Family labor supply, precautionary behavior, aggregate saving and employment

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Abstract

I study the impact of idiosyncratic earnings uncertainty on aggregate saving and employment in an economy populated by families consisting of two members. Families incur a fixed cost of participation when both members are employed. I argue that, because of market incompleteness and private information, the presence of this fixed cost can generate multiplicity of equilibrium. In particular there might be one equilibrium with high (female) employment and low savings and another one with low employment and high savings. The model suggests that aggregate saving and employment rates should be negatively correlated across countries. Finally, I present empirical evidence that supports both the partial and the general equilibrium predictions of the model.

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1 Introduction

The average household saving ratio over the last 30 years varies a lot across OECD countries. There are persistent differences in private saving rates across very similar economies, which traditional models of capital accumulation have a hard time explaining. This paper examines the impact of idiosyncratic earnings uncertainty on aggregate saving and employment, through the saving behavior and labor supply decisions of families allowing for within household interaction of labor supply choices. I model families as two member households, which exhibit prudence, and therefore have a precautionary saving motive. On the other hand, I depart from the traditional literature on precautionary savings and optimal inter-temporal consumption behavior by allowing for endogenous labor supply decisions in both the intensive and the extensive margin and I consider the general equilibrium implications of introducing within household heterogeneity.

I show analytically that if households are prudent the attachment of married women to the labor force is increasing in the level of household earnings uncertainty. Next, I examine the general equilibrium implications of the model for aggregate saving and (female) employment. I show that the existence of a non-convexity, introduced by the assumption that a family incurs a fixed cost of participation when both members of the household are simultaneously in the labor force, coupled with market incompleteness and private information, can generate multiplicity of equilibrium. In particular there might be one equilibrium with high (female) employment and low savings and another one with low employment and high savings. Therefore, the model predicts that aggregate saving and employment rates should be negatively correlated across countries.

The recurring theme in models of inter-temporal consumption/savings decision is consumption smoothing. Working more hours when productivity is low, to be able to keep the same level of consumption, seems to be at least as effective at smoothing consumption over time as engaging in precautionary savings, specially in the presence of borrowing constraints. Thus prudent households might do labor supply choices which are stirred by precautionary behavior. Moreover, my concern about modeling households' labor supply decisions, is driven by the drastic changes in work hours composition which occurred in the US over the past 30 years. Thus, McGratan and Rogerson (1998), report drastic reallocations of hours worked across gender, age and marital status groups, despite the relative constancy of aggregate work hours. I argue that this within family reallocations might be related to the well documented increase in earnings uncertainty, particularly among male workers. Therefore this paper is to a large extent motivated by the substantial increase in female labor market participation which in the US paralleled the increase in earnings uncertainty.

The corner-stone of this paper will be a model of family labor supply, which will capture within household extensive and intensive margin labor supply interactions, driven by precautionary behavior. In order to allow for both participation and hours choices among household members, I follow Cho and Rogerson (1987) and introduce within family labor supply decisions shaped by symmetric preferences and fixed costs of participation which are incurred when both members of the household are simultaneously in the labor force. Moreover, I pose the problem in such a way that female labor supply will act as a source of insurance against within family earnings fluctuations. This hypothesis has some empirical support. For example, Dynarski and Gruber (1997) find that families do a good job at smoothing consumption in the face of changes in the head's earnings. Moreover they find that a substantial amount of within family consumption smoothing is achieved through offsetting changes in other sources of family income, including spousal earnings.

The bridge between the partial equilibrium and the general equilibrium model is made through the labor market. In the general equilibrium model the wage difference across household members, interpreted as the gender wage gap, will be made endogenous[1]. Because the value for the firm of hiring a worker will depend on the attachment of the worker to the firm, firms will have to form rational expectations about the degree of attachment of the worker to the labor force. Hence the equilibrium employment rate will depend on the labor force exit rate of the second household member, which will be endogenous in the rational expectations general equilibrium.

Very few authors have investigated the interaction between earnings idiosyncratic uncertainty and household labor supply and savings. Notable exceptions are Marcet, Obiols-Homs and Weil (2003) and Pijoan-Mas (2004). These authors find that in a general equilibrium framework, and contrary to models with exogenous labor supply, the presence of uninsurable labor income risk might lead to less aggregate savings than under complete markets. Low (2004) analyses life cycle labor supply and savings in a partial equilibrium framework. He finds that when labor supply is flexible, consumption is smoother than when work hours are exogenous because individuals can work more hours instead of giving up consumption. Furthermore, he argues that making labor supply flexible has an ambiguous impact on the correlation between precautionary savings and earnings uncertainty, since on the one hand the cost of accumulating precautionary balances will be smaller, but on the other hand the value of precautionary wealth holdings will be less because households can now adjust labor supply to smooth consumption.

In a paper closely related to ours, Attanasio, Low and Sanchez-Marcos (2005) study the role of female labor supply as an insurance mechanism against idiosyncratic earnings uncertainty in a life cycle model of savings and labor supply. They find that increasing uncertainty increases participation rates and that this effect is larger when the households face exogenous borrowing constraints. I will further explore

[1] Jones, Manuelli and McGrattan (2003) find that small changes in the gender wage gap can explain the increase in the average hours worked by married women.

this hypothesis, providing both analytical and numerical results. I first show, in a partial equilibrium setting, that for prudent households the value of having both members participating in the labor force is increasing in the level of uncertainty. Next I study carefully the way within family labor supply adjustments and in particular extensive margin adjustments by the second household member affects precautionary balances and employment in a dynamic general equilibrium model with incomplete markets. The model suggests that a rise in equilibrium (female) employment rates, which translates in an increase in the number of two-earner families, should lead to a lower aggregate saving rate, as the variability of families income decreases.

Therefore, the paper delivers both micro level predictions which arise from the partial equilibrium analysis and aggregate implications. At the micro level, the model predicts that households whose head is exposed to more earnings uncertainty are more likely to have the second member of the family in the labor force. At the aggregate level, the model predicts a negative correlation between female participation rates in the labor force and aggregate saving. I present empirical evidence that supports both the partial and the general equilibrium predictions of the model. In particular the micro level empirical investigation will be carried through using household level data from the panel survey of income dynamics (PSID).

The remaining of the paper is organized as follows. Section 2 solves a two period partial equilibrium model which highlights the most important mechanisms I wish to explore and delivers analytical results which will carry through to the dynamic model. Next, in section 3, I model labor demand and I investigate a dynamic general equilibrium model. In section 4, I solve numerically for the model stationary competitive equilibrium. Finally, in section 5, I present relevant empirical findings which support the model predictions and section 6 concludes.

2 A Model of Saving and Family Labor Supply

I first consider a two period model of savings and family labor supply which should illustrate most of the mechanisms I want to study. A family is a partnership between two members, a husband (m) and a wife (f), which make an integrated choice over how much to consume and how many hours each member works in period one and period two. To model the preferences of each household, I follow Cho and Rogerson (1987) and in particular it is assumed that a family incurs a fixed cost of participation when both members of the household are simultaneously in the labor force. The husband and the wife have identical and separable preferences over consumption and labor supply, and household preferences are additive across members. The instantaneous utility of each household is given by

$$u(c_m) + u(c_f) + g(\ell_m) + g(\ell_f) - \phi \tag{2.1}$$

where

$$\phi = \begin{cases} 0 & if \;\; \ell_m \times \ell_f = 0 \\ \Phi > 0 & elsewhere \end{cases}$$

and $u(c)$ is increasing, strictly concave, $g'(\ell) < 0$, $g''(\ell) < 0$ and $g(0) = 0$. Moreover the following assumptions are made in order to establish the existence of an interior solution

$$\lim_{c \to 0} u'(c) = \infty \;, \qquad \lim_{c \to \infty} u'(c) = 0$$

$$\lim_{\ell \to 0} g'(\ell) = 0$$

The wage rate of the husband is normalized to one, and the wage rate of the wife will be $\lambda \leq 1$. λ, which will be made endogenous in the general equilibrium model of section 3, should be interpreted as the gender wage gap. Both members are endowed with one unit of time. Since the husband and the wife have identical preferences, the husband will always choose to participate in the labor force because of the wage gender differential, however, the choice of the wife will depend on the size of the fixed cost Φ. Moreover, because preferences are identical the consumption level chosen by both members will always be the same.

For simplicity I will assume that both the discount factor and the rate of return are zero and thus abstract from inter-temporal substitution. Family chooses in each period how much to consume and whether one or both members will be employed as well as how many hours each member works. Given the assumptions made about the existence of an interior solution, in the absence of the fixed cost both members of the household will choose to be employed. Finally, it is assumed that uncertainty enters additively the budget constraint of the household through a mean μ random variable ϵ [2].

The household solves the following problem

$$\max_{s,\ell_{1m},\ell_{1f}} \; 2u(c_1) + g(\ell_{1m}) + g(\ell_{1f}) - \phi_1 + E\left[2u(c_2) + g(\ell_{2m}) + g(\ell_{2f}) - \phi_2\right]$$

subject to

$$\begin{cases} 2c_1 + s = \ell_{1m} + \lambda \ell_{1f} \\ 2c_2 = s + \ell_{2m} + \lambda \ell_{2f} + \epsilon \end{cases}$$

where s denotes savings, c_i, $i = \{1,2\}$, denote consumption in period one and period two and ℓ_{ij}, $i = \{1,2\}$, $j = \{m,f\}$, are the period one and period two labor supply of the husband and of the wife. Finally ϕ_i is the fixed cost of participation

[2] The analytical results that follow do not depend on the assumption of separability in consumption and leisure of the utility function, which was made for simplicity. However they do rely on the assumption of additivity of uncertainty. The random component ϵ should be interpreted as a random component of earnings, independent of the choice of how many hours to work, made by each household member, at the given wage rate. Thus when I develop the general equilibrium version of the model I will consider the labor supply of the first member of the household exogenous and thus the random component will be interpreted as a shock to his productivity.

in the labor market by both members in period i, which is the only non-convexity in this problem.

Assuming an interior solution (or equivalently neglecting the fixed cost ϕ) and setting $\lambda = 1$ for ease of exposure, the necessary and sufficient conditions to solve this problem are

$$u'(c_1) = E\left[u'(c_2)\right] \tag{2.2}$$

$$u'(c_1) + g'(\ell_{1j}) = 0 \tag{2.3}$$

$$E\left[u'(c_2)\right] + E\left[g'(\ell_{2j})\right] = 0 \tag{2.4}$$

We want to investigate whether the choice to participate in the labor market by the wife might be driven by precautionary behavior. For this to be true, I have to show that the fixed cost required to force a corner solution rather than the interior solution is increasing in the level of uncertainty that the household is exposed to. Let $\left(s^*, \ell_{1m}^*, \ell_{1f}^*\right)$ be the optimal solution to the household problem in the absence of the fixed cost, and $\left(\bar{s}^*, \bar{\ell}_{1m}^*, 0\right)$ the constrained corner solution when a sufficiently large fixed cost is introduced in period one. I wish to show that

$$EV\left(s^*, \ell_{1m}^*, \ell_{1f}^*\right) - E\bar{V}\left(\bar{s}^*, \bar{\ell}_{1m}^*, 0\right)$$

is increasing in the level of uncertainty, where $EV(.)$ is the expected indirect utility function in the absence of the fixed cost, and $E\bar{V}(.\ )$ the constrained expected indirect utility. I therefore require conditions under which the introduction of uncertainty, or additional uncertainty through a zero mean spread, increases the value for the household of participation of both members in the labor force. Because the household choice is made over both the intensive and the extensive margin, investigating the impact of uncertainty on the choice of ℓ_{1f} requires studying the impact of uncertainty over the two margins.

I first study the intensive margin. That is, I require necessary and sufficient conditions for the introduction of uncertainty about future wealth to increase the current supply of labor in the absence of the fixed cost. Notice that, in the absence of the fixed cost, $\ell_{1m}^* = \ell_{1f}^* = \ell_1^*$ and $\ell_{2m}^* = \ell_{2f}^* = \ell_2^*$. Let ℓ^μ denote the level of labor supply that would be chosen by both household members without the fixed cost and if there was no uncertainty. Given the first order conditions, $\ell_1^* \geq \ell^\mu$ if and only if

$$E\epsilon - \mu = 0 \quad \Rightarrow \quad E_\epsilon u' \left(\frac{s + 2\ell_2 + \epsilon}{2} \right) \geq u' \left(\frac{s + 2\ell^\mu + \mu}{2} \right) \tag{2.5}$$

A necessary and locally sufficient (small risks) condition can be found by applying the diffidence theorem, developed by Gollier and Kimball (1996). For small risks, (2.5) will hold if and only if

$$u''' \left(\frac{s + 2\ell_2 + \epsilon}{2} \right) \left(\frac{\partial \ell_2}{\partial \epsilon} + \frac{1}{2} \right) \geq 0 \tag{2.6}$$

and application of the implicit function theorem on the equilibrium condition

$$u' \left(\frac{s + 2\ell_2 + \epsilon}{2} \right) + g' (\ell_2) = 0 \tag{2.7}$$

reveals that $-\frac{1}{2} < \frac{\partial \ell_2}{\partial \epsilon} < 0$. This implies that (2.6) is equivalent to

$$u''' (c) \geq 0 \tag{2.8}$$

And I have shown that for a prudent household, for whom the third derivative of the utility of consumption is positive, and in the absence of fixed costs, the current labor supply of both household members is increasing in the level of future earnings uncertainty. Therefore the current household labor supply is driven by precautionary behavior.

Lemma 1 *For a prudent household for whom $u''' (c) \geq 0$, the current supply of* *labor by both household members in the absence of the fixed cost is increasing in the level of uncertainty.*

It follows that, if only the intensive margin mattered, an increase in earnings uncertainty would increase the supply of labor by both household members. However I still have to consider the extensive margin problem. Because of the presence of a fixed cost, the households will only choose to have both members employed in period 1 if $\ell^*_{1f} \geq \mathcal{L}$, where $\mathcal{L}$ is a threshold value defined as

$$\mathcal{L} : 2u\left(\frac{2\mathcal{L} - s^*(\mathcal{L})}{2}\right) - 2u\left(\frac{\bar{\ell}^*_{1m} - \bar{s}^*}{2}\right) + 2g\left(\mathcal{L}\right) - g\left(\bar{\ell}^*_{1m}\right) + E_\epsilon\left[v\left(s^*, \epsilon\right) - v\left(\bar{s}^*, \epsilon\right)\right] = \Phi$$

where

$$v\left(s, \epsilon\right) = \max_{\ell_{2m}, \ell_{2f}} 2u\left(\frac{s + \ell_{2m} + \ell_{2f} + \epsilon}{2}\right) + g\left(\ell_{2m}\right) + g\left(\ell_{2f}\right) - \phi$$

And the household will only choose to have both members employed if

$$f\left(\ell^*_1\right) = 2u\left(\frac{2\ell^*_1 - s^*}{2}\right) - 2u\left(\frac{\bar{\ell}^*_{1m} - \bar{s}^*}{2}\right) + 2g\left(\ell^*_1\right) - g\left(\bar{\ell}^*_{1m}\right) + E_\epsilon\left[v\left(s^*, \epsilon\right) - v\left(\bar{s}^*, \epsilon\right)\right] \geq \Phi$$

Also, notice that $f(\ell^*_1)$ is an increasing function of ℓ^*_1 because application of the implicit function theorem on the equilibrium condition

$$u'\left(\frac{2\ell_1 - s}{2}\right) + g'\left(\ell_1\right) = 0 \tag{2.9}$$

reveals that

$$\frac{ds}{d\ell_1} = -\frac{u''\left(\frac{2\ell_1 - s}{2}\right) + g''\left(\ell_1\right)}{-\frac{1}{2}u''\left(\frac{2\ell_1 - s}{2}\right)} > 0$$

It thus follows from Lemma (1), and from the observation that given concavity of the indirect utility function

$$E_\epsilon\left[v\left(s^*, \epsilon\right) - v\left(\bar{s}^*, \epsilon\right)\right]$$

is increasing in the level of uncertainty, that an increase in undesirable risk increases the size of the fixed cost needed to prevent the household from having both family members employed. And I have shown that for a prudent household, for whom the third derivative of the utility of consumption is positive, the value of participation of both members in the labor force is increasing in the level of uncertainty. Therefore, in this setup, the choice to participate in the labor force is driven by precautionary behavior.

Proposition 1 *For a prudent household for whom* $u'''(c) \geq 0$ *the value of participation of both members in the labor force is increasing in the level of uncertainty.*

Finally, if I allow the household to have an initial endowment a_0, the size of the fixed cost needed to prevent the second member of the household to take a job is decreasing with the size of the initial endowment, because since ℓ^* is decreasing in wealth, the cost of constraining the household second member not to participate in a giving period will also be less if the household has a high initial endowment. This is not surprising because when wealth increases the solution to the household problem approaches the perfect foresight solution, and the impact of uncertainty diminishes, and therefore also the cost of constraining the household second member not to participate in the labor force is reduced.

Because wealth in the dynamic model, to be studied in the next section, is a state variable, the introduction of non-convexities of the sort presented here will give rise to households exhibiting non-monotone policy functions with important implications for both inference on the strength of the precautionary motive and aggregate saving as well as labor market equilibrium. Moreover, given that it only depends on the properties of the perfect foresight indirect utility function, our analysis carries through to the multiple period dynamic model, where the households problem has the following Bellman equation form[3]

$$V(a,\epsilon) = \max_{c,\ell_m,\ell_f \in \Gamma(a,\epsilon)} \left\{ 2u(c) + g(\ell_m) + g(\ell_f) - \phi + \beta E\left[V(a',\epsilon')\right] \right\}$$

In what follows I develop a dynamic general equilibrium model and investigate the aggregate implications of the model in an economy characterized by incomplete markets, and in particular I will make the gender wage gap λ and the female labor force participation rate endogenous. Hence, I first model the labor demand and next characterize a stationary competitive equilibrium, which will be characterized by an equilibrium wage gender gap and a stationary wealth distribution. The key result of the general equilibrium model are that aggregate saving and female employment are negatively correlated and that the gender wage gap will be less the more attached women are to the labor force. Thus, the non-convexity introduced by the fixed cost coupled with market incompleteness and private information will make possible the existence of multiple equilibrium. In particular there might be one equilibrium with high participation of married women and low savings and another one with low participation and high savings.

3 A Dynamic General Equilibrium Model

3.1 Labor Demand

To characterize general equilibrium, we need first to model the labor demand, coupled with a zero profits/free entry condition. I assume that labor is the only factor of production and moreover that there are constant returns to scale. This allows us to model a firm as a match between an employer and an employee. Firms compete for workers à la Bertrand and, given the equilibrium wage, workers choose how many hours ℓ to supply. In what follows I restrict the choice of ℓ to be either 0

[3]In the dynamic model I will assume that $\beta(1+r) < 1$, which implies that the household asset holdings will endogenously be bounded from above.

(non-participation) or 1 (participation). Moreover I assume that it will be always optimal for the first member of the household to supply one unit of labor. For this to be true we require the following assumption

Assumption 1 *Let* $V(a, \epsilon)$ *be the household value function. Let* $\bar{a} = \sup \{a \,\epsilon\, \mathcal{A}\}$ *be the supremo of wealth holdings and* $\bar{\epsilon}$ *the maximum of the support of the stochastic component of earnings. If* Φ *is zero, then* $V^{\ell_f=1}(\bar{a}, \bar{\epsilon}) > V^{\ell_f=0}(\bar{a}, \bar{\epsilon})$ *and the household will always choose to have both members employed.*

Assumption (1) allows us to carry forward proposition (1) when we only allow for extensive margin labor supply adjustments. There are no search frictions and no barriers to entry. However, in this otherwise perfectly competitive market, when a new match of a worker and a firm occurs, the marginal productivity of the worker will be $y - z$ and in the following periods and for as long as the match is kept, the marginal productivity will be y. This can be interpreted as firm specific human capital, which is entirely accumulated in the first period of the match. The problem of the firm is characterized by the following Bellman equations in discrete time

$$rJ^n = \ell(y - z - \omega) + (1 - p)(J^s - J^n) \tag{3.1}$$

$$rJ^s = \ell(y - \omega) + p\,(J^n - J^s) \tag{3.2}$$

where J^n is the value for the firm of creating a vacancy and J^s is the value for the firm of remaining in operation with the same worker as in the period before. The wage rate is given by ω and $0 < p < 1$ is the probability of separation of the match between the firm and the worker, which will be endogenous in the rational expectations general equilibrium. From (3.1) and (3.2) we get

$$J^s - J^n = \frac{\ell \times z}{1 + r}$$

and the value of a vacancy can be written as

$$rJ^n = \ell\left(y - \frac{p + r}{1 + r}\,z - \omega\right) \tag{3.3}$$

Finally, the free entry condition implies that in equilibrium J^n, the value of creating a vacancy, must be zero and therefore the equilibrium wage rate will be

$$\omega = y - \frac{p+r}{1+r}\, z \tag{3.4}$$

In our economy there will be two wage rules, one for workers of type m and another one for workers of type f because p , the probability of a match being destroyed, will differ according to the worker type. Because I have assumed that the first member of the household will always be part of the labor force, firms forming rational expectations about p_i, $i = \{m\ ,\ f\}$, will set $p_m = 0$. However, $p_f = P\,(\ell\,' = 0 \mid \ell > 0)$ will not be zero. Since the decision of the second household on the extensive margin is not a trivial one, because of the presence of the fixed cost ϕ, the firms will anticipate this when setting the wage rate. It follows that in equilibrium, the wage gap across the two types (the gender wage gap) will be

$$\lambda = \frac{\omega_f}{\omega_m} = \frac{y\,(1+r) - p_f - r}{y\,(1+r) - r}$$

and if we set $y = 1$, we obtain

$$\lambda = 1 - p_f \tag{3.5}$$

3.2 General Equilibrium

Characterization of a recursive competitive equilibrium for a dynamic heterogeneous agent model would require that we keep track of the wealth distribution because the equilibrium prices depend on the distribution of wealth and the forecast of agents about future prices depends on the law of motion for wealth. However, if the solution of the household's problem at given constant prices induces a stationary distribution of wealth, then a stationary equilibrium exists, because in our model economy there is no aggregate uncertainty and therefore, given a stationary distribution of asset holdings, prices will be constant. Our definition of stationary equilibrium is thus analogous to the one in Aiyagari (1994).

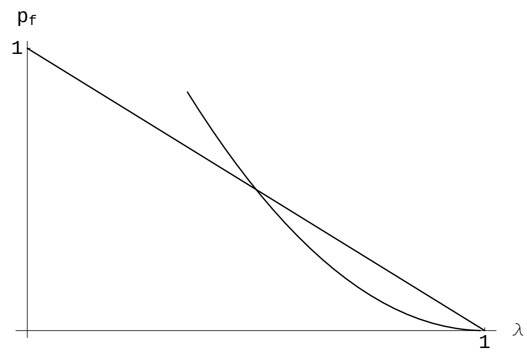

Figure 1: Determination of equilibrium λ and p_f

I assume household asset holdings is private information which firms cannot observe. Hence, the rational expectations forecast of firms about p_f will depend only on the expectations about next period prices and on the equilibrium law of motion of the wealth distribution. In the stationary equilibrium this will be time invariant. Thus, let

$$z = (1 + r)\ a + \epsilon$$

be the predetermined component of households' wealth and let $\bar{z}(\lambda)$ be the threshold level of wealth bellow which a household chooses to have both there members employed at the given price. The rational expectation of firms about p_f will be

$$p_f = \int_{\underline{z}}^{\bar{z}(\lambda)} P\left[z' \geq \bar{z}(\lambda)\ \mid z < \bar{z}(\lambda)\right] \times f_\lambda(z)\, dz \tag{3.6}$$

where $f_\lambda(z)$ is the stationary probability density function of z, which will depend on λ. Moreover, $\bar{z}(\lambda)$ is an increasing function of λ because the participation rate is increasing in λ. On the households side, there is a continuum (measure one) of two member families, indexed by $i \in \mathcal{I}$, that have identical preferences but whose earnings are subject to additive idiosyncratic shocks ϵ^i. For simplicity, I assume that the earnings idiosyncratic shocks are $i.i.d$ over time. Markets are incomplete, and the only asset in this economy are privately owned bonds which earn a net return r, which will be kept exogenous throughout the analysis.

Households discount future utility at rate β and $\beta(1+r) < 1$, which implies that absent uncertainty they would want to borrow against future consumption to finance current consumption. Moreover, I introduce an exogenous borrowing constraint[4] by imposing $a_t^i \geq 0 \; \forall \; t$. It follows that the optimization problem of the household can be expressed as[5]

$$V(a,\epsilon,\lambda) = \max_{c,\, l_f} \left\{ 2\ln(c) + g(1) + g(\ell_f) - \phi + \beta E\left[V(a',\epsilon',\lambda)\right] \right\}$$

subject to

$$a' + 2c = (1+r)\,a + \omega_m + \lambda\omega_m\ell_f + \epsilon$$
$$a' \geq 0$$
$$\phi = \begin{cases} 0 & if \;\; \ell_f = 0 \\ \Phi > 0 & elsewhere \end{cases}$$

A stationary competitive equilibrium relies on household behaving optimally given there wealth and prices (ω_m, λ), firms forming a rational expectation about p_f and a stationary wealth distribution $\Gamma(a)$.

Definition 1 *A stationary competitive equilibrium is defined by the pair* (λ, p_f)*, and a stationary wealth distribution* $\Gamma(a)$ *which arises from*

1. *Households optimal decision rules:* $c^*(a,\epsilon)$ *,* $\ell_f^*(a,\epsilon)$*.*

2. *Free entry of firms.*

3. *Firms forming rational expectations about* p_f*.*

[4]Carroll (1992, 1997 and 2004) shows that households for whom the discount rate is greater than the rate of return, that are prudent and that face possibly binding borrowing constraints will have a target buffer level of wealth.

[5]I have chosen u to be logarithmic so that preferences be consistent with a balanced growth path. No particular assumption was made for g because I focus on the extensive margin.

Figure 4 illustrates a possible shape for the locus defined by equation (3.6), which I will call the p_f-locus for ease of exposure. Notice that for low values of λ the locus will not be defined since no household will have both members employed. As λ becomes larger, $\bar{z}(\lambda)$ increases and therefore the unconditional probability of a household having both members in the labor force increases. This suggests that the p_f-locus should be decreasing however it does not ensure it because firms compute the probability conditional on the household being employed in the current period. In fact no analytical characterization of the p_f-locus is possible and therefore characterization of the stationary competitive equilibrium will be a numerical exercise, which I perform in the following section. The downward linear slope, which I will call λ-locus corresponds to equation (3.5). Clearly there may exist more than one equilibrium [6]. In particular, I show through numerical simulation that for many plausible parameterizations, two equilibriums will exist, one corresponding to a low λ and a very weak female attachment to the labor force (high p_f) and another one with a very strong attachment and a high λ[7]. Finally, notice that Φ is a free parameter which can be chosen in such way to always ensure the existence of an equilibrium. Thus, because $\beta(1+r) < 1$, the individual wealth holdings will be bounded and the wealth distribution has finite support. In particular there exists a $z^*(\lambda)$ such that for all $z \geq z^*(\lambda)$, $z' \leq z^*(\lambda)$ with probability one (Aiyagari [1994]). The following existence result can therefore be established

Lemma 2 *Given an appropriate choice of Φ, such that $\ell_f(z^*(1)) = 1$, there will always exist an equilibrium with full labor force participation and a zero gender wage gap, that is $\lambda = 1$.*

Proof: if Φ is such that $\ell_f(z^*(1)) = 1$ then, because participation is decreasing in

[6]Strictly speaking a situation with $p_f = 1$ and $\lambda = 0$ is a rational expectations equilibrium in this economy however it is not an interesting one as it implies zero female labor supply.

[7]It is remarkable to notice that the two equilibriums obtained can be Pareto ranked. In the equilibrium with high employment and low aggregate savings, firms are as well off as they would be in the low employment high savings equilibrium, because of the free entry condition. However, in the high employment equilibrium, households are better off because they solve the same inter-temporal problem but facing a looser budget constraint.

wealth, $\ell_f(z'(1)) = 1$ with probability one and hence the rational expectation of p_f will be zero and this will thus be an equilibrium.

In what follows I solve the model numerically and investigate the properties of equilibrium through simulation over a sequence of λs.

4 Model Computation and Calibration

To characterize equilibrium numerically, assuming that a stationary wealth distribution exists, we need to solve the household problem for a set of values for the gender wage gap λ. I solve the dynamic programming problem of the household by the method of discretization of the state space. The continuous process for the stochastic component of income is replaced by a discrete markov chain following Tauchen (1986). The household can hold a single asset a_t in discrete amounts chosen from the set $\mathcal{A}$, and the minimum amount is set to zero, as required by the liquidity constraint assumption. The presence of an upper bound for wealth holdings is an innocuous assumption given that $\beta(1+r) < 1$ and hence the ergodic wealth distribution will have finite support. The value function and the corresponding policy rules are found through value function iteration, until convergence is achieved[8].

I next simulate an economy inhabited by a large number ($N = 500$) of households over ($T = 200$) periods and compute the sample probability of an employed female worker to exit the labor force. The same exercise is performed over a dense grid

[8] The value function iteration is performed on the "conditional value functions" corresponding to each employment state. However, because of the presence of the fixed cost ϕ, it is not possible to rule out the existence of convex segments of the value function. As discussed in detail by Phelan and Townsend (1991), Hopenhayn and Nicolini (1997) and Lentz and Tranaes (2004) the solution to the household problem can be improved through the introduction of fair lotteries, which will only be part of the household optimal plan in the convex segments of the value function. The role of this lotteries would be to ensure concavity of the value function. However, as suggested by the literature mentioned above, the introduction of enough uncertainty allows to smooth away convexities, hence ruling out the purchase of lotteries.

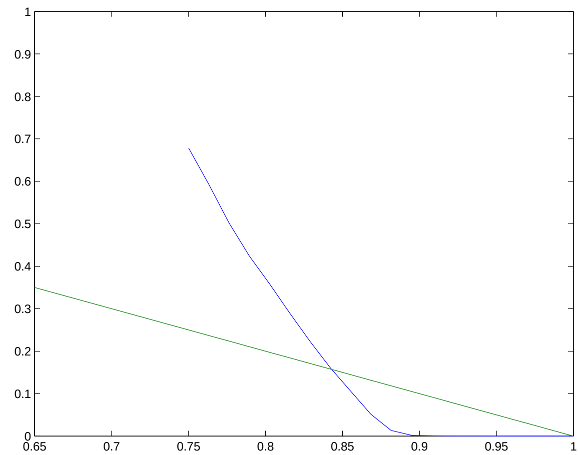

Figure 2: Numerical determination of equilibrium λ and p_f

of λs to find the p_f-locus. For λs not in the grid, the corresponding p_f is found through linear interpolation. To find an equilibrium solution we must compute for each λ the sample probability of a worker separation $\hat{p_f}(\lambda)$ which corresponds to

$$\hat{p_f}(\lambda) = \frac{\sum_{t=T-k}^{T} \sum_{i=1}^{N} I_{\left[\ell_{ft-1}^{i}=1\right]} \times I_{\left[\ell_{ft}^{i}=0\right]}}{\sum_{t=T-k}^{T} \sum_{i=1}^{N} I_{\left[\ell_{ft-1}^{i}=1\right]}} \tag{4.1}$$

where $k << T$ is chosen to be an integer number small enough to allow for convergence of the wealth distribution and I corresponds to an indicator function. Equation (4.1) is the numerical counterpart of equation (3.6). Next the equilibrium pair (λ, p_f) is found by solving the equation

$$\lambda = 1 - \hat{p_f}(\lambda) \tag{4.2}$$

Figure 5 shows a numerical example, for which two equilibriums exist. Here, R was 1.04, β was 0.95. As for the firms technology, y has been normalized to one and z is 0.2, which implies that a worker is 20 percent less productive in the first period of the match. As λ increases and because the ergodic wealth distribution

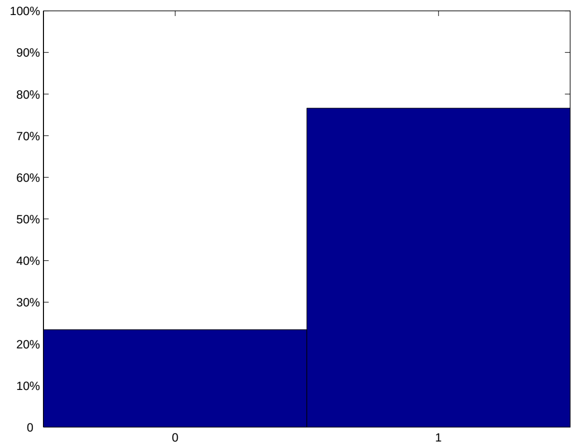

Figure 3: Equilibrium Female Employment

has finite support, we reach an equilibrium, for the chosen Φ ($\Phi = 0.25$), where everyone chooses to be part of the labor force. This in turn implies an equilibrium λ equal to one, and there will be no discrimination in the labor market. Of course this will not be the only possible equilibrium, as the p_f-locus and the λ-locus cross twice. There exists also an equilibrium with relatively low female attachment to the labor force and a positive wage gender gap.

In such an equilibrium, there will be a positive hazard rate for matches involving female workers and the wage gender gap will be given by this hazard rate. Figure 6 shows the equilibrium female employment rate which for the parametrization chosen will be of 77%. The greater the participation rate the lower will be the average marginal propensity to save. This is because the option value of holding precautionary balances decreases when a household has both members employed. Therefore there will be a negative correlation between aggregate saving and the female participation rate. Finally, figure 7 shows the wealth distribution. It has two different modes because the households have different buffer targets of savings according to the employment status of the second member. Moreover, wealthier households will not choose to have the second member employed and consquently they will have a higher marginal propensity to save out of earnings. This is an

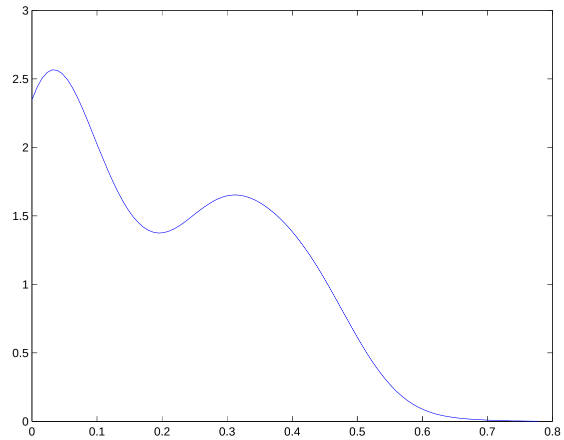

Figure 4: Wealth Distribution

interesting result, deserving further research, because it might provide a solution to endogenously generate enough skewness of the wealth distribution [9], a phenomena which is found in the data but that researchers working with this class of incomplete market models have found very difficult to replicate.

5 Some Empirical Evidence

The model introduced in this paper makes essentially two important predictions. The first one, which arises from the partial equilibrium component of the model, is expressed in proposition (1), according to which the value of participation of both members in the labor force is increasing in the level of uncertainty. We would therefore expect that married couples for whom the head is exposed to more earnings uncertainty should have more often the second member present in the labor force. This prediction is of course testable using micro level data on households. The second prediction of the model, which arises from the general equilibrium considerations, is that the aggregate personal saving rate should be greater in countries where female participation and employment is less.

[9]Chang and Kim (2006) examine the implication of within-household heterogeneity and family labor supply choices for cross-sectional earnings and wealth distributions.

5.1 Micro Level Evidence

To test the micro level predictions of the model we require information on married male heads of household earnings and earnings volatility as well as information on the within household employment choices. I wish to investigate the participation behavior of married women and in particular the impact of the husband's earnings uncertainty as well as the household wealth on the likelihood of the wife being in the labor force. The empirical investigation is performed on an eleven year (1981-1991) longitudinal sample of continuously married couples from the panel study of income dynamics (PSID).

5.1.1 The Household Level Data

The PSID is a longitudinal study of nearly 8000 US households, following the same families and individuals since 1968. The initial sample was made of roughly 5000 households, 3000 being representative of the US population and about 2000 being low-income families from the Survey of Economic Opportunities (SEO). Thereafter, both the original households and their split offs have been interviewed each year. The survey includes a variety of socioeconomic variables, including age, education, family structure and earnings. An important aspect of the PSID data is that the earnings questions are retrospective. The interviews are conducted in March, and the questions refer to earnings in the previous year. I date the observations according to the year corresponding to the earnings, instead of the year of the interview. Over our sample period, information about household wealth in the PSID was collected in 1984 and 1989. Because of this limitation I exploit the full sample to estimate the households head earnings uncertainty but just the 1989 cross-section to estimate a model of married women participation.

I have selected only continuously married couples whose household head was always part of the labor force in an attempt to match the concept of a household introduced in the preceding sections. Individuals (both women and men) are considered labor force participants if they report to be working, only temporarily lade-off or unemployed and looking for work. Following Hyslop (1999), I have kept both the random census subsample of families and the non random SEO subsample of families. Table 1 collects summary statistics on a group of relevant variables corresponding to the 1989 cross-section. The variable labeled *Head Avg Earnings* corresponds to 1981-1991 average earnings of the head. The sample is composed of 1281 households.

Table 1: Summary statistics

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**
Husband's Age	42.814	8.561	27	75
Wife's Age	40.738	8.419	25	74
% Participation	0.714	-	0	1
% Head Self-employed	0.184	-	0	1
# Children	1.529	1.229	0	6
# Children 1-2	0.108	0.33	0	2
# Children 3-5	0.191	0.43	0	2
# Children 6-13	0.887	0.934	0	5
Wealth (thousand $)	179.08	571.917	-224.77	14610
Head Avg Earnings (1981 thousand $)	25.602	21.98	1.814	374.593

5.1.2 Estimating Households Head Earnings Uncertainty

The most natural way to estimate household head earnings uncertainty was to exploit the 1981-1991 sample, and to estimate for each household head the volatility of the innovations to earnings. The appropriate measure of earnings for our purpose is total head labor income because I want to estimate labor market earnings uncertainty. We model the logarithm of earnings as an $I(1)$ process

$$\ln E_{t+1} = \ln E_t + \xi_{t+1} \tag{5.1}$$

The assumption that the logarithm of individual earnings has a unit root is one that is very much accepted in the literature. Moreover, notice that this representation is consistent with some sort of decomposition of the earnings process into a permanent and a transitory component, which has been extensively exploited by the literature (Meghir and Pistaferri [2004]). Finally, for each household, we take as a measure of heads earnings uncertainty the standard deviation of ξ.

5.1.3 An Empirical Model of Married Women Participation

Hyslop (1999) reports that women who are employed are better educated, have fewer dependent children and their husbands have a slightly lower income. Moreover participation is higher among black women. I obtain the same findings and, moreover, I find a positive correlation between the women likelihood to be on the labor force and the husband earnings uncertainty measure. Furthermore, I find that household wealth has a negative impact on the probability of a married woman being employed on a given year.

The econometric approach chosen is very simple. A probit specification is used. I estimate the model only on the 1989 cross-section. This way I do not worry about the identification of serial correlation and fixed-effects or other sources of unobserved heterogeneity. I control for individuals education, household number of dependent children, wife and husband age, individual race, husband occupation, household US state of residence, individuals' parents education and husband type of employment (employee or self-employed). I introduce two variables which proxy households head earnings uncertainty. The most important one is the volatility of the innovations to the household earnings process, $std(\xi)$, but the unemployment rate in the household county of residence is also included. Table 2 shows the estimation results.

As predicted by the partial equilibrium model, both measures of uncertainty have a positive impact on the likelihood of the wife being present in the labor force. And more importantly households head earnings volatility is found to have a significant impact. As for wealth, we also do not reject that both the household wealth and the head average earnings have a negative impact on married women participation. This reflects the income effect discussed in the literature but also is consistent with a second order effect corresponding to a decrease in precautionary labor supply when wealth increases. The empirical findings are strongly consistent with the micro level predictions of our model.

5.2 Aggregate Level Evidence

At the aggregate level, the prediction of our model is that there should be a negative correlation between an economy saving rate and female attachment to the labor force. This is because the option value of accumulating precautionary wealth balances is less when a household has both members employed. It follows that at the aggregate level, the greater is female employment, the less will be aggregate capital accumulation. This is a long run (steady state) prediction which is easily testable using cross-country data provided by the OECD. Figure 6 plots household personal saving rate against participation. The first three panels correspond to the 70's, 80's and 90's, respectively, and the last panel shows the plot corresponding to the overall sample averages. A negative relation between personal saving and female participation is evident. Table 4 reports the results for a single cross-section of variables averaged over time (the between estimator). Once more the results strongly support the predictions made by the model.

6 Conclusion

In this paper I have developed a heterogeneous agent dynamic general equilibrium model which jointly models aggregate saving and (female) employment. I hope that the paper will be a contribution towards explaining persistent differences in private saving rates across very similar countries, which challenge the most well established theories of saving.

I first showed analytically, in a partial equilibrium setting, that if households are prudent the attachment of married women to the labor force is increasing in the level of household earnings uncertainty. Next I showed that in general equilibrium, because of market incompleteness and private information and because firms are not willing to pay the same wage to workers with different degrees of attachment to the labor force, allowing for family extensive margin labor supply choices can lead to multiplicity of equilibrium. In particular we can have one equilibrium with high employment and low savings and another one with low employment and high savings. Furthermore, (female) employment and consequently the aggregate saving rate will depend on the gender wage gap, a parameter which is made endogenous in the general equilibrium model.

In the equilibrium with high employment and low aggregate savings, firms are as well off as they would be in the low employment/high savings equilibrium, because of the free entry condition. However, in the high employment equilibrium, households are better off because they solve the same inter-temporal problem but wages are higher because of the lower gender wage gap. Therefore, the multiple equilibriums can be Pareto ranked, and the paper thus offers insights useful for policy-makers.

The paper also delivers strong predictions which allows us to confront the model with the data. In particular, at the micro level the model predicts that households whose head is exposed to more earnings uncertainty are more likely to have the second member of the family in the labor force. At the aggregate level, the model predicts that employment and aggregate saving will be negatively correlated. This is because when equilibrium employment is higher, there will be a higher share of two earners households in the economy, which are less exposed to earnings uncertainty and therefore have lower saving rates. The empirical evidence presented supports the predictions of the model.

An important direction for further research is to move beyond stationary competitive equilibrium analysis and to examine the impact on aggregate fluctuations of market incompleteness in the setting described in this paper.

Table 2: Estimation results: Probit

Variable	Coefficient	(Std. Err.)
UNCERTAINTY : $std(\xi)$	0.322604†	(0.191687)
WEALTH	-0.000194†	(0.000111)
PERMANENT EARNINGS	-0.013813**	(0.004257)
COUNTY UNEMPLOYMENT	0.003488	(0.003225)
AGE	0.097289	(0.087071)
AGE^2	-0.001842†	(0.000943)
AGE (husband)	-0.020648	(0.084504)
AGE^2 (husband)	0.000695	(0.000893)
# KIDS $<$ 18	-0.006154	(0.066872)
# KIDS 1-2	-0.382039*	(0.183417)
# KIDS 3-5	-0.758033**	(0.140102)
# KIDS 6-13	-0.291627†	(0.152038)
EDUCATION	0.249400**	(0.039987)
EDUCATION MOTHER	-0.067489*	(0.032287)
EDUCATION FATHER	-0.011770	(0.028194)
BLACK	0.521363**	(0.153252)
NATIVE AMERICAN	1.262693*	(0.600524)
ASIAN	-0.291472	(0.492749)
OTHER	0.559110	(0.588861)
NO DISABILITY	0.058195†	(0.032758)
EMPLOYEE (husband)	0.266075†	(0.144206)
UNION (husband)	-0.019417	(0.130579)
N	1281	
Log-likelihood	-629.82828	
$\chi^2_{(73)}$	207.245686	

Significance levels : $\dagger$: 10% $\quad *$: 5% $\quad **$: 1%

The specification includes an intercept, dummy variables for husbands occupation and household US state

Table 3: Savings and female participation across OECD countries (1970-1999 average)

	Personal Saving	Female Participation
AUS	10.05	55.6
AUT	11.12	53.2
BEL	16.75	49.3
CAN	12.13	59.6
CHE	10.69	61.3
DNK	0.34	71.3
FIL	3.28	69.6
FR	13.33	55.8
GBR	8.60	60.5
GER	12.42	55.0
IRL	9.41	40.5
ITL	19.37	41.6
JAP	16.36	57.5
KOR	17.60	47.9
NED	5.93	46.8
NOR	2.71	64.2
PT	10.83	59.4
SP	11.01	37.6
SWE	5.65	73.5
US	7.93	62.2

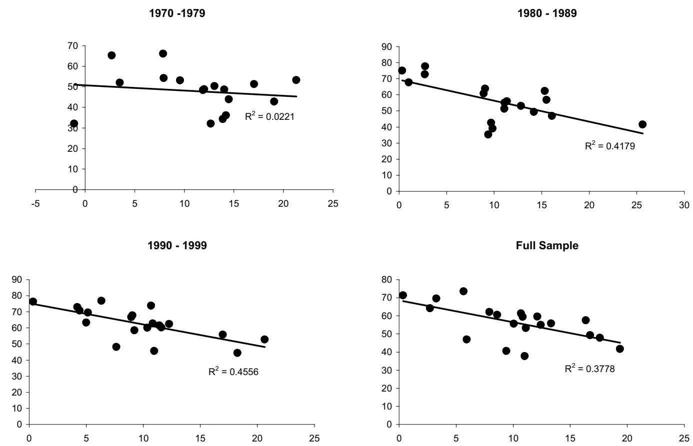

Figure 5: Participation and Aggregate Saving

Table 4: Between Estimator (full sample)

Variable	**Coefficient**	**(Std. Err.)**
PARTICIPATION	-0.294**	(0.085)
CONSTANT	27.329**	(4.969)
N	514	
Countries	20	
R^2	0.4	

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

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