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Petrella, Ivan and Santoro, Emiliano

Catholic University of Leuven, University of Copenhagen

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# Optimal Monetary Policy with Durable Consumption Goods and Factor Demand Linkages*

Ivan Petrella†
Catholic University of Leuven

Emiliano Santoro‡
University of Copenhagen

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**Abstract**

This paper deals with the implications of factor demand linkages for monetary policy design. We consider a dynamic general equilibrium model with two sectors that produce durable and non-durable goods, respectively. Part of the output of each sector serves as a production input in both sectors, in accordance with a realistic input-output structure. Strategic complementarities induced by factor demand linkages significantly alter the transmission of exogenous shocks and amplify the loss of social welfare under optimal monetary policy, compared to what is observed in standard two-sector models. The distinction between value added and gross output that naturally arises in this context is of key importance to explore the welfare properties of the model economy. A flexible inflation targeting regime is close to optimal only if the central bank balances inflation and value added variability. Otherwise, targeting gross output variability entails a substantial increase in the loss of welfare.

*JEL classification:* E23; E32; E52

*Keywords:* Input-Output Interactions, Durable Goods, Optimal Monetary Policy

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†Center for Economic Studies, Faculty of Business & Economics, Katholieke Universiteit Leuven, Naamsestraat 69, 3000 Leuven, Belgium. *E-mail*: Ivan.Petrella@econ.kuleuven.be.

‡Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen, Denmark. *E-mail*: emiliano.santoro@econ.ku.dk.

# Introduction

This paper deals with the implications of factor demand linkages for monetary policy design. We build a dynamic stochastic general equilibrium (DSGE) model with two sectors that produce durable and non-durable goods. The gross output of each sector serves either as a final consumption good, or as an intermediate input in both sectors, according to an input-output matrix calibrated on the US economy.

Introducing factor demand linkages into otherwise standard general equilibrium models with durable and non-durable consumption goods is of key importance. It is well documented that standard sticky-price models incorporating sectoral heterogeneity in price stickiness - usually in the form of sticky non-durable goods prices and flexible durables prices - cannot generate positive co-movement between sectors following a monetary policy innovation (Barsky et al., 2007). As argued by various contributions (see, among others, Aoki et al., 2004; Erceg and Levin, 2006; Barsky et al., 2007), co-movement between non-durable and durable consumption is an inherent feature of the US economy that multi-sector DSGE models need to be able to replicate. Recently, Bouakez, Cardia, and Ruge-Murcia (2008) and Sudo (2008) have shown that factor demand linkages are able to generate positive co-movement between non-durable and durable spending.[1] However, none of these papers takes a normative perspective. Input-output interactions are empirically relevant and play an important role in the transmission of shocks across sectors.[2] As such, they should be accounted for in designing the optimal monetary policy for a multi-sector framework. The key contribution of this paper is to explore how monetary policy should be pursued in a model with cross-industry flows of input materials. We show four main results.

First, the monetary authority cannot attain the Pareto optimal allocation consistent with the full stabilization of output and inflation, even in the absence of distortions in the labor market (imperfect labor mobility) and the goods market (monopolistic competition). The consumption of intermediate goods by both sectors imposes a more restrictive set of conditions to the full stabilization of the model economy in the face of exogenous shocks, compared to models where sectors of production do not employ input materials (e.g., Erceg and Levin, 2006) or are connected through a vertical trading chain (e.g., Huang and Liu, 2005).

Second, we explore optimal monetary policy under the assumption that the policy maker can credibly commit to a policy rule derived from the minimization of a social welfare function. We follow Rotemberg and Woodford (1998) and obtain a quadratic approximation to the utility function of the representative household. The welfare criterion balances, along with sectoral inflation variability, a preference to smooth durables accumulation and reduce fluctuations in aggregate consumption (or, equivalently, value

[1] Horvath (1998, 2000) and Carvalho (2009) show that cross-industry flows of input materials can reinforce the effect of sectoral shocks, generating aggregate fluctuations and co-movement between sectors, as originally hinted by Long and Plosser (1983). Kim and Kim (2006) show that a similar mechanism generates widespread co-movement of economic activity (e.g. in employment) across sectors. See also Hornstein and Praschnik (1997).

[2] Bouakez, Cardia, and Ruge-Murcia (2008) report evidence on the empirical relevance of input-output interactions. Dale Jorgenson's data on input expenditures by US industries show that materials (including energy) account for roughly 50% of outlays, while labor and capital account for 34% and 16%, respectively. The Input-Output accounts compiled by the Bureau of Labor Statistics (BLS) show that 70% of the material-input expenditures by the durables sector goes into goods produced by the nondurables sector. The converse proportion is around 10%, which is much smaller but still not negligible.

added). Given the natural distinction between consumption and production in the presence of input materials, it is no longer irrelevant whether the monetary authority targets the output gap or the consumption gap. This result has important implications for both the transmission of exogenous shocks and the selection of policy regimes as alternatives to the optimal policy under commitment.

Third, introducing factor demand linkages into an otherwise standard two-sector model amplifies the loss of social welfare and alters the transmission of shocks to the system, compared to the benchmark economy without input materials. A distinctive feature of the model is that a technology shock to either sector also affects potential output in the other sector, even if preferences over different types of consumption goods are separable. This results in an amplification of the consumption gap response in the sector hit by the shock and an attenuation in the other sector, compared to models where sectors are not related through input-output linkages. Furthermore, factor demand linkages imply that the relative price of non-durable goods not only affects the marginal rate of substitution between durable and non-durable consumption, but also exerts a positive (negative) impact on the real marginal cost in the durable (non-durable) goods sector. The relative magnitude of these effects depends on the off-diagonal elements in the input-output matrix. Consequently, the intermediate input channel modifies the transmission of positive sectoral cost-push shocks in two main respects, compared to what we conventionally observe in models without factor demand linkages: (i) the deflationary effect in the sector which is not hit by the shock is attenuated and (ii) the drop in the demand for both classes of consumption goods is amplified.

Fourth, we compare the welfare properties of the model under the optimal policy and various alternative loss functions. A flexible inflation targeting regime delivers a welfare loss close to that attained under the optimal policy. Most importantly, the central bank attains a smaller loss when fluctuations in aggregate (or core) inflation are balanced with those in real value added, compared to the loss induced by targeting gross output. We also consider the case of asymmetric price stickiness, which implies a natural divergence between core and aggregate inflation. Although such a difference is still relevant within our framework, targeting either core or aggregate inflation makes little difference in terms of welfare loss. By contrast, what matters is the term capturing real volatility: in this respect, targeting the consumption gap entails substantial benefits compared to targeting the production gap. Once again these results emphasize the distinction between consumption and production that naturally arises in this class of models.

Beyond reconciling conventional two-sector DSGE models with a realistic structure of the economy and with the empirical evidence on the transmission of monetary shocks, as in Bouakez, Cardia, and Ruge-Murcia (2008) and Sudo (2008), the key contribution of this paper is to detect important differences between the way monetary policy should be pursued and what is otherwise prescribed by the existing literature on multi-sector models without factor demand linkages. As in Erceg and Levin (2006), an interest rate rule targeting aggregate variables can closely approximate the optimal policy. However, targeting the production gap rather than the consumption gap may entail a substantial loss of welfare.

The remainder of the paper is laid out as follows: Section 1 introduces the theoretical setting; Section 2 reports the calibration of our model economy and shows how co-movement between durable and non-durable consumption emerges following a shock to an instrumental policy rule; Section 3 discusses the Pareto optimal outcome; Section 4 discusses the implementation of the optimal monetary policy under commitment and compares the resulting loss of social welfare with that attainable under a number of alternative policy regimes. We also consider optimal monetary policy under asymmetric degrees of competition and price stickiness between sectors. Section 5 concludes.

# 1 The Model

We develop a DSGE model with two sectors that produce durable and non-durable goods, respectively. The model economy is populated by a large number of infinitely-lived households. Each of these is endowed with one unit of time and derives utility from the consumption of durable goods, non-durable goods and leisure. The two sectors of production are connected through factor demand linkages.[3] Goods produced in each sector serve either as a final consumption good, or as an intermediate production input in both sectors. The net flow of intermediate goods between sectors depends on the input-output structure of the production side.

## 1.1 Producers

Consider an economy that consists of two distinct sectors producing durable (sector $d$) and non-durable goods (sector $n$). Each sector is composed of a continuum of firms producing differentiated products. Let $Y_t^n$ ($Y_t^d$) denote gross output of the non-durable (durable) goods sector:

$$Y_t^i = \left[ \int_0^1 \left( Y_{ft}^i \right)^{\frac{\varepsilon_t^i - 1}{\varepsilon_t^i}} df \right]^{\frac{\varepsilon_t^i}{\varepsilon_t^i - 1}}, \qquad i = \{n, d\} \tag{1}$$

where $\varepsilon_t^i$ denotes the time-varying elasticity of substitution between differentiated goods in the production composite of sector $i = \{n, d\}$. Each production composite is produced in the "aggregator" sector operating under perfect competition. It is possible to show that a generic firm $f$ in sector $i$ faces the following demand schedule:

$$Y_{ft}^i = \left( \frac{P_{ft}^i}{P_t^i} \right)^{-\varepsilon_t^i} Y_t^i, \qquad i = \{n, d\} \tag{2}$$

where $P_t^i$ is the price of the composite good in the i$^{th}$ sector. From (1) and (2) the relationship between the firm-specific and the sector-specific price is:

$$P_t^i = \left[ \int_0^1 \left( P_{ft}^i \right)^{1-\varepsilon_t^i} df \right]^{\frac{1}{1-\varepsilon_t^i}}, \qquad i = \{n, d\}. \tag{3}$$

Sectors are related by factor demand linkages. Part of the output of each sector serves as an intermediate input in both sectors. The allocation of output produced in the i$^{th}$ sector is such that:

$$Y_t^i = C_t^i + M_t^{in} + M_t^{id}, \qquad i = \{n, d\} \tag{4}$$

[3]Throughout the paper we will refer to factor demand linkages as indicating cross-industry flows of input materials. If a specific feature of the framework is essentially determined by the use of intermediate goods in the production process (i.e., inter-sectoral relationships are not essential) we will explicitly refer to input materials.

where $C_t^i$ denotes the amount of consumption goods produced by sector $i$, while $M_t^{in}$ ($M_t^{id}$) is the amount of goods produced in sector $i$ and used as input materials in sector $n$ ($d$).

The production technology of a generic firm $f$ in sector $i$ is:

$$Y_{ft}^i = Z_t^i \left[ \frac{\left(M_{ft}^{ni}\right)^{\gamma_{ni}} \left(M_{ft}^{di}\right)^{\gamma_{di}}}{\gamma_{ni}^{\gamma_{ni}} \gamma_{di}^{\gamma_{di}}} \right]^{\alpha_i} \left(L_{ft}^i\right)^{1-\alpha_i}, \qquad i = \{n, d\} \tag{5}$$

where $Z_t^i$ ($i = \{n, d\}$) is a sector-specific productivity shock, $L_{ft}^i$denotes the number of hours worked in the f$^{th}$ firm of sector $i$, $M_{ft}^{ji}$ ($j = \{n, d\}$) denotes material inputs produced in sector $j$ and supplied to firm $f$ in sector $i$. Moreover, $\gamma_{ij}$ ($i, j = \{n, d\}$) denotes the generic element of the ($2 \times 2$) input-output matrix, $\Gamma$, and corresponds to the steady state share of total intermediate goods used in the production of sector $j$ and supplied by sector $i$. The input-output matrix is normalized, so that the elements of each column sum up to one: $\sum_{j=\{n,d\}} \gamma_{nj} = 1$ (and $\sum_{j=\{n,d\}} \gamma_{dj} = 1$).

Note that production does not employ physical capital and that input materials supplied by the durable goods sector fully depreciate within the period. However, we allow consumers to store durable goods from which they derive a flow of services.[4]

Material inputs are combined according to a CES aggregator:

$$M_{ft}^{ji} = \left[ \int_0^1 \left(M_{kf,t}^{ji}\right)^{\left(\varepsilon_t^j - 1\right)/\varepsilon_t^j} dk \right]^{\varepsilon_t^j / \left(\varepsilon_t^j - 1\right)}, \tag{6}$$

where $\left\{M_{kf,t}^{ji}\right\}_{k \in [0,1]}$ is a sequence of intermediate inputs produced in sector $j$ by firm $k$, which are employed in the production process of firm $f$ in sector $i$.

Firms in both sectors set prices given the demand functions reported in (2). They are also assumed to be able to adjust their price with probability $1 - \theta_i$ in each period. When they are able to do so, they set the price that maximizes expected profits:

$$\max_{P_{ft}^i} E_t \sum_{s=0}^{\infty} (\beta\theta_i)^s \Omega_{t+s} \left[P_{ft+s}^i (1 + \tau_i) - MC_{ft+s}^i\right] Y_{ft+s}^i, \qquad i = \{n, d\} \tag{7}$$

where $\Omega_{t+s}$ is the stochastic discount factor (consistent with households' maximizing behavior, which is described in the next subsection), $\tau_i$ is a subsidy to producers in sector $i$, while $MC_{fs}^i$ denotes the marginal cost of production of firm $f$ in sector $i$. The optimal pricing choice, given the sequence $\left\{P_t^n, P_t^d, Y_t^n, Y_t^d\right\}$, reads as:

$$\overline{P}_{ft}^i = \frac{\varepsilon_t^i}{\left(\varepsilon_t^i - 1\right)(1 + \tau_i)} \frac{E_t \sum_{s=0}^{\infty} (\beta\theta_i)^s \Omega_{t+s} MC_{ft+s}^i Y_{ft+s}^i}{E_t \sum_{s=0}^{\infty} (\beta\theta_i)^s \Omega_{t+s} Y_{ft+s}^i}, \qquad i = \{n, d\}. \tag{8}$$

Note that assuming time-varying elasticities of substitution translates into sectoral cost-push shocks that allow us to account for sector-specific shift parameters in the supply

[4]In Bouakez, Cardia, and Ruge-Murcia (2008) and Sudo (2008) durables used as intermediate inputs are also modeled as fully depreciating goods on the production side. The BLS in the US publishes two different input-output tables: (i) the "input-use" table, which considers goods that fully depreciate in the same period they are produced, and are usually referred to as "materials" in the traditional KLEM setting; (ii) the "capital flow" table of the input-output accounts, that refers to input materials used as investment goods. Our $\Gamma$ is calibrated according to the "Input-use" table. It is important to stress that what we usually define as durables producing sectors have non-zero entries in the input-use matrix, even though the non-durables sector is still the main supplier of input materials.

schedules.

In every period each firm solves a cost minimization problem to meet demand at its stated price. The first order conditions from this problem result in the following relationships:

$$MC_{ft}^{i}Y_{ft}^{i}=\frac{W_{t}^{i}L_{ft}^{i}}{1-\alpha_{i}}=\frac{P_{t}^{n}M_{ft}^{ni}}{\alpha_{i}\gamma_{ni}}=\frac{P_{t}^{d}M_{ft}^{di}}{\alpha_{i}\gamma_{di}},\qquad i=\{n,d\}\,. \tag{9}$$

It is useful to express the sectoral real marginal cost as a function of the relative price, $Q_t = P_t^n/P_t^d$, and the sectoral real wage:

$$\frac{MC_t^n}{P_t^n}=\frac{\overline{\phi}^n\left(Q_t^{-\gamma_{dn}}\right)^{\alpha_n}\left(RW_t^n\right)^{1-\alpha_n}}{Z_t^n}, \tag{10}$$

where $RW_t^n = W_t^n/P_t^n$ is the real wage in sector $n$, and $\overline{\phi}^n$ is a convolution of the production parameters ($\overline{\phi}^i = \alpha_i^{\alpha_i}\left(1-\alpha_i\right)^{1-\alpha_i}$, $i=\{n,d\}$). Analogously, for the durable goods sector:

$$\frac{MC_t^d}{P_t^d}=\frac{\overline{\phi}^d\left(Q_t^{\gamma_{nd}}\right)^{\alpha_d}\left(RW_t^d\right)^{1-\alpha_d}}{Z_t^d}. \tag{11}$$

where $RW_t^d = W_t^d/P_t^d$.

From (10) and (11) it is clear that the relative price exerts a direct effect on the real marginal cost of each sector, whose magnitude depends on the size of the cross-industry flows of input materials.[5] Specifically, for the i$^{th}$ sector the absolute impact of $Q_t$ on $MC_t^i/P_t^i$ is related to the "importance" of the other sector as input supplier, i.e. on the magnitude of the off-diagonal elements in the input-output matrix ($\gamma_{nd}$ and $\gamma_{dn}$). This is a distinctive feature of the framework we deal with. By contrast, in traditional multi-sector models without factor demand linkages (e.g., Erceg and Levin, 2006), the relative price only affects the real marginal cost indirectly, through the marginal rate of substitution between different consumption goods.[6]

## 1.2 Consumers

Households derive income from working in the production sectors, investing in bonds, and from the stream of profits generated in the production sectors. Their preferences are defined over a composite of non-durable goods ($C_t^n$), an "effective" stock of durable goods ($\mathfrak{D}_t$), and labor ($L_t$). They maximize the expected present discounted value of

[5] In a multi-sector setting it is not realistic to consider the case of a diagonal input-output matrix, so that input materials are produced and employed within the same sector by means of a roundabout input-output structure. In such cases, a higher share of intermediate goods dampens the impact of the real wage on the real marginal cost, hereby increasing strategic complementarity in price-setting among firms in the same sector. In turn this may determine large output effects in the face of a disturbance to nominal spending (see Basu, 1995; Woodford, 2003, pp. 170-173). This effect is still at work within the general structure we envisage. In addition, cross-industry flows of input materials induce strategic complementarities between sectors (Horvath, 1998).

[6] Similarly, in a model with vertical input linkages, the relative price only exerts a direct effect on the real marginal cost of the final goods sector. Nevertheless, it can still be related to the real marginal cost of the intermediate goods sector through the marginal rate of substitution between consumption and leisure (see Huang and Liu, 2005; Strum, 2009).

their utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{H_t^{1-\sigma}}{1-\sigma} - \varrho \frac{L_t^{1+v}}{1+v} \right], \tag{12}$$

where $H_t = (C_t^n)^{\mu_n} \mathfrak{D}_t^{\mu_d}$ and $\beta$ is the discount factor, $\mu_n$ and $\mu_d (= 1 - \mu_n)$ denote the expenditure shares on non-durable and durable goods, $\sigma$ is the inverse of the intertemporal elasticity of substitution, $v$ is the inverse of the Frisch elasticity of labor supply. Durable goods are accumulated according to the following law of motion:

$$D_t = C_t^d + (1-\delta) D_{t-1}, \tag{13}$$

where $\delta$ is the depreciation factor. The effective stock of durables scales the effect of a quadratic cost of adjustment (see, e.g., Bernanke, 1985):[7]

$$\mathfrak{D}_t = D_t - \frac{\Xi}{2} \frac{(D_t - D_{t-1})^2}{D}, \quad \Xi \geq 0 \tag{14}$$

where $D$ denotes the steady state stock of durable consumption goods.

We assume that labor can be either supplied to sector $n$ or $d$ according to a CES aggregator:

$$L_t = \left[ \phi^{-\frac{1}{\lambda}} (L_t^n)^{\frac{1+\lambda}{\lambda}} + (1-\phi)^{-\frac{1}{\lambda}} \left(L_t^d\right)^{\frac{1+\lambda}{\lambda}} \right]^{\frac{\lambda}{1+\lambda}}, \tag{15}$$

where $\lambda$ denotes the elasticity of substitution in labor supply, and $\phi$ is the steady state ratio of labor supply in the non-durable goods sector over total labor supply (i.e., $\phi = L^n/L$). This functional form conveniently allows us to account for different degrees of labor mobility between sectors, depending on $\lambda$.[8] For $\lambda = 0$ labor is prevented from moving across sectors. For $\lambda = \infty$ workers devote all time to the sector paying the highest wage. Hence, at the margin, all sectors pay the same hourly wage and perfect labor mobility is attained. For $\lambda < \infty$ hours worked are not perfect substitutes. An interpretation of this is that workers have a preference for diversity of labor and would prefer working closer to an equal number of hours in each sector even in the presence of wage differences across sectors.[9] However, an important difference between (15) and the CES aggregator used by Horvath (2000) is that the former allows us to neutralize the impact of labor market frictions in the steady state.

The following sequence of (nominal) budget constraints applies:

$$\sum_{i=\{n,d\}} P_t^i C_t^i + B_t = R_{t-1} B_{t-1} + \sum_{i=\{n,d\}} W_t^i L_t^i + \sum_{i=\{n,d\}} \Psi_t^i - T_t, \tag{16}$$

[7] The inclusion of a cost of adjustment of the stock of durables allows us to obtain results in line with the empirical evidence on the behavior of durable consumption over the business cycle. King and Thomas (2006) show how the partial adjustment mechanism helps at accounting for the aggregate effects of discrete and occasional changes in durables consumption at the microeconomic level. Adda and Cooper (2000) provide evidence on the discrete nature of durables purchases at the individual level.

[8] Empirical evidence suggests that labor and capital are not perfectly mobile across sectors. Davis and Haltiwanger (2001) find limited labor mobility across sectors in response to monetary and oil shocks. Bouakez, Cardia, and Ruge-Murcia (2008) report evidence suggesting that perfect labor mobility across sectors, with its implication that sectoral nominal wages are the same (at the margin), is an imperfect characterization of the data.

[9] Horvath (2000) motivates a similar specification based on the desire to capture some degree of sector-specificity to labor while not deviating from the representative consumer/worker assumption. In a similar vein, we conveniently employ this mechanism to allow for imperfect labor mobility between sectors.

where $B_t$ denotes a one-period risk-free nominal bond remunerated at the gross risk-free rate $R_t$, and $T_t$ denotes a lump-sum tax paid to the government. The term $\Psi_t^n + \Psi_t^d$ captures the nominal flow of dividends from both sectors of production.

The following relationship can be retrieved from the first order conditions of consumers' utility maximization with respect to $L_t^n$ and $L_t^d$:[10]

$$\left(\frac{\phi}{1-\phi}\right)^{-\frac{1}{\lambda}} \left(\frac{L_t^n}{L_t^d}\right)^{\frac{1}{\lambda}} = \frac{W_t^n}{W_t^d}. \tag{17}$$

Notice that, as $\lambda \to \infty$ (and perfect labor mobility is attained), sectoral nominal wages are equalized.

### 1.3 The Government and the Monetary Authority

The government serves two purposes in the economy. First, it delegates monetary policy to an independent central bank. We assume that the short-term nominal interest rate is used as the instrument of monetary policy and that the policy maker is able to pre-commit to a time-invariant rule. We consider alternative specifications of the monetary policy rule, including both rules that can be regarded as reasonable characterizations of the recent historical experience (Section 2), and rules derived from an explicit optimization problem from the perspective of a benevolent central banker (Section 4).

The second task of the government consists of taxing households and providing subsidies to firms to eliminate distortions arising from monopolistic competition in the markets for both classes of consumption goods. This task is pursued via lump-sum taxes that maintain a balanced fiscal budget.

### 1.4 Market Clearing

Total production reads as:

$$Y_t = Y_t^n + Y_t^d. \tag{18}$$

The allocation of output produced by each sector requires that sectoral gross output is partly sold on the markets for consumption goods, while a proportion is sold on the markets for input materials. Therefore, (4) must be met in each sector.

It is important to recognize that $C_t^i$ and $Y_t^i$ are not equivalent in our setting. Specifically, $C_t^i$ can be interpreted as value added in the i$^{th}$ sector, while $Y_t^i$ is the sectoral gross output.[11] As sectoral gross output can be sold on both the intermediate goods market and the final goods market, total production is typically greater than real value added. Thus, according to our model economy, $C_t^i$ most closely matches the empirically relevant definition of value added (or GDP).

## 2 Solution and Calibration

To solve the model, we log-linearize behavioral equations and resource constraints around the non-stochastic steady state and then take the deviation from their counterparts un-

[10] These are reported in Appendix A.

[11] See also Nakamura and Steinsson (2008) on this distinction in multi-sector models with input materials.

der flexible prices. The difference between log-variables under sticky prices and their linearized steady state is denoted by the symbol "ˆ", while we use symbol "$*$" to denote percent deviations of variables in the efficient equilibrium (i.e., flexible prices and constant elasticities of substitution) from the corresponding steady state value. Finally, we use symbol "˜" to denote the difference between linearized variables under sticky prices and their counterparts in the efficient equilibrium.[12]

The model is calibrated at a quarterly frequency. We assume that the discount factor $\beta = 0.993$. We set $\sigma = 1$, a value in line with Ngai and Pissarides (2007) which implies separability in the utility derived from different consumption goods. The expenditure share on non-durable goods is $\mu_n = 0.682$. The inverse of the Frisch elasticity of labor supply ($v$) is set to 3, while $\lambda = 1$, which reflects limited labor mobility.[13] The production parameters $\alpha_n = \alpha_d = 0.6$, while the entries of the two-sector input-use matrix are such that $\gamma_{nn} = 0.899$ and $\gamma_{nd} = 0.688$,[14] according to the calibration of the US economy used by Bouakez, Cardia, and Ruge-Murcia (2008).[15] These values imply a positive net flow of input materials from the non-durable goods sector to the durable goods sector. The depreciation rate of the stock of durables is assumed at 2.5%.[16] In the baseline calibration we assume that the degree of nominal rigidity is the same across sectors, with $\theta_n = \theta_d = 0.75$. We will also allow for asymmetric degrees of price stickiness between the two sectors at different stages of the analysis. We assume that sectoral elasticities of substitution have a steady state value equal to 11. Finally, we set $\Xi = 600$, as in Erceg and Levin (2006).

As discussed above, the system features two sector-specific technology shocks, $z_t^n$ and $z_t^d$. The cost-push shocks, $\eta_t^n$ and $\eta_t^d$, are reduced-form expressions for the time-varying cost-shift parameters in the sectoral New Keynesian Phillips curves. Exogenous variables are assumed to follow a first-order stationary VAR with *iid* innovations and diagonal covariance matrix. We set the parameters capturing the persistence and variance of the productivity growth stochastic processes so that $\rho^{z^n} = \rho^{z^d} = 0.95$ and $\sigma^{z^n} = \sigma^{z^d} = 0.02$, respectively. These values are consistent with the empirical evidence showing that technology shocks are generally small, but highly persistent (see Cooley and Prescott, 1995; Huang and Liu, 2005). As to the cost-push shocks, we follow Jensen (2002), Walsh (2003) and Strum (2009), and assume that these are purely transitory, with $\sigma^{\eta^n} = \sigma^{\eta^d} = 0.02$.

## 2.1 Co-movement in the Face of a Monetary Policy Shock

Prior to exploring equilibrium dynamics under optimal monetary policy it is instructive to show how this model differs from one without factor demand linkages. To close the model at this stage, it is necessary to specify how the monetary authority sets the nominal

[12] Steady state conditions are reported in Appendix B. Appendix C reports the economy under flexible prices. The linearized system in extensive form is reported in Appendix D.

[13] This value is in line with the calibration proposed by Horvath (2000).

[14] This implies that the marginal impact of changes in the relative price on the real sectoral marginal cost is, in absolute value, higher for the durable goods sector, as $\alpha_n = \alpha_d$ and $\gamma_{nd} \gg \gamma_{dn}$.

[15] These shares have been computed using the table "The Use of Commodities by Industries" for 1992 produced by the BLS. Sudo (2008) shows that the matrix is fairly stable over time.

[16] As in Erceg and Levin (2006), this choice reflects that the durables sector in our model includes both consumer durables and residential investment, which have quarterly depreciation rates of about 5% and 0.75%, respectively, and that the expenditure share of consumer durables in the composite is about two-thirds.

rate of interest. In the first instance, and for illustration purposes only, we consider the following instrumental rule:[17]

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1 - \rho_r)\, \chi_\pi \pi_t + u_t^r, \quad \chi_\pi > 0 \text{ and } 0 \leq \rho_r \leq 1 \tag{19}$$

where $u_t^r$ is an $iid\,(0,1)$ monetary policy innovation. The interest rate-smoothing parameter, $\rho_r$, is set to 0.7, while $\chi_\pi = 1.5$. We assume that the monetary authority responds to a convex combination of the sector-specific rates of inflation. The corresponding price index is $\widetilde{p}_t = \phi \widetilde{p}_t^n + (1-\phi)\widetilde{p}_t^d$, where the weights assigned to sectoral prices are determined by the relative size of each sector.

Figure 1 reports the effects of a one-standard-deviation shock to the monetary policy rule, for various frequencies of adjustment in the price of durables (the degree of stickiness in the non-durable sector is kept constant at four quarters). Factor demand linkages induce co-movement in durable and non-durable goods spending. Durable and non-durable consumption decrease following the monetary contraction and gradually return to their equilibrium level thereafter.[18] Moreover, in accordance with the empirical evidence produced by Erceg and Levin (2006), the sensitivity of durable spending to the monetary innovation is larger than that of non-durable spending, despite the introduction of an adjustment cost of the effective stock of durables. This effect is more pronounced as the stickiness of durable goods prices increases.

When factor demand linkages are ruled out and prices in the durable goods sector are flexible, as reported in the right-hand panel of Figure 1, durable consumption increases, while non-durable consumption mirrors its path in the opposite direction.[19] This negative co-movement is induced by the assumption of price flexibility in the durables sector, as opposed to the assumption of price stickiness in the non-durables sector, which implies that the relative price of durables decreases following the initial monetary contraction. This economy behaves in line with the model developed by Barsky, House, and Kimball (2007), where the relative price only affects the marginal rate of substitution between different consumption goods.[20]

# 3 The Pareto Optimum

Removing sources of distortion in the labor market (imperfect labor mobility) and the goods market (monopolistic competition) represents a desirable situation for a benevolent central banker. At this stage of the analysis we are interested in understanding whether, after removing these distortions (in a variant economy without cost-push shocks), the monetary authority can attain a first best allocation where inflation and the output gap in both sectors are jointly stabilized. The answer to this question is negative for general

[17] See DiPace (2008) for a sensitivity analysis to different instrumental rules. Bouakez, Cardia, and Ruge-Murcia (2008) and Sudo (2008) produce analogous evidence through shocks to money supply.

[18] An alternative line of enquiry is followed by Monacelli (2009), who stresses the role of financial market imperfections in generating co-movement. DiCecio (2009), instead, argues that rigidity in nominal wages is the key friction underlying sectoral co-movement.

[19] We would obtain analogous conclusions if we were to assume that each sector employs only input materials that are produced within the same sector, i.e. by setting the off-diagonal elements of the input-output matrix ($\Gamma$) to zero. In this respect, the presence of cross-industry flows of input materials are crucial for generating co-movement.

[20] When input-output interactions are switched off, imperfect labor mobility is not enough to generate co-movement in the face of a monetary policy innovation (Bouakez et al., 2008).

parameter values and shock processes. The following proposition formalizes our results.

**Proposition 1** *In the model with sticky prices and perfect labor mobility across sectors, there exists no monetary policy that can attain the Pareto optimal allocation unless the shock buffeting the non-durable goods sector equals the one buffeting the durable goods sector, scaled by a factor* $\zeta = (1-\alpha_n)/(1-\alpha_d)$.[21]

***Proof.*** *Suppose there were a monetary policy under which the equilibrium allocation under sticky prices would be Pareto optimal. Then, in such an equilibrium, the gaps would be completely closed for every period. That is,* $\widetilde{rmc}_t^n = \widetilde{rmc}_t^d = 0$, $\forall t$. *It follows from the pricing conditions that* $\pi_t^n = \pi_t^d = 0$, $\forall t$. *Recall that the relative price evolves as:*

$$\widetilde{q}_t = \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^*.$$

*Since we also have that* $\Delta\widetilde{q}_t = 0$, *the equation above implies that* $\pi_t^n - \pi_t^d = \Delta q_t^*$. *It can be shown that:*[22]

$$q_t^* = \frac{(1-\alpha_d)\, z_t^n - (1-\alpha_n)\, z_t^d}{1+\varkappa},$$

*where*

$$\varkappa = \alpha_n\alpha_d(\gamma_{nn} + \gamma_{dd} - 1) - \alpha_n\gamma_{nn} - \alpha_d\gamma_{dd}.$$

*Therefore, it cannot be that* $\pi_t^n = \pi_t^d = 0$, *unless* $\Delta q_t^* = 0$, *which translates into:*

$$\frac{\Delta z_t^d}{\Delta z_t^n} = \frac{1-\alpha_d}{1-\alpha_n}.$$

■

It is useful to interpret this result in connection with some previous contributions in the literature on multi-sector models. Erceg and Levin (2006) suggest that in their two-sector economy the monetary authority is faced with a trade-off when trying to stabilize output in both sectors in the face of asymmetric technology shocks. In a similar vein, Huang and Liu (2005) emphasize that vertical trade linkages cause both aggregate output and the relative price to fluctuate in response to productivity shocks, unless these are identical, in which case only output would fluctuate. Therefore, the monetary authority is faced with a trade-off, as it can stabilize either the output gap or the relative price gap, but not both. In either case the impossibility to attain the Pareto optimum can be traced back to the presence of sector-specific (asymmetric) technology shocks that give rise to changes in the relative price. Since in the sticky-price equilibrium fluctuations in the relative price have an allocative role, the Pareto optimal allocation is not attainable. Were the technology shocks perfectly correlated (i.e., common) across sectors, no trade-off would arise.

When input materials are employed in both sectors, as in our setting, more restrictive conditions are required for the full stabilization of the model economy. In this case, not only sectoral shocks need to be perfectly correlated, but the production technologies need to be the same across sectors. Therefore, even in the presence of a common technology shock, different production technologies would prevent the attainment of the Pareto optimum.

[21] Allowing for imperfect labor mobility would only further constrain the ability of the monetary authority to neutralize technology shocks.

[22] See Appendix E for the derivation of the relative price in the efficient equilibrium with perfect labor mobility.

# 4 Optimal Monetary Policy

As shown in the previous section, the central bank cannot attain the Pareto optimal allocation even after different sources of distortion in the labor and the goods market are removed. Therefore, we turn our attention to policy strategies capable of attaining second best outcomes. We explore equilibrium dynamics under the assumption that the policy maker can credibly commit to a rule derived from the minimization of his objective function. The optimal policy consists of maximizing the conditional expectation of intertemporal household utility subject to private sector's behavioral equations and resource constraints, as discussed by Woodford (2003).[23]

To evaluate social welfare we take a second-order Taylor approximation to the representative household's lifetime utility.[24] Our procedure follows the standard analysis of Rotemberg and Woodford (1998), adapted to account for the presence of factor demand linkages. The resulting intertemporal social loss function reads as:

$$
\begin{aligned}
\mathcal{SW}_0 \approx\ & -\frac{U_H(H)\,H}{2}\Theta E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{\sigma-1}{\Theta}\left(\mu_n\widetilde{c}_t^n+\mu_d\widetilde{d}_t\right)^2\right. \\
& +\varsigma\left[\varpi\left(\pi_t^n\right)^2+(1-\varpi)\left(\pi_t^d\right)^2\right]+(1+\nu)\left[\omega\widetilde{c}_t^n+(1-\omega)\,\widetilde{c}_t^d\right]^2 \\
& \left.+S\left(\widetilde{d}_t-\widetilde{d}_{t-1}\right)^2\right\}+\text{t.i.p.}+O\left(\|\xi\|^3\right), \qquad (20)
\end{aligned}
$$

where:

$$
\begin{aligned}
S &= \mu_d\Theta^{-1}\Xi+(1-\delta)(1-\omega)\,\delta^{-2}, \\
\Theta &= \frac{\mu_n\left[1-\beta(1-\delta)\right]+\mu_d\delta}{1-\beta(1-\delta)}, \\
\varpi &= \phi\varepsilon^n\left(\kappa_n\varsigma\right)^{-1}, \\
\omega &= \frac{\mu_n\left[1-\beta(1-\delta)\right]}{\mu_n\left[1-\beta(1-\delta)\right]+\mu_d\delta}, \\
\varsigma &= \phi\varepsilon^n\left(\kappa_n\right)^{-1}+(1-\phi)\,\varepsilon^d\left(\kappa_d\right)^{-1}, \\
\kappa_i &= \frac{(1-\beta\theta_i)(1-\theta_i)}{\theta_i},\ i=\{n,d\}
\end{aligned}
$$

and t.i.p. collects the terms independent of policy stabilization, whereas $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher.

According to (20) the welfare criterion assumed by the central bank balances, along with sectoral inflation variability, fluctuations in aggregate consumption (or, equivalently, value added). This is a distinctive feature of the model under scrutiny, as the presence

[23] We pursue a "timeless perspective" approach, as in Woodford (1999). This involves ignoring the conditions that prevail at the regime's inception, thus imagining that the commitment to apply the rules deriving from the optimization problem had been made in the distant past. In this case, there is no dynamic inconsistency in terms of the central bank's own decision-making process. The system is solved for the evolution of the endogenous variables by relying on the common practice discussed, e.g., by Sims (2002).

[24] We assume that the shocks that hit the economy are not big enough to lead to paths of the endogenous variables distant from their steady state levels. This means that shocks do not drive the economy too far from its approximation point and, therefore, a linear quadratic approximation to the policy problem leads to reasonably accurate solutions. Appendix F reports the derivation of the quadratic welfare function.

of input materials implies a non-trivial distinction between output and consumption. Therefore, it is no longer irrelevant whether the central bank targets output or consumption gap variability. In the remainder of the paper we show that the distinction between output and consumption has important implications for the transmission of exogenous shocks under the optimal policy and the selection of an alternative policy regime that can generate a welfare loss close to that attained under the optimal policy.

Approximating the terms of social welfare associated with the durables sector also delivers a term reflecting a preference to smooth the accumulation of the stock of durable goods.[25] Assuming durables accumulation smoothing as a stabilization objective helps at counteracting the amplification effect of changes in the stock demand of durables on the flow demand of newly produced durable goods. The remaining weights of the time-varying terms in (20) can be interpreted as follows: (i) $\varsigma$ indexes the total degree of nominal stickiness in the economy and is inversely related to both $\kappa_d$ and $\kappa_n$; (ii) $\varpi$ accounts for the relative degree of price stickiness in the non-durable goods sector; (iii) $\omega$ is the relative weight of non-durable consumption over total consumption when durable goods are reported as a flow. This is an inverse function of $\Theta$. In turn, $\Theta$ depends on the degree of durability of goods produced in sector $d$. For $\delta = 0$ it reduces to $\mu_n$, whereas for $\delta = 1$ it reduces to one. Therefore, as the degree of durability increases, the weight attached to the non-durable consumption gap increases with respect to that attached to the durable term. Notice also that the relative importance of sector-specific inflation variability depends on the steady state ratio of labor supplied to the non-durable goods sector to the total labor force ($\phi$).[26] When input materials are not employed in the production process (i.e., $\alpha_n = \alpha_d = 0$) the loss function reduces to that obtained in traditional two-sector models where consumption and gross output are equalized (e.g., Erceg and Levin, 2006). Furthermore, for $\delta = 0$ we end up in the case considered by Woodford (2003, pp. 435-443).

How do factor demand linkages influence social welfare? The left-hand panel of Figure 2 reports the loss defined over the subspace of the production parameters $\alpha_n$ and $\alpha_d$. The right-hand panel of the figure reports analogous evidence under the assumption that technology shocks are the only source of exogenous perturbation. The general pattern suggests that welfare loss increases monotonically in the share of intermediate goods used to produce non-durable goods, whereas the share of input materials in the durable goods sector exerts a negligible impact. This differential impact can be ascribed to the non-durable goods sector being the largest sector and a net supplier of input materials in the model economy.

We are not only concerned, however, with the direct welfare implications of factor demand linkages, but also with central banks' potential misperception about their role in the production process. Neglecting cross-industry flows of input materials is likely to generate excess loss with respect to the welfare criterion consistent with correctly specified model economy. To address this issue we implement the optimal policy under the assumption that $\alpha_n = \alpha_d = 0$ . Table 1 reports the (percentage) excess loss under misperception with respect to the loss under the correctly specified production structure of the economy. As the actual intensity of use of input materials increases, excess loss can be substantial. Notice also that the marginal impact of misperceiving $\alpha_n$ is greater

[25] Details on the linear approximation of this term are available in the technical appendix. However, as discussed by Erceg and Levin (2006) this term makes a relatively minor contribution to the overall loss.

[26] When $\alpha_n = \alpha_d$ it follows that $\phi = \frac{L^n}{L} = \frac{Y^n}{Y^n+Y^d}$.

than that associated with $\alpha_d$. Once again, considering that the non-durables sector is a net supplier of input materials and the largest sector in our calibrated model economy is the key for interpretation of this result.

## 4.1 Impulse-response Analysis under Optimal Monetary Policy

Figure 3 reports equilibrium dynamics of the model economy following a one-standard-deviation technology shock in the non-durable goods sector.[27] The graphs in the left-hand panel refer to the model with factor demand linkages, whereas on the right-hand side we consider the model with no input materials. Inflation and interest rates are annualized. Symmetric nominal rigidity is assumed, with $\theta_n = \theta_d = 0.75$.[28]

A technology shock in the non-durable goods sector causes production of these goods to become relatively cheaper, thus increasing their production and consumption. However, their price is prevented from reaching the level consistent with flexible prices. This determines a negative non-durables consumption gap. As to the response of the central bank, the real interest rate (measured in units of non-durable goods) initially rises in the model with no input materials, thus preventing output (and consumption) in the non-durables sector from rising as much as it would do under flexible prices. Concurrently, the real rate of interest does not rise enough to prevent the output gap in the durables sector from rising too much. As discussed by Erceg and Levin (2006), keeping output at potential in the non-durable goods sector requires a "sharp and persistent fall" in the real interest rate. By contrast, a sharp rise in the policy instrument is required to keep output at potential in the durable goods sector. This is exactly what happens in the variant economy with no input materials. Conversely, in the model with factor demand linkages the real interest rate initially decreases, gradually converging to its equilibrium level thereafter. Keeping output at potential in the non-durable goods sector prevails over the alternative objective. This result is intimately related to the existence of factor demand linkages, which amplify the response of non-durables consumption under flexible prices, thus inducing a stronger drop in the consumption gap of the same sector. Moreover, cross-industry flows of input materials induce durable consumption under flexible prices to increase, thus helping to reduce the durables consumption gap. This endogenous mechanism is not at work in the model without factor demand linkages, in which case durables consumption under flexible prices is not affected by the shock as a result of setting $\sigma = 1$, which implies separability of households' preferences in durable and non-durable consumption.

It is worth recalling that, in the presence of input-output interactions between sectors, the relative price does not only exert a direct effect on the marginal rate of substitution between durable and non-durable consumption goods. As shown by equations (10) and (11), $Q_t$ also exerts a positive (negative) effect on the real marginal cost in the durable (non-durable) goods sector. A technology shock in the non-durables sector determines a positive relative price gap, which implies a substitution away from non-durable to durable

[27] The responses to a technology shock in the durable goods sector mirror those induced by a productivity shock in the non-durable goods sector in the opposite direction. For this reason, and for brevity of exposition, we skip their description. However, these results, as well as those in the case of a cost-push shock in the durables sector, are available upon request from the authors.

[28] As in Strum (2009) we opt for this choice to prevent the central bank from focusing exclusively on the stickier sector in the formulation of its optimal policy, as predicted by Aoki (2001). In the next subsections we draw implications from the model under asymmetric degrees of nominal rigidity across sectors.

consumption goods. Concurrently, the intermediate input channel is responsible for inducing higher inflationary pressures in the non-durables sector and a lower deflationary effect in the durables sector, compared to what is otherwise observed in the right-hand panel of Figure 3. The contraction in non-durables gross output is partially offset by the increase in the demand of non-durable intermediate goods from firms in the durables sector, which eventually results in lower deflationary pressures on the price of non-durables. Similarly, stronger inflationary pressures in the durable sector are induced by a production gap which is higher than that obtained by setting $\alpha_n = \alpha_d = 0$. Moreover, the positive relative price gap reinforces this effect on durables inflation through its influence on the real marginal cost. These effects, combined with the expansionary policy pursued by the central bank, determine rising inflationary pressures at the aggregate level.

Figure 4 reports equilibrium dynamics following a cost-push shock in the non-durables sector. A distinctive feature of the model with cross-industry flows of input materials is that the effect of the positive relative price gap on the marginal cost of firms producing durables partially counteracts the deflationary effect that operates through the conventional demand channel. Concurrently, the overall contractionary effect in consumption and production is magnified by the presence of factor demand linkages. This leads the central bank to pursue a weakly contractionary policy, initially accompanied by a negative real rate of interest. This policy reaction is also justified by the fact that changes in the relative price are channeled through the sectoral marginal costs and act as an endogenous attenuator of deflationary pressures in the sector which is not hit by the shock.

It is worth drawing attention to a subtle difference in the transmission of technology and cost-push shocks within this class of models. Sectoral technology shocks cause the consumption gaps in each sector to co-move negatively. The drop in the consumption gap of the sector that experiences the positive technology shock is compensated by a rise in the demand gap of intermediate goods from the other sector. Thus, each sector experiences opposite demand effects on the markets for the consumption and intermediate goods. By contrast, a sectoral cost-push shock determines a contraction of final goods consumption in both sectors. In turn, the contraction in the demand of both consumption goods causes a drop in the consumption of intermediate goods by both sectors, thus resulting in an even grater slump in the gross output.[29] The stark difference in the response of the output and consumption gaps to sectoral cost-push shocks has non-negligible implications for the implementation of the optimal policy and the choice of alternative policy regimes in the presence of a trade-off between inflation and output/consumption stabilization, as we show in Section 4.2.

## 4.2 Optimal Monetary Policy versus Alternative Policy Regimes

We now assess the loss of welfare under the optimal policy and various alternative policies. Alternative policy regimes admit simple loss functions, which are selected because of their suitability to be communicated to and understood by the public. We use the second-order welfare approximation (20) as a model-consistent metric. In each case we compute the expected welfare loss as the percentage of steady state aggregate consumption (and

[29] Importantly, imperfect labor mobility exacerbates this effect, increasing the wedge between consumption and production. When aggregate demand increases, as labor cannot flow across sectors without frictions, firms need to increase intermediate inputs by more than they would under the assumption of perfect labor mobility to meet the increased demand. Consequently, fluctuations in production and consumption are wider in the presence of imperfect labor mobility.

multiply the resulting term by 100).

We consider both strict and flexible inflation targeting regimes,[30] as well as consumption and output gap targeting. Inflation targeting regimes may target either core or aggregate inflation. In the first case the weights attached to the sectoral rates of inflation depend on the relative degree of price rigidity, as well as on the relative size of each sector and the degree of substitutability among differentiated goods. In the second case the weights attached to sectoral inflations only depend on the relative size of each sector. Flexible inflation targeting regimes balance fluctuations in core or aggregate inflation together with a term that penalizes fluctuations in aggregate consumption or gross output.

Both strict or flexible inflation targeting regimes aim at stabilizing the volatility of aggregate (or core) inflation and not the volatility of sectoral inflations separately. From a strategic viewpoint we are willing to understand whether the central bank can approximate the optimal policy outcome without taking the sectoral rates of inflation as separate objectives. In principle, this should enable the monetary authority to provide the public with a more intelligible target. Svensson (1997) stresses the importance of assuming intermediate targets which are highly correlated with the goal, easy to control, and transparent, so as to enhance communication to the public. In this sense, measures of overall inflation are more suitable than sectoral rates of inflation.

Table 2 reports the welfare loss under the optimal rule and various alternative policy regimes. The overall loss is disaggregated into the variability of each of the terms weighted in (20). To compare equilibrium paths under alternative regimes, we evaluate the associated loss by taking our second-order approximation to households' utility function as a benchmark. In the first instance both technology and cost-push shocks are assumed to buffet the model economy. Moreover, we assume that sectors are symmetric in the degree of nominal rigidity ($\theta_n = \theta_d = 0.75$): in this case $\pi_t^{core} = \pi_t^{agg}$. Later in this section we will relax this assumption. As expected on a priori grounds, whenever cost-push shocks are accounted for, a flexible inflation targeting regime performs nearly as well as the optimal policy.[31] Most importantly, the central bank attains a welfare loss closer to that under the optimal policy when fluctuations in aggregate (or core) inflation are balanced with those in the real value added (i.e., consumption), compared to the loss induced by controlling fluctuations in the gross output (i.e., production). Recall also from Section 4.1 that sectoral cost-push shocks typically induce higher variability in production than consumption, the reason being that a contraction (expansion) in the consumption of both sectors determines a contraction (expansion) in the demand of input materials and thus a higher drop (rise) in sectoral gross outputs. Therefore, assuming a welfare criterion that balances fluctuations in the rate of inflation with gross output variability misrepresents the actual trade-off faced by the central bank. Indeed, according to (20) inflation variability should be balanced with fluctuations in consumption rather than gross output.

Table 3 reports the relative performance of alternative policy regimes under different sources of exogenous perturbation. We consider both our benchmark model economy ($\alpha_n = \alpha_d = 0.6$), and the model with no input materials ($\alpha_n = \alpha_d = 0$). As observed at different stages of the analysis, deadweight loss is generally higher in the first case.

[30] The analytic specification of each regime is reported in Appendix G.

[31] Compared to the optimal policy, under this regime the central bank is more effective in stabilizing the sectoral rates of inflation, thus compensating, at least partially, the higher loss induced by the volatility in aggregate consumption and durables accumulation.

Once again, flexible inflation targeting outperforms other regimes. This is also the case when shocks to either sector are considered separately (see Table 3, columns 3 and 4). As noted by Woodford (2003, pp. 435-443), strict inflation targeting displays a "competitive" performance only when technology shocks are the unique source of perturbation to the system, whereas accounting for cost-push shocks entails a rather poor performance. However, in this case the loss attained under strict inflation targeting is much larger in the model with factor demand linkages, compared to that registered in the alternative scenario without input materials (see also Table 2, where both types of sectoral shocks are considered). In addition, it is worth pointing out that output gap targeting outperforms consumption gap targeting under either source of exogenous perturbation. In this case, reducing consumption gap volatility allows the central bank to control only part of the volatility in the marginal cost, whereas targeting the production gap would also account for the presence of factor demand linkages. In turn, sectoral inflation volatility also benefits from this effect (see Table 2, columns 3 and 4).

Table 4 reports the loss of welfare under asymmetric price stickiness, in the form of durables prices being more flexible than non-durables prices ($\theta_n = 0.75$, $\theta_d = 0.25$).[32] In this case core inflation differs from aggregate inflation, as discussed earlier. Considering core inflation targeting as an alternative to aggregate inflation targeting is somewhat related to a long-standing debate on the information (in terms of relative sectoral price stickiness) that the central bank can access when formulating its policy. Woodford (2003) shows that, in a two-sector model with no input materials, optimal commitment policy is nearly replicated by an inflation targeting regime, whereby the weights attached to sectoral inflations depend on the relative degree of nominal stickiness.[33] This result is robust across the two versions of the model economy and, as expected, it can only be replicated if we rule out sectoral cost-push shocks. However, when assessing flexible targeting regimes in the model with asymmetric price stickiness the dichotomy between aggregate and core inflation loses much of the usual appeal in terms of comparing welfare losses. Once again, what seems relevant and inherently connected with the presence of input materials is the distinction between output and consumption. In fact, a flexible inflation targeting regime balancing consumption and (either core or aggregate) inflation variability delivers a loss of welfare substantially lower than that attained under a loss function balancing output and (either core or aggregate) inflation variability.

## 4.3 Sectoral Asymmetries

We now examine the implications of allowing for asymmetric degrees of competitiveness and price stickiness between sectors for the optimal weighting of sectoral inflations and the resulting welfare properties of the model economy. Our exercise is performed by varying the sectoral Calvo parameters and the elasticities of substitution between goods produced within the same sector, under the assumption that their aggregate counterparts

[32] Bils and Klenow (2004) report a higher frequency of price adjustment for consumer durables than services. Erceg and Levin (2006) assume that non-durables and durables are equally sticky. By contrast, Barsky, House, and Kimball (2007) and Carlstrom and Fuerst (2006) develop models where durables prices are flexible and non-durables prices are sticky.

[33] Aoki (2001) shows that the welfare-theoretic loss function consistent with a multi-sector economy with heterogeneous degrees of price stickiness assigns higher weight on the inflation variability of sectors characterized by higher nominal stickiness. This provides a theoretical basis for seeking to stabilize an appropriately defined measure of "core" inflation rather than an equally weighted price index.

are fixed at the value consistent with the symmetric case.[34]

As the monetary policy authority operates in a timeless perspective, the resulting rule is optimal regardless of the nature of the (additive) exogenous disturbances that buffet the model economy, as discussed by Giannoni and Woodford (2003). Thus, it is possible to analyze the effect of sector-specific technology and cost-push shocks separately. Figure 5 summarizes the key results: the y-axis (x-axis) in each graph measures the stickiness gap (competitive gap), as in Lombardo (2006).[35]

On average, input-output interactions increase deadweight loss for different degrees of asymmetry in either dimension. This result holds both when only technology shocks are accounted for, as well as in the model with only cost-push shocks: in this case the loss is 15% to 55% higher than that attainable without input materials. Otherwise, when only technology shocks are considered, the loss of social welfare is 25% to 50% higher than that without input materials.

**Model with Technology Shocks -** We first consider technology shocks as the sole source of exogenous perturbation. Before exploring asymmetry under our baseline calibration, it is instructive to provide a gradual overview of the effects brought by each distinctive feature of the model.[36] We start by assuming a two-sector economy with sectors of equal size, no durability in either of the consumption goods, and no input materials (i.e., $\alpha_n = \alpha_d = 0$, $\delta = 1$ and $\mu_n = \mu_d = \omega = \phi$). The resulting loss function is concave over both dimensions of asymmetry. A hump-shaped pattern emerges over the subspace considered. As asymmetry increases in either dimension, overall unconditional variance decreases. This result is in line with Woodford (2003), Benigno (2004) and Lombardo (2006), and is closely related to the existence of a trade-off in the stabilization of sectoral rates of inflation.[37] When sectors have the same size and production characteristics, asymmetry in the degree of nominal stickiness (and/or in the competition gap) mitigates the trade-off, as the monetary authority can predominantly focus on controlling inflation variability in the stickier sector. The resulting deadweight loss decreases due to the combined effect of: (i) higher policy effectiveness against fluctuations in the sector with higher price rigidity and/or higher elasticity of substitution in demand, and (ii) lower price rigidity and/or lower elasticity of substitution in the demand of goods produced by the other sector. These effects imply lower inflation persistence and lower cross-sectional dispersion in prices, respectively. When input materials are introduced and sectors are of equal size ($\alpha_n = \alpha_d = 0.6$, $\delta = 1$ and $\mu_n = \mu_d$) the loss is still symmetric over the "nominal rigidity gap", but decreases in the gap between $\varepsilon^n$ and $\varepsilon^d$. Due to the presence of factor demand linkages, inefficiencies from nominal rigidities in either sector are partially passed onto the other sector. This effect helps at counteracting the influence

[34] We set $\overline{\theta} = 0.75$ and $\overline{\varepsilon} = 11$, as in the baseline calibration with symmetric sectoral nominal stickiness and elasticities of substitution in demand. Thus, we map the loss under an optimal monetary policy for different values of the "nominal rigidity gap" $(\theta_n - \theta_d)$ and the "competition gap" $\left(\varepsilon^n - \varepsilon^d\right)$, under the assumption that $\phi\theta_n + (1-\phi)\,\theta_d = \overline{\theta}$ and $\phi\varepsilon^n + (1-\phi)\,\varepsilon^d = \overline{\varepsilon}$.

[35] Benigno (2004) explores the impact of asymmetry in the degree of nominal stickiness on the loss of social welfare in the context of a monetary union. Lombardo (2006) shows how asymmetries in the degree of competition can exacerbate or mitigate the effects of asymmetric price rigidity.

[36] Due to space limitations, we only describe the deadweight loss under different parameterizations, rather than reporting the corresponding graphs. These are available, upon request, from the authors.

[37] Lombardo (2006) shows that when prices are set in a staggered fashion, the amount of 'output dispersion' generated by a given deviation of prices from their average depends positively on the elasticity of substitution between goods (i.e., the degree of competition in our model). Therefore, the country with the largest degree of competition is the one that generates the greatest cost of inflation.

of the nominal rigidity gap on social welfare. Moreover, if the competition gap is positive ($\varepsilon^n > \varepsilon^d$), then the weight attached to non-durable goods inflation rises linearly in $\varepsilon^n$. In this sense, the monetary authority has to pay more attention to inflation fluctuations in this sector (with respect to the symmetric case). In turn, provided that the non-durable goods sector is a net supplier of intermediate goods in the model economy, stabilizing $\pi_t^n$ helps to reduce fluctuations in the marginal cost gap of the durable goods sector, which is also affected by the relative price gap. When durability is accounted for ($\alpha_n = \alpha_d = 0.6$, $\delta = 0.025$ and $\mu_n = \mu_d$), an additional objective emerges in the central bank's welfare criterion, which reflects a preference for smoothing the accumulation of the stock of durables. This term amplifies the impact of nominal rigidities in the durable goods sector, thus causing the total deadweight loss to increase for $\theta_d > \theta_n$ and $\varepsilon^d > \varepsilon^n$.

Under our baseline calibration ($\alpha_n = \alpha_d = 0.6$, $\delta = 0.025$ and $\mu_n = 0.682$) asymmetry in the degree of nominal stickiness exerts a stronger marginal impact on welfare. For a given level of asymmetry in sectoral competition, optimal monetary policy places greater weight on the "stickier" sector. The minimum loss is attained when the economy is characterized by both the highest asymmetry in the degree of competitiveness and nominal stickiness, and specifically when the non-durable goods sector is more competitive and stickier. Note that, given the relative size of the two sectors, to keep aggregate stickiness at a fixed level, a marginal increase in $\theta_n$ has to be coupled with a more than proportional decrease in $\theta_d$. Analogous observations apply to the competition gap. Higher $\theta_n$ and $\varepsilon^n$ mean that the central bank penalizes inflationary pressures relatively more in the broader sector of the model economy.

**Model with Cost-Push Shocks -** In the remainder we rely on the baseline calibration reported in Section 2, while still considering the effects of varying the competition and the rigidity gap. The introduction of sectoral cost-push shocks generates a trade-off between output and inflation stabilization. Total welfare decreases in the degree of asymmetry in nominal stickiness and competition between sectors. This can be explained intuitively. As implied by (20), the Calvo parameters indexing the degree of nominal rigidity in either sector enter non-linearly in the term of loss associated with fluctuations in core inflation. By contrast, the elasticity of substitution between goods produced in either sector only affects the relative weight of core inflation, $\varsigma$. The trade-off between inflation and consumption stabilization is such that inflation volatility induced by sector-specific cost-push shocks is not completely stabilized by the monetary authority. Consequently, the contour map tracks the pattern of $\varsigma$ in the subspace defined over the asymmetry gaps. In particular, $\varsigma$ evolves convexically with respect to the nominal rigidity gap and increases (decreases) for $\theta_n$ greater (lower) than $\theta_d$.

**Model with Technology and Cost-Push Shocks -** As expected on a priori grounds, the analysis of the loss function in the presence of both sources of exogenous perturbation suggests that the monetary authority faces an easier task in the stabilization of the variability induced by the technology shocks, compared to its performance in the presence of cost-push shocks. As cost-push shocks produce a non-trivial trade-off between inflation and output stabilization, they have a predominant impact on welfare. This is clearly displayed in the last panel of Figure 5, where the aggregate loss is convex with respect to the nominal rigidity gap, while the effect of the competition gap is negative (positive) when the nominal rigidity gap is positive (negative).

# 5 Conclusions

We have integrated a horizontal input-output production structure into a dynamic general equilibrium model with two sectors that produce durable and non-durable goods. Part of the output produced in each sector is used as an intermediate input of production in both sectors, according to a realistic input-output structure of the economy. The resulting sectoral interactions have non-negligible implications for the formulation of policies aimed at reducing real and nominal fluctuations. A key role is played by the relative price of non-durable goods, which not only acts as an allocative mechanism on the demand side, through its influence on the user cost of durable goods, but also on the supply side, through its effect on the sectoral real marginal costs of production.

The presence of input materials implies a non-trivial difference between consumption (or, equivalently, value added) and gross output. Such a distinction proves to be of crucial importance at different stages of the analysis. In fact, the welfare criterion consistent with a second-order approximation to households' utility reveals that the policy maker is faced with the task of stabilizing fluctuations in sectoral inflation and aggregate value added, rather than gross output. Moreover, strategic complementarities induced by factor demand linkages amplify the loss of social welfare under the optimal policy and alter the transmission of shocks to the system, compared to what is commonly observed in otherwise standard two-sector models. For example, in the face of a sector-specific cost push shock the intermediate input channel acts as an endogenous stabilizer that attenuates the deflationary effect in the sector which is not hit by the shock.

These results show how accounting for a realistic feature of multi-sector economies, such as factor demand linkages, entails non-negligible differences with respect to policy prescriptions referring to frameworks that rule out cross-industry input-output interactions. The optimal policy can be closely approximated by a flexible inflation targeting regime. However, it is of crucial importance to target consumption gap variability rather than output gap variability. This strategy allows the central bank to avoid inducing additional loss emanating from inter-sectoral complementarities.

## References


Adda, J., and R. Cooper (2000): "Balladurette and Juppette: A Discrete Analysis of Scrapping Subsidies," *Journal of Political Economy*, 108(4), 778–806.

Aoki, K. (2001): "Optimal Monetary Policy Responses to Relative Price Changes," *Journal of Monetary Economics*, 48, 55–80.

Aoki, K., J. Proudman, and G. Vlieghe (2004): "House Prices, Consumption, and Monetary Policy: A Financial Accelerator Approach," *Journal of Financial Intermediation*, 13, 414–435.

Barsky, R. B., C. L. House, and M. S. Kimball (2007): "Sticky-Price Models and Durable Goods," *American Economic Review*, 97(3), 984–998.

Basu, S. (1995): "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," *American Economic Review*, 85(3), 512–31.

Benigno, P. (2004): "Optimal monetary policy in a currency area," *Journal of International Economics*, 63(2), 293–320.

Bernanke, B. S. (1985): "Adjustment costs, durables, and aggregate consumption," *Journal of Monetary Economics*, 15(1), 41–68.

Bils, M., and P. J. Klenow (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112(5), 947–985.

Bouakez, H., E. Cardia, and F. J. Ruge-Murcia (2008): "Durable Goods, Inter-Sectoral Linkages and Monetary Policy," Cahiers de recherche 0821, CIRPEE.

Carlstrom, C. T., and T. S. Fuerst (2006): "Co-movement in sticky price models with durable goods," Working Paper 0614, Federal Reserve Bank of Cleveland.

Carvalho, V. M. (2009): "Aggregate Fluctuations and the Network Structure of Intersectoral Trade," Mimeo, Universitat Pompeu Fabra.

Cooley, T. F., and E. C. Prescott (1995): *Economic Growth and Business Cycles*. Princeton University Press, in frontiers of business cycle research edn.

Davis, S. J., and J. Haltiwanger (2001): "Sectoral job creation and destruction responses to oil price changes," *Journal of Monetary Economics*, 48(3), 465–512.

DiCecio, R. (2009): "Sticky wages and sectoral labor comovement," *Journal of Economic Dynamics and Control*, 33(3), 538–553.

DiPace, F. (2008): "Revisiting the Comovement Puzzle: the Input-Output Structure as an Additional Solution," Birkbeck Working Papers in Economics and Finance 0807, Birkbeck, School of Economics, Mathematics & Statistics.

Erceg, C., and A. Levin (2006): "Optimal monetary policy with durable consumption goods," *Journal of Monetary Economics*, 53(7), 1341–1359.

Giannoni, M. P., and M. Woodford (2003): "Optimal Interest-Rate Rules: II. Applications," NBER Working Papers 9420, National Bureau of Economic Research, Inc.

Hornstein, A., and J. Praschnik (1997): "Intermediate inputs and sectoral comovement in the business cycle," *Journal of Monetary Economics*, 40(3), 573–595.

Horvath, M. (1998): "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks," *Review of Economic Dynamics*, 1(4), 781–808.

——— (2000): "Sectoral shocks and aggregate fluctuations," *Journal of Monetary Economics*, 45(1), 69–106.

Huang, K. X. D., and Z. Liu (2005): "Inflation targeting: What inflation rate to target?," *Journal of Monetary Economics*, 52(8), 1435–1462.

Jensen, H. (2002): "Targeting Nominal Income Growth or Inflation?," *American Economic Review*, 92(4), 928–956.

Kim, Y. S., and K. Kim (2006): "How Important is the Intermediate Input Channel in Explaining Sectoral Employment Comovement over the Business Cycle?," *Review of Economic Dynamics*, 9(4), 659–682.

King, R. G., and J. K. Thomas (2006): "Partial Adjustment Without Apology," *International Economic Review*, 47(3), 779–809.

Lombardo, G. (2006): "Inflation targeting rules and welfare in an asymmetric currency area," *Journal of International Economics*, 68(2), 424–442.

Long, J., and C. Plosser (1983): "Real business cycles," *Journal of Political Economy*, 91, 39–69.

Monacelli, T. (2009): "New Keynesian models, durable goods, and collateral constraints," *Journal of Monetary Economics*, 56(2), 242–254.

Nakamura, E., and J. Steinsson (2008): "Monetary Non-Neutrality in a Multi-Sector Menu Cost Model," NBER Working Papers 14001, National Bureau of Economic Research, Inc.

Ngai, L. R., and C. A. Pissarides (2007): "Structural Change in a Multisector Model of Growth," *American Economic Review*, 97(1), 429–443.

Rotemberg, J. J., and M. Woodford (1998): "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version," NBER Technical Working Papers 0233, National Bureau of Economic Research, Inc.

Sims, C. A. (2002): "Solving Linear Rational Expectations Models," *Computational Economics*, 20(1-2), 1–20.

Strum, B. E. (2009): "Monetary policy in a forward-looking input-output economy," *forthcoming, Journal of Money, Credit and Banking.*

SUDO, N. (2008): "Sectoral Co-Movement, Monetary-Policy Shock, and Input-Output Structure," IMES Discussion Paper Series 08-E-15, Institute for Monetary and Economic Studies, Bank of Japan.

SVENSSON, L. E. O. (1997): "Inflation forecast targeting: Implementing and monitoring inflation targets," *European Economic Review*, 41(6), 1111–1146.

WALSH, C. (2003): "Speed Limit Policies: The Output Gap and Optimal Monetary Policy," *American Economic Review*, 93(1), 265–278.

WOODFORD, M. (1999): "Commentary : how should monetary policy be conducted in an era of price stability?," *Proceedings*, pp. 277–316.

——— (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

# TABLES

TABLE 1: FACTOR DEMAND LINKAGES MISPERCEPTION

| TECH SHOCKS | | | | | COST PUSH SHOCKS | | | | | BOTH SHOCKS | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha_n/\alpha_d$ | 0 | 0.3 | 0.6 | 0.8 | $\alpha_n/\alpha_d$ | 0 | 0.3 | 0.6 | 0.8 | $\alpha_n/\alpha_d$ | 0 | 0.3 | 0.6 | 0.8 |
| 0 | 0 | 4.664 | 22.468 | 52.119 | 0 | 0 | 2.984 | 9.400 | 16.554 | 0 | 0 | 3.075 | 10.210 | 18.322 |
| 0.3 | 2.348 | 1.012 | 14.763 | 41.453 | 0.3 | 3.904 | 1.234 | 4.970 | 10.061 | 0.3 | 3.552 | 1.209 | 5.601 | 11.267 |
| 0.6 | 17.232 | 0.906 | 5.713 | 27.242 | 0.6 | 28.415 | 11.303 | 10.324 | 13.976 | 0.6 | 25.684 | 8.912 | 9.517 | 14.037 |
| 0.8 | 49.424 | 8.468 | 1.160 | 17.336 | 0.8 | 92.084 | 45.841 | 36.050 | 38.283 | 0.8 | 77.991 | 30.982 | 27.664 | 35.721 |

Note: We report the percentage excess loss under a misperception of the input-output structure with respect to the loss under the correctly specified production structure of the economy, for different shocks.

TABLE 2: WELFARE UNDER ALTERNATIVE POLICIES

| | Loss Components | | | | |
|---|---|---|---|---|---|
| | $\Delta\widetilde{d}_t$ | $\pi_t^n$ | $\pi_t^d$ | $x_t$ | Total |
| Optimal Policy | 0.0417 | 0.6477 | 0.2918 | 0.2274 | 1.2086 |
| Inflation Targeting | 0.5486 | 0.0785 | 0.3472 | 2.6662 | 3.6405 |
| Consumption Gap Targeting | 0.0234 | 1.6396 | 0.4026 | 0.0000 | 2.0656 |
| Production Gap Targeting | 0.0133 | 1.5380 | 0.3721 | 0.0067 | 1.9300 |
| Flexible Inflation Targeting (with Consumption) | 0.0476 | 0.6360 | 0.2900 | 0.2377 | 1.2113 |
| Flexible Inflation Targeting (with Output) | 0.0121 | 1.1501 | 0.3234 | 0.0243 | 1.5099 |

Note: $\Delta\widetilde{d}_t$ refers to the durable smoothing objective, and $x_t = \omega\widetilde{c}_t^n + (1-\omega)\,\widetilde{c}_t^d$. The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by $100$).

| TABLE 3: WELFARE UNDER ALTERNATIVE POLICIES AND DIFFERENT PRODUCTION STRUCTURES | | | | |
|---|---|---|---|---|
| MODEL WITH FACTOR DEMAND LINKAGES $(\alpha_n = \alpha_d = 0.6)$ | | | | |
| | Tech. Shocks | Cost Push Shocks | Dur. Sec. Shocks | Non-Dur. Sec. Shocks |
| Optimal Policy | 0.1741 | 1.0645 | 0.2686 | 0.9658 |
| Inflation Targeting | 0.1768 | 3.5025 | 0.3891 | 3.3110 |
| Consumption Gap Targeting | 0.2016 | 1.9235 | 0.2881 | 1.8400 |
| Production Gap Targeting | 0.1758 | 1.7903 | 0.2957 | 1.6749 |
| Flex. Inflation Targeting (with Cons.) | 0.1753 | 1.0654 | 0.2702 | 0.9665 |
| Flex. Inflation Targeting (with Prod.) | 0.1744 | 1.3664 | 0.2823 | 1.2581 |
| MODEL WITHOUT FACTOR DEMAND LINKAGES $(\alpha_n = \alpha_d = 0)$ | | | | |
| | Tech. Shocks | Cost Push Shocks | Dur. Sec. Shocks | Non-Dur. Sec. Shocks |
| Optimal Policy | 0.1156 | 0.6897 | 0.2854 | 0.5152 |
| Inflation Targeting | 0.1166 | 0.8662 | 0.3028 | 0.6778 |
| Output Gap Targeting | 0.1217 | 1.6498 | 0.4196 | 1.3670 |
| Flex. Inflation Targeting | 0.1163 | 0.6907 | 0.2856 | 0.5168 |

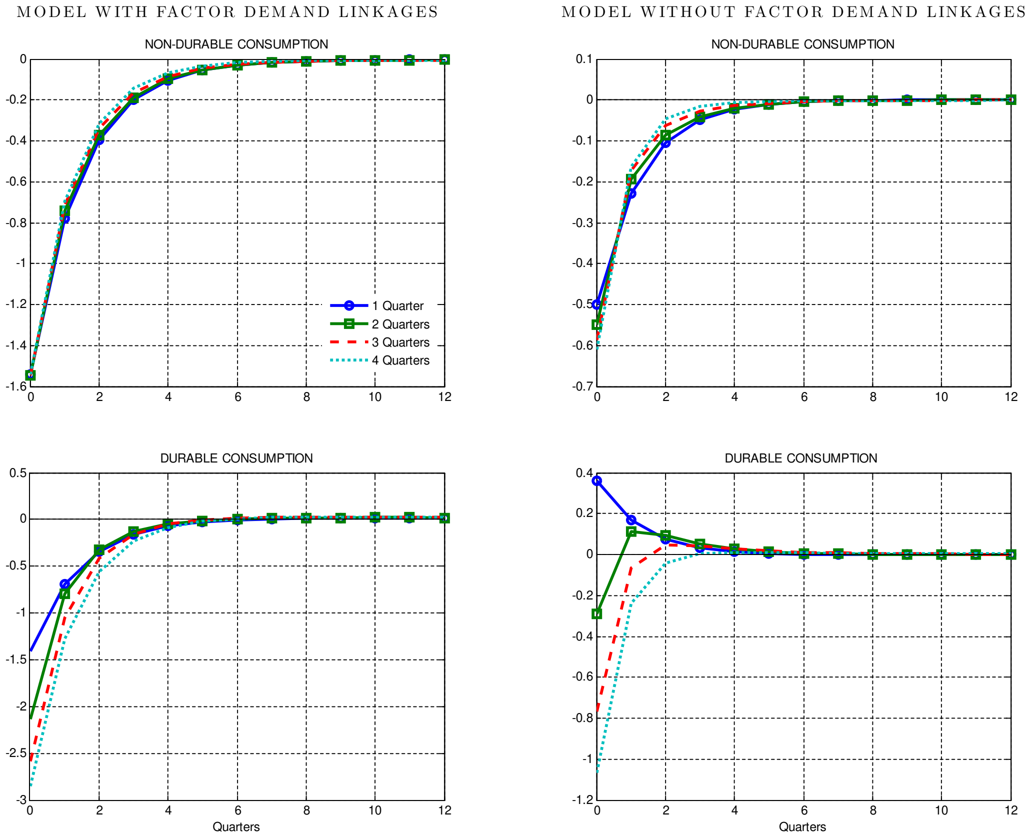
Note: The first two columns report the loss attributable to technology shocks and cost-push shocks generated in both sectors, respectively. The last two columns report the loss due to both shocks in either sector. The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by $100$).

TABLE 4: ASYMMETRIC STICKINESS

| | Tech. Shocks | Cost Push Shocks | Both Shocks |
|---|---|---|---|
| Optimal Policy | 0.0388 | 0.8648 | 0.9009 |
| Core Inflation Targeting | 0.0391 | 3.6864 | 3.7190 |
| Agg. Inflation Targeting | 0.2000 | 1.3171 | 1.4857 |
| Consumption Gap Targeting | 0.0408 | 1.6711 | 1.7106 |
| Production Gap Targeting | 0.0396 | 1.6575 | 1.6955 |
| Flex. Core Inflation Targeting (with Cons.) | 0.0402 | 0.8654 | 0.9036 |
| Flex. Agg. Inflation Targeting (with Cons.) | 0.0936 | 0.8757 | 0.9561 |
| Flex. Core Inflation Targeting (with Prod.) | 0.0396 | 1.1855 | 1.2235 |
| Flex. Agg. Inflation Targeting (with Prod.) | 0.0463 | 1.1517 | 1.1972 |

Note: We set the average duration of the non-durable goods' prices at 4 quarters, whereas we reduce the duration of durable goods prices to 1.3 quarters ($\theta_d = 0.25$). The welfare loss is computed as a percentage of steady state aggregate consumption (multiplied by $100$).

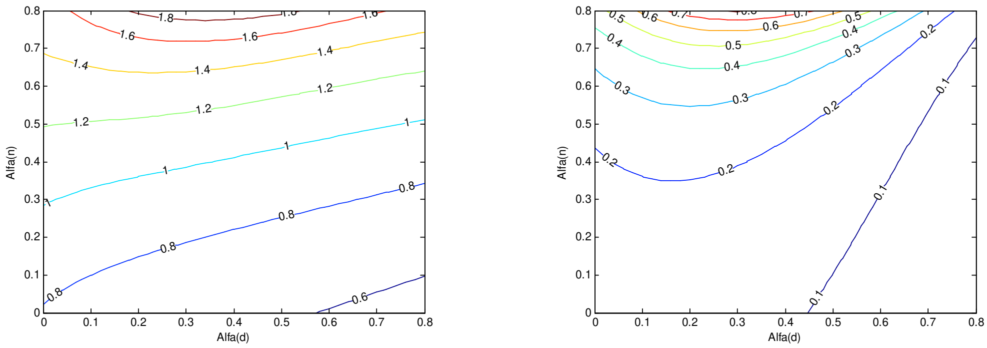
FIGURE 1: IMPULSE RESPONSES TO A MONETARY POLICY TIGHTENING




Note: We employ the following instrumental rule $\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1-\rho_r)\,\chi_\pi \pi_t + u_t^r$, where $u_t^r$ is an $iid\,(0,1)$ monetary policy innovation and the constant term involving the inflation target has been suppressed for simplicity. The interest rate smoothing parameter $\rho_r$ is set to $0.7$, while $\chi_\pi = 1.5$. We assume that the monetary authority responds to a convex combination of the sector-specific rates of inflation.

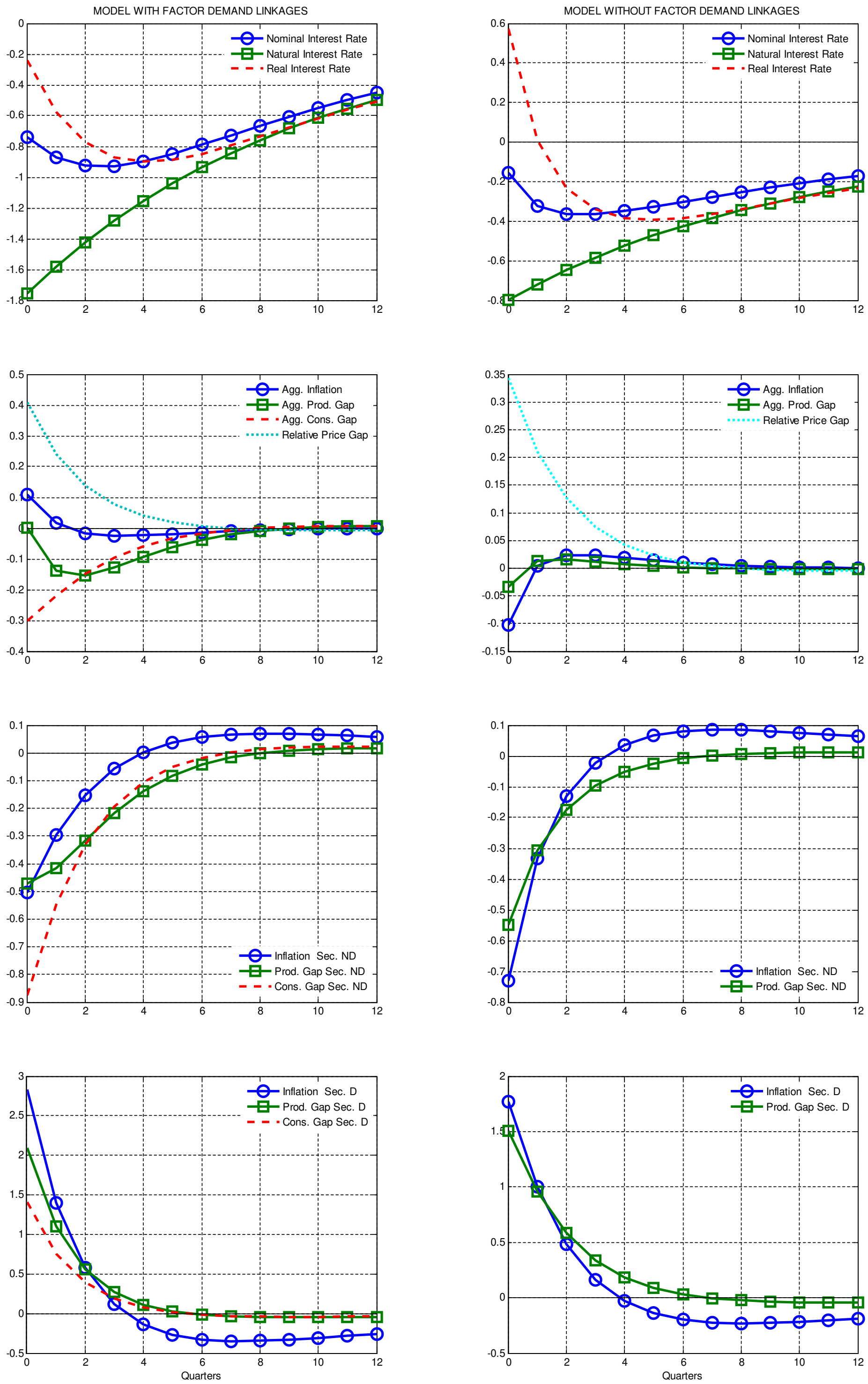
FIGURE 2: WELFARE UNDER OPTIMAL MONETARY POLICY FOR VARYING $\alpha_n$ AND $\alpha_d$

TECHNOLOGY AND COST-PUSH SHOCKS | ONLY TECHNOLOGY SHOCKS




Note: The left-hand panel of Figure 2 reports the welfare loss defined over the subspace of the production parameters in the two sectors when both technology and cost-push shocks buffet the model economy. The right-hand panel reports analogous evidence under the assumption that technology shocks are the only source of exogenous perturbation. The values of each contour line refer to the loss as a percentage of steady state aggregate consumption.

FIGURE 3: IMPULSE RESPONSES TO A TECHNOLOGY SHOCK IN THE NON-DURABLE GOODS SECTOR




Note: The relative price gap is re-scaled by a factor of four to account for the marked effect brought by factor demand linkages in the amplification of the sectoral shocks.

FIGURE 4: IMPULSE RESPONSES TO A COST-PUSH SHOCK IN THE NON-DURABLE GOODS SECTOR

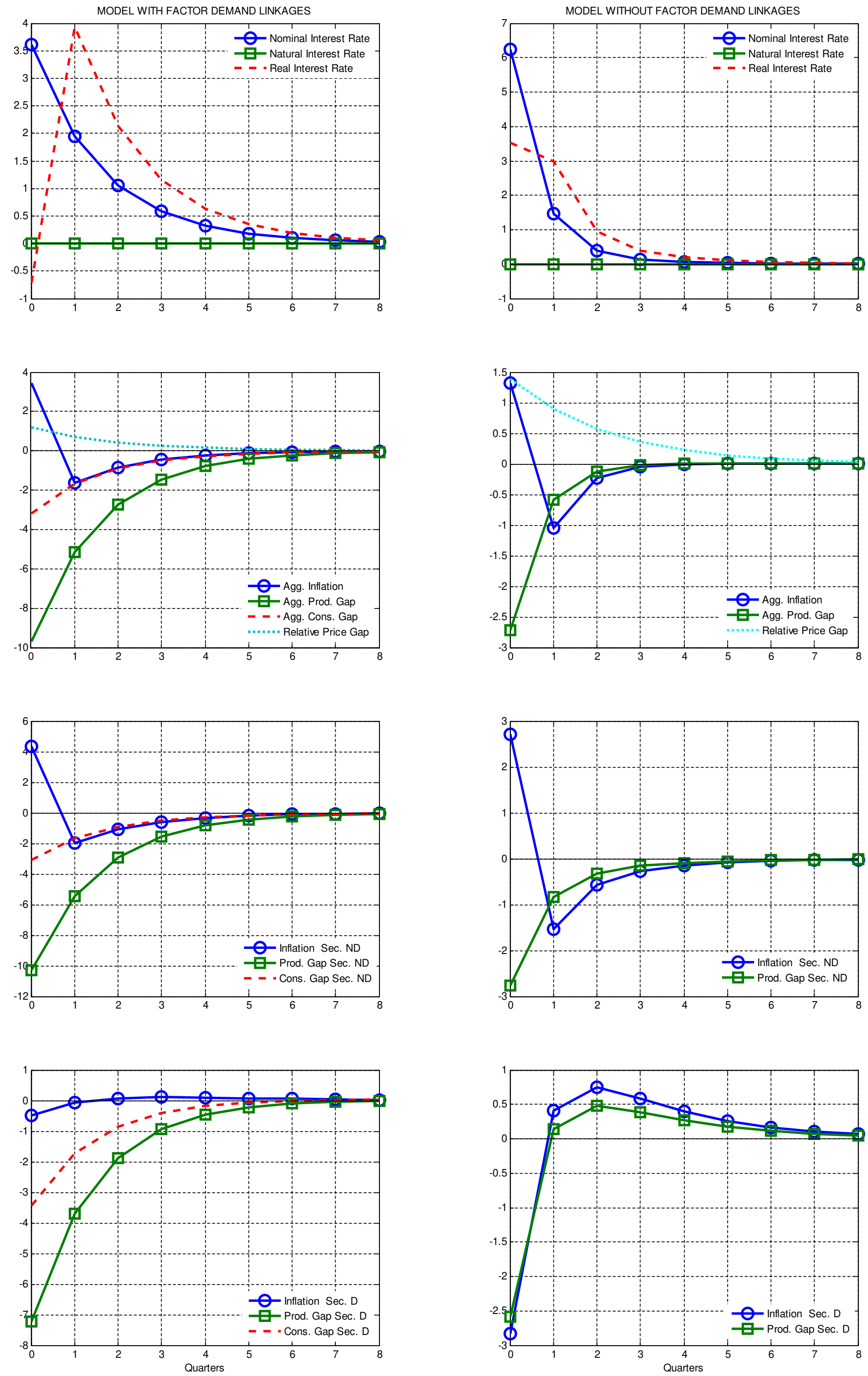
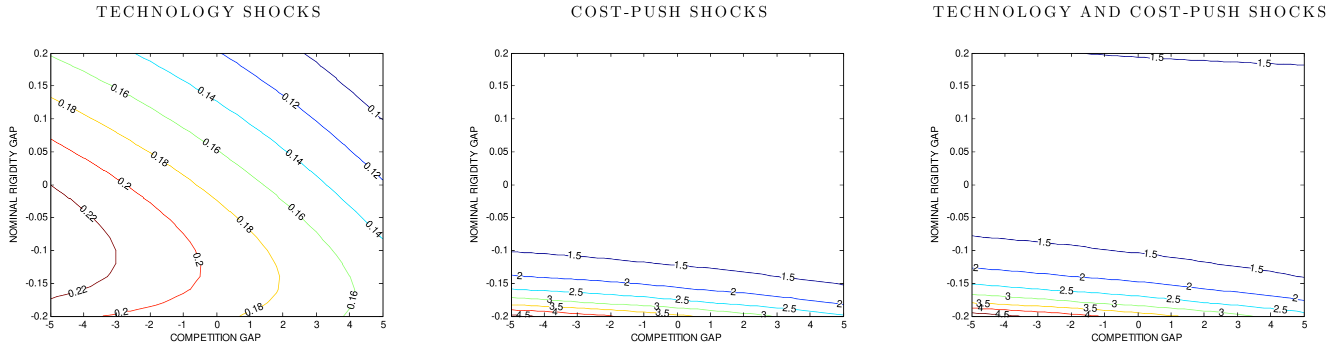


FIGURE 5: THE IMPACT OF ASYMMETRY ON WELFARE LOSS




Note: We set $\overline{\theta} = 0.75$ and $\overline{\varepsilon} = 11$, as in the baseline calibration with symmetric sectoral nominal stickiness and elasticities of substitution in demand. Thus, we map the loss under optimal monetary policy for different values of the "nominal rigidity gap" $(\theta_n - \theta_d)$ and the "competition gap" $\left(\varepsilon^n - \varepsilon^d\right)$, under the assumption that $\phi\theta_n + (1-\phi)\,\theta_d = \overline{\theta}$ and $\phi\varepsilon^n + (1-\phi)\,\varepsilon^d = \overline{\varepsilon}$. The values of each contour line refer to the loss as a percentage of steady state aggregate consumption.

# APPENDIX A: First Order Conditions from Households' Utility Maximization

Maximizing (12) subject to (13), (14), (15), and (16) leads to a set of first-order conditions that can be re-arranged to obtain:

$$\mu_n H_t^{1-\sigma}\left(C_t^n\right)^{-1} = \beta R_t E_t\left[\frac{\mu_n H_{t+1}^{1-\sigma}\left(C_{t+1}^n\right)^{-1}}{\Pi_{t+1}^n}\right], \tag{21a}$$

$$\frac{\mu_n H_t^{1-\sigma} P_t^d}{C_t^n P_t^n} = E_t\left\{\beta\left(1-\delta\right)\mu_n \frac{H_{t+1}^{1-\sigma} P_{t+1}^d}{C_{t+1}^n P_{t+1}^n} + \right. \tag{21b}$$

$$\left. + \frac{\mu_d H_t^{1-\sigma}}{\mathfrak{D}_t\left[1-\frac{\Xi}{D}\left(D_t - D_{t-1}\right)\right]^{-1}} + \beta\frac{\Xi}{D}\frac{\mu_d H_{t+1}^{1-\sigma}}{\mathfrak{D}_{t+1}\left(D_{t+1}-D_t\right)^{-1}}\right\},$$

$$W_t^n \frac{\mu_n H_t^{1-\sigma}\left(C_t^n\right)^{-1}}{P_t^n} = \varrho\phi^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}}\left(L_t^n\right)^{\frac{1}{\lambda}}, \tag{21c}$$

$$W_t^d \frac{\mu_n H_t^{1-\sigma}\left(C_t^n\right)^{-1}}{P_t^n} = \varrho\left(1-\phi\right)^{-\frac{1}{\lambda}} L_t^{v-\frac{1}{\lambda}}\left(L_t^d\right)^{\frac{1}{\lambda}}. \tag{21d}$$

# APPENDIX B: Some Useful Steady State Relationships

As in the competitive equilibrium real wage in each sector equals the marginal product of labor. Thus, we can derive the following relationship between the production in non-durable and durable goods in the steady state:

$$\frac{Y^n}{Y^d} = \frac{(1-\alpha_d)\phi}{\left(1-\alpha_n\right)\left(1-\phi\right)}Q^{-1}.$$

Furthermore, the following relationship between durable and non-durable consumption can be derived from the Euler conditions:

$$\frac{C^n}{C^d} = \left(1-\beta\left(1-\delta\right)\right)\frac{\mu_n}{\mu_d}\frac{1}{\delta}Q^{-1}.$$

Moreover, the following shares of consumption and intermediate goods over total production are determined for the non-durable goods sector:

$$\frac{C^n}{Y^n} = \frac{\left(1-\alpha_n\gamma_{nn}\right)\phi\left(1-\alpha_d\right) - \left(1-\alpha_n\right)\left(1-\phi\right)\alpha_d\gamma_{nd}}{\phi\left(1-\alpha_d\right)},$$

$$\frac{M^{nn}}{Y^n} = \alpha_n\gamma_{nn},$$

$$\frac{M^{nd}}{Y^n} = \frac{\left(1-\alpha_n\right)}{\phi}\frac{1-\phi}{1-\alpha_d}\alpha_d\gamma_{nd}.$$

Analogously, for the durable goods sector:

$$
\begin{aligned}
\frac{C^d}{Y^d} &= \frac{(1-\alpha_d\gamma_{dd})(1-\phi)(1-\alpha_n)-(1-\alpha_d)\,\phi\alpha_n\gamma_{dn}}{(1-\phi)(1-\alpha_n)}, \\
\frac{M^{dn}}{Y^d} &= \frac{1-\alpha_d}{1-\phi}\frac{\phi}{1-\alpha_n}\alpha_n\gamma_{dn}, \\
\frac{M^{dd}}{Y^d} &= \alpha_d\gamma_{dd}.
\end{aligned}
$$

These conditions prove to be crucial in the second-order approximation of consumers' utility to eliminate linear terms. Moreover, they allow us to derive the steady state ratio of labor supply in the non-durable goods sector over the total labor supply ($\phi$).

## The Relative Price in the Steady State

We consider the steady state condition for the marginal cost in the non-durable goods sector:

$$
\begin{aligned}
MC^n &= \Phi_n\left[(P^n)^{\gamma_{nn}}\left(P^d\right)^{\gamma_{dn}}\right]^{\alpha_n}(W^n)^{1-\alpha_n}, \\
\Phi_n &= \alpha_n^{\alpha_n}(1-\alpha_n)^{1-\alpha_n}.
\end{aligned}
$$

As in the steady state production subsidies neutralize distortions due to imperfect competition:

$$
\begin{aligned}
P^n &= MC^n \\
&= \Phi_n\left[(P^n)^{\gamma_{nn}}\left(P^d\right)^{\gamma_{dn}}\right]^{\alpha_n}(W^n)^{1-\alpha_n}.
\end{aligned}
$$

After some trivial manipulations it can be shown that:

$$
\Phi_n Q^{-\alpha_n\gamma_{dn}}(RW^n)^{1-\alpha_n} = 1.
$$

Analogously, for the durables goods sector:

$$
\Phi_d Q^{\alpha_d\gamma_{nd}}\left(RW^d\right)^{1-\alpha_d} = 1.
$$

Using the fact that in steady state $W^n = W^d = W$:

$$
\begin{aligned}
\frac{RW^n}{RW^d}Q &= 1, \\
\frac{\left(\Phi_n^{-1}Q^{\alpha_n\gamma_{dn}}\right)^{\frac{1}{1-\alpha_n}}}{\left(\Phi_d^{-1}Q^{-\alpha_d\gamma_{nd}}\right)^{\frac{1}{1-\alpha_d}}}Q &= 1.
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
Q &= \left(\Phi_n^{1-\alpha_d}\Phi_d^{-(1-\alpha_n)}\right)^{\frac{1}{\varphi}}, \\
\varphi &= (1-\alpha_n)(1-\alpha_d)+\alpha_n\gamma_{dn}(1-\alpha_d)+\alpha_d\gamma_{nd}(1-\alpha_n).
\end{aligned}
$$

Notice that, when $\alpha_n = \alpha_d = 1$:

$$
Q = \Phi_n\Phi_d^{-1}
$$

as in the case considered by Huang and Liu (2005) and Strum (2008).

# APPENDIX C: Equilibrium Dynamics in the Efficient Equilibrium

In this appendix we outline the solution method of the linear model under the efficient equilibrium. This is obtained when both prices are flexible and elasticities of substitution are constant. Let us start from the pricing rule under flexible prices:

$$\begin{aligned} P_t^{n^*} &= \frac{\Theta^n}{1+\tau_n} MC_t^{n^*} \\ &= \frac{\Theta^n}{1+\tau_n} \frac{\overline{\phi}^n \left[ \left( P_t^{n^*} \right)^{\gamma_{nn}} \left( P_t^{d^*} \right)^{\gamma_{dn}} \right]^{\alpha_n} \left( W_t^* \right)^{1-\alpha_n}}{Z_t^n} \end{aligned}$$

$$\begin{aligned} P_t^{d^*} &= \frac{\Theta^d}{1+\tau_d} MC_t^{d^*} \\ &= \frac{\Theta^d}{1+\tau_d} \frac{\overline{\phi}^d \left[ \left( P_t^{n^*} \right)^{\gamma_{nd}} \left( P_t^{d^*} \right)^{\gamma_{dd}} \right]^{\alpha_d} \left( W_t^* \right)^{1-\alpha_d}}{Z_t^d} \end{aligned}$$

where $\Theta^n$ and $\Theta^d$ denote the mark-up terms. In log-linear form the conditions above reduce to:

$$(1-\alpha_n)\, rw_t^{n*} = z_t^n + \alpha_n \gamma_{dn} q_t^* \tag{22}$$
$$(1-\alpha_d)\, rw_t^{d*} = z_t^d - \alpha_d \gamma_{nd} q_t^* \tag{23}$$

We now recall some conditions under flexible prices from the linearized system:

$$
\begin{aligned}
c_t^{d*} &= \frac{1}{\delta}d_t^* - \frac{1-\delta}{\delta}d_{t-1}^*, && (24)\\
rw_t^{n*} &= -\gamma c_t^{n*} - (1-\sigma)\,\mu_d d_t^* + \left[v\,(1-\phi) - \frac{1}{\lambda}\right] l_t^{d*} \\
&\quad + \left(\vartheta\phi + \frac{1}{\lambda}\right) l_t^{n*}, && (25)\\
l_t^{n*} &= \lambda\left(rw_t^{n*} - rw_t^{d*} + q_t^*\right) + l_t^{d*}, && (26)\\
y_t^{n*} &= \frac{C^n}{Y^n}c_t^{n*} + \frac{M^{nn}}{Y^n}m_t^{nn*} + \frac{M^{nd}}{Y^n}m_t^{nd*}, && (27)\\
y_t^{d*} &= \frac{C^d}{Y^d}x_t^* + \frac{M^{dn}}{Y^d}m_t^{dn*} + \frac{M^{dd}}{Y^d}m_t^{dd*}, && (28)\\
0 &= rw_t^{n*} + l_t^{n*} - y_t^{n*}, && (29)\\
0 &= rw_t^{d*} + l_t^{d*} - y_t^{d*}, && (30)\\
0 &= m_t^{nn*} - y_t^{n*}, && (31)\\
0 &= m_t^{nd*} + q_t^* - y_t^{d*}, && (32)\\
0 &= m_t^{dn*} - q_t^* - y_t^{n*}, && (33)\\
0 &= m_t^{dd*} - y_t^{d*}, && (34)
\end{aligned}
$$

where $\vartheta = \left(v - \frac{1}{\lambda}\right)$, $\gamma = (1-\sigma)\,\mu_n - 1$ and $\phi = \frac{L^n}{L}$. We substitute (22) and (23) into (28) and (29) respectively:

$$
\begin{aligned}
l_t^{n*} &= y_t^{n*} - \frac{1}{1-\alpha_n}z_t^n - \frac{\alpha_n\gamma_{dn}}{1-\alpha_n}q_t^*, && (35)\\
l_t^{d*} &= y_t^{d*} - \frac{1}{1-\alpha_d}z_t^d + \frac{\alpha_d\gamma_{nd}}{1-\alpha_d}q_t^*. && (36)
\end{aligned}
$$

We can use conditions (26), (27), and (31)-(34), to obtain:

$$
y_t^{n*} = \frac{C^n}{Y^n}c_t^{n*} + \frac{M^{nn}}{Y^n}y_t^{n*} + \frac{M^{nd}}{Y^n}\left(y_t^{d*} - q_t^*\right)
$$

and

$$
y_t^{d*} = \frac{C^d}{Y^d}c_t^{d*} + \frac{M^{dn}}{Y^d}\left(q_t^* + c_t^{g*}\right) + \frac{M^{dd}}{Y^d}y_t^{d*}.
$$

We can find a VAR solution to this system, so that we can express $y_t^{n*}$ and $y_t^{d*}$as a function of $c_t^{n*}$, $c_t^{d*}$ and $q_t^*$ :

$$
\mathbf{A}\left[\begin{array}{c} y_t^{n*}\\ y_t^{d*}\end{array}\right] = \mathbf{B}\left[\begin{array}{c} c_t^{n*}\\ c_t^{d*}\end{array}\right] + \boldsymbol{\Upsilon} q_t^*,
$$

where

$$\begin{aligned}
\mathbf{A} &= \begin{bmatrix} 1-\frac{M^{nn}}{Y^n} & -\frac{M^{nd}}{Y^n} \\ -\frac{M^{dn}}{Y^d} & 1-\frac{M^{dd}}{Y^d} \end{bmatrix} = \begin{bmatrix} \frac{C^n}{Y^n}+\frac{M^{nd}}{Y^n} & -\frac{M^{nd}}{Y^n} \\ -\frac{M^{dn}}{Y^d} & \frac{C^d}{Y^d}+\frac{M^{dn}}{Y^d} \end{bmatrix}, \\
\mathbf{B} &= \begin{bmatrix} \frac{C^n}{Y^n} & 0 \\ 0 & \frac{C^d}{Y^d} \end{bmatrix}, \\
\boldsymbol{\Upsilon} &= \begin{bmatrix} -\frac{M^{nd}}{Y^n} \\ \frac{M^{dn}}{Y^d} \end{bmatrix}.
\end{aligned}$$

Thus, we obtain:

$$\begin{bmatrix} y_t^{n*} \\ y_t^{d*} \end{bmatrix} = \mathbf{A}^{-1}\mathbf{B}\begin{bmatrix} c_t^{n*} \\ c_t^{d*} \end{bmatrix} + \mathbf{A}^{-1}\boldsymbol{\Upsilon} q_t^*,$$

or equivalently:

$$\begin{aligned}
y_t^{n*} &= \psi_1 c_t^{n*} + \psi_2 c_t^{d*} + \psi_5 q_t^*, \\
y_t^{d*} &= \psi_3 c_t^{n*} + \psi_4 c_t^{d*} + \psi_6 q_t^*.
\end{aligned}$$

Clearly, interdependence among sectors reflects the presence of cross-industry flows of input materials that imply $\psi_2, \psi_3, \psi_5, \psi_6 \neq 0$ and $\psi_1, \psi_4 \neq 1$. Plugging these expressions into (35) and (36) we obtain:

$$l_t^{n*} = \psi_1 c_t^{n*} + \psi_2 c_t^{d*} - \frac{1}{1-\alpha_n} z_t^n + \left(\psi_5 - \frac{\alpha_n \gamma_{dn}}{1-\alpha_n}\right) q_t^* \tag{37}$$

$$l_t^{d*} = \psi_3 c_t^{n*} + \psi_4 c_t^{d*} - \frac{1}{1-\alpha_d} z_t^d + \left(\frac{\alpha_d \gamma_{nd}}{1-\alpha_d} + \psi_6\right) q_t^* \tag{38}$$

Thus, we can substitute everything into (25) and (22):

$$\frac{1+v\phi}{1-\alpha_n} z_t^n + \xi_1 z_t^d = \xi_2 c_t^{n*} - (1-\sigma)\,\mu_d d_t^* + \xi_3 c_t^{d*} + \xi_4 q_t^*, \tag{39}$$

where:

$$\begin{aligned}
\xi_1 &= \frac{v(1-\phi)\lambda - 1}{(1-\alpha_n)\,\lambda}, \\
\xi_2 &= \frac{\lambda\,(v\phi\psi_1 - \gamma) + \psi_3\,[v(1-\phi)\lambda - 1]}{\lambda}, \\
\xi_3 &= \frac{\lambda v\phi\psi_2 + \psi_4\,[v(1-\phi)\lambda - 1]}{\lambda}, \\
\xi_4 &= -\frac{\alpha_n \gamma_{dn}\,(1+v\phi)}{1-\alpha_n} + \left[\frac{v(1-\phi)\lambda - 1}{(1-\alpha_d)\,\lambda}\left(\frac{\alpha_d\gamma_{nd}}{1-\alpha_d} + \psi_6\right) + \psi_5 v\phi\right].
\end{aligned}$$

In turn, we can plug (37), (38), (22) and (23) into (25):[38]

$$\xi_5 q_t^* = \frac{1}{1-\alpha_n} z_t^n - \frac{1}{1-\alpha_d} z_t^d - \frac{\psi}{(1+\lambda)}\left(c_t^{n*} - c_t^{d*}\right) \tag{40}$$

[38]It can be shown that $\psi_1 - \psi_3 = -(\psi_2 - \psi_4) = \left(\frac{M^{dn}}{C^d} + \frac{M^{nd}}{C^n} + 1\right)^{-1} = \psi < 1$.

where

$$\xi_5 = \frac{\psi_5 - \psi_6 - \lambda}{1+\lambda} - \left[\frac{\alpha_n \gamma_{dn}}{(1-\alpha_n)} + \frac{\alpha_d \gamma_{nd}}{(1-\alpha_d)}\right].$$

Conditions (39) and (40), together with the Euler conditions for the durable and the non-durable goods, and the law of accumulation for durable goods, allow us to determine a system of linear difference equations from which we derive equilibrium dynamics under flexible prices.

# APPENDIX D: Log-linear Economy

Here we report the log-linear economy in extensive form:

$$\begin{aligned}
\widetilde{c}_t^n &= \frac{1}{\gamma}\left(\widehat{r}_t - E_t\pi_{t+1}^n - rr_t^*\right) + E_t\widetilde{c}_{t+1}^n + \frac{(1-\sigma)\,\mu_d}{\gamma}E_t\Delta\widetilde{d}_{t+1}, \\
\widetilde{c}_t^n &= \frac{1}{\mu_n(1-\sigma)}\Bigg\{\left[1-\mu_d(1-\sigma)\right]\widetilde{d}_t + \frac{1}{1-\beta(1-\delta)}\left[(\mu_n(1-\sigma)-1)\,\widetilde{c}_t^n + \mu_d(1-\sigma)\,\widetilde{d}_t - \widetilde{q}_t\right] + \\
&\quad -\frac{(1-\delta)\,\beta}{[1-\beta(1-\delta)]}\left[(\mu_n(1-\sigma)-1)\,\widetilde{c}_{t+1}^n + \mu_d(1-\sigma)\,\widetilde{d}_{t+1} - \widetilde{q}_{t+1}\right] + \\
&\quad +\Xi\left(\widetilde{d}_t - \widetilde{d}_{t-1}\right) - \beta\Xi\left(\widetilde{d}_{t+1} - \widetilde{d}_t\right)\Bigg\} \\
\widetilde{c}_t^d &= \frac{1}{\delta}\widetilde{d}_t - \frac{1-\delta}{\delta}\widetilde{d}_{t-1}, \\
\widetilde{rw}_t^n &= -\gamma\widetilde{c}_t^n - (1-\sigma)\,\mu_d\widetilde{d}_t + \vartheta(1-\phi)\,\widetilde{l}_t^d + \left(\vartheta\phi + \frac{1}{\lambda}\right)\widetilde{l}_t^n, \\
\widetilde{l}_t^n &= \lambda\left(\widetilde{rw}_t^n - \widetilde{rw}_t^d + \widetilde{q}_t\right) + \widetilde{l}_t^d, \\
\pi_t^n &= \beta E_t\pi_{t+1}^n + \frac{(1-\beta\theta_n)(1-\theta_n)}{\theta_n}\widetilde{rmc}_t^n + \eta_t^n, \\
\pi_t^d &= \beta E_t\pi_{t+1}^d + \frac{(1-\beta\theta_d)(1-\theta_d)}{\theta_d}\widetilde{rmc}_t^d + \eta_t^d, \\
\widetilde{y}_t^n &= \alpha_n\gamma_{nn}\widetilde{m}_t^{nn} + \alpha_n\gamma_{dn}\widetilde{m}_t^{dn} + (1-\alpha_n)\,\widetilde{l}_t^n, \\
\widetilde{y}_t^d &= \alpha_d\gamma_{nd}\widetilde{m}_t^{nd} + \alpha_d\gamma_{dd}\widetilde{m}_t^{dd} + (1-\alpha_d)\,\widetilde{l}_t^d, \\
\widetilde{y}_t^n &= \frac{C^n}{Y^n}\widetilde{c}_t^n + \frac{M^{nn}}{Y^n}\widetilde{m}_t^{nn} + \frac{M^{nd}}{Y^n}\widetilde{m}_t^{nd}, \\
\widetilde{y}_t^d &= \frac{C^d}{Y^d}\widetilde{c}_t^d + \frac{M^{dn}}{Y^d}\widetilde{m}_t^{dn} + \frac{M^{dd}}{Y^d}\widetilde{m}_t^{dd}, \\
\widetilde{rmc}_t^n &= \widetilde{rw}_t^n + \widetilde{l}_t^n - \widetilde{y}_t^n, \\
\widetilde{rmc}_t^d &= \widetilde{rw}_t^d + \widetilde{l}_t^d - \widetilde{y}_t^d, \\
\widetilde{rmc}_t^n &= \widetilde{m}_t^{nn} - \widetilde{y}_t^n, \\
\widetilde{rmc}_t^d &= \widetilde{m}_t^{nd} + \widetilde{q}_t - \widetilde{y}_t^d, \\
\widetilde{rmc}_t^n &= \widetilde{m}_t^{dn} - \widetilde{q}_t - \widetilde{y}_t^n, \\
\widetilde{rmc}_t^d &= \widetilde{m}_t^{dd} - \widetilde{y}_t^d, \\
\widetilde{q}_t &= \widetilde{q}_{t-1} + \pi_t^n - \pi_t^d - \Delta q_t^*.
\end{aligned}$$

where $\gamma = (1-\sigma)\,\mu_n - 1$.

# APPENDIX E: Relative Price in the Efficient Equilibrium with Perfect labor Mobility

We now define the efficient equilibrium in the model with no frictions in both the goods and the labor market. On the labor market this condition, obtained for $\lambda \to \infty$, ensures that nominal salaries are equalized across sectors of the economy:

$$W_t^{n^*} = W_t^{d^*} = W_t^* . \tag{41}$$

Moreover, given the production subsidies that eliminate sectoral distortions due to monopolistic competition:

$$P_t^{n^*} = MC_t^{n^*} \qquad\qquad P_t^{d^*} = MC_t^{d^*}. \tag{42}$$

Conditions (41) and (42) imply that:

$$P_t^{n^*} = \left(\overline{\phi}^n\right)^{\frac{1}{1-\alpha_n\gamma_{nn}}} \left(P_t^{d^*}\right)^{\frac{\alpha_n\gamma_{dn}}{1-\alpha_n\gamma_{nn}}} \left(W_t^*\right)^{\frac{1-\alpha_n}{1-\alpha_n\gamma_{nn}}} \left(Z_t^n\right)^{-\frac{1}{1-\alpha_n\gamma_{nn}}} , \tag{43}$$

$$P_t^{d^*} = \left(\overline{\phi}^d\right)^{\frac{1}{1-\alpha_d\gamma_{dd}}} \left(P_t^{n^*}\right)^{\frac{\alpha_d\gamma_{nd}}{1-\alpha_d\gamma_{dd}}} \left(W_t^*\right)^{\frac{1-\alpha_d}{1-\alpha_d\gamma_{dd}}} \left(Z_t^d\right)^{-\frac{1}{1-\alpha_d\gamma_{dd}}} . \tag{44}$$

We then substitute (43) into (44) to eliminate $W_t^*$:

$$\left(P_t^{n^*}\right)^{\vartheta_n} = \Upsilon^{(1-\alpha_n\gamma_{nn})(1-\alpha_d)} \left(P_t^{d^*}\right)^{\vartheta_d} \left(Z_t^n\right)^{-(1-\alpha_d)} \left(Z_t^d\right)^{(1-\alpha_n)}$$

where

$$\Upsilon = \left(\overline{\phi}^n\right)^{\frac{1}{1-\alpha_n\gamma_{nn}}} \left(\overline{\phi}^d\right)^{-\frac{1}{1-\alpha_d}\frac{1-\alpha_n}{1-\alpha_n\gamma_{nn}}}$$

and

$$\vartheta_n = \vartheta_d = (1-\alpha_d)(1-\alpha_n\gamma_{nn}) + (\alpha_d\gamma_{nd})(1-\alpha_n).$$

Thus, after some trivial algebra we can show that the relative price reads as:

$$\begin{aligned} Q_t^* &= \frac{P_t^{n^*}}{P_t^{d^*}} = \Upsilon \left[\left(Z_t^n\right)^{-(1-\alpha_d)} \left(Z_t^d\right)^{1-\alpha_n}\right]^{\frac{1}{\varkappa+1}} \\ &= \Upsilon \left[\left(Z_t^n\right)^{-(1-\alpha_d)} \left(Z_t^d\right)^{1-\alpha_n}\right]^{\frac{1}{\varkappa+1}} . \end{aligned}$$

where

$$\varkappa = \alpha_n\alpha_d(\gamma_{nn} + \gamma_{dd} - 1) - \alpha_n\gamma_{nn} - \alpha_d\gamma_{dd}.$$

# APPENDIX F: Second-order Approximation of the Utility Function

Following Rotemberg and Woodford (1998), we derive a well-defined welfare function from the utility function of the representative household:

$$\mathcal{W}_t = U\left(C_t^n, D_t\right) - V\left(L_t\right).$$

We start from a second-order approximation of the utility from consumption of durable and non-durable goods:

$$\begin{aligned}
U\left(C_t^n, D_t\right) \approx\ & U\left(C^n, D\right) + U_{C^n}\left(C^n, D\right)\left(C_t^n - C^n\right) + \frac{1}{2}U_{C^nC^n}\left(C^n, D\right)\left(C_t^n - C^n\right)^2 && (45)\\
& + U_D\left(C^n, D\right)\left(D_t - D\right) + \frac{1}{2}U_{DD}\left(C^n, D\right)\left(D_t - D\right)^2 + \frac{1}{2}\Xi U_D\left(C^n, D\right)\left(D_t - D_{t-1}\right)^2 \\
& + U_{C^nD}\left(C^n, D\right)\left(C_t^n - C^n\right)\left(D_t - D\right) + O\left(\|\xi\|^3\right), && (46)
\end{aligned}$$

where $O\left(\|\xi\|^3\right)$ summarizes all terms of third order or higher. Notice that:

$$\begin{aligned}
U_D\left(C^n, D\right) &= \left(\mu_d C^n/\mu_n D\right) U_{C^n}\left(C^n, D\right),\\
U_{C^nC^n}\left(C^n, D\right) &= \left[\mu_n\left(1-\sigma\right) - 1\right]\left(C^n\right)^{-1} U_{C^n}\left(C^n, D\right),\\
U_{DD}\left(C^n, D\right) &= \left[\mu_d\left(1-\sigma\right) - 1\right]\left(\mu_d C^n/\mu_n D\right) U_{C^n}\left(C^n, D\right),\\
U_{C^nD}\left(C^n, D\right) &= \mu_d\left(1-\sigma\right) D^{-1} U_{C^n}\left(C^n, D\right).
\end{aligned}$$

As $\frac{C_t^n - C^n}{C^n} = \widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2$, where $\widehat{c}_t^n = \log\left(\frac{C_t^n}{C^n}\right)$ is the log-deviation from steady state under sticky prices, we obtain:

$$\begin{aligned}
U\left(C_t^n, D_t\right) \approx\ & U\left(C^n, D\right) + U_{C^n}\left(C^n, D\right) C^n\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right] + \\
& + \frac{1}{2}\left[\mu_n\left(1-\sigma\right) - 1\right] U_{C^n}\left(C^n, D\right) C\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right]^2 + \\
& + U_D\left(C^n, D\right) D\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right) + \frac{1}{2}\left[\mu_d\left(1-\sigma\right) - 1\right] U_D\left(C^n, D\right) D\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right)^2 + \\
& + \frac{1}{2}\Xi U_D\left(C^n, D\right) D\left(\widehat{d}_t - \widehat{d}_{t-1}\right)^2 + \\
& + \mu_d\left(1-\sigma\right) U_C\left(C^n, D\right) C^n\left[\widehat{c}_t^n + \frac{1}{2}\left(\widehat{c}_t^n\right)^2\right]\left(\widehat{d}_t + \frac{1}{2}\widehat{d}_t^2\right) + \text{t.i.p.} + O\left(\|\xi\|^3\right),
\end{aligned}$$

where t.i.p. collects terms independent of policy stabilization.

Next, we introduce a second-order approximation to the transition law for the stock of durables. This will substitute out the linear term for durables in the expression above (see Erceg and Levin, 2006). The law of motion reads as:

$$D_t = \left(1-\delta\right) D_{t-1} + X_t.$$

For a general function $F\left(Y, X\right)$ the second-order Taylor approximation can be written as:

$$
\begin{aligned}
F(Y,X) \approx\ & F_Y(Y,X)\,Yy + F_X(Y,X)\,Xx + \frac{1}{2}\left(F_X(Y,X)\,X + F_{XX}(Y,X)\,X^2\right)x^2 \\
& + \frac{1}{2}\left(F_Y(Y,X)\,Y + F_{YY}(Y,X)\,Y^2\right)y^2 + F_{YX}(Y,X)\,YXxy.
\end{aligned}
$$

Now, we can rewrite the accumulation equation as:

$$F(D_{t-1}, C_t^d) = \log\left[(1-\delta)\,D_{t-1} + C_t^d\right].$$

Therefore:

$$
\begin{aligned}
F_D &= \frac{(1-\delta)}{(1-\delta)\,D + C^d} = \frac{(1-\delta)}{(1-\delta)\,D + \delta D} = \frac{(1-\delta)}{D}, \\
F_{C^d} &= \frac{1}{(1-\delta)\,D + C^d} = \frac{1}{D}, \\
F_{DD} &= -\frac{(1-\delta)^2}{\left[(1-\delta)\,D + C^d\right]^2} = -\frac{(1-\delta)^2}{D^2}, \\
F_{C^dC^d} &= -\frac{1}{\left[(1-\delta)\,D + C^d\right]^2} = -\frac{1}{D^2}, \\
F_{DC^d} &= -\frac{1-\delta}{\left[(1-\delta)\,D + C^d\right]^2} = -\frac{1-\delta}{D^2}.
\end{aligned}
$$

Considering that in the steady state $C^d = \delta D$:

$$
\begin{aligned}
\widehat{d}_t \approx\ & \frac{(1-\delta)}{D} D\widehat{d}_{t-1} + \frac{1}{D}\delta D \widehat{c}_t^d + \\
& + \frac{1}{2}\left[\frac{(1-\delta)}{D}D - \frac{(1-\delta)^2}{D^2}D^2\right]\widehat{d}_{t-1}^2 + \\
& + \frac{1}{2}\left(\frac{1}{D}D - \frac{1}{D^2}D^2\right)\left(\widehat{c}_t^d\right)^2 - \frac{1-\delta}{D^2}\widehat{d}_{t-1}\widehat{x}_t \\
\approx\ & (1-\delta)\,\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \frac{(1-\delta)\,\delta}{2}\widehat{d}_{t-1}^2 + \frac{(1-\delta)\,\delta}{2}\left(\widehat{c}_t^d\right)^2 - \frac{(1-\delta)\,\delta}{2}\widehat{c}_t^d\widehat{d}_{t-1} \\
\approx\ & (1-\delta)\,\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \frac{(1-\delta)\,\delta}{2}\left(\widehat{d}_{t-1} - \widehat{c}_t^d\right)^2.
\end{aligned}
$$

Thus:

$$\widehat{d}_t \approx (1-\delta)\,\widehat{d}_{t-1} + \delta\widehat{c}_t^d + \psi_t, \tag{47}$$

where:

$$
\begin{aligned}
\widehat{\psi}_t &= \frac{(1-\delta)\,\delta}{2}\left(\widehat{c}_t^d - \widetilde{d}_{t-1}\right)^2 \\
&= \frac{(1-\delta)}{2\delta}\left(\widehat{d}_t - \widetilde{d}_{t-1}\right)^2.
\end{aligned}
$$

Now, let us iterate backward (47), to obtain:

$$\sum_{t=0}^{\infty}\beta^t\widehat{d}_t=\frac{1}{1-\beta\left(1-\delta\right)}d_0+\sum_{t=0}^{\infty}\beta^t\left[\frac{\delta}{1-\beta\left(1-\delta\right)}\widehat{c}_t^d+\frac{1}{1-\beta\left(1-\delta\right)}\widehat{\psi}_t\right].$$

In turn, the term on the RHS will replace the one on the LHS into the intertemporal loss function.

The next step is to derive a second-order approximation for labor disutility. Recall that:

$$\widehat{l}_t=\phi\widehat{l}_t^n+\left(1-\phi\right)\widehat{l}_t^d.$$

Therefore the second-order approximation reads:

$$\begin{aligned}V\left(L_t\right) &\approx V_L\left(L\right)L\left[\phi\widehat{l}_t^n+\left(1-\phi\right)\widehat{l}_t^d+\frac{\phi\left(1+2\nu\phi\right)}{2}\left(\widehat{l}_t^n\right)^2+\frac{\left(1-\phi\right)\left[1+2\nu\left(1-\phi\right)\right]}{2}\left(\widehat{l}_t^d\right)^2\right]+\\ &\quad +\text{t.i.p.}+O\left(\|\xi\|^3\right).\end{aligned}$$

After these preliminary steps, we need to find an expression for $\widehat{l}_t^n$ and $\widehat{l}_t^d$. Given the definition of the marginal cost, in equilibrium we get:

$$\begin{aligned}L_t^n &= \frac{\left(1-\alpha_n\right)MC_t^n}{W_t^n}\int_0^1 Y_{jt}^n dj=\frac{\left(1-\alpha_n\right)\overline{\phi}^n}{Z_t^n}\left(\frac{Q_t^{-\gamma_{dn}}}{RW_t^n}\right)^{\alpha_n}Y_t^n\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj,\\ L_t^d &= \frac{\left(1-\alpha_d\right)MC_t^d}{W_t^d}\int_0^1 Y_{kt}^d dk=\frac{\left(1-\alpha_d\right)\overline{\phi}^d}{Z_t^d}\left(\frac{Q_t^{\gamma_{nd}}}{RW_t^d}\right)^{\alpha_d}Y_t^d\int_0^1\left(\frac{P_{kt}^d}{P_t^d}\right)^{-\varepsilon_t^d}dk.\end{aligned}$$

Thus, we can report the linear approximation of the expressions above:

$$\begin{aligned}\widehat{l}_t^n &= -\alpha_n\gamma_{dn}\widehat{q}_t-\alpha_n\widehat{rw}_t^n-z_t^n+\widehat{y}_t^n+S_{nt},\\ \widehat{l}_t^d &= \alpha_d\gamma_{nd}\widehat{q}_t-\alpha_d\widehat{rw}_t^d-z_t^d+\widehat{y}_t^d+S_{dt},\end{aligned}$$

where:

$$S_{nt}=\log\left[\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj\right]\qquad S_{dt}=\log\left[\int_0^1\left(\frac{P_{kt}^d}{P_t^d}\right)^{-\varepsilon_t^d}dk\right]\tag{48}$$

If we set $\widehat{p}_{jt}^n$ to be the log-deviation of $\frac{P_{jt}^n}{P_t^n}$ from its steady state, which means that a second-order Taylor expansion of $\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj$ reads as:

$$\begin{aligned}\int_0^1\left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n}dj &\approx \int_0^1\left[1-\varepsilon^n\widehat{p}_{jt}^n-\varepsilon^n\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n+\frac{1}{2}\left(\varepsilon^n\right)^2\left(\widehat{p}_{jt}^n\right)^2\right]dj+O\left(\|\xi\|^3\right)\\ &= 1-\varepsilon^n\mathbf{E}_i\widehat{p}_{jt}^n-\varepsilon^n\mathbf{E}_i\widehat{p}_{jt}^n\widehat{\varepsilon}_t^n+\frac{1}{2}\left(\varepsilon^n\right)^2\mathbf{E}_i\left(\widehat{p}_{jt}^n\right)^2+O\left(\|\xi\|^3\right),\end{aligned}$$

where $\mathbf{E}_i \widehat{p}_{jt}^n \equiv \int\limits_0^1 \widehat{p}_{jt}^n dj$ and $\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 \equiv \int\limits_0^1 \left(\widehat{p}_{jt}^n\right)^2 dj$. At this stage, we need an expression for $\mathbf{E}_i \widehat{p}_{jt}^n$. Let us start from

$$P_t^n = \left[\int_0^1 \left(P_{jt}^n\right)^{1-\varepsilon_t^n} dj\right]^{\frac{1}{1-\varepsilon_t^n}},$$

which can be re-arranged as:

$$1 \equiv \int\limits_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n} dj.$$

Following the procedure above, it can be shown that:

$$\left(\frac{P_{jt}^n}{P_t^n}\right)^{1-\varepsilon_t^n} \approx 1 + (1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2}(1-\varepsilon^n)^2 \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Substituting this into the preceding equations yields:

$$0 = \int\limits_0^1 \left[(1-\varepsilon^n)\,\widehat{p}_{jt}^n - \varepsilon^n \widehat{p}_{jt}^n \widehat{\varepsilon}_t^n + \frac{1}{2}(1-\varepsilon^n)^2 \left(\widehat{p}_{jt}^n\right)^2\right] dj + O\left(\|\xi\|^3\right),$$

which reduces to:

$$\mathbf{E}_i \widehat{p}_{jt}^n = \frac{\varepsilon^n - 1}{2} \mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Thus:

$$\int\limits_0^1 \left(\frac{P_{jt}^n}{P_t^n}\right)^{-\varepsilon_t^n} dj = 1 + \frac{\varepsilon^n}{2} \mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 + O\left(\|\xi\|^3\right).$$

Now, notice that:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 = \mathbf{E}_i \left[\left(p_{jt}^n\right)^2 - 2p_{jt}^n p_t^n + \left(p_t^n\right)^2\right] + O\left(\|\xi\|^3\right),$$

where lower case letters denote the log-value of the capital letters. Here we can use a first-order approximation of $p_t^n = \int\limits_0^1 p_{jt}^n dj$, as this term is multiplied by other first-order terms each time it appears. With this, we have a second-order approximation:

$$\mathbf{E}_i \left(\widehat{p}_{jt}^n\right)^2 \equiv var_j p_{jt}^n.$$

Therefore, the second-order approximation can be represented as:

$$S_{nt} = \frac{\varepsilon^n}{2} var_j p_{jt}^n + O\left(\|\xi\|^3\right).$$

Analogous steps in the sector producing durable goods lead us to:

$$S_{dt} = \frac{\varepsilon^d}{2} var_k p_{kt}^d + O\left(\|\xi\|^3\right).$$

Following Woodford (2003, Ch. 6, Proposition 6.3), we can obtain a correspondence between cross-sectional price dispersions in the two sectors and their inflation rates:

$$
\begin{aligned}
var_j p_{jt}^n &= \theta_n var_j p_{jt-1}^n + \frac{\theta_n}{1-\theta_n} (\pi_t^n)^2 + O\left(\|\xi\|^3\right), \\
var_k p_{kt}^d &= \theta_d var_k p_{kt-1}^d + \frac{\theta_d}{1-\theta_d} \left(\pi_t^d\right)^2 + O\left(\|\xi\|^3\right).
\end{aligned}
$$

Iterating these expressions forward leads to:

$$
\sum_{t=0}^{\infty} \beta^t var_j p_{jt}^n = (\kappa_n)^{-1} \sum_{t=0}^{\infty} \beta^t (\pi_t^n)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{49}
$$

$$
\sum_{t=0}^{\infty} \beta^t var_k p_{kt}^d = (\kappa_d)^{-1} \sum_{t=0}^{\infty} \beta^t \left(\pi_t^d\right)^2 + \text{t.i.p.} + O\left(\|\xi\|^3\right), \tag{50}
$$

where

$$
\begin{aligned}
\kappa_n &= \frac{(1-\beta\theta_n)(1-\theta_n)}{\theta_n}, \\
\kappa_d &= \frac{(1-\beta\theta_d)(1-\theta_d)}{\theta_d}.
\end{aligned}
$$

After these preliminary steps, we can write $\mathcal{W}_t$ as:

$$
\begin{aligned}
\mathcal{W}_t \approx\ & U_{C^n}(C^n, D)\, C^n \left\{ \widehat{c}_t^n + \frac{1}{2}\left[\mu_n (1-\sigma)\right] (\widehat{c}_t^n)^2 + (\mu_d/\mu_n)\, \widehat{d}_t + \right. \\
& \left. + \frac{1}{2}\left[\mu_d (1-\sigma)\right] (\mu_d/\mu_n)\, \widehat{d}_t^2 + \mu_d (1-\sigma)\, \widehat{c}_t^n \widehat{d}_t + \frac{1}{2} \Xi\, (\mu_d/\mu_n) \left(\widehat{d}_t - \widehat{d}_{t-1}\right)^2 \right\} + \\
& - V_L(L)\, L \left\{ \phi \widehat{l}_t^n + (1-\phi)\, \widehat{l}_t^d + \right. \\
& \left. + \left(\frac{1+\nu}{2}\right) \left[ \phi^2 \left(\widehat{l}_t^n\right)^2 + (1-\phi) \left(\widehat{l}_t^d\right)^2 + 2\phi (1-\phi)\, \widehat{l}_t^n \widehat{l}_t^d \right] \right\} + \\
& + \text{t.i.p.} + O\left(\|\xi\|^3\right).
\end{aligned}
$$

We now consider the linear terms in $\mathcal{W}_t$, which are collected under $\mathcal{LW}_t$:

$$
\begin{aligned}
\mathcal{LW}_t =\ & \frac{U_{C^n}(C^n, D)\, C^n}{\mu_n} \left\{ \mu_n \widehat{c}_t^n + \mu_d \widehat{d}_t \right\} + \\
& - \left\{ V_L(L)\, L \phi \left(-\alpha_n \gamma_{dn} \widehat{q}_t - \alpha_n \widehat{rw}_t^n + \widehat{y}_t^n\right) + \right. \\
& \left. + (1-\phi) \left(\alpha_d \gamma_{nd} \widehat{q}_t - \alpha_d \widehat{rw}_t^d + \widehat{y}_t^d\right) \right\} + \\
& + \text{t.i.p.} + O\left(\|\xi\|^2\right).
\end{aligned}
$$

We substitute for the real wage from marginal cost expressions to get:

$$\begin{aligned}
\mathcal{LW}_t &= \frac{U_{C^n}(C^n, D)\, C^n}{\mu_n}\left\{\mu_n \widehat{c}_t^n + \mu_d \widehat{d}_t\right\} + \\
&\quad -V_L(L)\, L\phi\left(\frac{1}{1-\alpha_n}\widehat{y}_t^n - \frac{\alpha_n\gamma_{nn}}{1-\alpha_n}\widehat{m}_t^{nn} - \frac{\alpha_n\gamma_{dn}}{1-\alpha_n}\widehat{m}_t^{dn}\right) + \\
&\quad -V_L(L)\, L(1-\phi)\left(\frac{1}{1-\alpha_d}\widehat{y}_t^d - \frac{\alpha_d\gamma_{nd}}{1-\alpha_d}\widehat{m}_t^{nd} - \frac{\alpha_d\gamma_{dd}}{1-\alpha_d}\widehat{m}_t^{dd}\right) + \\
&\quad +\text{t.i.p.} + O\left(\|\xi\|^2\right).
\end{aligned}$$

After substituting the second-order approximation for the accumulation equation of durables we get:

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t \mathcal{LW}_t &= U_{C^n}(C^n, D)\, C^n \sum_{t=0}^{\infty}\beta^t\left\{\widehat{c}_t^n + \frac{\delta}{1-\beta(1-\delta)}\frac{\mu_d}{\mu_n}\widehat{c}_t^d\right\} + \\
&\quad -V_L(L)\, L\sum_{t=0}^{\infty}\beta^t\left\{\phi\left(\frac{1}{1-\alpha_n}\widehat{y}_t^n - \frac{\alpha_n\gamma_{nn}}{1-\alpha_n}\widehat{m}_t^{nn} - \frac{\alpha_n\gamma_{dn}}{1-\alpha_n}\widehat{m}_t^{dn}\right) + \right. \\
&\quad \left. +(1-\phi)\left(\frac{1}{1-\alpha_d}\widehat{y}_t^d - \frac{\alpha_d\gamma_{nd}}{1-\alpha_d}\widehat{m}_t^{nd} - \frac{\alpha_d\gamma_{dd}}{1-\alpha_d}\widehat{m}_t^{dd}\right)\right\} + \\
&\quad +\text{t.i.p.} + O\left(\|\xi\|^2\right).
\end{aligned} \tag{51}$$

Notice that the following linear approximations for the market clearing conditions hold:

$$\begin{aligned}
\widehat{y}_t^n &= \frac{1-\alpha_n}{\phi}\mu_n\widehat{c}_t^n + \alpha_n\gamma_{nn}\widehat{m}_t^{nn} + \frac{(1-\alpha_n)(1-\phi)}{\phi(1-\alpha_d)}\alpha_d\gamma_{nd}\widehat{m}_t^{nd}, \\
\widehat{y}_t^d &= \frac{\delta\mu_d(1-\alpha_d)}{(1-\phi)\left[1-\beta(1-\delta)\right]}\widehat{c}_t^d + \frac{(1-\alpha_d)\phi}{(1-\phi)(1-\alpha_n)}\alpha_n\gamma_{dn}\widehat{m}_t^{dn} + \alpha_d\gamma_{dd}\widehat{m}_t^{dd}.
\end{aligned}$$

It can be shown that, in the steady state, the following relationships hold:

$$V_{L^n}(L^n)\, L^n = \phi V_L(L)\, L \qquad V_{L^d}\left(L^d\right) L^d = (1-\phi)\, V_L L(L)$$

Moreover, the presence of production subsidies allows us to express the steady state marginal rate of substitution between labor supply and consumption of non-durable goods as:

$$\begin{aligned}
\frac{-V_{L^n}(L^n)}{U_{C^n}(C^n)} &= \frac{Y^n(1-\alpha_n)}{L^n}, \\
\frac{-V_{L^d}\left(L^d\right)}{U_{C^n}(C^n)} &= \frac{Y^d(1-\alpha_d)}{L^d Q}.
\end{aligned}$$

It is now convenient to express the marginal utility from non-durable consumption in terms of the marginal utility derived from total consumption:

$$U_{C^n}(C^n) = U_H(H)\, H\mu_n.$$

Therefore, we can re-write (51) as:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{L}\mathcal{W}_t &= U_H(H)H\sum_{t=0}^{\infty}\beta^t\left\{\left(\mu_n\widehat{c}_t^n+\frac{\delta\mu_d}{1-\beta(1-\delta)}\widehat{c}_t^d\right)+\right.\\&-\mu_n\left(\frac{C^n}{Y^n}\right)^{-1}(1-\alpha_n)\left[-\alpha_n\gamma_{dn}\widehat{q}_t-\alpha_n\widehat{rw}_t^n-z_t^n+\widehat{y}_t^n\right]+\\&\left.-\mu_n\left(\frac{C^n}{Y^d}\right)^{-1}(1-\alpha_d)Q^{-1}\left[\alpha_d\gamma_{nd}\widehat{q}_t-\alpha_d\widehat{rw}_t^d-z_t^d+\widehat{y}_t^d\right]\right\}+\\&+\text{t.i.p.}+O\left(\|\xi\|^2\right).\end{aligned}$$

It is now possible to show, given the linearized market clearing conditions in the two sectors, that $\sum_{t=0}^{\infty}\beta^t\mathcal{L}\mathcal{W}_t=0$. The linear term in $\mathcal{W}_t$ can therefore be dropped. Thus we are left only with second-order terms:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathcal{W}_t &\approx U_H(H)H\sum_{t=0}^{\infty}\beta^t\left\{\frac{1-\sigma}{2}\left(\mu_n\widehat{c}_t^n+\mu_d\widehat{d}_t\right)^2+\frac{1}{1-\beta(1-\delta)}\mu_d\widehat{\psi}_t+\frac{\mu_d}{2}\Xi\left(\widehat{d}_t-\widehat{d}_{t-1}\right)^2+\right.\\&-\frac{\Theta}{2}\left[\phi\varepsilon^n(\kappa_n)^{-1}(\pi_t^n)^2+(1-\phi)\varepsilon^d(\kappa_d)^{-1}\left(\pi_t^d\right)^2\right]+\\&\left.-\left(\frac{1+\nu}{2}\right)\Theta^{-1}\left[\mu_n\widehat{c}_t^n+\frac{\delta\mu_d}{1-\beta(1-\delta)}\widehat{c}_t^d\right]^2\right\}+\\&+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

where

$$\Theta=\left(\frac{C^n}{Y^n}\right)^{-1}\frac{(1-\alpha_n)\mu_n}{\phi}=\frac{\mu_n[1-\beta(1-\delta)]+\mu_d\delta}{1-\beta(1-\delta)}.$$

We next consider the deviation of social welfare from its Pareto-optimal level:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\widetilde{\mathcal{W}}_t &= \sum_{t=0}^{\infty}\beta^t\left(\mathcal{W}_t-\mathcal{W}_t^*\right)\approx\\&-\frac{U_H(H)H}{2}\Theta\sum_{t=0}^{\infty}\beta^t\left\{\frac{\sigma-1}{\Theta}\left(\mu_n\widetilde{c}_t^n+\mu_d\widetilde{d}_t\right)^2+\right.\\&+\left[\mu_d\Theta^{-1}\Xi+(1-\delta)(1-\omega)\delta^{-2}\right]\left(\widetilde{d}_t-\widetilde{d}_{t-1}\right)^2+\\&\left.+\varsigma\left[\varpi(\pi_t^n)^2+(1-\varpi)\left(\pi_t^d\right)^2\right]+(1+\nu)\left[\omega\widetilde{c}_t^n+(1-\omega)\widetilde{c}_t^d\right]^2\right\}+\text{t.i.p.}+O\left(\|\xi\|^3\right),\end{aligned}$$

where the following notation has been introduced:

$$\begin{aligned}\omega &= \frac{\mu_n[1-\beta(1-\delta)]}{\mu_n[1-\beta(1-\delta)]+\mu_d\delta},\\ \varpi &= \frac{\phi\varepsilon^n(\kappa_n)^{-1}}{\varsigma},\\ \varsigma &= \phi\frac{\varepsilon^n}{\kappa_n}+(1-\phi)\frac{\varepsilon^d}{\kappa_d}.\end{aligned}$$

# APPENDIX G: Alternative Policy Regimes

The following alternative period loss functions are considered in Section 4.3:

$$\begin{aligned}
\text{Strict inflation targeting} &: \widetilde{\mathcal{W}}_t^{IT} = \left(\pi_t^{IT}\right)^2, \\
\text{Gap targeting} &: \widetilde{\mathcal{W}}_t^{GT} = \left(\widetilde{x}_t^{GT}\right)^2, \\
\text{Flexible inflation targeting} &: \widetilde{\mathcal{W}}_t^{FIT} = \widetilde{\mathcal{W}}_t^{IT} + (1+\upsilon)\, \widetilde{\mathcal{W}}_t^{GT},
\end{aligned}$$

where

$$\begin{aligned}
\pi_t^{IT} &= \left\{\pi_t^{core}, \pi_t^{agg}\right\} \\
\pi_t^{core} &= \varpi \pi_t^n + (1-\varpi)\, \pi_t^d, \\
\pi_t^{agg} &= \phi \pi_t^n + (1-\phi)\, \pi_t^d,
\end{aligned}$$

and

$$\begin{aligned}
\widetilde{x}_t^{GT} &= \left\{\widetilde{x}_t^c, \widetilde{x}_t^p\right\} \\
\widetilde{x}_t^c &= \omega \widetilde{c}_t^n + (1-\omega)\, \widetilde{c}_t^d, \\
\widetilde{x}_t^p &= \phi \widetilde{y}_t^n + (1-\phi)\, \widetilde{y}_t^d.
\end{aligned}$$