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Lizarazo, Sandra

ITAM

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Sandra Valentina Lizarazo[†]

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Abstract

This paper develops a quantitative model of contagion of financial crisis and sovereign default for small open economies that cannot credibly commit to honor their international debts and have common risk averse investors. The existence of common investors with preferences that exhibit decreasing absolute risk aversion (DARA) generates financial links between the emerging economies' sovereign debt markets. These links help to explain the endogenous determination of credit limits, capital flows, and the risk premium in sovereign bond prices as function not only of the economy's fundamentals and the investors' characteristics (wealth and the degree of risk aversion), but more importantly as a function of the fundamentals of other emerging economies. Therefore this paper provides a theoretical formalization for the endogenous explanation of the contagion of financial crises. The model shows that whenever a country suffers a domestic shock that forces it to default, the associated decrease in investor's wealth will reduce her tolerance of risk; in turn her investments in other emerging economies will be reduced, producing a contagion of the crisis in those countries whose fundamentals are not solid. Also, even when a crisis in one country does not force that country to default, the domestic shock affects the overall riskiness of the investor's portfolio. Therefore the investor is forced to rebalance her portfolio. In this case the investor moves away from countries that are "too" risky towards countries that are relatively solid, exhibiting a behavior consistent with the observed phenomena denominated as "flight to quality". Quantitatively, the model is applied to the case of the Argentinean default of 2001 and the posterior contagion of the crisis to the neighboring country Uruguay. This application shows that the model presented in this paper is not only consistent with the business cycle behavior of the emerging economies considered; but it is also superior to models that do not contemplate financial links in several dimensions: i.) the model explains a larger proportion and volatility of the spread between sovereign bonds and riskless assets; ii.) the model explains endogenously the positive correlation between the economies' sovereign bonds spreads, debt flows and consumption, and iii.) the model exhibits the behavior observed in the data of higher volatility and comovement of the series of emerging economies during periods of volatility in financial markets prompted by a crisis in some emerging country.

Keywords: Contagion; Sovereign Debt; Financial Links; Default
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[†] Department of Economics, ITAM, Mexico, D.F. Email: slizarazo@itam.mx

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1 Introduction

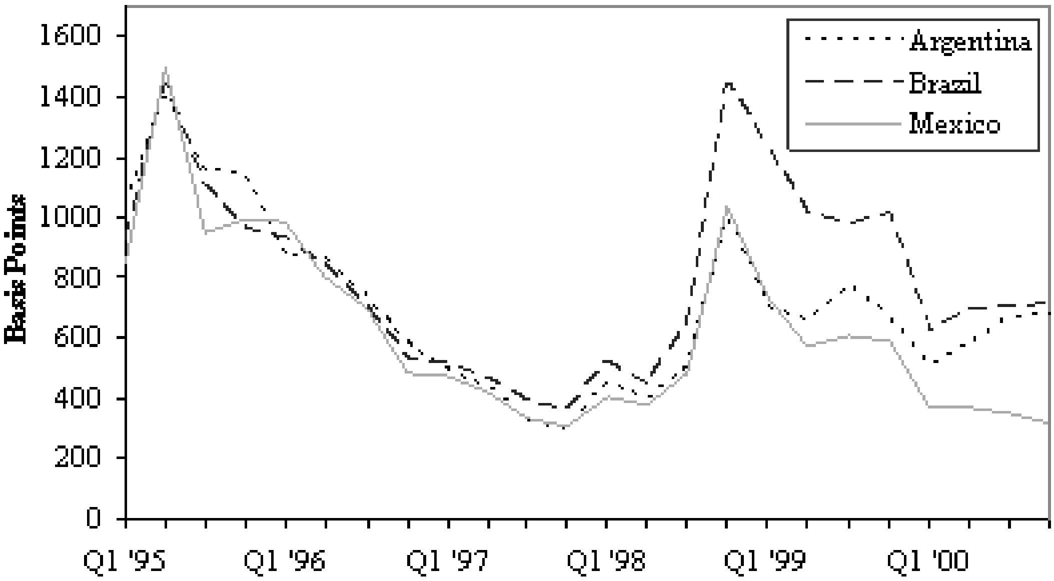
The last two decades of the 20th century can be described as a period of turbulence in international capital markets. During this period the world has witnessed many currency and financial crises. These crises can be characterized by the following stylized facts regarding emerging economies: a) the sudden loss of access to international capital markets, b) large reversal of the current account deficit, c) a collapse of domestic production and aggregate demand, and d) the simultaneous occurrence of these crises across countries. The simultaneity of crises can be seen in Figure 1, which shows spikes in sovereign bond spreads across several Latin American countries in the aftermath of the Mexican Crisis of 1994 and the Russian Crisis of 1998. The current paper is concerned with understanding why simultaneous financial crises occur and whether contagion spreads crises.

In the existing literature, the simultaneity of financial crises across economies can be explained by three different mechanisms, some of which are termed contagion and some of which are not: First, crises can be simultaneous when emerging economies are affected by a common shock, e.g., a shock to international interest rates, a shock to terms of trade, etc. Second, crises can be simultaneous when there exist fundamental links across emerging economies which facilitate the transmission of shocks from a ground zero country to other emerging economies. Examples of these types of links include financial links between countries that share investors and trade links between economies that are commercial partners or competitors. Third, crises might be simultaneous as a consequence of exogenous changes in investors' perceptions: when a crisis hits one country, the ground-zero country, investors may expect a higher default probability in non ground-zero countries. As a consequence, investors will reduce funds to non ground-zero countries and crisis spreads. While there is debate in the literature about which of these three mechanisms should properly be called contagion, the current paper defines contagion as the transmission of negative income shocks that cannot be explained by a common shock to several economies[1].

Identifying and explaining contagion has several important theoretical implications for

[1]Within the literature, some authors define contagion as the occurrence of simultaneous crises that is explained only by exogenous changes in the investors' perceptions. See, for example, Masson(1998), Dornbusch, Park, and Claessens (2000), Forbes (2000, 2001), and Forbes and Rigobon (2000). Other authors define contagion as the transmission of crises that can be explained either by fundamental links across emerging economies or by exogenous changes in investors' perceptions. See, for example, Corsetti et. al. (1998), Presenti, and Tille(2000), De Gregorio and Valdes (2000), Diao et. al. (2000), Edwards (2000a, 2000b), Edwards and Susmel (2000, 2001), Kodres, and Pritsker, (2002), Kyle, and Xiong, (2001), Lagunoff, and Schreft (2001), Paasche (2001), Pritsker (2000), Schinasi and Smith (1999), Valdes (1996), and Van Rijckeghem and Weder (1999).

Sovereign Bond Spreads: 1995-2000



Source: JP Morgan.

Figure 1: Sovereign Bond Spreads

the area of international finance. First, as discussed in Valdes (1996), contagion modifies the credit constraints faced by emerging economies. In general, credit constraints originate when problems of sovereign risk modify the formulation of emerging economies' dynamic optimal plans. When contagion is present, these constraints might vary through time for reasons beyond domestic fundamentals of a particular economy. The economic analysis of developing economies need to consider this possibility, and its impact on the formulation of their optimal dynamic plans. Second, as Goldstein and Pauzner (2001) argue, the existence of contagion modifies the extent to which diversification of risk is possible for investors. In the presence of contagion, the globalization of capital markets bears a cost due to the positive correlation between assets that would be otherwise independent. Contagion reduces the means to diversify risk. This reduction introduces questions about the optimal degree of openness of financial markets. Third, identifying the mechanisms that lead to contagion allows for more precise policy evaluations faced by international organizations. For example, if contagion is explained only by shifts in market sentiments, then information disclosure standards might be justified. On the other hand, if contagion is explained by financial links, then the imposition of some capital controls might be desirable. Additionally under the financial links explanation, country bailouts might be justified in order to reduce the impact of the crisis over the international investors' community and avoid crises of systemic proportions. Finally, if contagion is explained by trade links, bailouts would not be the best course of action. In this latter case, policies directed to enhancing trade diversification would be more effective.

The simultaneity of crises has mainly affected emerging countries as in the cases of the Latin American Debt crisis in 1982, the Mexican Crisis in 1994-1995, the East Asian Crises in 1997, and the Russian crisis in 1998[2].

These financial crises have inspired a large economic literature addressing the issue of contagion. However, despite the huge attention that the subject has received, the origin, severity, and extent of contagion of crises across economies has not been *endogenously* explained by any model. Instead, most models analyze contagion in the framework of a multiple equilibria world in which contagion results from an exogenous change in investors' perceptions. The disadvantage of this approach is that in most cases this framework is equally consistent with the occurrence of contagion and the non-occurrence of contagion. A few models, on the other hand, have attempted a general equilibrium approach to understand the role of links between countries in explaining contagion. But to generate contagion, these models impose an exogenous correlation between emerging economies' asset returns. In contrast, in the present work, emerging economies' returns are determined endogenously within the model.

Among the few general equilibrium models that have considered the role of economic links in contagion, to date, most models have focused on explaining contagion through trade links. These models are silent about the role of financial links. However, empirically trade links can only account for the severity and extension of contagion across economies that have strong trade linkages with one another. In most recent cases of contagion, trade links across infected countries were small or nonexistent. Even in the case where trade links were economically relevant, the links alone were not strong enough to account for the severity of contagion.

There are a few other shortcomings of the existing general equilibrium models: These models abstract from uncertainty and financial market imperfections[3]. However, in the

[2]The Mexican crisis in 1994 hit the economies of Argentina and Brazil. The Russian crisis spread to several countries in Latin America, and even developed countries were affected: The crisis spread to Brazil in 1999 and hit the US as a correction in asset prices.

[3]Only Paasche (2001) has considered financial market imperfections. In Paasche, endogenous credit constraints act as a propagation mechanism for contagion caused by trade links. His model, like all others

international finance literature, the inclusion of uncertainty and financial market imperfections has proved necessary to match the stylized facts of the emerging economies business cycle. Also, the models that consider financial links focus on investors—why investors act in a way that causes contagion, and how contagion affects those investors. However, these models do not take into account the effects of contagion on emerging economies' optimal plans.

In contrast, the model developed in the current paper emphasizes financial market links across countries in a dynamic stochastic general equilibrium setting where the stochastic processes of the emerging economies' assets are endogenously determined. In addition, the model in this paper undertakes a quantitative analysis of the impact of contagion on emerging economies. The model studies how contagion can explain co-movements in the price of emerging economy bonds, capital flows, output and consumption—beyond the level explained by a country's own fundamentals.

Within the present model, the framework is one of a set of small open economies with stochastic endowments. These small open economies have access to an international credit market populated by international investors. International investors are assumed to be risk averse, with preferences that exhibit a decreasing absolute risk aversion in wealth (DARA). Due to the fact that the number of countries is finite, international investors are not able to completely diversify the risk of their investment in any country. There is a problem of enforcement in the sense that international investors cannot force the small open economies to repay their debts. These economies repay because it is in their interest to do so: If any economy defaults it is temporarily excluded from the world asset market. Countries weigh the benefits and costs of default, and decide to repay or not. This context forces international investors to consider the risk of default when choosing their portfolio. Within the model, any type of reallocation of the international investors portfolio has effects over several countries at the same time. Therefore the risk of default is endogenously determined by the economy's own fundamentals and the fundamentals of other emerging economies: other economies' fundamentals determine the risk of default of those countries, which might modify the portfolio choice of the international investors, and therefore the availability of financial resources to any emerging economy.

Within this framework, income shocks to an emerging economy generate changes in the risk of default in that economy. Through financial links, these changes in turn impact other emerging economies. In this context, financial links generate contagion through two

of this type, is silent about the existence and importance of financial links across economies.

channels:

(i) The **Wealth** channel of contagion: If an income shock in the first country generate losses for international investors, for example when the shock forces the first country into default, then depending investors' preferences, the negative wealth effect of the shock may reduce investors' tolerance for risk. A reduction in tolerance for risk would make investors shift away from risky investments (countries) toward riskless (T-Bills). Countries that did not default or face an income shock nonetheless face a reduction in the amount of resources available to borrow from, and contagion occurs.

(ii) The **Portfolio Recomposition** channel of contagion**:** The risk of default might be correlated across countries because income processes are correlated, or solely because the possibility of contagion caused by financial links across countries (as the previously explained wealth channel, or trade links between the economies). In the case where default risk is positively correlated across countries, an increase in the risk of default in the first country generates, for a given level of wealth of the investors, a increase in the overall risk of their portfolio as investors expect other countries' risk of default to also increase. In this case, the change in the risk of default in one country modifies the optimal portfolio of international investors. As investors adjust their portfolios, countries which did not face an income shock nonetheless face a reduction in the amount of resources available to borrow from, and contagion occurs.

The model in this paper extends the literature in endogenous sovereign risk in order to consider sovereign bond markets in a multi-country framework.[4] For the case of small open economies, this type of model allows for an endogenous determination of the price of one period non-contingent discount bonds as a function of the economy's default risk. Further, through the consideration of financial links across economies, the default risk of any economy becomes a function not only of the domestic fundamentals but also a function of investors characteristics and the fundamentals of countries which share investors with the domestic country.

The main mechanism through which financial contagion occurs in the current paper, the wealth channel, is analyzed in the papers of Goldstein and Pauzner (2001), Kyle and Xiong (2001), and Lagunoff and Schreft (2001). These papers show that if investors' preferences

[4]Recent papers in the literature of endogenous sovereign risk include Aguiar and Gopinath (2006), Arellano (2008), Bai and Zhang (2006), Cuadra and Sapriza(2008), Lizarazo (2010),Hatchondo, Martinez and Sapriza (2008), Martinez and Hatchondo (2009), Mendoza and Yue (2008), and Yue (2006).

exhibit DARA, the optimal response of the investors to financial losses is to reduce their exposure to risky investments due to the reduction on their tolerance toward risk at lower levels of wealth.

The secondary mechanism through which financial contagion occurs in the current paper, the portfolio recomposition channel, is studied in the theoretical papers of Choueri (1999), Schinasi and Smith (1999) and Kodres and Pritsker (2002). Using a static partial equilibrium approach where the determination of asset returns is exogenous to the model, these papers highlight the fact that contagion might be successfully explained by standard portfolio theory: in order to reestablish the optimal degree of risk exposure in their portfolio after a negative shock to the return of the assets of some economy, it is optimal for investors to liquidate their holdings of assets with expected returns that exhibit some correlation with the expected return of the crisis country—whether this correlation is a consequence of contagion or just correlated fundamentals.

Several papers have empirically studied the role of financial links in explaining contagion. Kaminsky and Reinhart (1998) find that the probability of crisis for any country is the highest when another country that shares investors and/or lenders is in crisis. No other shared characteristic with the crisis country (e.g. region, macroeconomic practices, trade links, etc.) has as large an impact on the probability of crisis. Van Rijckeghem and Weder (1999) present evidence that for the Mexican, Asian, and Russian crises spill-overs through common bank lending were more significant in explaining contagion than trade linkages and macroeconomic similarities. Kaminsky, Lyons and Scmukler (1999) find evidence that individual share holders of open-end mutual funds followed contagion strategies in the case of the Russian crisis, and to a lesser degree in the Mexican crisis: individual investors in mutual funds (not mutual fund managers) sold securities from several emerging markets when a crisis affected one of the countries. In the aftermath of the Mexican, Asian, and Russian crises, Kaminsky Lyons and Schmukler (2000) find that when open-end mutual funds adjusted their portfolios they considered not only the degree of fragility in fundamentals of the economies, but also factors emphasized by financial channels of contagion—openness and liquidity of the markets, as well as the level of country risk of the economies. Kaminsky and Reinhart (2000) find that during the Russian-LCTM crises reductions in the risk exposure of investors' portfolios drastically reduced the liquidity of international capital markets and increased their volatility. Hernandez and Valdes (2001) find the presence of common lenders seems to explain almost all contagion episodes during the Asian, Russian, and Brazilian crises. Common lenders are also highly significant in explaining contagion in stock markets.

The results of the current paper are consistent with the empirical evidence regarding contagion as consequence of financial links.

First, since investors' preferences exhibit decreasing absolute risk aversion (DARA), they are able to tolerate more default risk when they are wealthier. So there is a positive correlation between lenders' wealth and their investment in emerging economies. Therefore both capital flows to emerging economies and the equilibrium price of sovereign bonds are increasing functions of investors' wealth levels[5]. This result implies that the current model can explain the high correlation of sovereign bond spreads and capital flows across emerging economies. Furthermore, the high correlation between investors' wealth and emerging economies financing conditions can account for the simultaneity of crises because a default by any economy is equivalent to a negative wealth shock to the investors. This shock is transmitted to other countries via the wealth channel of contagion.

Second, because the endogenous credit limits faced by the emerging economy are a function of investors' wealth and risk aversion, then when the probability of default increases for some foreign country, other countries' financing conditions change. When the probability of default for some foreign country increases, two opposing forces affect the financing situation of other emerging economies: On the one hand, a decrease in the price of the sovereign bonds of the foreign country constitutes an expected future negative wealth shock to the investors due to the higher associated probability that this country will default. This expected negative wealth shock increases the incentives of the other economies to default, and their default risk. On the other hand, an increase in default probabilities induces a substitution away from the assets of the economy whose risk increases strongly towards the assets of the economies whose risk does not increase too much. This effect would tend to increase the set of financial contracts available to some emerging economies. Given the opposing wealth and substituition effects, contagion occurs when the wealth effect dominates. In this case, the correlation of capital flows across emerging economies is positive. If the substitution effect dominates, "flight to quality" is observed: when several other countries are affected by financial crises emerging economies with robust fundamentals receive capital flows. The effect of the expected negative wealth shock will dominate the substitution effect whenever investors' wealth is sufficiently low, or their degree of risk aversion is sufficiently high, or the economies fundamentals are sufficiently weak. In the numerical simulations in

[5] This result is consistent with empirical findings which demonstrate a positive relation between proxies of investors wealth (like developed economies' GDP or stock indexes) and capital flows to emerging economies. It is also consistent with the empirical literature on the determination of sovereign credit spreads for emerging economies. See for example Goldberg (2001), Hernandez, Mellado and Valdes (2001), FitzGerald, and Krolzig (April 2003), Mody and Taylor (2004), Warther (1995), and Ferruci, Herzberg, Soussa, and Taylor (2004).

the present paper, the wealth effects dominate and contagion occurs.

Third, the likelihood of default in equilibrium for any emerging economy is a function not only of investors' characteristics and the economy's own fundamentals, but also of other emerging economies' fundamentals. In the numerical simulations in the present paper, default is more likely to be an equilibrium outcome when the fundamentals of other economies deteriorate.

The paper proceeds as follows: Section II documents some episodes of simultaneous crises across economies and discusses some empirical results which motivate the study of contagion; section III develops the model; section IV characterizes the equilibrium of the model; section V presents the numerical results of the paper; and section VI concludes.

2 Evidence Of Contagion: Time Clustered Crisis

International capital markets are highly volatile. A striking feature of this volatility is that most episodes of crisis within the last two decades have not been restricted to individual countries, or even regions. This time-clustering of crises has led economists to borrow from epidemiology the term contagion. Some examples of time clustered crises follow.

The Debt Crisis of 1982: Between 1979-1981 capital flows in the form of bank lending to Latin American countries reached about 6% of the region's GDP (their peak was $41 billion in 1981). In 1982, after a tremendous hike in international interest rates, Mexico declared a moratorium on its debt, and emerging markets around the world were excluded from voluntary capital markets and forced to run current account surpluses to pay their foreign debts. The crisis affected all countries in Latin America and spread to countries as far as Nigeria, the Philippines and Yugoslavia.

The Mexican Crisis in 1994: During the period 1992-1994, Mexico's current account deficit averaged more than 7% of the GDP. In 1995, after the crisis the country's current account was forced into balance, and the economy experienced a negative growth of 6.5%. During the two quarters following Mexican devaluation, international mutual funds reduced their average exposure to Brazil by approximately 5%. In Argentina, the central bank lost about a third of its liquid international reserves, and the banking system lost 18% of its deposits. Argentina's real GDP fell by almost 5% during 1995.

The Asian crises of 1997: During the mid-1990s flows into Indonesia, Malaysia, Korea, the Philippines and Thailand averaged more than US$40 billion per annum, with a maximum of US$70 billion in 1996. During the crisis period more than US$100 billion in short term debt bank loans were recalled from these same countries.

The Russian Default/Devaluation in August of 1998: By mid-August of 1998 a severe crisis began in Russia due to fiscal imbalances, the deterioration of the capital account, the fall in international prices of Russian exports, and huge losses of international reserves. This crisis spread to Argentina, Mexico, Venezuela, Brazil, Pakistan and South Africa. As noted in Kaminsky, Lyons and Schmukler (2000), following the Russian crisis, total capital inflows for Latin America diminished 35%, and short term portfolio flows—bonds, equity and bank lending—fell by 60%. Dornbush, Park and Claessens (2000) argue that the sharp reversal in capital flows to emerging economies after this crisis triggered recessions in many developing countries, and that in 1999 two fifths of the world economy experienced recession, with most GDP declines concentrated in the developing world.

As discussed in the introduction, the occurrence of simultaneous crises can be attributed to one of three type of reasons: i) monsoonal effects which are due to a common shock to several economies; ii) transmission of the crises through international fundamental economic links; iii) changes in international investors' perception about the outlook of an economy after a negative shock in some other economy.

Once the phenomena of time-cluster crises is observed, empirical tests are necessary to determined which of the previous reasons can explain the occurrence of simultaneous crises. The empirical literature on this subject is quite large, and evidence of contagion in sovereign bonds markets is considerable: according to Valdes (1996), during the period 1986-1994 there was increased co-movement between emerging economies' sovereign bond markets in periods of crisis; Baig and Goldfajn (1998) find that during the Asian crisis there was an increase in cross-country correlation among sovereign bond markets of the East Asian Economies affected by the crises; Edwards (1998) finds evidence of significant propagation of volatility from Mexico to Argentina bond markets during the Mexican Crisis; and Baig and Goldfajn (2000) find that during the Russian crisis there was contagion from Russia to Brazil Brady market.

3 The Model

The model is a discrete time, infinite horizon model. There are two types of agents in the model, $J < \infty$ representative agent small open economies, and infinite identical risk averse international investors. In each period, each of the emerging economies receives a stochastic endowment of tradable goods. The representative agents of these economies may smooth their consumption across periods by trading non-contingent discount bonds with the international investors but the emerging economies are not able to trade financial assets between them. For their part, investors may trade assets with the emerging countries or with industrialized countries. Thus the investors must choose an optimal allocation of their portfolio between the bonds of the emerging economies and bonds of the industrialized countries, denominated hereafter as T-Bills. By having only a finite number of small open economies, the possibilities of diversification for the investors are limited.

The market for T-bills, θ^{TB}, is not modeled explicitly. Debt contracts between the investors and industrialized countries are assumed to be enforceable, and investors are price takers in the market for T-Bills. The price of T-Bills, q^f, which is not determined endogenously in this context, is assumed to be deterministic. Therefore T-Bills are riskless assets.

Bonds of emerging economies, b_j, on the other hand, are risky assets because debt contracts between the investors and the emerging economies are not enforceable. As a consequence, there is a one sided commitment problem. While investors are able to commit to honor their debt obligations with the emerging economies, the representative agents of the emerging countries are not able to commit to honor their obligations with international investors. Therefore, in each period, the representative agent of each emerging economy compares the costs and benefits derived from the repayment of her obligations. The decision between repayment or default is made individually by each agent of each emerging economy. Each agent of any economy makes her decision, taking as given the decision of the other agents in her economy. However given that all agents in a given economy are identical and do not follow mixed strategies, it is possible to focus attention on the problem of the representative agent of each economy.

If any economy defaults, international investors are able to collude to punish her. As a consequence of default, it is assumed that investors will collude to temporarily exclude the defaulting country from financial markets. After defaulting in any subsequent period the country might re-enter international financial markets with an exogenously given probability θ. Since all investors behave in the same exact way, it is possible to focus on the representative international investor.

Both, the representative investor, and the representative agents of the economies take as given the price function of each emerging economy's non-contingent discount bonds, q_j.

As laid out here, the asset market is imperfect in three different ways. First, there is a one-sided commitment problem which implies that debt contracts with emerging economies are not enforceable. Second, markets are incomplete because the only traded assets are one period no-contingent bonds, and risk free T-Bills. Therefore the representative investor is not able to insure away the income uncertainty specific to the emerging countries. Third, the market structure of the financial market is non-competitive: investors form a cartel that colludes to punish any deviant investor or borrower.

In this model the state of the world is complex object defined as follows:

Definition 1 *The state of the world, $S = (s, W, \psi)$, is given by the realization of the emerging economies' fundamentals, $s = s_1 \times s_2 \times \ldots \times s_J$, the representative investor's asset position or wealth, W, and a probability measure ψ on $s \times W \times \Gamma$ where Γ is a borel σ-algebra. In this model $s_j = (b_j, y_j, d_j)$ where b_j is economy's j asset position, y_j is economy's j endowment, and d_j is a variable that describes if the economy j is in default or repayment state.*

3.1 International Investors

There are an infinite number of identical price-taking investors. Investors collude in order to punish any borrower that defaults on her debts or any investor that lends to a borrower who has previously defaulted, so that a defaulting country is temporarily excluded from the financial markets[6].

The representative investor is a risk averse agent whose preferences over consumption are defined by a constant relative risk aversion (CRRA) periodic utility function with parameter $\gamma^L > 0$. The investor has perfect information regarding the income process of the emerging economy, and in each period the investor is able to observe the realizations of this endowment.

The representative investor maximizes her discounted expected lifetime utility from consumption

$$\underset{c_t^L}{Max}\ E_{\ t} \sum_{t=0}^{\infty} \beta_L^t v\left(c_t^L\right) \tag{1}$$

where c^L is the investor's consumption. The period utility of this agent is given by $v(c^L) = \frac{\left(c^L\right)^{1-\gamma^L}}{1-\gamma^L}$. The representative investor is endowed with some initial wealth W_0, at time 0, and in each period, the investor receives an exogenous income X.

[6]The assumption of temporarily exclusion is supported by the data: Empirical evidence suggests that once a country defaults, that country is excluded from the credit market for an average of 5.4 years (Gelos, Sahay, and Sandleris 2003).

Because the representative investor is able to commit to honor her debt, she can borrow or lend from industrialized countries (which are not explicitly modeled here) by buying T-Bills at the deterministic risk free world price of q^f. The representative investor can also invest in non-contingent bonds of the emerging economy. These bonds have an endogenously determined stochastic price of q_j. In each period the representative investor faces the budget constraint

$$W + X = c^L + \sum_{j=1}^{J} d_j q_j \theta'_j + q^f \theta^{TB\prime} \tag{2}$$

where W is the investor's wealth at time t, θ'_j is the portfolio allocation to the emerging country and $\theta^{TB\prime}$ is the investor's allocation to the riskless asset. d_j is a variable that determines the default/repayment state of the emerging economy j in the current period:d_j is an indicator function that represents the emerging economy's j repayment/default decision in the current period. d_j takes the value of 1 when the small open economy j chooses to repay its debts, and takes a value of 0 otherwise.

It is assumed that investors cannot go short in their investments with emerging economies. Therefore whenever the emerging economy is saving, the representative international investor receives these savings and invests them completely in riskless bonds (T-Bills). The representative investor does not use these resources to go long in T-Bills. This assumption implies that $\theta'_j \geq 0$ for all t and j[7].

The law of motion of the representative investor's wealth is given by

$$W' = \sum_{j=1}^{J} d'_j \theta'_j + \theta^{TB\prime}. \tag{3}$$

The optimization problem that the representative investor faces can be described as one in which in each period t the representative international investor optimally chooses her portfolio according to her preferences in order to maximize her discounted expected lifetime utility from consumption, subject to her budget constraint, the law of motion of her wealth, and given W_0. This dynamic problem can be represented recursively by the

[7]This assumption does not seem to be inconsistent with reality. For example, while mutual funds are strictly restricted by The Investment Company Act in their ability to leverage or borrow against the value of securities in their portfolio, hedge funds and other types of investments face no such restrictions. Since international investments like hedge funds are not subject to these type of regulations, it seems reasonable to have the simplifying assumption that international investors are able to leverage the riskless asset, θ^{TB}, but must have a non-negative position on the emerging economy's asset.

Bellman Equation

$$V^L(S) = \max_{\{\theta'_j\}_{j=1}^J,\ \theta^{TB\prime}} v\left(c^L\right) + E\beta V^L\left(S'\right) \tag{4}$$

$$\begin{aligned}
s.t. \qquad W + X &= c^L + \sum_{j=1}^{J} d_j q_j \theta'_j + q^f \theta^{TB\prime} \\
W' &= \sum_{j=1}^{J} d'_j \theta'_j + \theta^{TB\prime} \\
\theta'_j &\geq 0 \qquad j = 1, \ldots, J \\
c^L &> 0 \\
W' &\geq \underline{\underline{W}}
\end{aligned} \tag{5}$$

Equation (5) corresponds to the "natural" debt limit discussed in Aiyagari (1994), which prevents the representative investor from running ponzi games.

Since the representative investor is not credit constrained (Equation (5)), the solution to the stochastic dynamic problem for the representative investor can be characterized by the following Kuhn-Tucker conditions:
For $\theta^{TB\prime}$

$$q^f v_{c^L}\left(c^L\right) = \beta_L E\left[v_{c^L}\left(c^{L\prime}\right)\right] + \left(\beta^L E\left[\varpi'\right] - q^f \varpi\right). \tag{6}$$

For θ'_j

$$q_j\left(v_{c^L}\left(c^L\right) + \varpi\right) d_j = \beta_L E\left[\left(v_{c^L}\left(c^{L\prime}\right) + \varpi'\right) d'_j\right] d_j. \tag{7}$$

where ϖ corresponds to the multiplier on the non-negativity constraint for the investors consumption c^L.

For X sufficiently large, the non-negativity constraints for the representative investor's consumption are not binding at any time (therefore $\varpi = \varpi' = 0$). Therefore the investor's optimization problem has an interior solution for the portfolio allocation.

When the non-negativity constraints are not binding the Kuhn-Tucker conditions for the investor's problem correspond to the following first order conditions:[8]

[8] Lizarazo (2010) discusses the issue of binding non-negativity constraints for the case of one emerging economy and risk averse investors. The results in this paper extend to the framework here: i)When these non-negativity constraints are binding in the current period, the current period portfolio allocation to the emerging economies implies lower levels of investment than the "optimal" ones; other things equal, the lower investment implies higher incentives to default by the emerging economies. ii) When these non-negativity constraints are binding only in future periods, the current period portfolio allocation to the

- For $\theta^{TB\prime}$

$$q^f v_{c^L}\left(c^L\right) = \beta_L E\left[v_{c^L}\left(c^{L'}\right)\right]. \tag{8}$$

- For θ_j'

$$q_j v_{c^L}\left(c^L\right) d_j = \beta_L E\left[v_{c^L}\left(c^{L'}\right) d_j{}'\right] d_j \tag{9}$$

According to (8), the investor chooses an allocation to the riskless asset such that the discounted expected marginal benefit of future consumption equals the marginal cost of current consumption. The set of J equations (9) determines the allocation of the resources of the representative investor to each one of the J emerging countries. Unless emerging country j is not in a default state, i.e. $d_j = 1$, emerging country j does not belong in the investment set of the international investors. If country j has not defaulted, then the jth equation(9) also equates the marginal cost of allocating wealth to bonds issued by emerging country j to the discounted expected marginal benefit of this investment. The benefit of this investment is realized only in those periods in which the emerging economy j optimally chooses to repay its debts $\left(d_j{}' = 1\right)$.

It is possible to manipulate equation (9) to get

$$\begin{aligned}
q_j &= \frac{\beta_L E\left[v_{c^L}\left(c^{L'}\right) d_j'\right]}{v_{c^L}\left(c^L\right)} \\
&= \beta_L \frac{Cov\left[v_{c^L}\left(c^{L'}\right) d_j'\right] + E v_{c^L}\left(c^{L'}\right) E d_j'}{v_{c^L}\left(c^L\right)} \\
&= \frac{\beta_L Cov\left[v_{c^L}\left(c^{L'}\right) d_j'\right]}{v_{c^L}\left(c^L\right)} + q^f\left(1-\delta_j\right) \\
&= \frac{\beta_L Cov\left[v_{c^L}\left(c^{L'}\right) d_j'\right]}{v_{c^L}\left(c^L\right)} + q_j^{RN}. \\
&= \zeta_j + q_j^{RN}. \qquad (10)
\end{aligned}$$

where $Ed_j'= 1-\delta_j$, and δ_j is the probability that the emerging economy will default in the next period and q_j^{RN} corresponds to the price of the emerging economy j's bonds that would equate the expected earnings of investing in the economy j's risky bonds to the earnings

emerging economies implies higher levels of investment than the "optimal" ones; other things equal, the higher investment implies lower incentives to default by the emerging economies.

obtained by investing in riskless bonds. Given emerging economy's default decisions for next period d', this price would prevail in a world with a risk neutral investor.

$$q_j^{RN} = q^f \left(1 - \delta_j\right)$$

ζ_j corresponds to an "excess" risk premium that sovereign bonds have to carry in order to induce risk averse investors to hold them. This term is the principal source of the transmission of crisis among countries that share investors. As shown in Lizarazo (2010), the main determinant of the "excess" risk premium ζ_j is the covariance term in equation (10). This covariance term is non-positive: $Cov\left[v_{c^L}\left(c^{L'}\right) d_j'\right] \leq 0$.[9]

From the set of equations (10), it can be seen that bond prices of any emerging economy are a function not only of the economy's own fundamentals but also of investors' risk aversion and wealth level (see Lizarazo (2010)). Additionally, and more importantly for the purpose of this model, bond prices of any emerging economy are also a function of the fundamentals of other emerging economies in the investor's portfolio: Because consumption of the representative investor is a function of her wealth, her risk aversion, and her investments in other economies, sovereign bond prices of the emerging economy j are also a function of those variables[10].

[9]When the emerging economy does not find it optimal to default next period in any state of the world, then $d_j{}' = 1$ for all states. Therefore $Cov\left[v_{c^L}\ \ c^{L'}\ \ d_j{}'\right] = 0$. On the other hand, when next period there exist states of the world in which the emerging economy would optimally choose to default, then for the states in which it is not optimal to default, $d_j{}' = 1$. In this case, next period wealth of the representative investor is given by

$$W' \mid \left[d_j{}' = 1 \quad = \theta_j{}' + \theta_j{}^{TB\prime}\right.$$

and next period wealth of the representative investor is given by

$$W' \mid \left[d_j{}' = 0 \quad = \theta^{TB\prime}\right.$$

It is obvious that

$$W' \mid \left[d_j{}' = 1 \quad > \ W' \mid \left[d_j{}' = 0\right.\right.$$

Therefore it must hold that

$$\left[c^{L'} \mid \left[d_j{}' = 1\ \right] \geq \left[c^{L'} \mid \left[d_j{}' = 0\ \right]\right.\right.$$

implying

$$\left[v_{c^L}\ \ c^{L'}\ \ \mid \left[d_j{}' = 1\ \right] \leq \left[v_{c^L}\ \ c^{L'}\ \ \mid \left[d_j{}' = 0\ \right]\right.\right.$$

As a consequence, for higher $d_j{}'$, we have lower $v_{c^L}\ \ c^{L'}$. Clearly for this case $Cov\left[v_{c^L}\ \ c^{L'}\ \ d_j{}'\right] < 0$.

[10]The way in which sovereign bond prices depend on investors' wealth and risk aversion is discussed in depth in Lizarazo (2010). Whenever investors are wealthier, the marginal cost for them in terms of utility of an additional unit of investment on sovereign bonds is relatively low. Taking as given the price of the bonds of an emerging economy j, it must hold that the net flow of resources to the economy $\left[\text{i.e., } -q_j b_j'\right.$ is increasing in investors' wealth and decreasing in investor's risk aversion. It will be evident once the emerging economies' problem is described that the benefits for the economy from paying her debts are also increasing

Because both investors' wealth and fundamentals of other emerging economies in the investors' portfolio have an effect on the determination of bond prices of some economy j, it is clear that sovereign bond prices across economies that share investors are jointly determined and therefore must be correlated.

The discussion on the way in which other countries fundamentals affect the determination of economy $j's$ bond prices, and debt flows will be postponed until the section on the characterization of contagion channels.

3.2 Sovereign Countries

The representative agent of each emerging economy j maximizes her discounted expected lifetime utility from consumption

$$\max_{\{c_{j,t}\}_{t=0}^{\infty}} \qquad E_{\tau} \sum_{t=0}^{\infty} \beta^t u\left(c_{j,\ t}\right) \tag{11}$$

where $0 < \beta < 1$ is the discount factor and $c_{j,\ t}$ is the j emerging economy's consumption at time t. The emerging economy's periodic utility takes the functional form

$$u(c_j) = \frac{c_j^{1-\gamma}}{1-\gamma}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

In each period, each economy j receives a stochastic stream of consumption goods y_j. This endowment is non-storable, and it is independently distributed across emerging economies; realizations of the endowment are assumed to have a compact support; and the endowment follows a Markov process drawn from probability space $(y_j, Y_j(y_j))$ with a transition function $f(y_j' \mid y_j)$.

In each period, based on the stochastic endowment y_j, the economy decides how much to consume c_j. The economy can consume $c_j > y_j$ by trading one period non-contingent discount bonds $b_j{}'$ at a price q_j with international investors. The economy may only trade bonds in period t if the economy is not in default state.

At any point in time, the emerging economies differ in their endowment and their asset position. Therefore it is necessary to have a probability measure ψ on $s \times W \times \Gamma$ where

in investors' wealth (decreasing in investor's risk aversion). Therefore incentives to default decrease with investors' wealth (increase with investor's risk aversion). In equilibrium sovereign economies bond prices are increasing in investors' wealth (decreasing in investor's risk aversion).

Γ is a borel σ-algebra. This probability measure is known by all the agents and its law of motion of the distribution is given by:

$$\psi' = H(\psi) \tag{12}$$

In equilibrium, the price of bonds of any emerging economy j is determined by both the investors and the emerging economies. As in other models of endogenous sovereign risk, as a consequence of the commitment problems on the side of the emerging economies, the price of the emerging economy's bond varies with the level of borrowing of those economies. If $b_j' \geq 0$, there is no risk of default on such a bond. In this case, the emerging economy's bond is identical to the bonds issued by industrialized markets; therefore, because the representative investor is a price taker, in equilibrium the bond price of a emerging economy with no default risk is the same as the bond price of industrialized countries. Consequently, the price of a bond with a positive face value is equal to the price of a T-Bill, so $q_j = q^f$.

If $b_j{}' < 0$ the emerging country j is borrowing. In this case, because emerging economies cannot bind themselves to honor their debts, the emerging country j might default next period. There might be values of $b_j{}' < 0$, for some given state of the world, S, such that the representative agent of the economy never finds it optimal to default. In this case the bonds issued by the emerging economy do not involve any default risk, and therefore $q_j = q^f$. However, for the same state of the world, S, some other values of $b_j{}' < 0$ might imply that the emerging economy will find it optimal to default on her debts in some states of the world next period S'. In this case, in order to induce international investors to buy the emerging economy's bonds, the price of such bonds needs to be lower than the price of a T-Bill, $q_j < q^f$. Finally, for the same state of the world, S, there might be values of $b_j{}' < 0$ such that once the debt is due the economy would not choose to repay in any state of the world next period, S'. In this case $q_j = 0$.

Therefore, the price of any emerging economy's bonds is a function not only of the state of the world, S, but also of $b_j{}'$.

The resource constraint of the emerging economy j is given by

$$c_j = y_j - (1 - d_j)\,\phi + d_j\left(b_j - q_j b_j'\right), \tag{13}$$

where d_j, which was defined in the investor's section, describes the state of economy with respect to participation in international financial markets. If $d_j = 1$, the economy is not in a default state. If $d_j = 0$, the emerging economy is in a state of default (either because she has defaulted on her debts in a previous period and has not regained access to financial markets or because she is defaulting on her debts in the current period); in this case, this country is in a state of temporary financial autarky. Once a country defaults, that country is temporarily excluded from access to the credit market, and the country remains in a state of default for a random number of periods. During the periods of exclusion from financial markets, the country is not able to smooth its consumption, and it is limited to consume its stochastic endowment minus some amount given by a function ϕ that defines the direct loss in terms of endowment that the country faces during the periods of exclusion from credit markets.

Under this framework, the optimization problem of the emerging country j can be represented recursively by the following Bellman equation

$$V_j(S) = \max\left\{V_j^C(S), V_j^D(S)\right\} \tag{14}$$

and

$$\begin{array}{rrcl} & V_j^C(S) & = & \max\limits_{c_j,\ b_j{}'} \ u\left(c_j\right) + \beta E V_j\left(S' \mid S\right) \\ s.t. & c_j & = & y_j + b_j - q_j b_j' \end{array} \tag{15}$$

where $V_j^C(S)$ is the value for economy j of not defaulting and $V_j^D(S)$ is the value of defaulting in the current period.

Definition 2 *The value for the emerging economy j of default is given by*

$$V_j^D(S) = u(y_j - \phi) + \beta E[\theta V_j^C(S' \mid S) + (1-\theta)V_j^D(S' \mid S)].$$

where θ is the exogenous probability of re-entry to the credit markets after defaulting, and ϕ is a function that defines the direct punishment to a defaulting country.

For the emerging country, the decision of default/repayment depends on the comparison between the continuation value of the credit contract, $V_j^C(S)$, versus the value of opting for financial autarky $V_j^D(S)$. The decision of current default/repayment takes the functional form:

$$d_{\ j} = \left\{ \begin{array}{ll} 1 & \text{if } V_j^C(S) > V_j^D(S) \\ 0 & \text{otherwise} \end{array} \right\} \tag{16}$$

Definition 3 *For a given level of wealth, W, and the fundamentals of other emerging economies in the investor's portfolio, the default set $D_j\left(b \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi\right)$ consists of* *the equilibrium set of y_j for which default is optimal when emerging economy's $j\prime s$ asset holdings are b_j:*

$$D_j(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi) = \left\{ y_j \in Y_j : V_j^C(S) \leq V_j^D(S) \right\}. \tag{17}$$

Equilibrium default sets, $D_j(b'_j \mid \{s'_k(S)\}_{k=1,k\neq j}^{J}, W'(S), \psi'(\psi))$, are related to equilibrium default probabilities, $\delta_j(S' \mid S)$, by the equation

$$\delta_j(S' \mid S) = E{d_j}'(S' \mid S) = \int_{D_j(b'_j \mid \{s'_k(S)\}_{k=1,k\neq j}^{J}, W'(S), \psi'(\psi))} f(y'_j \mid y_j) dy'_j. \tag{18}$$

If the default set is empty for ${b_j}'$, then for all realizations of the economy $j\prime s$ endowment ${d_j}' = 1$ and the equilibrium default probability $\delta_j(S' \mid S)$ is equal to 0. In this case, it is not optimal for the economy to default in the next period for any realization of its endowment, $Cov\left[v_{c^L}\left(c^{L\prime}\right){d_j}'\right] = 0$ and $q_j = q^f$. On the other hand, if the default set includes the entire support for the endowment realizations, i.e. $D_j(b'_j \mid \{s'_k(S)\}_{k=1,k\neq j}^{J}, W'(S), \psi'(\psi)) = Y_j$, then ${d_j}' = 0$ for all realizations of the economy's endowment. As a consequence, the equilibrium default probability $\delta_j(S' \mid S)$ is equal to 1, and $Cov\left[v_{c^L}\left(c^{L\prime}\right){d_j}'\right] = 0$, so $q_j = 0$.

Otherwise, when the default set is not empty but does not include the whole support for the endowment realizations $0 < \delta_j(S' \mid S) < 1$. In this case $Cov\left[v_{c^L}\left(c^{L\prime}\right){d_j}'\right] > 0$, so $q_j < q^f$.

Equations (10),(14),(16),(17) and (18) make clear that for the case of an economy that cannot commit to repayment, when there exist levels of b' in which the emerging economy finds it optimal to default in some states of the world, then the price of bonds depends not only on the emerging economy's fundamentals, the representative investor's level of wealth and risk aversion, but on the fundamentals of other emerging economies. This dependence of bond prices on the fundamentals of other emerging economies cannot be reproduced endogenously by models with risk neutral investors: in models of endogenous sovereign with risk neutral investors, the price of bonds of the economy depends only on the economy's own fundamentals.

4 Equilibrium

The recursive equilibrium for this model is defined as a set of policy functions for (i) the emerging economies' consumption $\{c_j(S)\}_{j=1}^{J}$,(ii) the emerging economy's asset holdings

$\left\{b_j'(S)\right\}_{j=1}^{J}$, (iii)the emerging economy's default decisions $\{d_j(S)\}_{j=1}^{J}$ and the associated default sets $\{D_j\left(b_j|W\right)\}_{j=1}^{J}$, (iv) the representative investor's consumption $c^L(S)$, (v) the representative investor's holdings of emerging economies' bonds $\left\{\theta_j'(S)\right\}_{j=1}^{J}$, (vi) the representative investor's holdings of T-Bills $\theta^{TB'}(S)$, (vii) the emerging economies' bond price functions $\{q(S,b')\}_{j=1}^{J}$, (viii) the law of motion of the distribution $H(\psi)$,(ix) an aggregate law of motion of the world economy $Q(S,\Gamma;\{b_j'\}_{j=1}^{J},W')$, and (x) an updating operator $T(\psi,Q(S\Gamma;\{b_j'\}_{j=1}^{J},W'))$ such that:

(i) Taking as given the representative investor's policies, the bond price functions $\{q_j(S,b')\}_{j=1}^{J}$, the law of motion of the distribution $H(\psi)$,and the aggregate law of motion of the world economy $Q(S,\Gamma;\{b_j'\}_{j=1}^{J},W')$, the emerging economies' consumption $\{c_j(S)\}_{j=1}^{J}$ satisfies the economies' resource constraints. Additionally, the economies' policy functions $\left\{b_j'(S)\right\}_{j=1}^{J}$, $\{d_j(S)\}_{j=1}^{J}$ and default sets $\{D_j\left(b|W\right)\}_{j=1}^{J}$ satisfy the optimization problem of the emerging economies.

(ii) Taking as given the emerging economy's policies, the bond price functions $\{q_j(S,b')\}_{j=1}^{J}$, the law of motion of the distribution $H(\psi)$,and the aggregate law of motion of the world economy $Q(S,\Gamma;\{b_j'\}_{j=1}^{J},W')$, the representative investor's consumption $c^L(s)$ satisfies the investor's budget constraint. Also, the representative investor's policy functions $\left\{\theta_j'(S)\right\}_{j=1}^{J}$ and $\theta^{TB'}(S)$ satisfy the optimization problem of the representative investor and the law of motion of the investor's wealth.

(iii) Bond prices reflect the emerging economies' probability of default and the risk premium demanded by the representative international investor; and these prices clear the market for all the emerging economies' bonds:

$$b_j\,'(S) = -\theta_j\,'(S) \quad \text{if } b_j\,'(S) < 0 \tag{19a}$$

$$0 = -\theta_j\,'(S) \quad \text{if } b_j\,'(s) \geq 0. \tag{19b}$$

Equations $(19a)$ and $(19b)$ imply that in equilibrium each emerging economy j and the representative investor agree on a financial contract, $b_j{}'$ and q_j, that is optimal for both agents.

(iv) The law of motion of the distribution $H(\psi)$, the aggregate law of motion of the world economy $Q(S,\Gamma;\{b_j'\}_{j=1}^{J},W')$, and the updating operator $T(\psi,Q(S\Gamma;\{b_j'\}_{j=1}^{J},W'))$ are such that

$$H(\psi) = T(\psi,Q(S,\Gamma;\{b_j{}'\}_{j=1}^{J},W')). \tag{20}$$

4.1 Characterization of Default Sets

The characterization of default sets is the characterization of incentives to default and therefore the characterization of endogenous default risk. In the model, default risk is a function of the emerging economy's fundamentals (the economy's endowment process and its asset position), the characteristics of the international investor (the investor's risk aversion and wealth), and the fundamentals of other countries, $k \neq j$, that share investors with country j. This section focus on the role that investors' characteristics and the own j emerging economy's fundamentals have on the determination of default incentives for economy j. The role of other economies' fundamentals in the characterization of default sets is analyzed in the section on contagion.

Maximum Credit Constraint and Maximum Safe Level of Debt In order to continue with the characterization of the default sets, it is necessary to define two concepts, the *maximum credit constraint* and the *maximum safe level of debt*. The *maximum credit constraint* is the maximum level of assets, $\underline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$, that is low enough such that no matter what the realization of the endowment, default is the optimal choice and $D_j(\underline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi) \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi) = Y_j$. In contrast, the *maximum safe level of debt* is the minimum level of assets $\overline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$ for which staying in the contract is the optimal choice for all realizations of the endowment. In this case, $D_j(\overline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi) \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi) = \emptyset$. Finally, because the value of the credit contract is monotonically decreasing in b_j,

$$\underline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi) \leq \overline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi) \leq 0$$

Proposition 1 *For any state of the world, S, the maximum credit constraint $\underline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$, and the maximum safe level of debt $\overline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$ are singled-valued functions.*

Proof. To define these concepts note that the stochastic process for the endowments have a compact support. Also note that, conditional on W, the fundamentals of other emerging economies in the investors' portfolio, and the distribution ψ, the value of the credit contract is monotonically decreasing in b_j. Monotonicity of the credit contract and compactness of the endowment support are sufficient conditions to guarantee that given the state of the world these critical values (i.e, endogenous credit constraint and maximum safe level of debt) are single-valued functions. ■

From the previous discussion, it is clear that given some current level of investors' wealth, the fundamentals of other emerging economies in the investors' portfolio, and the distribution ψ, any investments in the emerging economy's bonds in excess of $\underline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$ imply a probability of default equal to 1. These investments will have a price of 0. On the other hand, all investments in the emerging economy's bond of an amount lower than $\overline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$ imply a zero probability of default. These investments will have a price of q^f.

Default Sets and Risk Aversion of International Investors As discussed in Lizarazo (2010), the degree of investors' risk aversion is an important determinant of emerging economies' access to credit markets and of the risk of default. Lizarazo (2010) shows that the more risk averse are international investors, the higher is the default risk and the tighter is the endogenous credit constraint faced by all emerging economies.

Proposition 2 *For any state of the world, S, as the risk aversion of the international investor increases, the emerging economies' incentives to default increase.*

Proof. See Appendix. ■

The result in Proposition 2 is an extension of Lizarazo (2010) to the case of several economies that share international risk averse investors. (Lizarazo (2010) presents the case of one emerging economy that faces international risk averse investors.) Proposition 2 is consistent with empirical findings which characterize the role of investor's risk aversion in the determination of country risk and sovereign yield[11].

Default Sets and Investor's Wealth In the present model, the investor's wealth also affects the emerging economy's performance. This result is formalized in Proposition 3.

Proposition 3 *Default sets are shrinking in the assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b_j in some states y_j, given W_2 then default will be optimal for b_j for the same states y_j, given W_1. Therefore $D_j\left(b_j \mid W_2, \psi, \{s_k\}_{k=1,k\neq j}^{J}\right) \subseteq D_j\left(b_j \mid W_1, \psi, \{s_k\}_{k=1,k\neq j}^{J}\right)$*

[11]See, for example, Ferruci et. al. (2004), Mody and Taylor (2004), FitzGerald and Krolzig (2003), Cunningham, Dixon and Hayes (2001), Westphalen (2001), and Kamin and von Kleist (1999) and Arora and Cerisola (2001).

Proof. See Appendix. ■

The result in Proposition 3 is also an extension to the present context of a similar result in Lizarazo (2010). Proposition 3 is consistent with the findings of several empirical papers on the literature regarding the determinants of capital flows and sovereign bonds spreads of emerging economies[12].

Default Sets and the Asset Position of the Emerging Economy In the model, a highly indebted economy is more likely to default than an economy with lower debt. And as in models of the same type where lenders are risk neutral, default sets are shrinking in assets.

Proposition 4 *Default sets are shrinking in assets of the emerging economy. For all* $b_{j,1} < b_{j,2}$*, if default is optimal for* $b_{j,2}$ *in some states* y_j *, given* W*, and the asset position of other emerging economies in the investors' portfolio, then default will be optimal for* $b_{j,1}$ *for the same states* y_j*, given* W*,and the asset position of other emerging economies in the investors' portfolio. Therefore* $D_j\left(b_{j,2} \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi\right) \subseteq D_j\left(b_{j,1} \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi\right)$.

This result is analogous to the results in the previous literature on endogenous sovereign risk. [13] The main difference in the present paper is that the result is conditioned on the level of wealth of the representative investor, and the fundamentals of other emerging economies in the investors' portfolio. The result is consistent with the empirical literature on the determination of credit ratings and yield-bond spreads.[14]

Default Sets and Endowment Realization Default sets also depend on the realization of income. As in Arellano (2008), it is possible to show analytically that if the endowment process is $i.i.d.$ for given W and the fundamentals of the other economies in the investors' portfolio, then default incentives are stronger for lower levels of income. The numerical solution of the present model extends this result to the case in which the stochastic process of the endowments follows a Markov chain with persistence.

[12]See, for example, Warther (1995), Westphalen (2001), Kang et al (2003), FitzGerald and Krolzig (2003), Mody and Taylor (2004), and Ferruci et al (2004). Proposition 3 is also consistent with the evidence regarding financial contagion across countries who share investors. See, for example, Kaminsky and Reinhart (1998), Van Rijckeghem and Weder (1999), Kaminsky and Reinhart (2000) and Hernandez, and Valdes (2001).

[13]For similar results see for example Arellano (2008), and Aguiar and Gopinath(2006), and Lizarazo (2010).

[14]See for example, Cantor and Pecker (1996) and Cunningham, Dixon and Hayes (2001).

Proposition 5 *If the endowment process is i.i.d., default incentives are stronger the lower the endowment. For all* $y_{j,1} < y_{j,2}$ *if* $y_{j,2} \in D_j\left(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi\right)$ *then* $y_{j,1} \in D_j\left(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi\right)$.

The intuition for this result follows Arellano (2008). The main difference is that in the present context, the result is conditioned on the level of wealth of the investors and the fundamentals of other emerging economies in the investor's portfolio.

The logic behind this result follows from the fact that default is only optimal if under all feasible financial contracts the emerging economy experiences capital outflows. In the case of a recession, capital outflows are extremely costly in terms of the welfare of a risk averse agent (because the concavity of the periodic utility). Therefore at sufficiently low levels of the endowment realization, the credit market becomes a less effective tool for consumption smoothing than default.

This result is also consistent with the empirical literature on the determination of credit ratings and sovereign yields. In this literature, sovereign yield spreads increase when the economy's fundamentals deteriorate, mainly when GDP falls.

Additionally, this result implies that because default risk is counter-cyclical, domestic interest rates are also counter-cyclical. Counter cyclicality is consistent with the stylized facts of financial emerging markets (see Neumeyer and Perri (2004), and Uribe and Yue (2003)).

5 Contagion

This section characterizes the role of the fundamentals of foreign emerging economies in the determination of the incentives to default for the domestic emerging economy.

The domestic emerging economy's default sets also depend on the default/repayment decisions of other emerging economies in the investors' portfolios. A crisis in some foreign emerging economy k can be seen as a shock that changes the default/repayment decisions of that country, and therefore the bond prices of that country; therefore a crisis in emerging economy k has a wealth and a substitution effect over the optimal investor's portfolio allocation to other emerging economies.

First, the crisis in country k has a negative current or expected wealth effect in the investors. Because the investor's preference exhibit DARA, she would move away from risky emerging economies' assets towards safer assets; this effect corresponds to the channel previously referred to as the *wealth channel of contagion*.

Second, the crisis in country k generates substitution between different risky emerging economy assets in the investor's portfolio. The substitution effect of the crisis corresponds to the channel previously referred to as the *portfolio recomposition channel of contagion*.

More specifically, for the investor, whenever a crisis induces a default by country k a negative wealth shock occurs. In this case the wealth channel of contagion would propagate this negative shock to other risky emerging countries in the investor's portfolio. Furthermore the portfolio recomposition channel would amplify the negative effect of the wealth channel of contagion. In this situation contagion of the crisis in country k to other risky emerging economies in the investor's portfolio is observed.[15]

On the other hand, when the crisis in country k implies an increase in the probability of default of that country, the probability of a negative wealth shock to the investors in the future increases. The expected wealth effect would propagate the negative shock in country k to other risky countries in the investor's portfolio through the wealth channel. However, from the perspective of the international investor, assets of different emerging economies are substitutes, therefore the substitution effect of the shock to country k would have different effects in countries with weak fundamentals than in countries with solid fundamentals. For countries with sufficiently weak fundamentals the recomposition channel of contagion would amplify the effect of the wealth channel of contagion; for countries with sufficiently strong fundamentals the recomposition channel of contagion would mitigate or even reverse the effect of the wealth channel of contagion[16].

[15]As discussed previously, the effect of other economies' fundamentals in the domestic emerging economy bond price has its origin in the response of the excess risk premium to changes in foreign fundamentals. The response of the excess risk premium to those changes is dictated by the response to those same changes of the covariance term in the equation of the bond prices. The covariance term can be written as:

$$\begin{aligned} Cov[v_{c^L}(c^{L'})d_j{}'] &= \sum_{S'} \pi(S'|S) v_{c^L}(c^{L'}) d_j{}' - \sum_{S'} \pi(S'|S) v_{c^L}(c^{L'})(1-\delta_j(S')) \\ &= \sum_{S'} \pi(S'|S) v_{c^L}(c^{L'})(d_j{}'(S') - 1 + \delta_j{}'(S')). \end{aligned}$$

When in some state of the world, S, there are changes in the investor's future consumption, c^L, those changes would have effects on some economy $j's$ covariance term as long as $(d_j{}'(s') - 1 + \delta_j{}'(s'))$ is not 0 for all S', which in turns implies that δ_j is not either 0 or 1. Therefore only economies having the probability of default between 0 and 1 are affected by changes in the investor's future wealth and consumption. Countries with probability of default 1 or 0 are not risky by definition, so only risky countries would be affected by changes in other countries fundamentals.

[16]The definition of weak and strong fundamentals in this context is somewhat vague, and in a very

For countries with weak fundamentals, after observing an increase in the probability of default of country k, contagion is observed. However, for countries with strong fundamentals, after the crisis in country k, the recomposition channel of contagion may reverse the effect of the wealth channel of contagion. In this case, "flight to quality" occurs. "Flight to quality" corresponds to the phenomena when countries with relatively strong fundamentals receive capital inflows following a crises in some foreign emerging economy k.

For crises that do not imply an actual default by country k, but only an increase in the probability of default in that country, the result over a particular economy (contagion or flight to quality) would depend on the strength of the positive correlation between assets returns of economies that share investors that is generated by the wealth channel of contagion. For economies with sound fundamentals, if this positive correlation is not strong enough—i.e., investor's wealth is pretty high, or the investors are not too risk averse—the positive substitution effect of the crisis in country k might dominate its negative wealth effect. On the other hand, for economies with weak fundamentals, if either the wealth of the investors' is low, the expected wealth shock is large (i.e., the exposure of the investor to the country with problems is large), or the investor's risk aversion is high, then the portfolio recomposition channel will amplify the effects of the wealth channel due to the increase in the risk of some country k.

Wealth Channel of Contagion

Proposition 6 *There is a wealth channel of contagion. Proposition 3 implies that if economy k which is in the investor's portfolio defaults in her debts, incentives to default for economy j which is also in the investor's portfolio increase.*

Proof. See appendix. ■

The intuition of Proposition 6 is straight forward: a default by some emerging economy in the investors' portfolio is equivalent to a negative wealth shock. Therefore, from Proposition 3 incentives to default for other economies in the investors' portfolio increase as a consequence of the default by economy k. The result in proposition 6 is the foundation for the endogenous explanation of contagion of financial crises based on endogenous links across economies that share investors. This result corresponds to the Wealth Channel of contagion.

simplifying way corresponds to countries with relatively high default probability (and low bond prices) and countries with relatively low default probability (and high bond prices), respectively.

This result also implies that there is an expected negative wealth effect whenever the probability of default increases for some emerging country in the investors' portfolio. This expected wealth effect would tend to reduce the long-term benefits of maintaining access to international credit markets, and therefore would tend to increase the incentives to default for the other countries in the investors' portfolio.

The Recomposition Channel of Contagion

Proposition 7 *There is a recomposition channel of contagion. Whenever the probability of default of some country k increases, there is a recomposition of the investor's portfolio: the assets of emerging economies with weak fundamentals are substituted away by the assets of emerging economies with solid fundamentals or by T-Bills.*

Proof. See Appendix. ■

The intuition of Proposition 7 is as follows: whenever the probability of default of some country k increases, the wealth channel of contagion implies that the shock reduces the prices of the bonds of all economies in the investor's portfolio; however, concavity of the periodic utility function of the emerging economies implies that the increase in the incentives to default of the economies with weak fundamentals would be larger than the increase in the incentives to default of the economies with strong fundamentals. As a consequence, the reduction in the bond prices of the economies with weak fundamentals is larger than the reduction in the prices of the bonds of economies with strong fundamentals. Therefore economy k and the economies with weak fundamentals become relatively more risky than economies with strong fundamentals; and the substitution effect of the shock in k would induce a substitution in the investors' portfolio from assets of country k and economies with weak fundamentals towards assets of emerging economies with strong fundamentals and to riskless T-Bills.

Corollary 4 *If following a crisis in some foreign emerging economy k the overall risk of the investor's portfolio increases, then the incentives to default for all emerging economies increase.*

Proof. See Appendix. ■

If at the time of the crisis in country k all emerging economies have relatively weak fundamentals, the recomposition channel of contagion would amplify the contagion generated by the wealth channel of contagion. The intuition for this result comes from the fact that risk averse agents require an increasing risk premium in order to accept a higher level of risk. Therefore whenever their portfolio as a whole becomes more risky investors would ask for a larger risk premium from all the economies in order to allocate the same amount of resources to those countries as before. As a consequence, an increase in risk would reduce the benefits for all the economies to maintain participation in the credit markets, and would therefore increase incentives to default. In turn the overall risk of the investor's portfolio will increase even further.

Corollary 5 *For economies with strong fundamentals if the positive substitution effect of the crisis in some foreign economy k dominates the negative expected wealth effect of the crisis "flight to quality" occurs.*

Proof. See Appendix. ■

On the other hand, if at the time of the crisis in country k the domestic emerging economy has strong fundamentals,the recomposition channel of contagion would moderate or even reverse the effect generated by the wealth channel of contagion. This effect is consistent with the evidence of "flight to quality" observed during some periods of crises. [17]

From the previous discussion, it is clear that the fundamentals of the domestic emerging economy matter to determine if the wealth channel of contagion and the recomposition channel of contagion go in the same direction or in opposite directions, and therefore if the domestic economy experiences contagion or flight to quality. The numerical results of this paper suggest that when considering productivity shocks which are independently distributed across economies, a deterioration of the fundamentals of one economy causes an endogenous worsening of the situation of the other emerging economy.

The analysis in this section allows us to connect the possibility of observing contagion to the fragility of the fundamentals of an economy. In other words, given the investors' characteristics we should expect a higher probability of contagion for those economies with weaker fundamentals while economies with relatively strong fundamentals should experience "flight to quality."

[17] According to Kaminsky, Lyons, and Schmulker (2001) during the two first quarters after the Mexican crisis mutual fund flows to countries like Malaysia, Colombia, Poland and Czech Republic increased by more than 10%. During the two first quarters after the Thai crisis, mutual funds flows to countries like Venezuela, Slovak Republic and Sri Lanka increased by more than 5%. Finally, during the two first quarters after the Russian crisis, mutual funds flows to Mexico and Singapore increased by more than 5%.

5.1 Contagion at Equilibrium

In the current model, to observe contagion at equilibrium it is necessary to observe default at equilibrium, since only the possibility of default generates the wealth and portfolio recomposition channels of contagion. Therefore, to have contagion at equilibrium it must hold that beginning from an asset position b_j such that $D_j\left(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi\right) = \emptyset$, there exists a sequence of endowment shocks such that this economy ends up borrowing b'_j and $D_j\left(b_j \mid \{s_k\}_{k=1,k\neq j}^{J}, W, \psi\right) \neq \emptyset$.

In order to establish if contagion and default might be equilibrium outcomes of the model is necessary to determine if there exists such a $b'_j < \overline{b_j}\left(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi\right)$ for which by increasing its borrowing beyond the maximum safe level of debt the economy is able to increase its current capital inflows $-q_j(S, b'_j)b'_j$.

Following closely the analysis in Arellano (2008) and considering only the case in which the incentives to default are stronger for lower levels of the endowment, it is possible to define the conditional default boundary function $y_j^*(b_j | \{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$ as follows:[18]

Definition 6 *The conditional default boundary function* $y_j^*(b_j | \{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$ *corresponds to the endowment level* y_j^* *for a given level of debt* $b_j \in (\underline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi), \overline{b_j}(\{s_k\}_{k=1,k\neq j}^{J}, W, \psi))$ *that conditional on the fundamentals of the other economies in the investor's portfolio, and the representative investor's wealth makes the value of repayment and the value of default equal for the emerging economy:* $V_j^C(b_j, y_j^*, \{s_k\}_{k=1,k\neq j}^{J}, W, \psi) = V_j^D(y_j^*, \{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$.

Conditional on the fundamentals of the other economies in the investor's portfolio and the representative investor's wealth, $y_j^*(b_j | \{s_k\}_{k=1,k\neq j}^{J}, W, \psi)$ divides the space $\{y_j, b_j\}$ into the default and repayment regions. According to proposition 2, proposition 3, proposition 4 and corollary 4, the conditional default boundary is increasing in the investor's risk aversion, decreasing in the investor's wealth, decreasing in the emerging economy's assets and increasing in the overall risk of the investor's portfolio which is determined by the fundamentals of the other economies in the investor's portfolio.

Using the definition of bond prices in equation (10) and the definition of default probabilities in equation (18), it is possible to show that as in the case of risk neu-

[18] While theoretically it is possible to have the case in which incentives to default are stronger the higher is the endowment, the case for which incentives to default are stronger the lower is the endowment seems to be the empirically relevant case as long as the persistence of the endowment shocks is not excessively high.

tral investors, the equilibrium bond price $q_j(S,b_j')$ is a function of the default boundary $y_j^*(b_j'|\{s_k\}_{k=1,k\neq j}^J, W,\psi)$ and the distribution of shocks. Also, as discussed in Lizarazo (2010), the bond prices are functions of the investor's risk aversion γ^L and her wealth level W since these variables help to determine both the conditional default boundary $y_j^*(b_j'|\{s_k\}_{k=1,k\neq j}^J, W,\psi)$, and the *excess* risk premium included in the bond prices. More important for the study of contagion, the fundamentals of other economies also affect the bond prices through their effect on both the conditional default boundary, and the excess risk premium of the prices:

$$\begin{aligned} q_j(S,b_j') &= q^f[1-F(y_j^*(b_j'|\{s_k\}_{k=1,k\neq j}^J, W,\psi))] + \beta_L \frac{\left[Cov(v_{c^L}(c^{L'}(S)),\ d_j'(S))\right]}{v_{c^L}(c^L(S))} \\ &= \int_{y_j^*\left(b_j'|\{s_k\}_{k=1,k\neq j}^J, W,\psi\right)}^{Y} \beta_L \frac{v_{c^L}\left(c^{L'}(S)\right)}{v_{c^L}\left(c^L(S)\right)} f\left(y_j' \mid y_j\right) dy_j'. \end{aligned}$$

where F is the cumulative probability distribution of shocks.

Financial contracts $\{q_j(S,b_j'(S)), b_j'(S)\}$ observed at equilibrium change current consumption by the product $-q_j(S,b_j'(S))b_j'(S)$. As consequence of the result in proposition 4, the definition of the conditional boundary function, and the definition of equilibrium bond prices, as debt increases the equilibrium bond prices go to zero. Therefore it is possible to define the *endogenous borrowing limit* $b_j^*(S)$ as follows:

Definition 7 *The endogenous borrowing limit $b_j^*(S)$ is the level of debt for which $\pi_j \equiv -q_j(S,b_j^*(S))b_j^*(S)$ is such that*

$$\pi_j = \max_{b_j'} \left[-\left(q^f[1-F(y_j^*(b_j'|\{s_k\}_{k=1,k\neq j}^J, W,\psi))] + \frac{\beta_L[Cov(v_{c^L}(c^{L'}),\ d_j')]}{v_{c^L}(c^L)} \right) b_j' \right].$$

For any given state S, $b_j^*(S)$ is the endogenous borrowing constraint since for any $b_j' < b_j^*(S)$ $V_j^C(S,b_j') < V_j^C(S,b_j^*(S))$, and therefore $b_j' < b_j^*(S)$ cannot be optimal.

For any state S, the relevant risky region of the model is limited to contracts with $b_j' \in \left[b_j^*(S), \overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W,\psi)\right)$.

Proposition 8 *A necessary condition to observe contagion and default at equilibrium is that for some state S, the relevant risky region of the model is not empty. In other words,* *contagion and default are possible outcomes of the time series of the model* *only if* *there exists* $b_j^*(S)$ *such that :*

$$b_j^*(S) < \overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W, \psi).$$

In order to observe contagion and default at equilibrium the equilibrium price function cannot decrease "too fast" when assets decrease.

Given the speed at which bond prices decrease when the economy's assets decrease, the smaller is $\overline{b}(\{s_k\}_{k=1,k\neq j}^J, W, \psi)$, the higher is the chance that there exists $b_j^*(S) < \overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W, \psi)$. Intuitively, because investors must be compensated in order to induce them to take some default risk, this risk imposes an additional cost of borrowing for the emerging economy. For the borrower, the cost of borrowing beyond the maximum safe level must be paid over the total amount of resources borrowed, and not only over the marginal amount of borrowing. Therefore, the larger is the base over which this additional cost of borrowing has to be paid—i.e. the larger is the maximum safe level of borrowing—the higher is the cost of default risk and the lower is the likelihood that the economy would ever choose to borrow beyond the safe level of debt.

Lizarazo (2010) characterizes the roles of W and γ^L in the likelihood of observing default at equilibrium in a similar model of endogenous default risk with risk averse agents, and concludes that such a question cannot be answered unambiguously theoretically. The same results apply for this model. However, the quantitative results in this article establish that the possibility of observing contagion and default at equilibrium is largely independent of the level of wealth of the investor. The possibility of observing default is also independent of the degree of risk aversion of the investor, as long as the investor is risk averse—for a risk neutral investor contagion cannot be observed.

Role of $\{s_k\}_{k=1,k\neq j}^J$ in the determination of the existence of $b_j^*(S) < \overline{b_j}(\{s_k\}_{k=1,k\neq j}^J, W, \psi)$. First, corollary 4 establishes that the weaker are the fundamentals of the economies that share the investor, the less the economy is able to borrow and the lower is the maximum safe level of borrowing. Therefore, other things equal, if the other economies' fundamentals are weak, the cost of a change in the price of the bonds is felt over a smaller borrowing base. In this case, there is potentially more to gain from accepting a lower price for these bonds in order to further increase borrowing. Therefore this effect makes default and contagion more likely outcomes of the model.

Second, weak fundamentals of the other economies that share investors imply a larger overall risk in the investor's portfolio and therefore imply a larger response of $q_j\left(S, b_j'\right)$ to changes in the borrowing level. Other things equal, the more risky is the portfolio of the investor, the larger is the *excess* risk premium that she demands in order to take default risk.

These two effects go in opposite directions. Therefore, is not possible to establish theoretically how the equilibrium default probability of the model and the occurrence of contagion responds to changes in the fundamentals of the other economies in the investor's portfolio. The numerical simulations of the model performed here suggest that default and contagion are more likely whenever the other economies' fundamentals are weaker and default by those economies is expected.

6 Numerical Solution

As is well known, during 2001 Argentina faced one of the worst economic crisis in its history. The country was forced to default on $US100$ billion in external government debt (corresponding to nearly 37% of GDP) by the end of 2001. The crisis had strong real effects that extended into 2002: according to estimates by the IMF, Argentina's GDP fell by 4.4% during 2001, and by an additional 10.9% in 2002.

Uruguay, on other hand was facing economic problems since 1998. These problems were aggravated in 2001 by the outburst of cow foot-and-mouth disease which very negatively affected exports. On top of these preexisting conditions, the Argentinean crisis began, weakening confidence in Uruguay and prompting caution in consumers and investors. As a result, real demand fell, prompting an exchange rate depreciation of the Uruguayan peso. This depreciation generated a significative increase in the public debt to GDP ratio (this ratio went from 40% to 52%). According to IMF estimates, during 2001 Uruguay's GDP fell by 3.5%, and during 2002 Uruguay's GDP fell by and additional 7.1%. The fall in GDP in 2002 was due mainly to problems in Uruguay's financial sector which had strong financial links to Argentina: in early 2002 following Argentina's default Uruguay's financial sector experienced large dollar deposit outflows (these outflows exceeded 100 million per day in the month of July 2002), and Uruguay faced a rapid decline in its international reserves (Uruguay's international reserves fell from 3 billion dollars at the end of 2001 to 650 million by August 2002). Signaling the credit risk involved in Uruguay's external debt, during 2002, Uruguay's debt was downgraded by investment rating agencies.[19].

[19]Uruguay's deposit outflows were initially confined to non-residents but, as a result of problems of two local banks and the Argentina's crisis, deposit outflows accelerated in March 2002 and spread to resident

Table 1: Business Cycle for Argentina and Uruguay 1983:Q1-2002:Q1

Variable	mean(x)	std(x)	corr(x, y^{AR})	corr(x, r^{AR})	corr(x, y^{UR})	corr(x, r^{UR})	corr(x, y^{US})
Spread r^{AR}	12.67	5.42	-0.60		-0.44	0.18	-0.11
Trade-Balance TB^{AR}	0.58	1.83	-0.59	0.38	-0.47	0.05	0.08
Output y^{AR}	100	1.91			0.63	-0.24	-0.02
Consumption c^{AR}	78.32	1.94	0.93	-0.78	0.71	-0.63	0.60
Spread r^{UR}	8.53	0.86			-0.30		-0.25
Trade-Balance TB^{UR}	-10.67	3.15	-0.26	0.16	-0.48	0.09	-0.17
Output y^{UR}	100	1.64					0.49
Consumption c^{UR}	68.32	2.61	0.55	-0.31	0.86	-0.13	0.42

The simulation of the model in this paper analyzes the Argentinean default of 2001 and its implications on Argentina's neighbor Uruguay. Uruguay was the most affected country, by the spillovers of the Argentinean crisis. While capital flows to Latin America dipped sharply in the summer and fall of 2001 in response to the crisis in Argentina, by the end of the year they were recovering despite the expectation of Argentina's default. Contagion in the bond market had been significant until October 2001 with spreads throughout Latin America rising in step with the deterioration of Argentine credit risk. But after October the spreads in other Latin America bonds declined markedly. By March 2002 as spreads reached their lowest levels since April 1998, bond issuance remained healthy. Uruguay, however, with particulary close financial links to Argentina, suffered strong pressures on its currency. Because trade and financial links between Argentina and most of its other neighbors remained relatively limited, other Latin American countries were not as affected as Uruguay.[20] On the other hand, and in line with the previously discussed theoretical results of this article that establish that contagion occurs mainly in those countries with weak fundamentals, sound macroeconomic management in most of the region increased confidence of international investors. Finally, an environment of ample global liquidity favored large and relatively liquid markets such as Mexico and Brazil.

Tables 1 and 2 describe the relevant business cycle features for the periods under study: Table 1 describes the statistics for the entire period for which data is available; Table 2

depositors. In early August, the central Bank suspended operations of four private domestic banks and reprogrammed foreign currency time deposits in public banks by up to three years: authorities further established a fund designed to fully back existing foreign currency sight deposits at domestic banks. These measures helped to stop the deposit outflows for the remainder of the year. However in January of 2003, a new round of deposits outflows took place as gross official reserves fell to US540 million by end-February 2003 (equivalent to only half of the domestic reserve liabilities of the central bank)

[20]Only in Bolivia, Brazil, Chile, Paraguay and Uruguay does Argentina account for more than 10% of total trade. However, Argentina is not an important market for Latin American exports (1% of GDP of Bolivia, Brazil and Chile, 2% for Uruguay and 4% for Paraguay).

Table 2: Business Cycle for Argentina and Uruguay 2001:Q1-2001:Q4

Variable	mean(x)	std(x)	corr(x, y^{AR})	corr(x, r^{AR})	corr(x, y^{UR})	corr(x, r^{UR})	corr(x, y^{US})
Spread r^{AR}	22.26	13.59	-0.96		-0.80	0.52	-0.71
Trade-Balance TB^{AR}	2.13	2.11	-0.85	0.89	-0.94	0.58	-0.79
Output y^{AR}	95.6	2.98			0.84	-0.70	0.86
Consumption c^{AR}	76.99	3.71	0.98	-0.99	0.83	-0.60	0.77
Spread r^{UR}	9.53	1.45			-0.81		-0.88
Trade-Balance TB^{UR}	-9.30	2.17	-0.24	0.30	0.26	-0.12	0.19
Output y^{UR}	96.50	1.53					0.91
Consumption c^{UR}	100.46	1.88	0.88	-0.77	0.77	-0.63	0.91

describes the statistics for the year of the crisis. For the Argentinean output, consumption, and trade balance, the source of the data is the IFS. For the interest rate of Argentina the source is Neumeyer and Perri (2005). For Uruguay, the series for output, consumption and trade balance are constructed using the Uruguayan Central Bank quarterly and annual data on indexes and volume for these variables. Also, for Uruguay's interest rate, the source is the Uruguayan Central Bank; this rate corresponds to the domestic interest rate in loans[21].

The data for the business cycle statistics includes the period 1983:Q1-2001:Q4 for the all the Argentinean series except the consumption series which is only available for the period 1993:Q1-2001:Q4. For the Uruguayan series, the period for which the data are available corresponds to 1988:Q1-2001:Q4 for output, consumption, and trade balance, and to 1980:Q1-2001:Q4 for the interest rate. Therefore, the business cycle statistics for each variable correspond to the initial moment in which each of them is available to the fourth quarter of 2001. The correlations are taken for the common periods in which any pair of variables are available. Output and consumption for Argentina and Uruguay are seasonally adjusted and are in logs and filtered with the H-P filter. Argentina's and Uruguay's trade balances are reported as a percentage of their respective output. The interest spread is defined as the difference between the Argentinean and the Uruguayan interest rate and the yield of a 3 month U.S. T-Bill.

From the tables 1 and 2 it can be seen that both Argentina's and Uruguay's interest rates and trade balances are counter-cyclical, a fact which is well known for the case of emerging economies. More importantly for our analysis, the interest rates and trade balances of these two countries are also negatively correlated with their neighbor's GDP, and these negative correlations are larger during the period of the Argentinean crisis: the

[21]While the domestic rate is clearly not the interest rate in international loans, it should be positively correlated. Unfortunately there is no EMBI for Uruguay. And while it would be possible to calculate an implicit interest rate from Uruguay's debt service data, such data is available only annually.

Table 3: Contagion: Parameter Values

Parameter	Value
Emerging Economy's Mean Income $E[y]$	1
Std. Dev. Emerging Economy's Income $std[y]$	0.025
Autocorr. Emerging Economy's Income Process	0.945
Emerging Economy's Discount Factor β	0.953
Emerging Economy's Risk Aversion γ	2
Probability of re-entry τ	0.282
Critical level of output for asymmetrical output cost	$\hat{y} = 0.969E(y)$
Representative investor's Income X	0.01
Representative Investor's Discount Factor β^L	0.98
Representative investor's Risk Aversion γ^L	2
Risk Free Interest Rate $r^f = \frac{1}{q^f}$	0.017

correlation between Argentina's GDP and Uruguay's interest rate for the whole period of study is -0.24 and becomes -0.80 during the year of the crisis; the correlation between Uruguay's GDP and Argentina's interest rate for the whole period of study is -0.44 and jumps to -0.80 during the period of the Argentinean crisis. The behavior of the correlations of the countries' trade balances and their neighbor's GDP follows a similar pattern to the one observed for the correlations of the interest rates and the GDP's of the neighbor country.

For the analysis here, it is also important to notice that Argentina's and Uruguay's interest rates are positively correlated (0.18) and that this correlation is much larger during the period of the Argentinean crisis (0.52). Finally, the consumptions in these two countries are positively correlated with the neighbor country's GDP and the correlations are larger during the year of Argentina's crisis: the correlation between Argentina's consumption and Uruguay's GDP is 0.71 and 0.83 for the whole period and the crisis period respectively; the correlation between Uruguay's consumption and Argentina's GDP is 0.55 and 0.88 for the whole period and the crisis period respectively.

Given the assumption of the model of identical economies that only differ in the realizations of their endowments, and in order to facilitate comparison with the previous literature in the subject, the parameters considered for the simulation are chosen to replicate some features of the Argentinean economy, and are taken from the calibration for this economy in Arellano (2008). The parameters related to international investors are taken from Lizarazo (2010).

Table 3 shows the parameters of the numerical analysis of the model. The mean income of the emerging economy is normalized to 1. The coefficient of risk aversion of the economy is 2, a standard value considered in the business cycle literature. The free interest rate is set to 1.7%, to match the period under study to the quarterly US interest rate of a bond with a maturity of 5 years. The GDP is assumed to follow a log-normal AR(1) process $log(y_t) = \rho log(y_{t-1}) + \varepsilon^y$ with $E[\varepsilon^y] = 0$ and $E[\varepsilon^{y2}] = \sigma_y^2$. The values estimated by Arellano(2008) for the Argentinean economy are $\rho = 0.945$ and $\sigma_y = 0.025$. The shock is discretized into a Markov chain using the quadrature based procedure (Hussey and Tauchen (1991)). Following a default, there is an asymmetrical function for the output loss:

$$\phi(y) = \left\{ \begin{array}{ll} \widehat{y} & \text{if } y > \widehat{y} \\ y & \text{if } y \leq \widehat{y} \end{array} \right\} \tag{21}$$

with $\widehat{y} = 0.969E(y)$ which following Arellano (2008) targets a value of 5.53% for the average debt service to GDP ratio. The probability of re-entry to credit markets after defaulting is set at 0.282, which is consistent with the empirical evidence regarding the exclusion from credit markets of defaulting countries (see Gelos et al. (2002)), and which following Arellano (2008) targets a volatility of 1.75 for the trade balance. The discount factor is set a 0.953 which following Arellano (2008) targets a annual default probability of 3%.

The parameters for the international investors are as follows: the representative investor's discount factor is set to 0.98. As in Lizarazo (2010), if there were no uncertainty, the discount factor of the investors would pin-down the international risk free interest rate (i.e., $\frac{\beta^L}{q^f} = 1$); however, with uncertainty, in order to have a well defined distribution for the investor's assets, it is necessary to have a value of the discount factor such that $\frac{\beta^L}{q^f} < 1$. The value of $\beta^L = 0.98$ is the highest value in the range commonly used in business cycle studies of industrialized countries such that for an international interest rate of 1.7% the asset distribution of the investors is well defined.

Following Lizarazo (2010), the representative investor's coefficient of risk aversion is set at 2; the criteria to choose this parameter is to generate a mean spread for model that is as close as possible to the mean spread in Argentina for the period of study, which corresponds to 12.67%.

The representative investor receives a deterministic income of 1% of the emerging economy's mean income in each period. In Lizarazo (2010) this parameter is set to minimize the joint deviations between the observed long term excess return of the portfolio of an investor in sovereign debt markets and the observed sharpe-ratio of such a portfolio. According to Jostova (2006), if the investment strategy followed by the investor in sovereign debt markets is an active investment strategy (the adjustment in the shares allocated to risky and riskless investments is done based on short term shocks), then the annual average sharpe-ratio and the annual average excess return are 0.63 and 19.5%, respectively.

The parameter X affects the long term excess return of the portfolio of the investor and the volatility of those returns through its effect on the extent to which the investor is able to borrow from international credit markets. Small X implies less possibility of risk free borrowing by the investors, and therefore more difficulty for them in smoothing their consumption. As a consequence, investors receive large compensations for making risky investments, and obtain a large excess return of their portfolio. However, comparatively, the volatility of such portfolio is even larger. Relatively high volatility of the portfolio translates into a small sharpe-ratio for the investors' portfolio. In the data for the portfolios of the investors in sovereign debt markets, relatively large excess returns and large sharpe-ratios are observed. It is difficult for the model to simultaneously match these two observed phenomena: low values of X can generate larger excess returns but predict very low sharpe-ratios: on the other hand, high values of X can match the sharpe-ratio for the investor's portfolio but underestimate the average long term excess return of such portfolio.

6.1 Simulations

By considering the fundamentals of countries that share investors, the simulations presented here aim to replicate the following observed dynamics of sovereign yield spreads, and capital flows to emerging economies: i) the sovereign risk premium is high during recessions, or when the economy is highly indebted; ii) default is observed when the fundamentals of the economy deteriorate, iii) in periods previous to default, the economy experiences capital outflows and collapses in consumption, iv) capital flows and domestic interest rates across emerging economies are positively correlated, and iv) default is more likely to be observed when the fundamentals of other emerging economies deteriorate.

The model is simulated for two economies, Argentina and Uruguay, that are labeled as A and U. For each economy the endowment shock is discretized into a 5 state Markov chain and the asset positon of the economy is approximated by a 75 point grid. The investor's wealth level is approximated using a 10 point grid, over which the solution to the investor's problem is linearly interpolated.[22] The business cycles statistics of the model are derived as

[22] In both the discretization of the endowment realization and the asset position of the emerging economies there are important differences with the calibration in Arellano (2008): in that article the endowment realization has 21 states and the economy's asset position state variable has 200 states. For a model like the one here, with two economies, such a dimension in the economies' state variables is too large: for the

follows: The model is repeatedly simulated for $20,000$ periods. From these $20,000$ periods, sub-samples that have economy A staying in the credit market for 74 periods before going into a default are taken to compute the business cycles statistics of the two economies. This process is repeated 5000 times, and the busness cycle statistics are the average of the statistics derived from each of these repetitions.

The business cycle statistics of the data are compared with the statistics generated by the model. The results of the simulations are shown in Tables 4 for the whole period under study and 5 for the year of the crisis. In these tables the results of the model of contagion are compared with the results of a simulation of the model with risk neutral investors that has the same number of endowment shocks and the same points in the economies' grid for their asset position as the model of the contagion.

Table 4 and Table 5 show that in general terms the contagion model fits the business cycle statistics of Argentina and Uruguay relatively better than the model without financial links (i.e. the model with risk neutral investors). In both models a default occurs when economy's A GDP is on average 5.5% below its trend. In the contagion model, however, the effects of the default of country A on country U's business cycle statistics occur even when on average country U's income is not below its trend.

In the data, the spreads of Argentina are 12.67% for the whole period, and 22.26% during the crisis period (i.e., the year previous to a default episode). The contagion model generates a spread for the overall period of 5.7% and 12.7% for the crisis period, while the model without financial links only predicts a spread of 4.5% for the whole period and 4.9% for the crisis period. Unfortunately, regarding the volatility of the spreads, both models (the contagion and the one without financial links) under-predict this volatility. However the contagion model matches the near doubling of spread prior to the crisis.

The better matching of the spreads by the contagion model is not the result of a higher probability of default predicted by this model since both models predict a annual probability of default of 3.8%. Also, the higher spreads do not imply a contraction in the mean debt level that the contagion model supports in comparison to the model without financial links, since both models have an unconditional mean debt level for the whole period of 15.9%, and for both models the unconditional mean debt level for the period previous to the default

case of two countries, and taking in account the investor's wealth level a similar realization would imply $(21 \times 200) \times (21 \times 200) \times 10 = 176'400,000$ possible states for the whole model compared to $21 \times 200 = 4,200$ states in Arellano (2008). The version of the model in here has $(5 \times 750) \times (5 \times 750) \times 10 = 1'406,250$ possible states, still a much larger dimension than the model with only one economy and risk neutral investors.

Table 4: Contagion: Business Cycle Statistics for 74 periods before the default of country A

Business Cycles Statistics for the Contagion Model and the Model with Risk Neutral Investors											
	Contagion Model with 2 Countries							Model without Financial Links			
	Mean(x)	std(x)	corr(x,y^A)	corr(x,r^A)	corr(x,y^U)	corr(x,r^U)	corr(x,W)	Mean(x)	std(x)	corr(x,y^A)	corr(x,r^A)
Spread A	5.70	1.11	-0.06		-0.03	0.32	-0.34	4.5	0.6	-0.13	
Trade Balance A	0.4	0.77	-0.16	0.38	-0.01	0.05	0.10	0.4	0.76	-0.15	-0.10
Output A		1.94			0		0.35		1.94		
Consumption A		2.20	0.97	-0.04	0.01	-0.08	0.31		2.11	0.90	-0.14
	Mean unconditional Default Probability = 1.18							Mean unconditional Default Probability = 1.18			
	Mean unconditional Default Probability = 1.18 Mean unconditional Debt Level=-15.87 Mean conditional Debt Level=-11.86							Mean conditional Default Probability = 1.18 Mean unconditional Debt Level=-15.89 Mean conditional Debt Level=-15.89			

Table 5: Contagion: Business Cycle Statistics for 4 periods before the default of country A

Business Cycles Statistics for the Contagion Model and the Model with Risk Neutral Investors											
	Contagion Model with 2 Countries							Model without Financial Links			
	Mean(x)	std(x)	corr(x,y^A)	corr(x,r^A)	corr(x,y^U)	corr(x,r^U)	corr(x,W)	Mean(x)	std(x)	corr(x,y^A)	corr(x,r^A)
Spread A	12.40	0.81	-0.91		-0.80	0.88	-0.80	4.90	1.00	-0.20	
Trade Balance A	0.60	0.78	0.78	0.89	0.30	0.58	0.79	0.60	0.77	0.60	-0.22
Output A		5.19			0		0.85		5.19		
Consumption A		5.03	0.99	-0.90	0.29	-0.30	0.84		4.62	0.68	-0.25
	Mean unconditional Default Probability = 0.97							Mean unconditional Default Probability = 0.97			
	Mean unconditional Default Probability = 1.28 Mean unconditional Debt Level=-20.14 Mean conditional Debt Level=-12.20							Mean conditional Default Probability = 0.97 Mean unconditional Debt Level=-20.17 Mean conditional Debt Level=-20.20			

Table 6: Debt Securities Issue Abroad 1993(Q3)-2004(Q3)

Debt Securities Issued Abroad Correlations							
	Argentina	Brazil	Mexico	Russia	Philippines	Colombia	Venezuela
Argentina	1	0.98	0.95	0.94	0.95	0.98	0.88
Brazil		1	0.93	0.91	0.98	0.99	0.91
Mexico			1	0.90	0.88	0.90	0.81
Russia				1	0.91	0.91	0.90
Philippines					1	0.98	0.94
Colombia						1	0.90
Venezuela							1

Joint BIS-IMF-World Bank Statistics on External Debt

episode increases to 20.1%.[23]

It is also worth noticing that the probability of country U defaulting conditional that there is going to be a default episode by country A in the sample is 1.24% larger per year. This result of the model is consistent with the observed downgrading of the Uruguayan external debt by international credit rating agencies that occurred in light of the events of the Argentinean crisis. Also, for country U the mean debt level conditional on the fact that there is going to be a default in the sample is 11.9% for the whole period, and 12.2% for the year of the crisis, showing that what is going on with country A has important effects on country U's access to credit markets.[24] This effect of a country's fundamentals on other countries access to credit markets can be see in Table 6. Table 6 shows a very high and positive correlation for the debt securities issued abroad for the group of emerging countries that include Argentina, Brazil, Mexico, Russia, Philippines, Colombia, and Venezuela during the period 1993:Q3-2004:Q3.

Compared to the model without financial links, the contagion model does equally well in predicting the counter-cyclical behavior of the spreads and the trade balances, and the pro-cyclical behavior of consumption for the whole period. For some of some of these correlations, the contagion model does an even better job than the model without financial links during the periods of crisis . Additionally, the contagion model is able to explain the correlations between the economies' fundamentals and the wealth of the investors (proxied

[23]The larger probability of default in this model as well as the larger level of debt supported at equilibrium by the models in comparison to the results in Arellano (2008) for a very similar calibration of the economy might be explained by the results in Hatchondo and Martinez (2006) that suggest that the results of endogenous sovereign risk models are somewhat sensitive to the solution method employed for the model as well as the dimension of the grid used to expand the asset position of the emerging economies.

[24]The larger mean debt level that is observed during the periods of crisis can be explained by the fact that in order to smooth consumption with lower bond prices the emerging economies are forced to incur in higher levels of debt during periods of economic distress.

by the US's GDP) and the correlations between the fundamentals of the economies. The model without financial links predicts all this correlations to be zero.

For example, in the data for the period of the crisis, the correlation between Argentina's spread and its GDP is -0.90. For this same period, the correlation between the spread and the output predicted by the contagion model is -0.91; the model without financial links predicts this correlation to be only -0.20. Regarding the correlation between Argentina's trade balance and its GDP, for the whole period in the data, this correlation is -0.59; both models predict this relation to be negative and around -0.15. Unfortunately, for the period of the crisis, the correlations predicted by both models do not exhibit the counter-cyclical behavior observed in the data. One possible explanation for this result consists in that the agents in the model are doing as much consumption smoothing as possible during the periods previous to the crisis, while in the data, credit markets begin to close in the periods previous to the crisis.

With respect to the correlation between Argentina's consumption and its GDP this correlation is predicted relatively well by both models for the whole period— the correlation is 0.93 in the data, the contagion model predicts this correlation to be 0.97, and the model without financial links predicts it to be 0.90—but the contagion model predicts this correlation much better for the crisis period—in the data this correlation is 0.96, the contagion model predicts it to be 0.99 and the model without financial links predicts it to be 0.68.

Regarding the correlations between the fundamentals of economies A and U, clearly the contagion model is superior to the model without financial links because as long as the GDPs of the two countries are not correlated (this assumption allows us to focus on the role of financial links to explain contagion) the model without financial links predicts the correlations of the two economies' fundamentals to be zero.

For the case of the correlations between the GDP of the crisis country (A) and the other country (U) and the spread of the other country, the contagion model predicts the correct sign for the whole period for this correlation but largely underestimates it: in the data the correlation is -0.24 while the model predicts it to be -0.03; however for the period of crisis the contagion model does a good job at explaining this correlation in the sense that is consistent with an important increase in the correlation during the crisis period in relation to the whole period: in the data the correlation is -0.63 while the model predicts it to be -0.80.

Turning to the spreads of the two countries, for the whole period and during the crisis period, the contagion model is consistent with the observed positive correlation of the

Table 7: EMBI+ Correlations 1994(Q3)-2000(Q4)

EMBI+ Correlations							
	Argentina	Brazil	Mexico	Morocco	Nigeria	South Africa	Venezuela
Argentina	1	0.82	0.91	0.96	0.77	0.83	0.92
Brazil		1	0.78	0.83	0.70	0.49	0.85
Mexico			1	0.93	0.56	0.83	0.82
Morocco				1	0.65	0.78	0.90
Nigeria					1	0.49	0.77
South Africa						1	0.79
Venezuela							1

Source JP Morgan

spreads. The model is also consistent with the pattern observed in the data of a significant increase in the correlation during the period of crisis. The model over-predicts this correlation for the case of Argentina and Uruguay: in the data for the whole period the correlation between the spreads of this rates is 0.18 and the model predicts it to be 0.32; for the period of crisis the correlation is 0.52 in the data and the model predicts it to be 0.88. This over-prediction for the spreads of the interest rates of Argentina and Uruguay might be seen as a failure of the model, however, it is possible to notice by looking at Table 7 that the correlations predicted by the model are in line with the observe ones for the EMBis+ of pairs of countries like Argentina-Brazil, Argentina-Mexico, Argentina-Morocco, Argentina-Nigeria, Argentina-South Africa, and Argentina-Venezuela, whose average correlation was 0.87 during the period $1994 : Q3 - 2000 : Q4$.

The model is also able to reproduce relatively well the correlations between Argentina's and Uruguay's fundamentals and the wealth of the international investors (which in this context is proxy by the US's GDP). For example, according to the data for the whole period, there is a correlation of -0.10 and -0.54 between Argentina's spread and US's GDP during the whole period and the period of the crisis respectively, and the model predicts these correlations to be -0.34 and -0.80. Also, the correlation between investors' wealth and Argentina's consumption for the whole period and for the period of crisis is 0.60 and 0.72 respectively, and the contagion model predicts these correlation to be 0.31 and 0.84. Obviously, as discussed in Lizarazo (2010), the model without contagion links cannot reproduce this behavior.

In general, the results in here suggest that the framework presented in this paper can endogenously account for the transmission of crises across emerging economies. Furthermore, the inclusion of financial links across economies improves the quantitative features of comparable models of endogenous sovereign risk[25].

7 Conclusions

This paper presents a stochastic dynamic general equilibrium model of default risk that endogenizes the role of external factors in the determination of small open economies' incentives to default, sovereign bond prices, capital flows, and default episodes.

The empirical literature on international finance presents evidence that points to a very relevant role for the fundamentals of other emerging countries in the determination of sovereign credit spreads and capital flows to emerging economies. The model in this paper is the first model that endogenously determines sovereign bond prices and at the same time endogenously accounts for contagion of crises.

The endogenization of bond prices and contagion occurs in two ways. First, the consideration of enforcement problems in sovereign debt contracts allows default risk and default incentives to be endogenized; therefore sovereign bond prices can be determined endogenously by the model. Second, the assumption of decreasing absolute risk aversion for investors allows for endogenous financial links across economies that share investors. Together, these two elements build a framework that explains the contagion of crisis. The intuition for contagion is as follows: whenever a negative shock occurs in one country, this shock increases the risk associated with that country which implies an expected future negative wealth shocks for investors. Given decreasing absolute risk aversion, investors' tolerance toward risk decreases following the wealth shock, leading to a portfolio recomposition. Investors shift away from risky investments towards less risky ones.

This explanation for contagion is consistent with the observed behavior of international investors. Investors tend to pull away from other risky countries once one risky country goes into crisis. A testable implication of this explanation would be to test the correlation between the size of investors' losses and the extent of contagion: if the explanation offered in this article is correct, then a larger shock to investors' wealth should induce a more radical portfolio recomposition away from risky investments.

Qualitatively the results of the model are consistent with the empirical evidence of contagion from Argentina to Uruguay: First, sovereign spreads and capital flows to emerging

[25]By comparable it is meant models with the same dimension for the endowment shocks and the asset position of the economies.

economies are positively correlated across economies. Second, the fundamentals of foreign emerging economies affect the determination of domestic sovereign spreads and capital flows. Third, the average financing conditions of an economy are less favorable after other countries have defaulted.

While the model presented here presents obvious theoretical contributions to the existing literature on sovereign debt and contagion, the quantitative implementation of the model faces an important hurdle: the high dimension of the state space in the numerical solution of the model. In the current model, prices respond very strongly to changes in the economy's debt level (i.e. the price function is very steep). Therefore it is necessary to have very fine grids for the asset position of the economies in order to capture a great deal of the default action. This need, if satisfied, has an explosive effect on the dimensionality of the state space of the model.[26]

[26]While not explored in this paper, a logical way to reduce the steepness of the price function would be to consider longer term maturities for sovereign bonds as modeled in Arellano and Ramanayan (2008) and Hatchondo and Martinez (2009). Bonds with longer maturities should have a less steep pricing function because investors would care not only about the next period default risk but future period risk as well. Therefore sovereign bond spreads would be smoother since the spreads might be positive even if there is no chance that the economy will default in the next period.

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Appendix

The proofs that follow in this appendix are for the case in which the periodic utility function of the investor's has a CRRA representation and the probability for any emerging economy of coming back to the credit markets after defaulting is 0. Using the argument of continuity of the prices in the investor's wealth level might be possible to extent the results to the more general case in which the probability for a defaulting economy of coming back to credit markets is positive. However, whenever this is not possible, the numerical results of the exercise in the present article confirm the previous analytical results for the general case of temporary exclusion from credit markets after a default.

In what follows is important to remember the assumption that the representative investor does not go short in the emerging economy assets (whenever the emerging economy is saving the investor receives the savings and invest them completely in T-Bills). This assumption implies that in equilibrium $\theta_j' \geq 0$ and whenever $\theta_j' > 0$ then $b_j' < 0$. Then the more negative is b_j' the more an economy j is able to borrow from the investors.

Proposition 2 *For any state of the world, S, as the risk aversion of the international investor increases, the emerging economies' incentives to default increase.*

Proof. The investor's value function can be written as

$$V^L = E\sum_{t=\tau}^{\infty}\beta^{t-\tau}v\left(X + \theta_t^{TB} - q^f\theta_{t+1}^{TB} + \sum_{j=1}^{J} d_{j,t}\left[\theta_{j,t} - q_{j,t}\theta_{j,t+1}\right]\right).$$

Considering the case in which the economy has not defaulted in the current period (otherwise the investor will not invest in this economy in this period) and assuming an interior solution for the allocation to the emerging economy j asset

$$\phi\left(\theta_j'\right) = E\left\{-q_j v_c\left(c_L\left(\theta_j'\right)\right) + \beta v_c\left(c_L'\left(\theta_j'\right)\right)d_j'\right\} = 0.$$

If the periodic utility of the international investor is of the CRRA type and $\gamma_1^L < \gamma_2^L$, then there exists a concave function $\kappa(\cdot)$ such that $v_2\left(c;\gamma_2^L\right) = \kappa\left(v_1\left(c;\gamma_2^L\right)\right)$. If $\theta_{j,1}'$ is the optimal allocation when $\gamma^L = \gamma_1^L$, and $\theta_{j,2}'$ is the optimal allocation when $\gamma^L = \gamma_2^L$ then it holds that

$$\phi_1\left(\theta_{j,1}'\right) = E\left\{-q_j v_{1,c}\left(c_L\left(\theta_{j,1}'\right)\right) + \beta v_{1,c}\left(c_L'\left(\theta_{j,1}'\right)\right)d_j'\right\} = 0.$$
$$\phi_2\left(\theta_{j,2}'\right) = E\left\{-q_j v_{2,c}\left(c_L\left(\theta_{j,2}'\right)\right) + \beta v_{2,c}\left(c_L'\left(\theta_{j,2}'\right)\right)d_j'\right\} = 0.$$

Using $v_2\left(c;\gamma_2^L\right)=\kappa\left(v_1\left(c;\gamma_2^L\right)\right)$ it is possible to define

$$\phi_2\left(\theta'_{j,1}\right)=E\kappa'\left[v_1\left(\theta'_{j,1}\right)\right]\left\{-q_j v_{1,c}\left(c_L\left(\theta'_{j,1}\right)\right)+\beta v_{1,c}\left(c'_L\left(\theta'_{j,1}\right)\right)d'_j\right\}<0.$$

The last inequality comes from the fact that $\kappa'(\cdot)$ is positive and decreasing. The inclusion of this function in the previous equation implies that $\phi_2\left(\theta'_{j,1}\right)$ is lower than $\phi_2\left(\theta'_{j,2}\right)$ because $\kappa'(\cdot)$ gives little weight to the realizations of $d'_j=1$, and high weight to the realizations of $d'_j=0$. Therefore

$$\phi_2\left(\theta'_{j,2}\right)>\phi_2\left(\theta'_{j,1}\right).$$

The concavity of $V^L(\cdot)$ implies that given q_j and the risk of default, $\phi\left(\theta'_j\right)$,(represented by the expected realizations of d'_j) is a decreasing function. As a consequence

$$\theta'_{j,2}<\theta'_{j,1}$$

which in equilibrium implies $b'_{j,2}>b'_{j,1}$.

Then for any state of the world S, taking as given q_j and the risk of default (δ_j), a higher degree of risk aversion of the investor results in this agent allocating a lower proportion of her portfolio to each of the economies' sovereign bonds. Therefore, when the investor is less risk averse there are financial contracts that are available to each emerging economy which are not available when the investor is more risk averse. Consequently given q_j and δ_j

$$V_{j,1}^C\left(S;\gamma_1^L\right)\geq V_{j,2}^C\left(S;\gamma_2^L\right)$$

Since the utility of autarky for the emerging economies does not depend on the investor's risk aversion, it is clear that if default is optimal for economy j when the state of the world is given by S and $\gamma^L=\gamma_1^L$, then for the same state of the world S, default would be optimal if $\gamma^L=\gamma_2^L$. Additionally, because incentives to default would be higher whenever $\gamma^L=\gamma_2^L$ than if $\gamma^L=\gamma_1^L$, then at equilibrium $\delta_j\left(S,b_j{}';\gamma_2^L\right)>\delta_j\left(S,b_j{}';\gamma_1^L\right)$. Therefore $q_j\left(S,b_j{}';\gamma_2^L\right)<q_j\left(S,b_j{}';\gamma_1^L\right)$. In conclusion, for all states of the world incentives to default for each emerging economy become stronger when the investor's risk aversion is larger. ∎

Proposition 3 *Default sets are shrinking in the assets of the representative investor. For all $W_1<W_2$, if default is optimal for b_j in some states y_j, given W_2 then default will be optimal for b_j for the same states y_j, given W_1 therefore* $D_j\left(b_j\mid W_2,\psi,\{s_k\}_{k=1,k\neq j}^J\right)\subseteq D_j\left(b_j\mid W_1,\psi,\{s_k\}_{k=1,k\neq j}^J\right)$

Proof. From (9) if $W_1<W_2$ then for each economy j taking as given q_j and the level of default risk (δ_j)

$$b'_{j,1}>b'_{j,2}.$$

This inequality holds because decreasing absolute risk aversion implies that

$$v\left(X+W_1-q^f\theta_{t+1}^{TB}-\sum_{j=1}^{J}D_{j,t}q_{j,t}\theta_{j,t+1}\right)$$

is a concave transformation of

$$v\left(X+W_2-q^f\theta_{t+1}^{TB}-\sum_{j=1}^{J}D_{j,t}q_{j,t}\theta_{j,t+1}\right)$$

. So if $\theta'_{j,1}$ is the optimal allocation to emerging economy j assets when $W=W_1$, and $\theta'_{j,2}$ is the optimal allocation to this economy when $W=W_2$, defining

$$\begin{aligned} v_1\left(\theta_{j,1,t+1}\right) &= v\left(X+W_1-q^f\theta_{t+1}^{TB}-\sum_{j=1}^{J}D_{j,t}q_{j,t}\theta_{j,1,t+1}\right) \\ v_2\left(\theta_{j,2,t+1}\right) &= v\left(X+W_2-q^f\theta_{t+1}^{TB}-\sum_{j=1}^{J}D_{j,t}q_{j,t}\theta_{j,2,t+1}\right) \end{aligned}$$

then

$$\begin{aligned} \phi_1\left(\theta'_{j,1}\right) &= E\left\{-q_j v_{1,c}\left(c_L\left(\theta'_{j,1}\right)\right)+\beta v_{1,c}\left(c'_L\left(\theta'_{j,1}\right)\right)d'_j\right\}=0. \\ \phi_2\left(\theta'_{j,2}\right) &= E\left\{-q_j v_{2,c}\left(c_L\left(\theta'_{j,2}\right)\right)+\beta v_{2,c}\left(c'_L\left(\theta'_{j,2}\right)\right)d'_j\right\}=0. \end{aligned}$$

And because $v_1(c)=\kappa\left(v_2(c)\right)$

$$\phi_1\left(\theta'_{j,2}\right)=E\kappa'\left[v_2\left(\theta'_{j,2}\right)\right]\left\{-q_j v_{2,c}\left(c_L\left(\theta'_{j,2}\right)\right)+\beta v_{2,c}\left(c'_L\left(\theta'_{j,2}\right)\right)d'_j\right\}<0.$$

The previous inequality comes from the fact that $\kappa'(\cdot)$ is positive and decreasing. The inclusion of this function in the previous equation implies that $\phi_1\left(\theta'_{j,2}\right)$ is lower than $\phi_1\left(\theta'_{j,1}\right)$ because $\kappa'(\cdot)$ gives little weight to the realizations of $d'_j=1$, and high weight to the realizations of $d'_j=0$. Therefore

$$\phi_1\left(\theta'_{j,2}\right)<\phi_1\left(\theta'_{j,1}\right).$$

The concavity of $V^L(\cdot)$ implies that given q_j and the risk of default, $\phi\left(\theta'_j\right)$ is a decreasing function, and as consequence

$$\theta'_{j,2}>\theta'_{j,1}$$

which in equilibrium implies $b'_{j,2}<b'_{j,1}$.

Then for any state of the world S and taking as given q_j and the risk of default (δ_j), a lower level of investor's wealth would result in this agent allocating a lower proportion of her portfolio to the economy's sovereign bonds. Therefore, when the investor is more wealthy there are financial contracts that are available to the emerging economy that are not available when the investor is less wealthy. Consequently, given q_j and δ_j,

$$V_{j,1}^{C}\left(S;W_2\right) \geq V_{j,2}^{C}\left(S;W_1\right).$$

Because the utility of default for the emerging economy does not depend on the investor's wealth, it is clear that if for some state of the world, S, default is optimal when $W = W_2$, then for the same state of the world default would be optimal when $W = W_1$. Additionally, because incentives to default would be higher whenever $W = W_1$, than when $W = W_2$ at equilibrium $\delta_j\left(s, b'; W_1\right) > \delta_j\left(s, b'; W_2\right)$, and therefore $q_j\left(s, {b_j}'; W_1\right) < q_j\left(s, {b_j}'; W_2^L\right)$. Then, unambiguously for all states of the world, the emerging economy faces stronger incentives to default when the investor is less wealthy. ■

Proposition 6 *There is a wealth channel of contagion. Proposition 3 implies that if economy k which is in the investor's portfolio defaults in her debts, incentives to default for economy j which is also in the investor's portfolio increase.*

Proof. If economy k defaults in her debts with the investor, the wealth of this agent will be $(W \mid d_k = 0) = \theta^{TB'} + \sum_{m=1, m\neq k}^{J} \theta'_m$, which is lower than the wealth for economy k if she decides not to default, which is $(W \mid d_k = 1) = \theta^{TB'} + \sum_{m=1, m\neq k}^{J} \theta'_m + \theta'_k$. Therefore

$$V_j^C\Big(y_j, b_j, \{y_m\}_{m=1}^{J}, \{b_m\}_{m=1}^{J}, (W \mid d_k{=}1)\Big) > V_j^C\Big(y_j, b_j, \{y_m\}_{m=1}^{J}, \{b_m\}_{m=1}^{J}, (W \mid d_k{=}0)\Big)$$

which implies that emerging economy $j's$ incentives to default are larger when some economy k which shares investors defaults. ■

Proposition 7 *There is a recomposition channel of contagion. Whenever the probability of default of some country k increases there is a recomposition of the investor's portfolio: the assets of emerging economies with weak fundamentals are substituted away by the assets of emerging economies with solid fundamentals or by T-Bills.*

Proof. The following is the logical argument that would have to follow a formal proof.

In impact, an increase in the probability of default of a country k reduces the expected level of investors' consumption next period through its negative impact in the expected wealth of the investors. By Proposition 3 this shock should tend to, other things equal, decrease the prices of the bonds of all economies in the investor's portfolio through its effect in the covariance term of the bond price equation, even if the default probability of any country j is taken as given.

However, as discussed in Lizarazo (2008), this initial effect on the excess risk premium modifies the default repayment decision of the emerging economies. Because of the concavity of the periodic utility of the emerging economies, the impact in the incentives to default of the initial reduction in the bond prices generated by the expected decrease in the investor's wealth is stronger for economies with weaker fundamentals (i.e, those with lower lifetime utility and therefore larger default probability).

With this in mind, it is possible to label countries as having weak fundamentals if their default probability at the moment of the increase in the probability of default of country k is such that $\delta_j(S) > \delta^*(S)$, where $\delta^*(S)$ is a critical value for the default probability that separates the group of countries that share the investor between countries which bond prices respond strongly to the initial shock in economy k and countries which bond prices do not respond too strongly to the initial shock in k. Those countries for which bond prices do not respond too strongly to the initial shock in k can be labeled as countries with strong fundamentals.

Before the increase in the probability of default of country k the investor had an optimal portfolio allocation with the optimal overall exposure of this agent to risk. Then, after the shock to country k the riskiness of the current portfolio of the investor is larger to the optimal one. Therefore the investor must recompose its portfolio moving away of country k, and those countries which have become significatively more risky than before in response to the shock in k.

If the investor moves all the resources previously allocated to economy k and economies with $\delta_j(S) > \delta^*(S)$ to riskless assets (i.e., T-Bills), she *might* end up holding a portfolio with a risk exposure lower than the optimal. If such is the case, some resources previously allocated to economy k and economies with weak fundamentals might be allocated to economies with strong fundamentals (i.e., economies with $\delta_j(S) < \delta^*(S)$).

However, as is shown in Corollary 4 if initial impact of the expected wealth loss is very strong (either because the investor is very risk averse, or because it is initially poor, or because she has a large exposure to country k) such that all countries assets respond strongly to the shock in country k, then $\delta^*(S) > \delta_j(S)$ for all $j = 1, \ldots, J$, and no country would have strong enough fundamentals that allow it to get capital inflows after the crisis in k. ■

Corollary 1 *If following a crisis in some foreign emerging economy k the overall risk of the investor's portfolio increases, then the incentives to default for all emerging economies increase.*

Proof. The investor's value function can be written as

$$V^L = E\sum_{t=\tau}^{\infty}\beta^{t-\tau}v\left(X + \theta_t^{TB} - q^f\theta_{t+1}^{TB} + \sum_{j=1}^{J}D_{j,t}\left[\theta_{j,t} - q_{j,t}\theta_{j,t+1}\right]\right).$$

Assuming an interior solution for the allocation to the emerging economy j 's asset

$$\phi\left(\theta_j'\right) = ED_j\left\{-q_jv_c\left(c_L\left(\theta_j'\right)\right) + \beta v_c\left(c_L'\left(\theta_j'\right)\right)d_j'\right\} = 0.$$

Taking the risk of economy j as given, compare this first order condition for two scenarios: one in which the risk of economy k is relatively small such that $Ed_k' = 1 - \delta_L$ versus one in which the risk of economy k is relatively high such that $Ed_k' = 1 - \delta_H$, with $\delta_H > \delta_L$. Then it holds that

$$\phi_L\left(\theta_{j,L}'\right) = E_{|L}\left[D_j\left\{-q_jv_c\left(c_L\left(\theta_{j,L}'\right)\right) + \beta v_c\left(c_L'\left(\theta_{j,L}'\right)\right)d_j'\right\}\right] = 0.$$
$$\phi_H\left(\theta_{j,H}'\right) = E_{|H}\left[D_j\left\{-q_jv_c\left(c_L\left(\theta_{j,H}'\right)\right) + \beta v_c\left(c_L'\left(\theta_{j,H}'\right)\right)d_j'\right\}\right] = 0.$$

In this case, if the increase in the risk of emerging economy k generates an increase in the overall risk of the investors portfolio, then it must hold

$$\phi_H\left(\theta_{j,L}'\right) = E_{|H}\left[D_j\left\{-q_jv_c\left(c_L\left(\theta_{j,L}'\right)\right) + \beta v_c\left(c_L'\left(\theta_{j,L}'\right)\right)d_j'\right\}\right] > 0. \quad \text{(A-1)}$$

The previous inequality holds because an increase in the overall risk of the portfolio implies

$$E_{|H}\left[v_c\left(c_L'\left(\theta_{j,L}'\right)\right)d_j'\right] = \int\left[v_c\left(c_L'\left(\theta_{j,L}'\right)\right)d_j'\right]d\left(d_j' \mid d_{k,H}'\right)$$
$$> \int\left[v_c\left(c_L'\left(\theta_{j,L}'\right)\right)d_j'\right]d\left(d_j' \mid d_{k,L}'\right) = E_{|L}\left[v_c\left(c_L'\left(\theta_{j,L}'\right)\right)d_j'\right]$$

Equation $(A-1)$ implies that

$$\phi_H\left(\theta_{j,L}'\right) > \phi_H\left(\theta_{j,H}'\right)$$

The concavity of $V^L(\cdot)$ implies that given q_j, and d_j $\phi\left(\theta'_j\right)$ is a decreasing function, and as consequence

$$\theta'_{j,H} < \theta'_{j,L}$$

which in equilibrium implies $b'_{j,H} > b'_{j,L}$.

Then for any state of the world S, taking as given q_j, and d_j, a higher overall risk in the investor's portfolios would result in the investor allocating a lower proportion of her portfolio to economy $j\prime s$ sovereign bonds. Therefore, when the overall risk of the investor's portfolio is low there are financial contracts that are available to economy j that are not available when the risk of this portfolio is larger. Consequently given q_j

$$V^C_{j,1}(S_L) \geq V^C_{j,2}(S_H)$$

Because the utility of autarky for the emerging economies does not depend on the risk of other economies different to j, it is clear that if default is optimal for economy j if the state of the world is given by S_L, then default is also optimal for economy j when the state of the world is given by S_H. This holds true for any economy j, $j = 1, 2, \ldots, J$, $j \neq k$. To summarize, the incentives to default for all emerging economies become stronger when an increase in the risk of default of economy k causes and increase in overall risk of the investor's portfolio. ■

Corollary 2 *For economies with strong fundamentals if the positive substitution effect of the crisis in some foreign economy k dominates the negative expected wealth effect of such crisis "flight to quality" occurs.*

Proof. If the overall risk of the investor's portfolio does not increase in response to the increase in the probability of default of country k, then some resources that are move away from economy k and economies with weak fundamentals would be move towards economies with strong fundamentals (i.e., those that in proposition 7 have $\delta_j(S) < \delta^*(S)$). ■