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Abstract

In this paper we investigate the Smets & Wouters (2003a) DSGE model for Brazil, through a numerical simulation based on the Dynare code (developed by Cepremap). Impulse Response functions are presented and a Bayesian estimation is also conducted from the prior distributions of the parameters.

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1 Introduction

According to Sims (2003), the Dynamic Stochastic General Equilibrium (DSGE) model presented in Smets & Wouters (2003a) represents a very substantial advance, suggesting the possibility that DSGE models, combined with Bayesian methods of inference, may before long become the standard framework for macroeconomic policy modeling. He believes that the work breaks new ground, and has appropriately been carried out and disseminated quickly, despite some gaps in the description of the methodology and uncertainties about the results that should stimulate new research.

In monetary policy strategies geared towards maintaining price stability, conditional and unconditional forecasts of inflation and output play an important role. The Smets & Wouters' (2003a) work illustrates how modern sticky-price DSGE models, estimated using Bayesian techniques, can become an additional useful tool in the forecasting kit of central banks. Juillard et alli (2005) suggest that the development of a new generation of choice-theoretic models, as well as methods for evaluating alternative policy rules have made it feasible to re-examine the results of this literature from a new perspective.

Therefore, this work's main objective is to replicate Smets & Wouters (2003a) using Brazilian data. The recent popularity obtained by small-scale monetary business cycle models with sticky prices and wages (the New Keynesian or New Neoclassical Synthesis (NNS) models) in monetary analysis justifies our effort on verifying whether these models are able to explain the main features of the Brazilian macro data. As described in Smets & Wouters (2003a), the model contains many shocks and frictions. It features sticky nominal price and wage setting that allow for backward indexation, habit formation in consumption and investment adjustment costs, variable capital utilisation, fixed costs in production, among others. The stochastic dynamics is driven by ten orthogonal structural shocks. They are shocks on: productivity, labour supply, investment, preference, government spending, cost-push (given by the price and wage mark-up shocks and the equity premium shock) and monetary policy (given by the interest rate and the inflation objective shocks).

This study is organized as follows: Section 2 describes the theoretical model and the respective linearised version to be computed by Dynare program, Section 3 presents the parameters and its priors distributions. Section 4 shows the empirical results (impulse response functions and posterior distributions), and Section 5 summarizes the main conclusions.

2 Methodology

2.1 The DSGE model

The DSGE model is based on the work of Christiano, Eichenbaum and Evans (CEE, 2001) and Smets and Wouters (2003a). It contains many frictions and shocks that affect both nominal and real decisions. Some of them are already traditional on the literature, while others are motivated by recent research on the determinants of consumption, asset prices, investment and producivity. As examples of the latter, we have the inclusion of habit persistence in preferences for consumption, adjustment costs in investment and variable capital utiliz ation.

Let's start the model description by the household sector. There is a continuum of households indicated by the index τ. Each of them maximise a non-separable utility function with two arguments (consumption goods, C_t^τ, and labour effort, l_t^τ) over an infinite life horizon. The instantaneous utility function of each household is given by:

$$U_t^\tau = \left(\frac{1}{1-\sigma_c} \left(C_t^\tau - H_t\right)^{1-\sigma_c} + \varepsilon_t^L \right) \cdot \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} \left(l_t^\tau\right)^{1+\sigma_l} \right) \tag{2.1}$$

where σ_c is the coefficient of relative risk aversion of households (or the inverse of the intertemporal elasticity of substitution) and σ_l represents the inverse of the elasticity of work effort with respect to the real wage. We also consider the existence of a time-varying external habit variable, denoted by H_t. We'll assume that the external habit stock is proportional to aggregate past consumption:

$$H_t = hC_{t-1} \tag{2.2}$$

Observe that equation (2.1) also contains a preference shock, ε_t^L, which represents a shock to the labour supply. This shock is assumed to follow a first-order autoregressive process - AR(1) - with an IID-Normal error term: $\varepsilon_t^L = \rho_L \varepsilon_{t-1}^L + \eta_t^L$.

As for the labour effort, l_t^τ, each household is assumed to supply a complete set of differentiated labour types. Note that, in principle, households could work different amounts and earn different wage rates. Therefore, they could be heterogeneous with respect to consumption and asset holdings. However, a straightforward extension of arguments in Erceg, Henderson and Levin (2000) and Woodford (1996) establish that the existence of state contingent securities ensures that in equilibrium households are homogeneous with respect to consumption and asset holdings. We consider that each household has a monopoly power over the supply of its labour and will set the optimal wage accordingly.

The assumption that households act as price-setters in the labour market will result in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo. We assume that wages can only be optimally adjusted after some random "wage-change signal" is received. The probability that a particular household can change its nominal wage in period t is constant and equal to $1-\xi_w$. A household τ that receives such a signal in period t will set the new nominal wage, $\widetilde{w}_t^\tau$, taking into account the probability that it will not be re-optimized in a near future. Formally, the probability that it won't be able to reoptimize the wage after l periods is $(\xi_w)^l$. For the households that were unable to reoptimize, we consider the following adjustment for their wages:

$$W_t^\tau = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} (\overline{\pi}_t)^{1-\gamma_w} W_{t-1}^\tau \tag{2.3}$$

where γ_w is the degree of wage indexation to past inflation and $\overline{\pi}_t$ is the inflation objective of the central bank.

At last, we introduce a second preference shock, ε_t^b, affecting the dicount rate β. As we did with the shock to the labour supply, ε_t^L, this shock will also follow an AR(1) process with an IID-Normal error term: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$. Therefore, each household τ will maximize the following intertemporal utility function:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t \varepsilon_t^b U_t^\tau\right] = E_0\left[\sum_{t=0}^{\infty}\beta^t \varepsilon_t^b \left(\frac{1}{1-\sigma_c}\left(C_t^\tau - H_t\right)^{1-\sigma_c} + \varepsilon_t^L\right)\cdot \exp\left(\frac{\sigma_c - 1}{1+\sigma_l}\left(l_t^\tau\right)^{1+\sigma_l}\right)\right] \tag{2.4}$$

On its maximization problem, each household has to consider three constraints: the intertemporal budget constraint, the demand for labour and the capital acumulation equation. The first one is given by:

$$b_t \frac{B_t^\tau}{P_t} = \frac{B_{t-1}^\tau}{P_t} + Y_t^\tau - C_t^\tau - I_t^\tau \tag{2.5}$$

where b_t is the price of a bond (a one-period security) B_t. Equation (2.5) states that current income (Y_t^τ) and financial wealth $\left(\frac{B_{t-1}^\tau}{P_t} - b_t \frac{B_t^\tau}{P_t}\right)$ can be used for consumption (C_t^τ) and investment in physical capital (I_t^τ). In addition, household's total income is given by:

$$Y_t^\tau = W_t^\tau l_t^\tau + \left(r_t^k z_t^\tau K_{t-1}^\tau - \Psi\left(z_t^\tau\right) K_{t-1}^\tau\right) + Div_t^\tau \tag{2.6}$$

where z_t^τ is the capital utilisation rate and $\Psi\left(z_t^\tau\right)$ is the cost associated with it. That is, total income consists of three components: labour income ($w_t^\tau l_t^\tau$), the return on the real capital stock ($r_t^k z_t^\tau K_{t-1}^\tau$)

minus the cost associated with variations in the degree of capital utilisation ($\Psi(z_t^\tau) K_{t-1}^\tau$) and the dividends derived from the imperfect competitive intermediate firms (Div_t^τ). As in CEE (2001), it is assumed that the cost of capital utilisation is zero when capital utilisation is one ($\Psi(1) = 0$).

The demand for labour is given by[1]:

$$l_t^\tau = \left(\frac{W_t^\tau}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{2.7}$$

where aggregate labour demand, L_t, and the aggregate nominal wage, W_t, are given by the following Dixit-Stiglitz type aggregator functions[2]:

$$L_t = \left[\int_0^1 (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}} d\tau\right]^{1+\lambda_{w,t}} \tag{2.8}$$

$$W_t = \left[\int_0^1 (W_t^\tau)^{-\frac{1}{\lambda_{w,t}}} d\tau\right]^{-\lambda_{w,t}} \tag{2.9}$$

Furthermore, we will assume the following process for the parameter $\lambda_{w,t}$: $\lambda_{w,t} = \lambda_w + \eta_t^w$, where η_t^w is an IID-Normal shock. We will refer to this shock as the wage mark-up shock. Given equation (2.9), the law of motion of the aggregate wage index is given by:

$$(W_t)^{-1/\lambda_{w,t}} = \xi_w \left(\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} (\overline{\pi}_t)^{1-\gamma_w} W_{t-1}\right)^{-1/\lambda_{w,t}} + (1-\xi_w)(\widetilde{w}_t)^{-1/\lambda_{w,t}} \tag{2.10}$$

The third and last constraint is the capital accumulation equation. Households own the capital stock, a homogeneous factor of production, which they rent out to the firm-producers of intermediate goods at a given rental rate r_t^k. They can increase the supply of rental services from capital by two different ways: (1) investing in additional capital (I_t), which takes one period to be installed; or (2) changing the utilisation rate of already installed capital (z_t). On either ways, there are costs in terms of forgone consumption. The capital accumulation equation is given by:

[1] Equation (2.7) is derived from the problem of minimizing the costs with labour by the firm subject to equation (2.8). Formally,

$$\underset{l_t^\tau}{Min} \int_0^1 W_t^\tau l_t^\tau d\tau$$

$$s.t.\ \ L_t = \left[\int_0^1 (l_t^\tau)^{\frac{1}{1+\lambda_{w,t}}} d\tau\right]^{1+\lambda_{w,t}}$$

[2] W_t is the lowest cost of one unit of aggregate labour L_t

$$K_{t+1} = K_t(1-\tau) + \left[1 + \varepsilon_t^I - S\left(I_t/I_{t-1}\right)\right] I_t \tag{2.11}$$

where τ is the depreciation rate and the adjustment cost function $S(.)$ is a positive function of changes in investment. We'll assume that $S(.)$ equals zero in steady state with a constant investment level. We also introduce a shock to the relative efficiency of investment goods. This shock is equivalent to a shock in the relative price of investment versus consumption goods and takes up the investment specific technological shocks. The shock is assumed to follow an AR(1) process with an IID-Normal error term: $\varepsilon_t^I = \rho_I \varepsilon_{t-1}^I + \eta_t^I$. It is worth making one final comment about the variable z_t. One implication of variable capital utilisation is that it reduces the impact of changes in output on the rental rate of capital and, therefore, smoothes the response of marginal cost to fluctuations in output.

In summary, each household τ maximizes the intertemporal utility function given by (2.4) subject to the intertemporal budget constraint (2.5), the demand for labour (2.7) and the capital accumulation equation (2.11). Formally,

$$\begin{aligned}
&\underset{\{C_t^\tau, B_t^\tau, K_t, z_t^\tau\}_{t=0}^{\infty},\ \widetilde{w}_0}{Max} \quad E_0\left[\sum_{t=0}^{\infty} \beta^t \varepsilon_t^b \left(\tfrac{1}{1-\sigma_c}\left(C_t^\tau - H_t\right)^{1-\sigma_c} + \varepsilon_t^L\right) \cdot \exp\left(\tfrac{\sigma_c - 1}{1+\sigma_l}\left(l_t^\tau\right)^{1+\sigma_l}\right)\right] \\
&s.t. \quad b_t \frac{B_t^\tau}{P_t} = \frac{B_{t-1}^\tau}{P_t} + W_t^\tau l_t^\tau + \left(r_t^k z_t^\tau K_{t-1}^\tau - \Psi\left(z_t^\tau\right) K_{t-1}^\tau\right) + Div_t^\tau - C_t^\tau - I_t^\tau, \ \ \forall t \geq 0 \\
&\qquad l_t^\tau = \left(\frac{W_t^\tau}{W_t}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t, \ \ \forall t \geq 0 \\
&\qquad K_{t+1} = K_t(1-\tau) + \left[1 + \varepsilon_t^I - S\left(I_t/I_{t-1}\right)\right] I_t, \ \ \forall t \geq 0
\end{aligned}$$

where $\varepsilon_t^L = \rho_L \varepsilon_{t-1}^L + \eta_t^L$, $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, $\lambda_{w,t} = \lambda_w + \eta_t^w$ and $H_t = hC_{t-1}$. Moreover, note that when the household decides optimally on his wage, $\widetilde{w}_t$, he does so taking into account that he will be unable to re-optimize it after l periods with probability $(\xi_w)^l$. Remember that, in this case, the wage will be adjusted according to equation (2.3). Solving this equation recursively, we have:

$$\begin{aligned}
W_t^\tau &= \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} (\overline{\pi}_t)^{1-\gamma_w} W_{t-1}^\tau \\
&= \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} (\overline{\pi}_t)^{1-\gamma_w} \left(\frac{P_{t-2}}{P_{t-3}}\right)^{\gamma_w} (\overline{\pi}_{t-1})^{1-\gamma_w} W_{t-2}^\tau \\
&= \left(\frac{P_{t-1}}{P_{t-3}}\right)^{\gamma_w} (\overline{\pi}_t)^{1-\gamma_w} (\overline{\pi}_{t-1})^{1-\gamma_w} W_{t-2}^\tau \\
&= \left(\frac{P_{t-1}}{P_{t-3}}\right)^{\gamma_w} \prod_{s=1}^{s=t} (\overline{\pi}_s)^{1-\gamma_w} \widetilde{w}_0
\end{aligned}$$

The first-order conditions of the maximisation problem above results on the following equations:

$$E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{R_tP_t}{P_{t+1}}\right]=1 \tag{2.12}$$

where R_t is the gross nominal rate of return of bonds ($R_t = 1 + i_t = 1/b_t$) and λ_t is the marginal utility of consumption, that is given by: $\lambda_t = \varepsilon_t^b\left(C_t - H_t\right)^{-\sigma_c}\exp\left(\frac{\sigma_c-1}{1+\sigma_l}\left(l_t^\tau\right)^{1+\sigma_l}\right)$,

$$\frac{\widetilde{w}_t}{P_t}E_t\left[\sum_{i=0}^{\infty}\beta^i\xi_w^i\left(\frac{P_t}{P_{t+i}}\right)\left(\frac{P_{t+i-1}}{P_{t-1}}\right)^{\gamma_w}\left(\overline{\pi}_t\right)^{1-\gamma_w}\frac{l_{t+i}^\tau U_{t+i}^C}{1+\lambda_{w,t+i}}\right]=E_t\left[\sum_{i=0}^{\infty}\beta^i\xi_w^i l_{t+i}^\tau U_{t+i}^l\right] \tag{2.13}$$

where U_{t+i}^C is the marginal utility of consumption and U_{t+i}^l is the marginal disutility of labour,

$$Q_t = E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\left(Q_{t+1}\left(1-\tau\right)+r_{t+1}^k z_{t+1}-\Psi z_{t+1}\right)\right] \tag{2.14}$$

where Q_t is the Lagrange multiplier of the capital accumulation equation (and, therefore, it is the shadow-price of one unit of capital),

$$Q_tS'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}-\beta E_t\left[Q_{t+1}\frac{\lambda_{t+1}}{\lambda_t}S'\left(\frac{I_{t+1}}{I_t}\right)\frac{I_{t+1}}{I_t}\frac{I_{t+1}}{I_t}\right]+1=Q_t(1+\varepsilon_t^I) \tag{2.15}$$

$$r_t^k = \Psi'\left(z_t\right) \tag{2.16}$$

Equations (2.12) to (2.16) describes the optimal paths of, respectively, the consumption, the wage, the real value of capital, the investment and the rate of capital utilisation. Before moving on to the description of the firms' sector, it is worth making a final observation on the wage's equation (2.13). It shows that the nominal wage at time t of a household τ that is allowed to change its wage is set so that the present value of the marginal return to working is a mark-up over the present value of marginal cost (the subjective cost of working). When wages are perfectly flexible ($\xi_w = 0$), the real wage will be a mark-up (equal to $1 + \lambda_{w,t}$) over the current ratio of the marginal disutility of labour and the marginal utility of an additional unit of consumption.

After describing the household sector, we can proceed by describing the firms. In this economy, there is production of a single final good and a continuum of intermediate goods indexed by j, where $j \in [0, 1]$. The final-good sector is perfectly competitive, whereas the intermediate-good sector is characterized by monopolistic competition (each intermediate good is produced by a single firm).

The final good is produced using the intermediate goods in the following technology:

$$Y_t = \left[\int_0^1 \left(y_t^j \right)^{\frac{1}{1+\lambda_{p,t}}} dj \right]^{1+\lambda_{p,t}} \tag{2.17}$$

where y_t^j denotes the quantity of domestic intermediate good of type j that is used in final good's production, at date t. $\lambda_{p,t}$ is a stochastic parameter which determines the time-varying mark-up in the goods market. Shocks to this parameter will be interpreted as a "cost-push" shock to the inflation equation. We assume that $\lambda_{p,t} = \lambda_p + \eta_t^p$, where η_t^p is IID-Normal.

From the cost minimisations first order conditions, we can derive the following expression for the production of intermediate good j[3]:

$$y_t^j = \left(\frac{p_t^j}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t \tag{2.18}$$

where p_t^j is the price of the intermediate good j and P_t is the price of the final good. The latter is given by:

$$P_t = \left[\int_0^1 \left(p_t^j \right)^{-\frac{1}{\lambda_{p,t}}} dj \right]^{-\lambda_{p,t}} \tag{2.19}$$

The intermediate good, on the other hand, is produced by firm j using the following technology:

$$y_t^j = \left[\varepsilon_t^a \left(\widetilde{K}_{j,t} \right)^{\alpha} (L_{j,t})^{1-\alpha} \right] - \Phi \tag{2.20}$$

where ε_t^a is the productivity shock (assumed to follow an AR(1) process: $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$), $\widetilde{K}_{j,t}$ is the effective utilisation of the capital stock (given by $\widetilde{K}_{j,t} = z_t K_{j,t-1}$), $L_{j,t}$ is an index of different types of labour used by the firm given by equation (2.8) and Φ is a fixed cost.

Cost minimisation implies that:

$$\frac{W_t L_{j,t}}{r_t^k \widetilde{K}_{j,t}} = \frac{1-\alpha}{\alpha} \tag{2.21}$$

Equation (2.21) implies that the capital-labour ratio will be identical across intermediate goods producers and equal to the aggregate capital-labour ratio. The firms' marginal costs are given by:

[3] The derivation follows the same principle described in footnote 2.1.

$$MC_t = \frac{W_t^{1-\alpha}\left(r_t^k\right)^{\alpha}\left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right)}{\varepsilon_t^a} \tag{2.22}$$

This implies that the marginal cost, too, is independent of the intermediate good produced.

Nominal profits of firm j are then given by:

$$\pi_t^j = \left(p_t^j - MC_t\right)\left(\frac{p_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t - MC_t\Phi \tag{2.23}$$

Each firm j has market power in the market of its own good and maximises expected profits using a discount rate $(\beta\rho_t)$ which is consistent with the pricing kernel for nominal returns used by the shareholders-households: $\rho_{t+k} = \frac{\lambda_{t+k}}{\lambda_t}\frac{1}{P_{t+k}}$.

As in Calvo (1983), firms are not allowed to change their prices unless they receive a random "price-change signal". The probability that a given price can be re-optimised in any particular period is constant and equal to $1-\xi_p$. Following CEE (2001), prices of firms that do not receive a price signal are indexed to the weighted sum of last period's inflation rate and the inflation objective of monetary policy:

$$P_t = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p}(\overline{\pi}_t)^{1-\gamma_p} P_{t-1} \tag{2.24}$$

As observed in CEE (2001), the partial indexation on last period's inflation rate, rather than on the present inflation rate, is motivated by the fact that the latter was unable to generate sufficient inertia in inflation (see also Fuhrer and Moore (1995) and Gali and Gertler (1999)).

The Calvo price-setting mechanism can be interpreted as capturing firm's response to various costs of changing prices. The basic idea is that in the presence of these costs firms fully optimize prices only periodically, and follow simple rules for changing their prices at other times. The type of costs that we are referring to are those associated with optimization (some examples are: costs associated with information gathering, decision making, negotiation and communication). These costs are different from menu costs, which apply to all price changes. Zbaracki, Ritson, Levy, Dutta and Bergen (2000) provide some microeconomic evidence that costs associated with reoptimization are much more important than menu costs.

At last, equations (2.19) and (2.24) implies the following law of motion for the price index:

$$(P_t)^{-1/\lambda_{p,t}} = \xi_p \left(\left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} (\overline{\pi}_t)^{1-\gamma_p} P_{t-1} \right)^{-1/\lambda_{p,t}} + (1-\xi_p) \left(\widetilde{p}_t^j \right)^{-1/\lambda_{p,t}} \tag{2.25}$$

Finally, the profit maximisation by producers that are "allowed" to re-optimise their prices at time t results in the following first-order condition:

$$E_t \left[\sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} y_{t+i}^j \left(\frac{\widetilde{p}_t^j}{P_t} \left(\frac{P_t}{P_{t+i}} \right) \left(\frac{P_{t-1+i}}{P_{t-1}} \right)^{\gamma_p} (\overline{\pi}_t)^{1-\gamma_p} - (1+\lambda_{p,t+i}) MC_{t+i} \right) \right] = 0 \tag{2.26}$$

Equation (2.26) shows that the price set by firm j, at time t, is a function of expected future marginal costs. The price will be a mark-up over these weighted marginal costs. If prices are perfectly flexible ($\xi_p = 0$), the mark-up in period t is equal to $1 + \lambda_{p,t}$.

2.2 The linearised DSGE model

In this section we briefly describe the log-linearised DSGE model adopted in this paper. The ^ symbol above a variable denotes log deviations from steady state. The dynamics of aggregate consumption is given by:

$$\widehat{C}_t = \frac{h}{1+h} \widehat{C}_{t-1} + \frac{1}{1+h} E_t \widehat{C}_{t+1} + \frac{(\sigma_c - 1)}{\sigma_c (1+\lambda_w)(1+h)} \left(\widehat{l}_t - \widehat{l}_{t+1} \right)$$

$$- \frac{1-h}{(1+h)\sigma_c} \left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right) + \frac{1-h}{(1+h)\sigma_c} \widehat{\varepsilon}_t^b \tag{2.27}$$

Consumption $\widehat{C}_t$ depends on the ex-ante real interest rate $\left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right)$ and, with external habit formation, on a weighted average of past and expected future consumption. When $h = 0$, only the traditional forward-looking term is maintained. In addition, due to the non-separability of the utility function, consumption will also depend on expected employment growth $\left(\widehat{l}_t - \widehat{l}_{t+1} \right)$. When the elasticity of intertemporal substitution (for constant labour) is smaller than one ($\sigma_c > 1$), consumption and labour supply are complementary. Finally, remind that $\widehat{\varepsilon}_t^b$ represents a preference shock affecting the discount rate and is assumed to follow an AR(1) process with an IID-Normal error term.

The investment equation is given by:

$$\widehat{I}_t = \frac{1}{1+\beta} \widehat{I}_{t-1} + \frac{\beta}{1+\beta} E_t \widehat{I}_{t+1} + \frac{1/\varphi}{1+\beta} \left(\widehat{Q}_t + \widehat{\varepsilon}_t^I \right) \tag{2.28}$$

where φ depends on the adjustment cost function and β is the discount factor applied by the households. As discussed in CEE (2001), modelling the capital adjustment costs as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to various shocks including monetary policy shocks. A positive shock to the investment-specific technology, $\widehat{\varepsilon}_t^I$, increases investment in the same way as an increase in the value of the existing capital stock $\widehat{Q}_t$.

The corresponding Q equation is given by:

$$\widehat{Q}_t = -\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) + \frac{1-\tau}{1-\tau+\overline{r}^k}E_t\widehat{Q}_{t+1} + \frac{\overline{r}^k}{1-\tau+\overline{r}^k}E_t\widehat{r}_{t+1}^k + \eta_t^Q \tag{2.29}$$

where τ stands for the depreciation rate and $\overline{r}^k$ for the rental rate of capital so that $\beta = 1/(1-\tau+\overline{r}^k)$. The current value of the capital stock depends negatively on the ex-ante real interest rate, and positively on its expected future value and the expected rental rate. The introduction of a shock to the required rate of return on equity investment, η_t^Q, is meant as a shortcut to capture changes in the cost of capital that may be due to stochastic variations in the external finance premium. We assume that this equity premium shock follows an IID-Normal process. In a fully-fledged model, the production of capital goods and the associated investment process could be modelled in a separate sector. In such a case, imperfect information between the capital producing borrowers and the financial intermediaries could give rise to a stochastic external finance premium. For example, in Bernanke, Gertler and Gilchrist (1998), the deviation from the perfect capital market assumptions generates deviations between the return on financial assets and equity that are related to the net worth position of the firms in their model. Here, we implicitly assume that the deviation between the two returns can be captured by a stochastic shock, whereas the steady-state distortion due to such informational frictions is zero.

The capital accumulation equation becomes a function not only of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology shock:

$$\widehat{K}_t = (1-\tau)\,\widehat{K}_{t-1} + \tau\widehat{I}_{t-1} + \tau\widehat{\varepsilon}_{t-1}^I \tag{2.30}$$

With partial indexation to lagged inflation, the inflation equation becomes a more general specification of the standard New-Keynesian Phillips curve:

$$\widehat{\pi}_t - \overline{\pi}_t = \frac{\beta}{1+\beta\gamma_p}\left(E_t\widehat{\pi}_{t+1} - \overline{\pi}_t\right) + \frac{\gamma_p}{1+\beta\gamma_p}\left(\widehat{\pi}_{t-1} - \overline{\pi}_t\right) +$$

$$+\frac{1}{(1+\beta\gamma_p)}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\left[\alpha\widehat{r}_t^k+(1-\alpha)\widehat{w}_t-\widehat{\varepsilon}_t^a\right]+\eta_t^P \tag{2.31}$$

The deviation of inflation $\widehat{\pi}_t$ from the target inflation rate $\overline{\pi}_t$ depends on past and expected future inflation deviations and on the current marginal cost, which itself is a function of the rental rate on capital, the real wage $\widehat{w}_t$ and the productivity process, that is composed of a deterministic trend in labour efficiency γ and a stochastic component $\widehat{\varepsilon}_t^a$. When the degree of indexation to past inflation is zero ($\gamma_p = 0$), this equation reverts to the standard purely forward-looking Phillips curve. By assuming that all prices are still indexed to the inflation objective in that case, this Phillips curve will be vertical. Announcements of changes in the inflation objective will be completely neutral even in the short run. This is based on the strong assumptions that indexation habits will adjust immediately to the new inflation objective. With ($\gamma_p > 0$), the degree of indexation to lagged inflation determines how backward looking the inflation process is or, in other words, how much exogenous persistence there is in the inflation process. The elasticity of inflation with respect to changes in the marginal cost depends mainly on the degree of price stickiness. When all prices are flexible ($\xi_p = 0$) and the price-mark-up shock is zero, this equation reduces to the normal condition that in a flexible price economy the real marginal cost should equal one.

Similarly, the indexation of nominal wages results in the following real wage equation:

$$\widehat{w}_t=\frac{\beta}{1+\beta}E_t\widehat{w}_{t+1}+\frac{1}{1+\beta}\widehat{w}_{t-1}+\frac{\beta}{1+\beta}\left(E_t\widehat{\pi}_{t+1}-\overline{\pi}_t\right)-\frac{1+\beta\gamma_w}{1+\beta}\left(\widehat{\pi}_t-\overline{\pi}_t\right)+\frac{\gamma_w}{1+\beta}\left(\widehat{\pi}_{t-1}-\overline{\pi}_t\right)$$
$$-\frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w}\right)\xi_w}\left[\widehat{w}_t-\sigma_L\widehat{L}_t-\frac{1}{1-h}\left(\widehat{C}_t-h\widehat{C}_{t-1}\right)+\widehat{\varepsilon}_t^L\right]+\eta_t^w \tag{2.32}$$

The real wage $\widehat{w}_t$ is a function of expected and past real wages and the expected, current and past inflation rate where the relative weight depends on the degree of indexation γ_w to lagged inflation of the non-optimised wages. When $\gamma_w = 0$, real wages do not depend on the lagged inflation rate. There is a negative effect of the deviation of the actual real wage from the wage that would prevail in a flexible labour market. The size of this effect will be greater, the smaller the degree of wage stickiness ξ_w, the lower the demand elasticity for labour (higher mark-up λ_w) and the lower the inverse elasticity of labour supply (σ_L) or the flatter the labour supply curve.

The equalisation of marginal cost implies that, for a given installed capital stock, labour demand depends negatively on the real wage (with a unit elasticity) and positively on the rental rate of capital:

$$\widehat{L}_t=-\widehat{w}_t+(1+\Psi)\widehat{r}_t^k+\widehat{K}_{t-1} \tag{2.33}$$

where Ψ is the inverse of the elasticity of the capital utilisation cost function.

The goods market equilibrium condition can be written as:

$$\begin{aligned}\widehat{Y}_t &= (1-\tau k_y - g_y)\,\hat{C}_t + \tau k_y \hat{I}_t + \varepsilon_t^G \\ &= \phi\widehat{\varepsilon}_t^a + \phi\alpha\widehat{K}_{t-1} + \phi\alpha\Psi\widehat{r}_t^k + \phi\,(1-\alpha)\left(\widehat{L}_t + \gamma t\right)\end{aligned} \tag{2.34}$$

where k_y is the steady state capital-output ratio, g_y is the steady-state government spending-output ratio and ϕ is one plus the share of the fixed cost in production. We assume that the government spending shock follows a first-order autoregressive process with an IID-Normal error term: $\widehat{\varepsilon}_t^G = \rho^G\widehat{\varepsilon}_{t-1}^G + \eta_t^G$.

Finally, the model is closed by adding the following empirical monetary policy reaction function:

$$\widehat{R}_t = \overline{\pi}_t + \rho\left(\widehat{R}_{t-1} - \overline{\pi}_{t-1}\right) + (1-\rho)\left\{r_\pi\left(\widehat{\pi}_{t-1} - \overline{\pi}_{t-1}\right) + r_Y\left(\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^p\right)\right\} +$$

$$+r_{\Delta\pi}\left[\left(\widehat{\pi}_t - \overline{\pi}_t\right) - \left(\widehat{\pi}_{t-1} - \overline{\pi}_{t-1}\right)\right] + r_{\Delta Y}\left[\left(\widehat{Y}_t - \widehat{Y}_t^p\right) - \left(\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^p\right)\right] + \eta_t^R \tag{2.35}$$

The monetary authorities follow a generalised Taylor rule by gradually responding to deviations of lagged inflation from an inflation objective and the lagged output gap defined as the difference between actual and potential output. Consistently with the DSGE model, potential output $\widehat{Y}_t^p$ is defined as the level of output that would prevail under flexible price and wages in the absence of the three "cost-push" shocks (η_t^w , η_t^P , η_t^Q).

In practical terms, we expand the model consisting of equations (2.27) to (2.35) with a flexible-price-and-wage version in order to calculate the model-consistent output gap.The parameter ρ captures the degree of interest rate smoothing. In addition, there is also a short-run feedback from the current changes in inflation and the output gap. Finally, we assume that there are two monetary policy shocks: one is a temporary IID-Normal interest rate shock (η_t^R) also denoted a monetary policy shock; the other is a permanent shock to the inflation objective ($\overline{\pi}_t$) which is assumed to follow a non-stationary process $\overline{\pi}_t = \overline{\pi}_{t-1} + \eta_t^\pi$.

The dynamic specification of the reaction function is such that changes in the inflation objective are immediately and without cost reflected in actual inflation and the interest rate if there is no exogenous persistence in the inflation process. In the empirical exercise, we assume that this policy rule together with the process for the stochastic shocks is able to describe the behaviour of monetary authorities over the sample period. Especially for the euro area this is a strong assumption because

there was no unified monetary policy during most of the period under investigation. But even for the US, the hypothesis of a stable monetary policy rule over the sample period is frequently rejected in the literature and should be tested empirically. However, the presence of two types of monetary policy shocks distinguishes our exercise from many other studies on this topic. Equations (2.27) to (2.35) determine the nine endogenous variables: $\widehat{\pi}_t$, $\widehat{w}_t$, $\widehat{K}_{t-1}$, $\widehat{Q}_t$, $\widehat{I}_t$, $\widehat{C}_t$, $\widehat{R}_t$, $\widehat{r}_t^k$, $\widehat{L}_t$. of our model. The stochastic behaviour of the system of linear rational expectations equations is driven by ten exogenous shock variables: five shocks arising from technology and preferences ($\widehat{\varepsilon}_t^a$, $\widehat{\varepsilon}_t^b$, $\widehat{\varepsilon}_t^I$, $\widehat{\varepsilon}_t^L$, $\widehat{\varepsilon}_t^G$) which are assumed to follow an AR(1) process, three "cost-push" shocks (η_t^w , η_t^P , η_t^Q) which are assumed to follow a white-noise process and two monetary policy shocks ($\overline{\pi}_t$, η_t^R).

3 Parameters and Priors

We estimate equations (2.27) to (2.35) for Brazil using seven key macro-economic time series: output, consumption, investment, hours worked, real wages, prices and a short-term interest rate. The Bayesian estimation methodology is extensively discussed in Smets and Wouters (2002).

The Bayesian estimation methodology requires the specification of prior distributions over all the structural parameters of the log-linear DSGE model. Tables 2-6 (in appendix) present those prior distributions. For comparison with the results on the posterior distribution we report the mean and the standard error of the prior distribution.

According to Smets & Wouters (2004), the Bayesian estimation methodology contains five steps. In step 1 the linear rational expectations model is solved resulting in a state equation in the predetermined state variables. In step 2 the model is written in state space form by adding a measurement equation linking the seven observable variables to the vector of state variables. In step 3 the likelihood function is derived using the Kalman filter. Step 4 involves combining this likelihood function with a prior distribution over the parameters to form the posterior density function. The final step consists of numerically deriving the posterior distribution of the parameters using a Monte Carlo Markov Chain (MCMC) algorithm. The specific MCMC method used is the Metropolis-Hastings algorithm.

The persistence parameters of the AR(1) processes are assumed to follow a Beta distribution with mean of 0.70 and standard deviation of 0.2. The three mark-up shocks to prices, wages and equity prices are assumed to be IID white noise processes. All the innovations from the shocks are assumed to follow a Normal distribution with mean 0 and standard error of 2.0.

Five parameters are restricted to a point value prior to the estimation process. The discount rate β is set at 0.99, the depreciation rate τ is set at 0.05 (both on a quarterly basis). The value for τ was chosen according to Gomes, Pessôa and Veloso (2003). For the US, the capital income share in the Cobb Douglas production function α was fixed at 0.24 in Smets & Wouters (2003a). For the euro area, the production parameter was fixed at 0.3 in Smets & Wouters (2003b). For Brazil, we set $\alpha = 0.40$ based on Gomes, Pessôa and Veloso (2003). The rental rate of capital parameter $\overline{r}^k$ is obtained from $\beta = 1/\left(1 - \tau + \overline{r}^k\right) = 0.0601$.

The parameters describing the monetary policy rule are based on a standard Taylor rule: the long run reaction coefficient to inflation and the output-gap are described by a Normal distribution

with mean 1.5 and 0.3 and standard errors 0.3 for both of them. The persistence of the policy rule, determined by the coefficient on the lagged interest rate, is assumed to be Beta distributed around 0.7 with a standard error of 0.2. The prior on the short run reaction coefficients to inflation and output-gap changes reflect the assumptions of a gradual adjustment towards the long run. Their means are fixed at, respectively, 0.3 and 0.1 (with standard error of 0.3 for both).

The parameters of the utility function are distributed as follows. The prior on the intertemporal substitution elasticity is set at 1 (with a standard error 0.4), the habit parameter is assumed to fluctuate around 0.7 (with a standard error of 0.2) and the wage elasticity of labour supply is assumed to be around 2 (with a standard error of 0.75). The prior on the adjustment cost parameter for investment is set around 4 with standard error 1.5 and the capacity utilisation elasticity is set at 0.2 (standard error 0.1). The share of fixed costs in the production function is assumed to be distributed around 0.25 (the corresponding parameter is defined as 1.25). All these priors are described by a Normal distribution, with the exception of the habit persistence parameter, which follows a Beta distribution because it is restricted between 0 and 1.

Finally there are the parameters describing the price and wage setting. The Calvo probability is assumed to be around 0.7 for both prices and wages, corresponding to, aproximately, one-year average contract length. The degree of indexation on past inflation is set at 0.7, which corresponds to a significant coefficient (0.43) on the lagged inflation terms in the linearised inflation and wage equations.

4 Empirical Results

4.1 The Database

The linearised system of equations is estimated using a Bayesian methodology based on seven key macro-economic time series: output, consumption, investment, hours worked, real wages, prices and a short-term interest rate. All data are in quarter basis and are obtained from the Central Bank of Brazil (BCB), Institute of Applied Economic Research (IPEA), and Brazilian Institute of Geography and Statistics (IBGE). Our sample covers the period 1992.III to 2005.II (52 observations). The GDP, consumption and investment series are expressed in millions of 2005 Reais (deflated with IPCA). The inflation rate is measured in a standard approach by the IPCA price index, and the nominal interest rate is measured by the Over/Selic rate (equivalent to the U.S. Fed funds rate). Hours worked and real wages come from CNI (Confederação Nacional da Indústria) and were obtained from the IPEA website. The aggregate real variables are expressed per capita and all series are seasonally adjusted and demeaned. Consumption, investment, GDP, wages and hours/employment are expressed in 100xlog.

4.2 Dynare Code

To estimate the DSGE model using Bayesian techniques, we used the Dynare program. Dynare is a program for the simulation of rational expectation models. It is the result of research carried out at Cepremap (France) and is a collection of Matlab, Scilab or Gauss routines which solve, simulate and estimate non-linear models with forward looking variables. Dynare is a user oriented general program for the simulation of deterministic or stochastic models. For linear models, it implements a generalized Schur decomposition algorithm; for deterministic non–linear models, a Newton–type method and for stochastic non–linear models, a second order Taylor approximation algorithm.

The user writes the model and the computational tasks to be accomplished in a usual modelling language and in a text file. A parser then translates it in a Matlab program. In a deterministic setup, it can compute the anticipatory reaction and the inertial response of a model in the presence of one or several fully anticipated shocks, the response to unanticipated shocks, the convergence toward equilibrium or the transition between two equilibria.

In a stochastic context, Dynare computes one or several simulations corresponding to a random draw of the shocks. Dynare uses a second order Taylor approximation of the expectation functions,

and in version 3.0, it is possible to use Dynare to estimate model parameters either by maximum likelihood or using a Bayesian approach. Currently the development team of Dynare is composed of S. Adjemian, A. Benzougar, M. Juillard and O. Kamenik. In addition, several parts of Dynare use or have strongly benefited from publicly available programs by F. Collard, L. Ingber, P.Klein, S. Sakata, F. Schorfheide, C. Sims, P. Soederlind and R. Wouters. For further details, see the web page http://www.cepremap.cnrs.fr/dynare.

4.3 Posterior Estimates

Figures 1 to 3 makes a direct comparison between the prior and posterior distributions. Tables 3 through 6, on the other hand, shows the mean of the prior and posterior distribution for all the parameters. In addition, the standard deviation of the prior distribution and the 90% confidence interval over the mean of the posterior distribution are also presented.

Note that the posterior distributions obtained by the Metropolis-Hastings algorithm does not present a well-defined shape with an explicit mode. For this reason, analysing it's mean is probably more trustful that analysing it's mode. As an example, observe that the parameter ρ_b has a mode near 0. However, the probability that this parameter takes on a value near 1 is also considerable.

Some comments are worth making regarding the persistence parameters of the AR(1) processes (look on Table 3 for further details). All of them presents a posterior mean smaller that the mean of the prior, indicating that the persistence of the shocks are smaller than our prior beliefs. This difference is more evident on the persistence parameter of the productivity shock. Here, we have a drop from the pior mean to the posterior mean from 0.7 to 0.24. On Figure 1, you can see that the mode of the posterior distribution for ρ_a is very close to zero, meaning that the productivity shock could be modelled as a white noise rather than an AR(1) process.

Regarding the behavioural parameters, note that there is also a decrease on the mean of the consumption habit parameter h from 0.7 (on the prior distribution) to 0.52 (on the posterior distribution). Once again, there is evidence that there is less friction on our model compared to our prior beliefs. However, the coefficients on the investment adjustment cost and on the capital utilisation adjustment cost had their mean increased (specially the for the first). This could suggest a smoothing behaviour for the capital and for the investment, as can be verified on the impulse-response functions described on next section.

Analysing the price and wage parameters, we can verify that both the calvo parameters for price (ξ_p) and wage (ξ_w) decreased (the first dropped from a prior mean of 0.7 to a posterior mean of

0.54 and the latter decreased from 0.7 to 0.48). The indexation parameters (γ_p and γ_w) remained almost the same, with prior and posterior mean around 0.5.

Finally, there are the parameters of the monetary policy rule. There is a significant increase (from 0.7 to 0.84) on the lagged interest rate parameter (ρ), suggesting a smoothing path for the short-term interest rate. At last, observe that the posterior mean on the parameter $r_{\Delta Y}$ increased while the mean on $r_{\Delta\pi}$ decreased (both of them had the same prior mean of 0.3). We can interpret this as a monetary policy rule that, on the short run, gives relatively more importance to variations on the output gap than on variations on the inflation in respect to the inflation objective. On the other hand, the mean on the coefficient r_π is larger than on r_Y (although we already fixed the prior means on this way), indicating that, at the long run, the central bank is more concerned with the inflation compared to the output gap.

Figure 1

Prior distributions (in gray) and posterior distributions (in black)

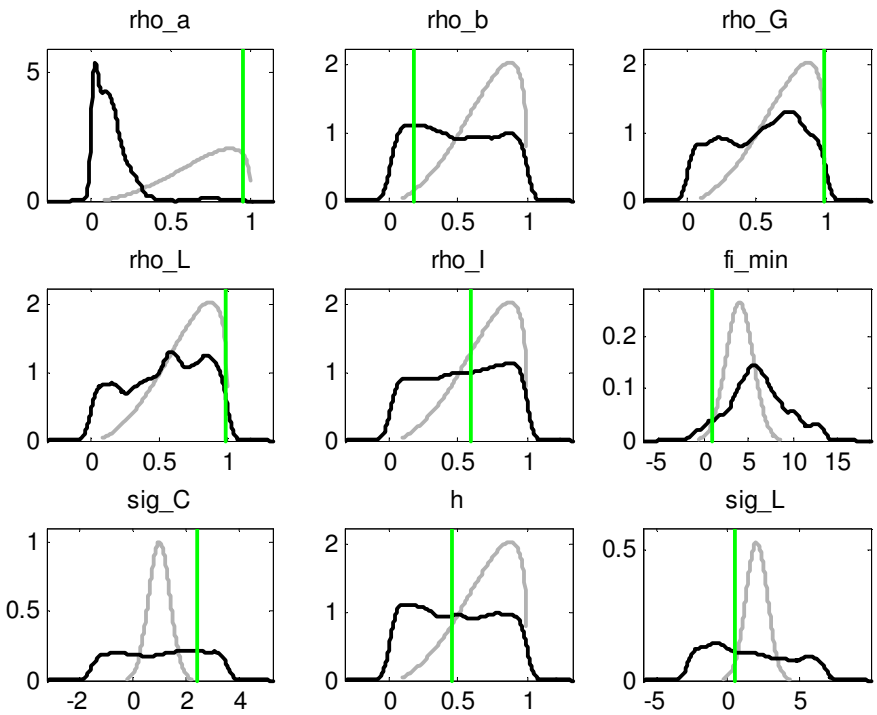


Figure 2

Prior distributions (in gray) and posterior distributions (in black) - cont.

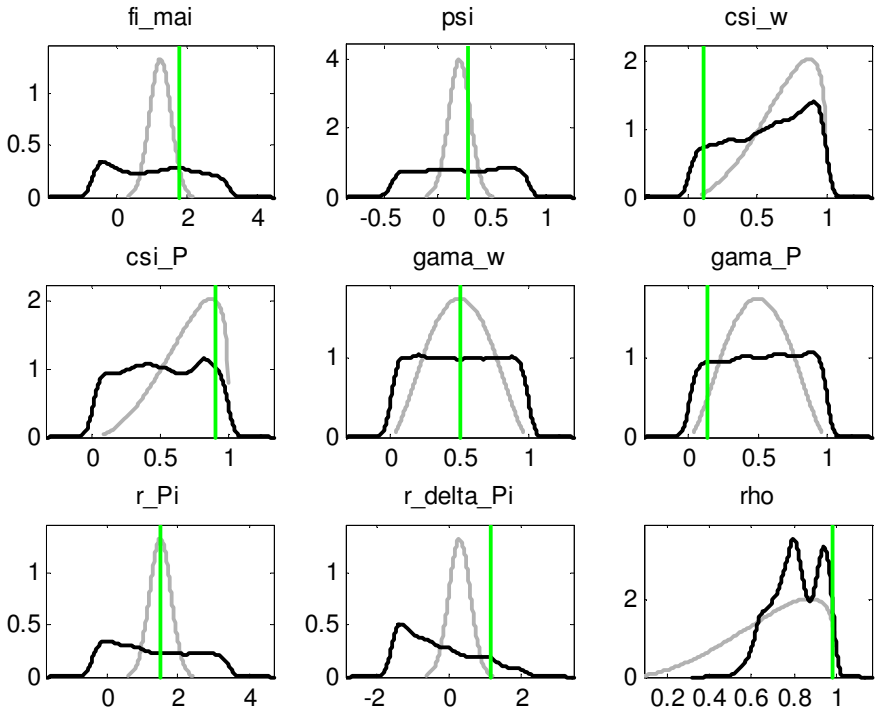
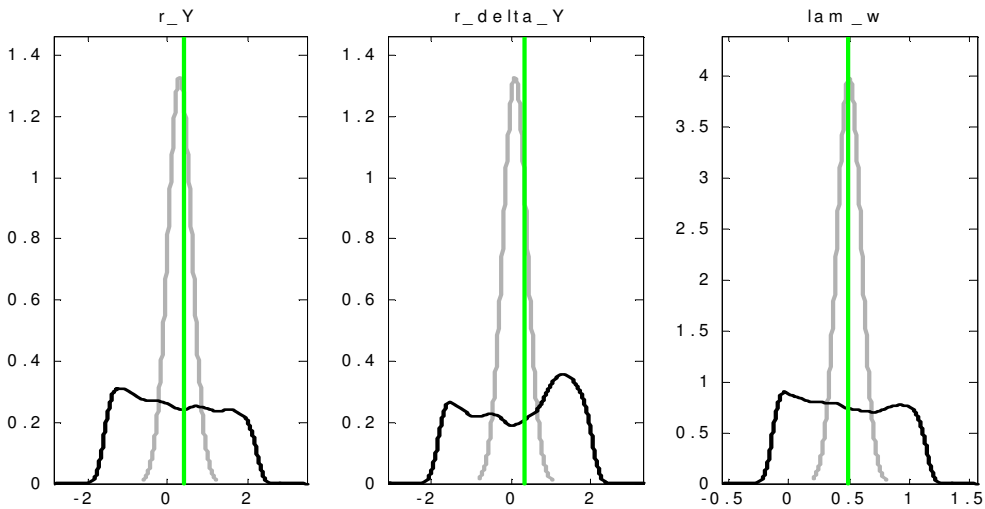


Figure 3

Prior distributions (in gray) and posterior distributions (in black) - cont.



4.4 Impulse-Response Functions (10 shocks)

The impulse responses to each of the structural (positive) shocks are generated using the parameters of the linearised DSGE model, estimated with Bayesian techniques. Overall, these effects are consistent with the impulse-response results presented in Smets& Wouters (2003a) and in Juillard et alli (2005).

Following a positive productivity shock (figure 4), output, consumption and investment rises, while employment falls, in contrast to the predictions of the standard RBC model without nominal rigidities. Due to the rise in productivity, the marginal cost falls on impact. As monetary policy does not respond strongly enough to offset this fall in marginal cost, inflation initially responds with a negative impact which vanishes after 10 quarters. According to Smets & Wouters (2003), a robust feature of the response to a positive productivity shock is that employment falls, which is mostly due to the sluggish response of the demand components due to habit formation and investment adjustment costs, accompanied by a deflationary effect.

Turning to the preference shock, figure 6 shows that a positive impact, while increasing consumption and output significantly, has a significant crowding-out effect on investment. Increased consumption demand puts pressure on the prices of production factors, putting upward pressure on the marginal cost and inflation. The strong crowding-out effects are also clear in response to a government spending shock (figure 12). In this case, both consumption and investment fall significantly and, while the rental rate on capital rises, real wages are not affected very much because of the greater willingness of households to work.

On the other hand, an inflation-target rate positive shock (figure 14) produces an increase in consumption, investment and output which gradually returns to zero, while the interest rate and inflation rate have a permanent positive effect due to the random walk nature of the inflation-target rate stochastic process. Figure 15 presents the effects of an interest rate shock. The temporary shock leads to a rise in the short-term interest rate and leads to a fall in output, consumption and investment. In line with the stylised facts following a monetary policy shock, real wages fall.

According to Smets & Wouters (2003a), the effects of a persistent change in the inflation objective are strikingly different in two aspects. First, there is no liquidity effect, as nominal interest rates start falling immediately as a result of the reduced inflation expectations. The presence of a liquidity effect following a monetary policy shock will depend on the persistence of the shock. Second, because the change in policy is implemented gradually and expectations have time to adjust, the output effects of the change in inflation are much smaller.

At last, note that both the capital and the investmest have a smooth and persistent path after a shock. As we observed in the previous section, this can be partly explained by the large coefficients estimated for the investment adjustment cost and on the capital utilisation adjustment cost.

Figure 4

Impulse-Response Function to a productivity shock (η_t^a) in a sticky price economy

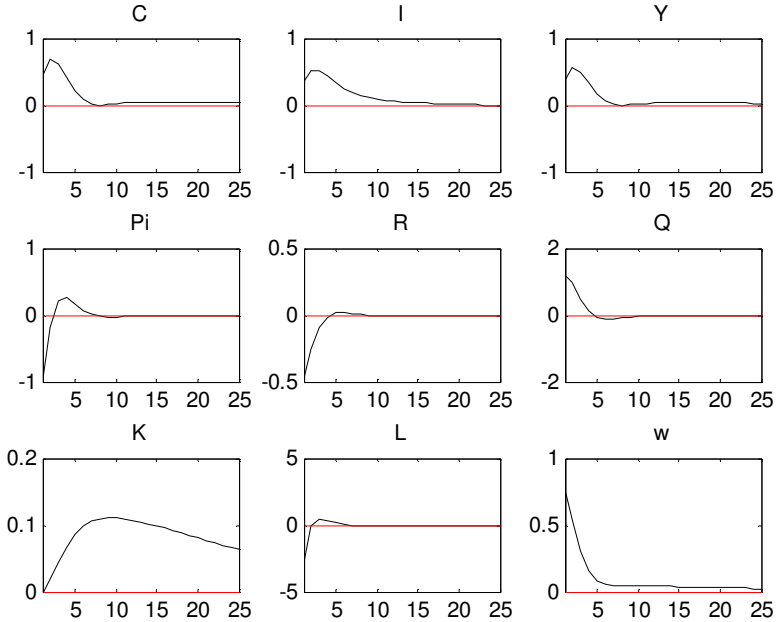


Figure 5

Impulse-Response Function to a productivity shock (η_t^a) in a flexible price economy

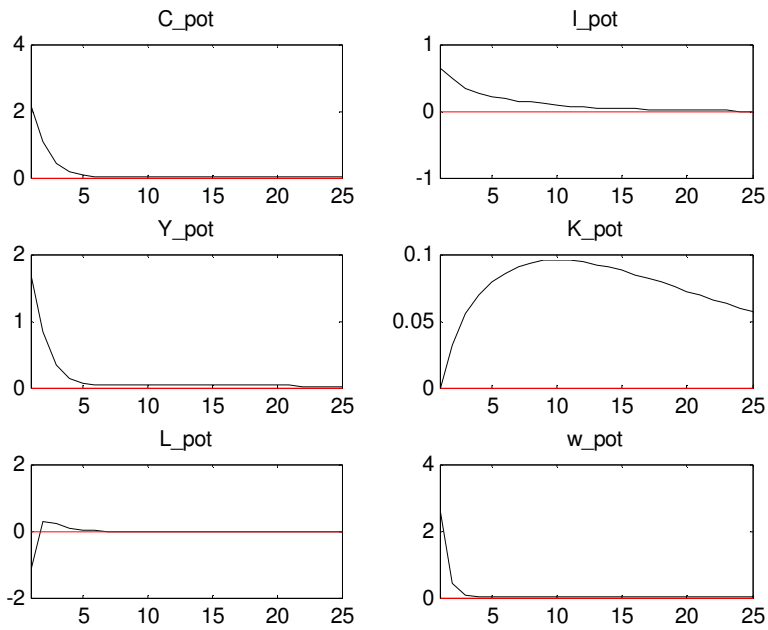


Figure 6

Impulse-Response Function to a consumer preference shock (η_t^b) in a sticky price economy

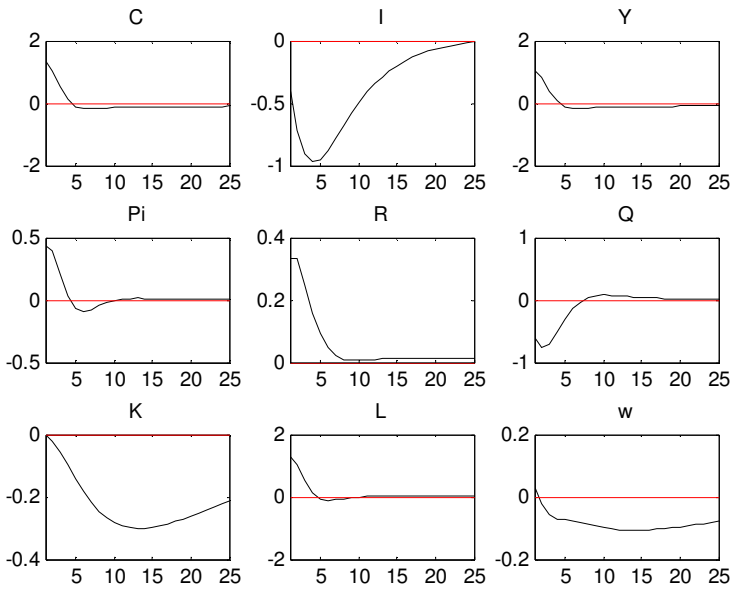


Figure 7

Impulse-Response Function to a consumer preference shock (η_t^b) in a flexible price economy

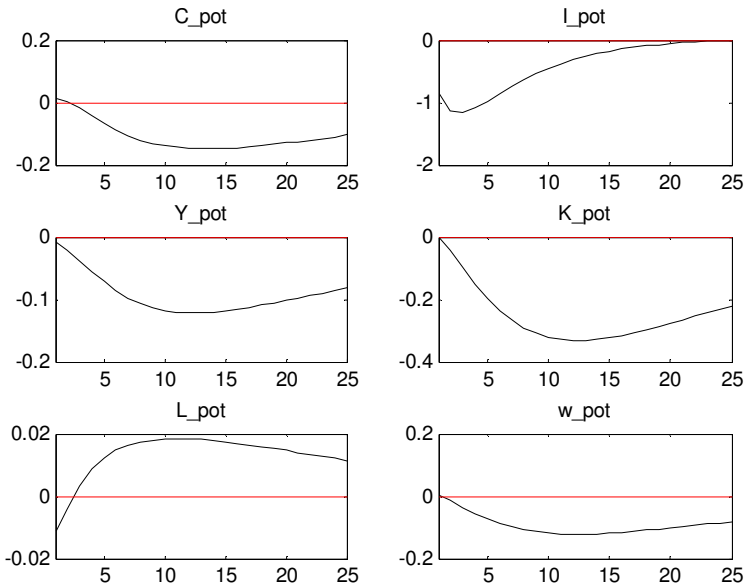


Figure 8

Impulse-Response Function to a investment shock (η_t^I) in a sticky price economy

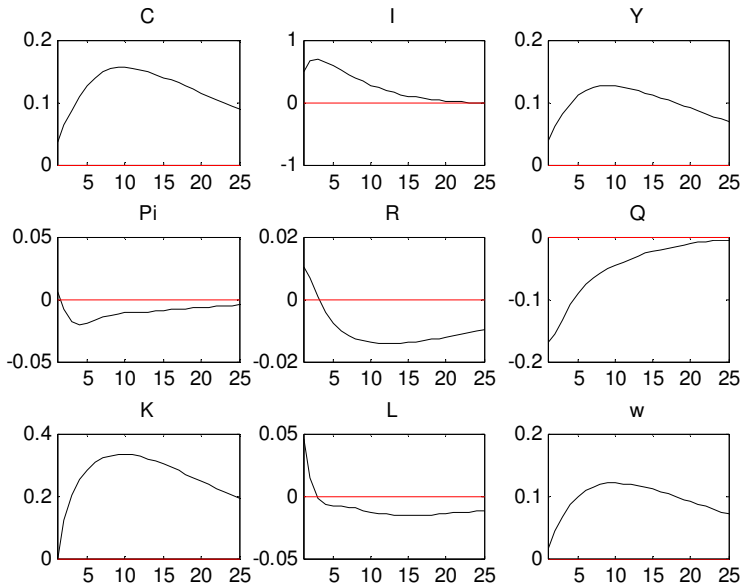


Figure 9

Impulse-Response Function to a investment shock (η_t^I) in a flexible price economy

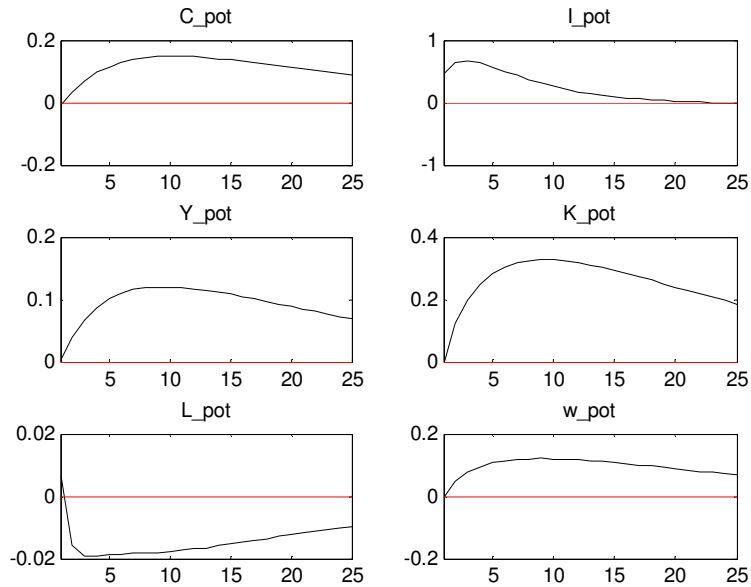


Figure 10

Impulse-Response Function to a labour supply shock (η_t^L) in a sticky price economy

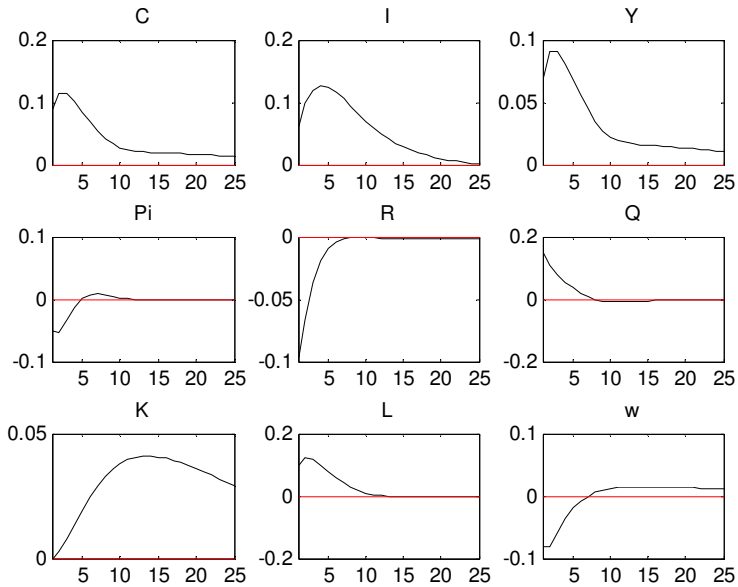


Figure 11

Impulse-Response Function to a labour supply shock (η_t^L) in a flexible price economy

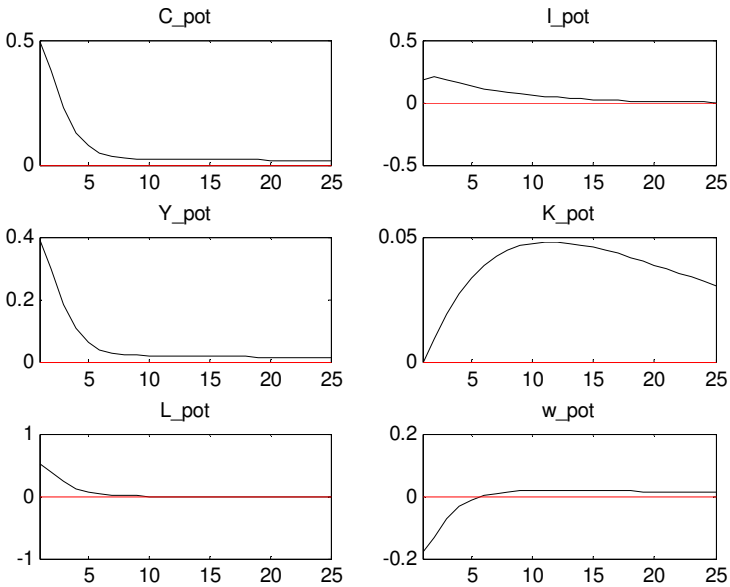


Figure 12

Impulse-Response Function to a government spending shock (η_t^G) in a sticky price economy

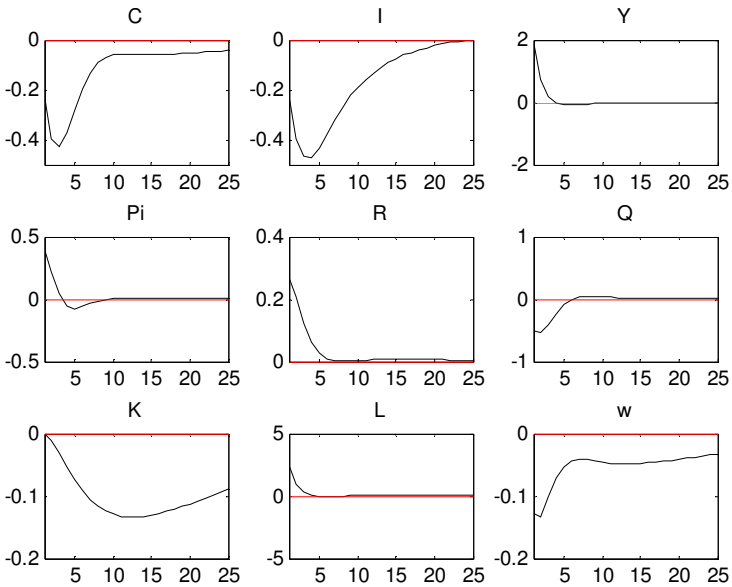


Figure 13

Impulse-Response Function to a government spending shock (η_t^G) in a flexible price economy

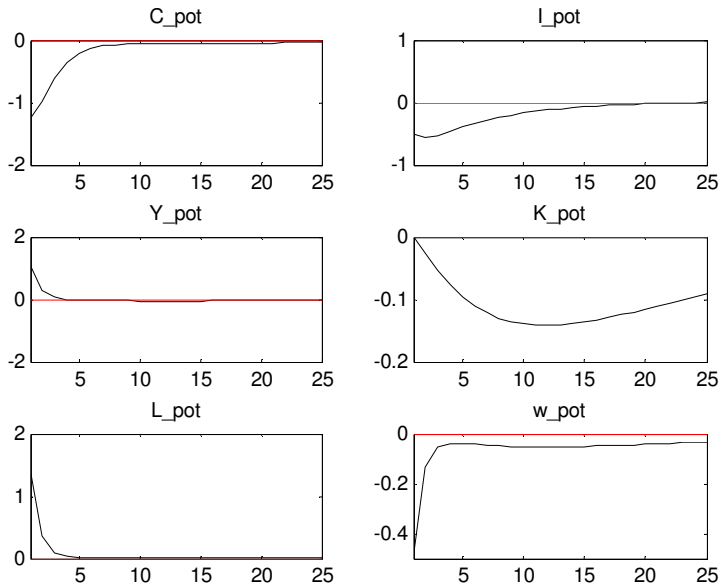


Figure 14

Impulse-Response Function to a inflation objective shock (η_t^π) in a sticky price economy

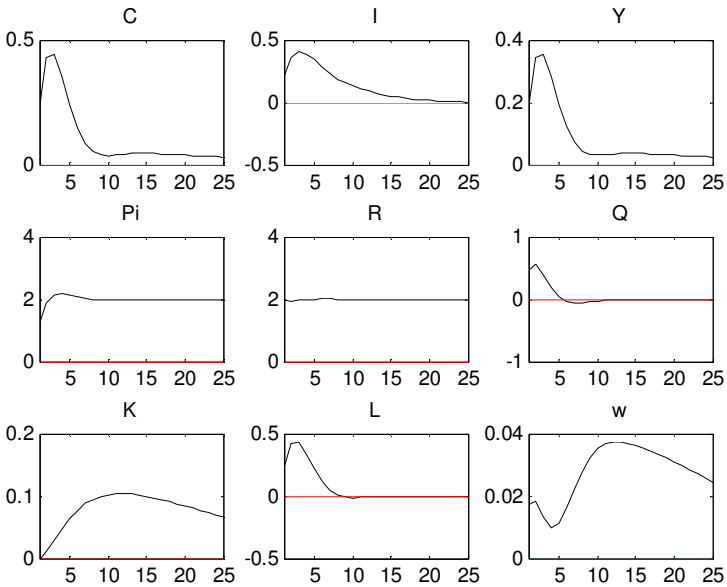


Figure 15

Impulse-Response Function to an interest rate shock (η_t^R) in a sticky price economy

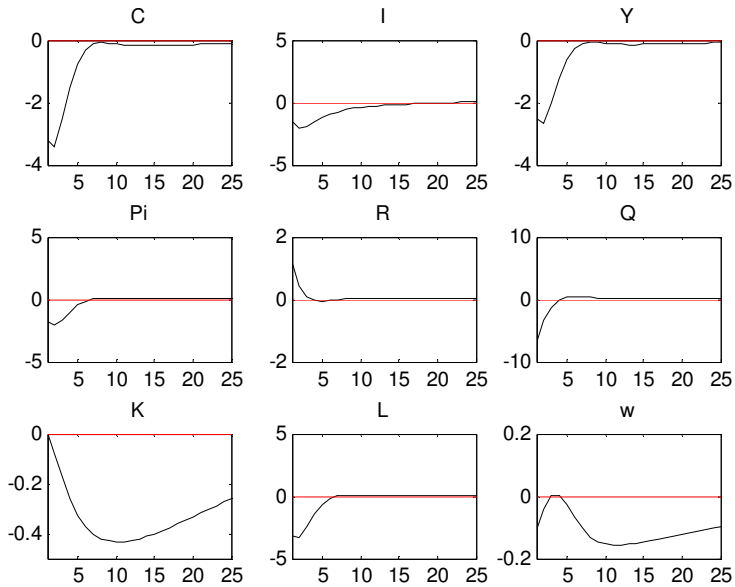


Figure 16

Impulse-Response Function to a price mark-up shock (η_t^P) in a sticky price economy

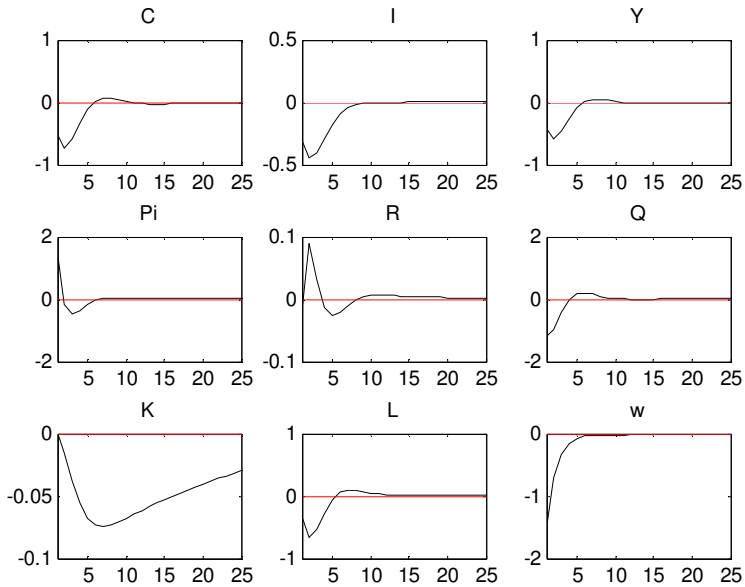


Figure 17

Impulse-Response Function to a wage mark-up shock (η_t^w) in a sticky price economy

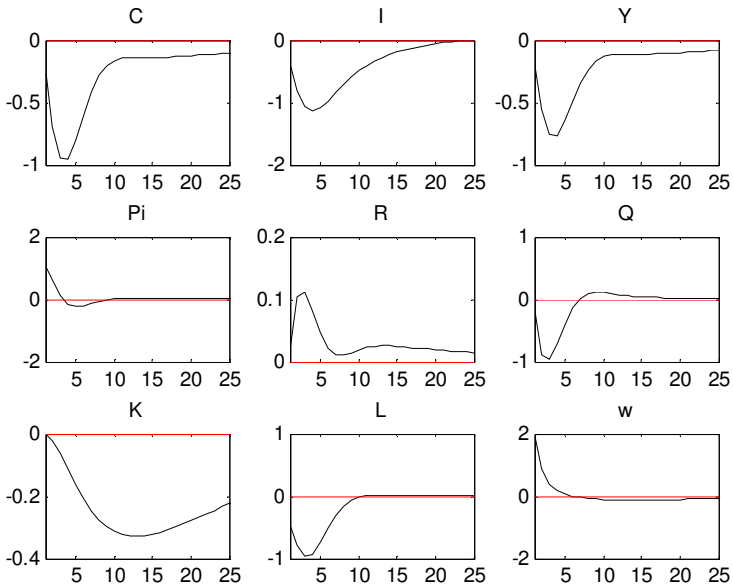
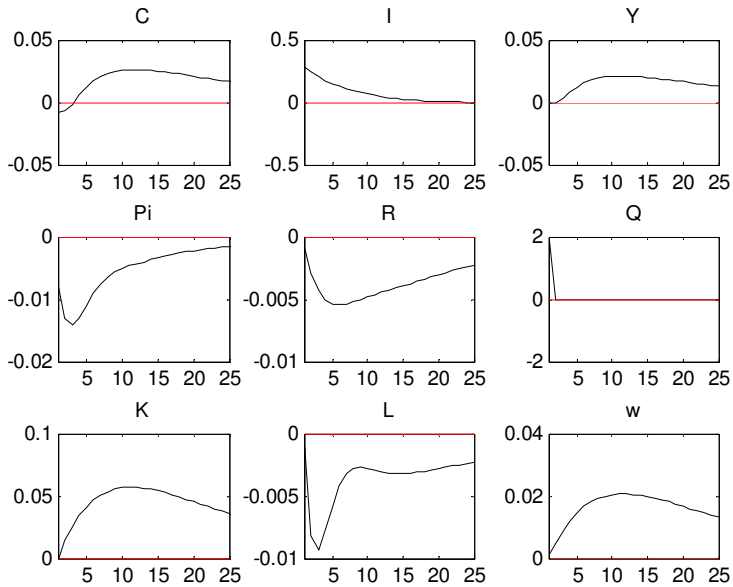


Figure 18

Impulse-Response Function to a equity premium shock (η_t^Q) in a sticky price economy



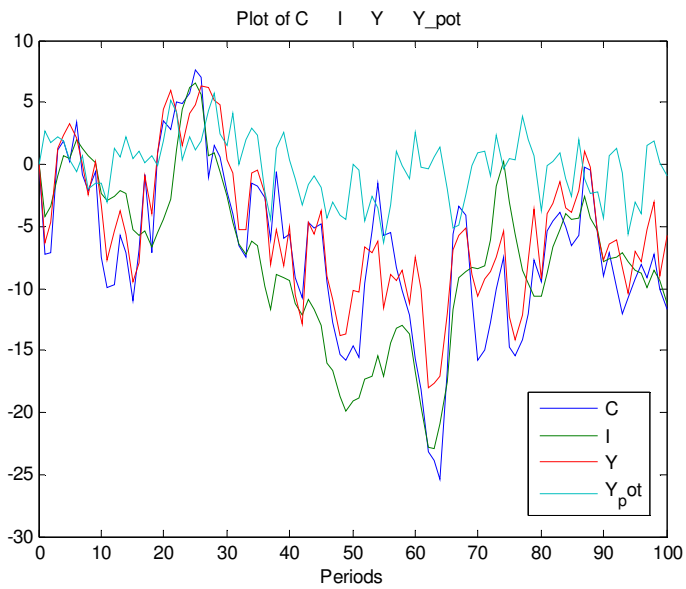
Regarding inflantionary effects due to the structural shocks, our results are summarised in Table 1. It should be noted the persistent response of inflation in Brazil to most of the shocks. For example, following a monetary policy shock, inflation reaches its new permanent level only after 7 quarters, while inflationary effects due to a productivity shock vanishes after 10 quarters.

Table 1 - Inflationary Persistence due to structural shocks

		Number of quarters to dissipate the inflationary effect	Graph scale
η^a	productivity shock	10	1.00
η^b	cons. pref. shock	14	0.50
η^I	investment shock	>25	0.05
η^L	labour supply shock	12	0.10
η^G	gov. spending shock	9	0.50
$\eta^{\overline{\pi}}$	inflation obj. shock	7 (permanent effect)	4.00
η^R	interest rate shock	6	5.00
η^P	price mark-up shock	6	2.00
η^w	wage mark-up shock	9	2.00
η^Q	equity premium shock	>25	0.02

Finally, figure 19 presents the results of a stochastic simulation of the model for 100 periods. The simulated endogenous variables are presented as log-deviations from it steady-state.

Figure 19 - Simulation of the DSGE model



5 Conclusions

In this work, we tried to replicate the model developed by Smets & Wouters (2003a) to Brazilian data. Although the posterior distributions calculated using the Metropolis-Hastings algorithm appeared to have an ackward shape, we could draw some interesting conclusions using the mean of these distributions. We observed that, in the light of evidence, the model seems to have less frictions, compared to our prior beliefs. The price and wage rigidities (represented by the parameters ξ_p and ξ_w) decreased when we compare the prior mean to the posterior mean. However, it is worth noting that the investment and capital utilisation adjustment costs had an increase on their means, suggesting that both the investment and the capital had a smoother path.

As we said in the introduction, the DSGE models have gained considerable popularity recently. The same thing can be said about use of Bayesian methods for inference. The latter is becoming an important instrument of analysis with the quick advance on computers verified on the past few years. The combination of DSGE models and Bayesian inference, therefore, has great perspectives of growth on the near future. However, there are still important gaps and uncertainties that still have to be solved.

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Appendix

A.1 Prior distribution for the parameters

Table 2 - Innovations of the Shocks

		Prior Distribution	*Mean*	*St. error*
η^a	productivity shock	normal	0.00	2.00
$\eta^{\overline{\pi}}$	inflation obj. shock	normal	0.00	2.00
η^b	cons. pref. shock	normal	0.00	2.00
η^G	gov. spending shock	normal	0.00	2.00
η^L	labour supply shock	normal	0.00	2.00
η^I	investment shock	normal	0.00	2.00
η^R	interest rate shock	normal	0.00	2.00
η^Q	equity premium shock	normal	0.00	2.00
η^P	price mark-up shock	normal	0.00	2.00
η^w	wage mark-up shock	normal	0.00	2.00

Table 3 - AR(1) Persistence of the Shocks

		Prior Distr.	*Mean*	*St. error*	*Post. Mean*	*Conf. Interval (90%)*	
ρ^a	productivity shock	beta	0.70	0.20	0.2415	0.0004	0.2465
$\rho^{\overline{\pi}}$	inflation obj. shock	-	1.00	-	1.00	-	-
ρ^b	cons. pref. shock	beta	0.70	0.20	0.4796	0.0005	0.8961
ρ^G	gov. spending shock	beta	0.70	0.20	0.5176	0.1131	1.0000
ρ^L	labour supply shock	beta	0.70	0.20	0.5039	0.1143	0.9991
ρ^I	investment shock	beta	0.70	0.20	0.5103	0.1055	0.9943

Table 4 - Behavioural parameters

		Prior Distr.	*Mean*	*St. error*	*Post. Mean*	*Conf. Interval (90%)*	
φ	invest. adj. cost	normal	4.00	1.50	5.9826	0.2156	11.0803
σ_C	consumption utility	normal	1.00	0.40	1.0629	-1.0174	3.5302
h	consumption habit	beta	0.70	0.20	0.5159	0.0004	0.8949
σ_L	labour utility	normal	2.00	0.75	1.5822	-2.7709	5.6304
ϕ	fixed cost	normal	1.25	0.30	1.1702	-0.6572	2.6785
Ψ	capital util. adj. cost	normal	0.20	0.10	0.2387	-0.3103	0.8284

Table 5 - Wage and Prices parameters

		Prior Distr.	*Mean*	*St. error*	*Post. Mean*	*Conf. Interval (90%)*	
ξ_w	calvo wages	beta	0.70	0.20	0.5421	0.1416	0.9999
ξ_P	calvo prices	beta	0.70	0.20	0.4810	0.0973	0.9828
γ_w	indexation wages	beta	0.50	0.20	0.5012	0.0329	0.9292
γ_P	indexation prices	beta	0.50	0.20	0.5092	0.1097	0.9978

Table 6 - Monetary policy function parameters

		Prior Distr.	*Mean*	*St. error*	*Post. Mean*	*Conf. Interval (90%)*	
r_π	inflation	normal	1.50	0.30	1.3328	-0.4073	2.9581
$r_{\Delta\pi}$	d(inflation)	normal	0.30	0.30	0.0864	-1.6077	1.2060
ρ	lagged interest rate	beta	0.70	0.20	0.8402	0.6411	0.9868
r_Y	output	normal	0.30	0.30	0.1350	-1.5900	1.7768
$r_{\Delta Y}$	d(output)	normal	0.10	0.30	0.2890	-1.7864	1.6859
λ_w	calvo employment	normal	0.50	0.10	0.4968	-0.1358	0.9968

A.2 Data File

```
// EPGE-FGV - Politica Monetaria (Prof. Marco Bonomo)
    // Estimacao Bayesiana do modelo DSGE de Smets&Wouters(2003)
    // para o Brasil, via Dynare (www.cepremap.cnrs.fr/dynare)
    // Autores: Hui Lok Sin & Wagner P. Gaglianone
    // Data: Janeiro-2006
    //
    // Fase B - Bayesiano

    var C,I,Q,K,Pi,w,L,R,Y, Pi_bar, C_pot,I_pot,Q_pot,K_pot,w_pot,L_pot,Y_pot, z_eps_a,z_eps_b,z_eps_I,z_eps_L,z_eps_G,
r_k, r_k_pot, rrf;
    varexo z_eta_a,z_eta_b,z_eta_I,z_eta_L,z_eta_Q,z_eta_P,z_eta_w,z_eta_R,z_eta_G,z_eta_Pi ;
    parameters ky,gy,tempo,h,sig_C,sig_L,lam_w,alfa,beta,gama,gama_P,gama_w,psi,fi_min,fi_mai,tau,rbar_k,csi_P,csi_w,rho,r_Pi,r_
    r_delta_Pi,r_delta_Y,rho_b,rho_I,rho_a,rho_L,rho_G,rho_Pi_bar;

    ky=0.4; // ad-hoc
    gy=0.2; // ad-hoc
    tempo=0;
    alfa=0.40;
```

```
beta=0.99;
tau=0.05;
rbar_k=0.06010; // beta=1/(1-tau+rbar_k)

// Média das Prior:

// rho_Pi_bar= 1;
// rho_a = 0.7;
// rho_b = 0.7;
// rho_G = 0.7;
// rho_L = 0.7;
// rho_I = 0.7;
// rho_l_hat = 0.7;
// fi_min = 4.00;
// sig_C = 1.00;
// h = 0.71;
// sig_L = 2.00;
// fi_mai = 1.25;
// psi = 0.20;
// csi_w = 0.75;
// csi_P = 0.75;
// gama_w = 0.77;
// gama_P = 0.77;
// r_Pi = 1.50;
// r_delta_Pi= 0.30;
// rho = 0.75;
// r_Y = 0.13;
// r_delta_Y = 0.06;
// lam_w = 0.50;

// Média das Posteriors;

rho_Pi_bar= 1;
rho_a = 0.2415;
rho_b = 0.4796;
rho_G = 0.5176;
```

```
rho_L = 0.5039;
rho_I = 0.5103;
fi_min = 5.9826;
sig_C = 1.0629;
h = 0.5159;
sig_L = 1.5822;
fi_mai = 1.1702;
psi = 0.2387;
csi_w = 0.5421;
csi_P = 0.4810;
gama_w = 0.5012;
gama_P = 0.5092;
r_Pi = 1.3328;
r_delta_Pi= 0.0864;
rho = 0.8402;
r_Y = 0.1350;
r_delta_Y = 0.2890;
lam_w = 0.4968;

model;

C = (h/(h+1))*C(-1) + (1/(h+1))*C(+1) + ((sig_C-1)/(sig_C*(1+lam_w)*(1+h)))*(L-L(+1)) - ((1-h)/((1+h)*sig_C))*(R-
Pi(+1))
+ ((1-h)/((1+h)*sig_C))*(z_eps_b);
I = (1/(1+beta))*I(-1) + (beta/(beta+1))*I(+1) + (1/(fi_min*(1+beta)))*(Q+z_eps_I);
Q = -(R-Pi(+1)) + ((1-tau)/(1-tau+rbar_k))*Q(+1) + (rbar_k/(1-tau+rbar_k))*r_k(+1) + z_eta_Q;
K = (1-tau)*K(-1) + tau*I(-1) + tau*z_eps_I(-1);
Pi-Pi_bar = (beta/(1+beta*gama_P))*(Pi(+1)-Pi_bar) + (gama_P/(1+beta*gama_P))*(Pi(-1)-Pi_bar) + (1/(1+beta*gama_P))
*((1-beta*csi_P)*(1-csi_P)/csi_P)*(alfa*r_k+(1-alfa)*w-z_eps_a-(1-alfa)*gama*tempo) + z_eta_P;
w=(beta/(1+beta))*w(+1)+(1/(1+beta))*w(-1)+(beta/(1+beta))*(Pi(+1)-Pi_bar)-((1+beta*gama_w)/(1+beta))*(Pi-Pi_bar)+
(gama_w/(1+beta))*(Pi(-1)-Pi_bar)-((1/(1+beta))*(1-beta*csi_w)*(1-csi_w)/((1+(1+lam_w)
*sig_L/lam_w)*csi_w))*(w-sig_L*L-((1/(1-h))*(C-h*C(-1)))+z_eps_L) + z_eta_w;
L = -w + (1+psi)*r_k + K;
Y = fi_mai*z_eps_a + fi_mai*alfa*K + fi_mai*alfa*psi*r_k + fi_mai*((1-alfa)*(L + gama*tempo)) - (fi_mai-1)*gama*tempo;
R = Pi_bar + rho*(R(-1)-Pi_bar(-1)) + (1-rho)*( r_Pi*(Pi(-1)-Pi_bar(-1)) + r_Y*(Y(-1) - Y_pot(-1) )) + r_delta_Pi*((Pi-
Pi_bar)
```

```
-(Pi(-1)-Pi_bar(-1))) + r_delta_Y*((Y-Y_pot)-(Y(-1) - Y_pot(-1) )) + z_eta_R;
Pi_bar = Pi_bar(-1)*(rho_Pi_bar) + z_eta_Pi;
z_eps_a = z_eps_a(-1)*(rho_a) + z_eta_a;
z_eps_b = z_eps_b(-1)*(rho_b) + z_eta_b;
z_eps_I = z_eps_I(-1)*(rho_I) + z_eta_I;
z_eps_G = z_eps_G(-1)*(rho_G) + z_eta_G;
z_eps_L = z_eps_L(-1)*(rho_L) + z_eta_L;
C_pot = (h/(h+1))*C_pot(-1) + (1/(h+1))*C_pot(+1) + ((sig_C-1)/(sig_C*(1+lam_w)*(1+h)))*(L_pot-L_pot(+1))
- ((1-h)/((1+h)
*sig_C))*(rrf) + ((1-h)/((1+h)*sig_C))*(z_eps_b);
I_pot = (1/(1+beta))*I_pot(-1) + (beta/(beta+1))*I_pot(+1) + (1/(fi_min*(1+beta)))*(Q_pot+z_eps_I);
Q_pot = -(rrf) + ((1-tau)/(1-tau+rbar_k))*Q_pot(+1) + (rbar_k/(1-tau+rbar_k))*r_k_pot(+1);
K_pot = (1-tau)*K_pot(-1) + tau*I_pot(-1) + tau*z_eps_I(-1);
w_pot-sig_L*L_pot-((1/(1-h))*(C_pot-h*C_pot(-1)))+z_eps_L = 0 ;
// zeramos o termo da equacao do salario, conforme S&W, para Y_pot
L_pot = -w_pot + (1+psi)*r_k_pot + K_pot;
Y_pot = fi_mai*z_eps_a + fi_mai*alfa*K_pot + fi_mai*alfa*psi*r_k_pot + fi_mai*((1-alfa)*(L_pot + gama*tempo))
- (fi_mai-1)
*gama*tempo;
(alfa*r_k_pot+(1-alfa)*w_pot-z_eps_a-(1-alfa)*gama*tempo) = 0; // zeramos termo da Curva de Phillips no caso de
Y_potencial
Y = (1-tau*ky-gy)*C + tau*ky*I + gy*gama*tempo + z_eps_G;
Y_pot = (1-tau*ky-gy)*C_pot + tau*ky*I_pot + gy*gama*tempo + z_eps_G;
end;

// initval;
// tempo=0;
// end;

steady;
check;

estimated_params;

// rho_Pi_bar, normal_pdf, 0.99, 0.01;
rho_a, beta_pdf, 0.7, 0.20;
```

```
rho_b, beta_pdf, 0.7, 0.20;
rho_G, beta_pdf, 0.7, 0.20;
rho_L, beta_pdf, 0.7, 0.20;
rho_I, beta_pdf, 0.7, 0.20;
fi_min, normal_pdf, 4.00, 1.500;
sig_C, normal_pdf, 1.00, 0.400;
h, beta_pdf, 0.70, 0.200;
sig_L, normal_pdf, 2.00, 0.750;
fi_mai, normal_pdf, 1.25, 0.300;
psi, normal_pdf, 0.20, 0.100;
csi_w, beta_pdf, 0.70, 0.20;
csi_P, beta_pdf, 0.70, 0.20;
gama_w, beta_pdf, 0.50, 0.20;
gama_P, beta_pdf, 0.50, 0.20;
r_Pi, normal_pdf, 1.50, 0.30;
r_delta_Pi, normal_pdf, 0.30, 0.30;
rho, beta_pdf, 0.70, 0.20;
r_Y, normal_pdf, 0.30, 0.30;
r_delta_Y, normal_pdf, 0.10, 0.30;
lam_w, normal_pdf, 0.50, 0.10;
// gama, normal_pdf, 0.40, 0.10;
end;

shocks;
var z_eta_a = 4.00;
var z_eta_b = 4.00;
var z_eta_I = 4.00;
var z_eta_L = 4.00;
var z_eta_R = 4.00;
var z_eta_Pi= 4.00;
var z_eta_G = 4.00;
var z_eta_Q = 4.00;
var z_eta_P = 4.00;
var z_eta_w = 4.00;
end;
```

```
stoch_simul(dr_algo=0,irf=25,periods=100,simul,simul_seed=123) C I Y Pi R Q K L w;
// stoch_simul(dr_algo=0,irf=25,periods=100,simul,simul_seed=123) C_pot I_pot Y_pot K_pot L_pot w_pot;
rplot C I Y Y_pot;
varobs C I Y L w Pi R ;
estimation(datafile=Dados_Brazil,lik_init=2,linear,mh_replic=200000,mh_nblocks=2,mh_jscale=0.8);

// rplot C I Y;
// dynatype(ResultBrazil1);
// dynasave(ResultBrazil2);
```