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# Information Technology and the Rise of Household Bankruptcy\*

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## Abstract

Several studies attributed the rise of household bankruptcy in the past two decades to the decline of social stigma associated with default. Stigma explanations, however, cannot account for the increase of credit availability during this period. I try to explain both of these facts as a result of a more informative credit rating technology.

I study an adverse selection environment where borrowers are heterogeneous with respect to their cost of default. Creditors have access to a rating technology which provides an exogenous signal about borrowers' default costs. Equilibrium contracts subject each borrower to a credit limit such that the creditors' expected profit, conditional on the signal about the borrower's default cost, is zero. As the exogenous signal becomes more informative, the credit market will provide higher credit limits for borrowers with high default costs, and lowers credit limits of borrowers with low costs of default. Hence a more informative signal allows those with high default costs to borrow more, making them more likely to default, while decreasing borrowing and default by those with low default costs.

Using Simulated Method of Moments, I estimate the model parameters to match the increases in the average consumer credit card limit, the average unsecured consumer debt level and the spread of the credit limit distribution from 1992 to 1998 using the Survey of Consumer Finance's data. The model does well in matching the targeted moments and can account for one third to half of the increase in the number of bankruptcy filings from 1992 to 1998.

Keywords: Consumer Bankruptcy, Information and Market Efficiency, Rating Agencies.

JEL Classification: G14, E44, K35, E21.

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# 1 Introduction

The paper provides an *informational* explanation for three main trends observed in the US consumer credit market since 1980s: the rise of *household bankruptcy*, the increase in *unsecured consumer debt*, and the rise of *availability of unsecured consumer credit*. Improvement in creditors' information for assessing borrowers' *default cost* and the resulting change in *credit allocation* is the main focus of this paper. Heterogeneity in borrower default costs not only generates heterogeneous propensity to default, but also heterogeneous utilization of available credit. Therefore, when creditors offer more informed *credit contracts*, credit availability and borrowing increase. More interestingly, better allocation of credit allows more borrowers to accumulate large enough unsecured debt levels to find it optimal to default. Hence, in addition to higher credit availability and indebtedness, better informed credit contracts could result in higher default frequencies and larger levels of discharged debt.

Household bankruptcy filings have been increasing in the US for the past quarter of a century. In 1984, 0.33% of American households filed for bankruptcy. The number of filers rose to 0.93% of households in 1991 and continued to increase up to 1.41% in 2004.<sup>1</sup> This trend can also be spotted in the number of Canadian bankruptcy filers (Livshits, MacGee and Tertilt [19]), suggesting that the increase should not be solely attributed to legal changes in the US.

During this period, households' access to unsecured credit (mainly through credit cards) flourished. While in 1989, 56% of households had access to credit cards and 29% of households carried a positive balance on their accounts. Fifteen years later credit card access rose to 72% and 40% of American households were carrying debt on their accounts (the latter are called *revolvers* in the literature).<sup>2</sup> Moreover, the average credit card debt of revolvers increased from \$1,830 in 1989 to \$3,300 in 2004.<sup>3</sup> But households were not just borrowing more subject to the same credit limits. During this period the average credit card limit available for an American household more than doubled; they rose from \$7,100 in 1989 to \$15,200 in 2004.<sup>4</sup>

The importance of credit card debt in a household's decision to file for bankruptcy has been well documented (see for example Domowitz and Sartain [11] as well as Sullivan, Warren and Westbrook [25].) Therefore, understanding the dynamics behind the expansion of credit card availability and its usage is critical for study of the rise in the number of household bankruptcies.

Barron and Staten [5] document that expansion of the credit card industry would not be possible without rapid improvements in information technology and credit rating technologies. In 1997 credit bureaus issued some 600 million reports about credit seekers, (Padilla and Pagano [23]), and in the following decade credit scores produced by the Fair Isaac and Company, known as FICO scores, became the industry's standard tool for assessing borrowers' credit worthiness. Edelberg [13] shows that creditors increasingly used risk-based pricing of interest rates in consumer loan markets during the mid-1990s, and Berger [6] reports that improvement in the lending capacity was due to improvements in information technology used by the banks. Moreover, Musto [22] documents the importance of creditors' information on borrowers' riskiness for extending credit.

This paper tries to explain the rise in the number of bankruptcy filings as a result of an improvement in the credit market's assessment of borrowers' riskiness. This might sound counter intuitive at first. When creditors separate borrowers according to their riskiness, they will tighten credit supply for riskier borrowers, which will make them less likely to default. However, safer borrowers will receive higher borrowing limits which allows them to borrow more and, in turn, can result in more default. This is because even safer borrowers, *ceteris paribus*, are more likely to default when they carry higher debt levels.

If the rating technology does not work well and the credit market lacks information on borrowers riskiness, then it will have difficulty in differentiating different types of borrowers. I call this case the "*pooling case*". The

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<sup>1</sup>Just before the sweeping changes to America's bankruptcy code took effect in 2005, the number of bankruptcy filers jumped to 1.55% of American households. Unsurprisingly, the number of filers plummeted after the change went into effect. Recent data suggest the number of filers is picking up again.

<sup>2</sup>20% of households (69 percentage of revolvers) were carrying more than \$500 debt in 1989. This fraction rose to 30% of households (75 percentage of revolvers) in 2004.

<sup>3</sup>All dollar amounts are in 1989 constant prices.

<sup>4</sup>A household's credit card limit is the sum of limits on all of the household's credit cards.

equilibrium supply of credit will be so tight that safer borrowers do not borrow much, and therefore will not pay much to cover the credit market's losses from lending to riskier borrowers.<sup>5</sup> Now, suppose the rating technology improves and the credit market obtains information on borrowers' riskiness, which could be used to differentiate different types of borrowers. I call this the "*separating case*". Differentiating between borrowers, the market will cut back credit supply for riskier borrowers. But compared with the pooling case, the reduction will be small and their default rate will not fall significantly. On the other hand, creditors will largely extend credit supply credit for safer borrowers, who are now separated from riskier ones. Safer borrowers will be able to borrow much more than the pooling case and hence will default more frequently.<sup>6</sup> I call this mechanism the *informational* explanation for the rise of household bankruptcy.

There are different sources of heterogeneity amongst borrowers, but this paper focuses on the heterogeneity in default costs. Obviously other heterogeneities, like differences in borrowers' income processes, have also clear implications for heterogeneity in borrowing and default decisions. However, this paper abstracts from the other heterogeneities and assumes a reduced form pecuniary default cost.<sup>7</sup> The default cost not only affects borrowers' default decision, but is also determinant of their borrowing pattern. More specifically, when a borrower has accumulated enough debt to find defaulting a possible future outcome, different default costs, and hence default probabilities, translate into different marginal costs of borrowing. Therefore, in addition to having heterogeneous probability of default, borrowers with different default costs show heterogenous *marginal propensity to borrow* out of an available credit line.

The heterogeneity in marginal propensity to borrow is essential for the increase in the average credit availability after switching from the pooling case to the separating case. That is, although riskier borrowers' debt levels and default probabilities are very elastic with respect to credit availability, safer borrowers could have a low and relatively inelastic debt level and default probabilities in the pooling case. Hence, after switching from the pooling case to the separating case, there will be a small decline in credit availability for riskier borrowers, while credit will become much more available for safer borrowers, generating a net increase in the average credit availability.

More specifically, I study a model in which borrowers start with a temporary low income level, but expect their income to switch permanently to a higher level in future. However, there is uncertainty about the time of switching as well as the level of the change. While waiting for realization of their permanent income, borrowers can borrow up to a credit limit determined by a competitive credit market. Depending on how long they wait until realization of their permanent incomes, borrowers can accumulate different levels of unsecured debt. Once a borrower realizes her permanent income, depending on the debt amount and the realized level of the permanent income, as well as her default cost, she chooses to repay her debt, or default.

Despite being quite stylized, the model does a good job in showing the importance of information on borrowers' riskiness for credit allocation. In a simple quantitative exercise, the model generates an increase in credit availability, unsecured debt and default rate, as a result of switching from the pooling case to the separating case. It also generates dispersion in credit availability and unsecured debt, with more debt accumulated by safer borrowers. Since most of the increase in default is due to *safer* borrowers, and they only default if their debt level is relatively high, the model also generates an increase in debt discharge rate. All these changes are qualitatively consistent with the trends observed in the US data. Some of the changes generated by this stylized model are even quantitative consistent with the data, showing the importance of the underlying mechanism addressed by this paper. In particular, the model can generate one third to half of the increase in bankruptcy filings from 1992 to 1998, while accounting for the increase in the average unsecured consumer debt, the rise of credit availability and dispersion of credit allocations.

The literature provides other explanations for the rise of household bankruptcies. The common explanation

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<sup>5</sup>For example, if riskier borrowers are much more riskier than safer ones and there are a lot of them in the pool of borrowers, borrowing and default will be mostly done by riskier ones and safer borrowers will be effectively inactive.

<sup>6</sup>Borrowers' responsiveness to terms of credit contracts, and specifically credit limits, is well documented by Gross and Souleles [16].

<sup>7</sup>Note that with constant relative risk aversion, all pecuniary default costs could be represented by a non-pecuniary cost, as long as they are proportional to the defaulter's consumption. Most increases in defaulters' cost of living, like higher renting cost, car insurance rate etc., are proportional to living costs.

attributes it to the fall of “stigma” attached to bankruptcy. Gross and Souleles [15] report that *ceteris paribus*, a credit card holder in 1997 was almost 1 percentage point more likely to declare bankruptcy than a card holder with identical risk characteristics in 1995. Fay, Hurst and White [14] report that even after controlling for state and time fixed effects, households are more likely to file for bankruptcy if they live in districts with higher aggregate bankruptcy filings rates.

The stigma explanation, however, has counterfactual implications for credit availability and equilibrium debt levels. If borrowers become less reluctant to default on their debt, then shouldn't creditors restrain their supply of credit? and wouldn't this result in less borrowing rather than more? Athreya [1] and Livshits et al. [19] have noted that the decline in stigma alone would lead to a counterfactual decline in the ratio of unsecured debt to income. To account for the rise of consumer debt level they suggest a reduction in the transaction costs of lending.<sup>8</sup> Livshits et al. [19] use a combination of decline in stigma and fall in transaction costs to explain the changes in filings and the ratio of unsecured debt to income.<sup>9</sup>

But a fall in credit transaction costs cannot explain the observed spread in the distribution of consumer credit card limits. While from 1992 to 1998, when bankruptcy filings rose significantly, the average American household's credit card limit increased from \$7,200 to \$12,800, the standard deviation of the cross sectional distribution of credit limits rose from \$8,200 in 1989 to \$15,700 in 2004.<sup>10</sup> That is, the distribution of credit card limits did not just shift rightward, its spread also increased. That is, the rise of credit limit has been larger for some households than others. Gross and Souleles [15] report that creditors extended the larger lines to less risky accounts, suggesting that the spread of credit supply is mostly associated with the improvement in risk assessment.

Using a combination of the rise of stigma and fall of transaction costs to address different trends of the consumer credit industry ignores an important innovation of this industry: the consumer credit risk rating. This paper studies the implication of this innovation on the trends of this industry.<sup>11</sup> Specifically, I examine how the credit limit and debt distributions as well as the number of bankruptcy filings differ in a market where creditors have information about the their borrowers' riskiness from a credit market where they do not.

Recently four papers have addressed the increase in household bankruptcy from informational perspectives in environments with *ex ante* heterogeneous borrowers. In Sanchez [24], borrowers are endowed with heterogeneous income processes, and the improvement in information is modeled by a decline in the cost of *monitoring*. Although the model generates an increase in credit availability and average debt, it does not generate the observed dispersion of these two variables in the US data. Moreover, given that the distinction between safer and riskier borrowers is in their income processes, after separating safer borrowers from riskier ones, safer borrowers borrow more than the pooling case due to lower cost of borrowing. However, they would not borrow more than riskier borrowers and would not default on larger levels of debt. Hence the model cannot generate the dispersion of unsecured debt as well as the increase in debt discharge rate, as observed in the data.

Drozd and Nosal [12] do not directly address improvement in information. But their model for lowering credit solicitation cost in a search environment with varying borrower characteristics, is an informational explanation at heart. That is, if credit contracts are updated more frequently, they will reflect borrowers' characteristics more accurately. Similar to my model, they also assume credit card contracts to be combination of credit limits and interest rates.<sup>12</sup> However, in their model borrowers default if and only if they reach their credit limits. This is counterfactual: a significant fraction of borrowers are at their limit but do not default and a lot of borrowers default before reaching their credit limits.<sup>13</sup> Moreover, the model does not generate enough dispersion in credit availability and also fails to increase the debt discharge rate.

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<sup>8</sup>Athreya [1] also uses the same reduction to generate the rise in filings, which leads to a significantly higher debt to income ratio than the observed level in the data.

<sup>9</sup>Another possible explanation for more bankruptcy filings could be the rise of “uncertainty” in households' income and emergency expenses. This explanation implies a similar counterfactual decline in credit provision. Moreover, Livshits et al. [19] find its effect on the rise of filing numbers insignificant.

<sup>10</sup>Credit limits are reported in 1989 dollars

<sup>11</sup>Chatterjee, Corbae and Rios-Rull [9] provide a model with dynamic updating of creditors' beliefs about borrowers' creditworthiness that they associate with credit scores. Their paper, however, does not address the industry's trends, which is the main focus of my paper.

<sup>12</sup>Mateos-Planas and Rios-Rull [21] show how the *credit line* contract (combination of an interest rate and a credit limit) arises endogenously in the equilibrium.

<sup>13</sup>At least partly to show their “*good faith*” to their bankruptcy judge.

Livshit, MacGee and Tertilt [20] provide a stylized model to show how more informed signals about borrowers' income process allow creditors to offer more and better specified credit contracts, when offering contract is costly. Their model captures dispersion of credit contracts along the interest rate dimension, while accounting for the rise of bankruptcy filings. However, they focus on increase in credit availability along the *extensive margin*, and the *democratization of credit*. This was the main trend observed during 1980s and early 1990s but not since early 1990s. As I show in the next section of this paper the most striking feature of data since early 1990s is the increase in bankruptcy while borrowers received more credit along the *intensive margin* and could accumulate and default on larger debts. Moreover, since the main source of heterogeneity is in borrowers' income process, the informational improvement of their model only generates increase in the default rate but fails to generate increase in the debt discharge rate. That is, the additional new defaulters are riskier than the defaulters under the limited information case and hence default on even smaller debt levels.

Athreya, Tam and Young [2] provide a rigorous quantitative accounting for contribution of different sources of borrowers' heterogeneity to the rise of bankruptcy. Their paper, confirms the importance of heterogeneity in default cost for explaining the observed trends in the US data over the other sources of borrower heterogeneities. Moreover, unlike the other two quantitative models they use a life cycle model and show the importance of *unfulfilled expectation of permanent income* for most of defaulters, a feature captured by my model's simple income process. However, they model credit contracts as one period maturing bonds. Beside computational complications, their setup forces borrowers to default in response to short lived bad shocks, which increases incentive to default artificially. That is, "credit tightens exactly during the period which it is most needed." My paper's simple credit arrangement avoids this counterfactual incentive to default, generating a more realistic default decision despite using a more stylized model than theirs.

Section (2) provides some facts from the US data on the number of household bankruptcy filings, the distributions of credit card limits and debts, as well as changes in these distributions across time. Section (3) describes a model of borrowers' demand for credit, the responsiveness of their demand to credit contracts, in particular credit limits, and the change in propensity to default when credit supply increases. Then the model is used to show how a more informed credit market supplies more credit, while increasing the dispersion of credit supply. Section (4) provides a simple quantitative example and shows the model does well in explaining the rise of credit supply, consumer unsecured debt and the number of household bankruptcies. Section (5) concludes.

## 2 Data and Motivation

Households can file for bankruptcy under chapter 7 or 13. Under chapter 7, their unsecured debt such as credit card debt, installment loans, medical bills and damage claims are discharged, and filers lose all of their assets above their state specific exemption levels.<sup>14</sup> Under chapter 13, filers must propose a plan to repay a portion of their debts from their future income, but do not lose their assets. Since households have the right to choose between the chapters, they are only obliged to use future earnings to repay debt to the extent they would repay under chapter 7. Those who file under chapter 13 are allowed to file again under chapter 7, but the chapter 7 filers cannot file for another 6 years. The bankruptcy flag remains in a filer's credit history for 10 years (see Musto [22].)

Approximately 70% of those who seek bankruptcy protection file under chapter 7 and two third of those who file under chapter 13 ended up filing again under chapter 7.<sup>15</sup> This paper, however, does not distinguish between filing under the two chapters and studies a notion of bankruptcy similar to filing under chapter 7.

Figure (1) shows the number of bankruptcy filings by American households in the past two decades.<sup>16</sup> Except three short periods of 1992-94, 1997-2000 and 2003-04, bankruptcy filings have been increasing during this two decades. From 1994 to 1997 bankruptcy filings rose by 63% during a period of robust economic expansion.<sup>17</sup>

<sup>14</sup>Exemption levels differ across the US states.

<sup>15</sup>See Li and Sarte [18] for an elaborated study of bankruptcy filers' choice of chapter.

<sup>16</sup>The percentage of filers for the 1984-95 period are reported from Fay et al. [14]. The number of filings for the 1995-2005 period are from [www.uscourts.gov](http://www.uscourts.gov) and the number of households for this period are from [www.census.gov](http://www.census.gov).

<sup>17</sup>Gross and Souleles [15] study bankruptcy and payment delinquency of credit card holders during this period.

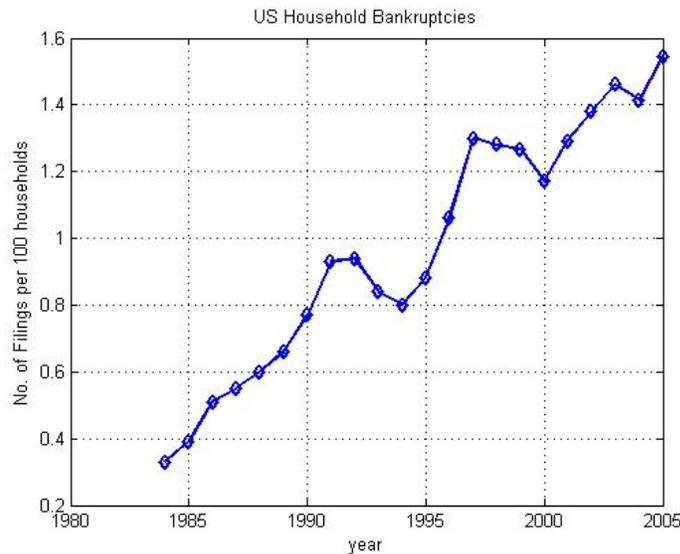


Figure 1: US Household Bankruptcies

Using the Survey of Consumer Finance (SCF), I find 11% of American households had at least once filed for bankruptcy in their lives by 2004, and from those who had filed, 69.4% of them had filed in the past 10 years. That is, more than 7% of American households had a bankruptcy flag on their credit history in 2004.

Availability of credit cards and their usage have also risen in the past two decades. Figure (2) and table (1) report the average credit limit for those who had access to credit cards and the fraction of population with credit card access from the SCF 1989-2004. The fraction of the population with positive credit card limit, which I call the *extensive margin* of credit supply, rose almost 17%. The average credit limit for card holders, which I call the *intensive margin* of credit supply, more than doubled.<sup>18</sup> Just from 1992 to 1998, the intensive margin of credit supply increased by a factor of 79%.

Households also borrowed more on their credit cards. In 1989, 29% of households were revolvers: carrying positive debt on their credit cards. By 2004 the fraction of revolvers rose to 40%. Revolvers' average credit card debt almost doubled in this period and went from \$1,828 in 1989 to \$3,295 in 2004. Just from 1992 to 1998, revolvers' debt increased by a factor of 59%. Table (1) reports the average debt level of revolvers and households with access to credit cards. The average debt level of revolvers remains almost two times as large as the average debt level of general card holders, revolvers and non-revolvers combined.

But the increase of average credit limits and debt levels does not thoroughly summarize the changes in the distributions of these two variables. The standard deviation of the cross section of credit limits and debt levels also doubled from 1989 to 2004. This observation is critical for the approach of this paper.

Figure (3) depicts the empirical distributions of credit limits in 1992 and 1998 from SCF.<sup>19</sup> As it can be easily noted, the distribution shifted rightward. But the shift was not caused by a uniform extension of credit supply to all card holders. The increase of credit limit was larger for some households than others. To illuminate this point, figure (3) also depicts a counterfactual distribution, which is made by uniformly increasing the credit limits in 1992 to match the average credit limit of 1998. Although the credit limit distribution of 1998 and the counterfactual distribution both have the same average, the 1998 distribution is more spread out. The uneven extension of credit limit is also documented by Gross and Souleles ([15]). More interestingly they report that creditors extended larger lines for safer accounts and provided less extension for riskier accounts.

Credit card contracts usually consist of a credit limit and an interest rate. Table (1) reports the average and

<sup>18</sup>Limits are given in 1989 dollars.

<sup>19</sup>Reported in 1989 dollars.

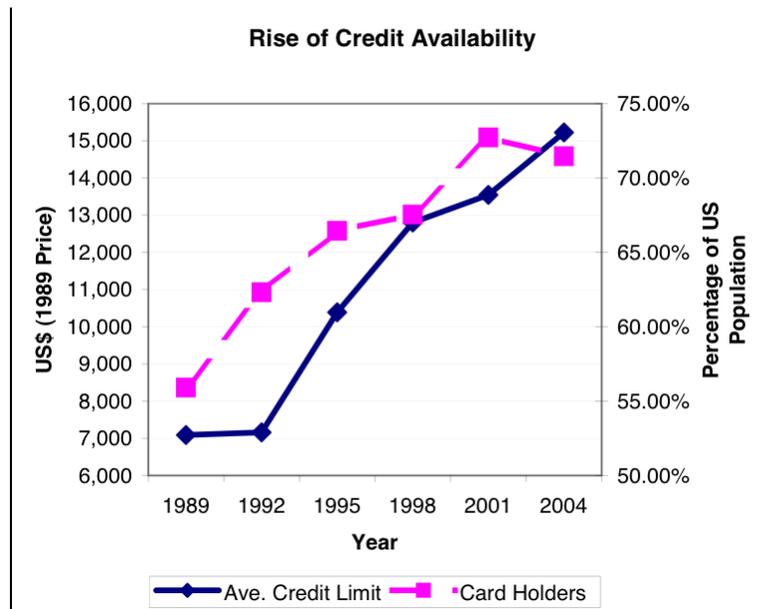


Figure 2: Rise Of Credit Availability

standard deviation of credit card interest rates for 1995-2004.<sup>20</sup> From 1995 to 2001, when the Bank Prime Loan Rate (MPRIME) fluctuated between 8.00% and 9.50%, the average of credit card interest rate remained around 14.5%, and its standard deviation rose almost one percentage point from 4.29% in 1995 to 5.24% in 2001.<sup>21</sup> The average of credit card interest rates in 2004 decreased to 11.49% while MPRIME dropped to 4.00% – 5.00%. The variation of credit card interest rates across households also rose. Specifically, the standard deviation increased to 6.42%.

The simultaneous rise in the spreads of credit limits and interest rates, indicates that creditors have started to offer more differentiated credit terms to their borrowers. Variation in credit limits, however, has increased far more than that of interest rates, especially prior to 2004. While the standard deviation of limits rose by a factor of 68% from 1995 to 2001, the standard deviation of interest rates increased by a factor of 22%.<sup>22</sup> This fact motivates my focus on changes in credit limits rather than variation in interest rates.

Gross and Souleles [16] study borrowers’ response to credit supply and report an average “marginal propensity to consume (MPC) out of liquidity” ( $dDebt/dLimit$ ) in the range of 10 – 14%. Their study finds that MPC is significant even for borrowers well below their limits. Average MPC of 14% implies a \$790 increase in the average credit debt for the observed \$5,645 increase of the average credit limit from 1992 to 1998. The actual average increase of debt level is \$671 for this period, suggesting that the rise of credit card debt could be mostly attributed to the increase of credit supply.

According to Gross and Souleles [16], the long-term elasticity of debt to the interest rate is approximately  $-1.3$ . Although the SCF does not report interest rates for 1992, the implied change of the average debt level due to the change of interest rates from 1995 to 1998 is \$56, while the actual average credit card debt rose \$2,412. This fact again supports this paper’s focus on the quantity aspect of credit supply, namely credit limit, rather than the price of credit, namely interest rate.<sup>23</sup>

So far, I have reported the credit card limit and debt for an average household. But how about the credit

<sup>20</sup>Other than a general consumer credit interest rate reported for 1983, the SCF did not collect the credit card interest rates prior to 1995.

<sup>21</sup>MPRIME is reported from the Board of Governors of the Federal Reserve System.

<sup>22</sup>Stickiness of credit card interest rates have been studied by Ausubel [3] and Callem and Mester [8].

<sup>23</sup>Another challenge for studying the effect of interest rate on debt level lies in the fact that credit card prices contain other dimensions like cash back rates, flyer mileages and other point programs on which no data is available from the SCF.

		1989	1992	1995	1998	2001	2004
Cred. Lim	Mean	7,092	7,157	10,390	12,802	13,548	15,223
	Debt > 0	7,125	6,579	9,832	11,505	11,964	13,643
Cred. Lim	Std.	11,296	8,223	13,151	17,861	22,055	20,911
	Debt > 0	9,624	7,204	11,233	15,696	21,645	18,066
Cred. Debt (Card Holders)	Mean	954	1,025	1,346	1,696	1,453	1,851
	Debt > 0	1,828	1,947	2,404	3,098	2,707	3,295
Cred. Debt (Card Holders)	Std.	2,120	2,303	3,076	3,979	4,172	4,246
	Debt > 0	2,648	2,878	3,788	4,958	5,390	5,228
Interest Rate	Mean	–	–	14.51	14.45	14.36	11.49
	Debt > 0	–	–	14.14	14.48	14.20	11.81
Interest Rate	Std.	–	–	4.29	4.63	5.24	6.42
	Debt > 0	–	–	4.46	5.04	5.62	6.63
Card Holders		55.91%	62.32%	66.45%	67.54%	72.72%	71.46%
Revolvers(Debt > 0)		29.18%	32.83%	37.21%	36.97%	39.01%	40.14%

Table 1: Summary of US Households' Credit Cards

card limit and debt level of those who file for bankruptcy? The households who report bankruptcy filing in the SCF, have usually finished their legal processes and their debt levels are discharged. Moreover, after filing for bankruptcy, credit cards are cancelled so no information is available from the SCF on the filers' limits. Not being a panel dataset, the SCF does not allow me to observe the credit card limit and debt of defaulters just before filing for bankruptcy.<sup>24</sup> Therefore, this paper uses the financial description of bankruptcy filers from Sullivan, Teresa and Westbrook [25].

Table (2) summarizes the distribution of credit card debt listed in bankruptcy filings in 1991 and 1997 reported by Sullivan et al. [25]. Since the SCF data collection and report takes approximately one year, the data on filers' credit card debt corresponds to 1992 and 1998 data from the SCF. The table also reports the ratio of credit card debt to income for bankruptcy filers. The average credit card debt of filers rose by a factor of 24%, and the median increased by a factor of 35%, suggesting that the distribution not only shifted rightward but also spread out. The increase in the ratio of credit card debt to income is even higher. The average of the ratio of credit card debt to income for filers rose 44% while the median increased by 51%.

Credit card debt(Ratio to income)	1991	1997
Mean	\$10,193(.531)	\$12,608(.767)
s.d.	\$13,751(.755)	\$15,380(1.154)
25th percentile	\$2,702(.122)	\$3,864(.167)
median	\$6,112(.310)	\$8,262(.469)
75th percentile	\$12,807(.645)	\$14,188(.874)

Table 2: Credit Card Debt Listed in Bankruptcy (in 1989 dollars) From Sullivan et al.

This data suggests those who were filing for bankruptcy in 1997 were defaulting on much higher credit card debts. The next section provide a framework to study how those defaulters could get access to more credit, through higher credit card limits, and accumulate larger debts before defaulting on them.

<sup>24</sup>I tried to use the PSID, but that dataset does not report credit limits in exact amounts.

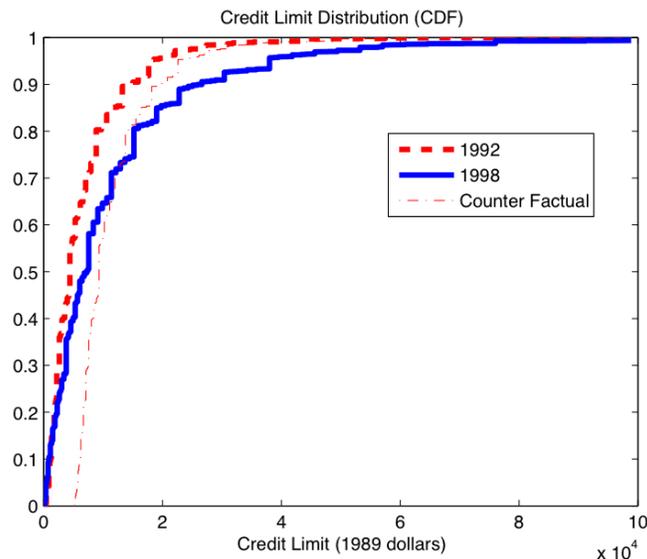


Figure 3: Distribution of Credit Limits

### 3 Model

Credit rating agencies usually use a borrower's credit history to assess her creditworthiness. Hence borrowers should potentially take the effect of their borrowing/payment decisions on their future terms of credit contracts, into consideration. The natural method of modeling the credit market would be employing dynamic signaling models; these models, however, are very difficult to analyze.<sup>25</sup> This paper takes a simple approach to model the improvement in credit risk rating. Creditors receive a public signal about borrowers' types when credit contracts are offered. I use more informative signals as a proxy for the improvement of the credit rating technology. This paper abstracts away from how credit scores are developed and just focuses on the informational content of signals when a household starts borrowing on its credit cards.

Musto [22] documents that creditors change their supply of credit as they lose information on the creditworthiness of borrowers due to the removal of the bankruptcy flag from their credit histories ten years after the filing date. This paper tries to study what happens when creditors become more informed about the creditworthiness of borrowers due to a better credit rating technology.

It is important to notice that the notion of safe and risky are relative. If a safe borrower, one with a high default cost, accumulates a large debt, she may be more likely to default than a risky borrower, one with a low cost of default, who has accumulated a small debt. The nature of costs associated with default are not the focus of this paper. These costs could have different pecuniary and non-pecuniary forms. An essential assumption of the paper is the heterogeneity of these costs across borrowers. That is, households with identical debts can make different decisions about filing for bankruptcy. Obviously, as debt rises all households become more likely to default. This paper tries to study the implication of this heterogeneity for credit allocation and default.

As noted by Calem, Gorday and Mester [7], credit card balances show high persistence with yearly autocorrelation of 0.90. That is, households use credit cards for medium- or long-term financing rather than for short-term or unanticipated liquidity shocks. This fact is exploited by this paper in modeling household borrowing needs.

<sup>25</sup>For an example of a model with dynamic updating of creditors' beliefs about borrowers' creditworthiness see Chatterjee et al. [9]. Extension of credit over time in their model depends on the evolution of household credit scores (or Bayesian posteriors of household type). Effectively their contracts ration through price rather than quantity limits, which is the focus of my paper. As documented in the previous section, credit limits are an important part of credit contracts and have experienced larger variations over time.

Instead of using a Markovian income process with high persistence, I assume households start with an initial income, then at a random time they switch to their permanent income level, which is likely to be higher. In this framework, households are not using their credit lines to smooth their consumption whenever they receive a short-term income or liquidity shock. Instead, credit lines are used for long-term financing, an implication consistent with the data.<sup>26</sup> Moreover, this setup allows for a very simple link between credit supply and unsecured household debt. Depending on the size of available credit limit and how long it takes before realizing her permanent income, a borrower can accumulate different levels of debt. After receiving her permanent income, she can consider repaying her debt, or simply incurring her default cost and walking away from her debt obligations. Obviously, unfulfilled expectation of permanent income and low cost of default raise borrowers' incentive to default.

An average American household has about four credit cards, with usually different terms of contract (i.e. different credit limits and interest rates.) However, they tend to carry their revolving unsecured debt on a credit card which offers the lowest interest rate. I assume each borrower is only allowed to have a single credit contract with a creditor of her choice. Without this assumption, the agent could be offered a continuum of contracts with incremental credit limits and increasing interest rates. In that case, the agent would start borrowing from the contract with the lowest interest rate and as her debt rose she would use the contracts with higher interest rates. This approach is identical to offering borrowers a menu of interest rates for different debt levels.<sup>27</sup>

Although the framework is general enough to allow for contracts to vary in both of credit limit and interest rate dimensions, in order to simplify the model and focus on the credit limit dimension of credit card contracts, I assume all of the offered contracts have the same interest rate, which is above the risk free rate.<sup>28</sup> The assumption is made particularly to focus on the response of consumer unsecured debt to changes in credit card limit. Moreover, as explained in the previous section, most of the heterogeneities in the credit card contracts during 1990s were along the credit limit dimension and variations in the interest rates were much more limited. This makes the model consistent with the US data during 1990s.

In the following subsections, I first describe the environment, then the household's problem will be studied. The creditor's problem and existence of equilibrium will conclude this section.

### 3.1 Environment

Consider a continuous time economy populated by a unit measure of infinitely live agents denoted by  $i \in (0, 1)$ . Agents discount future at rate  $\beta$  and their instantaneous utility from consumption is given by a strictly increasing and strictly concave function  $u(\cdot)$ . There is also a competitive market of risk neutral creditors with access to funds at rate  $r = \beta$ , who can only lend at rate  $\rho \geq r$ .

At the beginning of the economy each agent  $i$  realizes her risk type  $\theta(i) \in [0, 1]$  which is private information. There is a *Rating Technology* which sends a public signal  $\tilde{\theta}(i)$  about each agent's type  $\theta(i)$ . The joint distribution of types and signals, denoted by  $\psi(\theta, \tilde{\theta})$ , is public information.

Agents receive two streams of incomes. At the beginning, they receive their *initial* stream of income,  $y^I > 0$ , until a random switching time governed by a Poisson process with the arrival rate  $\delta$ . Once the switching time arrives, agents draw their publicly observable *permanent* income,  $y^P$ , from a distribution  $F(\cdot)$ . Afterward, they will receive a certain stream of  $y^P$  units of income for the rest of their lives. Assume the support of  $y^P$  is uniformly bounded away from zero<sup>29</sup> and  $F(\cdot)$  does not have any mass point. Moreover, we assume  $E[y^P] \gg y^I$ .

<sup>26</sup>For an elaborate model of credit card usage with Markovian income process, see Chatterjee, Corbae, Nakajima and Rios-Rull [10]

<sup>27</sup>Chatterjee et al. [10] uses this approach.

<sup>28</sup>In an earlier version of the paper, I had assume variable credit limits and interest rates. However, in order to highlight the role of credit scores and prevent the creditors to sort borrowers based on their choice of contract, the timing of the environment was different. In particular, borrowers would choose their contracts before realizing their types. In the current version, however, borrowers know their types when they choose their contracts. Nevertheless, since there is only one instrument available for the creditors, namely credit limit, they cannot use offering different contracts to sort borrowers with different types.

<sup>29</sup>That is  $\exists \epsilon > 0$  such that  $F(\epsilon) = 0$ .

Agents can borrow from the creditors while waiting for the realization of their permanent income. Lending contracts can only be made at the beginning of the economy after receiving the public signals about the agents' types. Lending contracts are constrained to have a fixed interest rate  $\rho \geq r$ , but credit limits are determined by the competitive credit market, based on the public signals on the agents' types. Once the contracting phase is over and an agent receives a credit limit  $L$ , the creditor is committed to the contract until the agent realizes her permanent income. In the meantime, she can accumulate debt,  $D$ , at the interest rate  $\rho$  up to the credit limit  $L$ .

At any point in time, the agents are allowed to exercise their *bankruptcy* option. If an agent files for bankruptcy, all of her debt will be forgiven but she cannot borrow from the credit market anymore. Moreover, after filing for bankruptcy, agent  $i$  can only consume  $\theta(i)$  fraction of her income from that time on. Once an agent  $i$  realizes her permanent income  $y^P(i)$ , while having  $D$  units of debt, she chooses either to file for bankruptcy and consumes  $(\theta(i) \cdot y^P(i))$  units from that point on, or to repay her debt and consumes  $(y^P(i) - r \cdot D)$  units for the rest of her life.

To summarize, at the beginning of the economy the competitive credit market receives signals on the agents' risk types, and offers credit contracts which consist of a fixed interest rate but signal specific credit limits.<sup>30</sup> Then, agents start using their credit lines until they realize their permanent income. At any point agents can default on their debt which will cause them to lose a type-dependent fraction of their income for the rest of their lives.

### 3.2 Agent's Problem

After realization of their types,  $\theta$ , and receiving the offered credit contracts, which we denote by a pair of credit limit and interest rate,  $(L, \rho)$ , agents decide on how much to borrow/pay on their credit lines, and whether to file for bankruptcy or not. In particular, at the beginning of time an agent can choose  $b(t)$ , the amount of borrowing/payment on her credit line if she does not switch to her permanent income by time  $t$ . Also, she can decide whether to default on her debt at time  $t$  if she has not realized  $y^P$  by that time. Since agents can only default once, we can denote the time of filing for bankruptcy by  $T^*$ . That is, if the agent's income does not switch to  $y^P$  by time  $T^*$ , she defaults at that time. Obviously the agent can choose to not default on her debt before realization of her permanent income, in which case we have  $T^* = \infty$ .

The sequential problem for a type  $\theta$  agent with initial income  $y^I$  who is offered contract  $(L, \rho)$  is given by:

$$V((\theta, y^I); (L, \rho)) = \max_{b(t), T^*} \left\{ \begin{array}{l} \int_0^{T^*} e^{-(\delta+\beta)t} [u(y^I + b(t)) + \delta W(D(t); \theta)] dt \\ + \int_{T^*}^{\infty} e^{-(\delta+\beta)t} [u(\theta y^I) + \delta W^D(\theta)] dt \end{array} \right\} \quad (1)$$

where the debt level at time  $t < T^* \leq \infty$ , denoted by  $D(t)$ , must satisfy the credit limit constraint:

$$D(t) = \int_0^t e^{\rho(t-\tau)} b(\tau) d\tau \leq L. \quad (2)$$

$W(D; \theta)$  is the expected value of realizing the permanent income for a type  $\theta$  agent with debt level  $D$ , and  $W^D(\theta)$  is the expected value of realizing the permanent income for a type  $\theta$  agent who has defaulted on her debt and filed for bankruptcy before realizing her permanent income.

In particular, the expected value of realizing the permanent income after default is:

$$W^D(\theta) = \frac{1}{\beta} \int u(\theta y^P) dF(y^P). \quad (3)$$

When a type  $\theta$  agent with  $D$  units of debt realizes her publicly observable permanent income  $y^P$ , since  $r \geq \beta$ , she has no incentive to borrow from the credit market without the intention of defaulting on it. Hence the creditors

<sup>30</sup>Note that the contracting phase does not involve any dynamic decision making.

will not allow her to use the remaining part of her credit line after she realizes her permanent income. Thus the agent has to decide either to pay back her debt or to default on it. If the agent files for bankruptcy, the present value of her utility from consuming  $\theta$  fraction of her income stream is:

$$\frac{1}{\beta}u(\theta y^P).$$

If the agent decides to pay back her debt, since there is no uncertainty about her future income for the competitive credit market, the charged interest rate will be set at  $r$ . Then she should choose an stream of payments,  $p(t)$ , to maximize the present value of her utility given by:

$$\max_{p(t)} \int_0^\infty e^{-\beta t} u(y^P - p(t)) dt,$$

subject to  $\dot{D}(t) = rD(t) - p(t)$ . The Hamiltonian for this problem is given by:

$$\mathcal{H} = e^{-\beta t} u(y^P - p) + \lambda(rD - p)$$

which yields the solution  $\dot{p}(t) = (r - \beta) \frac{u'(y^P - p(t))}{u''(y^P - p(t))}$ . In the case of  $r = \beta$ , the solution is given by  $p = rD$ , and hence the present value of the agent's utility is given by:

$$\frac{1}{\beta}u(y^P - rD).$$

In this case, if  $y^P - rD \geq \theta y^P$  the agent will choose to consolidate her debt at the interest rate  $r$  and pay it back. Otherwise, she will default on her debt and consume  $(\theta \cdot y^P)$  for the rest of her life. Therefore for  $r = \beta$  we have:

$$W(D; \theta) = \frac{1}{\beta} \left[ \int_0^{\frac{rD}{1-\theta}} u(\theta \cdot y^P) dF(y^P) + \int_{\frac{rD}{1-\theta}}^\infty u(y^P - rD) dF(y^P) \right]. \quad (4)$$

Overall, the agents have three decisions to make: (i) whether to default or pay back their debt after realizing their permanent income, (ii) whether to default or not before realizing the permanent income, that is to set  $T^* < \infty$  or  $T^* = \infty$ , and (iii) the sequence of borrowing/payment  $b(t)$ .

If an agent decides to default before realizing her permanent income, she does not do so before using all of her available credit limit, or otherwise she can continue borrowing and default later.<sup>31</sup> Let's denote the time of reaching the credit limit by  $T$ . Then if the agent defaults before realizing her permanent income, that is  $T^* < \infty$ , then  $T = T^*$ . Later we will show, under certain conditions,  $T$  is finite. That is, even if  $T^* = \infty$  and the agent does not default before realizing her permanent income, she will reach her credit limit in finite time, unless she realizes her permanent income before reaching her credit limit.

Let's denote the agent's borrowing at the time of reaching her credit limit by  $b^* = b(T)$ . If she does not default at the limit, she has to pay the interest charge of her debt to satisfy the credit limit, that is  $b^* = -\rho L$ . Therefore, if the agent does not default after reaching her credit limit, the continuation value equals:

$$\frac{1}{\delta + \beta} [u(y^I - \rho L) + \delta W(L; \theta)].$$

But if she defaults, the continuation value will be

$$\frac{1}{\delta + \beta} [u(\theta \cdot y^I) + \delta W^D(\theta)].$$

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<sup>31</sup>Notice that even if she realizes her permanent income the option of default is still available for her.

So the agent defaults at the credit limit only if:

$$[u(\theta \cdot y^I) + \delta W^D(\theta)] > [u(y^I - \rho L) + \delta W(L; \theta)]. \quad (5)$$

Now if (5) holds and the agent defaults at the limit, then  $b^* = b(T) = b(T^*) \geq -\rho L$ . In general:

**Lemma 1** *Borrowing/payment at the credit limit,  $b^* = b(T) \geq -\rho L$ , satisfies:*

$$u'(y^I + b^*) \leq \frac{[u(y^I + b^*) + \delta W(L; \theta)] - [u(\theta y^I) + \delta W^D(\theta)]}{\rho L + b^*}, \quad (6)$$

with equality if  $b^* > -\rho L$ . Moreover, if (5) holds then (6) uniquely determines  $b^*$ .

**Proof.** See the Appendix. ■

In short, the left hand side of (6) is the marginal utility of consumption for an agent just before she reaches her credit limit. The numerator of the right hand side is the difference between the stream of utility before and after default, and the denominator is the rate of debt increase (or equivalently the rate of approaching the limit as the time of default.) Overall the right hand side of (6) is the marginal cost of approaching the event of default due to increasing consumption.

Given the agent's borrowing/payment and default decisions at her credit limit, let's study her borrowing decision before reaching the limit. The Hamiltonian for the agent's problem (1) before reaching the limit is given by:

$$\mathcal{L} = e^{-(\delta+\beta)t} [u(y^I + b) + \delta W(D; \theta)] + \lambda [\rho D + b], \quad (7)$$

and the optimal solution must satisfy:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \rho D + b = \dot{D} \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial D} = e^{-(\delta+\beta)t} \delta W(D; \theta) + \rho \lambda = -\dot{\lambda} \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial b} = e^{-(\delta+\beta)t} u'(y^I + b) + \lambda = 0. \quad (10)$$

We have the solution for  $b(T)$  from (6) which implies the  $\lambda(T)$  from (10). Now solving for  $\lambda$  backward from (9) and then substituting it in (10) for  $t < T$  we have:

$$\begin{aligned} u'(y^I + b(t)) &= e^{-(\delta+\beta-\rho)(T-t)} u'(y^I + b(T)) \\ &\quad - \int_t^T \delta e^{-(\delta+\beta-\rho)(\tau-t)} W_D(D(\tau); \theta) d\tau. \end{aligned} \quad (11)$$

The left hand side of (11) is the marginal utility from increasing debt at time  $t$ . The right hand side gives us the two marginal costs of increasing debt. The first expression is the marginal cost of getting closer to the credit limit, and hence being credit constrained. The second expression is the marginal cost of debt if the agent realizes her permanent income before reaching her credit limit.

Figure (4) helps to understand the two marginal costs. If the agent increases her borrowing for a small period of time but otherwise leaves it unchanged, then she will reach the credit limit sooner. This is depicted by the altered debt path reaching the credit limit sooner than the original debt oath. As a result, the agent's consumption has to decline for a period just before the time she used to reach her credit limit. On the other hand if the agent

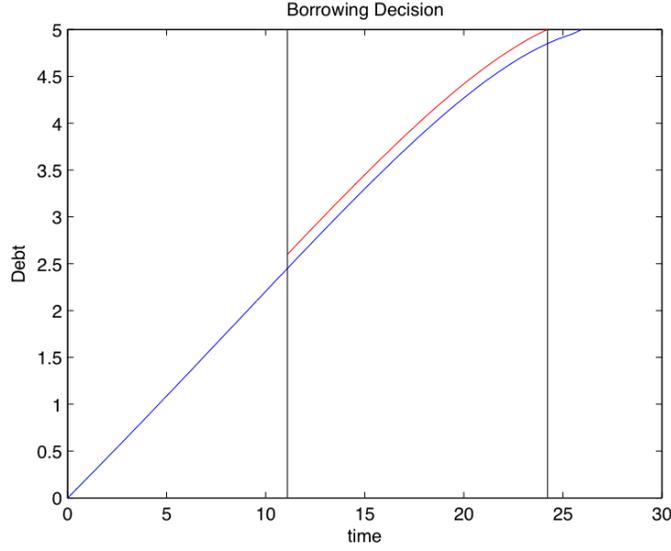


Figure 4: Change of Borrowing Decision

realizes her permanent income before reaching the limit, with the altered borrowing, she will carry more debt which will be more costly to pay back.

Notice that we can also consider borrowing/payment,  $b$ , as a function of debt,  $D$ , and the credit contract  $(L, \rho)$ . That is  $b(t) = b(D(t); (L, \rho))$ . Then by taking the derivative of (10) with respect to time and then substituting for  $\dot{\lambda}$  from (9) we have:

$$\frac{db(D; (L, \rho))}{dD} = \frac{\dot{b}}{\dot{D}} = \frac{(\delta + \beta - \rho)u'(y^I + b) + \delta W_D(D; \theta)}{u''(y^I + b)(\rho D + b)}. \quad (12)$$

Using  $b(L; (L, \rho)) = b^*$  from (6) as the boundary condition, we can find  $b(D; (L, \rho))$  for  $\forall D < L$  by solving the differential equation (12).<sup>32</sup> Moreover, we can use this approach to state some properties of the borrowing/payment function.

**Lemma 2**  $b(D; (L, \rho))$  is continuous in debt level,  $D$ , credit line,  $L$ , and interest rate,  $\rho$ .

**Proof.** By construction  $b(L; (L, \rho)) = b^*$  is continuous in  $L$  and  $\rho$ , from (6). The continuousness of  $b(D; (L, \rho))$  for  $D < L$  follows from continuity of the solution for the differential equation (12). ■

**Lemma 3** For  $L_1 < L_2$ , if the agent's solution for (1) finds it optimal to increase her debt up to her credit limit, then:

$$b(L_1; (L_1, \rho)) < b(L_1; (L_2, \rho)).$$

**Proof.** See the Appendix. ■

**Theorem 4** If the agent's solution for (1) finds it optimal to increase her debt up to her credit limit, then  $b(D; (L, \rho))$  is strictly increasing in  $L$ .

<sup>32</sup>I use this method to compute the borrowing/payment function in the quantitative exercise of the next section.

**Proof.** Suppose not. Then  $\exists D^*$  and  $L_1 < L_2$ , such that

$$b(D^*; (L_1, \rho)) \geq b(D^*; (L_2, \rho)).$$

Since  $b(L_1; (L_1, \rho)) < b(L_1; (L_2, \rho))$  then by continuity  $\exists D^{**}$  such that

$$b(D^{**}; (L_1, \rho)) = b(D^{**}; (L_2, \rho)).$$

But in that case (12) implies

$$b(D; (L_1, \rho)) = b(D; (L_2, \rho)) \quad \forall D \geq D^{**}$$

which contradicts  $b(L_1; (L_1, \rho)) < b(L_1; (L_2, \rho))$ . ■

The theorem states that when the agents receive higher credit limits they will accumulate more debt. This point can be seen in figure (5). With the credit limits depicted in this figure, agents do not default after reaching the credit limit and pay back the interest charge of their debt, waiting for the realization of their permanent income. Notice that with higher credit limit it takes longer for the agent to reach the limit. Moreover, as time passes and agents accumulate debt, their borrowing declines. Following (11) there are two factors contributing to the curbing of their borrowing. First, as debt rises the agent gets closer to the credit limit which makes her borrowing constrained. Second, the marginal cost of repayment after realization of permanent income is increasing in debt level. These two effects can make borrowing a decreasing function of debt. The first factor is always in place, but the second one may not. Specially, once the agent chooses to default on her debt, despite the increasing marginal cost of repayment, the increasing propensity to default can decrease the marginal cost of debt.

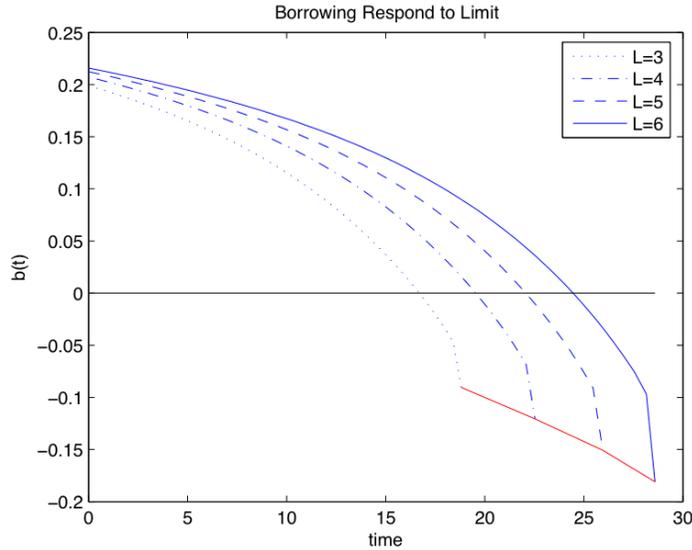


Figure 5: Borrowing with Different Credit Limits

Figure (6) shows borrowing patterns for two credit limits. With the lower limit, the agent does not default after reaching her credit limit. But if she receives the higher credit limit, she will default on her debt once she reaches this higher limit.<sup>33</sup> This figure also depicts another important response of borrowing to credit limit. As the agent approaches her credit limit she curbs her borrowing due to the first factor described above. However,

<sup>33</sup>The figure shows that there are times when the agent borrows more with the lower credit limit. This is because by that time, with the higher credit limit, the agent has accumulated more debt. More specifically,  $b(D; (L_1, \rho)) < b(D; (L_2, \rho))$  still holds for  $L_1 < L_2$ .

at the beginning of her borrowing period, as time passes and her debt rises, the agent might actually increase her borrowing. This is because the second factor explained above is not in place. That is the marginal cost of debt after realization of permanent income is actually declining, due to the rise of propensity to default.

Notice that, as long as the agent is not going to default on her debt after realization of her permanent income, the marginal cost of debt after realization of permanent income is an increasing function of debt, due to the increasing marginal cost of debt repayment.<sup>34</sup> But if the probability of default after realization of permanent income is positive, as debt rises the probability of repayment also falls. The latter effect could decrease the marginal cost of debt after realization of permanent income. We summarize these effects in the following lemma.

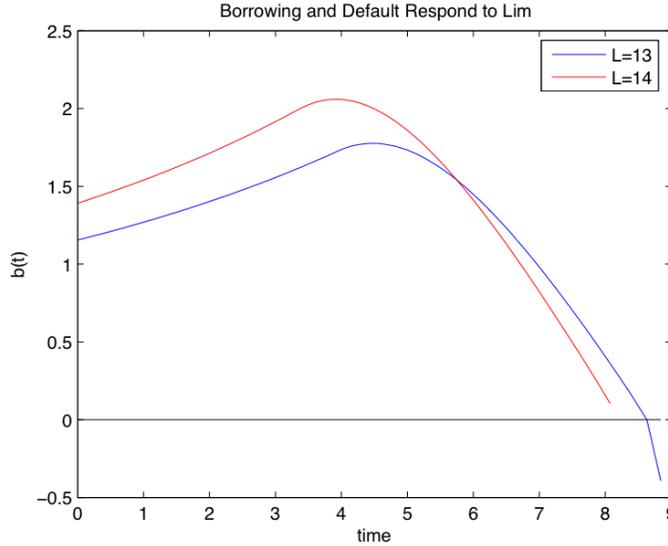


Figure 6: Borrowing and Default with Different Credit Limits

**Lemma 5** *If  $r = \beta$  and  $F(\cdot)$  is concave, then  $W_D(D; \theta)$  is decreasing in  $D$  for  $D \in [0, \frac{(1-\theta)y^P}{r}]$  and increasing for  $D > \frac{(1-\theta)y^P}{r}$ .*

**Proof.** See the Appendix. ■

Notice that so far we have simply assumed agents continue to accumulate debt until reaching their credit limits. But what if the revolving interest rate,  $\rho$ , is too high, or the expected rise in the permanent income is too small? Would agents ever reach their credit limits then? In particular if the solution of (12) is such that  $\exists D < L$  where  $b(D; (L, \rho)) = -\rho D$ , then the agent stops borrowing after reaching to the debt level  $D$ . The following lemma provides a sufficient condition for optimality of borrowing up the credit limit.

**Lemma 6** *For  $r = \beta$  and concave  $F(\cdot)$ , if  $(\delta + \beta - \rho)u'(y^I) + \delta W_D(\frac{(1-\theta)y^P}{r}; \theta) > 0$ , then the agent does not stop increasing her debt before reaching her credit limit.*

**Proof.** See the Appendix. ■

Notice that this condition is independent of the credit limit,  $L$ . However, it depends on the contract's interest rate,  $\rho$ . Obviously it also depends on the initial income,  $y^I$ , the inverse of the expected length of the borrowing

<sup>34</sup>We assume the utility function is strictly concave.

period,  $\delta$ , the permanent income distribution,  $F(\cdot)$ , and the cost of default  $(1 - \theta)$ . Although the previous lemma provides a sufficient condition for the agents' borrowing to continue until their credit limit, it is silent about whether the agents actually reach their limits or not. The next theorem answers this question.

**Theorem 7** For  $r = \beta$  and concave  $F(\cdot)$ , if  $(\delta + \beta - \rho)u'(y^I) + \delta W_D(\frac{(1-\theta)y^P}{r}; \theta) > 0$ , then  $\exists T$  such that  $\forall t \geq T$  we have  $D(t) = L$ . That is, if the agents do not realize their permanent income for a long enough period, then they will reach their credit limits.

**Proof.** Lemma 6 guarantees that for any debt level  $D < L$ , the agent increases her debt level above it. Suppose the agent does not reach the credit limit, then for a large enough  $t$  such that  $D(t) \approx L$  and  $b(t) \approx -\rho L$ , (11) implies:

$$\begin{aligned} u'(y^I + b(t)) &= - \int_t^\infty \delta e^{-(\delta+\beta-\rho)(\tau-t)} W_D(D(\tau); \theta) d\tau \\ &\approx \frac{-\delta}{\delta + \beta - \rho} W_D(L; \theta), \end{aligned}$$

which contradicts  $(\delta + \beta - \rho)u'(y^I - \rho L) + \delta W_D(L; \theta) > 0$ . ■

Next we study the effect of the agents' riskiness on their borrowing behavior. In particular, we show that due to lower cost of default for riskier agents, they face a lower marginal cost of borrowing, which in turn induces them to borrow more. First we need to study the effect of the agents' riskiness on their borrowing at their credit limits.

**Lemma 8** Borrowing at the limit,  $b(L; (L, \rho), \theta)$ , is increasing in the agent's riskiness,  $\theta$ , and strictly increasing if default is optimal at the limit.

**Proof.** Following lemma 1, if default is not optimal at the limit then  $b(L; (L, \rho), \theta) = -\rho L$ . If default is optimal, then following (6),  $b(L; (L, \rho), \theta)$  is governed by:

$$u'(y^I + b) \cdot (\rho L + b) - u(y^I + b) + u(\theta y^I) - \delta \{W(L; \theta) - W^D(\theta)\} = 0.$$

Therefore:

$$\frac{db}{d\theta} = - \frac{y^I u'(\theta y^I) + \delta \int_{\frac{rL}{1-\theta}}^\infty y^P u'(\theta y^P) dF(y^P)}{u''(y^I + b) \cdot (\rho L + b)} > 0,$$

where the inequality follows from  $u''(\cdot) < 0 < u'(\cdot)$  and  $b(L; (L, \rho), \theta) > -\rho L$ . ■

The proof relies on the benefit of postponement of default decision until realization of the permanent income being a decreasing function of the agent's riskiness. That is  $\frac{d}{d\theta} \{W(L; \theta) - W^D(\theta)\} < 0$ . Next we use (12) to study the effect of the agents' riskiness on their borrowing before reaching their credit limit.

**Theorem 9** Borrowing,  $b(D; (L, \rho), \theta)$ , is increasing in the agent's riskiness,  $\theta$ .

**Proof.** See the appendix. ■

The proof relies on the marginal cost of debt after realization of permanent income, being lower for riskier agents. The lower marginal cost, induces riskier agents to borrow more and accumulate debt at a faster pace.

So far, we have studied the effect of the credit limit on borrowing and debt, but not the default rate. From (5) the probability of defaulting on  $D$  units of debt after realization of permanent income, is given by:

$$F\left(\frac{rD}{1-\theta}\right). \quad (13)$$

As depicted in figure (5), when an agent is offered a larger credit limit, she will accumulated more debt. Since the time of switching to permanent income is exogenous and independent of debt level, the probability of having more debt at the time of switching, and therefore the probability of default, is higher with a larger credit limit.

The agents might also default when they reach to their credit limits before realization of their permanent incomes. This is shown in figure (6). In this comparison the higher credit limit not only causes the agent to accumulate more debt by the time of switching to her permanent income, but also causes her to reach the credit limit sooner and then default on her debt. In summary, a larger credit limit induces the agents to borrow more, and hence makes them more likely to default. Moreover, riskier agents, who have lower cost of default, not only are more likely to default on their debt after realization of their permanent incomes, but also accumulate more debt before switching to their permanent incomes.

### 3.3 Creditors' Problem

In the previous subsection we studied decision rules of a type  $\theta$  agent with the initial income  $y^I$  and a credit contract  $(L, \rho)$ . The creditors take the agents' decision rules as given. Given the agents' decision rule as a function of the offered credit contract, the creditors' expected profit from offering a contract  $(L, \rho)$  to a type  $\theta$  agent with the initial income  $y^I$  is given by:

$$\Pi((\theta, y^I); (L, \rho)) = \int_0^{T^*} e^{-(\delta+r)t} [-b(t) + \delta D(t)(1 - F(\frac{rD(t)}{1-\theta}))] dt,$$

where  $b(t)$  and  $T^*$  are the borrowing and default time decisions which solve (1) for a type  $\theta$  agent with the initial income  $y^I$  who is offered a credit contract  $(L, \rho)$ . Moreover  $D(t)$  is the implied debt amount from (2). Notice that when a type  $\theta$  agent switches to her permanent income with debt level  $D(t)$ , the probability of repaying her debt is  $1 - F(\frac{rD(t)}{1-\theta})$  from (13).

When a creditor in the competitive credit market offers a credit contract, the only available information is the public signal  $\tilde{\theta}$  from the Rating Technology. Denoting the conditional probability of drawing type  $\theta$  given the signal  $\tilde{\theta}$  by  $\psi(\theta|\tilde{\theta})$ , a creditor's expected profit from offering credit contract  $(L, \rho)$  to an agent with a rating signal  $\tilde{\theta}$  is:

$$\Pi(\tilde{\theta}; (L, \rho)) = \int \Pi((\theta, y^I); (L, \rho)) d\psi(\theta|\tilde{\theta}). \quad (14)$$

Recall that when the agents receive their credit contracts, they have realized their private information about their true risk types,  $\theta$ . However, the offered credit contracts can only vary in their credit limits,  $L$ , and they all offer the same fixed interest rate,  $\rho$ .<sup>35</sup> It is easy to show that all agents (weakly) prefer contracts with higher credit limits. Therefore, the equilibrium credit limit for an agent with public rating signal  $\tilde{\theta}$  is given by:

$$L_{\tilde{\theta}} = \max \left\{ L \mid \Pi(\tilde{\theta}; (L, \rho)) = 0 \right\}. \quad (15)$$

Notice that when the signals are not very informative and hence agents with different types are locked into the same contract, then some types generate positive profit which is used to compensate the losses made on the other types. Figure (7) shows how a creditor's profit changes as she increases the offered credit limit,  $L$ , for a fix interest rate  $\rho > r$ . The figure depicts profit from two types with different default costs. As the limit increases the creditor's profit rises, since agents can borrow more but yet do not find it optimal to default on their debt. Hence

<sup>35</sup>In an earlier version of the paper, creditors could choose the interest rate as well. However, in order to avoid market failure due to asymmetric information, I had assumed that the agents do not learn their types until after choosing their credit contracts.

for  $\rho > r$ , the creditor's expected profit rises. For a large enough credit limit, the riskier agents accumulate enough debt to find it optimal to default, and the creditor's profit from them declines as the credit limit rises. However, the safer agent still generates more profit. As the credit limit increases further, eventually the safer agents also accumulate enough debt to find the default option optimal. This makes the expected profit from them to fall as well. Next section provides sufficient conditions under which the solution for the creditor's problem (15) exists.

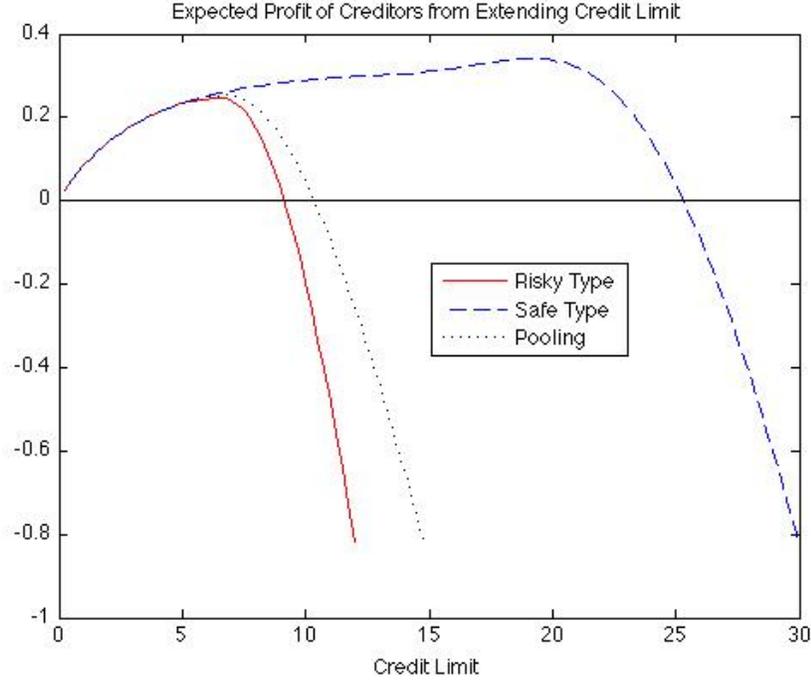


Figure 7: Creditors' Profit

### 3.4 Equilibrium Existence

In this part we show the existence of equilibrium, which is characterized by the solution of the creditors' problem (15) for offering credit contract  $(L, \rho)$  to an agent with the rating signal  $\tilde{\theta}$ .

**Lemma 10**  $\Pi((\theta, y^I); (L, \rho))$  is continuous in  $L$  and  $\rho$ .

Proof follows from the continuity of the decision rule  $b$  and lack of mass points in  $F(\cdot)$ <sup>36</sup>.

**Lemma 11** If the support of  $\psi(\cdot|\tilde{\theta})$  is bounded away from zero, the utility function  $u(\cdot)$  is unbounded from above and the expected present value of all future income for an agent with signal  $\tilde{\theta}$  is bounded then:

$$\lim_{L \rightarrow \infty} \Pi(\tilde{\theta}; (L, \rho)) < 0 \quad \forall \rho.$$

<sup>36</sup>Note that, when the supplied credit limit is such that the agent is indifferent between default or staying at the limit after reaching the credit limit, then creditor's profit depends on the fraction of agents who default after reaching the limit. Therefore  $\Pi((\theta, y^I); (L, \rho))$  is a correspondence of  $L$ , however, a continuous one.

**Proof.** See the Appendix. ■

The proof first shows for any  $\rho \geq r$  as the credit limit increases, so does the expected value of the contracts for the agents with signal  $\tilde{\theta}$ . That is,  $\lim_{L \rightarrow \infty} V(\tilde{\theta}; (L, \rho)) \rightarrow \infty$ . Then uses the fact that with  $\Pi(\tilde{\theta}; (L, \rho)) \geq 0$  the expected utility  $V^I(\tilde{\theta}; (L, \rho))$  is bounded from above.

**Theorem 12** *If the support of  $\psi(\cdot|\tilde{\theta})$  is bounded away from zero, the utility function  $u(\cdot)$  is unbounded from above and the expected present value of all future income for an agent with signal  $\tilde{\theta}$  is bounded, then the creditors' problem (15) has a unique solution.*

**Proof.** By definition, for a given interest rate  $\rho$ , the agents' expected value  $V(\tilde{\theta}; (L, \rho))$  is increasing in  $L$ . This is because the agents can always opt out and do not use their credit limits. Let's  $L^*(\tilde{\theta}; \rho)$  denotes the largest  $L$  such that  $\Pi(\tilde{\theta}; (L, \rho)) = 0$ . Since  $\Pi(\tilde{\theta}; (0, \rho)) = 0$  and  $\Pi(\tilde{\theta}; (L, \rho))$  is continuous in  $L$ , the previous lemma guarantees the existence and uniqueness of  $L^*(\tilde{\theta}; \rho)$ , which solves (15). ■

Note that for  $\rho > r$ , if  $E(y^p) \gg y^I$ , then for small enough  $\epsilon > 0$ ,  $V(\tilde{\theta}; (\epsilon, \rho)) > 0$  and  $\Pi(\tilde{\theta}; (\epsilon, \rho)) > 0$ , and therefore  $L^*(\tilde{\theta}, \rho) > 0$ .

## 4 Discussion and Quantitative Example

As stated in the previous section, when the creditors offer credit contracts the only available information is the exogenous signal from the Rating Technology. Therefore, the contracts in this environment are only conditional on the rating signals. That is, agents with identical signals will receive contracts with identical terms. However, agents with different types but the same contract may choose different borrowing patterns. Moreover, heterogeneity in the arrival times and the realized levels of permanent income, lead the agents with identical contracts and risk types to accumulate different amounts of debt and make different default decisions.

I assume credit contracts are all subject to a fixed interest rate  $\rho > r$ , and the only difference among types is in their default costs. In this case the only variation across contracts for different signals will be in their credit limits. Suppose different types are pooled together by the Rating Technology, which is possible if the signals are not very informative about the agents' types. In the equilibrium, the agents will receive a credit limit which sets the creditors' expected profit from the pooling contract equal to zero. The agents with a low cost of default will generate negative profit which will be compensated by the positive profit generated by the agents with high cost of default. Notice that although agents know their types when the contracts are offered, since all agents (weakly) prefer contracts with higher limits and contracts have a fixed interest rates, in this environment, the creditors cannot separate different types of agents within the pool of the agents with the identical rating signals by offering them different contracts.

Moreover, since the agents with high default costs are more likely to pay back their debt, the marginal cost of borrowing is higher for them relative to the agents with low default costs. Therefore, with the same credit limit the high default cost agents accumulate lower levels of debt, and the responsiveness of their debt level to an increase of their credit limit is also smaller. That is, they have lower propensity to utilize their credit limits. If the credit limit is increased above the equilibrium level, then loans taken by the low default cost agents which will make negative expected profit will increase more than loans taken by the high default cost agents which will make positive expected profit. This prevents the creditors from raising the credit limit in the pooling case.

Now suppose different types are separated by the Rating Technology, which is possible if the signals are informative about the agents' types. While in the pooling case there is cross subsidization across types, in the separating case the expected profit from the high default cost type cannot be used to subsidize the losses made on the low default cost type. In that case, the equilibrium supply of credit will increase significantly for the high default cost agents after switching to the separating case. Since the interest rate is assumed to be fixed, the competitive creditors extend the credit limit for the high default cost agents up to a level where some of them increase their debt large enough to find default optimal.

In the next subsection, I provide a quantitative example of the model to account for the increase in the average credit limit and credit card debt, as well as the rise in the number of bankruptcies observed from 1992 to 1998. In this quantitative example the mechanism which matches the data can be thought of as an increase in credit supply along the intensive margin and not the extensive margin. Afterward, I will provide an explanation for the increase of credit supply along the extensive margin.

## 4.1 Quantitative Example

Most of the households with positive credit limits do not borrow on their credit cards (see the last row of table (1)). Therefore I restrict my attention to revolvers, who have positive debt on their credit cards. The model predicts that households first increase their credit card debt before realizing their permanent income, then either default on their debt or pay it back. The SCF is not a panel dataset so I could not use it for direct study of the dynamics of household credit card debt accumulation and de-accumulation. The SCF, however, reports households' answer to the following question:

Thinking only about Visa, Mastercard, Discover, Optima and store cards, do you almost always, sometimes, or hardly ever pay off the total balance owed on the account each month?

Roughly speaking, half of the revolvers answer “they hardly ever pay off the total balance.” This group of households are those who are accumulating credit card debt and I call them *A-revolvers*. The last row of table (3) reports the fraction of A-revolvers from all households in the SCF 1992 and 1998. The first and third rows report the average ratios of credit limit and credit card debt to annual income for A-revolvers.<sup>37</sup> The standard deviations of the distributions of ratios of credit limit and credit card debt to income for A-revolvers are also reported. The next row reports the fraction of A-revolvers who filed for bankruptcy, assuming all filers are also A-revolvers.

Since the fraction of A-revolvers in the population increased, looking at the ratio of defaulters to A-revolvers underestimates the rise of bankruptcy filings. However, the results of this quantitative exercise are not sensitive to using the ratio of defaulters to the average number of A-revolvers across two years.

A-revolvers	1992	1998
Av. (Cred Lim/Income)	25.97 %	42.48%
Std. (Cred Lim/Income)	41.38 %	77.50%
Av. (Cred Card Debt/Income)	10.66%	16.95%
Std. (Cred Card Debt/Income)	17.61%	35.46%
Default Rate	5.86%	7.39%
A-revolvers/Population	16.05%	17.31%

Table 3: Target Moments

For this exercise I use the constant relative risk aversion utility function, that is  $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ . The switching process from the initial income to the permanent income follows a Poisson process with parameter  $\delta$  for all types. The initial income is fixed for all types and the permanent income is drawn from a truncated exponential distribution:

$$F(y^P) = 1 - e^{-\eta(y^P - \underline{y}^P)}$$

for all types. Finally I assume there are two types,  $0 < \theta_L < \theta_H < 1$ , with the distribution  $\mu(\theta_L) = 1 - \mu(\theta_H)$ .

In order to highlight the role of informational change, I will consider two cases. In the first one, the Rating Technology's signal, contains no information on the agents' types. Therefore, we will have complete pooling of

<sup>37</sup>This figures are reported after dropping less than half a percentage of the subsample who report zero or negative income. I also tried the exercise with the average credit limit and credit card debt of A-revolvers divided by their average annual income. The results are very similar.

the two types in this case. In the second case, I assume the signal is fully informative. Hence, in this case we will have complete separation and each type will receive a different credit contract.

I try to match the moments generated by the model in the pooling case to the data moments from 1992, and the moments generated by the model in the separating case to the data moments from 1998. Clearly the Rating Technology provided some information on households' risk types in 1992, and did not provide full information in 1998. In particular, households were not offered a single credit card contract in 1992 as the model suggests in the pooling case. Instead, a distribution of credit card limits were supplied by the creditors in 1992. However, the spread of the distribution of credit limits increased from 1992 to 1998. In this quantitative exercise I try to account for the increase of the spread of the distribution of credit limits as a result of a more informative Rating Technology.<sup>38</sup>

In the pooling case, both types are offered a single credit limit,  $L_P$ , while in the separating case two credit limits,  $L_S^{\theta_L}$  and  $L_S^{\theta_H}$ , are offered depending on the agent's type  $\theta \in \{\theta_H, \theta_L\}$ . The 1992 data provides a distribution of credit limits. To study the increase of the spread of the credit limit distribution from 1992 to 1998, suppose any credit limit  $L$  from the distribution of credit limits in 1992 was replaced by two credit limits  $\frac{L_S^{\theta_L}}{L_P} L$  and  $\frac{L_S^{\theta_H}}{L_P} L$ , with weights  $\mu(\theta_L)$  and  $\mu(\theta_H)$ , in the distribution of credit limits for 1998. In this case, the following would hold for the coefficients of variation for these three distributions:

$$CV(L_{1998})^2 + 1 = (CV(L_{1992})^2 + 1)(CV(L_S)^2 + 1) \quad (16)$$

where  $L_{1992}$  and  $L_{1998}$  are the distribution of credit limits in 1992 and 1998,  $L_S$  is the distribution of credit limits in the separating case, and  $CV(\cdot) = \frac{\sigma(\cdot)}{\mu(\cdot)}$  is the coefficient of variation.

Taking the time unit to be 3 months, I calibrate  $\beta = r = \ln(1.01)$  to be consistent with the 4% average annual growth rate. I set  $\rho = \ln(1.03)$  to be consistent with a 12% interest charge on credit lines.<sup>39</sup> I set the coefficient of risk aversion to be equal to 1. I set  $y^I$  to be the average quarterly income of A-revolvers from the SCF which is \$10,000.<sup>40</sup>

$$\frac{\beta}{\ln(1.01)} \quad \frac{\rho}{\ln(1.03)} \quad \frac{\sigma}{1} \quad \frac{y^I}{\$10,000}$$

I estimate the three parameters related to permanent income, plus the three parameters related to the default cost and its distribution, to match six targeted moments. Four of the moments are the average ratios of credit limits to income and credit card debt to income for 1992 and 1998. The fifth targeted moment is the coefficient of variation of the credit limits in the separating case implied from 1992 and 1998 data using (16). This moment captures the increase in the spread of the credit limit distribution from 1992 to 1998. Two estimations are reported. In first one, the last targeted moment is the default rate of the A-revolvers in 1992, but the exercise does not target the default rate of the A-revolvers in 1998. In the second exercise, the sixth targeted moment is the default rate of the A-revolvers in 1998, but 1992 default rate is not targeted.

An identity weight matrix is used to minimize the percentage deviation of the moments generated by the model, from the targeted data moments. This provides us consistent estimates for the parameters. To generate the

<sup>38</sup>If the Rating Technology sent signals about certain risk characteristics of borrowers in 1992, by 1998 the signals still contained the information about those characteristics. However, as the Rating Technology became more informative, it could provide some additional information about the risk characteristics of borrowers. We can interpret the switch from the pooling case (uninformative signal) to the separating case (fully informative signal) as the provision of additional information by the Rating Technology on borrowers' characteristics.

<sup>39</sup>Notice that the average credit card interest rate in this period is around 14.50 – 15.00%. However, since the model generates the equilibrium credit limits by equalizing the creditors' profit to zero, we should consider the creditors' operational costs. I approximate this cost to be 3% from the difference between the Bank Prime Loan Rate and the Federal Fund Rate.

<sup>40</sup>Since I use log utility and the target moments are the ratios relative to income, the initial income could be set without loss of generality.

targeted moments by the model, I compute the borrowing/payment paths backward, using the 4th order Runge Kutta method for the ordinary differential equation (12). Given the simplicity of the model, the targeted moments are computed pretty fast for each set of parameters, despite having three zero profit conditions.

The estimated values are reported in table (4) and the moments generated from the model are reported in tables (5) and (6) (the targeted values are inside the parentheses).

Default Target Year	$\delta$	$y^P$	$\eta$	$\theta_H$	$\theta_L$	$\mu(\theta_H)$
1992	0.1502	4.5710	0.0708	0.9883	0.9722	0.5043
1998	0.1326	5.1576	0.0795	0.9890	0.9740	0.5293

Table 4: Estimated Parameters

The estimated parameters for the permanent income, implies that on average households' permanent income is about 75% to 90% larger than their initial income, and it takes around one and a half years before switching to permanent income. Generally speaking, this is consistent with the characteristics of households' income processes during financial distresses.

		Pooling (Data 1992)	Separating (Data 1998)
Target	Av. (Cred Lim/Income)	25.90% (25.97%)	42.95% (42.48%)
Moments	Av. (Cred Card Debt/Income)	10.67% (10.66%)	16.71% (16.95%)
	Default Rate	5.86% (5.86%)	
	Coefficient of Variation		47.04% (47.25%)
Overidentifying	Default Rate		6.42% (7.39%)
Moments	Cred Card Debt/Income of Defaulter	61.23% (53.10%)	120.08% (76.70%)

Table 5: Generated Moments by targeting 1992 default rate

		Pooling (Data 1992)	Separating (Data 1998)
Target	Av. (Cred Lim/Income)	25.96% (25.97%)	42.51% (42.48%)
Moments	Av. (Cred Card Debt/Income)	10.66% (10.66%)	16.95% (16.95%)
	Default Rate		7.39% (7.39%)
	Coefficient of Variation		47.22% (47.25%)
Overidentifying	Default Rate	6.61% (5.86%)	
Moments	Cred Card Debt/Income of Defaulter	53.51% (53.10%)	102.04% (76.70%)

Table 6: Generated Moments by targeting 1998 default rate

I estimate six parameters to match six moments from the data. However, in the first exercise, the default rate of 1998, and in the second exercise the default rate of 1992, are not amongst the targeted moments and hence can be used to test for consistency of the model. According to the data, the default rate by A-revolvers rose from 5.86% in 1992 to 7.39% in 1998. In the first exercise, going from the pooling case to the separating case, the model generates an increase of default rate from 5.86% to 6.42%, which can account for more than 37% of the increase in bankruptcy filings in the data. In the second exercise, going from the separating case to the pooling case, the model generates a decrease of default rate from 7.39% to 6.61%, which can account for 51% of change

in bankruptcy filings. Given the simple structure of the model and the fact that the exercise did not target the increase in the number of bankruptcy filings, the results are appealing.

The ratio of credit card debt to income generated by the model and reported from data are also provided in the last row of tables (5) and (6) ( the data moments are from Sullivan et al. [25]). In both exercises, the average credit card debt of a defaulter generated by the model in the pooling case is close to the reported data. However, the debt to income ratio of defaulters in the separating case generated by the model is much higher than the reported data. In the second exercise (i.e. targeting the default rate of 1998) defaulters’ debt to income ratio almost doubles by switching from the pooling case to the separating case, while Sullivan et al. [25] report only 44% increase in this ratio. I should emphasize that these two moments were not targeted in the estimation process, and the fact that the model gets close enough to them, especially for the pooling case, is striking.

Tables (7) and (8) report limit, debt, default rate and average debt of a defaulter for both types in both cases generated in each exercise. In the pooling case, agents with the high default cost (i.e. the safe type) borrow very little and do not default at all. Due to their small debt, they do not compensate the creditors very much, for the losses made on the riskier agents. Hence, when they are separated, borrowing and default by the riskier agents do not change significantly. However, in the separating case, the safer types are offered a much larger credit limit. Therefore, they can accumulate larger debts which results in more defaults by the “safer” agents.

		Pooling	Separating
Low Default Cost $\theta_H$ (Risky)	Lim/Income	25.90%	22.93%
	Debt/Income	15.17%	12.40%
	Def Rate	11.63%	7.75%
	Def Debt/Income	61.23%	57.37%
High Default Cost $\theta_L$ (Safe)	Lim/Income	25.90%	63.37%
	Debt/Income	6.09%	21.10%
	Def Rate	0.00%	5.08%
	Def Debt/Income	–	183.87%

Table 7: Generated Moments for Different Types (targeting 1992 default rate)

		Pooling	Separating
Low Default Cost $\theta_H$ (Risky)	Lim/Income	25.96%	23.58%
	Debt/Income	15.15%	12.74%
	Def Rate	12.48%	8.77%
	Def Debt/Income	53.51%	53.76%
High Default Cost $\theta_L$ (Safe)	Lim/Income	25.96%	63.80%
	Debt/Income	5.61%	21.68%
	Def Rate	0.00%	5.85%
	Def Debt/Income	–	159.70%

Table 8: Generated Moments for Different Types (targeting 1998 default rate)

The model does relatively well in accounting for the rise of credit card debt of bankruptcy filers, a.k.a. the debt discharge rate. This is because the safer types only default when their debt level is really high. Therefore, the rise of default in the separating case, goes hand in hand with the rise of the debt discharge rate.

Finally I use the estimated parameters to generate two counterfactuals motivated by the stigma explanation of the rise of bankruptcy, each corresponding to one of the above exercises. In the first one, instead of changing

the information structure from pooling to separating, I keep the pooling structure but increase the fraction of high risk types to generate an increase of bankruptcy filings equal to the separating case. This is done, while keeping the estimated values of the other parameters in the first exercise unchanged. This goal is attained by increasing the fraction of type  $\theta_H$ , high risk, agents from  $\mu(\theta_H) = 50.4\%$  to  $\mu(\theta_H) = 63.1\%$ , which raises the default rate from 5.86% to 6.42%. However, the higher fraction of risky agents, decreases the equilibrium credit limit to 24.79% of income, increases the average debt level only to 11.16% of income, and the debt to income ratio of defaulters slightly decreases from 61.23% to 59.79%. All of these changes contradict the significant increasing trends observed in the data.<sup>41</sup>

In the second counterfactual, keeping the separating informational structure, I reduce the fraction of high risk agents, to generate a decrease in bankruptcy filings similar to the decrease in bankruptcy filings generated by switching from separating to pooling case in the second informational change exercise. This is attained by decreasing  $\mu(\theta_H) = 52.93\%$  to 26.0%, which would reduce bankruptcy filings from 7.39% down to 6.61%. Again I keep the estimated values of the other parameters unchanged. The lower fraction of risky agents, increases the average credit limit to 53.34% of income, raises the average debt level to 19.35% of income and increases the average debt to income ratio of defaulters from 102.04% to 131.38%. All of these changes contradict even the direction of changes observed in data by going from 1998 back to 1992.<sup>42</sup>

## 4.2 Rise of Extensive Margin

The model can also explain the rise of credit supply along the extensive margin. As we noted earlier, the fraction of households with access to credit cards rose from 56% in 1989 to 72% in 2004. Assume there is a type who incurs no cost of default, that is  $\theta = 1$ . Supply of credit to this type only generates a loss for the creditors and the loss is increasing in the credit limit. Now suppose the Rating Technology pools a small fraction of another type  $\theta^* < 1$  agents, who have a positive default and therefore do not default on small debts, with the type  $\theta = 1$  agents. Then in the pooling case, no credit will be supplied to the type  $\theta^*$  agents who are pooled with type  $\theta = 1$  agents. But when the Rating Technology separates the two types, the type  $\theta^*$  agents will receive a positive credit limit.

## 5 Last Thoughts

This paper was the first attempt in providing an informational explanation for the rise of household bankruptcy. I simultaneously account for the increase of credit supply and the corresponding increase in average credit card debt. The extension of credit supply follows from separating borrowers with different degrees of riskiness. The rise of bankruptcy is explained by the increase in the availability of credit for the borrowers with high default costs, which allows them to accumulate more credit card debt. While these borrowers are less likely to default on small debts, they are more likely to default when they accumulate large amounts of unsecured debt. Hence, the debt discharge rate rises as well.

Using the simulated method of moments I provide a simple quantitative example of matching the average credit limit and debt levels, as well as the increase in the spread of the credit limit distribution. The model accounts for about one third to half of the increase in the default rate, which is quite appealing given the simple stylized structure of the model. This result shows the importance of the heterogeneity of borrowers' default costs over other sources of heterogeneities like variation in income processes.

Understanding the rise of household bankruptcy has important policy implications. If this rise was due to the decline of bankruptcy stigma, the policy response should have been a tighter bankruptcy code to increase the cost

<sup>41</sup>To generate an increase in the default rate similar to the increase observed in 1998 data, 7.39%, we should increase  $\mu(\theta_H) = 50.4\%$  to  $\mu(\theta_H) = 89.0\%$ . In this case the equilibrium credit limit decreases to 23.36% of income, the average debt level slightly increases to 12.05% of income, and the debt to income ratio of bankruptcy filers slightly decreases to 57.92%.

<sup>42</sup>To generate a decrease in the default rate similar to 1992 level, 5.86%, the fraction of risky agents should be reduced to 0.05%. This change would increase the average credit limit to 63.60% of income, raise the average debt level to 21.63% of income and increase the average debt to income ratio of defaulters from to 159.16%.

of bankruptcy. The 2005 change of bankruptcy code tried to accomplish this goal. But if the rise of bankruptcy filings resulted from a more informed credit market, then tightening of the bankruptcy code was not necessarily required. However, the model of this paper is too stylized to be used for determining the optimal bankruptcy code.

Finally, Huggett [17] and Aiyagari [4] pointed out the importance of credit limits for households' precautionary saving motives and therefore the aggregate price of capital. This paper does not address the saving decision of households, but tackles the question of how credit limits are allocated, which has an important role for saving decisions. But, households who borrow on their credit cards tend to belong to the bottom 80% of the wealth distribution and their contribution to the aggregate productive capital stock is not significant. However, access to credit could have an important implication for the entrepreneurial activities at the household level which should not be omitted in a thorough welfare analysis of the consumer credit market.

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## Appendix

**Lemma 1** Borrowing/payment at the credit limit,  $b^* = b(T) \geq -\rho L$ , satisfies:

$$u'(y^I + b^*) \leq \frac{[u(y^I + b^*) + \delta W(L; \theta)] - [u(\theta y^I) + \delta W^D(\theta)]}{\rho L + b^*},$$

with equality if  $b^* > -\rho L$ . Moreover, if (5) holds then (6) uniquely determines  $b^*$ .

**Proof.** If (5) does not hold, i.e. the agent does not find it optimal to default at the credit limit, then  $b^* = -\rho L$  which satisfies (6). If (5) holds and the agent defaults at the limit, then (6) uniquely determines  $b^*$ . To see this point note that (6) can be rearranged as:

$$u'(y^I + b^*)(\rho L + b^*) - [u(y^I + b^*) + \delta W(L; \theta)] + [u(\theta y^I) + \delta W^D(\theta)] = 0. \quad (17)$$

If (6) holds, then left hand side of (17), which is decreasing in  $b^*$  due to the concavity of  $u(\cdot)$ , is positive for  $b^* \geq -\rho L$ . Derivative of the left hand side of (17) with respect to  $b^*$  is  $u''(y^I + b^*)(\rho L + b^*)$ , which is negative for  $b^* \geq -\rho L$ . Moreover, since  $u(\cdot)$  is strictly concave as  $b^* \rightarrow \infty$ , the left hand side approaches zero. Then by continuity (6) has a unique solution.

The optimality of the unique solution for (17) follows from using calculus of variation for borrowing amount  $\mathbf{b}$  when  $D = L - \epsilon$  for a very small  $\epsilon > 0$ . Suppose the agent wants to maximize her utility for the next  $\overline{\Delta t}$  periods, where  $\overline{\Delta t}$  is small enough. Also, suppose she will definitely default afterward. If the agent borrows a constant stream of  $\mathbf{b}$  before reaching the credit limit, when she will default, then it takes approximately  $\Delta t = \frac{\epsilon}{\rho D + \mathbf{b}}$  units of time to reach the credit limit. So the agent will approximately receive a value equal to:

$$\left(\frac{\epsilon}{\rho D + \mathbf{b}}\right)[u(y^I + \mathbf{b}) + \delta W(L; \theta)] + \left(\overline{\Delta t} - \frac{\epsilon}{\rho D + \mathbf{b}}\right)[u(\theta y^I) + \delta W^D(\theta)].$$

Taking the derivative of this expression with respect to  $\mathbf{b}$  and setting it equal to zero yields (17). ■

**Lemma 3** For  $L_1 < L_2$ , if the agent's solution for (1) finds it optimal to increase her debt up to her credit limit, then:

$$b(L_1; (L_1, \rho)) < b(L_1; (L_2, \rho)).$$

**Proof.** If  $b(L_1; (L_1, \rho)) = -\rho L_1$ , that is the agent does not default at the limit, then since the agent increases her debt level up to the limit we should have  $b(L_1; (L_2, \rho)) > -\rho L_1$ .

If  $b(L_1; (L_1, \rho)) < -\rho L_1$ , that is the agent defaults at credit limit  $L_1$ , then from (5) it follows that she also defaults at credit limit  $L_2$ , therefore  $b(L_1; (L_1, \rho))$  and  $b(L_2; (L_2, \rho))$  are both governed by (17). From (17) it follows that:

$$\frac{db^*}{dL} = \frac{-\rho u'(y^I + b^*) + \delta_\theta W_D(L; \theta)}{u''(y^I + b^*)(\rho L + b^*)}. \quad (18)$$

Now comparing (18) and (12), it follows that

$$\frac{db^*}{dL} > \left. \frac{db(D; (L, \rho))}{dD} \right|_{D=L},$$

hence by continuity of  $b^*$  and  $b(D; (L, \rho))$ , it follows that  $b(L_1; (L_1, \rho)) < b(L_1; (L_2, \rho))$ . ■

**Lemma 5** If  $r = \beta$  and  $F(\cdot)$  is concave, then  $W_D(D; \theta)$  is decreasing in  $D$  for  $D \in [0, \frac{(1-\theta)y^P}{r}]$  and increasing for  $D > \frac{(1-\theta)y^P}{r}$ .

**Proof.** For  $D \in [0, \frac{(1-\theta)y^P}{r}]$ , concavity of  $W(D; \theta)$  follows from concavity of  $u(\cdot)$  since the agent does not default after realizing her permanent income. For  $D > \frac{(1-\theta)y^P}{r}$  we have:

$$\begin{aligned}
W_{DD}(D; \theta) &= \frac{d}{dD} \left\{ - \int_{\frac{rD}{1-\theta}}^{\infty} u'(y^P - rD) dF(y^P) \right\} \\
&= \frac{r}{1-\theta} u' \left( \frac{\theta r D}{1-\theta} \right) F' \left( \frac{rD}{1-\theta} \right) + r \int_{\frac{rD}{1-\theta}}^{\infty} u''(y^P - rD) dF(y^P) \\
&= \frac{r}{1-\theta} \cdot u' \left( \frac{\theta r D}{1-\theta} \right) F' \left( \frac{rD}{1-\theta} \right) + r u'(y^P - rD) F'(y^P) \Big|_{y^P = \frac{rD}{1-\theta}}^{y^P = \infty} - r \int_{\frac{rD}{1-\theta}}^{\infty} u'(y^P - rD) d^2 F(y^P) \\
&= \frac{\theta r}{1-\theta} \cdot u' \left( \frac{\theta r D}{1-\theta} \right) F' \left( \frac{rD}{1-\theta} \right) - r \int_{\frac{rD}{1-\theta}}^{\infty} u'(y^P - rD) d^2 F(y^P) \\
&> 0
\end{aligned}$$

where the inequality follows from concavity of  $F(\cdot)$  and  $u'(\cdot) > 0$ . ■

**Lemma 6** For  $r = \beta$  and concave  $F(\cdot)$ , if  $(\delta + \beta - \rho)u'(y^I) + \delta W_D(\frac{(1-\theta)y^P}{r}; \theta) > 0$  then the agent does not stop increasing her debt before reaching her credit limit.

**Proof.** Using the previous lemma, if  $(\delta + \beta - \rho)u'(y^I) + \delta W_D(\frac{y^P(1-\theta)}{r}; \theta) > 0$ , then it is guaranteed that  $\frac{\delta + \beta - \rho}{\delta} \cdot u'(y^I) + W_D(D; \theta) > 0$  for  $\forall D > 0$ .

Suppose the agent stops increasing her debt level above  $\bar{D}$ , that is  $\rho \bar{D} + b(\bar{D}) = 0$ , while  $\bar{D} < L$ . In this case the marginal benefit from increasing the borrowing amount is  $(\delta + \beta)u'(y^I + b(\bar{D}))$  and the marginal cost is  $\rho u'(y^I + b(\bar{D})) - \delta W_D(\bar{D}; \theta)$ . The concavity of  $u(\cdot)$  guarantees  $(\delta + \beta - \rho)u'(y^I + b(\bar{D})) + \delta W_D(\bar{D}; \theta) > 0$ , hence it is optimal to increase  $b(\bar{D})$  and therefore the debt level. ■

**Theorem 9** Borrowing,  $b(D; (L, \rho), \theta)$ , is increasing in the agent's riskiness,  $\theta$ .

**Proof.** First, notice that the marginal cost of debt level after realization of the permanent income,  $-W_D(D; \theta)$ , is decreasing in riskiness,  $\theta$ . That is:

$$W_{D,\theta}(D; \theta) = \frac{r^2 D}{(1-\theta)^2} \cdot u \left( \frac{\theta r D}{1-\theta} \right) \cdot F' \left( \frac{r D}{1-\theta} \right) \geq 0.$$

Lemma (8) guarantees that  $b(L; (L, \rho), \theta)$  is increasing in  $\theta$ . Next we prove that  $b(0, (L, \rho), \theta)$  is also increasing in  $\theta$ . Define  $V(D; (L, \rho), \theta)$  as the value of having debt level  $D < L$  with a contract  $(L, \rho)$ , while waiting for the realization of the permanent income by a type  $\theta$  agent. By definition, we have:

$$\begin{aligned}
V(D; (L, \rho), \theta) &= \frac{1}{\beta + \delta} \{ u(y^I + b(D)) + \delta W(D; \theta) + (b(D) + \rho \cdot D) V_D(D; (L, \rho), \theta) \} \\
&= \frac{1}{\beta + \delta} \{ u(y^I + b(D)) + \delta W(D; \theta) - (b(D) + \rho \cdot D) u'(y^I + b(D; (L, \rho), \theta)) \}, \quad (19)
\end{aligned}$$

where the second equality follows from the optimality of the choice of  $b(D; (L, \rho), \theta)$ . That is,  $V_D(D; (L, \rho), \theta) + u'(y^I + b(D; (L, \rho), \theta)) = 0$  for  $D < L$ . Using (19) for  $D = 0$ , we have:

$$V(0; (L, \rho), \theta) = \frac{1}{\beta + \delta} \{u(y^I + b(0; (L, \rho), \theta)) + \delta W(0, \theta) - b(0; (L, \rho), \theta)u'(y^I + b(0; (L, \rho), \theta))\} \quad (20)$$

Notice that, since the support of the permanent income distribution,  $F(\cdot)$ , is bounded above zero, agents do not default on zero debt level. Therefore  $W(0, \theta)$  is independent of  $\theta$ , hence we have:

$$\frac{d}{d\theta} \{V(0; (L, \rho), \theta)\} = \frac{1}{\beta + \delta} \{-b(0; (L, \rho), \theta)u''(y^I + b(0; (L, \rho), \theta))\} \cdot \frac{d}{d\theta} \{b(0; (L, \rho), \theta)\}. \quad (21)$$

Since the cost of default is decreasing in  $\theta$ , and therefore the agents with a high  $\theta$  can derive a higher utility by following the borrowing schedule of the agents with a low  $\theta$ , by construction,  $V(0; (L, \rho), \theta)$  is increasing in  $\theta$ . Following (21), and by the concavity of  $u(\cdot)$ ,  $b(0; (L, \rho), \theta)$  will be also increasing in  $\theta$ .

Now suppose  $\exists \tilde{D} > 0$  and  $\theta_1 < \theta_2$  such that  $b(D, (L, \rho), \theta_1) > b(\tilde{D}, (L, \rho), \theta_2)$ . Since  $b(0, (L, \rho), \theta_1) \leq b(0, (L, \rho), \theta_2)$ , there exist  $0 \leq D^* \leq \tilde{D}$  such that  $b(D^*, (L, \rho), \theta_1) = b(D^*, (L, \rho), \theta_2)$ . Without loss of generality, we can assume  $D^*$  is the supremum of such  $D^*$ s, and by continuity of  $b(\cdot, \theta_1)$  and  $b(\cdot, \theta_2)$  in debt level, we still have  $b(D^*, (L, \rho), \theta_1) = b(D^*, (L, \rho), \theta_2)$ . But then using (12) and the fact that  $-W_D(D; \theta)$  is decreasing in  $\theta$ , we have:

$$\frac{db(D^*, (L, \rho), \theta_1)}{dD} < \frac{db(D^*, (L, \rho), \theta_2)}{dD}$$

which contradicts  $b(\tilde{D}, (L, \rho), \theta_1) > b(\tilde{D}, (L, \rho), \theta_2)$ . ■

**Lemma 11** If the support of  $\psi(\cdot|\tilde{\theta})$  is bounded away from zero, the utility function  $u(\cdot)$  is unbounded from above and the expected present value of all future income for an agent with signal  $\tilde{\theta}$  is bounded then:

$$\lim_{L \rightarrow \infty} \Pi(\tilde{\theta}; (L, \rho)) < 0 \quad \forall \rho.$$

**Proof.** Let's define

$$V(\tilde{\theta}; (L, \rho)) = \int V((\theta, y^I); (L, \rho)) d\psi(\theta|\tilde{\theta}).$$

First we show for any  $\rho \geq r$  we have  $\lim_{L \rightarrow \infty} V(\tilde{\theta}; L) \rightarrow \infty$ . Next we show if  $\Pi(\tilde{\theta}; (L, \rho)) \geq 0$  then  $V(\tilde{\theta}; (L, \rho))$  is bounded from above.

For any credit limit  $L$ , consider the plan of borrowing  $b = \frac{\rho L}{e^\rho - 1}$  during  $t \in [0, 1]$ , and then defaulting. Also always defaulting after realization of the permanent income. This plan delivers an expected present value of lifetime utility equal to

$$\int \frac{1}{\delta + \beta} \left[ (1 - e^{-(\delta + \beta)})u(y^I + \frac{\rho L}{e^\rho - 1}) + e^{-(\delta + \beta)}u(\theta y^I) + \delta V^D(\theta) \right] d\psi(\theta|\tilde{\theta}).$$

This is a lower bound for  $V(\tilde{\theta}; (L, \rho))$ , and If the support of  $\psi(\cdot|\tilde{\theta})$  is bounded away from zero, this lower bound goes to infinity as  $L \rightarrow \infty$ , due to the unboundedness of  $u(\cdot)$ . Hence  $V(\tilde{\theta}; (L, \rho))$  is unbounded as  $L \rightarrow \infty$ .

Let's  $\bar{y}_{\tilde{\theta}}$  denotes the stream of incomes which has the same present value as the expected present value of all future income for an agent with the rating signal  $\tilde{\theta}$  at interest rate  $r$ . That is

$$\bar{y}_{\tilde{\theta}} = \int \int \frac{1}{r + \delta} (ry^I + \delta y^P) dF(y^P) d\psi(\theta|\tilde{\theta}).$$

Due to the concavity of  $u(\cdot)$ , if  $\Pi(\tilde{\theta}; (L, \rho)) \geq 0$  then  $V(\tilde{\theta}; (L, \rho)) \leq \frac{1}{\beta} u(\bar{y}_{\tilde{\theta}})$ . Since  $V(\tilde{\theta}; (L, \rho))$  is bounded from above when  $\Pi(\tilde{\theta}; (L, \rho)) \geq 0$  and is unbounded when  $L \rightarrow \infty$ , we conclude  $\lim_{L \rightarrow \infty} \Pi(\tilde{\theta}; (L, \rho)) < 0$  ■