Technology transfer in a linear city with symmetric locations

Bouguezzi, Fehmi

LEGI and Faculty of Management and Economic Sciences of Tunis

2010

Technology transfer in a linear city with symmetric locations

Fehmi Bouguezzi[1]

LEGI and Faculty of Management and Economic Sciences of Tunis

Abstract

This paper compares patent licensing regimes in a Hotelling model where firms are located symmetrically and not necessary at the end points of the city. I suppose that one of the firms owns a process innovation reducing the marginal unit cost. This patent holding firm will decide to sell a license or not to the non innovative firm and will choose, when licensing, between a fixed fee or a royalty. The key difference between this paper and other papers is that here I suppose that firms are not static and can move along the linear city symmetrically. I find that when there is no licensing, Nash equilibrium exists only when innovation is non drastic. I also find that royalties licensing is better than fixed fee licensing when innovation is small. When the innovation is intermediate I find that fixed fee is better than a royalty. The paper shows that a fixed fee is not better than a non licensing regime independently of the innovation size and the optimal licensing regime is royalties when innovation is small. Finally, I show that a patent holding firm should not license its innovation when it is intermediate or drastic.

Key words: *Hotelling model, Technology transfer, Patent licensing*
Classification JEL : D43, D 45, L13

[1] *Email : fehmi_ bouguezzi@yahoo.fr*
Address: Loboratory of Management and Industrial Economics , Polytechnic School of Tunisia. El Khawarezmi Rd B.P. 743, 2078 La Marsa - Tunisia. Phone. (+216)71774611 - Fax (+216)71748843

Introduction

Several authors studied patent licensing and transfer of innovation. Wang (1998) and (2002) compared licensing regimes in a Cournot duopoly and then in a differentiated Cournot oligopoly and find that optimal licensing regime depends on the size of the innovation (drastic or non drastic [2]). Kamien, Oren and Tauman (1992) studied optimal licensing regime for a cost reducing innovation when innovative firm is outside of the market. Cohen and Morrison (2004) focused on spillovers in the US food manufacturing industry across states and from agricultural input supply and consumer demand and find average and marginal cost effects in the spatial and industry dimensions that affect location decisions. Mai and Peng (1999) discussed cooperation and competition between firms in a Hotelling spatial model with differentiation. Piga and Theoloky (2005) supposed that R&D spillovers depend on firm's location which means that spillovers increase when firms are close the each others. They show that distance between firm's location increases with the degree of product differentiation. Osborne and Pitchik (1987) studied optimal locations of two competing firms in a Hotelling's model and find that they choose locations close to the quartiles of the market. Paci and Usai (2000) investigate the process of spatial agglomeration of innovation and production activities in an econometric analysis of 85 industrial sectors and 784 Italian local labor systems and find that technological activities of a local industry influence positively innovations of the same sectors in contiguous areas. Alcacer and Chung (2007) examine firms' location choices expecting differences in firm's strategies of new entrants into the United States from 1985 to 1994 and find that firms favor locations with academic innovation activity. Alderighi and Piga (2009) investigate properties of two types of cost reducing restrictions that guarantee the existence of equilibrium in pure strategies in Bayesien spatial models with heterogeneous firms. Poddar and Sinha (2004) studied technology transfer in a Hotelling model where firms are located at the end points of the linear city and find, for an insider patentee, that royalty licensing is optimal when innovation is non drastic while no licensing is the best when innovation is drastic. Matsumura and Matsushima (2008) studied the relationship between licensing activities and the locations of the firms. They find that licensing activities following R&D investment always lead to the maximum differentiation between firms and the mitigation of price competition. Long and Sonbeyran (1998) supposed in a Hotelling model that spillovers depend on the distance between firms and find that agglomeration can be optimal. They also find that geographical dispersion in a two dimensional plane is another possible outcome. Hussler, Lorentz and Rond (2007) supposed that, in a Hotelling model, absorptions capacities of the firms are function of their internal R&D investment

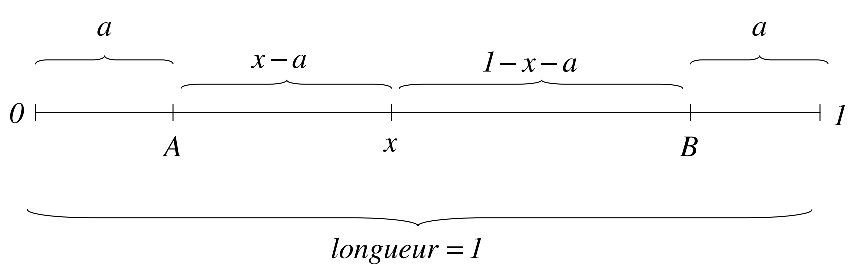
[2] Arrow (1962) was the first to introduce the analysis of the innovation drasticity. An innovation is called drastic when the patent owner become a monopoly.

and firms determine endogenously the maximum level of knowledge spillovers they might absorb. They find that knowledge spillovers are maximum if firms are located symmetrically and tend to agglomeration in the center of the linear city when transportation cost increase. Pinkse, Slade and Brett (2002) investigate the nature of price competition among firms producing differentiated products and competing in markets that are limited in extent through an econometric study of the US wholesale gasoline markets and find that competition is highly localized. Alderighi and Piga (2008) considered a Salop model with heterogeneous costs and find that cost heterogeneity increases welfare and induce less excessive entry.

This paper studies fixed fee and royalty licensing in a linear model where firms are not necessary at the end points of the city like in Poddar and Sinha (2004) bur are located symmetrically. The paper shows that Nash equilibrium exists if and only if firms are located with respect to some conditions depending on the cost reducing innovation.

1 Model

Let's suppose a linear city with a long l and two firms A and B producing homogeneous goods and located symmetrically on the city. Let's suppose too that firm A is located at a and firm B at $l - a$ $(a < \frac{l}{2})$.



To compare patent licensing regimes, I suppose that firm A owns a patented cost reducing innovation allowing to reduce the unit marginal cost by ε which measures the size of the innovation and depends on the investment on R&D by innovative firm. Consumers are uniformly distributed on the linear city (interval $[0, l]$)and each one pay a linear transport cost equal to td (t is the transport unit cost and d the distance between the consumer and the firm). The innovative firm will choose between two licensing regimes : a fixed fee licensing where non innovative firm must pay an amount of money not depending on the quantity produced in exchange of the use of the license or a royalty licensing where non innovative firm must pay a fixed rate on each quantity produced using the new technology. Game stages are as follows: in the first stage, the two firms choose their locations. In the second stage, decides to license its innovation or not and the fixed fee or the royalty to apply and in the third and last stage, the two firms compete in prices. To calculate demand functions of the two firms, we must find the location of the marginal consumer where its utility function when buying the product of the firm A is equal to its utility when buying the product of the firm B. The utility of each consumer depends negatively of the transportation cost and the price of the product.

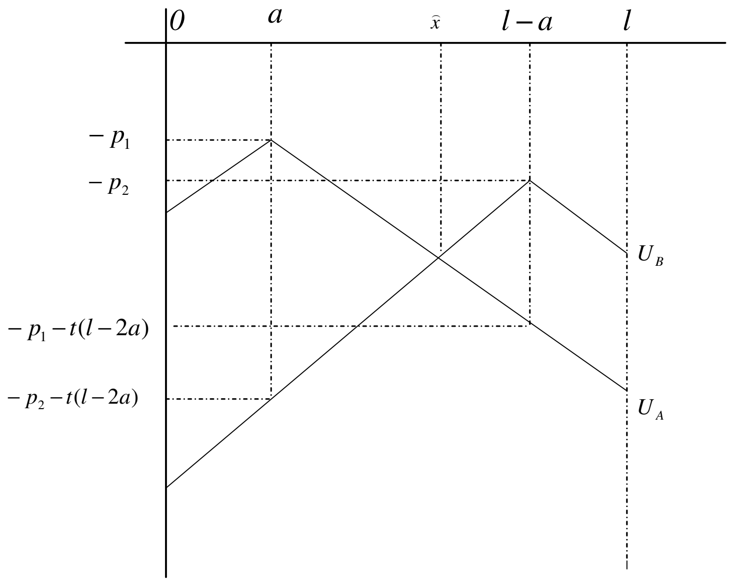
$U_A = -p_1 - t\,|x-a|$ and $U_A = -p_2 - t\,|l-x-a|$

The utility function of a consumer located at x and buying the firm A product is :

$$U_A = \begin{cases} -p_1 - t\,(a-x) & \text{if } x < a \\ -p_1 - t\,(x-a) & \text{if } x > a \end{cases}$$

The utility function of a consumer located at x and buying the firm B product is :

$$U_B = \begin{cases} -p_2 - t\,(l-x-a) & \text{if } x < l-a \\ -p_2 - t\,(x+a-l) & \text{if } x > l-a \end{cases}$$



In the interval $x \in [a, l-a]$, the location of the marginal consumer is $\tilde{x}$ and verifies :$U_A = U_B \Longleftrightarrow \tilde{x} = \frac{p_2 - p_1 + tl}{2t}$

Demand function of the firm A is :

$$D_A = \begin{cases} l & if \quad p_1 \in INT_1^A \\ \tilde{x} & if \quad p_1 \in \ INT_2^A \\ 0 & if \quad p_1 \in INT_3^A \end{cases} \Longleftrightarrow D_A = \begin{cases} l & if \quad p_1 \in INT_1^A \\ \frac{p_2-p_1+tl}{2t} & if \quad p_1 \in \ INT_2^A \\ 0 & if \quad p_1 \in INT_3^A \end{cases}$$

where $INT_1^A = [c_1, p_2 - t(l-2a)]$, $INT_2^A = [p_2 - t(l-2a), p_2 + t(l-2a)]$ and $INT_3^A = [p_2 + t(l-2a), +\infty]$

Demand function of the firm B is :

$$D_B = \begin{cases} 0 & if \quad p_2 \in INT_1^B \\ l - \tilde{x} & if \quad p_2 \in INT_2^B \\ l & if \quad p_2 \in INT_3^B \end{cases} \Longleftrightarrow D_B = \begin{cases} 0 & if \quad p_2 \in INT_1^B \\ \frac{p_1-p_2+tl}{2t} & if \quad p_2 \in INT_2^B \\ l & if \quad p_2 \in INT_3^B \end{cases}$$

where $INT_1^B = [p_1 + t(l-2a), +\infty]$, $INT_2^B = [p_1 - t(l-2a), p_1 + t(l-2a)]$ and $INT_3^B = [c_2, p_1 - t(l-2a)]$

Profits of the innovative and non innovative firms are :

$$\pi_A = \begin{cases} (p_1 - c_1)\, l & if \quad p_1 \in INT_1^A \\ (p_1 - c_1)\, \frac{p_2 - p_1 + tl}{2t} & if \quad p_1 \in \ INT_2^A \\ 0 & if \quad p_1 \in INT_3^A \end{cases}$$

$$\pi_B = \begin{cases} 0 & if \quad p_2 \in INT_1^B \\ (p_2 - c_2)\, \frac{p_1 - p_2 + tl}{2t} & if \quad p_2 \in INT_2^B \\ (p_2 - c_2)\, l & if \quad p_2 \in INT_3^B \end{cases}$$

To find a Nash equilibrium, the prices of the two firms p_1 and p_2 must verify this inequality : $|p_1 - p_2| \leq t(l - 2a)$. In fact, the profit of the firm A is not positive in the interval INT_3^A and to make a positive profit, firm A should choose a price p_1 verifying $p_1 < p_2 + t(l - 2a)$.Also, firm B realize a non positive profit in the interval INT_1^B and should choose a price p_2 verifying $p_2 < p_1 + t(l - 2a)$. We show finally that a Nash equilibrium exists in the interval INT_2^A (or INT_2^B).

Profits maximization in respect of prices gives:

$$\begin{cases} \frac{\partial \pi_A}{\partial p_1} = \frac{1}{2t}\left(p_2 - 2p_1 + tl + c_1\right) \\ \frac{\partial^2 \pi_A}{\partial p_1^2} = -\frac{1}{t} < 0 \end{cases} \quad \text{and} \quad \begin{cases} \frac{\partial \pi_B}{\partial p_2} = \frac{1}{2t}\left(p_1 - 2p_2 + tl + c_2\right) \\ \frac{\partial^2 \pi_B}{\partial p_2^2} = -\frac{1}{t} < 0 \end{cases}$$

We find at the equilibrium :

$$\begin{cases} \frac{\partial \pi_A}{\partial p_1} = 0 \\ \frac{\partial \pi_B}{\partial p_2} = 0 \end{cases} \iff \begin{cases} p_1 = \frac{1}{2}\left(p_2 + tl + c_1\right) \\ p_2 = \frac{1}{2}\left(p_1 + tl + c_2\right) \end{cases} \implies \begin{cases} p_1^* = tl + \frac{1}{3}\left(2c_1 + c_2\right) \\ p_2^* = tl + \frac{1}{3}\left(c_1 + 2c_2\right) \end{cases}$$

The optimal profit functions of firms A and B at the equilibrium are :

$\pi_A^* = \frac{1}{2t}\left(tl + \frac{1}{3}\left(c_2 - c_1\right)\right)^2$ and $\pi_B^* = \frac{1}{2t}\left(tl + \frac{1}{3}\left(c_1 - c_2\right)\right)^2$

Demand functions are :

$D_A = \tilde{x} = \frac{1}{2t}\left(tl + \frac{1}{3}\left(c_2 - c_1\right)\right)$ if $p_1 \in \ INT_2^A$

$D_B = l - \tilde{x} = \frac{1}{2t}\left(tl - \frac{1}{3}\left(c_2 - c_1\right)\right)$ if $p_2 \in \ INT_2^B$

2 No licensing

In this regime, innovative firm profit alone from its innovation while non innovative firm uses the old technology. Denoting by c_1 and c_2 marginal unit costs of respectively firm A and firm B, we can write: $c_1 = c - \varepsilon$ and $c_2 = c$. Replacing in firms equilibrium profits, we find:

$\pi_A = \frac{1}{2t}\left(tl + \frac{1}{3}\varepsilon\right)^2$ and $\pi_B = \frac{1}{2t}\left(tl - \frac{1}{3}\varepsilon\right)^2$

Price equilibrium are :

$p_1^* = tl + c - \frac{2}{3}\varepsilon$ and $p_2^* = tl + c - \frac{1}{3}\varepsilon$

We can see in p_1^* and p_2^* that the use of the new technology allows firm A to buy its product at a price $|p_1^* - p_2^*| = \frac{1}{3}\varepsilon$ lower than firm B price. this difference in prices depends on the size of innovation ε which will decide if the non innovative firm will leave or not the market. In fact, firm B using the old technology make a non negative profit when $p_2^* > c \Longleftrightarrow \varepsilon < 3tl$. We can see that equilibrium price of firm B exceeds its production unit cost c when innovation is non drastic ($\varepsilon < 3tl$). To have a Nash Equilibrium, we suppose that innovation of firm A is not drastic to avoid having a monopoly on the linear city since for $\varepsilon \geq 3tl$ we have $p_2 < c$ and $\pi_B = 0$.

The profit of firm A includes three functions: an affine function on the interval INT_1^A, a parabolic function on the interval INT_2^A and a null function on the interval INT_3^A. To have a Nash equilibrium on the interval INT_2^A, the maximum profit of firm A on the interval INT_2^A must be higher than its maximum profit on the interval INT_1^A.

Firm A optimal profit on INT_2^A is $\pi_A^{Int2} = \frac{1}{2t}\left(tl + \frac{1}{3}\varepsilon\right)^2$ and the optimal profit on INT_1^A is $\pi_A^{Int1} = \left(2at + \frac{2}{3}\varepsilon\right) l$

$$\pi_A = \begin{cases} (p_1 - c_1)\, l & Si \quad p_1 \in INT_1^A \\ (p_1 - c_1)\, \frac{p_2 - p_1 + tl}{2t} & Si \quad p_1 \in \ INT_2^A \\ 0 & Si \quad p_1 \in INT_3^A \end{cases} \Longrightarrow \pi_A^* = \begin{cases} \left(2at + \frac{2}{3}\varepsilon\right) l & Si \quad p_1 \in INT_1^A \\ \frac{1}{2t}\left(tl + \frac{1}{3}\varepsilon\right)^2 & Si \quad p_1 \in \ INT_2^A \\ 0 & Si \quad p_1 \in INT_3^A \end{cases}$$

A Nash equilibrium exists if $\pi_A^{Int2} > \pi_A^{Int1} \Longleftrightarrow a < \frac{l}{4} + \frac{\varepsilon}{36t^2 l}\left(\varepsilon - 6tl\right)$.

Proposition 1 *When there is no licensing, a Nash equilibrium exists when innovation is non drastic and when firms are located symmetrically on the linear city and verifying $a < \frac{l}{4} + \frac{\varepsilon}{36t^2 l}\left(\varepsilon - 6tl\right)$. When innovation is drastic, the non innovative firm, using the old technology, leave the linear city.*

Let's now study the effect of the innovation size on firm locations. We denote by $a_{\max} = \frac{l}{4} + \frac{\varepsilon}{36t^2 l}(\varepsilon - 6tl)$ the maximum distance between one firm and the nearest end point of the city. Calculating the derivative of the maximum location with respect to ε we find:

$\frac{\partial a_{\max}}{\partial \varepsilon} = \frac{1}{18lt^2}(\varepsilon - 3lt) < 0$ when $\varepsilon < 3tl$

we see that, when the size of innovation increase, the maximum location decrease which means that firms come closer to the end points of the city.

Proposition 2 *When innovative firm do not license its innovation, more the innovation size increase more firms, placed symmetrically on the linear city, become closer to the end points.*

3 Fixed fee licensing

In this regime, firm B can use the new technology in exchange of the payment of a fixed fee denoted by F to the patent holding firm. The maximum amount that firm A can choose is equal to the increase of firm B profit when using the new technology. $F = \pi_B^F - \pi_B^{PL} - \alpha$ with $\alpha \longrightarrow 0$ to be sure that firm B will accept to buy the license.

Firm A and firm B production unit costs are $c_1 = c_2 = c - \varepsilon$. Replacing in the profit functions we find : $\pi_A = \frac{1}{2t}(tl)^2$ and $\pi_B = \frac{1}{2t}(tl)^2$

Fixed fee amount is equal to :

$F = \frac{\varepsilon}{6t}\left(2tl - \frac{1}{3}\varepsilon\right) - \alpha$

Total revenue of the patent holding firm is:

$\Pi_A^F = \pi_A + F = \frac{1}{2t}(tl)^2 + \frac{\varepsilon}{6t}\left(2tl - \frac{1}{3}\varepsilon\right) - \alpha = \frac{1}{2t}\left(\left(tl + \frac{1}{3}\varepsilon\right)^2 - \frac{2}{9}\varepsilon^2\right) - \alpha$

Proposition 3 *In a Hotelling model where firms are located symmetrically and with a patented cost-reducing innovation owned by one firm, we find that fixed fee licensing is always lower than no licensing independently of the innovation size.*

PROOF. [Preuve] $\Pi_A^F - \pi_A^{PL} = -\frac{1}{9t}\varepsilon^2 - \alpha < 0$.

4 Royalty licensing

In the royalty regime, the cost-reducing innovation is sold to the non innovative firm in exchange of a royalty amount depending on the production made with the use of the new technology. The amount of royalties is proportional to the demand of firm B and equal to $r\,(l-\tilde{x})$. Firm B will accept to buy the license in this regime only when it will allow it to increase its no licensing profit which means that r must be in the interval $]0,\varepsilon[$ unless this licensing regime will not be important to study.

Production unit costs of firms A and B are respectively $c_1 = c-\varepsilon$ and $c_2 = c-\varepsilon+r$. Replacing in the profit functions we find :

$$\pi_A = \frac{1}{2t}\left(tl+\frac{1}{3}r\right)^2 \text{ and } \pi_B = \frac{1}{2t}\left(tl-\frac{1}{3}r\right)^2$$

Maximizing firm A profit with respect to r we find :

$\frac{\partial \pi_A}{\partial r} = \frac{1}{9t}\,(r+3lt) > 0$. Since $r \in]0,\varepsilon[$ then π_A is maximal when r is maximal which means that $r^* = \varepsilon - \alpha\ (\alpha \to 0)$

The profits of the two firms are :

$$\pi_A = \frac{1}{2t}\left(tl+\frac{1}{3}\varepsilon\right)^2 \text{ and } \pi_B = \frac{1}{2t}\left(tl-\frac{1}{3}\varepsilon\right)^2$$

Total revenue of the patent holding firm is :

$$\Pi_A^r = \pi_A + r\,(l-\tilde{x}) = \frac{1}{2t}\left(tl+\frac{1}{3}\varepsilon\right)^2 + \frac{\varepsilon}{2t}\left(tl-\frac{1}{3}\,(2c+\varepsilon)\right)$$

Proposition 4 *Royalty licensing is better than no licensing when innovation is small ($\varepsilon < 3tl-2c$).*

PROOF. $\Pi_A^r - \pi_A^{PL} = \frac{\varepsilon}{6t}\,(3tl-2c-\varepsilon)$

$\Pi_A^r > \pi_A^{PL}$ if $\varepsilon < 3tl-2c$ and $\Pi_A^r < \pi_A^{PL}$ if $\varepsilon > 3tl-2c$■

Comparing firm A total revenue when licensing with a fixed fee and its total revenue when with a royalty licensing, we find that (when $\alpha \to 0$) $\Pi_A^r - \Pi_A^F = \frac{\varepsilon}{18t}\,(9tl-6c-\varepsilon)$. We notice that $\Pi_A^r > \Pi_A^F$ if $\varepsilon < 9tl-6c$ and $\Pi_A^r < \Pi_A^F$ if $\varepsilon > 9tl-6c$.

Proposition 5 *Royalties licensing can be better than fixed fee licensing when innovation is small or intermediate ($\varepsilon < 9tl-6c$). However, a fixed fee is better for the patent holding firm than royalties when innovation is strong ($\varepsilon > 9tl-6c$).*

Optimal licensing regimes for the patent holding firm are as follows :

$$\begin{cases} r \succ PL \succ F & \mathit{if}\ 0 < \varepsilon < 3tl - 2c \\ PL \succ r \succ F & \mathit{if}\ 3tl - 2c < \varepsilon < 9tl - 6c \\ Pl \succ F \succ r & \mathit{if}\ \varepsilon > 9tl - 6c \end{cases}$$

The optimal licensing regime when $\varepsilon < 3tl - 2c$ is royalties while no licensing become better when $\varepsilon > 3tl - 2c$. However, royalty licensing is an optimal strategy for the patent holding firm only when we have two firms on the market. Le's remember that to be in a Nash equilibrium, firm A and firm B must choose their prices such that $|p_1 - p_2| \leq t(l - 2a)$ unless one of them make a non positive profit. So firm A will keep its price in this interval only when it has not interest to deviate, which means that it makes a profit in this interval greater than its profit in the other interval.

$$\Pi_A^r = \begin{cases} \left(\frac{2}{3}\varepsilon + 2at\right) l & \mathit{if} \quad p_1 \in INT_1^A \\ \frac{1}{2t}\left(tl + \frac{1}{3}\varepsilon\right)^2 + \frac{\varepsilon}{2t}\left(tl - \frac{1}{3}(2c + \varepsilon)\right) & \mathit{if} \quad p_1 \in INT_2^A \\ 0 & \mathit{if} \quad p_1 \in INT_3^A \end{cases}$$

$$\Pi_A^r\left(INT_2^A\right) > \Pi_A^r\left(INT_1^A\right) \Longleftrightarrow a < \frac{l}{4} - \frac{\varepsilon}{18t^2 l}\left(\varepsilon - (9tl - 3c)\right)$$

Proposition 6 *Royalty licensing is optimal for the innovative firm when innovation is small ($\varepsilon < 3tl - 2c$) and when it is located in the interval $[0, \frac{l}{4} - \frac{\varepsilon}{18t^2 l}(\varepsilon - (9tl - 3c))[$. For other locations or innovation size, no licensing become the optimal licensing strategy.*

5 Conclusion

We studied in this in this model optimal licensing strategies for an innovative firm on a Hotelling linear city. We find that the size of the innovation has an effect on firms equilibrium locations. when firms are located symmetrically, we find that an increase in the innovation size make firms more close to the end points of the city. We also find that a fixed fee licensing is always lower than no licensing regime and in this licensing, the non innovative firm leaves the market when innovation is drastic. In a comparison between fixed fee and royalties, we find that fixed fee is better than royalties when innovation is intermediate or strong. Finally, we show that a Nash equilibrium exists for a royalty licensing when innovation is small and for specific firm locations. In the other cases, the patent holding firm should benefit alone of its innovation and become a monopoly.

References

[1] Alcácer, J. and Chung, W., (2007) Location Strategies and Knowledge Spillovers, Management Science Vol. 53, No. 5, May 2007, pp. 760–776

[2] Alderighi, M. and Piga, C.,The Circular City with Heterogeneous Firms (July, 13, 2008). Available at SSRN: http://ssrn.com/abstract=1159381

[3] Alderighi, M. and Piga, C., (2009) , On cost restrictions in spatial competition models with heterogeneous firms , Department of Economics , Discussion Paper Series.

[4] Arrow, K., (1962). Economic welfare and the allocation of resources for inventions. In: Nelson, R. (Ed.), The Rate and Direction of Inventive Activity. Princeton University Press, Princeton.

[5] Cohen, J.P. and C.J. Morrison Paul, 2004, Agglomeration Economies and Industry Location Decisions: The Impacts of Spatial and Industrial Spillovers

[6] Hussler C., Lorentz A., Ronde P. (2007), "Agglomeration and endogenous absorptive capacities: Hotelling revisited", Jena Economic Research Papers, WP2007-12.

[7] Long, N.G., Soubeyran, A., 1998. R&D spillovers and location choice under Cournot rivalry. Pacific Economic Review 3, 105– 119.

[8] Mai, C.-C. and Peng, S.-K., 1999. Cooperation vs. competition in a spatial model. Regional Science and Urban Economics 29, 463–472.

[9] Matsumura and Matsushima (2008) On patent licensing in spatial competition with endogenous location choice

[10] Osborne, M. and Pitchik, C. (1987), equilibrium in a hotelling's model of spatial competition, Econometrica 55 (1987), pp. 911–922

[11] Paci, R. Usai, S. (2000) Externalities, knowledge spillovers and the spatial distribution of innovation, GeoJournal (vol. 4, 2000) special issue "Learning and Regional Development: Theoretical Issues and Empirical Evidence". Ed. Ron Boschma

[12] Piga, C., Theotoky, J. (2005) Endogenous R&D spillovers and locational choice , Regional Science and Urban Economics 35 (2005) 127– 139

[13] Pinkse, Slade, C Brett (2002), Spatial price competition a semiparametric approach, - Econometrica, 2002

[14] Poddar, S. and Sinha, U.B., (2004). On patent licensing in spatial competition. Economic Record 80, 208–218.

[15] Hussler C., Lorentz A., Ronde P. (2007), "Agglomeration and endogenous absorptive capacities: Hotelling revisited", Jena Economic Research Papers, WP2007-12.

[16] Wang, X.H., 1998. Fee versus royalty licensing in a Cournot duopoly model. Economics Letters 60, 55–62.

[17] Wang, X.H., 2002. Fee versus royalty licensing in a differentiated Cournot duopoly. Journal of Economics and Business 54, 253–266.