Making dynamic modelling effective in economics

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Making Mathematics Effective in Economics

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1. Introduction

The aim of this article is to discuss both empirical and complexity/computational limitations in dynamics, especially limitations that have implications for economics. In order to make my essay more useful for economists, I begin by emphasizing the grounding of mathematical laws of nature in the four basic space-time invariance principles: translations, rotations, time translations, and Galilean transformations. Given the laws of dynamics, predictions mathematically take the form of continuous trajectories. I describe how physicists have started with continuum ideas and then have made finite precision predictions for chaotic dynamical systems in order to compare theory with the results of measurement, which are of very limited precision. I point out that the idea of solvability does not distinguish integrable from nonintegrable (chaotic or complex) dynamics, and locate where computational limitations begin to become interesting in the digitalization of a nonlinear dynamical system for computation. I also demonstrate the danger of existence proofs of equilibrium in the absence of dynamics[1]. In other words, I follow the sequence from empirical data through inferred dynamics and limitations on finding solutions, to the appearance of ideas of complexity in empirical data and dynamics.

But this essay is not restricted to deterministic dynamics. Using stochastic dynamics, which is generally required empirically by markets, a new empirically based model of financial market dynamics is my central contribution. Here, we do not force-fit a preconceived model to the data, rather, we deduce the quantitative form of the noise from the market data. This is unusual, and as a consequence the volatility enters in a natural way, as is demanded by data. The new model leads us to the idea of market instability, and a prescription for falsifiability of the idea of the famous Invisible Hand. Finally, I ask where complexity enters market dynamics, and then suggest a new analogy from cell biology for inventions and market growth. Along the way, I offer an observation about ‚emergence'.

My choice of the sequence of themes described above is encouraged by my overlap of interest with key topics that have also been discussed by Kumaraswamy Velupillai: the unreasonable effectiveness of mathematics in physics and the unreasonable ineffectiveness of mathematics in economics (Velupillai, 2003, 2005), the question of the right way to digitize a dynamical system for computation (Velupillai, 2003, 2004),

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[1] Without stating extra conditions, both stable and unstable equilibria may be either

falsifiability (Velupillai, 2003), and the appearance of computational complexity in nonlinear dynamics (Velupillai, 2000). These are all central questions for economic theory.

2. Symmetry Principles are the Basis of Mathematical Law

Data collection and analysis are central to physics. Data collection in the attempt to describe the motion of bodies began with the ancient astronomers, who used epicycles to describe planetary orbits. Epicycles are not necessarily wrong, because they don't define a specific dynamical model: in modern terms, epicycles are truncations of Fourier series expansions of orbits (Hoyle, 1973).

Physics, the mathematical description of empirically discovered laws of nature, began with Archimedes' discovery of the conditions for static equilibrium. Galileo and Kepler revived the Archimedian tradition in the seventeenth century and provided the empirical discoveries from which Newton was able to formulate nature's dynamics mathematically in a very general way (Barbour, 1989). Newtonian predictions have been verified with high decimal precision at macroscopic length and time scales where, on the one hand, light speed doesn't matter, and on the other where quantum phase coherence has been destroyed.

Why can we discover strict mathematical laws of inanimate nature, and why haven't we discovered corresponding time-

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computable or noncomputable.

invariant mathematical laws of socio-economic behavior? Wigner (1967) posed and answered both these questions in his beautiful essays on symmetry and invariance, where he identifies the basis of the seemingly unreasonable effectiveness of mathematics in physics.

Following Galileo and Kepler, scientists have discovered mathematical laws obeyed by nature via repeatable, identical experiments (physics, chemistry, genetics) and observations (astronomy). The foundation for the invariance of experimental results performed at different locations and times and in different states of motion lies in the symmetry principles that form the basis of Newtonian mechanics: mathematical laws of nature are invariant under translations, rotations, time translations, and transformations among Galilean/inertial frames.

Newton's Second Law describes a classical mechanical system

$$\frac{dp}{dt} = F$$

(1)

where $p=(p_1,\ldots,p_f)$ = mdX/dt is the momentum vector, m is a body's mass, X is its position, and F is the force. If the force F transforms like a Cartesian vector, F'=RF, then Newton's law is covariant under Galilean transformations, spatial translations and rotations, and time translations. When F=0 then we have Galilean invariance of solutions p(t), which is the mathematical basis for repeatable, identical experiments. In this case X is a coordinate in n dimensional configuration space, and the earth is approximately an inertial frame for experiments performed on a time scale much less than a day (McCauley, 1997a).

Globally integrable dynamical systems, like Keplerian orbits, reflect the basic space-time invariance principles through the standard conservation laws taught in every introductory physics text: conservation of momentum, angular momentum, and energy. Without the four underlying symmetry principles, empirical observations would generally depend on absolute time, absolute position and orientation, and absolute velocity. In a hypothetical universe without local invariance principles, even if there were underlying mathematical laws of motion we would be unable to extract them from observational data. E.g., Galileo first inferred special cases of Newton's First and Second Laws and the local law of gravity from very simple trajectories. A local law is a solution that holds for short enough space-time intervals, like the integrated form of Newton's First Law (p=constant, where p is momentum). A global solution, like a Keplerian orbit, holds over an unbounded space-time region. Differential equations describing both continuous symmetries (generators of Lie groups) and dynamics (eqn. (1), e.g.) are examples of local laws of motion. The universal applicability of the local law (1) lies in its grounding in the four space-time invariance principles. Global solutions of local laws of motion, if they appear at all in observed data, are solutions that hold approximately for very long times. Strictly seen, in mathematics, global solutions would hold for all possible times, past and future. But what about holism, which is advocated in some quarters today?

No mathematical law of motion can be deduced from empirical data unless a large part of the world can be neglected, to zeroth order, so that the most dominant features of nature can be studied in isolation and the rest can be described via interaction forces, perhaps only perturbatively. This is ‚reductionism', the division of the world into ‚dynamical system plus initial conditions' and ‚environment, where the initial conditions are effectively ‚lawless' (Wigner, 1967). The idea of holism is an illusion: if every part of the world were strongly coupled to every other part of the world, then we could discover little or nothing systematic about the world.

Aside from the known laws of physics, there are also models of motion that are not necessarily obeyed by any observed phenomena e.g., the neo-classical economic model or a complex adaptable model of Darwinism. Whatever the origin, empirical or postulated, every mathematical model that can be written down is a form of reductionism (the renormalization group method in statistical physics, valid at order-disorder transitions, reduces phenomena at a critical point approximately to symmetry and dimension). Quantum theory, the law of nature at very small length scales explains chemistry via atoms and molecules. Cell biology attempts to reduce observed phenomena to very large, complicated molecules, to genes and DNA, proteins, and cells. Proponents of self-organized criticality try to reduce the important features of nature to the equivalent of sand grains and sand piles via the hope for an underlying universality principle. Network enthusiasts likewise hope to reduce many interesting phenomena to nodes and links (Barabasi, 2002). The weakness in the latter two programs is that there are no known universality principles for driven-dissipative systems far from thermal equilibrium, except at the transition to chaos.

The isolation of cause and effect, the standard method used by an auto mechanic or radio repairman to repair a defective system, is basically the method of science. There is no escape from reductionism of one form or another. Given a successful model, meaning one that correctly describes a particular set of data and predicts new phenomena, one can then try perturbatively to take into account previously neglected interactions, but one cannot imagine taking into account everything. Falsifiabilty via empirical data is a scientific necessity. The idea of falsifiability is not a new idea. Karl Popper simply put into words what hard science since Galileo has practiced. In physics, a new model will not be accepted if it only describes phenomena that are already understood: a new model must also make empirically falsifiable new predictions. For example, Newtonian mechanics was used to predict the existence of a ‚new' planet before Neptune was discovered.

I will return to Wigner's theme in parts 5 and 6 below. First, some results from nonlinear dynamics and a way to compare dynamics predictions with real data, e.g. with time series obtained from experiment or observation, in order to focus more toward our eventual discussion of computability limitations.

3. Solvable Deterministic Dynamics

Since p=mdX/dt where $X=(X_1, \ldots, X_f)$ is position and m is mass, we can rewrite Newton's laws as a flow in a 2f dimensional phase space,

$$\frac{dx}{dt} = V(x) \qquad (2)$$

where phase space is a flat inner product space so that the n=2f axes labeled by $(x_1,\ldots,x_n)$ can be regarded as Cartesian and V(x) is an n-component time-independent velocity field in phase space. E.g., for a one degree of freedom Newtonian system dp/dt=F we have a two dimensional phase space where $x_1=X$, $x_2=p$, with $dx_1/dt=p=V_1$ and $dx_2/dt=F=V_2$.

Flows that preserve the Cartesian volume element $d\Omega = dx_1 \ldots dx_n$ are defined by $\nabla \cdot V = 0$ (conservative flows) while driven dissipative-flows correspond to $\nabla \cdot V \neq 0$, where ∇ denotes the Cartesian gradient in n dimensions. The expectation of stable equilibria in a neo-classical or any other supply-demand model would require a driven-dissipative system, e.g.

The condition for a phase flow is that for any initial condition x_o the solution $x_i(t) = U(t)x_{io}$ has no *finite* time singularities, singularities of flows at finite times are not permitted on the real axis in the complex time plane. The time evolution operator U(t) then exists and defines a one parameter transformation group for all real finite times t, with the inverse operator given by $U^{-1}(t) = U(-t)$, so that one can in principle integrate backward in time, $x_{oi} = U(-t)x_i(t)$ as well as forward, for both conservative and for driven-dissipative flows. That the time evolution operator U(t) has an inverse means that there is no diffusion; the variable x behaves deterministically, not stochastically. In part 7 we will relax this restriction to include stochastic dynamics.

In deterministic dynamics, one should distinguish between the ideas of solvable and integrable vs. nonintegrable. The later category includes chaotic and complex motions, and is where interesting computability limitations enter in a natural way. Any flow, even a chaotic or complex one, has a unique, well-defined solution (is solvable) so long as the velocity field V(x) satisfies a Lipshitz condition with respect to the n variables x_i. If, in addition, the velocity field V(x) is analytic in those variables then the power series locally defining the time evolution operator $U(t)=e^{tL}$,

$$x_i(t) = x_{io} + t\,(Lx_i)_o + t^2\,(L^2x_i)_o/2 + \ldots . \quad (3)$$

The infinitesimal generator is $L = V \cdot \nabla$, and (3) has a nonvanishing radius of convergence, so that the solution of (2) can in principle be defined by power series combined with analytic continuation for *all* finite times (Poincaré, 1993). The radius of convergence of (3) is typically small and unknown. Unless one can determine the singularities of (2) in the complex time plane, one does not know when and where to continue analytically. Therefore, in practice, we cannot expect to solve nonintegrable dynamical systems more than locally, for only very short time intervals. This is a restriction on predictability that precedes any computability limitations that follow from our next considerations.

An error-free way to digitize (2) for computation (McCauley 1987, 1993) is to use algorithms for computable numbers for all initial conditions and control parameters in the local solution (3). If the dynamical system is chaotic or complex, then one cannot compute very far into the future by using fixed precision. The precision of the computation must be increased systematically after a typically small number of time steps according to the demands made by the largest Liapunov exponent of (3). This is easy to understand: the same is required in order to compute solutions y(t) of dy/dt=y for large times, where the solution exhibits unbounded motion with a positive Liapunov exponent. This equation provides us with the simplest example of exponential instability of nearby orbits, the butterfly effect. If one computes the solution y(t) mod 1 at equally spaced discrete times, then the motion is bounded (lies on a circle) and one obtains a Bernoulli shift, the simplest chaotic dynamical system. So avoiding making a mistake while calculating •2 is an example of beating the butterfly effect.

If one only uses a limited precision method, like floating point arithmetic, then the only known test for numerical accuracy of the solution is to integrate (2) forward in time, and then integrate backward again in order to try to recover the initial condition to within at least one decimal place. If you cannot recover a single digit of the assumed initial condition, then you have integrated illegally too far forward in time. Below, we will discuss a method of predicting chaotic trajectories with controlled precision that does not require integration forward in time. The emphasis there is on the use of finite precision, with systematically increasing precision, which reflects computability.

A chaotic or complex system is always locally integrable but cannot be globally integrable. Integrable and nonintegrable systems are defined and discussed in a simple but incomplete way in McCauley (1997a). Arbitrary velocity fields V(x) generally define nonintegrable systems, and deterministic chaos typically occurs. Galileo's parabolic orbits are examples of local integrability of a Newtonian system. Kepler's planetary orbits are examples of global integrability for a two-body problem. But a Kepler orbit assumes that the solar system consists only of the sun and a single planet. Over extremely long times, there is evidence that the Newtonian dynamics of interacting planets in the solar system is nonintegrable and chaotic (Sussman and Wisdom, 1992).

The Newtonian three-body problem is chaotic for arbitrary initial conditions.

4. Computing Chaotic Dynamics with Controlled Precision

Measurement always means finite precision, and generally with at best few decimal place accuracy. E.g., market prices are specified only to within a few decimal places ($101.32. e.g.). In physics we are more concerned with making limited precision predictions correctly than with the more stringent requirement of computability. However, the phase space of chaotic systems is characterized by dense sets of unstable periodic and unstable quasiperiodic orbits, which are nontrivial to compute to within any controlled degree of decimal accuracy over long times.

Rather than study local power series solutions of differential equations digitized for computation, it is theoretically more convenient to study a corresponding discrete map, like a stroboscopic map or a Poincaré map. Such maps are always invertible because flows defined by ordinary differential equations are perfectly time reversible. Examples of iterated maps derived from differential equations and physical systems are given in McCauley (1987, 1997a).

The main ideas about maps can be most easily described by using a one-dimensional chaotic map of the unit interval, $x_n=f(x_{n-1})$, which necessarily has a multi-valued inverse and therefore cannot occur rigorously as a Poincaré map of a phase flow. To begin, discretize the map for computation in some integer base μ of arithmetic (expand the initial condition and all control parameters in base μ). In this case, if we choose μ •e^{λ} where λ is the map's positive Liapunov exponent for the class of initial conditions considered, then the meaning of λ is that we have to increase the precision of a forward time iteration at the rate of about one digit per iteration of the map in order to compute without making any error. Here, as in the digitization of a system of differential equations for computation, we meet the idea of the initial condition as program encoded in base μ, and the digitized dynamical system (the map) as computer. Most chaotic dynamical systems perform only trivial computations like ‚read and shift', add, multiplicaty, and combinations thereof. If one iterates such a system forward in discrete time n without increasing the precision at each time step, then one soon makes an error that ruins the computation. This is the most fundamental limitation on predicting the future for a chaotic map on a computer. In nature, we know the initial data to only within a few digits, and this leads to a similar limitation in collecting observational data. A positive Liapunov exponents is the condition for 'mixing', which is the condition for the applicability of the methods of statistical physics. Mixing also occurs in fluid turbulence, doe to an unstable cascade of eddies.

A positive Liapunov exponent reflects local orbital instability. Deterministic chaos means local orbital instability combined with global (Poincaré) recurrence of initial conditions. We can calculate both the unstable periodic and quasiperiodic orbits that characterize chaotic systems by iterating the map backward in time, where the Liapunov exponent contracts rather than expands intervals (errors). By iterating the unit interval, the phase space of a one dimensional map, backward in time n, the multi-valuedness of the map's inverse generates a tree that provides us with both the symbolic dynamics and the generating partition of the map (Cvitanovic et al, 1988; Cvitanovic et al', 2003; Feigenbaum, 1988; McCauley, 1993, 1997a). The generating partition, a hierarchy of sets of intervals or length scales, is peculiar to the map, but the symbol sequences are universal for an entire class of topologically related maps: the symbol sequences are invariant under continuous transformations of the map. The unstable periodic orbits are organized on the tree. Here, we generally do not need the more refined idea of computability because the tree provides us with a hierarchy of finite precision descriptions of the fractal attractor or repeller, and one can compare these finite precision descriptions with empirical observations, if observational data are accurate enough. The latter condition is nontrivial to satisfy.

Kepler's neutrally stable orbits of period one were used by Newton to discover the inverse square law of gravity. Prior to that, the neutrally stable parabolic trajectories discovered by Galileo reflected local versions of Newton's First and Second Laws of Motion (Heisenberg (1930), discovered systematically that he needed noncommuting operators in order to describe atomic spectral data). In the eighties, unstable periodic orbits extracted from time series near the transition to fluid turbulence were used to try to discover an underlying map that generates the transition to soft turbulence. That effort makes sense because there are well-defined universality classes of dynamical systems at the transition to chaos via period doubling, a bifurcation sequence described asymptotically by the renormalization group method. The effort to discover the universality class of maps from empirical data met serious nonuniqueness problems because of the very limited precision of the data (Chhabra et al, 1988): to determine the generating partition unambiguously, one needs very high precision in the data. This is a main point that I will return to in parts 7 and 9 below.

To what extent can the empirically observed time series of a particular market be used to infer the underlying dynamics? This question is of central importance for economics, where market dynamics have not yet been deduced empirically beyond finance theory. This latter asertion may well raise the ire of many economists, and so I will explain it in part 7 below.

5. Local versus Global Expectations in Dynamics

Symmetry not only plays the key role in forming the basis for the discovery of laws of motion from empirical data, it also plays a central role when one searches for solutions of dynamical equations. A globally integrable dynamical system in n dimensions has simple solutions that are globally valid, because the system has n global conservation laws that restrict the motion to rectilinear motion via a coordinate transformation based on those conservation laws. Global conservation laws reflect global symmetries in the n dimensional phase space.

If we discuss deterministic market models

$$\frac{dp}{dt} = \varepsilon(p)$$

(4)

then $p = (p_1,\ldots p_n)$ is the price, and the vector field $\varepsilon(p) = D(p)-S(p)$ is the excess demand. This does not approximate a mathematical rule for a market unless supply S and demand D are deduced *empirically* from that market. Typically, as in neo-classical theory, the functions S(p) and D(p) are merely modeled without paying attention to what real markets are doing. For two outstanding exceptions, see Osborne (1977) and Maslov (2000).

Before going further, let me emphasize that there is only one definition of equilibrium that is dynamically correct: dp/dt=0, or excess demand vanishes. Contrary to confusion rampant in the economics and finance literature (see, e.g., McAdam and Hallett, 2000), a limit cycle is not an equilibrium, nor is a strange attractor. Neither a Wiener nor lognormal stochastic process is defines equilibrium. More than seven diferent misuses of the term "equilibrium" are identified in McCauley (2004). Dynamic equilibria are defined by the vanishing of excess demand vanishes at one mor more points p*, $\varepsilon(p^*)=0$. In statistical equilibrium *all* averages of moments of the price p vanish, $d\langle p^n\rangle/dt=0$ for all values of n, which is the same as saying that the price distribution is time independent (stationary), or that the Gibbs entropy of the distribution is constant. We can achieve clarity of thought in finance and economics taking care to be precise in our mathematical definitions. Or, as an early Pope is reputed to have said, "One should tell the truth even if it causes a scandal" (Casanova, 1997).

In deterministic market modeling, we should expect no global conservation law other than the ‚Walras law' that defines the budget constraint and confines the motion to a sphere in phase space. The budget constraint reflects the symmetry of the price sphere in phase space (price space), but motion on the n-1 dimensional price sphere is typically nonintegrable. Arrow (1958) discovered interesting but humanly unattainable conditions for the mathematical 'existence' of an equilibrium point (perfect foresight combined with total conformity: all agents have the same expectations into the infinite future), but equilibrium points on the price sphere are unstable for the typical case of nonintegrability.

We know from nonlinear dynamics and general relativity that global integrability of local laws of motion (differential equations, iterated maps) is the rare exception. For global integrability of a dynamical system (2) in an n dimensional phase space, there must be enough symmetry that there are n-1 time independent global conservation laws that restrict the motion of the n-dimensional phase flow to trajectories that are topologically equivalent to rectilinear line motion at constant speed. Typically, in dynamics, one must almost always settle for local integrability. This means, as I have emphasized above, that, *even if we would know a correct deterministic dynamics describing a market*, then we could not hope in practice to calculate solutions that would be correct over large time intervals.

In mainstream economics, the neo-classical equilibrium model is taught as if it would be useful for understanding markets (Mankiw, 2000; Varian, 1992), but market equilibrium is not an empirically established fact, and the stability of the theoretically predicted equilibrium is anyway unknown (Kirman, 1988). For the model to be useful, it would be necessary that real markets could be described perturbatively by starting with the neo-classical model as a zeroth order approximation. Nothing of the sort has been achieved, or likely will ever be achieved. Still, the model has been used by the IMF, the World Bank, the E.U. and the U.S. Treasury as the theoretical basis for imposing extreme free market financial requirements on nations in the drive toward globalization via deregulation and privatization (Stiglitz, 2002). Here, an equilibrium point that cannot even be shown to be mathematically stable is deduced from a falsified model (McCauley, 2004) and is assumed to apply worldwide over significant time scales.

In reality, globalization via deregulation and privatization is a completely uncontrolled experiment whose outcome cannot be known in advance. The empirical evidence from The Third World is against the idea that globally uniform local rules and requirements yield either locally stable results or approximately uniform economic growth (see Stiglitz, 2002, for many qualitative examples of market instability). Certainly, there is no empirical evidence, or theoretical evidence from nonlinear or stochastic dynamics, to support such an idea. Furthermore, the example of biology tells us that, for survival, it is redundance and error correction ability, not efficiency, that matters.

Three results about the neo-classical model beyond Arrow's seem to me to be remarkable. Sonnenschein (1973) showed that there is no theoretical basis via aggregation or averaging for a neo-classical macroeconomic supply-demand model: either any curve or no curve at all may follow from aggregation. Contrast this with classical equilibrium statistical physics, which predicts thermodynamics uniquely via averaging. Radner (1968) has made the very interesting speculation that liquidity, the demand for money and financial markets, arises from computational limitations and other forms of uncertainty, although Radner apparently did not have in mind either a Turing machine or a clear idea of what he meant by his phrase ‚computational capacity' while writing his paper. Apparently unaware of Radner's speculation, Bak et al (1999) tried but failed to show how money might emerge from the addition of noise to optimizing behavior. Osborne (1977) showed that the neo-classical supply-demand model is falsified both microeconomically and macroeconomically. I regard Osborne, also the father of the lognormal price model in financial markets (Cootner, 1964), as the first econophysicist.

The neo-classical equilibrium model is a zero entropy (or perfect knowledge) model. Real markets reflect inherently finite entropy effects like production, consumption, and decision-making. Even without the observations made by Osborne, the neo-classical model is falsified unless one can find empirical evidence from at least one real market for stability and equilibrium. So instead of arguing that 'the neo-classical model is ideal' and the data are 'hard to describe' (no physicist will give any weight to such an argument), we must ask what the unmassaged market data can teach us. Or, more poetically, let us ask not what we can do for the data (give it a massage, or attack it with a specific model in mind, e.g.), let us ask instead what the data can teach us. It must be emphasized that the approach of the physicist is not at all the method of the econometrician (Granger, 1999): instead of having limited, preconceived models in mind, we deduce the stochastic model from the data (see McAdam and Hallet (2000) for a good example an attempt to force preconceived notions on the data). I illustrate this program in part 7, where I will argue that real market data are not at all hard to fit accurately by using dynamical models. To the contrary, market data are too easy to fit:

lack of uniqueness in empirically based modeling is the real problem that we face. Even if we would restrict our considerations to agent based trading models, we will not be able to escape the nonuniqueness.

6. Can Economic Dynamics Emerge from Market Data?

Given enough symmetry principles obeyed by prices, we should in principle be able to discover mathematical laws obeyed by markets. We will discuss two recently discovered invariance principles for markets below. But there is a fundamental difference between economic motions, like price changes (or GNP growth), and laws of nature.

Unlike natural law, acting on human expectations creates all of economic behavior. Without actions determined by our brains, wishes, and actions, markets and prices would not exist. Nature, e.g., stars, planets, DNA, and atoms are not invented and manipulated in that way. Mathematical laws of nature are beyond human invention, intervention, and convention. Without human agreement and/or regulation, in contrast, markets and prices do not even exist. Given that human decisions and actions create markets and money, to what extent can we hope to discover an approximately correct dynamics of markets? And bear in mind that nonuniqueness due to limited precision in data analysis can lead us not to a single model, but to some (non universality) class of models.

Consider a distribution of markets for a single asset, like gold or globalized autos (Ford, Toyota, GM, VW, or BMW, e.g.) on the face of the earth. The price g(p,t) or returns density f(x,X,t)=g(p,X,t)dp/dx depends not just on price p (or returns x) but on location X as well. By return, I mean the logarithmic return $x=\ln p(t)/p_o$ where p_o is some initial or reference price and g is a conditional probability density, a 'Green function' in the language of physics. Therefore, the ‚no-arbitrage' principle is equivalent to the assumption of rotational invariance (McCauley, 2004) of the price density (or translational invariance in a tangent plane containing two separate nearby markets, like Berlin and Frabkfort). The absence of arbitrage is a purely geometric statement that guarantees nothing other than that the probability distribution of the asset is independent of position X. In particular, ‚no-arbitrage' has nothing to do whatsoever with market equilibrium. Market equilibrium is equivalent to time translational invariance of the price distribution: in equilibrium, g(p,t) is independent of t.

There is also a weak form of ‚Galilean invariance' in markets. Consider a returns density f(x,t) for a single market in a single asset, like a stock (or, if you prefer, like housing in a particular region). By starting with the empirical histograms for that market, we can deduce a model of the returns distribution for vanishing expected return $R = \langle x \rangle = 0$ (Gunaratne and McCauley, 2002). If ‚Galilean invariance' holds, then the density for a finite return R is given by replacing x by $x-R\Delta t$ in the density f(x,t). In finance theory, because of nontrivial volatility D(x,t) in the stochastic differential equation describing the market, Galilean invariance is ‚broken' by the assumption of a riskfree hedge whenever the local volatility is x-dependent, as is the case with the empirical finance distribution (McCauley, 2004). No other invariance principles are known for markets. From this standpoint, we should not think of dx/dt=0 (or dp/dt=0) only as market equilibrium, but rather more generally as a weak analog of Galilean invariance. The analog is weak because „x" is logarithmic return instead of the position X of a particle in space-time.

For relatively slow markets like cars and housing, where trades occur on a time scale of days or longer instead of seconds, one can always use discrete time dynamics instead of the continuous time version, which is at best a mathematical convenience. Stochastic rather than deterministic dynamics are indicated, because there is no evidence for local integrability in market statistics at the shortest times (on the order of a second for heavily traded financial markets, e.g.).

Given this incompleteness, the absence of enough symmetry principles to pin down dynamical laws in economics, what can we do? The answer is the same as if there would be enough invariance principles to pin down real mathematical laws: we can study the available data for a specific market and try to extract a dynamical model that reproduces that data. In this case, we know in advance that we are modeling data for a particular market in a particular era, and that any model is expected to fail at some unknown time in the future. It should eventually fail as a result of a ‚surprise'. Surprises are the essence of complexity and are discussed in part 9 below. Therefore, it's essential that the model has few enough empirically known parameters to be falsifiable, otherwise one cannot know when the market has shifted in a complex/fundamental way. Such is the failure of the complicated, unenlightening, many-parameter models used by global banks (Bass, 1991).

I have indicated above how such a discovery program has been carried out empirically where deterministic chaotic dynamics apply. In part 9, I will discuss the challenge that we would face in trying to analyze time series reflecting deterministic complex dynamics, even if the underlying dynamical model would be simple. In part 7, we describe how to model financial markets empirically correctly using stochastic dynamics, and also will describe the difficulty in trying to extend the same program to nonfinancial markets.

Financial markets differ from other markets mainly in that many trades are made very frequently, even on a time scale of seconds, so that very good data are available for the falsifiability of few-parameter models. For houses or cars, e.g., the time scale for a large number of trades is much greater so that the data are much more sparse. Such markets are far less liquid and vary much more from one locale to another. This is main difference between financial and most nonfinancial markets. Because of the abundance of adequate and reliable data, financial markets provide the best testing ground for both new and old ideas. Financial markets exhibit the interesting characteristics of economic systems in general: growth and ‚the business cycle'(see

Goodwin (1993) for a discussion of these characteristics). When we speak of the ‚business cycle', a field where both stochastic (Cootner, 1964) and nonequilibrium nonlinear deterministic models were considered rather early (Velupillai, 1998), we no longer expect to discover any periodicity. We now understand it instead as volatility, or ‚fat tails', combined with lack of stationarity in the market distribution (the market distribution is simply the set of histogbrams obtained from real market data). Stationarity is another name for time invariance. Nonstationarity means that market entropy increases without limit. The nontrivial local volatility required to generate fat tails in the absence of stationarity is introduced in part 7. Market entropy is defined in part 8, where market instability is illustrated and discussed.

My answer to the title of this section is that nonfinancial market histrograms obtained from time series should be studied and modeled empirically in the spirit of financial markets, as is described in the next section.

7. Empirically Based Models of Financial Markets

In a stochastic description of markets the excess demand ε(p,t) is modeled by drift plus noise

$$\frac{dp}{dt} = \varepsilon(p) = rp + p\sqrt{d(p,t)}\,\frac{dB}{dt} \quad (5)$$

where dB is a Wiener process, so that dB/dt is white noise, and $d(p,t)p^2$ is the price diffusion coefficient. But we can more systematically write (5) as a stochastic differential equation

$$dp = rpdt + p\sqrt{d(p,t)}dB \qquad (6)$$

and use Ito calculus. The stochastic differential equation for the returns variable $x=\ln p(t)/p_o$ is given by Ito calculus as

$$dx = (r - D(x,t)/2)dt + \sqrt{D(x,t)}dB \qquad (7)$$

where the returns diffusion coefficient is D(x,t)=d(p,t). We can understand the returns diffusion coefficient D(x,t)=d(p,t) as the local volatility (McCauley, 2004), where the global (or globally averaged) volatility is $\sigma^2 = <(x-<x>) = \Delta t$, the mean square fluctuation in the return x.

The first quantitative description of stock market returns was proposed by the physicist turned finance theorist M.F.M. Osborne (Cootner, 1964), who plotted price histograms based on Wall St. Journal data in order to try to deduce the empirical distribution of stock prices. He inferred that asset returns do a random walk, so that prices are distributed lognormally. The lognormal price distribution is generated by the stochastic differential equation

$$dp = rpdt + \sigma_p pdB$$
(8)

with variable price diffusion coefficient $d(p)p^2=(\sigma_p p)^2$. The corresponding returns distribution is Gaussian and is generated by

$$dx = (r - \sigma_p{}^2/2)dt + \sigma_p{}^2 dB$$
(9)

where the returns diffusion is constant. One can invent various agent-based trading models that generate Gaussian or other sorts of returns. This is only part of the nonuniqueness that is faced in modeling empirical data.

Osborne's stochastic model is Markovian, so that the Hurst exponent H in $\sigma^2 =<(x-<x>) = \Delta t^{2H}$ is H=1/2. We know from data analysis that H=O(1/2) (Mantegna and Stanley, 2000), but whether H=.4, .5, or .6 is impossible to distinguish empirically. The choice H=1/2 yields models obeying the ‚efficient market hypothesis', which means simply that the market is very hard to beat: for H=1/2 there are no long time correlations in the market. A Hurst exponent H•1/2 implies fractional Brownian motion and yields long-time correlations that could, in principle, be exploited for profit. The stochastic integral that generates fractional Brownian motion is defined and discussed in McCauley (2004).

The Black-Scholes (1973) model of option pricing assumes Osborne's Gaussian returns model, but without having referenced Osborne. The Black-Scholes model is based on only two empirically measureable parameters, σ and r, and so is falsifiable. In fact, the model has been falsified: the empirical density of returns has fat tails $f(x,t) \bullet x^{-\mu}$ (fig. 1) for large returns x (Dacorogna et al, 2001), where μ is a nonuniversal scaling exponent that varies from market to market.

We do not know if the Gaussian returns model was ever accurate historically, because the data before the era of computerization are too sparse for a reliable test. Traders ca. 1990 claimed that the model no longer priced options correctly after 1987 (traders had once used Texas Instrument hand calculators with B-S programmed in, to price options on the trading floor). Whether there was a fundamental market shift to fat tails after 1987 (a surprise characteristic of complexity), or whether trading behavior changed, we simply do not know.

To a first approximation, financial data since at least 1990, for small to moderate returns, are also not approximately Gaussian but are instead exponentially distributed (fig. 2). The exponential distribution is generated by a Markovian model with nontrivial local volatility (diffusion coefficient D(x,t))

$$D(x,t) = \begin{matrix} b^2(1+\nu(x-\delta), x>\delta \\ b'^2(1-\gamma(x-\delta), x<\delta \end{matrix}$$

(10)

where b and b' are constants, ν, γ are proportional to $1/\bullet\Delta t$, and δ depends on Δt and defines the peak of the returns density. When

‘Galilean invariance’ holds then δ=RΔt. This local volatility yields a Brownian–like average (or global) volatility $\sigma^2 \bullet \Delta t$ at long times. The exponential model prices options correctly without the need for fudge-factors like ‚implied volatility‘ that characterize financial engineering based on a Gaussian returns model. Fat tails in returns are generated by including in (10) a term quadratic in $(x-\delta)/\bullet t$, as is shown in fig. 3. So financial data can be reproduced falsifiably by a simple stochastic model.

In a stochastic model, dB(t) is a Wiener process, but •D(x,t)dB(t) is not a Wiener process if the diffusion coefficient D depends on x (noise terms where D depends on t alone, and not on x, are equivalent to Wiener processes by a time transformation). This is the main point, that the formk of the diffusion coefficient D(x,t), that defines the noise term •D(x,t)dB(t) in dynamics, should be deduced empirically. The alternative would be to assume a dynamical model that tries to impose a preconceived diffusion coefficient on the data. Our program is to respect the noise and therfore first to discover the form of the empirical distribution. Then, we determine the time dependence of the distribution’s parameters from the data, and use that information to deduce a dynamical model: plugging the empirical distribution into a stochastic equation allows one to solve the ‘inverse problem’ to find the diffusion coefficient that generates the observed distribution (McCauley, 2004). This is analogous to the way that Newton deduced the inverse square law of gravity from Kepler’s orbits. The method of the economist, in contrast, is typically to assume a stochastic model and then try to aextract a best fit of parameter values for that model from the data. That is, a postulated model is used in an attempt to force fit to the data. E.g., the Real Business Cycle (RBC) model assumes a particular form for the noise term. In contrast with RBC, we deduce the form of the noise term from the data. This is physically significant: the noise term reflects what the 'noise traders' are doing. The noise term that would describe a stochastic model of the GNP would reflect the nature of the noise in the economy.

Had we restricted ourselves to the assumptions made in Granger (1999) or Granger and Newbold (1974, 1986) or to any method recommended in the economics literature, then we could not even have come close to the dynamics discovery presented in this section. The same can be said about the method proposed by Crutchfield (1994), which we discuss in part 9. The dynamics of the exponential model described above is completely new: aside from the Levy distribution, neither physics, finance, nor economics has previously yielded a nonstationarity dynamical model of volatility/fat tails without assuming stochastic volatility. But the volatility of the data requires that the diffusion coefficient depends on both 'position' x and time t. Now for another important aspect.

There is nonuniqueness in the choice of time dependence of γ, ν that can be used to fit finance market data. Given the nonuniqueness faced in extracting chaotic dynamics from data, this is not a surprise. In applying the new model to option pricing, we found (McCauley, 2004) that we have the unwarranted luck that the nonuniqueness doesn't matter on time scales much less than a hundred years. Normally, one should not expect such luck.

The main aim of economic theory in our era should be to match the success of the empirical description of financial markets for at least one nonfinancial market. Toward that end, ideas of stability and equilibrium in economics should either be verified empirically or else completely abandoned as guiding theoretical principles. This takes us to the interesting idea of the falsifiability of Adam Smith's Invisible Hand.

8. Searching for Adam Smith's Stabilizing Invisible Hand

Adam Smith's Invisible Hand is an idea of price changes near a stable equilibrium, that supply in a free market should rise to meet demand and tend to equilibrate. Stable markets could exhibit only small fluctuations about equilibrium, or at least near a steady state. Smith's idea can be described mathematically as a stationary process in stochastic dynamics, one where the Gibbs entropy of the market

$$S(t) = - \int f(x,t) \ln f(x,t) dx \qquad (11)$$

becomes asymptotically constant as t increases. Both the average return and global/average volatility of a stationary process are constants. That is, stable markets are both stationary and nonvolatile, but lack of volatility does not imply either equilibrium or stability in a model. Neither of these conditions is satisfied by financial markets, which instead are both nonstationary and volatile. Neither the (nonvolatile) Osborne-Black-Scholes lognormal model nor the (volatile) exponential model describes a stationary process. Instead, both of these models describe unstable dynamics with ever increasing market entropy (McCauley, 2004). Financial markets cannot be understood by using equilibrium ideas.

If we could locate equilibrium in a real market then we could define 'value' meaningfully. 'Value' would simply be the equilibrium price. The lack of equilibrum in market data means that value does not exist as an unambiguous idea, only price exists uniquely (to within arbitrage). Therefore, assertions that an asset is either undervalued or overvalued are subjective, but wishful thinking acted on collectively can lead to big price swings, as in the phenomenon of 'momentum investing' and the stock market bubble of the last decade. This psychological condition, the inability to know 'value', likely contributes to both nonstationarity and fat tails. One can imagine noise traders changing their minds frequently, and so trading frequently because they're very uncertain of the 'value' of a financial holding like a stock or bond. This proposition might be tested via an agent based trading model. An interesting exercise would be to introduce a trading model where equilibrium 'exists' mathematically but is in some sense noncomputable (could be simply NP-complete, not necessarily Turing (1936) noncomputable) and see what would be the effect on the agents' behavior. The liquidity bath term •D(x,t)dB(t) in (7), which is not a Wiener process when D(x,t) depends on x (even though dB(t) is always Wiener), approximates the effect of these 'noise traders'. In the language of statistical physics, equations (6) and (7) provide us with an analog of a mean field approximation to a complex system of interacting agents. Real agents have PC's or Macs, high computational capability, but generally can't do any worthwhile calculations when trading because they can't distinguish knowledge from noise, and can only make guesses about future prices. I know this from experience. I'm an amateur trader, and so is my wife. Trading done by professionals in markets is the closest we can come to an analog of performing experiments in physics.

Above, we have assumed that financial markets can be treated statistically independently of other markets. This is not strictly true but reflects the approximation whereby global finance drives other markets. For correlated assets, the Gibbs entropy requires the density of returns of those assets, and that density doesn't decouple into statistically independent factors. However, a diagonalization of correlation matrix of the Capital Asset Pricing Model leads to eigenvectors representing sectors (Plerou et al, 1999), and suggests that we might try to study different sectors approximately statistically independently.

Adam Smith's Invisible Hand is a falsifiable proposition: one need only test a set of price or returns data for a given market for asymptotic stationarity, or at least for lack of growth and lack of volatility (McCauley, 2004). The problem that one faces is that typical nonfinancial markets have such sparse data that reliable testing is difficult or even impossible (too easy to fit by completely wrong models), but that is no ground for teaching the falsified and completely unrealistic neo-classical model. Instead, empirically based models should be taught. Such models will likely be computable, even if the dynamical behavior described would be undecideable. Because of nonuniqueness in extracting models from data, e.g., I expect that GNP data should be relatively easy to fit by using nonstationary, volatile models. To date, there is no convincing evidence from empirical data that any known market is asymptotically stationary, and market volatility is rather common. Instead, the known price and return distributions spread without limit as time increases.

Existence proofs of equilibrium in the absence of dynamics are common in the study of economic models. I now give an example that shows the danger inherent in an existence proof of equilibrium in the absence of dynamics that would show how such equilibrium could be attained.

Consider Osborne's model of lognormal market prices (8) where $d(p,t)=\sigma_p^2$=constant. If we would take $r<0$, negative expected return, then the drift term would provide us with an example of a restoring force, an example of the Invisible Hand (McCauley, 2004). Does the Invisible Hand pull the market toward equilibrium? The corresponding Fokker-Planck equation describing the price distribution is

$$\frac{\partial g}{\partial t} = -r\frac{\partial}{\partial p}(pg) + \frac{\sigma_p^2}{2}\frac{\partial^2}{\partial p^2}(p^2 g)$$

(12)

and, indeed, has a very simple equilibrium solution g(p). However, the time dependent solution of (12), the lognormal density g(p,t), spreads without limit as t increases and never reaches statistical equilibrium. In particular, the second moment $<p^2>$ increases without limit. The reason that equilibrium is not approached is that the spectrum of the Fokker-Planck operator defined by (12) is continuous, not discrete. Imposing finite limits on p, price controls, would yield a discrete spectrum so that statistical equilibrium would then follow asymptotically.

Suppose we would try to approximate the lognormal model (12) by making the uncontrolled approximation $p^2 \bullet <p^2>$ in the price diffusion coefficient $\sigma_p^2 p^2$. The second moment $<p^2>$ is time-dependent, but if we systematically define a new time variable t' via integration, then we obtain the equivalent stochastic differential equation

$$dp = rpdt' + cdB$$

(13)

where, because c is constant and r<0, the uncontrolled approximation (13) describes a stationary process, the Smoluchowski-Ornstein-Uhlenbeck process. In this case, the time-dependent density g(p,t) approaches an equilibrium Gaussian density $g_{eq}(p)$ as t goes to infinity, whereas the correct density described by (12) is nonstationary. Were financial markets described by a stochastic process like (13), then The Invisible Hand (negative returns combined with d(p) = constant) would always push the market toward statistical equilibrium. The model (13) describes real data in physics (where p is the speed of a colloidal particle in Brownian motion a heat bath), but not at all in finance.

Again, the challenge put to economists and econophysicists is to find at least one market where there is empirical evidence for the stabilizing Invisible Hand. By this, I do not mean assuming a stationary process and then force-fitting it to the data, an illegal proceedure that is often done in finance. I mean: deduce the form of the noise term from the data, and show that the variance does not spread as time increases, at the very least. Nonstationary methods have been used to analyze economic data, but nonstationarity alone is not enough, assuming a time dependent local volatility is not enough: fat tails, or 'volatility', cannot be correctly described by such an assumption. In general, the data will imply a diffusion coefficient that depends not merely on time but on returns as well (such models have not been studied in physics either!). This is the lesson of our finance model described above, a lesson that is not reflected by any known data analysis in economics to date.

Financial markets, including option pricing, have been accurately described by very simple stochastic dynamics, so where's the complexity? The question of complexity is a question of computational limitations or intractability. The highest degree of computational complexity is that of a Turing machine (Feynman, 1996; Velupillai, 2000).

We expect that markets are not merely stochastic but are also in some yet to be defined sense complex. Can the empirically observed time series of a complex system be used to infer the underlying dynamics? I return next to deterministic dynamics, leaving it to others to combine the ideas of stochasticity and complexity in an empirically useful or at least theoretically satisfying way. My excuse is that

> "… the human brain is a rather limited intellectual tool, better suited to hunting rabbits than doing mathematics, …"
>
> David Ruelle (2004)

In what follows, I assume that all functions are Turing computable and that computable numbers are used as control parameters and initial conditions. I want to avoid the trivial noncomputability of the measure one set of numbers that can be defined to ‚exist' in the continuum, but cannot be generated algorithmically.

9. Complexity in Physics, Biology, and Economics

I consider here only two ideas of maximal computational complexity. Other ideas of complexity are the Chomsky hierarchy of languages, NP-completeness, and a bewildering zoo of finer degrees of computational intractability (http://www.complexityzoo.com/). To date, we have no physically or biologically motivated definitions of complexity, in spite of the fact that cell biology provides us with numerous examples of complexity. Our everyday computers are an example of complexity and can be described mathematically as Newtonian electro-mechanical machines.

First, consider algorithmic complexity. This is the notion that the dynamics consists in simply writing down a sequence, starting from some initial condition, and that there is no algorithm that compresses the rule for generating the sequence. The continued fraction expansion of $x(x+2)=1$ to generate •2-1 is not algorithmically complex, e.g. A nontrivial example of algorithmic complexity is perhaps given by the rules for generating scalefree networks (Barabasi, 2002): starting from an already present node, one links to another node where the new node is drawn from a certain probability distribution. So far, these ideas have not been well enough developed to use them to describe money transfers on networks in any empirically correct way. The digit expansions of rational numbers are not examples of algorithmically complex patterns, because every rational is described by a very short algorithm: divide an integer P by another integer Q.

A cautionary note: the shortest algorithm that generates the statistics or pattern is not necessarily the one that explains the real time evolution of the phenomena. An example is the use of selfaffine fractals to generate pictures of mountain landscapes. In that case, the simple dynamics used to generate the art (Barnsley, 1988) gives us no insight into the physics/geology of mountain formation. Maybe algorithmic complexity comes closest to describing our nonhabitual/nonrecurring decisions in everyday life.

This leads us to a second, extremely interesting way to generate maximal complexity (Moore, 1990, 1991). Low dimensional iterated maps that are equivalent to Turing machines provide examples. These dynamical systems have no attractors, no symbolic dynamics/no generating partition, and so exhibit no scaling laws that would inform us of behavior at smaller length scales in terms of observed dynamics at larger length scales. Instead, ‚surprises', new unforeseen behavior, are possible at all length scales. By length scales, I think here of the hierarchy of coarsegrainings defined by the generating partition in a chaotic system, where one looks in finer and finer detail at the dynamics, increasing the precision of the microscope, so to speak. Without symbolic dynamics and the corresponding generating partition, it is not clear how or even if a Turing-equivalent dynamical system could be extracted from time series. The output of Moore's maps must already be included as a subset of binary expansions of numbers, so how can we understand and distinguish that class of nonperiodic digital patterns, number theoretically?

Mutations of viruses and bacteria to new forms provide an example of the surprises characteristic of complexity. In markets, the complexity may appear in two ways: first, the expected return r can change suddenly due to market psychology, or liquidity can dry up in a crash. On a longer time scale, the entire market distribution may change its form due to factors/surprises beyond our horizon of expectations. In this respect, it would be of interest to know if financial returns statistics exhibited fat tails before 1987.

In everyday life, surprises are regarded as something 'external', arising from factors not taken into account in our attempt to forecast a sequence of events. In Moore's iterated maps, the surprises arise *internally* from the system's dynamics. That every highly complicated computer program has bugs may be an example of the surprises of complexity: you can only discover the bugs by running the program. In order to imagine how Moore's surprises could enter into our finance market model, we must consider the entire system composed of fluctuating asset price (described to zeroth order by (6)) and the liquidity bath, which finance theory assumes to remain unchanged. The analogy of the liquidity bath with the heat bath for a Brownian particle is described in McCauley (2004). In a financial market, the appearance of a surprises may cause the liquidity bath to dry up suddenly, as in a market crash. In that case, (6) and (7) do not apply: a liquidity drought is not a Wiener, exponential, or any other continuous time stochastic process, it is more approximately the complete absence of the noise traders. In order to try to describe surprises mathematically, one could try to model the interacting system of agents trying to set prices, avoiding assuming the liquidity bath/Brownian motion approximation explicitly. But then one likely faces the computational complexity of a neural net equivalent to a Turing machine. Siegelman (1995) has suggested the equivalence of Moore's maps with neural nets. In any case, we are not used to the idea that surprises are generated *within* the system, especially for low-dimensional deterministic dynamics.

Continuing with Moore's maps, for a deterministic dynamical system with universal computational capability a classification into topologic universality classes is impossible. Given an algorithm for the computation of an initial condition to as many digits as computer time allows, nothing can be said in advance about the future *either statistically or otherwise* because the future is computationally undecideable. *This maximum degree of computational complexity occurs in low dimensional nonintegrable conservative Newtonian dynamics*. In particular, billiard ball dynamics exhibit positive Liapunov exponents and provide us with an example of a chaotic system that is mixing (Cvitanovic et al', 2003). But billiard balls can also be used to compute reversibly and universally (Fredkin and Toffoli, 1982). Such a method of computation would be impractical because the positive Liapunov exponents magnify errors in initial conditions of the billiard balls, messing up the computation.

Molecular biology is apparently largely about complexity at the cellular and molecular (DNA-protein) level. E.g., the thick, impressive, and heavy text by Alberts et al (2002) is an encyclopedia of cell biology, but displays no equations. Again, with no equations as an aid, Weinberg (1999) describes the 5-6 independent mutations required to produce a metastasizing tumor. All these impressive biological phenomena may remind us more of the results of a complicated computer program than of a dynamical system, and have all been discovered reductively by standard isolation of cause and effect in controlled, repeatable experiments. We might learn something about complexity ‚physically' were we able to introduce some useful equations into Alberts et al. The Nobel Prize winning physicist-turned-biophysicist Ivar Giæver (1999) has remarked on the difference between biology and physics texts: "Either they are right or we are right, and if we are right then we should put some equations in that text."

Many economists and econophysicists would like to use biological analogies in economics, but the stumbling block is the complete absence of a dynamical systems description of biological evolution. Instead of simple equations, we have simple objects (genes) that behave like symbols in a complicated computer program. Complex adaptable mathematical models notwithstanding, there exists no mathematical description of evolution that is empirically correct at the macroscopic or microscopic level. Schrödinger (1944), following the track initiated by Mendel[2] that eventually led to the identification of the molecular structure of DNA and the genetic code, explained quite clearly why evolution can only be understood mutation by mutation at the molecular level of genes. Mendelism provides us with a falsifiable example of Darwinism, at the cellular level, the only precise definition of biological evolution, there being no falsifiable model of Darwinism at the macroscopic level. That is, we can understand how a cell mutates to a new form, but we do not have a picture of how a fish evolves into a bird. That is not to

[2] It may be of some interest that Mendel was trained in the Galilean method: he studied and taught physics in Vienna. He did not get an academic position, and so retrested to Brnn and studied peas.

say that it hasn't happened, only that we don't have a model that helps us to imagine the details, which must be grounded in complicated cellular interactions that are not yet understood. Weinberg (1999) suggests that our lack of understanding of cellular networks also limits our understanding of cancer, where studying cellular interactions empirically will be required in order to understand how certain genes are turned on or off.

The terms 'emergence' and 'self-organization' are not precisely defined, they mean different things to different people. I shamelessly confess that I have never understood what people have in mind, other than symmetry-breaking and pattern formation at a bifurcation in dynamics, when they claim that a system 'self organizes'[3]. Some researchers who study complex models mathematically expect to discover new, ‚emergent' dynamics for complex systems, but so far no one has produced an empirically relevant or even theoretically clear example. See Lee (2004) for a readable account of some of the usual ideas of self-organization and emergence. Crutchfield and Young (1990), Crutchfield[4] (1994) and others have emphasized the interesting idea of nontrivial computational capability appearing/emerging in a dynamical system due to bifurcations. This doesn't present us with any new dynamics, it's simply about computational

[3] Hermann Haken (1983), at the Landau-Ginzburg level of nonequilibrium statistical physics, provided examples of bifurcations to pattern formation via symmetry breaking. All subsequent writers have used 'self-organized' as if the term would be self-explanatory, even when there is no apparent symmetry breaking. Is a deterministic or noisy stable equilibrium point or limit cycle (or other invariant set without escape) an example of self-organization? If so, then maybe we don't need the fancy phrase. According to Julian Palmore (1964): if you can't define your terms precisely then you don't know what you're talking about.

[4] My essay is completely contrary to the postmodernist philosophical outlook expressed, especially in part I, of Crutchfield's 1994 paper.

capability appearing within already existing dynamics at a bifurcation to chaos or beyond. Crutchfield assumes a generating partition and symbolic dynamics, but Moore has shown that we have to give up that idea for dynamics with Turing-equivalent complexity. Another weakness in Crutchfield is the restriction of noise to stationary processes. That won't work for market data, or for realistic market models either. There is, in my opinion, another weakness in that program: if we would apply that proposed method of discovery to Galilean and Keplerian orbits, then we would discover only trivial automata reflecting orbits of period zero and one. Newton did considerably better, and we've done better in finance theory, so there must be more to the story. One can argue: the scheme wasn't invented to discover equations of motion, it was invented as an attempt to botanize complexity. In that case, can the program be applied to teach us something new and unexpected about empirical data? Why doesn't someone try to apply it to market data? Crutchfield's scheme is in any case far more specific than whaat proposed by Mirowski (2002) proposed in a similar vein.

Given the prevailing confusion over 'emergence', I seize the opportunity to offer an observation to try to clarify at least one point: whatever length and time scales one studies, one first needs to discover approximately *invariant* objects before one can hope to discover any possible new dynamics[5]. The 'emergent dynamics', if such dynamics can be discovered, will be the dynamics of those objects. Now, what many complexity theorists hope and expect is

[5] E.g., a cluster, like suburbanization in a city (Lee, 2004), is not an example of an approximately invariant object, because the cluster changes on significantly on the length and time scale that we want to study it.

that new dynamical laws beyond physics will somehow emerge statistically-observationally at larger length and time scales, laws that cannot be derived systematically from phenomena at smaller length scales. A good example is that many Darwinists would like to be able to ignore physics and chemistry altogether and try to understand biological evolution macroscopically, independently of the mass of details of genetics, which have emerged from controlled experiments and data analysis.

Continuing with my seized opportunity, consider the case of cell biology, where the emergent invariant objects are genes. Genes constitute a four-letter alphabet used to make three letter words. From the perspective of quantum physics, genes and the genetic code are a clear example of emergent phenomena. With the genetic code, we arrive at the basis for computational complexity in biology. Both DNA and RNA are known to have nontrivial computational capability (Adelman, 1994; Bennett, 1982; Lipton, 1989). One can think of the genes as ‚emergent' objects on long, helical molecules, DNA and RNA. But just because genes and the code of life have emerged on a long one dimensional tape, we do not yet know any corresponding new dynamical equations that describe genetics, cell biology, or cancer. So far, one can only use quantum or classical mechanics, or chemical kinetics, in various different approximations to try to calculate some aspects of cell biology.

My main conclusion is that 'e*mergence' does not guarantee the appearance of new laws of motion*. Apparently, invariant objects can emerge without the existence of any simple new dynamics to describe those objects. Genes obey simple rules and form four letter words but that, taken alone, doesn't tell us much about the consequences of genetics, which reflect the most important possible example in nature of computational complexity: the evolution from molecules to cells and human life.

At a more fundamental level, genes obey the laws of quantum mechanics in a heat bath, with nontrivial intermolecular interactions. I emphasize that Schrödinger has already explained why we should not expect to discover statistically based laws that would evolution at the macroscale. So I am not enthusiastic about the expectation that new ‚emergent' laws of motion will be discovered by playing around with nonempirically inspired computer models like ‚complex adaptable systems'. I think that we can only have hope of some success in economics, as in chemistry, cell biology and finance, by following the traditional Galilean path and sticking close to the data. E.g., we can thank inventive reductionist methods for the known ways of controlling or retarding cancer, once it develops. At the same time, it would certainly be interesting to have a falsifiable complex adaptable model, if that is possible.

Thinking of examples of emergence in physics, the Newtonian level, mass and charge are invariant. The same objects are invariant in quantum theory, which obeys exactly the same local space-time invariance principles as does the Newtonian mechanics, and gives rise to the same global conservation laws[6].

[6] Integrable systems, like the hydrogen atom, whether classical or quantum can be solved by direct use of the conservation laws (this is the method of every text). In a nonintegrable system like the helium atom, a three body problem (and chaotic), that is impossible.

We do not yet understand how Newtonian mechanics ,emerges' from quantum mechanics in a self-consistent mathematical way. Similarly, we do not understand why genes should behave like elements of a classical computer, while the DNA molecule requires quantum mechanics for its formation and description. The famous quantum measurement problem is unsolved, so we do not understand mathematically *within quantum theory* how *quantum phase coherence* is destroyed. Quantum phase coherence must be destroyed in order that a Newtonian description, or classical statistical mechanics, becomes valid as a mathematical limit as Planck's constant vanishes. One can make arguments about the destruction of phase coherence via external noise in the heat bath defined by the environment, but this path only begs the question. However, this incompleteness in understanding does not reduce our confidence in either classical or quantum mechanics, because all known observations of the motions of masses and charges are described correctly to within reasonable or high decimal precision at the length scales where each theory applies. One point of mesoscopic physics is to study the no man's land between the quantum and classical limits.

I end this essay by suggesting a simpleminded biological analogy. The creation of new markets depends on new inventions and their exploitation for profit. Mathematical invention has been described psychologically by Hadamard (1945). Conventional ideas of psychology completely fail to describe the solitary mental act of invention, whether in mathematical discovery, or as in the invention of the gasoline engine or the digital computer. Every breakthrough that leads to a new invention is a ‚surprise‘, something emerging from within the system (the system includes human brains and human actions) that was not foreseen. A completely new product is based on an invention. The creation of a successful new market, based on a new product, is partly analogous to an epidemic: the disease spreads seemingly uncontrollably at first, and then eventually meets limited or negative growth. The simplest mathematical model of creation that I can think of would be described by the growth of a ‚tree‘, where new branches (inventions or breakthroughs) appear suddenly without warning. This is not like a search tree in a known computer program. Growth of any kind is a form of instability, and mathematical trees reflecting instability do appear in nature, in the turbulent eddy cascade e.g., but in that case the element of ‚surprise‘ is missing.

Summarizing, I've discussed the Galilean method in physics and finance and have suggested that it be applied in economics. Computability of a model is certainly necessary, but empirically motivated models are necessary beforehand if mathematics is to be made effective in general economics, as it has become in the specific area of finance. Empirically based models will likely be computable in the Turing sense. Market time series and histograms are, of course, of limited value in predicting the future: they reflect in coarse fashion how we've been behaving economically. The future in socio-economic phenomena is to some unknown degree undecidable and can't be known in advance, even statistically. Using market statistics as a basis for prediction assumes that tomorrow will be statistically like yesterday. If we've modeled carefully, as in finance, then this assumption may not get us into hot water so long as there are no surprises. Insurance companies make money by assuming that the future will be like the past statistically, and lose money when it isn't.

Of course, one can also make nonempirically based mathematical or even nonmathematical models, and assert that if we assume this and that, then we expect that such and such will happen. That sort of modeling activity is not necessarily completely vacuous, because new socio-economic expectations can be made into reality by acting strongly enough on wishes or expectations: a model can be enforced or legislated, e.g. Both communism (implemented via bloody dictatorships) and globalization via deregulation and privatization (implemented via legislation, big financial transfers, and supragovernmental[7] edict) provide examples. In any case, models based on real market statistics are useful for confronting the pie in the sky claims of ideologues and other true believers with the coarse face of reality. Instability and surprises are good examples of market reality in our era.

Acknowledgement

I'm grateful to Vela Velupillai for having given me the chance to summarize and present my ideas about econophysics in an economics book that is devoted to ideas of computability. I hope that the reader will at least find my discussion entertaining, if not

[7] Examples of powerful supragovernmental organizations are the IMF, the world Bank,the World trade Organization, and the European Union.One can argue that the U.S. Federal Reserve Bank has comparable influence.

stimulating. I regret that I was not aware of the many interesting papers in computable economics before writing parts of chapters 2 and 7 of my recent book „Dynamics of Markets“, where I mistakenly attributed notions of computability limitations in general equilibrum theory to Radner and Kirman instead of Alain Lewis. I'm also grateful to Kevin Bassler, Hassan Masum, and Roy Radner for very helpful correspondence, and to my very helpful, critical home editor, Cornelia Küffner, for reading and suggesting improvements in the manuscript. Vela Velupillai helped me to sharpen my presentation in the end by reading the manuscript and posing challenging questions from the economists' viewpoint. My answers to many of his questions are written into the text. Gemunu Gunaratne's original unpublished discovery of the exponential distribution in finance and turbulence is described in McCauley (2004). My earlier view about economic dynamic models, expressed in McCauley (1997a,b), has been corrected by my recent work: in 1997, I did not expect that the parameters in an empirically deduced market model could be few enough in number, or would stand still long enough, for the model to be falsifiable. That viewpoint was written before I had understood the success during the 1980's of the Osborne-Black-Scholes model in finance.

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