Defining extreme volatility events at the SP 500 Index

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Working Paper

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In this paper we estimated not-overlapped monthly historic standard deviations of the S&P 500 Index returns for the period 1950 – 2009, then using extreme value theory we defined extreme volatility events and introduced an alternative "fear scale" that is compared with the "fear index".

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I. S&P 500 Index Volatility

We estimated daily returns of S&P 500 Index from January 4th, 1950 to December 31th, 2009, downloaded from Yahoo Finance, for a total 15,096 observations.

Figure I depicts S&P 500 Index monthly volatility, measured as not-overlapped standard deviation (using Excel STDEV function) of Index's returns recorded in a calendar month.

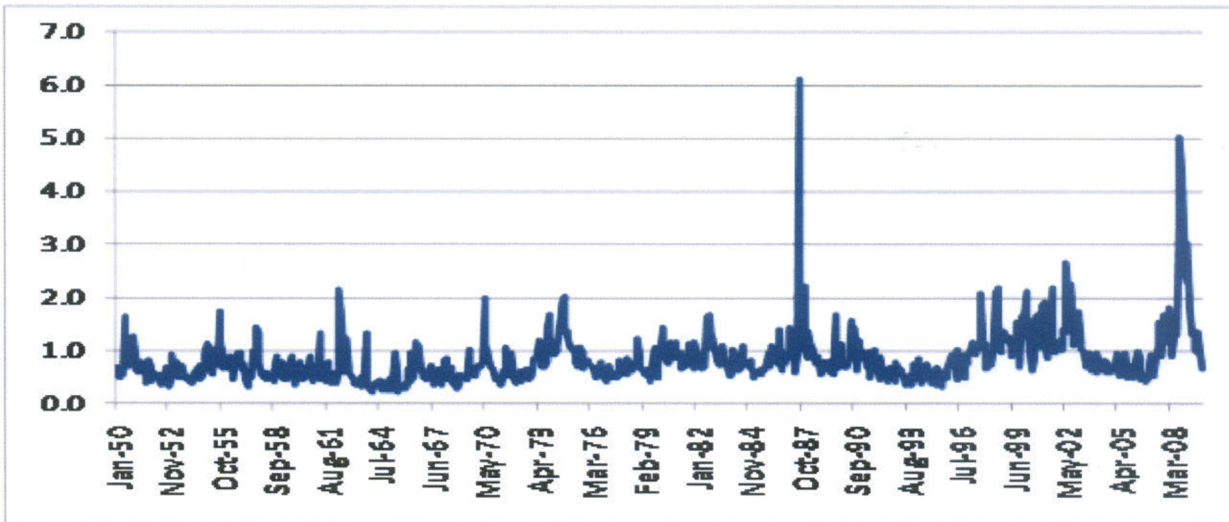

Fig. I. S&P 500 Index Monthly Volatility

Next table summarized main data statistics:

Historical	No. Observations	Mean	St. Dev	Min	Max
S&P 500 Index St.Dev	720	0.829	0.502	0.192	6.119

We analyzed registered volatility values to define extreme events using extreme value theory, specifically the peak over threshold (POT) method, instead of F-test (e.g. Dolde et. al. (2002)).

II. Extreme Events

Extreme events occur when a risk takes values from the tail of its distribution (McNeil, 1999).

Let $X = (X_1, \ldots, X_n)$ be independent identically distributed random variables with a unknown distribution function F.

Then for a large enough threshold μ, the conditional distribution of $Y = (X_i - \mu \mid X_i > \mu)$, under the Balkema and de Haan - Pickands Theorem, is approximately:

$$H(y) = \quad 1 - (1 + y\,\tfrac{\xi}{\sigma}\,)^{-1/\xi}\,) \qquad \text{for } \xi \neq 0$$

To apply peak over threshold method, first a threshold μ must be selected, and then a generalized pareto distribution (GPD) is estimated.

Using the extRemes Toolkit developed by Eric Gilleland, within statistical software R, we conducted a visual inspection of mean excess and computed GPD.

Figure 2 shows Mean Residual Life plot, looking for lowest threshold where plot is nearly linear.

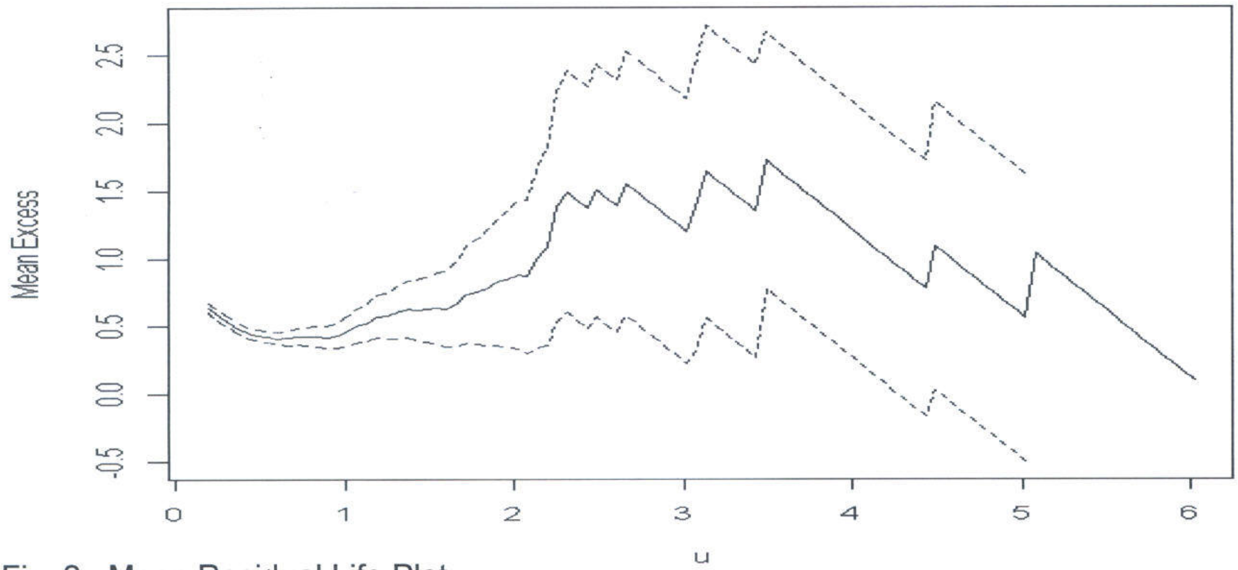

Fig. 2. Mean Residual Life Plot

A threshold of 1.0% volatility was selected, for a 165 exceedances of threshold given a GPD with parameters (μ): 1.0, (σ): 0.33015 and (ξ): 0.30041.

III. Fear Scale

The return level R_k^n is the level expected, on average, to be exceeded in one out of *k* periods of length *n*.

The return period is the amount of time expected to wait for particular return level to be exceed; return period is the inverse of the probability of an event (e.g. a called "100 years event" has a 1% probability of exceed the record level in a given year).

For a generalized pareto distribution, the k year return level is defined:[1]

$$R_k \approx \mu + \frac{\tilde{\sigma}}{\tilde{\xi}}([k^* n_y{}^* Pr(X>\mu)]^{\xi} - 1) \quad \text{for} \quad \tilde{\xi} \neq 0$$

where μ is the defined threshold, $\tilde{\sigma}$, and $\tilde{\xi}$ are the parameters of the GPD, n_y is the number of observations per year, and Pr (X>μ) is equal to number of exceedances of threshold (*Nu*) divided by total number of observations (*N*).

Figure 3 presents the return level plot from the GDP estimated in the previous section.

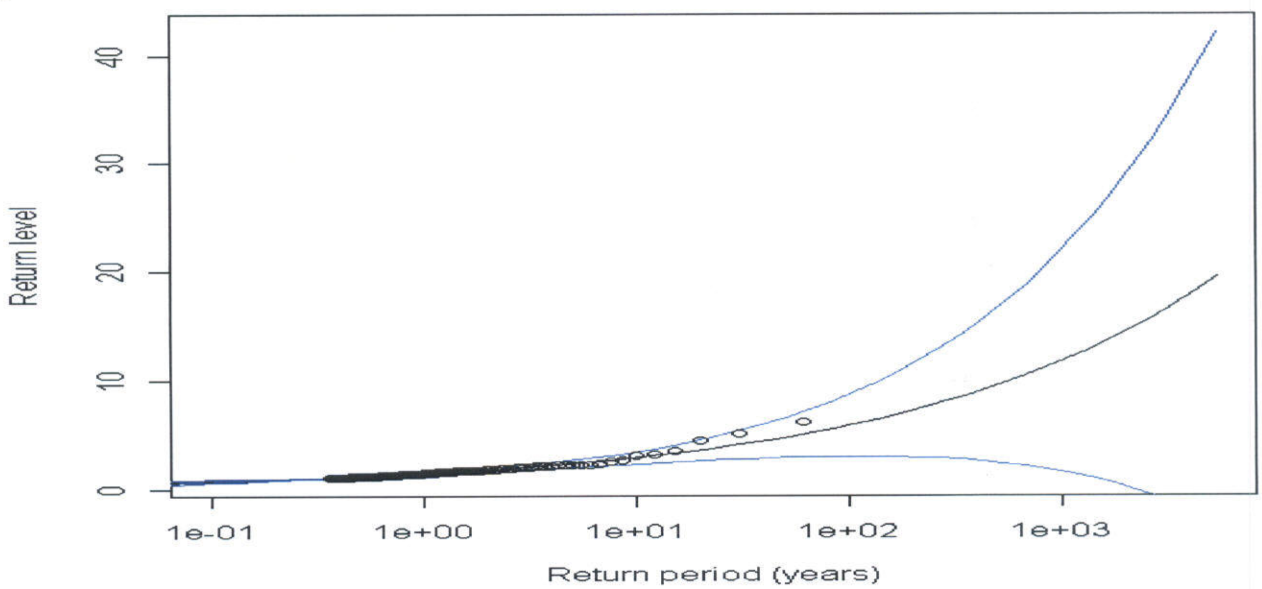

Fig. 3. Return Level Plot

[1] Under extreme value theory, value at risk at a high confidence level α (VaR_α) is equivalent to a particular k year return level (R_k) given:

$$k \approx 1 / [\, 1 - \alpha^{ny} \,]$$

For a GPD, the value at risk at a confidence level α is defined:

$$VaR_\alpha \approx \mu + \frac{\tilde{\sigma}}{\tilde{\xi}}\left(\left[\frac{Nu}{N*(1-\alpha)}\right]^{\xi} - 1\right) \quad \text{for} \quad \tilde{\xi} \neq 0$$

We defined a 100, 20 and 10 years event (1%, 5%, and 10% probability of exceed the record level in a given year) as extreme volatility events used as limits of a fear scale, for a named panic, fear and alarm level, respectively. The obtained limits values are: 5.84%m 3.56% and 2.88%.

S&P 500 Index Fear Scale

Level	St. Dev. Of Montly Returns
Panic >=	5.8%
Fear >=	3.6%
Alarm >=	2.9%

The analysis of the S&P 500 Index monthly volatility from 1950 to 2009, computed only one panic scenario, October 1987 that includes Black Monday of October 19th, 1987 where the largest one-day percentage decline in stock market's history of 22.9% was recorded, two fear scenarios, October 2008 and November 2008, and three additional alarm scenarios, September 2008, December 2008 and March 2009, registered during the financial crisis.

Finally, we compared (see Figure 4) the fear scale level of the S&P 500 Index monthly volatility for the period 1990 – 2009 against the maximum value recorded in a calendar month of the Volatility Index (VIX)[2] for the same period, given that a high value of the named fear index represent a greater degree of market uncertainty; consistent results seems to be obtained.

[2] For further information about the index visit www.cboe.com.

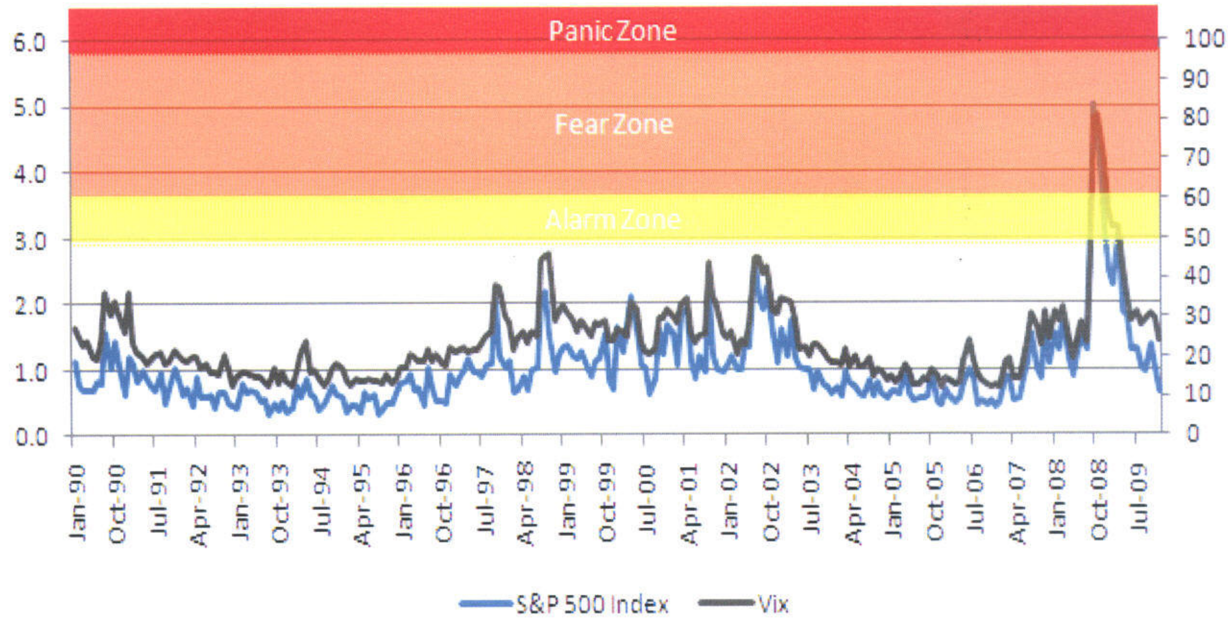

Fig. 4. Fear Scale and Fear Index

Future lines of research could apply a "fear scale" to evaluate extreme volatilities in others markets (i.e. commodities and foreign exchange) and the potential use of this instrument as part of technical analysis.

References

Dolde, W., Saad, R., Tirtiroglu, D. (2002). Evidence that Extreme Volatility in Stock Prices is Associated with Extraordinary Reported News Items.

McNeil, A. J. (1999). Extreme Value Theory for Risk Managers. *Internal Modelling and CAD II*. RISK Books, 93-113.