, the literature also investigated the relationship between the introduction of public pensions, together with the private system of old-age backing, and economic development. Nishimura and Zhang (1992, 1995) and Cigno (1995) – by assuming only backward altruism (i.e. from children to parents) – show that introducing a social security system with a tax equal to the transfer from the young to the old may increase utility, but since their models only deal with a partial equilibrium context, they do not address the economic growth issue. Zhang and Zhang (1995) – again only by assuming backward altruism – and Wigger (1999) – by assuming both backward and forward altruism at the same time (i.e. from parents to children) –, found, by

[1] "It has been observed that at certain stage of economic growth and development a nation starts to consider the introduction of social security programs. In developing nations, there has been increasing discussion in popular press of introducing social security systems." (Zhang and Zhang, 1995, p. 441).

removing the partial equilibrium context and introducing a production side with endogenous growth features, that the rate of growth of GDP per worker may, in some cases,[2] increase along with the introduction of public PAYG pensions.

Unlike previous studies, in this paper, in order to capture the rather realistic idea that the "pleasure" for own children exists also in the absence of public pensions, the decision to have children in such a case is not only based on the old-age-support motive but also on altruism towards children. Moreover, the assumption of children as a capital good, on which all this literature is based, is quite different from that at the basis of the choices of fertility in the modern approach (i.e. the new home economics, see Becker, 1960), where both the number and quality of children affect parents' utility.[3] In order to be in line with the new home economics approach, in this paper we enter the number of children into the parents' utility. Therefore, the "love" for children exists in both the presence and absence of social security. In the former case the same fraction of the income the young workers is collected by the public pension system in order to publicly provide backing for old-age people, while in the latter case young people support their parents with a fraction of their income, which is, as in Bental (1989) and in Morand (1999), exogenously given and may also be thought as owing to cogent social norms.

Although the love for children remains unchanged in both cases, the incentive to have children is larger in the absence of PAYG pensions because in such a case the size of the old age support

[2] More in detail, while in Zhang and Zhang (1995) the rate of economic growth in the case of social security is always larger than in the case of privately provided old-age backing, in Wigger (1999, p. 627) "small sized public pensions stimulate per capita income growth but further increases in public pensions reduce it."

[3] For simplicity, in this paper we abstract from the quality of children, which would imply also the study of the dynamics of human capital, so raising the complexity of the analysis. However, this important extension, in order to be more close to the new home economic approach, is left for future research. Moreover, it should also be noted that in the new home economics approach also the welfare of offspring sometimes enters parents' utility (Becker and Barro, 1988), but such dynastic models have the property that the "effects of changes in taxes are negated via changes in bequests, and so are ill suited to analyzing social security or publicly funded education." (Pecchenino and Pollard, 2002, p. 149).

depends on the parents' own number of children rather than the economy-wide average fertility rate (as in the case of PAYG pensions) and, hence, the higher the size of the inter-generational transfer is the lower the net cost of bearing offspring would be. In other words, instead of investigating the effects of introducing PAYG pensions by keeping the assumption of backward altruism unchanged, we assume forward "weak" altruism (love for children) in both cases, so differing from Nishimura and Zhang (1992, 1995), Cigno (1995) and Zhang and Zhang (1995). We also differ from Wigger (1999) who, on the contrary, simultaneously assumes the existence of both types of altruism as well as private and public alternatives for supporting consumption of old-age people. Moreover, and more important, we differ from all these models because we *separately* compare the two alternatives for supporting the old-aged rather than consider them together, in order to better capture the effects of the historical evolution from intra-family to public old-age support systems on the neoclassical economic growth and economic stability.

This paper shows that the shift from a private system of old-age insurance to a public system of social security in an economy with overlapping generations (OLG) and endogenous fertility, always increases GDP per worker. Moreover, it is also shown, under myopic foresight, the preferences for old-age consumption and children are crucial for determining economic stability in the case of intra-family transfers, while such parameters play no role in the case of public pensions (where stability depends only on technology and the size of the pension provision). The reason is that in the first case individuals take into account the fact that it is the number of own children rather than the average fertility rate in the whole economy to determine the size of the benefit received when old. This implies that in the first case the economy is always more prone to an unstable outcome, and rather counter-intuitively, the lower the love for children, the higher the risk of cyclical instability. Therefore, in particular, economies with low preference for children and high degree of parsimony are largely beneficiated in terms of stability by the public rather than private intra-family provision of support for old-age consumption.

The remainder of the paper is organised as follows. In Section 2 we briefly describe the features common to a general equilibrium economy with both private old-age backings and public PAYG pensions. In Section 3 (4) we analyse and discuss the steady state and dynamical effects of the public pension provision (private intra-family old-age support). In Section 5 we compare the two systems of social security and present a numerical experiment. Section 6 concludes.

## 2. The economy

### *2.2. Firms*

Identical firms act competitively on the market. At time $t$, final goods ($Y_t$) are produced with a Cobb-Douglas constant returns to scale technology in aggregate capital ($K_t$) and labour ($L_t = N_t$ in equilibrium), that is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, with $0 < \alpha < 1$ and $A > 0$. Production per worker, therefore, is $y_t = Ak_t^{\alpha}$ where $k_t := K_t / N_t$ and $y_t := Y_t / N_t$.

Since capital totally depreciate at the end of each period and the price of final output is normalised to unity, profit maximisation implies:

$$r_t = \alpha A k_t^{\alpha-1} - 1, \quad (1)$$

$$w_t = (1-\alpha) A k_t^{\alpha}, \quad (2)$$

that is capital and labour are paid their respective marginal productivities.

### *2.2. Individuals*

Individuals are identical and live in a general equilibrium OLG closed economy. Life is divided into childhood and adulthood. In the former period, individuals do not make economic decisions. In the latter period, they work and bear children when young, while being retired when old. Young individuals of generation $t$ ($N_t$) inelastically supply their whole time endowment of measure one on the labour market and receive a unitary wage income at the competitive rate $w_t$. This income is used to consume ($c_{1,t}$), to save ($s_t$) and to bear children. As regards child-rearing activities, we assume that parents devote a positive fixed amount of resources $e$ to take care of each child, so that the cost of raising $n_t$ children is $e n_t$.

Preferences of individuals of generation $t$ over young-aged consumption, old-aged consumption and the number of children[4] are described by the following lifetime logarithmic utility function:

$$U_t = \ln\left(c_{1,t}\right) + \beta \ln\left(c_{2,t+1}\right) + \gamma \ln\left(n_t\right), \tag{3}$$

where $0 < \beta < 1$ is the subjective discount factor and $\gamma > 0$ the taste for children.

## 3. Public PAYG pensions

In this section we assume that a public unfunded system of social security exists to transfer resources from the young to the old in every period. The rules of the PAYG scheme imply that current workers finance pensions to current pensioners. At time $t$, therefore, the balanced-budget pension accounting rule in per worker terms reads as

$$p_t = \tau w_t n_{t-1}, \tag{4}$$

the left-hand side being the pension expenditure and the right-hand side the tax receipt, where $0 < \tau < 1$ is the fixed contribution rate and $n_{t-1}$ the average fertility rate in the whole economy at time $t-1$.

The budget constraints of both working and retirement periods faced by an individual started working at $t$ in a PAYG-taxed economy read, respectively, as:

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[4] A similar logarithmic utility function including, in accord with the new home economics, the *number* of children as an argument, is used, amongst many others, by Eckstein and Wolpin (1985), Galor and Weil (1996), van Groezen et al. (2003) and van Groezen and Meijdam (2008).

$$c_{1,t} + s_t + en_t = w_t(1-\tau), \tag{5}$$

$$c_{2,t+1} = \left(1 + r^e_{t+1}\right)s_t + p^e_{t+1}. \tag{6}$$

Eq. (5) says that when young, wage income – net of the contribution levied by the government to finance the benefit to current pensioners – is divided between material consumption, savings and the (fixed) cost of bearing $n_t$ descendants. Eq. (6), instead, reveals that consumption possibilities when old are constrained by both the amount of resources saved when young (plus the expected interest accrued at the rate $r^e_{t+1}$) and the expected pension benefit $p^e_{t+1}$.

The representative individual of generation $t$ wishes to choose how much to save out of her disposable income as well as how many children to raise in order to maximise the lifetime utility index Eq. (3) subject to Eqs. (5) and (6). The first order conditions for an interior solution are, therefore, the following:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1}{\beta} = 1 + r^e_{t+1}, \tag{7}$$

$$\frac{c_{1,t}}{n_t} \cdot \gamma = e. \tag{8}$$

Eq. (7) equates the marginal rate of substitution between consumption in working and retirement periods to the expected interest rate determined on the capital market. Eq. (8), instead, equates the marginal rate of substitution between consuming when young and raising children to the expected marginal cost of bearing an extra child. Since individuals do not take into account the government pension accounting rule Eq. (4) when deciding on both savings and fertility, the marginal cost of child rearing is independent of the size of the publicly provided inter-generational transfer, which is determined on the basis of the economy-wide number of children.

Therefore, using the first order conditions together with the individual budget constraints as well as the one-period forward pension accounting rule Eq. (4), the demand for children, young-aged consumption, old-aged consumption and the saving rate in a PAYG-taxed economy are respectively determined as:

$$n_t = \frac{\gamma w_t (1-\tau)}{(1+\beta+\gamma)e - \gamma\tau \frac{w^e_{t+1}}{1+r^e_{t+1}}}, \tag{9}$$

$$c_{1,t} = \frac{w_t(1-\tau)}{1+\beta+\gamma} + \frac{\tau \frac{w^e_{t+1}}{1+r^e_{t+1}} n_t}{1+\beta+\gamma} = \frac{w_t(1-\tau)e}{(1+\beta+\gamma)e - \gamma\tau \frac{w^e_{t+1}}{1+r^e_{t+1}}}, \tag{10}$$

$$c_{2,t+1} = \beta\left(1+r^e_{t+1}\right)\left[\frac{w_t(1-\tau)}{1+\beta+\gamma} + \frac{\tau \frac{w^e_{t+1}}{1+r^e_{t+1}} n_t}{1+\beta+\gamma}\right] = \frac{\beta\left(1+r^e_{t+1}\right) w_t (1-\tau)e}{(1+\beta+\gamma)e - \gamma\tau \frac{w^e_{t+1}}{1+r^e_{t+1}}}, \tag{11}$$

$$s_t = \frac{\beta w_t(1-\tau)}{1+\beta+\gamma} - \frac{1+\gamma}{1+\beta+\gamma}\tau \frac{w^e_{t+1}}{1+r^e_{t+1}} n_t = \frac{w_t(1-\tau)}{(1+\beta+\gamma)e - \gamma\tau \frac{w^e_{t+1}}{1+r^e_{t+1}}}\left(\beta e - \gamma\tau \frac{w^e_{t+1}}{1+r^e_{t+1}}\right). \tag{12}$$

Eqs. (9)-(12) deserve some comments. In an economy with public PAYG pensions, a rise in the contribution rate $\tau$ affects the demand functions and the saving rate in the following way. As regards fertility, young-aged and old-aged consumptions, from Eqs. (9)-(11) it is easy to see that the effect is twofold: (*i*) a negative effect due to a reduction in the disposable income when young, and (*ii*) a positive effects due to a rise in the pension benefit received when old, both weighted by the preference parameters. As regards savings, Eq. (12) reveals that a rise in $\tau$ negatively affects savings through two channels: an intra-generational effect due to a lower disposable income when young, and an inter-generational effect exerted by the rise in the present value of the expected pension benefit weighted by the fertility rate times a coefficient smaller than unity due to the effects of preferences.

### *3.1. Equilibrium with myopic foresight in a PAYG-taxed economy*

Since the future expected value of factor prices is included in all previous equations, then in order to close the model it is necessary to specify the type of expectations formation of individuals with respect to the future values of both the wage and interest rate.

The two extreme cases used to study the dynamics of a general equilibrium economy are (*i*) myopic expectations and (*ii*) rational expectations (see, e.g., de la Croix and Michel, 2002).

In this paper we exclusively focus on studying the dynamics an economy under myopic foresight for the very interesting dynamical features that the model presents in that case. The assumption of myopic expectations is rather usual in literature (see Michel and de la Croix, 2000), and a certain level of myopia seems to be inherent with the life-cycle context in presence of social security schemes (see Pecchenino and Pollard, 2005).[5]

In particular, if individuals are short-sighted, then they expect the future values of both the interest and wage rates to depend on the current value of the stock of capital per worker, that is

$$\begin{cases} 1 + r^{e}{}_{t+1} = \alpha A k_t^{\alpha-1} \\ w^{e}{}_{t+1} = (1-\alpha) A k_t^{\alpha} \end{cases}. \tag{13}$$

Now, given the government budget Eq. (4) and knowing that population evolves according to $N_{t+1} = n_t N_t$, the equilibrium condition in goods and capital markets, which is given by the equality between investments and savings, can be written in per worker terms as

$$n_t k_{t+1} = s_t . \tag{14}$$

Combining Eqs. (9), (12), (13) and (14) we obtain the following linear difference equation describing the dynamic evolution of capital in a PAYG-taxed economy:

$$k_{t+1} = \frac{\beta e}{\gamma} - \tau \mathrm{B}_1 k_t . \tag{15}$$

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[5] The essence of a social security system might be based on an "intrinsic" myopia of agents when young, as the illuminating words of Samuelson (1975, p. 543) reveal: "Many social security systems, like the New Deal U.S. system, may be deemed most valuable precisely because the myopia ignored by the present models does in fact prevail. People live miserably in old age because they do not realize when young what are the consequences of their private saving habits. So by democratic fiat, they paternalistically impose on themselves a within-life pattern of consumption that favors old age at the expense of young. Precisely because of the myopia that makes paternalism optimal…"

Eq. (15) shows that the dynamic evolution of capital in an economy with public PAYG pensions depends on two parts, which have an opposite effect on capital accumulation: (*i*) a component which is constant because of the hypothesis of fixed cost of children (the first term on the right-hand side of Eq. 15), determined as the preference for old-age consumption times the fixed cost of child bearing divided by the taste for children, and (*ii*) the inter-generational transfer component due to the existence of public pensions (the second term on the right hand side of Eq. 15), which negatively affects capital accumulation because it represents a reduction in the disposable income when young as well as a reduced need of savings for sustaining old-age consumption and, hence, results in a lower saving rate (a crowding out effect). In particular, given the myopic foresight hypothesis, the negative component (*ii*) is a linear function of the capital stock at time $t$, and its size is determined as the contribution rate weighted by a coefficient that measures the relative importance between the share of labour and the share of capital in technology.

Steady-state implies $k_{t+1} = k_t = k^*$.[6] Therefore,

$$k^*_{PAYG} = \frac{\beta e \alpha}{\gamma[\alpha + \theta(1-\alpha)]}. \tag{16}$$

Analysis of Eqs. (15) and (16) gives the following proposition:

**Proposition 1**. *In a PAYG-taxed economy the dynamics of capital is the following.*

*(1) Let* $0 < \alpha < 1/2$ *hold. Then* $\hat{\tau} < 1$*, and:*

*(1.1) if* $0 < \tau < \hat{\tau}$*, the non-monotonic dynamics of capital is globally stable towards* $k^*_{PAYG}$*;*

*(1.2) if* $\tau = \hat{\tau}$*, a flip bifurcation emerges;*

*(1.3) if* $\hat{\tau} < \tau < 1$*, the non-monotonic dynamics of capital is globally unstable.*

[6] Note that the steady state is the same irrespective of the expectations formation on the future values of factor prices (see Michel and de la Croix, 2000).

*(2) Let* $1/2 < \alpha < 1$ *hold. Then* $\hat{\tau} > 1$ *and the non-monotonic dynamics of capital is globally stable towards* $k^*_{PAYG}$ *for any* $0 < \tau < 1$*,*

*where*

$$\hat{\tau} = \hat{\tau}(\alpha) := \frac{1}{B_1}. \tag{17}$$

**Proof**. Differentiating Eq. (15) with respect to $k_t$ gives:

$$\frac{\partial k_{t+1}}{\partial k_t} = -\tau B_1 < 0. \tag{18}$$

From Eq. (18) we conclude that the dynamics of capital is always non-monotonic.

*Non-monotonic dynamics: stability analysis in a PAYG-taxed economy*

The condition $\frac{\partial k_{t+1}}{\partial k_t} \gtrless -1$ implies:

$$\frac{\partial k_{t+1}}{\partial k_t} = -\tau B_1 \gtrless -1 \Rightarrow \tau \lessgtr \hat{\tau}, \tag{19}$$

where $\tau = \hat{\tau}$ (defined by Eq. 17) is the value of the contribution rate below (beyond) which the dynamics of capital is globally stable (unstable). In particular, $\hat{\tau} < 1$ ($\hat{\tau} > 1$) for any $0 < \alpha < 1/2$ ($1/2 < \alpha < 1$).

Therefore,

(*i*) if $0 < \alpha < 1/2$ then $\hat{\tau} < 1$ and (*1.1*) $-1 < \frac{\partial k_{t+1}}{\partial k_t} < 0$ for any $0 < \tau < \hat{\tau}$, (*1.2*) $\frac{\partial k_{t+1}}{\partial k_t} = -1$ if and only if $\tau = \hat{\tau}$, and (*1.3*) $\frac{\partial k_{t+1}}{\partial k_t} < -1$ for any $\hat{\tau} < \tau < 1$. This proves point (1);

($ii$) if $1/2 < \alpha < 1$ then $\hat{\tau} > 1$ and $-1 < \frac{\partial k_{t+1}}{\partial k_t} < 0$ for any $0 < \tau < 1$. This proves point (2). **Q.E.D.**

The dynamics of capital in an economy with public PAYG pensions is the result of two counterbalancing forces when the stock of capital varies: (*i*) a constant component, and (*ii*) a negative crowding out effect on capital accumulation due to the financing of public pensions. As previously discussed, with myopic foresight, this latter effect is a linear function of the capital stock with a coefficient determined by the contribution rate $\tau$ multiplied by the ratio between the share of labour and the share of capital. When production is relatively labour-oriented (i.e. the capital share $\alpha$ is low) and the size of the inter-generational transfer is high enough, the negative effect (*ii*) is high and, hence, the dynamics may become cyclically unstable. This holds because with $\alpha$ being low enough, the weight of the wage relative to that of the interest rate in the component (*ii*) is high and then, when the stock of capital increases the weight of (*ii*) raises while the component (*i*) is kept unchanged, and this in turn implies that the slope of the phase map becomes steeper and the dynamics tends to become unstable.

Moreover, from Eq. (17) we have the following proposition:

**Proposition 2**. *In a PAYG-taxed economy, an exogenous positive shock on the output elasticity of capital ( $\alpha$ ) [the taste for children ( $\gamma$ ), the individual degree of thriftiness ( $\beta$ )] acts as an economic stabiliser [is neutral for stability].*

**Proof**. The proof immediately follows from $\frac{\partial \hat{\tau}}{\partial \alpha} = \frac{1}{(1-\alpha)^2} > 0$, $\frac{\partial \hat{\tau}}{\partial \gamma} = \frac{\partial \hat{\tau}}{\partial \beta} = 0$. **Q.E.D.**

## 4. Private old-age support

In this section we concentrate on the case of an economy where private intra-family, rather than public, transfers from young to old members exist as a means of old-age insurance. In particular, we assume that the members of the younger generation at $t$ voluntarily devote a fixed[7] fraction $0 < \eta < 1$ of wage income to support material consumption of the elderly, so that $\eta w_t$ is the cost to each young and $\eta w^e_{t+1} n_t$ is the expected benefit to the elderly. Therefore, the budget constraints of both working and retirement periods of an individual started working at $t$ can now be written as

$$c_{1,t} + s_t + en_t = w_t(1-\eta), \tag{20}$$

$$c_{2,t+1} = \left(1 + r^e_{t+1}\right)s_t + \eta w^e_{t+1} n_t . \tag{21}$$

Eq. (20) is the same as Eq. (5) with the only difference that now the fraction $\eta$ of wage income is voluntarily devoted to support old-aged consumption. Eq. (21), instead, reveals that consumption when old is determined as the proceeds of savings plus the expected benefit voluntarily lavished when young.

Therefore, maximisation of the utility function Eq. (3) subject to Eqs. (20) and (21) gives the following first order conditions:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1}{\beta} = 1 + r^e_{t+1} , \tag{22}$$

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[7] The assumption of an exogenously given fraction of wage income as a transfer to the old is in accord, for instance, with Bental (1989) and Morand (1999), and is made, again in accord with these papers, in order to simplify the analysis (more in detail, Bental (1989) assumes a fixed lump-sum transfer, while Morand (1999) considers, as in the present model, a fixed fraction of wage as a transfer to the elderly). For the sake of precision, we point out that Ehrlich and Lui (1991) and Azariadis and Drazen (1991) consider the size of the transfer as an endogenous variable. Moreover, it should be noted that all these papers implicitly assumed that the implicit contract concerning intra-family transfers between parents and children will be honoured. The emerging of time consistency and self-enforcement problems of this contract are discussed, for instance, by Ehrlich and Lui (1991).

$$\frac{c_{1,t}}{n_t}\cdot\gamma = e - \eta\frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}. \tag{23}$$

Eq. (22) is the same as Eq. (7). In contrast, Eq. (23) – which equates the marginal rate of substitution between consuming when young and raising children to the expected marginal cost of bearing an extra child – is different from the corresponding Eq. (8) in the case of public pensions. Indeed, since individuals now take into account the fact that the size of the private inter-generational transfer is determined on the basis of their own number of children rather than the economy-wide average fertility rate, the cost of children now is reduced by the present value of the expected benefit received when old, and the higher the fraction of wage income devoted to finance old-age consumption, the higher such a benefit and, hence, the lower the net marginal cost of raising an additional child.

Therefore, the demand for children, young-aged and old-aged consumption and the saving rate in an economy with private old-age support are respectively determined as

$$n_t = \frac{\gamma w_t(1-\eta)}{(1+\beta+\gamma)\left(e-\eta\frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}\right)}, \tag{24}$$

$$c_{1,t} = \frac{w_t(1-\eta)}{1+\beta+\gamma}, \tag{25}$$

$$c_{2,t+1} = \frac{\beta(1+r^e{}_{t+1})w_t(1-\eta)}{1+\beta+\gamma}, \tag{26}$$

$$s_t = \frac{\beta w_t(1-\tau)}{1+\beta+\gamma} - \eta\frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}n_t = \frac{w_t(1-\eta)}{(1+\beta+\gamma)\left(e-\eta\frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}\right)}\left[\beta e-(\beta+\gamma)\eta\frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}\right]. \tag{27}$$

From Eq. (24) it is easy to see that a rise in the (intra-family) contribution rate $\eta$ affects fertility in a twofold way. First, it reduces fertility owing to a lower disposable income when young. Second, it raises fertility due to the inter-generational transfer effect. *Ceteris paribus*, the size of the (negative) former effect is the same as that of an economy with public PAYG pensions (see Eq. 24), while that of the (positive) latter effect is quite different; in particular, since individuals now internalise the externality of children (because the benefit when old is computed on the basis of the individual, rather than economy-wide, fertility rate), the size of the inter-generational transfer effect is higher (because the marginal cost of raising an extra child is lower) and weighted by both the individual degree of thriftiness and taste for children.

As regards consumption, Eqs. (25) and (26) clearly show that now only an intra-generational negative effect on young-aged and old-aged consumption exists when $\eta$ raises, as given by the reduction in the disposable income of the young workers. Since when young individuals know that they will receive a higher benefit when old, then they decide to give birth to a higher number of descendants, and the inter-generational effect of the private transfer system, therefore, is nil. As a consequence, the negative effect of the private transfer system on savings is different than that of the public one. In fact, as Eq. (27) shows, *ceteris paribus*, the reduction in the disposable income is the same as that would be obtained with PAYG pensions, but the size of the inter-generational effect now is higher: in particular, as can easily be ascertained by looking at the numerator of the saving function Eq. (27) versus that of Eq. (12), the negative effect of the inter-generational transfer on savings is weighted by the sum of the parameters representing the preference for old-aged consumption and the taste for children rather than only by the latter parameter.

Therefore, comparison of Eqs. (9) and (24), and Eqs. (12) and (27), makes clear the reason why fertility and savings are different depending on whether a public or private transfer system exists as a means of old-age insurance. In particular, as regards fertility it can easily be shown, *ceteris paribus* as regards the contribution rate as well as all the other variables and parameters of the problem, that the fertility rate in an economy with private old-age support is always higher than that of an economy with public PAYG pensions, while the savings rate is higher when the latter transfer system is in place.[8] This fact will obviously give rise to differences in both the steady-state and

[8] The proof is not reported here for economy of space.

dynamical effects of the private and public transfer systems, which will be deeply discussed in Section 5.

*4.1. Equilibrium with myopic foresight in an economy with private old-age support*

Market-clearing in goods and capital market is still determined by Eq. (14). Therefore, substituting out $n_t$ and $s_t$ from Eqs. (24) and (27) into Eq. (14), and then using Eq. (13), the dynamic path of capital accumulation in an economy with private old-age support is described by the following linear difference equation:

$$k_{t+1} = \frac{\beta e}{\gamma} - \eta \mathrm{B}_1 \mathrm{B}_2 k_t , \tag{28}$$

where $\mathrm{B}_1 := \frac{1-\alpha}{\alpha}$ and $\mathrm{B}_2 := \frac{\beta+\gamma}{\gamma} > 1$.

Eq. (28) is similar to Eq. (15). The only difference is that the negative effect on capital accumulation given by the (private) inter-generational transfer component (the second term on the right hand side of Eq. 28), is now reinforced by a factor $\mathrm{B}_2$ greater than unity, and this difference seems to work as a *destabilising* device for the economy, as shown below.

The steady-state stock of capital per worker, therefore, is

$$k^*_{OAS} = \frac{\beta e \alpha}{\alpha\gamma + \eta(1-\alpha)(\beta+\gamma)}.[9] \tag{29}$$

From Eqs. (28) and (29) the following proposition holds:

**Proposition 3**. *In an economy with private intra-family transfers as a means of old-age insurance, the dynamics of capital is the following.*

[9] The subscript $OAS$ means “old-age support”.

*(1) Let* $0 < \alpha < \hat{\alpha}$ *hold. Then* $\hat{\eta} < 1$*, and:*

*(1.1) if* $0 < \eta < \hat{\eta}$*, the non-monotonic dynamics of capital is globally stable towards* $k^{*}_{OAS}$*;*

*(1.2) if* $\eta = \hat{\eta}$*, a flip bifurcation emerges;*

*(1.3) if* $\hat{\eta} < \eta < 1$*, the non-monotonic dynamics of capital is globally unstable.*

*(2) Let* $\hat{\alpha} < \alpha < 1$ *hold. Then* $\hat{\eta} > 1$ *and the non-monotonic dynamics of capital is globally stable towards* $k^{*}_{OAS}$ *for any* $0 < \eta < 1$*,*

*where*

$$\hat{\eta} = \hat{\eta}(\alpha, \beta, \gamma) := \frac{1}{\mathrm{B}_1 \mathrm{B}_2}, \tag{30}$$

$$\hat{\alpha} = \hat{\alpha}(\beta, \gamma) := \frac{\beta + \gamma}{\beta + 2\gamma}, \quad 1/2 < \hat{\alpha} < 1. \tag{31}$$

**Proof**. Differentiating Eq. (28) with respect to $k_t$ gives:

$$\frac{\partial k_{t+1}}{\partial k_t} = -\eta \mathrm{B}_1 \mathrm{B}_2 < 0. \tag{32}$$

From Eq. (32) it is easy to see that the dynamics of capital is always non-monotonic.

*Non-monotonic dynamics: stability analysis in an economy with private old-age support*

The condition $\frac{\partial k_{t+1}}{\partial k_t} \gtrless -1$ implies:

$$\frac{\partial k_{t+1}}{\partial k_t} = -\eta \mathrm{B}_1 \mathrm{B}_2 \gtrless -1 \Rightarrow \eta \lessgtr \hat{\eta}, \tag{33}$$

where $\eta = \hat{\eta}$ (defined by Eq. 30) is the value of $\eta$ below (beyond) which the dynamics of capital is globally stable (unstable). In particular, $\hat{\eta} < 1$ ($\hat{\eta} > 1$) for any $0 < \alpha < \hat{\alpha}$ ($\hat{\alpha} < \alpha < 1$), where $1/2 < \hat{\alpha} < 1$ is defined by Eq. (31).

Therefore,

(*i*) if $0 < \alpha < \hat{\alpha}$ then $\hat{\eta} < 1$ and (*1.1*) $-1 < \frac{\partial k_{t+1}}{\partial k_t} < 0$ for any $0 < \eta < \hat{\eta}$, (*1.2*) $\frac{\partial k_{t+1}}{\partial k_t} = -1$ if and only if $\eta = \hat{\eta}$, and (*1.3*) $\frac{\partial k_{t+1}}{\partial k_t} < -1$ for any $\hat{\eta} < \eta < 1$. This proves point (1);

(*ii*) if $\hat{\alpha} < \alpha < 1$ then $\hat{\eta} > 1$ and $-1 < \frac{\partial k_{t+1}}{\partial k_t} < 0$ for any $0 < \eta < 1$. This proves point (2).

**Q.E.D.**

Proposition 3 can easily be interpreted as follows. The dynamics of capital with private old-age support is the result of two counterbalancing forces when the stock of capital varies: (*i*) a positive component which remains constant, and (*ii*) a negative effect due to the existence of the present value of the private intra-family inter-generational transfer (which depends on the ratio between the labour share and the capital share) multiplied by the destabilising factor $\mathrm{B}_2$. The difference of the capital dynamics in the two old-age support cases is grounded on the role played by this composite preference parameter, and it is due to the fact that in an economy with private old-age support individuals know that the benefit they will receive when old is computed on the basis of their own number of children. This, in turn, implies that the cost of children is lower than with PAYG pensions, so that fertility is higher and the saving rate is lower (because the present value of inter-generational transfer component is higher).

Moreover, the analysis of Eq. (30) gives the following proposition:

**Proposition 4**. *In an economy with private intra-family transfers as a means of old-age insurance, an exogenous positive shock on the output elasticity of capital ( $\alpha$ ), the taste for children ( $\gamma$ ) [the individual degree of thriftiness ( $\beta$ )] acts as an economic stabiliser [de-stabiliser].*

**Proof**. The proof immediately follows from $\frac{\partial \hat{\eta}}{\partial \alpha} = \frac{\gamma}{(1-\alpha)^2(\beta+\gamma)} > 0$, $\frac{\partial \hat{\eta}}{\partial \gamma} = \frac{\alpha\beta}{(1-\alpha)(\beta+\gamma)^2} > 0$ and $\frac{\partial \hat{\eta}}{\partial \beta} = \frac{-\alpha\gamma}{(1-\alpha)(\beta+\gamma)^2} < 0$. **Q.E.D.**

## 5. Private intra-family old-age insurance versus public PAYG pensions

In the previous sections fertility, savings, the steady state capital and the (myopic foresight) dynamical process of capital accumulation have been investigated for each old-age support system.

Therefore, we are now able to answer to the question of whether and how the (neoclassical) economic growth as well as the economic stability are changed with the substitution of the private old-age support system with the public pension system.

The aim of this section is to compare the steady states and the dynamic adjustment processes under the two alternative inter-generational transfer systems analysed and discussed in the previous sections, *ceteris paribus* as regards the size of the (private or public) contribution rate (i.e. letting $\eta = \tau$ ).[10]

As regards the steady states, the analysis of Eq. (16) and (29) gives the following proposition:

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[10] The assumption of the equality between the tax rate and gift rate, in order to compare the results of the two models, is also made by Nishimura and Zhang (1992, 1995), Cigno (1995), Zhang and Zhang (1995), but in their papers the level of gift rate is "optimally" chosen by the government. Indeed, in the present paper we abstract from optimality issues, but the above procedure to choose the value of $\eta = \tau$ would leave unchanged our results.

**Proposition 5**. *Let* $\eta = \tau$ *hold. Then* $k^*_{PAYG} > k^*_{OAS}$.

**Proof**. Using Eqs. (16) and (29) it is straightforward to verify that Proposition 5 holds for any $\eta = \tau$. **Q.E.D.**

As regards stability, the analysis of Eqs. (19) and (33) gives the following proposition:

**Proposition 6**. *Let* $\eta = \tau$ *hold. Then the risk of cyclical instability in a PAYG-taxed economy is always lower than in an economy with private intra-family old-age insurances.*

**Proof**. Comparison of Eqs. (19) and (33) implies $\hat{\eta} < \hat{\tau}$ since $B_2 > 1$. Moreover, since $1/2 < \hat{\alpha} < 1$ then we may conclude that the parametric region where cyclical instability is possible in a PAYG-taxed economy is always lower than that of an economy with private old-age insurances for any $\eta = \tau$. **Q.E.D.**

The reason why capital accumulation is higher with PAYG pensions rather than with private intra-family transfers is the following. With a private system of old-age support, individuals know that they will receive the benefit when old on the basis of their own number of children raised when young, rather then the average fertility rate in the whole economy, and this therefore tends to reduce the marginal cost of child rearing as compared with an economy with public PAYG pensions, so that the fertility rate in a PAYG-taxed economy is lower than that of an economy with private intra-family transfers. Moreover, substitution of a private inter-generational transfer system with a public system of social security increases savings, *ceteris paribus* as regards the size of the inter-generational transfer. Since, capital accumulation is the result of the ratio between savings and fertility, it results to be higher than that obtained under private intra-family old-age backing.

Finally, as shown in Propositions 1 and 3, the relative weight of the inter-generational transfer component in capital accumulation in an economy with a private system of old-age insurance is higher than that of a PAYG-taxed economy, and, hence, the risk of cyclical instability in higher because the slope of the capital accumulation locus is higher in that case.

### *5.1. A numerical illustration*

When the output elasticity of capital is larger than one-half, Proposition 1 above ensures that with public PAYG pensions stability is "strong" in the sense that the economy can never be destabilised whatever the size of the pension system. To the extent that capital shares and contribution rates are not too low, therefore, we may conclude that economies under PAYG systems enjoy good health as regards stability concerns.

A numerical illustration may easily show that the "magnitude" of the stabilising role of PAYG pensions as a substitute of the intra-family old age support is significant in realistic economies.

For illustrative purposes, we observe: (*i*) the share of physical capital ($\alpha$) for each of G7 economies, from Bernanke and Gürkaynak (2001), (see Table 1, also reported in Pecchenino and Pollard (2005, p. 458); (*ii*) following rather common estimates, an average contribution rate to the PAYG system, for instance in Europe, of about 16 per cent as well as an expected higher future contribution rate around 28 per cent by the year 2040.[11]

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[11] For instance, Liikanen (2007, p. 4) argued that the "pension contributions in Europe would rise from their present level of around 16% of aggregate wages to around 28% by the year 2040. Japan, which starts out from a lower base, would end up at approximately the same level. As regards the US the contribution rate is lower, and probably increasing: "… the payroll tax rate would have to increase 48 percent (from today's 10.6 percent to 15.7 percent) to finance projected benefits with the present pay-as-you-go structure." (Feldstein, 2005, p. 35).

**Table 1**. Capital shares, $\alpha$, for the G7 countries.

| Canada | France | Germany | Italy | Japan | U.K. | U.S. |
|---|---|---|---|---|---|---|
| 0.32 | 0.26 | 0.31 | 0.29 | 0.32 | 0.25 | 0.26 |

It is easy to see that, for instance in the case of an average capital share around 0.30 as well as an average contribution rate of about 0.16, the stability condition (Eq. 19) is largely satisfied in that the potential destabilising factor is $\tau \mathrm{B}_1 = 0.3733 << 1$, and even for the less favourable case, e.g., as regards the capital share, the U.K. with $\alpha = 0.25$, and assuming a future expected high contribution rate of 28 per cent, such a condition is safely satisfied (now $\tau \mathrm{B}_1 = 0.84 < 1$). Therefore several economies are practically stable with a certain reliability. This example, although only illustrative, shows that with actual values of both the size of PAYG pensions (as measured by the contribution rate $\tau$) and output elasticity of capital, the most part of developed countries would enjoy a "strong" stability of the economy.

In contrast, things would be dramatically different if the support for the old-aged were privately provided within the family rather than publicly organised. In order to better understand this point, we now "calibrate" the model with private old-age support by choosing, in addition to the standard parameters discussed above (i.e., an average value of the capital share $\alpha = 0.30$ and a contribution rate in both public and private systems $\tau = \eta = 0.16$), other plausible parameters. As regards preferences, we posit a value of $\beta$ (the standard measure of the individual's impatience to consume) around 0.6, in accord with Žamac (2007, p. 628),[12] while also assuming a value of parents' taste for children $\gamma = 0.20$. Furthermore, we posit $A = 2$ and $e = 0.1$. We this parameter set we obtain an equilibrium value of the number of children around the replacement level (e.g. 2.1 children per couple) and a percentage cost of child rearing of about 30 per cent of the equilibrium

[12] Following Žamac (2007), this assumption is derived by using the one-year estimate from Auerbach and Kotlikoff (1987) of 0.98, and a broad definition of each period as representing about 27 years, which translates to $\beta = 0.60$

competitive wage, which are rather realistic. In this case it is easy to verify that the stability condition Eq. (33) is largely violated. Indeed, we find that $\eta \mathrm{B}_1\mathrm{B}_2 = 1.4933 > 1$. In particular, the magnitude of the *destabilising factor* $\mathrm{B}_2$ in an economy with private intra-family transfers is very large, in this example equal to $4$, which is more than ten times greater than the factor $\eta \mathrm{B}_1 = 0.3733$. A private old-age support system instead of public PAYG pensions, therefore, magnifies in an extraordinary way the instability which is always generated by the presence of an inter-generational transfer system.

To sum up we may conclude that a rather plausible economy, which would be strongly stable under PAYG pensions, becomes highly unstable under private intra-family old backing. A shift of the obligation for elderly income support to the state seems to be an option not only for promoting economic growth but also ensuring a “strong” economic stability.

**Conclusions**

This paper investigates the effects of two historical alternatives as a means of old-age insurance, i.e., voluntary intra-family transfers from young to old members versus pay-as-you-go public pensions, on the dynamical features of a neoclassical overlapping generations growth model, by also assuming, in line with the new home economics, that children are a desirable good. The shift from the former to the latter old-age insurance system has been commonly observed through the passages between the stages of development, especially between developing and developed countries.

The results shows that, *ceteris paribus* as regards the fraction of the wage that the young people devote (either privately or publicly through taxation) to their parents, a public pension system rather than a private old-age support leads to a higher neoclassical economic growth. Therefore the introduction of a social security system in place of the private intra-family old age support system, is always, under the assumptions of the present paper, GDP-improving. Moreover, it is shown, under myopic expectations, that such an introduction plays a very important stabilising role, and such a role may be effective in many realistic cases where, otherwise, the alternative private system of inter-generational transfers would easily generated the perils of an economic instability. In particular, it has been noted that, while in the case of public pensions only technology matters for stability, in the alternative case of private intra-family old-age support, also the preference parameters (high propensity to consume when old and low taste for children) play a crucial role as a destabilising factor.

Our findings suggest that the introduction of a public system of social security as a substitute to private intra-family insurances in order to guarantee the financial help in the old-age period, may be justified by relevant motivations which have been not so far deeply scrutinised: in fact, in addition to many reasons pointed out by the previous literature (e.g. Samuelson, 1975), such an introduction achieves (*i*) a higher GDP per worker, and (*ii*) a "safer" economic stability.

**References**

Auerbach, A.J., Kotlikoff, L.J., 1987. Dynamic Fiscal Policy. Cambridge University Press, Cambridge.

Azariadis, C., Drazen, A., 1991. Demographic transition in a dual economy. UCLA Working Paper.

Becker, G.S., 1960. An economic analysis of fertility. In Demographic and Economic Change in Developing Countries. National Bureau Committee for Economic Research, Ed., Princeton, NJ: Princeton University Press.

Becker, G.S. Barro, R.J., 1988. A reformulation of the economic theory of fertility. Quarterly Journal of Economics 103, 1–25.

Bental, B., 1989. The old age security hypothesis and optimal population growth. Journal of Population Economics 1, 285–301.

Bernanke, B.S., Gürkaynak, R.S., 2001. Is growth exogenous? Taking Mankiw, Romer, and Weil seriously. NBER Working Paper no. 8365.

Cigno, A., 1995. Public pensions with endogenous fertility: Comment on Nishimura and Zhang. Journal of Public Economics 57, 169–173.

Cigno, A., Werding, M., 2007. Children and Pensions. MIT Press, Cambridge.

de la Croix, D., and Michel, P., 2002. A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations. Cambridge University Press, Cambridge.

Diamond, P., 1965. National debt in a neoclassical growth model. American Economic Review 55, 1126–1150.

Eckstein, Z., Wolpin, K.I., 1985. Endogenous fertility and optimal population size. Journal of Public Economics 27, 93–106.

Ehrlich, I., Lui, F.T., 1991. Intergenerational trade, longevity, and economic growth. Journal of Political Economy 99, 1029–1059.

Feldstein, M., 2005. Structural reform of social security, Journal of Economic Perspectives 19, 33–55.

Galor, O., Weil, D.N., 1996. The gender gap, fertility, and growth. American Economic Review 86, 374–387.

Groezen, B. van, Meijdam, L., 2008. Growing old and staying young: population policy in an ageing closed economy. Journal of Population Economics 21, 573–588.

Groezen, B. van, Leers, T., Meijdam, L., 2003. Social security and endogenous fertility: pensions and child allowances as Siamese twins. Journal of Public Economics 87, 233–251.

Liikanen, E., 2007. Population ageing, pension savings and the financial markets. Bank of International Settlement Review 53, 1–6.

Michel, P., de la Croix, D., 2000. Myopic and perfect foresight in the OLG model. Economics Letters 67, 53–60.

Morand, O., 1999. Endogenous fertility, income distribution and growth. Journal of Economic Growth 4, 331–349.

Nishimura, K., Zhang, J., 1992. Pay-as-you-go public pensions with endogenous fertility. Journal of Public Economics. 48, 239–258.

Nishimura, K., Zhang, J., 1995. Sustainable plans of social security with endogenous fertility. Oxford Economic Papers 47, 182–194.

Pecchenino, R.A., Pollard, P.S., 2002. Dependent children and aged parents: funding education and social security in an aging economy. Journal of Macroeconomics 24, 145–169.

Pecchenino, R.A., Pollard, P.S., 2005. Aging, myopia, and the pay-as-you-go public pension systems of the G7: A bright future? Journal of Public Economic Theory 7, 449–470.

Samuelson, P.A., 1975. Optimum social security in a life-cycle growth model. International Economic Review 16, 539–544.

Wigger, B.U., 1999. Pay-as-you-go financed public pensions in a model of endogenous growth and fertility. Journal of Population Economics 12, 625–640.

Žamac, J., 2007. Pension design when fertility fluctuates: The role of education and capital mobility. Journal of Public Economics 91, 619–639.

Zhang, J., Zhang, J., 1995. The effects of social security on population and output growth. Southern Economic Journal 62, 440–450.