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Abstract

Multi-product firms dominate production activity in the global economy. There is widespread evidence showing that large corporations improve their efficiency by increasing the scale of their operations; this objective can be realized either by consistently investing in R&D or by expanding the product range. In this paper, we explore the implications of this fact by embedding multi-product firms in a General Equilibrium model of endogenous growth. We analyze an economy with oligopolistic firms that carry out in-house R&D programs in order to achieve cost-reducing innovations. Market structure is endogenous in the model and is jointly determined by the number of firms and the number of product varieties per firm. Both economies of scope and scale characterize the economic environment. We show that the market equilibrium involves too many firms (too much inter-firm diversity) and too few products per firm (too little intra-firm diversity); moreover, we find out that the total number of products and productivity growth are inefficiently low under laissez-faire. The nature of these distortions is discussed in detail.

KEYWORDS: imperfect competition, multi-product firms, endogenous growth, R&D

1 Introduction

Worldwide evidence shows that firms build their market position not only by investing in Research and Development (R&D) but also by expanding their product lines. Production activity is dominated by large multi-product enterprises;[1] to give an idea of the importance of this phenomenon, Bernard, Jensen and Schott (2005a) observe that firms with more than one product line generate over 90% of U.S. manufacturing output and account for over 95% of U.S. exports. Despite this empirical evidence, most of the macroeconomic literature has not shed enough light on this issue. For instance, the Chamberlinian monopolistic competition model, with the well-known Dixit-Stiglitz formulation (Dixit and Stiglitz, 1977), has been extensively used in growth theory: under this formulation, each firm is restricted to produce only one product and strategic interactions between firms are not taken into account.[2] The monopolistic competition model provides a good representation of industries characterized by a large number of small firms producing only one product; however, it is not suitable for markets with enterprises producing a large set of varieties.

In this paper, we develop an R&D-based growth model that extends the Dixit-Stiglitz framework of product differentiation to allow for multi-product firms and strategic interactions.

On the demand side, we consider a nested CES utility function for the differentiated products; at this regard, we assume that varieties produced by a single firm are better substitutes for one another than varieties produced by different firms. On the supply side, producers choose the size of their product range (the level of product proliferation) and engage in process innovation (in-house R&D) in order to lower their production costs; technological progress is measured by the average rate of cost reduction. Firms compete in the product market under the Bertrand mode of competition by fixing the prices of the varieties produced. The economic environment presents both economies of scope and scale as we model both firm-level and variety-level fixed costs;[3] a

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[1]Although firms may decide to specialize and produce a single variety, mono-product firms represent the exception rather than the rule (Teece, 1994).

[2]On this point, Lancaster (1990) observes: "An important *limitation* on the Dixit-Stiglitz and other neo-Chamberlinian models is that firms make *no product choice* − it is as though each firm, as it enters the group, is assigned a product by random choice (without replacement) from an urn containing blueprint for all possible products", (Lancaster, 1990, page 194). (The italics are ours.)

[3]In our model, in fact, the presence of the firm-level fixed cost allows each firm to produce several different products with lower unit costs than if they had been produced in independent firms. The reader is referred to Bayley and Friedlaender (1982) for an extensive

free entry condition determines the mass of firms. The mass of firms and the mass of varieties per firm jointly characterize market structure and are endogenously determined in equilibrium together with the rate of economic growth.

The model-setting considered in the present paper belongs to the class of *creative accumulation* models that have started to become popular in growth theory after the contributions of Smulders and van de Klundert (1995) and Peretto (1996).[4] During the last decade, these authors have developed a novel framework which makes explicit the link between industrial organization (IO) and growth theory and is suitable for capturing the interdependence between market structure and economic growth. In these models, the mass of firms changes in response to market and technology conditions and is endogenously determined; in addition, market structure affects the returns to innovation of profit-seeking firms and plays a crucial role in explaining the economic performance of the economy. It is important to observe that, in these models market structure is measured only by the mass of firms which can be supported in equilibrium; these authors deal with mono-product firms and, consequently, the degree of product variety is determined simply by the mass of firms. We contribute explicitly to this literature because we break the identity between the mass of varieties and the mass of firms by shedding light on the choice of product range; the latter represents an important component of firm size and, together with the mass of firms, determines market structure in our model.[5]

The analysis of multi-product firms presents some interesting features that are worth mentioning here. First, when a firm introduces a new variant of its product line, there is a trade-off between the benefit of attracting a larger number of customers and the cost of *cannibalizing* its existing products. Sec-

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treatment of the economies of scope.

[4]An important growth model that takes market structure seriously is the one of Thompson (2001); differently from Smulders and van de Klundert (1995) and Peretto (1996), the structure of the model in Thompson (2001) is stochastic and is suitable to capture firm's heterogeneity.

[5]Since the product range represents an important determinant of firm size, the level of firm's proliferation plays a role in one of the two "Schumpeterian hypotheses", that is the one regarding the relation between innovation and firm size. Product diversification is an argument that is often put forward in support of the role of large enterprises in innovation: in fact, a large multi-product firm can realize a higher yield on the resources devoted to R&D projects. The expected profit of R&D effort is positively affected by the degree of firm's diversification, it is argued, because a widely diversified enterprise is better able to exploit its research output and appropriate the returns associated with technological activities (see, e.g., Kamien and Schwartz, 1982, Chapters 2 and 3). A remarkable, seminal contribution to the understanding of firm growth through product diversification is given by Penrose (1959).

ond, each multi-product firm tries to internalize demand linkages between all the varieties it produces. Finally, firms can use their product range as an instrument to mitigate competition by affecting the pricing-production decisions of rivals; the reason why it happens is that an increase in the level of a firm's proliferation reduces the prices of other firms. Naturally, all these effects are not at work under the standard assumption of single-product firms and are crucial in the comparison between social optimum and decentralized economy. At this regard, we highlight a welfare cost implicit with imperfect competition and multi-product firms; the magnitude of this distortion is endogenous and is the result of the interdependence between market structure and economic growth.

In Proposition 1, we show that the market equilibrium involves too many firms (over-entry) and too few products per firm (too narrow product range) with respect to the social optimum; in addition, we find out that the mass of product varieties is inefficiently low under *laissez-faire*. Intuitively, the divergence between the decentralized economy and the social optimum can be explained as follows. Entrants do not take into account in their decisions that entry lowers the profits of all firms because of a decline in market share for each firm. Since there is interdependence between the entry decision of a potential competitor and the choice of product range, a new entrant leads to a contraction of existing firms' product ranges and this, in turn, induces entry. Consequently, the market results in excessive firms' entry and too little product variety. Variety-level fixed costs play a crucial role in generating the sub-optimality result of this economy: entry lowers firms' profits and incumbents reduce their costs by choosing fewer varieties. In fact, when the product range is narrower, the burden of proliferation fixed costs that each firm has to sustain becomes smaller. In contrast, the social planner internalizes the fact that more entry reduces profits of incumbents. Consequently, in the social optimum entry is lower; higher profits support higher proliferation fixed costs, and therefore, more variety.

In Proposition 2, we focus on the growth performance of the decentralized economy and we show that there is insufficient growth under *laissez-faire*. The reason why this occurs is that increasing returns to scale in R&D are internal to the firm and the rate of growth depends on the scale of the R&D program of the individual firm. Since the mass of firms is high in the decentralized equilibrium, the amount of R&D per firm is low. This leads to lower average R&D which reduces the rate of growth. In the social optimum, instead, average R&D is larger and economic growth is higher. Our analysis suggests that market structure is excessively fragmented in the decentralized equilibrium; this reduces the ability of firms to apply resources to innovation and hurts

economic growth.

It is interesting to compare these welfare results with the ones obtained in some related papers. Van de Klundert and Smulders (1997) develop a growth model with two sectors, a high-tech sector with differentiated goods and a traditional sector with a homogeneous good. The economy is affected by three distortions. First, oligopolistic firms face a price elasticity of demand that is lower than the elasticity of product substitution. Second, there is distortionary pricing due to the fact that high-tech firms, differently from traditional firms, set prices higher than marginal cost in this two-sector economy. Third, there are knowledge spillovers across firms. Though no clear conclusion emerges with respect to the mass of product varieties, the authors show that economic growth is too low under *laissez-faire*; in our model we also find that firms under-invest in R&D but, differently from van de Klundert and Smulders (1997), the result is not generated by technological spillovers between firms.

In order to focus on the effect produced by strategic interactions on growth and welfare, Peretto (1999) develops a growth model with only one sector and abstracts from knowledge spillovers between firms.[6] In the normative analysis of his model, the author finds out that the market provides too much variety and too little growth. Our model, that shares with Peretto (1999) the absence of knowledge spillovers and likewise is based on a one-sector economy, shows that allowing firms to produce more than one variety changes significantly the welfare analysis; in fact, R&D under-investment is now accompanied by too little product variety.

The rest of the paper is organized as follows. Section 2 reviews some related literature on multi-product firms in IO and international trade: here, we discuss the main contributions and outline the links with the present paper. Section 3 lays out the growth model we use in the analysis. Sections 4 and 5 develop respectively the welfare analysis and the comparison between the market equilibrium and the social optimum. Finally, Section 6 gives the conclusions and suggests further questions for research. All the main calculations are relegated in two Appendices.

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[6]In the one-sector economy, in fact, oligopolistic pricing does not give rise to distortions across industries, since all goods are priced in the same way.

2 Related literature

2.1 The industrial organization literature

A part from some notable exceptions by Anderson and de Palma (1992, 2006), Ottaviano and Thisse (1999), and more recently Ju (2003), most of the literature on IO treats the determination of the equilibrium mass of firms and the choice of product range as two separate issues. Some papers deal with product line selection by multi-product firms while keeping both the number of firms and the number of varieties per firm exogenous (see, e.g., Brander and Eaton, 1984). Other papers deal with the formation of multi-product firms while taking the total number of product varieties fixed (see, e.g., Wolinsky, 1986; Shaked and Sutton, 1990). Finally, some works explore the choice of the size of product range in differentiated markets while holding the number of firms constant (see, e.g., Raubitschek, 1987; Champsaur and Rochet, 1989).

We differ from these papers because market structure is endogenous in our analysis; in fact, the equilibrium mass of firms and the mass of product varieties per firm are jointly determined. At this regard, we outline two interesting features of our model. First, we allow for intra-firm competition because each firm co-ordinates its pricing decisions across all the varieties that it produces in order to maximize overall profits. When a firm creates a new variety, there is a reduction in the demand for all the other substitute varieties the firm produces (the *cannibalization effect*); therefore, each producer has to internalize competition within its product line. Second, we assume that firms produce a non-negligible set of varieties and take into account the effects of their pricing decisions on the industry's price index, while taking the prices of all other firms as given (the *strategic interaction effect*);[7] this means that we also allow for inter-firm competition. Firms behave like oligopolists and not as monopolistic competitors as in the standard Dixit-Stiglitz framework.[8]

Intra-firm decision coordination and inter-firm competition are two key-features of Anderson and de Palma (1992, 2006) and Ottaviano and Thisse (1999); these authors propose different models to study the performance of multi-product firms. Anderson and de Palma (1992) analyze a nested multi-nomial logit model that has been extensively used in empirical studies in IO. Ottaviano and Thisse (1999) consider a quadratic utility model which generates a linear demand system. Anderson and de Palma (2006), recently, revisit their seminal contribution within a general nested demand structure.

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[7]In this regard, we follow Yang and Heijdra (1993).

[8]In oligopoly, in fact, firms are *large actors* and interact in a more strategic way than in the case of monopolistic competition.

Our model, instead, shares the same consumers' preferences of Ju's model (2003); however, it is important to observe that Ju (2003) explores only the comparative-static properties of the nested CES Dixit-Stiglitz model without undertaking any welfare analysis.[9]

We notice that, with the exception of Ottaviano and Thisse (1999), all these works are based on partial equilibrium models. In the present paper, instead, we develop our analysis into a general equilibrium (GE) setting; we remark this difference by mentioning the advantages of using such a structure. First, a GE framework makes clear that firm compete not only for sales on the product market, but also for resources. Second, in such a setting the determination of demand and saving is endogenous. Finally, a GE setting guarantees that feedback effects from growth to market structure and vice versa, working mostly through the labor market, are fully captured.[10]

2.2   The international trade literature

In the international trade literature, theoretical work has started shedding light on multi-product firms only recently. Baldwin and Ottaviano (2001) examine the determinants of foreign direct investment and model trade patterns between two multi-product multinationals into a partial equilibrium setting. In their two-country model, the authors consider one multi-product enterprise per country and show how multinational firms have the incentive to place some of their factories abroad in order to mitigate the *cannibalization effect* which any given variety produces upon other varieties manufactured by the same enterprise.

Bernard, Redding and Schott (2005b) introduce multi-product firms into a GE model of comparative advantage and firm heterogeneity. The authors show that trade liberalization encourages firms to focus on their core competencies, reducing the range of products manufactured, and increasing the range of products exported. Movements of resources across product lines within firms generate a new source of welfare gain from trade, and provide an additional source of reallocation in response to trade liberalization.

In a related paper, Eckel and Neary (2005) analyze how multi-product firms

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[9]Allanson and Montagna (2005) propose a model which shares many features of the one proposed by Ju (2003) in order to explore the proposition that a shift from a fragmented market structure to a more concentrated equilibrium is induced by industry shakeout. The main difference between these two works is that Allanson and Montagna (2005) rule out strategic interactions in firms' pricing and product range decisions.

[10]For further discussion of the advantages of using a GE setting, the reader is referred to Peretto (1999).

react to trade liberalization; in particular, the authors focus on the intra-firm adjustments that take place in a multi-product firm after a change in the economic environment (for instance a change in factor prices). The analysis proposed by Eckel and Neary features how these intra-firm adjustments affect the demand for labor and, related to that, explains how induced changes in the wage rate influence the optimal product range.

Differently from these works, we do not deal with trade but with innovation; in-house R&D and accumulation of firm-specific knowledge are the key-features of the growing economy in the present paper.

3 The model

3.1 Basic framework

Our analysis is based on a *creative accumulation* model which is inspired by the works of van de Klundert and Smulders (1997) and Peretto (1999); these authors have developed GE growth models characterized by strategic interactions in the production (pricing) decisions of firms competing on the product market. We also allow for strategic interactions between firms; additionally, our model presents a second crucial feature, that is the full internalization of the *cannibalization effect* by multi-product firms.

In the following, we present the model; first, we describe consumers' preferences (the demand side), and then, we focus on technology (the supply side).

3.1.1 Consumption

We analyze a one-sector economy populated by a fixed amount, L, of identical individuals who supply labor services and consumption loans in competitive labor and capital markets. The typical consumer is endowed with one unit of labor that is supplied inelastically. He chooses consumption C to maximize lifetime utility:

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} \frac{C(\tau)^{1-\sigma}}{1-\sigma} \, \mathrm{d}\tau, \tag{1}$$

subject to the usual flow budget:

$$\dot{A}(t) = r(t)A(t) + W(t) + D(t) - E(t),$$

where $\rho > 0$ is the rate of time preference, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $A(t)$ is assets holding, $r(t)$ is the rate

of interest, $W(t)$ is the wage rate, $D(t)$ is dividends, $E(t) \equiv C(t)q(t)$ is total expenditure and $q(t)$ is the aggregate consumption price index. The only financial asset available to individuals is ownership shares of firms (stocks). In the economy, consumers own firms in equal shares and receive profits as dividends. Throughout the paper, we take labor as *numéraire* and normalize the wage rate to unity.

$C(t)$ is a composite good that is differentiated in two dimensions; there are m multi-product firms that are producers of differentiated goods:

$$C = \left(\int_{i=0}^{m} x_i^{\frac{\theta-1}{\theta}} \, \mathrm{d}i \right)^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{2}$$

where θ is the across-firms elasticity of substitution and the composite good of firm i, x_i, is:

$$x_i = \left(\int_{j=0}^{n_i} x_{ij}^{\frac{\delta-1}{\delta}} \, \mathrm{d}j \right)^{\frac{\delta}{\delta-1}}, \delta > 1, \tag{3}$$

where x_{ij} is the production of variety j by firm i.[11] In Eq. (3), one can observe that firm i produces a mass of varieties, n_i; the elasticity of substitution between these differentiated goods is δ. The product varieties are grouped into nests with the degree of substitutability between varieties within nests being higher than the one between nests, that is $\delta > \theta$; all the varieties in a nest are produced by the same firm. At this regard, it is interesting to observe that the literature on multi-product firms have considered two alternative industry configurations (see, e.g., Brander and Eaton, 1984). In the first, denoted as the *market segmentation* case, each nest $i \in [0, m]$ corresponds to a firm and a typical manufacturer produces a mass of n_i close substitute varieties of the good. In the alternative case, denoted as *market interlacing*, each nest $i \in [0, m]$ consists of varieties produced by different firms, with each manufacturer producing less closely related products; in this case the typical nest i is occupied by a mass n_i of firms. The specific characteristics of the product market under analysis are crucial for the choice between these two alternative industry configurations. In this paper, we assume that the products of a firm are perceived by consumers to be closer substitutes to each other than to those of other firms and, therefore, we analyse the *market segmentation* case.[12]

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[11]One can observe that even if we consider a continuum of firms, the multidimensionality of the firm's product range implies that each manufacturer is likely to behave as a *large actor* that strategically operates on the product market.

[12]This model aims to describe mature industries in which the trademark is the primary

The solution for the optimal expenditure plan is represented by the standard Euler equation which describes the savings policy and gives the optimal time path of consumption:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left(r - \rho - \frac{\dot{q}}{q}\right),$$ (4)

where r is the nominal rate of interest.

The representative consumer decides the consumption of each variety within nest i and therefore, maximizes x_i subject to the expenditure constraint on the products of firm i, $\int_{j=0}^{n_i} p_{ij}x_{ij}\,\mathrm{d}j \leq E_i$. The resulting demand function for each variety j in nest i is given by:

$$x_{ij} = \frac{E_i}{p_{ij}^{\delta} q_i^{1-\delta}},$$ (5)

where the price index that corresponds to firm i, q_i, can be written as:

$$q_i = \left(\int_{j=0}^{n_i} p_{ij}^{1-\delta}\,\mathrm{d}j\right)^{\frac{1}{1-\delta}}.$$ (6)

Finally, the consumption decision over each nest is taken and the representative consumer maximizes C subject to the budget constraint on composite goods, $\int_{i=0}^{m} q_i x_i\,\mathrm{d}i \leq E$; this gives:

$$x_i = \frac{E}{q_i^{\theta} q^{1-\theta}},$$ (7)

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locus of differentiation with other specific product attributes being of secondary importance; Allanson and Montagna (2005, page 591) make a similar assumption by stating that substitutability between varieties is higher within nests than between nests; Anderson and de Palma (1992, page 265) also assume that products within a group are more similar than products belonging to different groups. From the empirical side, it is important to outline that both the degree of product differentiation and firm differentiation are unobserved to the econometrician; consequently, the econometric procedure estimates both product and store differentiation as unknown parameters. The empirical IO literature analyzes the issue of whether we should expect greater differentiation (less substitutability) within a particular firm or among different firms; according to Richards and Hamilton (2005), the specificity of the market taken into account seems to be crucial in this respect. The restaurant trade is an example of a segmented market; in fact, customers first choose which restaurant to patronise and then select specific items from the menu. On the contrary, in the supermarket example, retailers sell products that fill many different needs, with each store selling roughly similar types of products; therefore, substitution across stores is likely to be greater than within stores.

where the aggregate consumption price index, q, equals:

$$q = \left(\int_{i=0}^{m} q_i^{1-\theta} \, \mathrm{d}i \right)^{\frac{1}{1-\theta}}. \tag{8}$$

Since $E_i = q_i x_i$ and there are L identical consumers, Eqs. (5) and (7) give the aggregate demand of variety j produced by firm i:

$$X_{ij} = L x_{ij} = \frac{LE}{q^{1-\theta} p_{ij}^{\delta} q_i^{\theta-\delta}}. \tag{9}$$

The demand for the individual variety depends negatively on its price and positively on both the nest-level and industry-level price indexes.

To conclude, we observe that the representative consumer behavior may be treated as the outcome of a three-stage utility maximization procedure. In the first stage, the consumer optimally allocates consumption expenditure over time. In the second and third stages, consumption decisions are made respectively over the varieties within each nest and over nests.

3.1.2 The production of goods and R&D activity

Each firm is characterized by a productivity level that changes over time through a cost reduction process driven by the accumulation of firm-specific knowledge.

Labor input is used in the production of each differentiated good and in the R&D activity. The production of a unit of good X_{ij} requires labor l_{xij} and firm-specific knowledge h_i, that is:

$$X_{ij} = l_{xij} h_i. \tag{10}$$

R&D is an in-house activity and results in productivity increases; we do not model spillovers of the fruits of R&D, so that firm-specific knowledge is completely tacit. For firm i, knowledge simply accumulates according to the following equation:

$$\dot{h}_i = \vartheta l_{ri} h_i, \tag{11}$$

where $\vartheta > 0$ is the research productivity parameter and l_{ri} is labor input employed in research activities by firm i. We observe that $\dot{h}_i$ is the flow of product-specific knowledge generated by an R&D project. The R&D technology exhibits increasing returns to scale to knowledge and labor and constant returns to scale to knowledge, the accumulated factor. This ensures that constant growth is feasible in the steady-state.

The choice of this R&D technology prevents us from studying some interesting issues related to the presence of knowledge spillovers across firms. We could introduce these technological externalities in the model but this would complicate the welfare analysis without adding to the basic insights. Since the objective of the present paper is to focus on the presence of strategic interactions in an oligopolistic environment populated by multi-product firms, we prefer to keep knowledge spillovers out from the analysis.[13]

3.2 Incumbent firms

Multi-product firms face the same production and R&D technologies and demand schedules; they start out with the same knowledge level and behave non-cooperatively on the product market. In the short-run, market structure can be characterized by a given mass of firms m. We model firms' decisions as a two-stage game. In the first stage firms choose the mass of varieties to produce, and so they determine the size of their product range. In the second stage they compete in price and choose the amount of research labor. In order to determine the short-run market equilibrium, we solve the model by backward induction using the subgame Nash perfect equilibrium concept. Free-entry, instead, determines the long-run equilibrium of the economy.

3.2.1 Second stage of the game

Firms need a fixed amount of labor, equal to l_f, for promoting each variety produced; in addition, a fixed amount of labor, equal to l_k, is necessary to sustain production at any level of product proliferation. The size of each firm's product range is limited by the variety-level fixed cost l_f which gives rise to scale economies at the variety level. On the contrary, the firm-level fixed cost l_k is related to firm-specific activities and generates economies of scope by providing an incentive for the firm to produce a large mass of varieties.[14] The instantaneous profits of firm i at time τ can be written as:

$$\pi_i(\tau) = \int_{j=0}^{n_i} \left[p_{ij}(\tau) - \frac{1}{h_i(\tau)} \right] X_{ij}(\tau) \, \mathrm{d}j - l_{ri}(\tau) - n_i l_f - l_k.$$

An incumbent firm i chooses the price p_{ij} and the research labor l_{ri} for every variety $j \in [0, n_i]$ for a given level of proliferation n_i in order to maximize the

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[13]We refer to van de Klundert and Smulders (1997) for a welfare analysis dealing with this issue.

[14]Marketing, management services and distribution are examples of firm-specific activities.

discounted present value at time t of the flow of profits:

$$\Pi_i(t) = \int_t^\infty R(\tau)\pi_i(\tau) \, \mathrm{d}\tau, \tag{12}$$

where $R(\tau) \equiv e^{-\int_t^\tau r(s) \, \mathrm{d}s}$ is the cumulative discount factor. This maximization problem is subject to the demand schedules Eq. (9), the production technology Eq. (10) the research technology Eq. (11), $h_i(t) = \bar{h} > 0$ (the initial knowledge level is given and equal for all firms), $h_i(\tau)$ for all $\tau \geqslant t$ given for all $j \neq i$ (the firm takes as given the R&D paths of its rivals), and $\dot{h}_i(\tau) \geqslant 0$ for $\tau \geqslant t$ (knowledge accumulation is irreversible). Since consumers own firms in equal shares, asset holdings $A(t)$ are equal to $\int_{i=0}^m \Pi_i(t)/L \, \mathrm{d}i$.

We assume that firms act as Bertrand-Nash competitors and internalize the so-called *cannibalization effect*; more precisely, each producer, when setting the prices of its own varieties, takes into account that a reduction in the price of one of the varieties that it produces generates a fall in the demand for all its other varieties. Moreover, since producers are not small relative to the size of the market, they take into account the effects of their pricing decisions on the industry's price index, while taking the prices of all other firms as given. A Nash equilibrium in prices emerges, with each firm choosing a pricing rule for each variety within its nest.

In Appendix A3 it is shown that a typical firm i charges the same price for all the varieties within its nest, that is $p_{ij} = p_i$ for every $j \in [1, n_i]$:

$$p_i = \frac{1}{h_i} \frac{[\theta - \epsilon_i(\theta - 1)]}{(\theta - 1)(1 - \epsilon_i)}, \tag{13}$$

where ϵ_i, the market share of firm i, is equal to:

$$\epsilon_i = \frac{q_i x_i}{E} = \left(\frac{q_i}{q}\right)^{1-\theta} = \frac{n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}}{\left(\int_{i=0}^m n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta} \, \mathrm{d}i\right)}.$$

Given the pricing strategy Eq. (13), the Lerner index of market power, which determines the magnitude of the mark-up over marginal cost, can be easily derived, that is:

$$\frac{p_i - 1/h_i}{p_i} = \frac{1}{\theta - \epsilon_i(\theta - 1)},$$

meaning that the market power of a firm is lower the smaller is its market share.[15]

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[15]This index provides an appropriate measure of *competition toughness*, as defined by Sutton (1991): the lower is the mark-up rate, the tougher is the competition on the product market.

Now, we turn at the accumulation process of firm-specific knowledge. Each firm invests in R&D in order to improve its level of productivity; Appendix A3 shows that the rate of return to R&D investment equals the following expression:

$$r_{\mathrm{R\&D}} = \vartheta \int_{j=0}^{n_i} l_{xij} \, \mathrm{d}j. \tag{14}$$

One can observe that the right hand side of Eq. (14) represents the effect of innovation on the cash flow and gives the cost-reducing effect of knowledge accumulation.

3.2.2 First stage of the game

In the first stage of the game, firm i, anticipating the subsequent price competition, determines the level of proliferation n_i taking as given the mass of competitors. Firms play a Nash game with each other; this means that, when a typical firm i chooses its product range, it takes as given the product ranges of all other firms.

Now, given the pricing strategy Eq. (13), the gross profits of firm i can be written as:[16]

$$\pi_i = LE \left[1 - \frac{\theta}{(\theta - 1)} \frac{1}{p_i h_i} \right] - l_{ri} - n_i l_f - l_k. \tag{15}$$

Firm i maximizes this expression with respect to n_i. The first order condition (FOC) for the profit-maximizing choice of product line, $\partial \pi_i / \partial n_i = 0$, can be written as:

$$LE \frac{\theta}{(\theta - 1)} \frac{1}{p_i^2 h_i} \frac{\partial p_i}{\partial n_i} - l_f = 0. \tag{16}$$

The benefit of expanding the product range is given by the first term on the left hand side of Eq. (16): the creation of a new product increases the firm's market share, leading to higher profits and larger incentives to create new products. However, more products involve higher proliferation costs; the second term on the left hand side of Eq. (16), in fact, gives the cost of creating one more variety.

3.2.3 Entry

As mentioned above, the market is characterized by free-entry; in addition, we assume that firms present zero scrap value. Now, differentiating firm's value

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[16]For the details of this calculation, look at Appendix A4.

Eq. (12) with respect to time and rearranging terms yields:

$$r = \frac{\pi_i}{\Pi_i} + \frac{\dot{\Pi}_i}{\Pi_i}, \tag{17}$$

which is a perfect-foresight, no-arbitrage condition for the equilibrium in the capital market. It says that the rate of return to firm ownership is equal to the rate of return to a riskless loan of size Π_i. The two terms on right hand side of Eq. (17) are respectively the ratio between profits and the firm's stock market value and the capital gain (loss) from the stock appreciation (depreciation); the sum of these two terms gives the rate of return to firm ownership.

Firms consider entry when $\Pi_i > 0$ and exit when $\Pi_i < 0$. Since entry and exit are not costly, the mass of firms is a variable that is free to jump and an instantaneous equilibrium with free entry and exit exists whenever $\Pi_i = 0$, which implies $\dot{\Pi}_i = 0$. Multiplying Eq. (17) by Π_i gives the instantaneous zero-profit condition, $\pi_i = 0$, for all values of the interest rate, r; this condition determines the long-run equilibrium mass of firms.

3.2.4 Symmetry

From now on, we restrict our attention on the symmetric equilibrium. Before proceeding, we discuss in some detail the nature of this equilibrium. In the dynamic game described above, firms that enter the market commit to time-paths of pricing, product range, R&D spending and knowledge accumulation; the Nash equilibrium of this game is given by the first order conditions of all active firms. Assuming that firms start out with the same knowledge level implies that the model is symmetric in the fundamentals that firms face and in the starting values of the state variables. Since the dynamic behavior characterizing each firm is governed by identical equations and boundary conditions, the rate of knowledge accumulation is the same for all the firms; this guarantees the symmetry of the Nash equilibrium at all times. Since the firm's Hamiltonian is linear in R&D investment, the bang-bang strategies yield that firms jump to their symmetric steady state (immediate convergence) if investment is unconstrained. In GE investment is constrained by the saving behavior of households; this implies that it is necessary to study the firm-level transitional dynamics to the symmetric steady state if firms start out with different initial knowledge stocks. Since the microeconomic dynamics of the Nash equilibrium are not crucial to the aggregate analysis we perform in this paper, assuming that firms start out with the same knowledge level is needed to simplify the analysis.

Now, imposing symmetry across firms means that each producer presents the same market share, that is $\epsilon_i = 1/m$ for every $i \in [1, m]$. This implies that, in the second stage of the game, the price of each differentiated good can be written as:

$$p = \frac{1}{h} \frac{[(m-1)\theta + 1]}{(\theta - 1)(m - 1)}, \tag{18}$$

where h is the level of knowledge that is identical across firms. Given this pricing rule, the Lerner index of market power amounts to $m/[(m-1)\theta + 1]$. In addition, plugging Eq. (18) into Eq. (15) makes instantaneous profits equal to:

$$\pi = \frac{LE}{[(m-1)\theta + 1]} - l_r - nl_f - l_k, \tag{19}$$

where l_r and n are respectively the research investment and the product range of a typical firm. The term $LE/[(m-1)\theta + 1]$ represents the *gross-profit effect*; it is equal to revenue per firm, LE/m, times the Lerner index of market power, $m/[(m-1)\theta + 1]$. One can observe that the latter tends to $1/\theta$ as m becomes infinitely large: this means that if firms are atomistic, markups become exogenous as in monopolistic competition.[17] The *gross-profit effect* is decreasing in the mass of firms m because the market share and the markup are lower the larger is m; in addition, it is decreasing in the inter-firm elasticity of substitution θ because oligopoly markup is lower the higher is θ.

Turning to the first stage of the game, under symmetry the FOC for the problem of maximization of firms' profits with respect to the level of product proliferation becomes:[18]

$$\frac{LE}{[(m-1)\theta + 1]} \cdot \frac{(\theta - 1)(m - 1)}{(\delta - 1)m} \cdot \frac{m\theta(m-1)}{[m\theta(m-1) + \theta - 1]} = nl_f \tag{20}$$

On the left hand side of Eq. (20) we individuate two effects: the *gross-profit effect* and the *expansion effect*. The latter effect consists of two terms. The second term on the left hand side of (20) depends positively on the rivals' total market share $(m-1)/m$; in fact, there are increasing possibilities to *steal business* from the rivals through product proliferation when the population of firms gets larger. We observe that this term is lower than one; this is because in the choice of its product range, each firm takes into account that the introduction of a new variety raises the firm's own price and is detrimental for the sales of the firm's existing varieties (*cannibalization effect*). The third

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[17]As we will see, this issue is important at the light of the fact that many feed-backs of the model are based on the endogenous nature of markups.

[18]Look at Appendix A4 for the calculations.

term on the left hand side of (20) captures the effect of a change in the mass of varieties of a producer on the pricing decisions of other firms; obviously, it is increasing in the mass of firms m because the effect produced on the rivals' behavior is negligible when the mass of competitors is large. This term is also lower than one. The reason is that an increase in the level of a firm's proliferation reduces the prices of other firms; consequently, each producer tends to under-expand its product range in order to limit price competition in the second stage (*strategic interaction effect*).

Rearranging Eq. (20), we write the mass of varieties per firm as:

$$n = \frac{LE\,(m-1)^2\,(\theta-1)\,\theta}{l_f\,(\delta-1)\,[(m-1)\theta+1]\,[m\theta(m-1)+\theta-1]}. \tag{21}$$

In the symmetric short-run equilibrium, where the mass of producers, m, is given, the level of product proliferation is inversely related to the fixed cost associated with the launching of a new variety, l_f. An increase in the intra-firm elasticity of substitution between varieties, δ, has a negative effect on n; the reason is that the importance of the *cannibalization effect*, which drives down the level of firm's proliferation, grows as a firm's varieties become more substitute. Conversely, an increase in the inter-firm elasticity of substitution, θ, influences positively the size of product range. In order to stress this point, we observe that two forces emerge as θ gets larger. From one side, the *gross-profit effect* decreases; the market becomes more competitive and markups get smaller by reducing firms' returns to expand their product range. From the other side, the *expansion effect* increases: the effectiveness of expanding the level of firm's proliferation and stealing customers from other firms is higher the larger is the inter-firm elasticity of substitution θ. Simple computation shows that the second force always dominates the first one leading firms to increase the level of product proliferation. Finally, n tends to zero when m tends to infinity meaning that each firm wants to sell a single product (formally, a zero measure set of varieties) when the mass of firms becomes very large; this implies that the two dimensions of diversity (intra-firm and inter-firm) are substitutes.

Under symmetry, the growth rate of firm-specific knowledge takes a very simple expression; using Eq. (11), it amounts to:

$$g = \vartheta l_r \tag{22}$$

By using Eq. (14), the rate of return to R&D activity becomes:

$$r_{\text{R\&D}} = \vartheta n l_x, \tag{23}$$

where l_x represents the amount of labor needed to produce one variety; it is important to observe that firm size is equal to nl_x and depends positively on the product range n. Equation (23) shows that the larger is firm size, the larger is the return to R&D activity; intuitively, the benefit of undertaking a cost-reducing innovation within a firm depends positively on the mass of varieties upon which such an innovation can be implemented.

To conclude, we plug Eq. (22) into Eq. (19); imposing $\pi = 0$ gives the instantaneous zero-profit condition:

$$\frac{LE}{[(m-1)\theta + 1]} - \frac{g}{\vartheta} - nl_f - l_k = 0. \tag{24}$$

In a zero-profit equilibrium, firms just break even; each producer spends all its cash flow, net of variable and fixed labor costs, on R&D activity.

3.3   General equilibrium

In order to discuss the main results of the paper, we focus on the balanced growth path (BGP) state where the growth rate of labor productivity and the allocation of workers between activities are constant. The GE of the model is determined as follows. We first impose labor and capital market clearing; these conditions yield an equilibrium relation denoted as CME. Next, we build a locus where labor market clears and firms choose their profit-maximizing mass of varieties; this relation is labeled as PR. Both curves, CME and PR, describe the BGP of the economy when the mass of firms is exogenous. Finally, we derive an equation, denoted as ZP, by using jointly the zero-profit condition and the labor market clearing condition.[19] The BGP of the economy with an endogenous mass of firms is given by the crossing of these three loci.

3.3.1   Labor and capital market equilibrium

To get an expression for employment in production, we substitute the pricing strategy Eq. (18) into the cost function Eq. (10), using the demand schedule Eq. (9), and aggregating over firms. This gives the aggregate quantity of production labor L_x:

$$L_x = mnl_x = \frac{(\theta - 1)(m-1)}{[(m-1)\theta + 1]} LE. \tag{25}$$

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[19]CME, PR and ZP stand for capital market equilibrium, product range and zero profit respectively.

Denoting aggregate R&D as L_r, labor market clearing requires that $L = L_x + L_r + mnl_f + ml_k$; using Eqs. (22) and (25), the labor market clearing condition can be written as:

$$\frac{L}{m} = \frac{(\theta - 1)(m - 1)}{[(m - 1)\theta + 1]}\frac{LE}{m} + \frac{g}{\vartheta} + nl_f + l_k. \tag{26}$$

Now, we turn to the capital market. In this market, each agent lends financial resources to firms and obtains a rate of return to savings that has to be equal to the rate of return to investment in order to ensure capital market clearing.

Let us consider the rate of return to R&D first. Using Eq. (25) into Eq. (23), one can write $r_{\text{R\&D}}$ as:

$$r_{\text{R\&D}} = \vartheta\frac{(\theta - 1)(m - 1)}{[(m - 1)\theta + 1]}\frac{LE}{m}.$$

It is important to note that the effect of cost reduction on cash flow can be decomposed into two terms: (i) the *gross-profit effect* discussed above (the term $LE/[(m - 1)\theta + 1]$); and (ii) the *business-stealing effect* which gives the increase in market share achieved by undertaking R&D (the term $\vartheta(\theta - 1)(m - 1)/m$). Differently from the *gross-profit effect*, the *business-stealing effect* increases with the mass of firms m; the reason is that there are increasing possibilities to *steal business* from the rivals' total market share when the mass of firms gets larger. The *business-stealing effect* is increasing in the inter-firm elasticity of substitution θ because the effectiveness of a cost-reduction in attracting customers is higher the more homogeneous are the products offered by different firms.

In order to determine the rate of return to savings, we proceed as follows. We get the growth rate of consumption expenditure by plugging the savings policy Eq. (4) into $\dot{E}/E = \dot{C}/C + \dot{q}/q$, that is:

$$\frac{\dot{E}}{E} = \frac{r - \rho}{\sigma} - \frac{\dot{q}}{q}\frac{(1 - \sigma)}{\sigma}.$$

Since per capita expenditure is constant on the balanced growth path, we set $\dot{E} = 0$ into the previous equation; in addition, we require that $\dot{q}/q = -g$, which means that the aggregate consumption price index decreases at the constant rate g.[20] This gives the rate of return to savings:

$$r_{\text{savings}} = \rho + (\sigma - 1)g. \tag{27}$$

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[20]The reader can easily verify that $\dot{q}/q = -g$ by using Eqs. (6), (8) and (18). We observe that this occurs because labor is the *numéraire* in the model. If, alternatively, the aggregate consumption good C was set as *numéraire*, then wages would be exponentially growing at a rate equal to g.

Finally, we equalize $r_{\text{R\&D}}$ to r_{savings}, that is:

$$\vartheta \frac{(\theta - 1)(m - 1)}{[(m-1)\theta + 1]} \frac{LE}{m} = \rho + (\sigma - 1) g, \tag{28}$$

which represents the capital market clearing condition.

3.3.2   Equilibrium loci

As mentioned above, we build three equilibrium loci that are useful for characterizing the BGP of the economy. These three curves are obtained in such a way to eliminate spending per capita, E. The intersection of these three loci, CME, PR and ZP, gives the growth rate $\bar{g}$, the mass of firms $\bar{m}$, and the mass of varieties per firm $\bar{n}$ in the market equilibrium.

The CME locus   This locus is obtained from the labor and the capital market clearing conditions, Eqs. (26) and (28) respectively. This gives:

$$g = \frac{\vartheta}{\sigma} \left(\frac{L}{m} - nl_f - l_k \right) - \frac{\rho}{\sigma}. \tag{CME}$$

The CME curve is a locus of allocations where all the markets clear; however, since Eqs. (21) and (24) have not been used to derive CME, this locus describes equilibria for a given mass of firms m and varieties per firm n. As the CME locus shows, the rate of innovation is negatively related to the mass of firms m and the mass of varieties per firm n. The reasons for that are the following. Increases in m make the market more *fragmented*; this reduces firm size and leads to a lower rate of return to R&D. Large values of n increase the burden of proliferation costs limiting the ability to devote resources to R&D (through the labor market-clearing condition). In both cases, the rate of growth is reduced.

The PR locus   This locus is built by requiring that the labor market clears and the product range is the one that maximizes firms' profits. From Eqs. (21) and (26), we get:

$$n = \frac{\vartheta \frac{L}{m} - g - \vartheta l_k}{\vartheta l_f [(\delta - 1) G(m) + 1]}, \tag{PR}$$

where $G(m) = \frac{m\theta(m-1)+\theta-1}{m\theta(m-1)}$. The PR locus gives the mass of varieties per firm for given growth rate g and mass of firms m. Once again, as in the case of the CME curve, more *fragmented* markets and a large innovation rate reduce

the level of product proliferation. In addition, it is important to remark that increases in the intra-firm elasticity of substitution δ and in the inter-firm elasticity of substitution θ make the product range narrower. When δ takes larger values, in fact, the varieties produced within each single firm become more homogeneous and the *cannibalization effect* produces a contraction of the product range. An increase in θ, instead, makes product ranges more homogeneous and results in a decline in the Lerner index of market power. This leads firms to choose a smaller level of product proliferation (*strategic interaction effect*): the product range, in fact, is used as an instrument to mitigate price competition.[21]

The ZP locus We derive the ZP locus by crossing the zero-profit condition (24) with the labor market clearing condition (26). We get:

$$g = \vartheta \frac{L}{m} \frac{1}{M\left(m\right)} - \vartheta n l_f - \vartheta l_k, \qquad \text{(ZP)}$$

where $M(m) = \frac{\theta(m-1)+1}{m}$. The ZP locus gives the mass of firms for a given growth rate g and level of product proliferation n. We observe that the innovation rate is inversely related to the size of product range, the mass of firms and the inter-firm elasticity of substitution θ. The reasons for that are the following. First, if the mass of varieties per firm gets larger, proliferation costs increase; in order not to incur losses, firms have to choose a lower rate of innovation. Next, whenever the mass of incumbents becomes smaller, profit margins rise; the consequent increase in the market power allows firms to sustain higher R&D costs. Finally, if product ranges become more homogeneous, there is a decline in the market power and the *gross-profit effect* goes down; therefore, only a smaller innovation rate is sustainable.

Before proceeding with the analysis of the model, it is useful to characterize the short-run equilibrium by studying the relation between the mass of firms m and the rate of growth g, taking the mass of firms as given. We perform this objective by plugging n from (PR) into (CME) and rearranging terms;[22] it yields:

$$g = \frac{\left[\vartheta \frac{L}{m} - \vartheta l_k - \rho - \frac{\rho}{(\delta-1)G(m)}\right]}{\left[\frac{\sigma-1}{(\delta-1)G(m)} + \sigma\right]}.$$

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[21]At this regard, the reader can easily check that $G(m)$ increases in θ.

[22]It is important to stress that in order to derive this relationship, we do not use the ZP locus that plays a crucial role in the determination of the long-run equilibrium of the economy under free-entry; in our model, in fact, the mass of firms m is determined endogenously by a process which drives net profits to zero.

One can easily check that there is an inverse relationship between g and m; to explain this result, it is important to take into account two effects. First, a larger mass of firms causes a fall in average R&D; this is due to the fact that less labour resources can be devoted to R&D in more fragmented markets. Second, we observe that the effect of a change in the mass of varieties of a producer on the pricing decisions of other firms is large when m is small; this implies that each firm tends to use consistently its product range to limit price competition in the second stage. Since producers under-expand their product range by the *strategic interaction effect*, more resources can be invested in R&D activities and, consequently, the rate of growth is larger.

3.3.3   Stability of the equilibrium

Following Evans, Honkapohja and Romer (1998), we investigate the stability properties of the model by analyzing the dynamics of the economy away from the steady state when agents have to learn about the location of the equilibrium. Appendix A.5. shows that a parametric restriction has to be introduced in order to have a stable equilibrium with a positive growth rate. This restriction gives rise to Assumption 1:

Assumption 1   The firm-level fixed cost has to satisfy the following inequality, that is:

$$l_k > \frac{\rho}{\vartheta(\sigma - 1)}.$$

The role played by the firm-level fixed cost in Assumption 1 can be explained as follows. In order to have a stable solution, the rate of return to R&D has to be larger than the rate of return to savings when the growth rate is below its equilibrium value; such a situation, in fact, provides the incentive to raise the rate of innovation until the achievement of an equilibrium, where the two rates of return are equal. As it is explained more in detail in Appendix A.5., the rates of return to R&D and savings are both increasing functions of the rate of innovation; in addition, the firm-level fixed cost affects positively firm size and the rate of return to R&D through the zero-profit condition. This implies that, when the growth rate is low and the effect of g on $r_{\text{R\&D}}$ and r_{savings} is negligible, the firm-level fixed cost has to be high enough in order to make the realized rate of return on R&D investment larger than the rate of return to savings.

In the following analysis we assume that this Assumption holds.
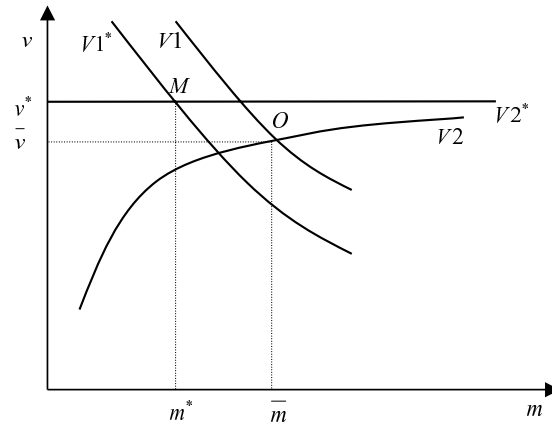
M = monopolistic competition = Social Optimum

O = oligopoly

Figure 1: Decentralized economy and Social Optimum

3.3.4 Market structure and growth

Now, we reduce the three-dimensional system, given by CME, PR and ZP, to a two-dimensional system in the mass of firms, m, and the total mass of product varieties the market produces, denoted by $v = m \times n$. Combining ZP with CME and ZP with PR gives respectively:

$$v = \frac{L}{(\sigma - 1)l_f} \left[\frac{\sigma}{M(m)} - 1 \right] - \frac{m}{l_f} \left[l_k - \frac{\rho}{\vartheta(\sigma - 1)} \right], \qquad \text{(V1)}$$

$$v = \frac{L \left[1 - \frac{1}{M(m)} \right]}{l_f (\delta - 1) G(m)}. \qquad \text{(V2)}$$

We draw these two relations, V1 and V2, in the (m, v) plane. Since $M'(m) > 0$ and $G'(m) < 0$, the V1 and V2 loci are respectively decreasing and increasing in m.[23] In Figure 1, the intersection of these two relations gives the equilibrium values for v and m.

It is important to observe that, when the mass of competitors becomes very large, $M(m)$ and $G(m)$ tend respectively to θ and 1; this means that the V1 locus shifts down while the V2 locus becomes flat. Our model converges to

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[23]Observe that in the V1 locus $l_k > \frac{\rho}{\vartheta(\sigma-1)}$ by Assumption 1.

monopolistic competition because strategic interactions across multi-product firms disappear.

Once the masses of firms and product varieties are determined, the equilibrium mass of varieties, $\bar{n}$, is simply obtained as $\bar{v}/\bar{m}$. Plugging $\bar{m}$ and $\bar{n}$ into the CME locus yields the equilibrium growth rate, $\bar{g}$.

Before turning our attention to the social welfare analysis, it is worth studying the impact of the population size L on the main economic variables and check for the scale effect in the model. Now, an increase in the labor force L shifts the V1 and V2 curves upward, so that the total mass of product varieties in the market equilibrium $\bar{v}$ raises. Since we are not able to know the sign of the change in $\bar{m}$ by means of a graphical treatment of the model, we perform some comparative-static exercises, whose details are contained in Appendix A.6.. Here, we show that an increase in population raises the mass of firms, the mass of varieties per firm and the growth rate in equilibrium. An interesting property of our model is that this (positive) scale effect vanishes asymptotically; in fact, we find out that as L increases and $\bar{m}$ becomes large, growth becomes independent of the size of the labor force because the growth rate of the oligopolistic economy tends to the one of monopolistic competition:

$$\frac{\vartheta l_k\,(\delta-1)\,(\theta-1) - \rho(\delta-\theta)}{\sigma\delta - 1 - \theta\,(\sigma+\delta-2)}.$$

The direct implication of this result is that growth is not explosive if population grows exponentially.

4 Welfare

A social planner seeking to achieve the first best maximizes utility of a typical consumer evaluated under symmetry.

Using Eqs. (2), (3), (10), instantaneous utility C can be written as $C = l_x h m^{\frac{\theta}{\theta-1}} n^{\frac{\delta}{\delta-1}}$. Assuming that the social planner has access to a large set of instruments, the social planning solution can be described as the result of choosing the sequences of l_r, m and n to maximize the lifetime utility function subject to the accumulation technology of firm-specific knowledge $\dot{h} = \vartheta l_r h$ and the labor market constraint Eq. (26). From this maximization problem (Appendix B1) we get three relations, CME*, PR* and ZP*; they represent respectively the counterparts of CME, PR and ZP in the decentralized economy.

CME* gives the optimal growth rate g for given mass of firms m and

varieties per firm n:

$$g = \frac{\vartheta}{\sigma} \left(\frac{L}{m} - nl_f - l_k \right) - \frac{\rho}{\sigma}. \tag{CME*}$$

PR* gives the optimal mass of varieties per firm n for given mass of firms m and growth rate g:

$$n = \frac{\vartheta \frac{L}{m} - g - \vartheta l_k}{\vartheta l_f \delta}. \tag{PR*}$$

Finally, ZP* gives the optimal mass of firms m for given growth rate g and mass of products per firm n:

$$g = \vartheta \left(\frac{1}{\theta} \frac{L}{m} - nl_f - l_k \right). \tag{ZP*}$$

The growth rate g^*, the mass of firms m^*, and the mass of varieties per firm n^* in the social optimum are obtained by the intersection of these three loci, CME*, PR* and ZP*.

5 Discussion

In this section we compare the market equilibrium with the social optimum. We first focus on the market structure and, by means of a graphical representation of the two solutions, we show that the market equilibrium is characterized by an excessive mass of firms (too much inter-firm diversity) offering too few varieties, both individually (too little intra-firm diversity) and in total. Then, we look at the innovation rates of the two configurations, market economy and social optimum; at this regard, we find that firms under-invest in R&D under *laissez-faire.*

5.1 Market Structure

We build the counterparts of the V1 and V2 loci for the social optimum; combining ZP* with CME* and ZP* with PR* gives respectively:

$$v = \frac{L}{(\sigma - 1)l_f} \left(\frac{\sigma}{\theta} - 1 \right) - \frac{m}{l_f} \left[l_k - \frac{\rho}{\vartheta(\sigma - 1)} \right], \tag{V1*}$$

$$v = \frac{L \left(1 - \frac{1}{\theta} \right)}{l_f \left(\delta - 1 \right)}. \tag{V2*}$$

We observe that the V1* locus is decreasing in m; V2*, instead, is a horizontal line. The intersection of these two curves gives the values of v and m in

the social optimum. It is straightforward to check that V1 lies always above V1* because $M(m) < \theta$; in addition, V2 lies always below V2* and tends asymptotically to it for very large values of m. We observe that in a model of monopolistic competition the V1 and V2 loci coincide respectively with V1* and V2*; this means that in a monopolistically competitive economy, the equilibrium market structure coincides with the one chosen by the social planner. As Figure 1 shows, $v^* > \bar{v}$ and $m^* < \bar{m}$; this implies that $n^* > \bar{n}$. In other terms, in the decentralized economy there is an excessive mass of firms offering too few varieties, both individually and in total. This result is summarized in the following Proposition:

Proposition 1 *In presence of multi-product enterprises, the market equilibrium involves too many firms and too few products per firm with respect to the social optimum. In addition, the total mass of varieties in the decentralized economy is inefficiently low.*

The intuition behind Proposition 1 may be obtained by shedding light on the effects that a new firm's entry produces on the incumbents' behavior. First, there is the well-known *business stealing* externality because an individual firm does not internalize the profit reductions that its operation imposes on other firms when it decides whether to enter an industry; clearly, this represents a tendency toward over-entry.[24] Then, we have the *consumer surplus* externality consisting in the fact that an entrant is not able to extract the whole surplus associated with the production of its product range; consequently, this force pushes toward under-entry. Finally, there is an additional externality whereby an entrant generates a contraction of the product ranges of existing firms;[25] this is a tendency toward insufficient product variety per firm and over-entry. The net effect of these forces leads to excessive entry, too narrow product ranges, and too low product variety.

It is interesting to observe that our findings are in accordance with the ones obtained by Anderson and de Palma (1992) for the nested multinomial logit demand model (see Proposition 3 at page 270).[26]

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[24]For the *business stealing effect*, the reader is referred to Mankiw and Whinston (1986).

[25]For firms, the net value of an additional variety in the product range is zero via the FOC for the profit-maximizing choice of product line. On the contrary, the contraction of the product ranges involves a loss in the consumer surplus because the social value of the variants that are not more produced is strictly positive.

[26]Differently from us, Anderson and de Palma (1992) develop their analysis within a partial equilibrium framework and, above all, do not deal with innovation and economic growth.

5.2 Growth

To get the intuition of the forces at work when we compare the innovation rate of the market equilibrium with the one realized by the social planner, we now focus on the CME locus since it coincides with its counterpart CME*. We have shown two results: (i) too much inter-firm diversity, that is $m^* < \bar{m}$, and (ii) too little intra-firm diversity, that is $n^* > \bar{n}$. Now, the first result leads to R&D under-investment in the decentralized economy (plug (i) into CME); too many firms operating in the market equilibrium reduce the innovation rate below the optimum (*market fragmentation*). The second result, instead, leads to over-investment in R&D (plug (ii) into CME); too few varieties per firm in the market equilibrium reduce the burden of the proliferation costs and, consequently, a larger number of workers in the decentralized economy can be employed in R&D activities. Since the algebraic solution for $\bar{g}$ is quite complicated, it is not straightforward to make a direct comparison between $\bar{g}$ and g^*; we focus on this issue in Appendix B2 where it is shown that the innovation rate in the market economy is insufficiently low. This result is summarized in the following Proposition:

Proposition 2 *In presence of multi-product enterprises, the incentive to innovate is lower in the market economy than in the social optimum; therefore, there is too little growth under laissez-faire.*

Proof. See Appendix B2. ∎

Some intuitive insight into Proposition 2 may be obtained by observing that, in this model, the rate of cost reduction depends on the scale of the R&D activity undertaken by each individual firm; dynamic increasing returns are internal to the firm and economic growth depends on average R&D, not aggregate R&D. Now, the larger mass of firms realized in the market equilibrium induces dispersion of R&D resources; this limits exploitation of scale economies internal to the firm and slows down growth. In other words, the excessive fragmentation of market structure in the decentralized economy reduces the ability to apply resources to innovation and represents the source of the economic under-performance in the market equilibrium.

As previously observed, as the mass of firms becomes large, the V1 and V2 curves become similar to their counterparts V1* and V2*; the market solutions for m, v and n coincide with the ones chosen by the social planner. Now, plugging these results into the CME locus gives the same growth rate in two configurations, market economy and social optimum. Therefore, we

can conclude that a monopolistically competitive economy achieves Pareto optimality in all the regards: R&D investment, firms' entry, and provision of product varieties.

5.3   Comparison with Peretto (1999)

Here, we compare the normative results of our model with the ones found by Peretto (1999) (see Result 4 in Peretto, 1999, page 192).

In Peretto (1999), the price elasticity of demand perceived by oligopolistic firms is lower than the elasticity of product substitution. This distortion, which represents the source of the divergence between the social optimum and the decentralized equilibrium, makes competition softer and leads to a lower rate of return to R&D. Since firms spend too little on R&D, the rate of return to entry becomes too high and too many producers enter the market. Therefore, the economy described by Peretto (1999) is Pareto inefficient because there are insufficient growth and excessive variety; moreover, as the number of firms goes to infinity, the price elasticity of demand converges to the elasticity of product substitution and the decentralized equilibrium tends to the social optimum.

As already mentioned in the Introduction, in our model the market provides globally too little variety, although there are over-entry into the industry and insufficient growth like in Peretto (1999). At this regard, we observe that in Peretto (1999) the result of excessive entry translates into a too large product variety because there exists a one-to-one correspondence between firm and variety; in fact, firms produce a single variety and there is no *cannibalization effect* at work in his model. In our paper, instead, over-entry into the industry does not lead to excessive variety because we allow each firm to produce more than one variety; in fact, in presence of multi-product firms, the entry decision of a potential competitor and the choice of product range are interdependent: a new entrant leads to a contraction of existing firms' product ranges and this, in turn, induces entry. This explains why in our set-up, firms offer too little product varieties, both individually and in total, although the market accommodates too many enterprises. However, convergence to the social optimum occurs also in our model, because the *cannibalization effect* and the *strategic interaction effect*, that are responsible for the under-expansion of firms' product ranges, vanish as the mass of firms goes to infinity.

6   Conclusions

In this paper, we develop a general equilibrium model of endogenous growth in order to analyze an economy with multi-product firms which carry out in-house R&D programs intended for the achievement of cost-reducing innovations. Multi-product enterprises compete *à la* Bertrand on the product market. Each firm co-ordinates its pricing decisions across all the varieties produced in order to maximize overall profits. In doing so, firms internalize competition within their product lines; at this regard, our analysis identifies the *cannibalization effect* which any given product generates upon other product lines of the same firm. In addition, we allow for inter-firm competition; in fact, we model multi-product firms as *large actors* which interact strategically on the product market. In such a framework, the mass of firms as well as the mass of product varieties per firm are endogenous and jointly determine market structure in equilibrium.

In the comparison between social optimum and decentralized economy, we show that the market equilibrium involves too many firms and too few products per firm; in addition, we find that under *laissez-faire* the total mass of varieties and productivity growth are inefficiently low. Therefore, in our model under-investment in R&D is accompanied by under-provision of product varieties; the latter result occurs both at the firm level and at the level of the whole economy. In absence of technological externalities, such as knowledge spillovers across firms, our findings are driven by the fact that firms interact strategically on the product market. Multi-product enterprises choose inefficiently narrow product ranges in order to internalize the *cannibalization effect* and relax price competition: this exacerbates excessive entry of firms since the choice of product range and the entry decision of a potential competitor are interdependent. The resulting market fragmentation limits the ability to invest resources in R&D and, consequently, the market provides too little growth. Decentralization of the first-best social optimum thus requires subsidies to promote the creation of product varieties and R&D activities by individual firms; in addition, an entry fee increasing the cost of entry is welfare improving.[27] At this regard, it is interesting to shed light on some recent studies whose normative results are consistent with the policy implications of our model. Starting from the seminal contribution of Chamberlin (1933), a great deal of theoretical research shows that free entry can lead to social

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[27]In order to achieve the optimal number of industry participants, one can think of a different setting where the only form of industry regulation is given by the determination of the number of operating licenses that are awarded. In our model, entry restrictions increase welfare.

inefficiency (see, e.g., Dixit and Stiglitz, 1977; Mankiw and Whinston, 1986; Anderson, de Palma and Nesterov, 1995). From the empirical side, Berry and Waldfogel (1999) recently quantify inefficiency associated to free entry by considering data from the radio industry in the United States. The empirical results show a large degree of business stealing by new stations, which in turn implies a large welfare loss (to market participants) from free entry. The authors find out that the there is excessive entry into the radio industry and welfare loss from free entry, as opposed to the social optimum, is about 40% of industry revenue, which suggests that entry restrictions are welfare improving. Mowery and Rosenberg (1989) provide evidence on the fact that excessive fragmentation of market structure limits ability to apply resources to innovation. The authors show how a period of *market restructuring* occurred across all U.S. industrial sectors at the beginning of the 20th century; the increased market control stimulated innovative activity by allowing the accumulation of resources by the firm. There were, in fact, a widespread expansion of R&D departments and a general increase of innovative activity.

The idea that entry fees to discipline entry could be introduced by governments to achieve social optimality is also consistent with the conclusions of some recent papers. As concerning the literature on multi-product firms, Anderson and de Palma (1992, 2006) show that the market system attracts too many firms with too few products per firm; this conclusion suggests that entry fees and subsidies to promote product varieties increase welfare. As regards the literature on economic growth and R&D, in Peretto (1999) the Pareto inefficiency of the economy is characterized by insufficient growth and excessive entry, which implies that an R&D subsidy and an entry fee increasing the cost of entry are necessary to restore Pareto optimality. In two companion papers, Etro (2005, 2006) develop a Schumpeterian growth model where incumbent patentholders are leaders in the patent races, invest in R&D, and enjoy a monopolistic position that is partially persistent. The author describes an equilibrium characterized by an inefficient bias toward too small firms in the market for innovation; in these papers, Etro shows that the social optimum can be achieved with two policy tools, an R&D subsidy, which optimally allocates resources between investors and an entry fee, which targets the optimal number of firms.[28]

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[28]In Etro (2005), the author outlines that the dynamic inefficiency of the growth process is able to explain why a country with an industrial structure characterized by small enterprises does not achieve efficient results. The author observes that this conclusion is interesting at the light of the low growth performance of countries that lack large and innovative corporations. Etro concludes that this is the case of many European countries, and in particular Italy, whose industrial structure is characterized by a great number of small

Despite we simultaneously model the accumulation of firm-specific knowledge, the entry decisions of new firms and the product choices of multi-product firms, the framework developed in this paper remains still tractable and is suitable for studying other interesting issues that deserve to be further investigated. In the current paper, for instance, we abstract from knowledge spillovers which affect the appropriability of the returns from innovation and potentially play a role in the determination of market structure. In presence of multi-product firms, in fact, market structure may range from *concentrated equilibria* (in which either one or a small mass of firms each produce many varieties) to *fragmented equilibria* (in which a large mass of firms produce either one or a small range of varieties): an issue, that we plan to address in future research, regards the analysis of how changes in the technological appropriability and opportunity conditions contribute to the appearance of the various possible equilibria.

Appendix A

A.1. Calculations for the demand elasticities

Transforming the demand function X_{ij} (9) into logarithms, we have:

$$\ln X_{ij} = (\theta - 1) \ln q + \ln LE - \delta \ln p_{ij} - (\theta - \delta) \ln q_i.$$

Given Eq. (6), it is easy to show that $\partial \ln q_i / \partial \ln p_{ij} = (p_{ij}/q_i)^{1-\delta}$. Therefore, the effect of p_{ij} on the market price index q is:

$$\frac{\partial \ln q}{\partial \ln p_{ij}} = \frac{\partial \ln q}{\partial \ln q_i} \frac{\partial \ln q_i}{\partial \ln p_{ij}} = \left(\frac{q_i}{q}\right)^{1-\theta} \left(\frac{p_{ij}}{q_i}\right)^{1-\delta}.$$

We have the following expressions for the elasticities:

$$\frac{\partial \ln X_{ij}}{\partial \ln p_{ij}} = -\delta - (\theta - \delta) \left(\frac{p_{ij}}{q_i}\right)^{1-\delta} + (\theta - 1) \left(\frac{q_i}{q}\right)^{1-\theta} \left(\frac{p_{ij}}{q_i}\right)^{1-\delta}, \qquad \text{(A.1)}$$

$$\frac{\partial \ln X_{ik}}{\partial \ln p_{ij}} = - (\theta - \delta) \left(\frac{p_{ij}}{q_i}\right)^{1-\delta} + (\theta - 1) \left(\frac{q_i}{q}\right)^{1-\theta} \left(\frac{p_{ij}}{q_i}\right)^{1-\delta}, \text{ for } k \neq j. \text{ (A.2)}$$

and medium size enterprises whose innovative capacity is quite limited. It is interesting to observe that our model shares the same normative conclusions of Etro (2005) although the growth set-up is quite different.

These calculations are important for firm's pricing strategy; each producer, in fact, internalizes the effects of competition both within its product line and between firms within the industry.

A.2. Dynamic optimization problem

The Current Value Hamiltonian for the maximization problem of firm i is:

$$\mathcal{H}_i = \int_{j=0}^{n_i} (p_{ij} X_{ij} - l_{xij}) \; \mathrm{d}j - l_{ri} - n_i l_f + \mu_i \vartheta l_{ri} h_i,$$

where p_{ij} and l_{rij} are the control variables (for every $j \in [0, n_i]$), h_i the state variable, μ_i the co-state variable associated to the dynamic constraint. Let us consider variety j for firm i; the optimality conditions are:

$$\frac{\partial \mathcal{H}_i}{\partial p_{ij}} = 0 \Rightarrow X_{ij} + \int_{j=0}^{n_i} \left(p_{ik} - \frac{1}{h_i} \right) \frac{\partial X_{ik}}{\partial p_{ij}} \; \mathrm{d}j = 0, \qquad (A.3)$$

$$\frac{\partial \mathcal{H}_i}{\partial l_{ri}} = 0 \Rightarrow 1 = \mu_i \vartheta h_i, \qquad (A.4)$$

$$\dot{\mu}_i = r\mu_i - \frac{\partial \mathcal{H}_i}{\partial h_i} \Rightarrow r\mu_i = \frac{\partial \pi_i}{\partial h_i} + \mu_i \vartheta l_{ri} + \dot{\mu}_i, \qquad (A.5)$$

$$\lim_{t \to \infty} R(t) \mu_i h_i = 0. \qquad (A.6)$$

We interpret economically conditions Eqs. (A.3)-(A.6).

Condition (A.3) tells us how firm i sets the variety j's price. To derive this condition, the effect of variety j's price upon the demand schedules of all the other varieties manufactured by firm i is taken into account.

Condition (A.4) regards the decision of investing in R&D targeted at the accumulation of firm-specific knowledge: more precisely, it says that the value of the marginal product of labor engaged in R&D activities has to be equal to its marginal cost.

Condition (A.5) represents a no-arbitrage condition telling that it is equivalent to invest an amount of money equal to μ_i on the capital market, receiving a return equal to $r\mu_i$, or in the creation of new firm-specific knowledge: in this case, there is a positive effect of the knowledge on profits, a positive effect on the accumulation of further knowledge and the capital gain $\dot{\mu}_i$.

Condition (A.6) is the usual transversality condition; it requires that at the end of the planning horizon firm-specific knowledge has no value.

A.3. Omitted details in the solution of the second stage of the game

Now, we use the results of the dynamic firm's optimization problem to determine the optimal pricing strategy and the return to R&D activity. This corresponds to solving the second stage of the game.

Using Eqs. (A.1) and (A.2), Eq. (A.3) becomes:

$$X_{ij} - \frac{\delta X_{ij}}{p_{ij}}\left(p_{ij} - \frac{1}{h_i}\right) +$$
$$- \int_{k=0}^{n_i} \frac{X_{ik}}{p_{ij}}\left(p_{ik} - \frac{1}{h_i}\right)\left[\theta - \delta - (\theta - 1)\left(\frac{q_i}{q}\right)^{1-\theta}\right]\left(\frac{p_{ij}}{q_i}\right)^{1-\delta} \, dk = 0, \quad \text{(A.7)}$$

which, by using Eq. (9), can be rewritten as:

$$\frac{LE}{q^{1-\theta}}q_i^{1-\theta} - \delta\frac{LEq_i^{1-\theta}}{q^{1-\theta}}\frac{\left(p_{ij} - \frac{1}{h_i}\right)}{p_{ij}} = \int_{k=0}^{n_i} X_{ik}\left(p_{ik} - \frac{1}{h_i}\right)\left[\theta - \delta - (\theta - 1)\left(\frac{q_i}{q}\right)^{1-\theta}\right] \, dk.$$

One can observe that the right hand side of this equation is the same for all $j \in [0, n_i]$. This implies that $p_{ij} = p_i$ for all $j \in [0, n_i]$; consequently, the term $(p_{ij}/q_i)^{1-\delta}$ in Eq. (A.7) becomes equal to $1/n_i$. Using Eqs. (6) and (8), we get:

$$\left(\frac{q_i}{q}\right)^{1-\theta} = \frac{n_i^{\frac{1-\theta}{1-\delta}}p_i^{1-\theta}}{\left(\int_{i=0}^{m} n_i^{\frac{1-\theta}{1-\delta}}p_i^{1-\theta} \, di\right)}. \quad \text{(A.8)}$$

Then, Eq. (A.7) can be simplified as:

$$p_i = \frac{1}{h_i}\frac{[\theta - (\theta - 1)\epsilon_i]}{(\theta - 1)(1 - \epsilon_i)}, \quad \text{(A.9)}$$

where $\epsilon_i = \left(\frac{q_i}{q}\right)^{1-\theta}$.

Finally, we determine the rate of return to R&D investment; this can be obtained from Eqs. (A.4) and (A.5) as follows. We divide both sides of eq. (A.5) by μ_i to get:

$$r = \frac{1}{\mu_i}\frac{\partial \pi_i}{\partial h_i} + \vartheta l_{ri} + \frac{\dot{\mu}_i}{\mu_i} = \vartheta h_i\frac{\partial \pi_i}{\partial h_i} + \vartheta l_{ri} - \frac{\dot{h}_i}{h_i},$$

where we use the fact that $\mu_i = 1/(\vartheta h_i)$ according to Eq. (A.4). In the previous equation, the term $\partial \pi_i/\partial h_i$ equals:

$$\frac{\partial \pi_i}{\partial h_i} = -\int_{j=0}^{n_i} \frac{\partial l_{xij}}{\partial h_i} \, dj = \int_{j=0}^{n_i} \frac{X_{ij}}{h_i^2} \, dj,$$

since $l_{xij} = X_{ij}/h_i$ by Eq. (10); therefore, the rate of return to R&D can be written as:

$$r = \vartheta h_i \int_{j=0}^{n_i} \frac{X_{ij}}{h_i^2} \, \mathrm{d}j + \vartheta l_{ri} - \frac{\dot{h}_i}{h_i} = \vartheta \int_{j=0}^{n_i} l_{xij} \, \mathrm{d}j + \vartheta l_{ri} - \frac{\dot{h}_i}{h_i}.$$

The rate of return to R&D presents three components. First, there is the effect of innovation on the cash flow (the term $\vartheta \int_{j=0}^{n_i} l_{xij} \, \mathrm{d}j$). Second, it is important to observe the effect of R&D on the process of knowledge accumulation (the term ϑl_{ri}). In fact, investing in R&D allows firms to increase their level of knowledge; this contributes to improve the efficiency of production and stimulates the process of accumulation of firm-specific knowledge. Finally, we have the change in the value of knowledge (the term $-\dot{h}_i/h_i$); this term captures the fact that if the value of knowledge increases (decreases) over time, investing in R&D becomes more (less) attractive.

By using Eq. (11), the expression for the rate of return to R&D simplifies and becomes:

$$r = \vartheta \int_{j=0}^{n_i} l_{xij} \, \mathrm{d}j,$$

which corresponds with Eq. (14) in the text.

A.4. Omitted details in the solution of the first stage of the game

In the following, we look at the first stage of the game that consists in determining the level of proliferation per firm, given the decisions taken in the second stage.

Eq. (A.9) implies that the inverse of the mark-up can be written as:

$$\frac{p_i}{p_i - 1/h_i} = \theta - \epsilon_i(\theta - 1). \tag{A.10}$$

By using Eqs. (6), (8), (9), (10), (A.9) and (A.10), the gross profits of firm i can be written as:

$$
\begin{aligned}
\pi_i &= n_i(p_i X_i - l_{xi}) - l_{ri} - n_i l_f - l_k \\
&= LE \frac{\epsilon_i \left(p_i - \frac{1}{h_i} \right)}{p_i} - l_{ri} - n_i l_f - l_k \\
&= LE \left[1 - \frac{\theta}{(\theta - 1)} \frac{1}{p_i h_i} \right] - l_{ri} - n_i l_f - l_k.
\end{aligned}
$$

Differentiation of π_i with respect to n_i yields the first order condition:

$$\frac{\partial \pi_i}{\partial n_i} = LE\frac{\theta}{(\theta-1)}\frac{1}{p_i^2 h_i}\frac{\partial p_i}{\partial n_i} - l_f = 0. \tag{A.11}$$

Summing (A.10) over i gives:

$$\int_{i=0}^{m} \frac{p_i}{p_i - 1/h_i} \, \mathrm{d}i = 1 + (m-1)\theta.$$

Differentiating this equation with respect to n_i yields:

$$\int_{k=0}^{m} \frac{\partial p_k}{\partial n_i} \, \mathrm{d}k = 0. \tag{A.12}$$

Under symmetry, $p_i = p$, $h_i = h$ and $n_i = n$ for every i; then, Eq. (A.10) becomes:

$$\frac{p}{p - 1/h} = \theta - \frac{(\theta-1)}{m}. \tag{A.13}$$

and Eq. (A.12) can be written as:

$$(m-1)\frac{\partial p_j}{\partial n_i} + \frac{\partial p_i}{\partial n_i} = 0. \tag{A.14}$$

By rearranging Eq. (A.10), we get:

$$\frac{\frac{p_i}{p_i-1/h_i} - \theta}{\frac{p_j}{p_j-1/h_j} - \theta} = \frac{\epsilon_i}{\epsilon_j} = \frac{n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta}}{n_j^{\frac{1-\theta}{1-\delta}} p_j^{1-\theta}}.$$

Cross-multiplying the terms of this equation and then differentiating with respect to n_i gives:

$$\left[\frac{1/h_j}{(p_j - 1/h_j)^2} n_i^{\frac{1-\theta}{1-\delta}} p_i^{1-\theta} + (1-\theta)\left(\frac{p_i}{p_i - 1/h_i} - \theta\right) n_j^{\frac{1-\theta}{1-\delta}} p_j^{-\theta} \right] \frac{\partial p_j}{\partial n_i}$$
$$= \left[\frac{1/h_i}{(p_i - 1/h_i)^2} n_j^{\frac{1-\theta}{1-\delta}} p_j^{1-\theta} + (1-\theta)\left(\frac{p_j}{p_j - 1/h_j} - \theta\right) n_i^{\frac{1-\theta}{1-\delta}} p_i^{-\theta} \right] \frac{\partial p_i}{\partial n_i}$$
$$+ \frac{(\theta-1)}{(\delta-1)}\left(\frac{p_j}{p_j - 1/h_j} - \theta\right) n_i^{\frac{\delta-\theta}{1-\delta}} p_i^{1-\theta}.$$

This equation, combined with Eq. (A.14), becomes:

$$\frac{m}{(m-1)}\left[\frac{1/h}{(p - 1/h)^2} n^{\frac{1-\theta}{1-\delta}} p^{1-\theta} + (1-\theta)\left(\frac{p}{p - 1/h} - \theta\right) n^{\frac{1-\theta}{1-\delta}} p^{-\theta} \right] \frac{\partial p_i}{\partial n_i}$$
$$= -\frac{(\theta-1)}{(\delta-1)}\left(\frac{p}{p - 1/h} - \theta\right) n^{\frac{\delta-\theta}{1-\delta}} p^{1-\theta}.$$

Finally, using Eq. (A.13) into the previous equation gives:

$$\frac{\partial p_i}{\partial n_i} = \frac{(\theta - 1)}{(\delta - 1)} \frac{p}{n} \left[\frac{m - 1}{m\theta(m - 1) + \theta - 1} \right]. \tag{A.15}$$

One can observe that an increase in the level of a firm's proliferation raises the firm's own price and reduces the prices of other firms according to Eq. (A.14).

Plugging $\partial p_i / \partial n_i$ from Eq. (A.15) into the first order condition Eq. (A.11) yields Eq. (20) in the text. The profit-maximizing mass of varieties produced by each firm is:

$$n = \frac{LE\,(m - 1)^2\,(\theta - 1)\,\theta}{l_f\,(\delta - 1)\,[(m - 1)\theta + 1]\,[m\theta(m - 1) + \theta - 1]}.$$

A.5. Stability of the equilibrium

Combining Eq. (21) with Eq. (25) yields:

$$nl_f = nl_x \frac{m\theta(m - 1)}{(\delta - 1)\,[m\theta(m - 1) + \theta - 1]}. \tag{A.16}$$

Plugging Eqs. (A.16) and (25) into the zero profit condition (24) gives:

$$nl_x = \left(\frac{g}{\vartheta} + l_k \right) \left[\frac{1}{M(m) - 1} - \frac{1}{(\delta - 1)\,G(m)} \right]^{-1}.$$

where firm size nl_x depends positively on the rate of growth g and the firm-level fixed cost l_k. The reason is that R&D expenditure, which is a sunk cost that is borne at each moment in time, and the firm-level fixed cost affect negatively incumbent's profits and lead to a smaller mass of the active firms (larger firm size) through the zero-profit condition.

Now, the expression for firm size can be used into Eq. (23) to obtain the rate of return to R&D:

$$r_{\text{R\&D}} = \vartheta \left(\frac{g}{\vartheta} + l_k \right) \left[\frac{1}{M(m) - 1} - \frac{1}{(\delta - 1)\,G(m)} \right]^{-1}. \tag{A.17}$$

It is interesting to observe that this rate of return depends positively on the rate of growth and the firm-level fixed cost. As Eq. (27) shows, the rate of return on savings is also increasing in the rate of innovation.

Eqs. (27) and (A.17) are depicted in the space (r, g); in Figure 2 we label these two relations RC and RF respectively. Consider a situation in which
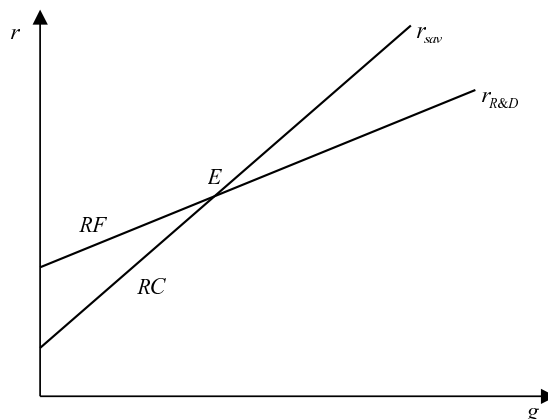
Figure 2: Stability of the equilibrium

the growth rate is below the equilibrium growth rate. In such a situation, the realized rate of return on R&D activity exceeds the rate of return on consumers' savings. Observing this, firms realize that they can invest in projects with a higher rate of return than required by consumers; at the same time, consumers can save more at a higher rate of return than required. This gives rise to an adjustment process of expectations that ends only when the equilibrium is achieved; we observe that the opposite occurs when the growth rate is above the equilibrium growth rate. Obviously, this logic applies only when the RC locus intersects the RF locus from below; in fact, if this does not occur, the adjustment process takes the economy away from the equilibrium. Therefore, stability requires that the RC locus is steeper than the RF locus, that is $\sigma - 1 > \left[\frac{1}{M(m)-1} - \frac{1}{(\delta-1)G(m)} \right]^{-1}$. Moreover, the growth rate is positive when the intercept on the vertical axis of the RF locus is higher than the one of the RC locus; this occurs when $\left[\frac{1}{M(m)-1} - \frac{1}{(\delta-1)G(m)} \right]^{-1} > \frac{\rho}{\vartheta l_k}$. Consequently, we apply the following parameter restriction:

$$\sigma - 1 > \left[\frac{1}{M(m) - 1} - \frac{1}{(\delta - 1)\, G(m)} \right]^{-1} > \frac{\rho}{\vartheta l_k}, \tag{A.18}$$

which gives rise to Assumption 1:

$$l_k > \frac{\rho}{\vartheta(\sigma - 1)}.$$

A.6. Some comparative-static exercises

In this Appendix, we study the impact of the population size L on the main economic variables and check for the scale effect in the model. We reduce the three-dimensional system, given by CME, PR and ZP, to a two-dimensional system in the mass of firms, m, and the growth rate, g. Combining ZP with CME and ZP with PR gives respectively:

$$g = \vartheta \frac{L}{m} \left(1 - \frac{1}{M}\right) \frac{1}{(\sigma - 1)} - \frac{\rho}{(\sigma - 1)}, \qquad (A.19)$$

$$g = \vartheta \frac{L}{m} \left[\frac{1}{M} + \frac{1}{M(\delta - 1)G} - \frac{1}{(\delta - 1)G}\right] - \vartheta l_k, \qquad (A.20)$$

where $M = \frac{\theta(\bar{m}-1)+1}{\bar{m}}$ and $G = \frac{\bar{m}\theta(\bar{m}-1)+\theta-1}{\bar{m}\theta(\bar{m}-1)}$. Totally differentiating Eqs. (A.19) and (A.20) with respect to L and rewriting the result in a matrix form yield:

$$\Gamma \cdot \begin{bmatrix} \frac{dm}{dL} \\ \frac{dg}{dL} \end{bmatrix} = \begin{bmatrix} \frac{\vartheta}{m}\left(1 - \frac{1}{M}\right)\frac{1}{(\sigma-1)} \\ \frac{\vartheta}{m}\left[\frac{1}{M} + \frac{1}{M(\delta-1)G} - \frac{1}{(\delta-1)G}\right] \end{bmatrix},$$

where Γ is equal to:

$$\Gamma \equiv \begin{bmatrix} \vartheta\frac{L}{m^2}\left(1 - \frac{1}{M}\right)\frac{1}{(\sigma-1)} - \vartheta\frac{L}{m}\frac{M'}{M^2}\frac{1}{(\sigma-1)} & 1 \\ \vartheta\frac{L}{m^2}\left[\frac{1}{M} + \frac{1}{M(\delta-1)G} - \frac{1}{(\delta-1)G}\right] + \vartheta\frac{L}{m}\left[\frac{M'}{M^2} + \frac{M'}{M^2(\delta-1)G} + \frac{G'}{M(\delta-1)G^2} - \frac{G'}{(\delta-1)G^2}\right] & 1 \end{bmatrix}.$$

Applying the Cramer's rule, we have:

$$\frac{d\bar{m}}{dL} = \frac{|\Gamma_1|}{|\Gamma|},$$

$$\frac{d\bar{g}}{dL} = \frac{|\Gamma_2|}{|\Gamma|},$$

where Γ_i is a transformed matrix with the solution column replacing column i of matrix Γ. After some computations, we get:

$$\begin{aligned} |\Gamma| &= -\vartheta\frac{L}{m^2}\left[\frac{1}{M} + \frac{1}{M(\delta-1)G} - \frac{1}{(\delta-1)G} - \left(1 - \frac{1}{M}\right)\frac{1}{(\sigma-1)}\right] + \\ &\quad -\vartheta\frac{L}{m}\left[\frac{M'}{M^2} + \frac{M'}{M^2(\delta-1)G} - \frac{G'}{(\delta-1)G^2}\left(1 - \frac{1}{M}\right) + \frac{M'}{M^2}\frac{1}{(\sigma-1)}\right], \end{aligned}$$

$$|\Gamma_1| = -\frac{\vartheta}{m}\left[\frac{1}{M} - \frac{1}{(\delta-1)G}\left(1 - \frac{1}{M}\right) - \left(1 - \frac{1}{M}\right)\frac{1}{(\sigma-1)}\right],$$

$$|\Gamma_2| = -\vartheta \frac{L}{m} \frac{M'}{M^2} \frac{1}{(\sigma-1)} \frac{\vartheta}{m} \left[\frac{1}{M} - \frac{1}{(\delta-1)G} \left(1 - \frac{1}{M}\right)\right] +$$
$$- \frac{\vartheta}{m}\left(1 - \frac{1}{M}\right)\frac{1}{(\sigma-1)}\left\{\vartheta\frac{L}{m}\left[\frac{M'}{M^2} + \frac{M'}{M^2(\delta-1)G} - \frac{G'}{(\delta-1)G^2}\left(1 - \frac{1}{M}\right)\right]\right\}.$$

It is easy to check that the determinants of matrices Γ, Γ_1 and Γ_2 are smaller than zero under the parameter restriction (A.18). Consequently, we get that $\frac{d\bar{m}}{dL} > 0$ and $\frac{d\bar{g}}{dL} > 0$; this means that an increase in population raises the mass of firms and the growth rate. However, this (positive) scale effect vanishes asymptotically. We show this feature of the model by expressing the equilibrium growth rate $\bar{g}$ as a function of the equilibrium mass of firms $\bar{m}$, $\bar{g}(\bar{m})$; in fact, the intersection of CME, PR, and ZP gives:

$$\bar{g}(\bar{m}) = \frac{G\vartheta l_k \left(\delta - 1\right)\left(M - 1\right) - \rho\left[1 - M + G(\delta - 1)\right]}{\sigma - 1 + G\sigma\left(\delta - 1\right) - M\left[\sigma - 1 + G(\delta - 1)\right]}.$$

Now, as L increases and $\bar{m}$ becomes large, we have that M and G tend respectively to θ and 1. Consequently, the growth rate of the oligopolistic economy $\bar{g}(\bar{m})$ converges to the one of monopolistic competition:

$$\frac{\vartheta l_k \left(\delta - 1\right)\left(\theta - 1\right) - \rho(\delta - \theta)}{\sigma\delta - 1 - \theta\left(\sigma + \delta - 2\right)}.$$

The main implication of this result is that growth becomes independent of the size of the labor force if L is allowed to grow exponentially.

Finally, we study the impact of L on $\bar{n}$; we rewrite (V1) and (V2) as:

$$n = \frac{\frac{L}{m}}{(\sigma - 1)l_f}\left(\frac{\sigma}{M} - 1\right) - \frac{1}{l_f}\left[l_k - \frac{\rho}{\vartheta(\sigma - 1)}\right], \qquad (A.21)$$

$$v = \frac{\frac{L}{m}\left(1 - \frac{1}{M}\right)}{l_f\left(\delta - 1\right)G}. \qquad (A.22)$$

Totally differentiating Eqs. (A.21) and (A.22) with respect to L and rewriting the result in a matrix form yield:

$$\Sigma \cdot \begin{bmatrix} \frac{dm}{dL} \\ \frac{dn}{dL} \end{bmatrix} = \begin{bmatrix} \frac{\frac{1}{m}}{(\sigma-1)l_f}\left(\frac{\sigma}{M} - 1\right) \\ \frac{\frac{1}{m}\left(1 - \frac{1}{M}\right)}{l_f(\delta-1)G} \end{bmatrix}$$

where $\Sigma \equiv \begin{bmatrix} \frac{\frac{L}{m^2}}{(\sigma-1)l_f}\left(\frac{\sigma}{M} - 1\right) + \frac{M'}{M^2}\sigma\frac{\frac{L}{m}}{(\sigma-1)l_f} & 1 \\ -\frac{L}{l_f(\delta-1)}\frac{\left[-\frac{1}{m^2}\left(1 - \frac{1}{M}\right) + \frac{M'}{M^2}\frac{1}{m}\right]G - \frac{G'}{m}\left(1 - \frac{1}{M}\right)}{G^2} & 1 \end{bmatrix}$. Applying the Cramer's rule, we get:

$$\frac{d\bar{m}}{dL} = \frac{|\Sigma_1|}{|\Sigma|},$$

$$\frac{d\bar{n}}{dL} = \frac{|\Sigma_2|}{|\Sigma|},$$

where Σ_i is a transformed matrix with the solution column replacing column i of matrix Σ. After some computations, we get:

$$|\Sigma| = \frac{L}{ml_f}\left\{\frac{1}{m}\left[\frac{1}{(\sigma-1)}\left(\frac{\sigma}{M}-1\right)-\frac{1}{(\delta-1)G}\left(1-\frac{1}{M}\right)\right]+\right.$$
$$\left.+\frac{M'}{M^2}\left[\frac{1}{(\delta-1)G}+\frac{\sigma}{(\sigma-1)}\right]-\frac{G'}{(\delta-1)G^2}\left(1-\frac{1}{M}\right)\right\},$$

$$|\Sigma_1| = \frac{1}{ml_f}\left[\frac{1}{(\sigma-1)}\left(\frac{\sigma}{M}-1\right)-\frac{1}{(\delta-1)G}\left(1-\frac{1}{M}\right)\right],$$

$$|\Sigma_2| = \frac{L}{m^2(\delta-1)Gl_f^2}\left[\frac{M'}{M^2}-\frac{G'}{(\sigma-1)G}\left(1-\frac{1}{M}\right)\left(\frac{\sigma}{M}-1\right)\right].$$

One can check that the determinants of matrices Σ, Σ_1 and Σ_2 are larger than zero under the parameter restriction (A.18). This implies that $\frac{d\bar{n}}{dL} > 0$ and $\frac{d\bar{m}}{dL} > 0$ (as confirmed by the previous comparative-static exercise); we conclude that an increase in population raises the mass of varieties produced by each firm.

Appendix B

B.1. Social optimum

Under symmetry within each firm and across firms and using the labor market clearing condition, the current value Hamiltonian can be written as follows:

$$\mathcal{H} = \frac{\left(l_x h m^{\frac{\theta}{\theta-1}} n^{\frac{\delta}{\delta-1}}\right)^{1-\sigma}}{1-\sigma} + \mu\vartheta l_r h = \frac{\left[\left(\frac{L}{mn}-\frac{l_r}{n}-l_f-\frac{l_k}{n}\right)hm^{\frac{\theta}{\theta-1}}n^{\frac{\delta}{\delta-1}}\right]^{1-\sigma}}{1-\sigma} + \mu\vartheta l_r h,$$

where l_r, m and n are the control variables, h the state variable, μ the co-state variable associated to the dynamic constraint. The optimality conditions are:

$$\frac{\partial\mathcal{H}}{\partial l_r} = 0 \Rightarrow \frac{C^{-\sigma}m^{\frac{\theta}{\theta-1}}n^{\frac{1}{\delta-1}}}{\vartheta} = \mu, \tag{B.1}$$

$$\frac{\partial\mathcal{H}}{\partial m} = 0 \Rightarrow \frac{L}{mn} = \theta\left(\frac{l_r}{n}+l_f+\frac{l_k}{n}\right), \tag{B.2}$$

$$\frac{\partial\mathcal{H}}{\partial n} = 0 \Rightarrow \frac{L}{mn} = \delta l_f + \frac{l_r}{n}+\frac{l_k}{n}, \tag{B.3}$$

$$\dot{\mu} = \rho\mu - \frac{\partial \mathcal{H}}{\partial h} \Rightarrow \rho\mu = C^{-\sigma}\left[\left(\frac{L}{mn} - \frac{l_r}{n} - l_f - \frac{l_k}{n}\right)m^{\frac{\theta}{\theta-1}}n^{\frac{\delta}{\delta-1}}\right] + \mu\vartheta l_r + \dot{\mu}, \tag{B.4}$$

$$\lim_{t\to\infty} \mu e^{-\rho t} h = 0. \tag{B.5}$$

Differentiating Eq. (B.1) with respect to time gives:

$$\frac{\dot{\mu}}{\mu} = -\sigma\frac{\dot{C}}{C} + \frac{\theta}{\theta-1}\frac{\dot{m}}{m}. \tag{B.6}$$

Using Eqs. (B.1) and (B.4), we get:

$$\frac{\dot{\mu}}{\mu} = \rho - \vartheta\frac{L}{m} + \vartheta n l_f + \vartheta l_k. \tag{B.7}$$

Now, differentiating C with respect to time yields:

$$\frac{\dot{C}}{C} = \frac{\theta}{\theta-1}\frac{\dot{m}}{m} + \frac{\dot{l}_x}{l_x} + \frac{\dot{h}}{h}. \tag{B.8}$$

Since the allocation of labor and the mass of producers are constant, Eqs. (B.6)-(B.8) give:

$$g = \frac{\vartheta}{\sigma}\left(\frac{L}{mn} - l_f - \frac{l_k}{n}\right) - \frac{\rho}{\sigma}, \tag{B.9}$$

which represents the optimal growth rate g for a given mass of firms m and varieties per firm n since Eqs. (B.2) and (B.3) have not been used to derive (B.9). It corresponds with the CME* locus in the text.

Using Eq. (22) into Eq. (B.3) yields:

$$n = \frac{\vartheta\frac{L}{m} - g - \vartheta l_k}{\vartheta l_f \delta}, \tag{B.10}$$

which gives the optimal mass of products per firm n for a given growth rate g and mass of firms m. It corresponds with the PR* locus in the text.

Finally, plugging (22) into (B.2) yields:

$$g = \vartheta\left(\frac{1}{\theta}\frac{L}{m} - n l_f - l_k\right), \tag{B.11}$$

that gives the optimal mass of producers m for a given growth rate g and mass of products per firm n. Relation (B.11) corresponds with the ZP* curve in the text.

B.2. Proof of Proposition 2

In this Appendix, we show that $\bar{g} < g^*$. A direct comparison between $\bar{g}$ and g^* is not possible because the solution for $\bar{g}$ is quite complicated. For this reason, we follow a different procedure; we express $\bar{g}$ as a function of $\bar{m}$ (first step), and then we show that $\bar{g}(\bar{m}) < g^*$ (second step).

As we saw above, the intersection of CME, PR, and ZP gives the equilibrium growth rate $\bar{g}$ as a function of the equilibrium mass of firms $\bar{m}$:

$$\bar{g}(\bar{m}) = \frac{G\vartheta l_k\,(\delta-1)\,(M-1) - \rho\,[1 - M + G(\delta-1)]}{\sigma - 1 + G\sigma\,(\delta-1) - M\,[\sigma - 1 + G(\delta-1)]}. \tag{B.12}$$

By using ZP*, CME*, and PR*, we get the growth rate in the social optimum g^*:

$$g^* = \frac{\vartheta l_k\,(\delta-1)\,(\theta-1) - \rho(\delta-\theta)}{\sigma\delta - 1 - \theta\,(\sigma + \delta - 2)}.$$

Now, we calculate the derivative of $\bar{g}(\bar{m})$ with respect to $\bar{m}$:

$$\frac{d\bar{g}(\bar{m})}{d\bar{m}} = \frac{(\delta-1)\,[-G'(1-M)^2 + M'G^2(\delta-1)]\,[\vartheta l_k\,(\sigma-1) - \rho]}{\{\sigma - 1 + G\sigma\,(\delta-1) - M\,[\sigma-1+G(\delta-1)]\}^2},$$

which is positive since $G' < 0$, $M' > 0$ and $[\vartheta l_k\,(\sigma-1) - \rho] > 0$ by Assumption 1. Since the growth rate of the oligopolistic economy $\bar{g}(\bar{m})$ is increasing in $\bar{m}$ and converges to the one of social optimum g^* when $\bar{m}$ becomes infinitely large (M and G tend respectively to θ and 1 when $\bar{m}$ goes to infinity), we conclude that $g^* > \bar{g}$.

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