Conflict and Conflict Management with Interdependent Instruments and Asymmetric Stakes,(The Good-Cop and the Bad-Cop Game)

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Raul Caruso

Institute of Economic Policy
Università Cattolica del Sacro Cuore di Milano

raul.caruso@unicatt.it

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Abstract

This paper considers a partial equilibrium model of conflict where two asymmetric, rational and risk-neutral opponents clash in order to redistribute a divisible prize in their favour. Differently from common contest models agents have the option of choosing a second instrument to affect the outcome of the conflict. The second instrument is assumed to capture a complex bundle of Conflict Management Procedures (CMPs). Through comparative statics, different scenarios are studied. A Potential Settlement Region (PSR) is presented as the set of all possible settlement points. First, the role of asymmetry in the evaluation of the contested stake has been underlined. The agent with the lower evaluation will expend efforts in conflict management only when the asymmetry is extremely large. When agents are asymmetrical both in evaluation of the stake and in fighting abilities, there is also a smaller PSR. Once the destruction parameter is considered, agents clearly also take into account the opportunity cost of the conflict and enlarge a PSR. Finally, throughout the paper, the concept of entropy has been applied as a tool for the measurement and evaluation of conflict and conflict management.

KEYWORDS: Conflict, Contest, Conflict management, Asymmetry in evaluation, Statistical entropy, Uncertainty.

JEL CODE: D7, D72, D74, D74, D82.

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CONFLICT AND CONFLICT MANAGEMENT WITH INTERDEPENDENT INSTRUMENTS AND ASYMMETRIC STAKES

(THE GOOD-COP AND THE BAD-COP GAME)

I. Background of the Model

This paper considers a partial equilibrium model of conflict where two asymmetric, rational and risk-neutral opponents clash in order to redistribute a divisible stake in their favour. In recent economic literature, Jack Hirshleifer pioneered the work on modelling conflict, whose foundations are in Hirshleifer (1987, 1988, 1989). The economic theory of conflict[1] rests to a large extent upon the assumption that agents involved in conflict interactions have to choose an optimal level of efforts or resources devoted to the unproductive activity of conflict. A significant element in economic theory of conflict is that investing resources in conflict is necessarily detrimental for welfare. This is central to theory of conflict as well as to theory of rent-seeking and contests. Given the partial-equilibrium framework adopted in this work, the analysis produced can be generalized to all these theoretical categories[2]. However, conflicts, contests and rent-seeking can be considered directly unproductive activities (DUP) in the spirit of the definition provided by Bhagwati (1982), who proposes a general taxonomy for a broader range of economic activities representing ways of making profit despite being directly unproductive. According to this view, such activities yield pecuniary returns but do not produce goods and services which enter a utility function, either

[1] In more recent years several studies extended Hirshleifer's basic model. See among others: Grossman (1991/1998), Skaperdas (1992), Garfinkel (1990/1994), Grossman and Kim (1995), Skaperdas and Syropoulos (1996/1997), Neary (1997a), Anderton et al. (1999), Anderton (1999/2000), Garfinkel and Skaperdas (2000), Alesina and Spolaore (2003/2005), Dixit (2004), Spolaore (2004), Caruso (2006).

[2] Outlining in depth the differences between contest, rent-seeking and conflict models goes far beyond the subject of this work. However, a few lines could suggest some interesting insights. The main differences between conflict models, rent-seeking, and contest models are that the former are generally general equilibrium models. This means that conflict models involve a trade-off between productive and unproductive activities and the prize (or the rent) of the contest is endogenous. The stake of the conflict is interpreted as a joint production which depends on the productive efforts of agents. At the same time, the cost function is represented by the foregone production. In such a construction the greater the number of the agents, the greater the 'pie' to be shared. In rent-seeking and contest models, the prize (or the rent) is given exogenously. In such a case, even if the number of contestants becomes larger the rent does not change. Moreover, rent-seeking and contest models can involve unconstrained optimization, whereas conflict models necessarily imply constrained optimization. Neary (1997b) and Hausken (2005) propose a comparison of conflict and contest models along these lines. Another point of interest in modelling conflicts is that it is often designed as being twice costly. In fact, in conflict agents can retain an *ex-ante* perception of losses. This is often formally captured through a destruction parameter (commonly bounded between zero and the unity). Two other remarkable points that would deserve a further deep attention are (i) voluntary participation (ii) the existence of a referee.

directly or indirectly through increased production or availability to the economy of goods that enter a utility function. This is the rationale behind the labelling *directly unproductive profit-seeking activities* (DUP). This paper is intended to extend the literature on this subject dealing with two main points:

(i) the existence of a second type of effort (instrument) to win the conflict;
(ii) an asymmetry in the evaluation of the stake of the conflict.

A story which immediately recalls the intuitions of this work is the story of *The Good-Cop and Bad-Cop Game*. Consider two cops arresting a suspect. Imagine also that they lack sufficient evidence to convict him. Then, they have to spend efforts in order to induce prisoners to confess. Next, as usually happens in American movies, in the questioning room cops have to play the good-cop and bad-cop game. The bad cop has to appear more aggressive, rude and less conciliatory. He would send exactly what students of strategy would define a 'credible threat'. On the other hand, the good cop has to appear less rude and more conciliatory expounding the advantages of confessing. The Cops' dilemma will be how much efforts in both behaviours should be spent. Of course, the outcome of questioning will depend upon the interdependent impact of complement instruments, rudeness and persuasion. On the other hand, the suspect has to choose whether to confess or to stick to his presumption of innocence. At the same time, and by sticking, the suspect can 'signal' his or her willingness to cooperate through partial openings.

This story highlights the existence of two interdependent instruments which can be used by rational agents in order to win a conflict. Analysing the realm of violent conflicts, it would be a great mistake to think of many conflicts as an exclusively violent activity. Most conflicts involve remarkable bargaining efforts between the antagonists. Beyond violence, as applied when sending actual or potential threats, agents are used to applying other instruments to successfully end the contest. During a war, for example, the exploitation of actual violence is often interlinked with diplomatic efforts. In international interactions the exploitation of potential or actual violence cannot be disentangled from partial openings and cooperative behaviours. Diplomatic negotiations are often conducted while troops are deployed on the battlefield. This also does link with the famous idea of 'carrot-and-stick' strategy.

In other words, it is reasonable to assume that agents can use different instruments in order to pursue their own maximum utility. Of course, any of the instruments used is interdependent with the others. Therefore, the outcome of the contest will arise from the interaction of such different instruments. In this view, the standard one-instrument models, commonly adopted in literature, can be considered as a special case of multi-instruments models.

Therefore, the limiting assumption of this paper is that once involved in conflict interactions, agents face the option of choosing also a second instrument in order to improve the outcome of the conflict. Thus, in the continuation of this work I will refer to the second instrument as a complex bundle of *Conflict Management Procedures* (CMPs).

A CMP can take different shapes. It can involve, among others, elements of communication, negotiation and signalling. Under the assumption of complete information, the second instrument must be perfectly observable. Thus,

(i) the use of a second instrument needs not to be "payoff-irrelevant": it must have a direct impact on both agents' payoffs;
(ii) the second instrument must also be costly. There is no room for *cheap talk*. In fact, what is needed is a "credibility –cost". Under the assumption of complete information, an observable costly effort is assumed to be also credible;
(iii) investment in conflict management must be irrevocable;
(iv) the two instruments must be complements.

In the theory of contest the use of a second instrument is not a novelty, although such approach has not been deeply developed.[3] In particular, this paper is relative to a model proposed by Epstein and Hefeker (2003). They model a contest where, in order to win, each opponent can use two instruments. In such a model the use of two instruments is intended to improve each agent's own performance. The authors show that the use of two instruments strengthens the player with the higher stake, decreases the relative rent dissipation and decreases total expenditure if the parties are sufficiently asymmetric. They interpret this second instrument as a complementary effort which might create an advantage for one of the opponents.

As noted above, I will interpret the second instrument[4] as a complex bundle of *Conflict Management Procedures* (hereafter CMPs for brevity).

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[3] Baik and Shogran (1995), study a contest between players with unknown relative ability. Under the assumption of decreasing aversion to uncertain ability agents are allowed to expend resources in order to reduce such uncertainty through spying. Konrad (2003), enriches a model of rent-seeking considering the interaction between two types of efforts: (i) the standard rent-seeking efforts to improve their own performance in the view of winning a prize; (ii) a sabotaging effort in order to reduce the effectiveness of other agents' efforts. In this model, sabotage is targeted towards a particular rival group and reduces this group's performance. The point of interest is that through sabotage a group can increase its own probability of winning the prize as well as the other contestants'. Thus, the model predicts that sabotage disappears whenever the number of contestants becomes large. Caruso (2005b) presents two different models of contest with two instruments. The analysis is applied to sport contests in order to consider the phenomena of match-fixing and doping. Arbatskaya and Mialon (2005) analyse in depth the equilibrium properties of a two-instruments contest model and compare the results to those attainable in standard one-instrument models.

[4] I am aware that a simple objection would be related to the nature of this second instrument. It could be argued that in a conflict situation both opponents could choose whether to use or not two different types of bombs, grenades or other offensive devices. This could be true of course. However, this would relate to the dimension of the technology of conflict itself. In the

Therefore, it would appear reasonable to distinguish between ‘pure conflict’ efforts and ‘conflict management’ efforts.

However, it must be underlined that the two instruments are intended to be complements. Namely, the outcome of the conflict depends upon the mixed effect of violence and negotiation. This means that opponents do no give up their willingness to pursue the maximum possible payoff. Then, it would be interesting to verify whether, given the existence of conflict, agents can also retain an incentive to manage, and maybe to solve, it.

The main goal presented in this paper will be represented by the identification of a *Potential Settlement Region* (hereafter PSR for brevity) as the set of possible peaceful agreements. The limiting hypothesis is that a settlement region is feasible if and only if both agents choose to negotiate, namely to expend efforts in the second instrument.

This seems to be a quite realistic assumption. No settlement region appears to be feasible if one agent has no incentive to negotiate. In such a region, agents can reach an agreement but the model does not suggest whether they are likely to do it. One might say that this approach does fit more with the opportunity and incentives of opening talks or negotiations. This would imply that the existence of CMPs does not necessarily lead to a peaceful settlement but it is a characteristic feature of conflict itself.

The remainder of the paper is organised as follows: in the first part the basic hypothesis and formal definitions are presented. In the second part, a basic model allowing for the second instrument is presented. The third part is focused on the issue of measurement. In particular, entropy is presented as an alternative tool for measurement of conflict and conflict management. In the following sections other extensions will deal with (a) the existence of an asymmetry in abilities; (b) the existence of a weighted sharing rule. Finally, the last section summarises the results and provides suggestions for future research.

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literature of conflict, technological features of conflict are usually captured through a decisiveness parameter attached to the functional form of CSF adopted.

II. The Basic Model

There are two risk-neutral agents indexed by $i=1,2$. They conflict over a positive and divisible stake denoted by $X_i \in (0,\infty), i=1,2$. Agents are identical in abilities but at the same time they have different evaluations of the stake in the conflict. Then, it is possible to write that $X_i \neq X_j, \forall i \neq j, i, j=1,2$.

Assuming a divisible stake clearly implies that this is not a winner-takes-all contest. The stake of the conflict can be interpreted in different ways. It might be for example a contested natural resource, a territory or a homogenous input. It is also assumed that the evaluation of the stake is different. In particular, agent 1 has a higher evaluation than agent 2. Let $\delta \in (0,1)$ denote the degree of asymmetry between the stakes of the two agents, namely $\exists \delta \in (0,1)\, s.t. X_2 = \delta X_1$. There is common knowledge about such hypotheses.

Under the assumption of risk-neutrality, agents interpret the outcome of the non-cooperative conflict game as deterministic. That is, given the assumption of risk-neutrality agents are indifferent between conflict and sharing the stake in accordance with the winning probabilities. Then, the conflict is supposed to give each party control over a positive fraction of the contested stake in order to maximise its payoff. Let $z_i \in (0,\infty), i=1,2$ denote the positive amount of violent efforts and $h_i \in [0,\infty), i=1,2$ denote the CMP efforts.

The core assumption is that agents behave rationally and choose the elements of their strategy sets in order to maximize their own expected payoff. It is assumed that violence and appropriation constitute the first option for both agents. Hence, by means of comparative statics, agents will choose to use a second instrument if and only if the attainable payoffs will be greater than in the standard 'one-instrument' contest mechanism. In other words, here I also maintain that two different regimes can be studied. Under a first regime – say 'Pure Conflict' – violent and appropriative efforts represent the only instrument, whereas under a second regime – say 'Conflict Management' – both agents can use both instruments.

Let π_i^o, π_i^t for $i=1,2$ denote the payoff achievable under the standard contest and the payoff achievable using the two instruments respectively. In particular, in their general form the payoff functions in general form can be written as $\pi_i^o = \pi_i^o(z_i, z_j, X_i, X_j)$ and $\pi_i^t = \pi_i^t(z_i, z_j, h_i, h_j, X_i, X_j)$ respectively. Then, each agent will choose to use the second instrument if and only if $\pi_i^t > \pi_i^o, i=1,2$.

In the remainder of the paper, I will define it as *Conflict Management Condition* (CMC). Whenever it holds, it would be possible to say that a willingness to negotiate emerges. To summarise formally:

$$\begin{cases} z_i > 0 \quad h_i > 0 \quad \Leftrightarrow \quad \pi_i^T > \pi_i^o \quad i = 1,2 \\ z_i > 0 \quad h_i = 0 \quad \Leftrightarrow \quad \pi_i^T < \pi_i^o \quad i = 1,2 \end{cases} \tag{c.1}$$

In simpler words, agents will also choose the second instrument if – and only if – it reinforces the effect of the first instrument. That is, agents will devote resources to negotiations if and only if this behaviour appears to guarantee a higher payoff.

As expounded above, I am interested in the case where both agents choose to use the second instrument. In this case there is room to identify a potential settlement region. Then, another limiting hypothesis is that a settlement region is feasible if and only if both agents choose to be involved in a CMP. This seems to be a quite realistic assumption. No settlement region is feasible if one agent has no incentive to negotiate.[5]

To summarise, formally in the two-agents scenario, a PSR does exist if and only if:

$$\begin{cases} \pi_1^T > \pi_1^o \\ \pi_2^T > \pi_2^o \end{cases} \tag{c.2}$$

Since according to (c.2) the payoff with two instruments must exceed the attainable payoff with only one instrument for both agents, this also means that there exists a positive value $\gamma_i = \pi_i^T - \pi_i^o, i = 1,2$. Then, the PSR can be defined as the set of all the positive values[6] for γ, namely $PSR \equiv \{(\gamma_i, \gamma_j) \in \Re : \gamma_i > 0, \gamma_i > 0, i = 1,2, i \neq j\}$.

As noted above, a partial equilibrium model of conflict with an exogenous prize is not technically distinguishable from the standard rent-seeking model. Thus, the cornerstone of this class of models is the Contest Success Function[7] (hereafter CSF for brevity). In particular, the outcome of the conflict is determined through a CSF. It summarises the relevant aspects of

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[5] Note that such a condition is likely to hold in this limiting two-agents scenario. It is reasonable to think that in a n-agents scenario it might be relaxed.

[6] A more realistic assumption would imply a positive value, say $\varepsilon \in (0, \infty)$, such that the PSR can be defined as the set of all $(\gamma + \varepsilon)$. Whatever the arbitrary value chosen for ε, it would imply that negligible positive value for γ are ruled out. In other words, this means that each agent is willing to negotiate if and only if the difference between the expected payoffs is not negligible.

[7] The Contest Success Function is a mathematical relation that links the outcome of a contest and the efforts of the players. It is actually a founding pillar of many models. Selective seminal contributions are by Tullock (1980), O'Keeffe et al. (1984) and Rosen (1986). Dixit (1987) develops a general framework for contests using the general properties of logit functions. Hirshleifer (1989) focuses on a different form for the CSF: the ratio form and logit form. See then Skaperdas (1996) and Clark and Riis (1998) for a basic axiomatization.

what Hirshleifer defines the *technology* of conflict. In particular, even if the CSF can take different forms, the *ratio* form of the CSF[8] is used here.

$$p_i = \frac{z_i}{z_i + z_j} \qquad \text{for } i = 1,2 \text{ and } j \neq i \qquad (1)$$

Equation (1) is twice differentiable and follows the conditions below:

$$\begin{cases} p(0,0) \equiv 1/2 \\ p_1 + p_2 = 1 \\ \partial p_i / \partial z_i > 0 \quad \partial p_i / \partial z_j > 0 \\ \partial^2 p_i / \partial^2 z_i \leq 0 \quad \partial^2 p_i / \partial^2 z_j \geq 0 \end{cases} \qquad (1.1)$$

The functional form adopted in equation (1) implies that the conflict is not decisive, namely there is no preponderance of a party over the other. This is of course a limiting assumption, even if many conflicts fall in this category. Under the assumption of risk-neutrality the outcome of the CSF denotes the proportion of appropriation going to agent i for $i = 1,2$. Eventually, the payoff function is given by:

$$\pi_i^o = \frac{z_i}{z_i + z_j} X_i - z_i \qquad (2)$$

Each agent will maximise its own payoff with respect to its own level of violent efforts. This yields to the first order conditions:

$$\frac{\partial \pi_i^o}{\partial z_i} = \frac{X_i z_j}{(z_i + z_j)^2} - 1 = 0, i = 1,2, i \neq j \qquad (3)$$

Solving the first order condition, the equilibrium (denoted by stars superscripted) choices of violent efforts are given by:

$$z_i^{*O} = \frac{X_i^2 X_j}{(X_i + X_j)^2}, i = 1,2; i \neq j \qquad (4)$$

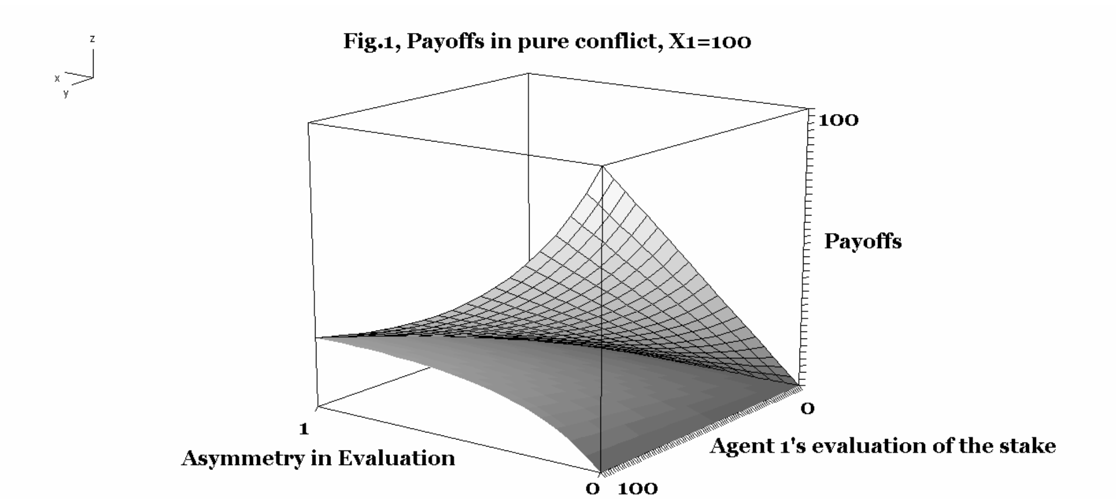
In equilibrium the payoffs for agent i are given by:

[8] Hirshleifer (1989) analyses the different impact of two different function form for CSF: the *ratio form* and the *logistic form*. In the first case, the contest outcome depends upon the ratio of the efforts applied, whilst in the second case it depends upon the difference between the resources committed.

$$\pi_i^{*O} = \frac{X_i^3}{\left(X_i + X_j\right)^2}, i = 1,2; i \neq j \qquad (5)$$

Recall that $X_2 = \delta X_1$ by assumption. Therefore, the agent with a higher evaluation of the stake of the conflict will arm more, namely $z_1^{*O} > z_2^{*O}$. At the same time, payoff for agent 1 is greater than payoff for agent 2, namely $\pi_1^{*O} > \pi_2^{*O}$. In other words, the agent with a higher evaluation of the stake in equilibrium is capable of getting a higher payoff by means of a higher level of violence.

Plot 1 shows the relationship between the payoffs of both agents and the degree of asymmetry in evaluation of the stake (with an arbitrary value attached to the agent 1's evaluation of the stake). As the asymmetry in evaluation decreases, the difference between the attainable payoffs decrease as well. Payoffs equal when there is no asymmetry in the evaluation of the stake.



III. Investing in Conflict Management

Consider now the option of a second instrument. Parties commit themselves to the use of a second instrument in order to affect the outcome of the contest. As mentioned above, the basic model presented hereafter follows and partly modifies the one proposed in Epstein and Hefeker (2003). The ordinary Contest Success Function is modified in order to allow for a second instrument. The two instruments are assumed to be complementary to each other. Then, the use of the second instrument would strengthen the effect of the first instrument. The CSF becomes:

$$p_i = \frac{z_i\left(h_i+1\right)}{z_i\left(h_i+1\right)+z_j\left(h_j+1\right)}, i \neq j, i=1,2 \tag{6}$$

Eventually, the payoff function for each agent becomes:

$$\pi_i^T = \frac{z_i(h_i+1)}{z_1(h_1+1)+z_2(h_2+1)} X_i - z_i - h_i \tag{7}$$

And follows the conditions below:

$$\begin{cases} \dfrac{\partial \pi_i^T}{\partial z_i} > 0 & \dfrac{\partial^2 \pi_i^T}{\partial z_i} < 0 \\ \dfrac{\partial \pi_i^T}{\partial z_j} < 0 & \dfrac{\partial^2 \pi_i^T}{\partial z_j} > 0 \\ \dfrac{\partial \pi_i^T}{\partial h_i} > 0 & \dfrac{\partial^2 \pi_i^T}{\partial h_i} < 0 \\ \dfrac{\partial \pi_i^T}{\partial h_j} < 0 & \dfrac{\partial^2 \pi_i^T}{\partial h_j} > 0 \end{cases} \tag{7.1}$$

Also in this case, a Nash-Cournot behaviour for both agents is assumed. Therefore, each party maximizes its own payoff. The first order conditions for maximization are:

$$\begin{aligned} \frac{\partial \pi_i}{\partial z_i} &= \frac{X_i z_j \left(h_i+1\right)\left(h_j+1\right)}{\left(h_i z_i + h_j z_j + z_i + z_j\right)^2} - 1 = 0 \\ \frac{\partial \pi_i}{\partial h_i} &= \frac{X_i z_i z_j \left(h_j+1\right)}{\left(h_i z_i + h_j z_j + z_i + z_j\right)} - 1 = 0 \\ i &= 1,2, i \neq j \end{aligned} \tag{8}$$

Solving the four first order conditions for both agents yields the equilibrium level both for violent appropriation and CMP efforts:

$$\begin{cases} z_1^{T*} = \dfrac{\delta^2}{\left(\delta^2+1\right)^2} X_1 \qquad h_1^* = X_1 \dfrac{\delta^2}{\left(\delta^2+1\right)^2} - 1 \\ \\ z_2^{T*} = \dfrac{\delta^3}{\left(\delta^2+1\right)^2} X_1 \qquad h_1^* = X_1 \dfrac{\delta^3}{\left(\delta^2+1\right)^2} - 1 \end{cases} \tag{9}$$

Note that $z_i^{*T} > 0, i = 1{,}2$ if and only if

$$h_i > 0 \Leftrightarrow X_1 > \frac{\left(\delta^2+1\right)^2}{\delta^2} \tag{10}$$

That is, in order to have a positive investment in conflict management the value of the stake must be relatively large. also note that the agent with a higher evaluation of the stake arms more than the opponent ($z_1^{*T} > z_2^{*T}$). Another point of interest is that the difference of both instruments exactly equals. That is, in formal terms, $z_1^{*T} - z_2^{*T} = h_1^* - h_2^*$. Then, in order to verify whether the critical points $\left(z_1^{T*}, z_2^{T*}, h_1^*, h_2^*\right)$ represent a maximum it is possible to consider the Hessian matrices for both agents. In the appendix are reported the results. The analysis shows that $\left(z_1^{T*}, z_2^{T*}, h_1^*, h_2^*\right)$ does constitute only a local max.

Using also $X_1 = \delta X_2$, the equilibrium payoff for both agents can be expressed in terms of X_1 and given by:

$$\pi_1^{*T} = \frac{\delta^4 + \delta^2\left(2 - X_1\right) + X_1 + 1}{\left(\delta^2+1\right)^2} \tag{11.1}$$

$$\pi_2^{*T} = \frac{X_1\delta^3\left(\delta^2 - 1\right) + \delta^4 + 2\delta^2 + 1}{\left(\delta^2+1\right)^2} \tag{11.2}$$

Given $\delta \in (0{,}1)$, it would be simple to verify that $\pi_1^T > \pi_2^T$. That is, the agent with a higher evaluation of the stake of the conflict is able to achieve a higher payoff. Consider agent 2's payoff. Plot 2 clearly shows that there is a large range where agent 2's payoffs turn negative. Plot 3 also shows that agent 2' payoffs are higher than the opponent. In particular, look at some numerical examples presented in Table 1. It sheds light over the fact that agent 2 will have an incentive to expend efforts in the second efforts only when the asymmetry in the evaluation of the stake is really large. Otherwise, there is no room for investing resources in conflict management. Therefore, a potential settlement region is feasible only in the presence of a large asymmetry. In sum, it would be possible to write:

Proposition 1: *When agents are identical in abilities and retain different valuations of the stake, the agent with the lower evaluation will expend efforts in conflict management only when the asymmetry is extremely large. Only in such a case a potential settlement region can be established.*

Fig.2, Agent 2's payoffs

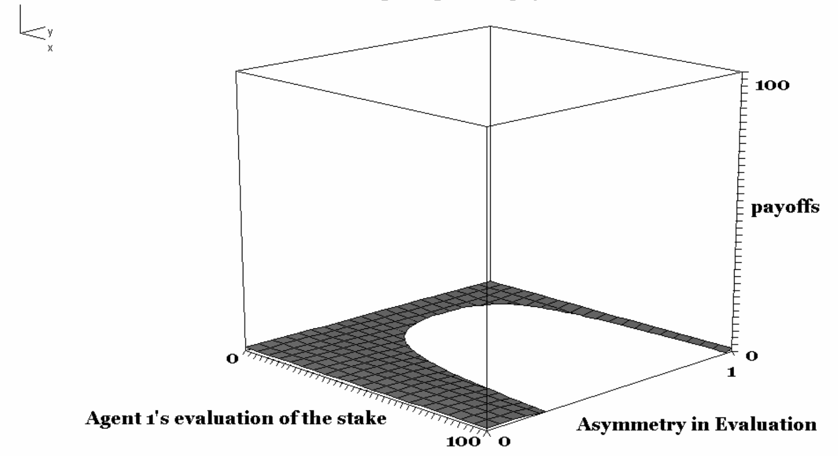


Fig. 3, payoffs for both agents

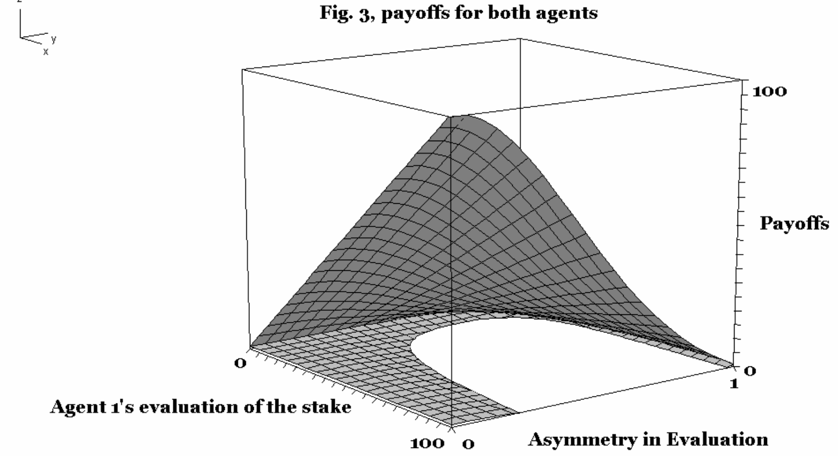


Table 1. Comparison of Payoffs

		Pure Conflict		Conflict Management	
X_1	δ	π_1^{pc}	π_2^{pc}	π_1^T	π_2^T
100	.10	83	0	**98**	**1**
100	.15	76	0	**94**	**1**
100	.20	69	1	90	0
100	.25	64	1	84	0
100	.30	59	2	78	-1
100	.35	55	2	71	-2
100	.40	51	3	63	-3
100	.45	48	4	56	-4
100	.50	44	6	49	-5
100	.55	42	7	42	-6
100	.60	39	8	36	-6
100	.65	37	10	30	-7
100	.70	35	12	24	-7
100	.75	33	14	19	-7
100	.80	31	16	14	-6
100	.85	29	18	10	-5
100	.90	28	20	7	-3
100	.95	26	23	4	-1
250	.10	207	0	**244**	**1**
250	.15	189	1	235	0
250	.20	174	1	223	-1
250	.25	160	3	209	-2
250	.30	148	4	192	-4
250	.35	137	6	175	-6
250	.40	128	8	157	-9
250	.45	119	11	139	-12
250	.50	111	14	121	-14
250	.55	104	17	104	-16
250	.60	98	21	88	-18
250	.65	92	25	72	-19
250	.70	87	30	58	-19
250	.75	82	34	46	-18
250	.80	77	40	34	-16
250	.85	73	45	24	-13
250	.90	69	50	15	-10
250	.95	66	56	8	-5

IV. Measuring Conflict through Statistical Entropy

Conflict is susceptible to measurement. In the standard partial equilibrium contest theory the resources expended do constitute the social cost of contest. Then, recall the optimal choices of violent efforts. It would be possible to write that the total cost under pure conflict is given by:

$$TC^{o} = z_1^{*} + z_2^{*} = \frac{\delta^2 \left(\delta + 1\right)}{\left(\delta^2 + 1\right)} X_1 \qquad (14)$$

Recalling (9) the total cost of contest when both agents expend efforts in a second instrument is given by:

$$TC^{T} = z_1^{*T} + h_1^{*} + z_2^{*T} + h_2^{*} = 2\left(X_1 \frac{\delta^2 \left(\delta + 1\right)}{\left(\delta^2 + 1\right)^2} - 1 \right) \qquad (15)$$

Define $M = \left(\delta^2 \left(\delta + 1\right)\right) / \left(\delta^2 + 1\right)^2$ for compactness. Then, it is possible to write that $TC^T > TC^O$ for $X_1 > \left(2 / M\right)$.

Then a contest with two instruments would be more detrimental for welfare than a contest with only one instrument. It might be maintained that devoting resources to CMPs is wasteful and welfare-immiserizing. This result, of course, is sensitive to the functional form adopted. Hence, establishing a PSR would be welfare-immiserizing. In such a narrow sense, however, a pure conflict scenario would be paradoxically preferable. Establishing a PSR would be less efficient than pure conflict. The latter may seem a desirable scenario. Of course, this kind of conclusion would be sensitive to the modelling adopted but such a ‘positive’ impact would not be theoretically excluded from the start.

It is clear that such a measurement could be unsatisfactory. The analysis needs to be carried further beyond. It would be also reasonable to identify a complementary measure for conflict and conflict management. An appealing idea for a more useful evaluation can be related to those of disorder and randomness. In fact, since conflict is a destructive interaction between two or more parties, it seems reasonable to consider also the degree of uncertainty it spreads. In actual violent appropriative conflicts uncertainty about the final outcome does clearly constitute a characteristic element that should be considered while developing devices to solve the conflict itself.

The measure of uncertainty as the degree of disorder can be captured through the concept of *entropy*. In communication theory and physical sciences entropy is commonly adopted as a measure of the degree of disorder, uncertainty or randomness in a system.[9] The famous reference is the work of

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[9] Consider among others some applications of entropy to social sciences: The Nobel graduate in physic Dennis Gabor in Gabor and Gabor (1958) applied entropy to the measurement of

Shannon and Weaver (1949) which posed the quantitative foundations of information theory. In such a framework, entropy is defined as:

$$E(p_1,...,p_n) = -k\sum_{i=1}^{n} p_i \ln p_i, \qquad (16)$$

where k is an arbitrary constant which can be set to unity without loss of generality.[10] Note that, following the prevailing literature, p_i can be interpreted in two different ways. First, it can represent a probability. Secondly, it can represent a share of some total quantity. Then, this flexible interpretation does fit well with the assumption of risk-neutrality and the following properties of the CSF.

The greatest disorder would occur when all outcomes have the same probability, i.e. $p_i = 1/n$ for $i = 1,...n$. The degree of disorder is given by: $E(1/n,...,1/n) = k \ln n$. For instance, in the limiting case of $n = 2$ and $k = 1$ the degree of disorder will be given by $E = \ln(2)$. Then, consider the pure-conflict case when agents use only one instrument. Thus, in such a case it would be simple to demonstrate that entropy is given by:

$$E^O(p_1, p_2) = \frac{(\delta+1)\ln(\delta+1) - \delta \ln(\delta)}{(\delta+1)} \qquad (17)$$

Consider now the case of conflict management. In such a case the entropy is given by:

$$E^T(p_1, p_2) = \frac{(\delta^2+1)\ln(\delta^2+1) - 2\delta^2 \ln(\delta)}{(\delta^2+1)} \qquad (18)$$

It would not be difficult to show that $E^O > E^T$ for $\delta \in (0,1)$. This means that in presence of efforts devoted to conflict management the degree of disorder is lower. In particular, the point of interest is that as the asymmetry in evaluation decreases the degree of disorder and turbulence increases. This point sheds further light upon the results of the foregoing sections. It had been showed that as the asymmetry decreases, agents have no longer incentives to invest in any CMPs. Then, as the incentives to conflict increase the degree of disorder increases. In particular, as the degree of asymmetry approaches the unity, the difference in the degree of disorder decreases.

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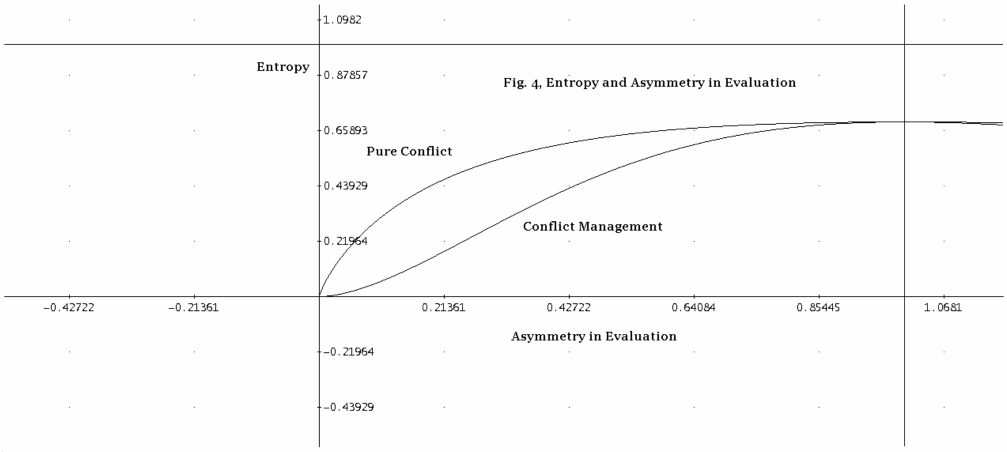
social and economic freedom. Entropy has been also proposed as a measure of competitiveness and diversification in market structure: see Attaran and Zwick (1989) and Horowitz and Horowitz (1968).

[10] The form adopted here is the one presented in Campiglio (1999), ch.4, and Liossatos (2005).



In order to refine better the use of entropy for measurement of conflicts it would be also useful to introduce the concept of *relative entropy*. Relative entropy is defined as the ratio of the actual to the maximum entropy in a system. Relative entropy does not give any information about the degree of disorder That is, it would be useful to recognize the extent to which the degree of disorder approaches the maximum level attainable. In formal terms it is possible to write the relative entropy as: $RE = E / Ln(n)$. Then, relative entropy for pure conflict and conflict management respectively will be:

$$RE^{o}_{PC}(p_1, p_2) = \frac{(\delta+1)\ln(\delta+1) - \delta\ln(\delta)}{(\delta+1)\ln(2)} \tag{19}$$

$$RE^{T}_{CM}(p_1, p_2) = \frac{(\delta^2+1)\ln(\delta^2+1) - 2\delta^2\ln(\delta)}{(\delta^2+1)\ln(2)} \tag{20}$$

The relative entropy ratio would range from a value of zero for no entropy to a value of one when the maximum degree of entropy is attained.

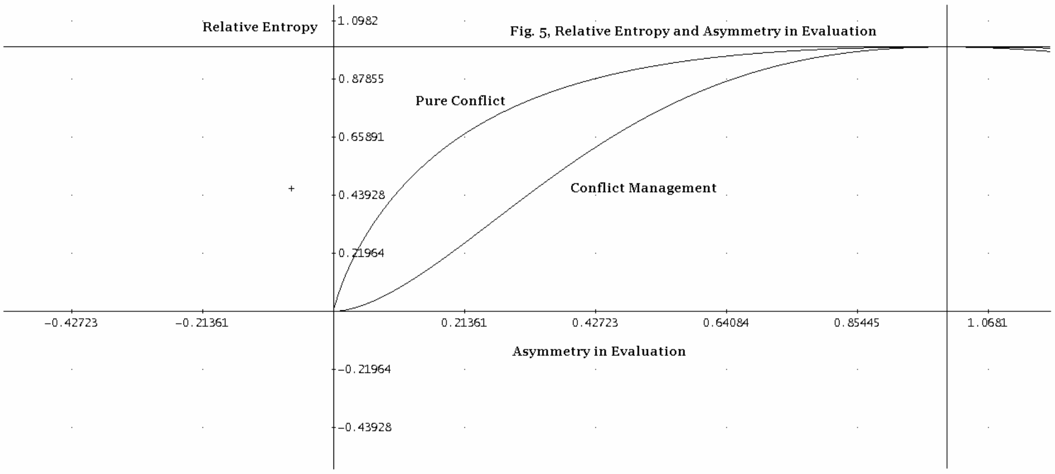


Table 2 presents the calculations for entropy and relative entropy respectively.

Table 2. Entropy and Relative Entropy

	Pure Conflict		Conflict Management	
Asymmetry in Evaluation	Entropy	Relative Entropy	Entropy	Relative Entropy
0.05	0.19	0.28	0.02	0.03
0.15	0.39	0.56	0.11	0.15
0.25	**0.50**	**0.72**	**0.22**	**0.32**
0.35	0.57	0.83	0.34	0.50
0.45	0.62	0.89	0.45	0.65
0.55	0.65	0.94	0.54	0.78
0.65	0.67	0.97	0.61	0.88
0.75	0.68	0.99	0.65	0.94
0.85	0.69	1	0.68	0.98
0.95	0.69	1	0.69	1

The figures clearly show that entropy is lower in the presence of CMPs. At the same time, it is worth noting that whenever agents expend resources in conflict management, the system fails to achieve its maximum possible degree of entropy at a relatively lower rate. In sum it would be possible to write:

Proposition 2: *When agents are asymmetrical in evaluation of the stake and identical in fighting abilities, the conflict management scenario appears to be less turbulent than the pure conflict scenario.*

Although entropy appears to be an appealing concept to evaluate conflicts and contests, some points should be highlighted. First, a remarkable point of interest which would deserve further attention is related exactly to the functional form of CSF adopted. In particular, if entropy is used as a measure of the degree of disorder it would be clear that it will depend directly upon (i) the technology of conflict; (ii) the number of contestants; (iii) the abilities of contestants; (iv) the consideration of a perceived *ex-ante* destruction parameter and (v) the existence of institutional constraints or noises. In other words, this also means that in partial equilibrium models the impact of incentives (that is the value of the contested stake) to conflict or to settle does not affect the degree of disorder.

The result of this section also raises questions about a trade-off between the welfare losses and the degree of disorder. There could be equilibria where a lower degree of disorder could be attainable with a higher waste of resources. Consider for example the case of $\delta = .15$. In such a case both (c.1) and (c.2) hold (that is a PSR can be established) and the degree of entropy is lower in conflict management scenario. However, the social waste of resources is higher than in pure conflict scenario. This simple consideration would represent a crucial point for a future research agenda. A trade-off between the wasting of resources and the degree of turbulence clearly emerges.

A third necessary point in order to avoid a common misunderstanding is related to the Second Law of Thermodynamics. This states that the total entropy tends to increase over time approaching a maximum value. However, the Second Law of Thermodynamics holds only for isolated systems. Needless to say, any social or international conflict do not occur in an isolated system. It constantly interacts with the environment. Hence, the Second Law of Thermodynamics would not hold for conflict interactions.

V. Heterogenous Abilities

Up to this point I considered the simplest case never occurring in reality, namely two agents with identical abilities. Of course, in reality agents are heterogeneous. Then, in order to evaluate the impact of heterogenous abilities, let $a \in (0,\infty)$ denote a positive weight capturing the degree of asymmetry in abilities between contestants. It will enter the CSF which becomes:

$$p_i\left(a, z_i, h_i\right) = \frac{az_i\left(h_i + 1\right)}{az_i\left(h_i + 1\right) + z_j\left(h_j + 1\right)} \tag{21}$$

It is clear that when $a \in (0,1)$, the first agent is less-endowed in abilities with respect to the opponent. When $a \in (1,\infty)$, the first agent is more powerful and more endowed in abilities than agent two. When $a = 1$, the two agents are identical in abilities.

Consider first the case of pure conflict. Then agents will use only the first instrument. The CSF reduces to:

$$\begin{aligned} &p_i\left(a, z_i\right) = \frac{az_i}{az_i + z_j} \\ &i \neq j, i = 1,2 \end{aligned} \tag{22}$$

Eventually the payoff functions become:

$$\begin{aligned} &\pi_i^a = \frac{az_i}{az_i + z_j} X_i - z_i \\ &i \neq j, i = 1,2 \end{aligned} \tag{23}$$

Using the ordinary process of maximization the equilibrium payoffs for the agents are:

$$\pi_1^a * = \frac{a^2}{\left(a + \delta\right)^2} X_1 \tag{24.1}$$

$$\pi_2^a * = \frac{\delta^3}{\left(a + \delta\right)^2} X_1 \tag{24.2}$$

A first point of interest is that – differently from the simplest case – the agent with a higher evaluation of the stake does not always attain a higher payoff. In fact, it is possible to demonstrate that:

$$\pi_1^{a*} > \pi_2^{a*} \Leftrightarrow a > \delta^{3/2} \tag{25}$$

since $\delta \in (0,1)$, it is clear that when $a < 1$ – namely, the agent with a higher evaluation of the stake is less endowed in fighting abilities – the agent with a lower evaluation of the stake is able to attain a higher payoff. Indeed, there is an *ability effect* counterbalancing the *incentive effect* which does depend upon the evaluation of the stake.

Consider now the option for the second instrument. The payoff functions become now:

$$\pi_1^{aT} = \frac{az_1(h_1+1)}{az_1(h_1+1)+z_2(h_2+1)} X_1 - z_1 - h_1 \tag{26.1}$$

$$\pi_2^{aT} = \frac{z_2(h_2+1)}{az_1(h_1+1)+z_2(h_2+1)} \delta X_1 - z_2 - h_2 \tag{26.2}$$

which follows the conditions presented in (7.1) and also:

$$\begin{cases} \dfrac{\partial \pi_1^a}{\partial a} > 0 \\ \dfrac{\partial^2 \pi_1^a}{\partial a} < 0 \\ \dfrac{\partial \pi_2^a}{\partial a} < 0 \\ \dfrac{\partial \pi_2^a}{\partial a} > 0 \end{cases} \tag{27}$$

The first order conditions for a maximum are given by:

$$\frac{\partial \pi_i^a}{\partial z_i} = \frac{az_j X_i (h_1+1)(h_2+1)}{\left[az_1(h_1+1)+z_2(h_2+1)\right]^2} - 1 = 0 \tag{28.1}$$

$$\frac{\partial \pi_i^a}{\partial h_i} = \frac{aX_i z_i z_j (h_j+1)}{\left[az_1(h_1+1)+z_2(h_2+1)\right]^2} - 1 = 0 \tag{28.2}$$

Solving the four first order conditions for both agents yields the equilibrium level both for violent appropriation and CMP efforts:

$$
\begin{cases}
z_1^{a*} = \dfrac{a\delta^2}{\left(\delta^2 + a\right)^2} X_1 & h_1^{a*} = X_1 \dfrac{a\delta^2}{\left(\delta^2 + a\right)^2} - 1 \\[2ex]
z_2^{a*} = \dfrac{a\delta^3}{\left(\delta^2 + a\right)^2} X_1 & h_1^{a*} = X_1 \dfrac{a\delta^3}{\left(\delta^2 + a\right)^2} - 1
\end{cases}
\tag{29}
$$

Note that also in this case $z_i^{a*} > 0, i = 1,2$ and that:

$$
\begin{aligned}
&h_1^{a*} > 0 \Leftrightarrow X_1 > \frac{\left(\delta^2 + a\right)^2}{a\delta^2}; \\
&h_2^{a*} > 0 \Leftrightarrow X_1 > \frac{\left(\delta^2 + a\right)^2}{a\delta^3}.
\end{aligned}
\tag{30}
$$

that is, in order to have a positive effort in the second instrument it is necessary to have a relatively large value for the stake. Eventually the payoffs are given by:

$$
\begin{aligned}
&\pi_1^{a*}\left(z_1^*, z_2^*, h_1^*, h_2^*\right) = X_1 \frac{a\left(a - \delta^2\right)}{\left(\delta^2 + a\right)^2} + 1; \\
&\pi_2^{a*}\left(z_{1,}^* z_2^*, h_1^*, h_2^*\right) = X_1 \frac{\delta^3\left(\delta^2 - a\right)}{\left(\delta^2 + a\right)^2} + 1.
\end{aligned}
\tag{31}
$$

However, also in this case it is necessary to verify whether $\pi_1\left(z_1^*, z_2^*, h_1^*, h_2^*\right)$ and $\pi_2^{a*}\left(z_1^*, z_2^*, h_1^*, h_2^*\right)$ constitute an optimum. Computations presented in the appendix show that (31) does constitute only a local max.

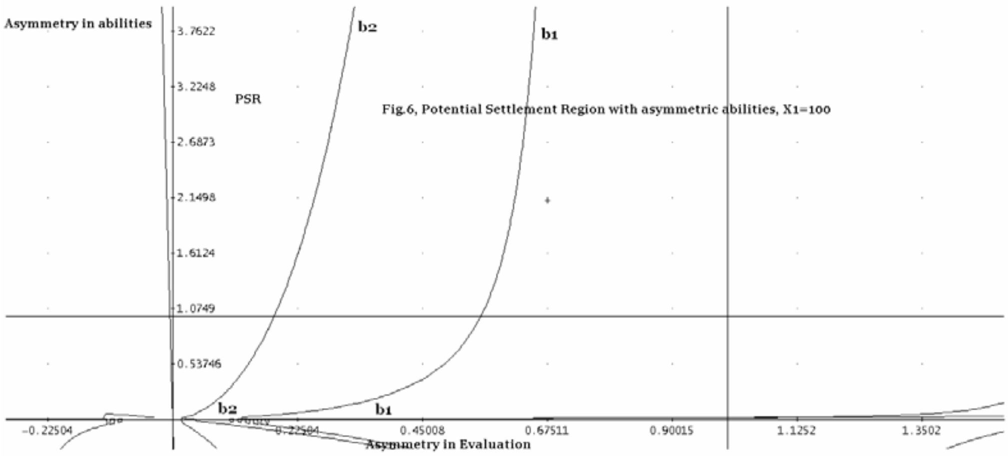
Comparing the equilibrium level of agents’ payoffs it is possible to write that:

$$
\pi_1^{a*} > \pi_2^{a*} \Leftrightarrow a > \delta^2 \tag{32}
$$

that is the agent with a higher evaluation of the stake retains a higher level of payoff if and only if the degree of asymmetry in abilities is higher than a certain level. Otherwise, when agent 1 is not sufficiently endowed he will attain a lower payoff than the opponent. Also in this case, there is an ability effect counterbalancing the incentive effect which does depend upon the evaluation of the stake. Through a comparison of (25) and (30) it is worth noting that in pure conflict the condition allowing for higher payoffs accruing to agent 2 is stricter.

As in the previous sections, to check for the existence of a PSR a comparison between equilibrium payoffs is needed. Then, recall (24) and (31). The plot below shows which area in the (a,δ) parameter space constitutes a PSR. That is, the graph shows areas where payoffs accruing to the agents under conflict management are greater than those attainable in a pure conflict scenario. All the points on the left of $b2b2$ curve constitute a PSR. The points on the left of curve $b1b1$ would represent the region where the CMC condition hold only for agent 1.

In particular, it does appear clear that when agent 1 is less endowed in abilities (namely when $a<1$) the room for settlement is smaller compared with the case when agent 1 is more endowed in abilities.



Due to the analytical complexity I present some numerical examples in the table below. Figures in bold denote the room for a PSR. It does appear that – *ceteris paribus* – when agent 1 is less endowed in abilities there is smaller room for a PSR. As agent 1 becomes stronger in abilities, the room for a PSR slightly enlarges. In sum it could be maintained:

Proposition 3: *When agents are asymmetrical both in evaluation of the stake and in fighting abilities then: whenever the agent with the higher evaluation of the stake is less endowed in fighting abilities there is a smaller room for a PSR.*

Table 3. Payoffs with asymmetrical abilities

			Pure Conflict		Conflict Management	
X_1	δ	a	π_1^a	π_2^a	π_1^{aT}	π_2^{aT}
100	.05	.3	**73.5**	**0.1**	**98.5**	**1.0**
100	0.1	.3	**56.3**	**0.6**	**91.5**	**0.7**
100	.15	.3	44.4	1.7	81.0	0.1
100	.2	.3	36.0	3.2	68.5	-0.8
100	.25	.3	29.8	5.2	55.2	-1.8
100	.3	.3	25.0	7.5	42.4	-2.7
100	.35	.3	21.3	10.1	30.8	-3.3
100	.4	.3	18.4	13.1	20.8	-3.2
100	.45	.3	16.0	16.2	12.6	-2.5
100	.5	.3	14.1	19.5	6.0	-1.1
100	.55	.3	12.5	23.0	0.8	1.1
100	.6	.3	11.1	26.7	-3.1	4.0
100	.65	3	10.0	30.4	-6.0	7.4
100	.7	.3	9.0	34.3	-8.1	11.4
100	.75	.3	8.2	38.3	-9.6	15.9
100	.8	.3	7.4	42.3	-10.5	20.7
100	.85	.3	6.8	46.4	-11.1	25.8
100	.9	.3	6.3	50.6	-11.4	31.2
100	.95	.3	5.8	54.9	-11.5	36.7
100	0.05	0.9	**89.75**	**0.01**	**100.17**	**0.99**
100	0.1	0.9	**81.00**	**0.10**	**97.73**	**0.89**
100	0.15	0.9	**73.47**	**0.31**	**93.80**	**0.65**
100	0.2	0.9	66.94	0.66	88.60	0.22
100	0.25	0.9	61.25	1.18	82.36	-0.41
100	0.3	0.9	56.25	1.88	75.38	-1.23
100	0.35	0.9	51.84	2.74	67.93	-2.19
100	0.4	0.9	47.93	3.79	60.27	-3.22
100	0.45	0.9	44.44	5.00	52.65	-4.23
100	0.5	0.9	41.33	6.38	45.23	-5.14
100	0.55	0.9	38.53	7.91	38.19	-5.87
100	0.6	0.9	36.00	9.60	31.61	-6.35
100	0.65	0.9	33.71	11.43	25.57	-6.50
100	0.7	0.9	31.64	13.40	20.10	-6.28
100	0.75	0.9	29.75	15.50	15.20	-5.66
100	0.8	0.9	28.03	17.72	10.87	-4.61
100	0.85	0.9	26.45	20.05	7.07	-3.14
100	0.9	0.9	25.00	22.50	3.77	-1.24
100	0.95	0.9	23.67	25.05	0.93	1.07

100	0.05	1.2	**92.2**	**0.0**	**100.4**	**1.0**
100	0.1	1.2	**85.2**	**0.1**	**98.5**	**0.9**
100	0.15	1.2	**79.0**	**0.2**	**95.5**	**0.7**
100	0.2	1.2	73.5	0.4	91.5	0.4
100	0.25	1.2	68.5	0.7	86.6	-0.1
100	0.3	1.2	64.0	1.2	81.0	-0.8
100	0.35	1.2	59.9	1.8	74.9	-1.6
100	0.4	1.2	56.3	2.5	68.5	-2.6
100	0.45	1.2	52.9	3.3	61.9	-3.6
100	0.5	1.2	49.8	4.3	55.2	-4.6
100	0.55	1.2	47.0	5.4	48.7	-5.6
100	0.6	1.2	44.4	6.7	42.4	-6.5
100	0.65	1.2	42.1	8.0	36.4	-7.1
100	0.7	1.2	39.9	9.5	30.8	-7.5
100	0.75	1.2	37.9	11.1	25.6	-7.7
100	0.8	1.2	36.0	12.8	20.8	-7.5
100	0.85	1.2	34.3	14.6	16.5	-6.9
100	0.9	1.2	32.7	16.5	12.6	-6.0
100	0.95	1.2	31.2	18.5	9.1	-4.8
100	0.05	2	**95.181**	**0.003**	**100.626**	**0.994**
100	0.1	2	**90.703**	**0.023**	**99.512**	**0.951**
100	0.15	2	**86.533**	**0.073**	**97.687**	**0.837**
100	0.2	2	**82.645**	**0.165**	**95.195**	**0.623**
100	0.25	2	79.012	0.309	92.093	0.288
100	0.3	2	75.614	0.510	88.452	-0.181
100	0.35	2	72.431	0.776	84.352	-0.787
100	0.4	2	69.444	1.111	79.875	-1.524
100	0.45	2	66.639	1.518	75.108	-2.377
100	0.5	2	64.000	2.000	70.136	-3.321
100	0.55	2	61.515	2.559	65.038	-4.327
100	0.6	2	59.172	3.195	59.891	-5.360
100	0.65	2	56.960	3.911	54.762	-6.382
100	0.7	2	54.870	4.705	49.709	-7.354
100	0.75	2	52.893	5.579	44.783	-8.236
100	0.8	2	51.020	6.531	40.027	-8.991
100	0.85	2	49.246	7.561	35.471	-9.585
100	0.9	2	47.562	8.668	31.141	-9.987
100	0.95	2	45.964	9.852	27.055	-10.169

Since the parameter enters directly the CSF, it is useful compute the degree of entropy. For sake of brevity I only report equations for relative entropy that are:

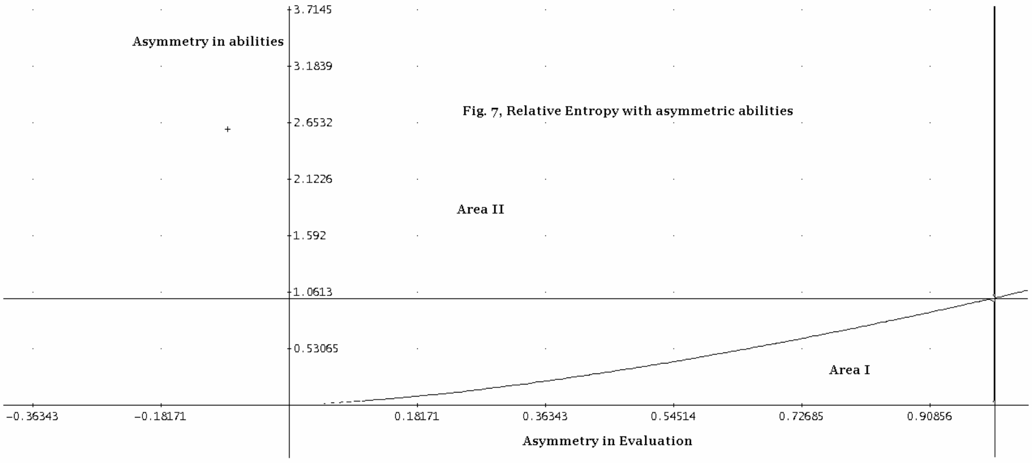
$$RE_a^{pc}(p_1, p_2) = \frac{(\delta + a)\ln(\delta + a) - a\ln(a) - \delta\ln(\delta)}{(\delta + a)\ln(2)} \qquad (33)$$

$$RE_{CM}^{a}(p_1, p_2) = \frac{(\delta^2 + a)\ln(\delta^2 + a) - a\ln(a) - 2\delta^2\ln(\delta)}{(\delta^2 + a)\ln(2)} \qquad (34)$$

A point of interest is the level of entropy attainable in presence of asymmetry in fighting abilities. Differently from the scenario where agents were identical in abilities, it is possible to verify that $RE_{CM}^{a} > RE_{PC}^{a}$ for specific combinations of the given parameters. In particular, when the agent with the higher evaluation of the stake has a lower fighting ability (namely when $a < 1$) the level of entropy is higher in the case of conflict management with respect to the pure conflict case when the asymmetry in the evaluation of the stake is not so large (see plot 7 and see figures in bold in the following table).

By contrast, when $a > 1$ the pure conflict scenario is undoubtedly more turbulent than conflict management scenario. In the graph, area II contains all the points where $RE_{PC}^{a} > RE_{CM}^{a}$. In other words, conflict management appears to lead to a less turbulent scenario even if there is a fighting preponderance of a party.

This result is not trivial and hardliners would not appreciate it. In fact, this contrasts the common belief according to which a power imbalance can lead to a more stable scenario where a party acquiesces to a credible threat sent by a stronger opponent. That is, a stable domination of one party over another does not seem to emerge. However, area II also contains points where $a < 1$. In particular, it would be possible to show that the degree of entropy is greater in pure conflict if and only if $a > \delta^{3/2}$. To enrich the meaning of this outcome, consider also that with $a > \delta^{3/2}$ both agents would invest more in arms. However, results of table 3 show that there is no room for negotiating under those conditions. To summarise, it is possible to write:



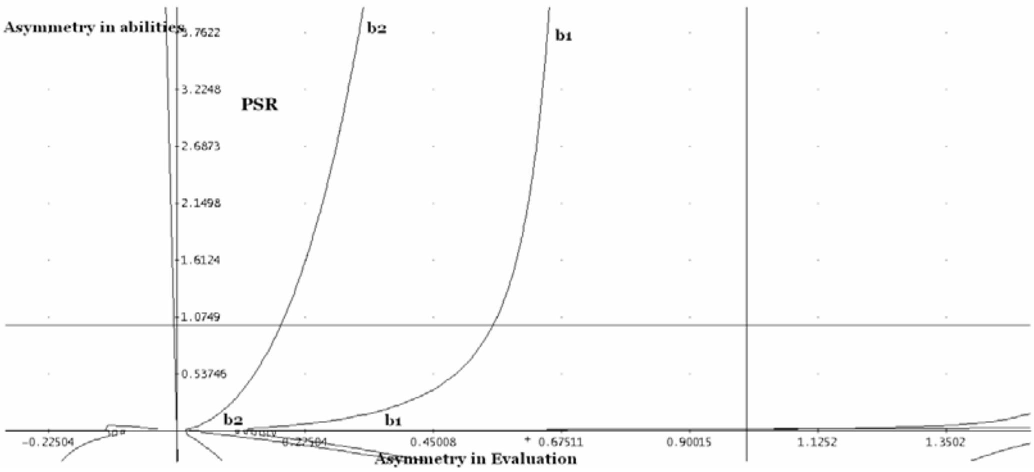
Proposition 4: *When agents are asymmetrical both in evaluation of the stake and in fighting abilities, then: (i) when the agent with the higher evaluation of stake is also the more endowed in abilities the pure conflict scenario has always a greater degree of entropy than the conflict management scenario; (ii) when the agent with the higher evaluation of stake is also the less endowed in abilities, as the asymmetry in evaluation decreases, the conflict management scenario appears to be more turbulent than the pure conflict scenario.*

Table 4, Entropy and Relative Entropy with asymmetric abilities

		Pure Conflict		Conflict Managment	
Asymmetry in Evaluation	Asymmetry in abilities	Entropy	Relative Entropy	Entropy	Relative Entropy
.05	.25	.451	.650	.056	.080
.25	.25	.693	1,000	.500	.722
.5	.25	.637	.918	.693	1,000
.75	.25	.562	.811	.617	.890
.95	.25	.512	.738	.523	.755
.05	.5	.305	.439	.031	.045
.25	.5	.637	.918	.349	.503
.5	.5	.693	1,000	.637	.918
.75	.5	.673	.971	.691	.998
.95	.5	.644	.929	.651	.940
.05	1,5	.143	.206	.012	.018
.25	1,5	.410	.592	.168	.242

.5	1,5	.562	.811	.410	.592
.75	1,5	.637	.918	.586	.845
.95	1,5	.668	.963	.662	.955
.05	2	.115	.165	.010	.014
.25	2	.349	.503	.136	.196
.5	2	.500	.722	.349	.503
.75	2	.586	.845	.526	.759
.95	2	.628	.907	.620	.894
.05	4	.067	.096	.005	.008
.25	4	.224	.323	.079	.115
.5	4	.349	.503	.224	.323
.75	4	.436	.629	.373	.539
.95	4	.489	.705	.478	.689

Finally, overlapping the graph and using as arbitrary value $X_1 = 100$ it is possible to show that in correspondence of a PSR the efforts in conflict management lead also to a lower degree of entropy.



VI. Destructive Conflict and Existence of a Sharing Rule

This section introduces two extensions. First, conflict is supposed to be destructive, namely only a fraction of the contested stake will be attained by agents. In simpler words, agents are aware that the higher the level of violent efforts, the lower is the contestable stake. This is easier to understand in a general equilibrium framework. In such a case, since agents would be supposed to split their own resources endowment between 'butter' and 'guns', a destruction parameter denoted by $\beta \in (0,1)$ captures the expected foregone fraction of the positive stake due to the violent activity. In other words, as β increases, the conflict becomes less and less destructive. The destruction parameter can be interpreted as an *ex-ante* perceived evaluation of conflict losses. Both agents share the same perception of expected destruction. Albeit unrealistic, such a strict limiting assumption prevails in literature and, for analytical simplicity, I shall keep this throughout.

Secondly, agents divide the stake according with a particular rule of division. In fact, following and extending Nitzan (1991), Garfinkel and Skaperdas (2000) and Caruso (2006) it is assumed that each agent's share of the 'pie' will be a weighted combination of two possible rules: (i) the CSF and (ii) a symmetric split-of-surplus rule of division. The latter would correspond to the appropriate axiomatic outcome as indicated in bargaining literature. The relative weights are determined by the destruction parameter. According to this construction, the stake of the contest can be disposed in one of two ways: through conflict management or through a peaceful and predefined division. Then the payoff functions become:

$$\pi_i^s = \left[\frac{\beta z_i (h_i + 1)}{z_i (h_i + 1) + z_j (h_j + 1)} + \frac{(1-\beta)}{2} \right] X_i - z_i - h_i, i = 1,2, i \neq j \tag{35}$$

the first order conditions for maximization are:

$$\frac{\partial \pi_i^s}{\partial z_i} = X_i \frac{\beta z_j (h_1 + 1)(h_2 + 1)}{(h_1 z_1 + h_2 z_2 + z_1 + z_2)^2} - 1 = 0 \tag{36.1}$$

$$\frac{\partial \pi_i^s}{\partial h_i} = X_i \frac{\beta z_i z_j (h_j + 1)}{(h_1 z_1 + h_2 z_2 + z_1 + z_2)} - 1 = 0 \tag{36.2}$$

Solving the first order conditions and using $X_2 = \delta X_1, \delta \in (0,1)$ the optimal choices of both violent efforts and conflict management efforts are given by:

$$\begin{cases} z_1^{*s} = \beta \dfrac{\delta^2}{\left(\delta^2+1\right)^2} X_1 & h_1^{*s} = \beta \dfrac{\delta^2}{\left(\delta^2+1\right)^2} X_1 - 1 \\ z_2^{*s} = \beta \dfrac{\delta^3}{\left(\delta^2+1\right)^2} X_1 & h_2^{*s} = \beta \dfrac{\delta^3}{\left(\delta^2+1\right)^2} X_1 - 1 \end{cases} \tag{37}$$

Eventually the payoffs accruing to the agents are:

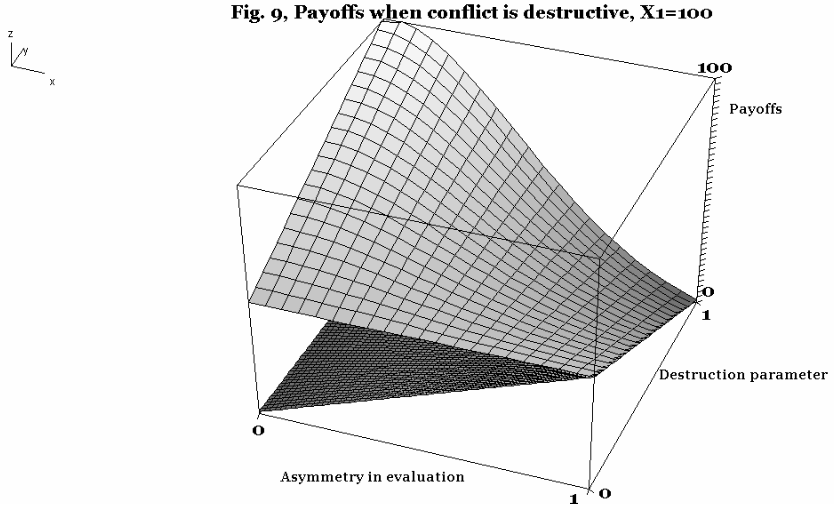
$$\pi_1^{s*} = \frac{X_1}{2}\left[1 - \beta \frac{\left(\delta^2-1\right)^2}{\left(\delta^2+1\right)^2}\right] + 1 \tag{38}$$

$$\pi_2^{s*} = \frac{\delta X_1}{2}\left[1 + \beta \frac{\left(\delta^2-1\right)^2}{\left(\delta^2+1\right)^2}\right] + 1 \tag{39}$$

However, also in this case it is necessary to verify whether $\pi_1\left(z_1^*, z_2^*, h_1^*, h_2^*\right)$ and $\pi_2^{a*}\left(z_1^*, z_2^*, h_1^*, h_2^*\right)$ constitute an optimum. The computations presented in the appendix show that it does constitute a local max. (please see the appendix).

Since $\delta \in (0,1)$, It is clear that $\pi_1^{s*} > \pi_2^{s*}$. The plot below shows the payoff accruing to the agents.

Fig. 9, Payoffs when conflict is destructive, X1=100



As in the previous sections, through comparative statics it is possible to evaluate whether agents are willing to commit themselves to manage the conflict. In a pure conflict scenario equilibrium payoffs are:

$$
\begin{cases} \pi_1^{o*} = \dfrac{\beta}{(\delta+1)^2} X_1 \\ \pi_2^{o*} = \beta \dfrac{\delta}{(\delta+1)^2} X_1 \end{cases} \tag{40}
$$

Thus, according to the argument of this paper, a PSR does exist if and only if:

$$
\frac{X_1}{2}\left[1-\beta\frac{(\delta^2-1)^2}{(\delta^2+1)^2}\right]+1>\frac{\beta}{(\delta+1)^2}X_1 \tag{41}
$$

$$
\frac{\delta X_1}{2}\left[1+\beta\frac{(\delta^2-1)^2}{(\delta^2+1)^2}\right]+1>\beta\frac{\delta^3}{(\delta+1)^2}X_1 \tag{42}
$$

As in the previous sections, consider an arbitrary value for the contested stake, namely $X_1 = 100$ and use (41) and (42) as strict equalities. The plot below shows that a potential settlement region does exist. In particular, the PSR is given by the area under the curve $a2a2$.

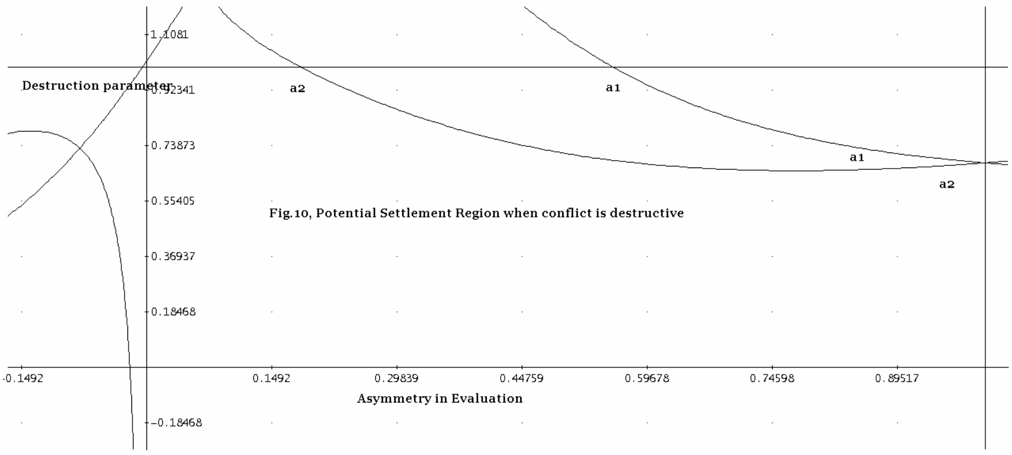


Fig.10, Potential Settlement Region when conflict is destructive

The plot clearly shows that a large PSR is feasible under some combinations of the destruction parameter and the degree of asymmetry in evaluation. The numerical examples presented in table 5 explicit this result. Once the destruction parameter is considered, agents clearly also take into account the opportunity cost of the conflict. That is, the perceived loss due the destructive interaction affects the optimal choice of both agents. Then, it favours an investment in conflict management in order to reach a peaceful settlement. However, there is a contrasting effect between the asymmetry in the evaluation of the stake and the destruction parameter. In particular, as the conflict is supposed to be less and less destructive, the room for a settlement vanishes. In sum, it would be possible to write:

Proposition 5. *if agents are asymmetric in evaluations of the stake and retain an equal ex-ante perceived evaluation of conflict losses, then (i) a potential settlement region does exist and it depends upon different combinations of perceived destruction and the asymmetry in the evaluation of the stake; (ii) as the conflict becomes less and less destructive, a potential settlement region is no longer feasible.*

Take into consideration the level of entropy. It does depend directly upon the mixed effect of both the level of asymmetry and the destruction parameter. For sake of brevity I only report equations for relative entropy that are:

$$RE_{PC}^{o}(p_1, p_2) = -\frac{\beta\left(\left(\delta+1\right)\ln\left(\frac{\beta}{\delta+1}\right)+\delta\ln\left(\delta\right)\right)}{\left(\delta+1\right)\ln(2)}. \tag{43}$$

$$RE_{CM}^{s}(p_1, p_2) = -\frac{\beta\left(\left(\delta^2+1\right)\ln\left(\frac{\beta}{\delta^2+1}\right)+2\delta^2\ln\left(\delta\right)\right)}{\left(\delta^2+1\right)\ln(2)}. \tag{44}$$

Through a comparison of (43) and (44) it is clear that $RE_{PC}^{o} > RE_{CM}^{s}$. Namely, the conflict management scenario exhibits a lower level of turbulence. However, as δ approaches to unity, the level of both entropy and relative entropy converge. As the conflict becomes less and less destructive the entropy reaches its maximum when the evaluations of the stake almost equal. In sum,

Proposition 6. *if agents are asymmetric in evaluations of the stake and retain an equal ex-ante perceived evaluation of conflict losses, then (i) the conflict management scenario appears to be less turbulent than the pure conflict scenario*; (ii) *as the conflict becomes less destructive the degree of entropy reaches its max when evaluation of the stake are slightly different.*

Table 5. Payoffs when Conflict is Destructive

			Pure Conflict		Conflict Management with sharing rule	
X_1	δ	β	π_1^o	π_2^o	π_1^s	π_2^s
100	0.05	0.3	27.21	0.00	66.79	3.76
100	0.1	0.3	24.79	0.02	66.15	5.51
100	0.15	0.3	22.68	0.08	65.14	7.25
100	0.2	0.3	20.83	0.17	63.79	8.95
100	0.25	0.3	19.20	0.30	62.17	10.62
100	0.3	0.3	17.75	0.48	60.35	12.26
100	0.35	0.3	16.46	0.71	58.41	13.87
100	0.4	0.3	15.31	0.98	56.42	15.49
100	0.45	0.3	14.27	1.30	54.44	17.13
100	0.5	0.3	13.33	1.67	52.53	18.83
100	0.55	0.3	12.49	2.08	50.73	20.59
100	0.6	0.3	11.72	2.53	49.08	22.46
100	0.65	0.3	11.02	3.03	47.61	24.45
100	0.7	0.3	10.38	3.56	46.33	26.58
100	0.75	0.3	9.80	4.13	45.26	28.86
100	0.8	0.3	9.26	4.74	44.39	31.32
100	0.85	0.3	8.77	5.38	43.74	33.96
100	0.9	0.3	8.31	6.06	43.29	36.78
100	0.05	0.5	45.35	0.01	76.64	3.25
100	0.1	0.5	41.32	0.04	75.56	4.49
100	0.15	0.5	37.81	0.13	73.84	5.68
100	0.2	0.5	34.72	0.28	71.54	6.81
100	0.25	0.5	32.00	0.50	68.78	7.86
100	0.3	0.5	29.59	0.80	65.67	8.84
100	0.35	0.5	27.43	1.18	62.34	9.78
100	0.4	0.5	25.51	1.63	58.90	10.69
100	0.45	0.5	23.78	2.17	55.47	11.63
100	0.5	0.5	22.22	2.78	52.13	12.63
100	0.55	0.5	20.81	3.46	48.95	13.72
100	0.6	0.5	19.53	4.22	46.00	14.96
100	0.65	0.5	18.37	5.04	43.32	16.38
100	0.7	0.5	17.30	5.93	40.93	18.00
100	0.75	0.5	16.33	6.89	38.84	19.85
100	0.8	0.5	15.43	7.90	37.07	21.95
100	0.85	0.5	14.61	8.97	35.61	24.31
100	0.9	0.5	13.85	10.10	34.45	26.94

100	0.05	0.9	81.63	0.01	96.34	2.25
100	0.1	0.9	74.38	0.07	94.38	2.45
100	0.15	0.9	68.05	0.23	91.24	2.56
100	0.2	0.9	62.50	0.50	87.04	2.52
100	0.25	0.9	57.60	0.90	82.00	2.34
100	0.3	0.9	53.25	1.44	76.31	2.01
100	0.35	0.9	49.38	2.12	70.20	1.58
100	0.4	0.9	45.92	2.94	63.87	1.10
100	0.45	0.9	42.81	3.90	57.53	0.62
100	0.5	0.9	40.00	5.00	51.33	0.22
100	0.55	0.9	37.46	6.23	45.40	-0.01
100	0.6	0.9	35.16	7.59	39.84	-0.03
100	0.65	0.9	33.06	9.08	34.73	0.24
100	0.7	0.9	31.14	10.68	30.11	0.85
100	0.75	0.9	29.39	12.40	26.01	1.83
100	0.8	0.9	27.78	14.22	22.43	3.21
100	0.85	0.9	26.30	16.15	19.35	5.01
100	0.9	0.9	24.93	18.17	16.77	7.25

Table 6. Entropy and Relative Entropy when Conflict is Destructive

		Pure Conflict		Destructive conflict with Sharing rule	
β	δ	Entropy	Relative Entropy	Entropy	Relative Entropy
0.3	0.05	0.42	0.60	0.37	0.53
0.3	0.1	0.45	0.65	0.38	0.55
0.3	0.15	0.48	0.69	0.39	0.57
0.3	0.2	0.50	0.72	0.41	0.59
0.3	0.25	0.51	0.74	0.43	0.62
0.3	0.3	0.52	0.75	0.45	0.64
0.3	0.35	0.53	0.77	0.46	0.67
0.3	0.4	0.54	0.78	0.48	0.69
0.3	0.45	0.55	0.79	0.50	0.72
0.3	0.5	0.55	0.80	0.51	0.74
0.3	0.55	0.56	0.80	0.52	0.76
0.3	0.6	0.56	0.81	0.53	0.77
0.3	0.65	0.56	0.81	0.54	0.78
0.3	0.7	0.56	0.81	0.55	0.80
0.3	0.75	0.57	0.82	0.56	0.80
0.3	0.8	0.57	0.82	0.56	0.81
0.3	0.85	0.57	0.82	0.57	0.82
0.3	0.9	0.57	0.82	0.57	0.82
0.3	0.95	0.57	0.82	0.57	0.82
0.3	1	0.57	0.82	0.57	0.82
0.5	0.05	0.44	0.64	0.36	0.51
0.5	0.1	0.50	0.72	0.37	0.54
0.5	0.15	0.54	0.78	0.40	0.58
0.5	0.2	0.57	0.83	0.43	0.62
0.5	0.25	0.60	0.86	0.46	0.66
0.5	0.3	0.62	0.89	0.49	0.71
0.5	0.35	0.63	0.91	0.52	0.75
0.5	0.4	0.65	0.93	0.55	0.79
0.5	0.45	0.66	0.95	0.57	0.83
0.5	0.5	0.66	0.96	0.60	0.86
0.5	0.55	0.67	0.97	0.62	0.89
0.5	0.6	0.68	0.98	0.64	0.92
0.5	0.65	0.68	0.98	0.65	0.94
0.5	0.7	0.69	0.99	0.66	0.96

0.5	0.75	0.69	0.99	0.67	0.97
0.5	0.8	0.69	1.00	0.68	0.98
0.5	0.85	0.69	1.00	0.69	0.99
0.5	0.9	0.69	1.00	0.69	1.00
0.5	0.95	0.69	1.00	0.69	1.00
0.9	0.05	0.27	0.39	0.11	0.16
0.9	0.1	0.37	0.53	0.14	0.21
0.9	0.15	0.44	0.64	0.19	0.27
0.9	0.2	0.50	0.72	0.24	0.35
0.9	0.25	0.55	0.79	0.30	0.43
0.9	0.3	0.58	0.84	0.35	0.51
0.9	0.35	0.61	0.88	0.41	0.58
0.9	0.4	0.63	0.91	0.46	0.66
0.9	0.45	0.65	0.94	0.50	0.73
0.9	0.5	0.67	0.96	0.55	0.79
0.9	0.55	0.68	0.98	0.58	0.84
0.9	0.6	0.69	1.00	0.61	0.89
0.9	0.65	0.70	1.01	0.64	0.93
0.9	0.7	0.70	1.02	0.66	0.96
0.9	0.75	0.71	1.02	0.68	0.99
0.9	0.8	0.71	1.03	0.70	1.01
0.9	0.85	0.72	1.03	0.71	1.02
0.9	0.9	0.72	1.04	0.71	1.03
0.9	0.95	0.72	1.04	0.72	1.04

CONCLUDING REMARKS

This paper analysed the incentives for risk-neutral agents of investing in conflict management in a contest under different conditions. Through comparative statics different scenarios have been studied. A Potential Settlement Region (PSR) is interpreted as the set of all positive differences between payoffs received in the alternative scenarios for both agents.

First, the role of asymmetry in the evaluation of the contested stake has been underlined. When agents with identical abilities retain a different evaluation of the stake, the agent with the lower evaluation will expend efforts in conflict management only when the asymmetry is extremely large. Only in such a case a potential settlement region can be established. By contrast, when agents are asymmetrical both in evaluation of the stake and in fighting abilities there is a shrinkage of a PSR. In fact, whenever the agent with the higher evaluation of the stake is less endowed in fighting abilities there is a smaller room for a PSR. Once the destruction parameter is considered, agents clearly also take into account the opportunity cost of the conflict. That is, the perceived loss due the destructive interaction affects the optimal choice of both agents. Then, it favours an investment in conflict management in order to reach a peaceful settlement. In such a case, a PSR appeared to be extremely large while compared with those of the previous scenarios considered.

As a novelty of this work, I would quote the use of concept of entropy as a tool for measurement of conflict. Following the common neoclassical approach, investing in conflict management would be welfare-immiserizing. In such a narrow sense, however, a pure conflict would be preferable to a scenario where agents invest resources in conflict management. Establishing a PSR would be less efficient than pure conflict. An appealing idea for a more useful evaluation can be related to those of disorder and randomness. In fact, since conflict is a destructive interaction between two or more parties, it seemed to me reasonable to consider the degree of uncertainty it spreads. In actual violent appropriative conflicts uncertainty about the final outcome does clearly constitute a characteristic element that should be considered while developing devices to solve the conflict itself. It has been showed that the level of entropy also depends on the level of the asymmetry in the evaluation of the stake. In particular, the point of interest is that as the asymmetry in evaluation decreases the degree of disorder and turbulence increases. In particular, in presence of efforts devoted to conflict management the degree of disorder is lower.

The discussion related to the concept of entropy recalls to mind the debate, famous among students of international relations during the Cold War, about the stability of systems grounded on deterrence. In such a view, deterrence would be a stable system thanks to the existence of a credible threat. The results of this paper firmly contrasts this idea. A threat system (namely the ‘pure conflict’ scenario) is more turbulent than ‘conflict management’ scenario. However, future research on this point could contribute to this enduring debate.

The analysis paves the way for several extensions. In particular, remarkable points deserving further extension are the impact of a larger time horizon and the setting of a learning process. The model expounded in this work is a timeless model. Nevertheless, consider a possible application to a multi-period interaction. Assume for example that a dynamic interaction involves a learning process. Then imagine that such a learning process can modify the asymmetry in evaluation. Consider for example that valuations of the stake converge over time. Furthermore you can also imagine that some peculiar features of agents modify (consider among others: production function, access to market, investment in new technologies etc). In such a case, in a future period (say $t+n$), the asymmetry in evaluation can decrease, namely $\delta_{t+n} > \delta_t$. In such a case a settlement could be no longer possible. Parties could prefer a pure conflict.[11]

Moreover, for a future research agenda, consider that the CSF is used as a fundamental building block of several broader models. Applying the crucial modification of the CSF allowing for a second instrument can have an impact on the results emerging in these analyses.

Last but not least, what I would also claim as a remarkable point of interest is the relationship with bargaining. The outcome of this work partly contrasts with Thomas Schelling’s famous statement according to which “*conflict is a bargaining situation.*” The results of the model show that conflict *can* evolve in a bargaining situation. It does if - and only if - some conditions are fulfilled. Specifically conditions (c.1) and (c.2) must hold. The first ensures that agents have an incentive to negotiate. The latter suggests that a bargaining space does exist when both agents spend efforts to negotiate. Even if conflict can be considered as an enduring situation it could be maintained that parties must have an adequate incentive to negotiate. Otherwise they can choose to spend their efforts only in violent means. Then, my interest in this story is that it also provides also information about how bargaining takes shape. In other words, bargaining cannot be taken for granted. This statement also can be considered a matter of perspective if you consider that hardliners are used to saying that bargaining arises only as a failure of conflict. Many still believe that perfect conflicts are something other than the exploitation of actual violence, or whatever destructive efforts.

Finally, this line of theoretical analysis, which considers conflict and conflict management intertwined from the beginning, can have remarkable implications for the designing of economic policies in societies where conflict is a characteristic element. Consider for instance, the case of post-conflict societies, some LDC countries or mafia-infiltrated states.

[11] See on this point the intuitions presented in Arrow (1995).

PLOTS AND TABLES

PLOTS

TABLES

APPENDIX

Throughout this appendix I shall check whether the critical points for a maximum computed constitute a global max, namely a NE. Thus, I have to check whether $\pi_1\left(z_1^*, z_2^*, h_1^*, h_2^*\right) \geq \pi_1\left(z_1, z_2^*, h_1, h_2^*\right), \forall\left(z_1, h_1\right) \in A$ and $\pi_2\left(z_1^*, z_2^*, h_1^*, h_2^*\right) \geq \pi_2\left(z_1^*, z_2, h_1^*, h_2\right), \forall\left(z_2, h_2\right) \in A$.

In order to check where the candidate critical points $\left(z_1^{T*}, z_2^{T*}, h_1^*, h_2^*\right)$ represent a maximum it is useful to compute the Hessian matrices for both agents. Let me denote $X_1 = X$ and $z_i = z^T, i = 1,2.$ for notational simplicity. First, I compute the payoff function for agent 1 $\pi_1\left(z_1, h_1, z_2^*, h_2^*\right)$. The payoff function becomes:

$$\pi_1\left(z_1, h_1, z_2^*, h_2^*\right) = \frac{z_1\left(h_1+1\right)\left(\delta^4+2\delta^2+1\right)^2 X}{\delta^6 X^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)} - h_1 - z_1 \tag{9.1}$$

And the Hessian matrix is given by:

$$H_1\left(z_1, h_1, z_2^*, h_2^*\right) = \begin{pmatrix} \dfrac{\partial \pi_1}{\partial z_1 z_1} & \dfrac{\partial \pi_1}{\partial h_1 z_1} \\ \dfrac{\partial \pi_1}{\partial z_1 h_1} & \dfrac{\partial \pi_1}{\partial h_1 h_1} \end{pmatrix} =$$

$$= \begin{pmatrix} -\dfrac{2\delta^6 X^3\left(h_1+1\right)^2\left(\delta^4+2\delta^2+1\right)^4}{\left[\delta^6 X^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]^3} & \dfrac{\delta^6 X^3\left(\delta^4+2\delta^2+1\right)^2\left[\delta^6 X^2 - z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]}{\left[\delta^6 X^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]^3} \\ \dfrac{\delta^6 X^3\left(\delta^4+2\delta^2+1\right)^2\left[\delta^6 X^2 - z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]}{\left[\delta^6 X^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]^3} & -\dfrac{2\delta^6 X^3 z_1^2\left(\delta^4+2\delta^2+1\right)^4}{\left[\delta^6 X^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\left(h_1+1\right)\right]^3} \end{pmatrix}$$

(9.2)

Note that the Hessian matrix is symmetric. Let H_{1k} denote the k_{th} order leading principal submatrix of $H_1\left(z_1^T, z_2^T, h_1^*, h_2^*\right)$ for $k = 1,2$. The determinant of the kth order leading principal minor of $H_1\left(z_1^T, z_2^T, h_1^*, h_2^*\right)$ is denoted by $\left|H_{1k}\right|$. The leading principal minors alternate signs as follows:

$$\begin{aligned} &\left|H_{11}\right| < 0, \\ &\left|H_{12}\right| > 0 \Leftrightarrow \delta^6 X^2 - \left[3z_1\left(\delta^8+4\delta^6+6\delta^4+4\delta^2+1\right)\left(h_1+1\right)\right] < 0. \end{aligned} \tag{9.3}$$

Then I compute the payoff function for agent 2 $\pi_2\left(z_1^*, h_1^*, z_2, h_2\right)$,

$$\pi_2\left(z_1^*, h_1^*, z_2, h_2\right) = \frac{z_2\left(h_2+1\right)\left(\delta^4+2\delta^2+1\right)^2 \delta X}{\delta^4 X^2 + z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)} - h_2 - z_2 \tag{9.4}$$

And the Hessian matrix is given by:

$$H_2\left(z_1^*,h_1^*,z_2,h_2\right)=\begin{pmatrix}\frac{\partial\pi_2}{\partial z_2 z_2} & \frac{\partial\pi_2}{\partial h_2 z_2}\\ \frac{\partial\pi_2}{\partial z_2 h_2} & \frac{\partial\pi_2}{\partial h_2 h_2}\end{pmatrix}=$$

$$=\begin{pmatrix} -\frac{2\delta^5X^3\left(h_2+1\right)^2\left(\delta^4+2\delta^2+1\right)^4}{\left[\delta^4X^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]^3} & \frac{\delta^5X^3\left(\delta^4+2\delta^2+1\right)^2\left[\delta^4X^2-z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]}{\left[\delta^4X^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]^3}\\ \frac{\delta^5X^3\left(\delta^4+2\delta^2+1\right)^2\left[\delta^4X^2-z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]}{\left[\delta^4X^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]^3} & -\frac{2\delta^5X^3z_2^2\left(\delta^4+2\delta^2+1\right)^4}{\left[\delta^4X^2+z_2\left(\delta^4+2\delta^2+1\right)^2\left(h_2+1\right)\right]^3}\end{pmatrix}$$

(9.4)

Also in this case, let H_{2k} denote the k_{th} order leading principal submatrix of $H_1\left(z_1^*,z_2,h_1^*,h_2\right)$ for $k=1,2$. The determinant of the kth order leading principal minor of $H_2\left(z_1^*,z_2,h_1^*,h_2\right)$ is denoted by $\left|H_{2k}\right|$. The leading principal minors alternate in sign as follows:

$$\begin{aligned}&\left|H_{21}\right|<0,\\ &\left|H_{22}\right|>0\Leftrightarrow\delta^4X^2-\left[3z_2\left(\delta^2+1\right)^4\left(h_2+1\right)\right]<0.\end{aligned}\qquad(9.5)$$

since the Hessian matrices are not negative semidefinite it is necessary to deepen the analysis in order to show whether the critical points $\left(z_1^{T*},z_2^{T*},h_1^*,h_2^*\right)$ represent a global max. Then I compute the limits of both agents' payoffs. For the first agent we have:

$$\lim_{h_1 \to 0} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = \frac{z_1 X\left(\delta^4 + 2\delta^2 + 1\right)^2}{\delta^6 X^2 + z_1\left(\delta^4 + 2\delta^2 + 1\right)^2} - z_1$$

$$\lim_{h_1 \to \infty} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\infty$$

$$\lim_{z_1 \to 0} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -h_1 \tag{9.6}$$

$$\lim_{z_1 \to \infty} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\infty$$

I do the same for agent 2.

$$\lim_{h_2 \to 0} \pi_2\left(z_1^*, z_2, h_1^*, h_2\right) = \frac{z_2 \delta X\left(\delta^4 + 2\delta^2 + 1\right)^2}{\delta^4 X^2 + z_2\left(\delta^4 + 2\delta^2 + 1\right)^2} - z_2$$

$$\lim_{h_2 \to \infty} \pi_2\left(z_1^*, z_2, h_1^*, h_2\right) = -\infty$$

$$\lim_{z_2 \to 0} \pi_2\left(z_1^*, z_2, h_1^*, h_2\right) = -h_1$$

$$\lim_{z_2 \to \infty} \pi_2\left(z_1^*, z_2, h_1^*, h_2\right) = -\infty$$

(9.7)

therefore for both agents it is still necessary to check for $h_i = 0, i = 1, 2$. Consider first the payoff function of agent 1:

$$\pi_1\left(z_1, z_2^*, 0, h_2^*\right) = \frac{z_1\left(\delta^4+2\delta^2+1\right)^2 X}{\delta^6 X^2 + z_1\left(\delta^4+2\delta^2+1\right)^2} - z_1 \tag{9.8}$$

$$\frac{\partial \pi_1}{\partial z_1} = \frac{\delta^6 X^3\left(\delta^4+2\delta^2+1\right)^2}{\left[\delta^6 X^2 + z_1\left(\delta^4+2\delta^2+1\right)^2\right]^2} - 1 = 0 \tag{9.9}$$

$$z_1 = -\frac{\delta^3 X^{3/2}\left(\delta^3 X^{1/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^4+2\delta^2+1\right)^2}$$

(9.10)

the payoff is:

$$\pi_1 = \frac{\left(\delta^3 X^{1/2} - \delta^4 - 2\delta^2 - 1\right) X}{\left(\delta^4+2\delta^2+1\right)^2}$$

(9.11)

then, compare (11.1) and (9.11):

$$\frac{\left(\delta^2+1\right)^2 - X\left(\delta^2-1\right)}{\left(\delta^2+1\right)^2} > \frac{\left(\delta^3 X^{1/2} - \delta^4 - 2\delta^2 - 1\right) X}{\left(\delta^4+2\delta^2+1\right)^2}$$

(9.12)

Due to the analytical complexity I present a plot in a (δ, X) space. The vertical axe corresponds to $\delta = 1$. The shaded area in the plot below shows the region where the critical points $\left(z_1^{T*}, z_2^{T*}, h_1^*, h_2^*\right)$ represent an optimum.

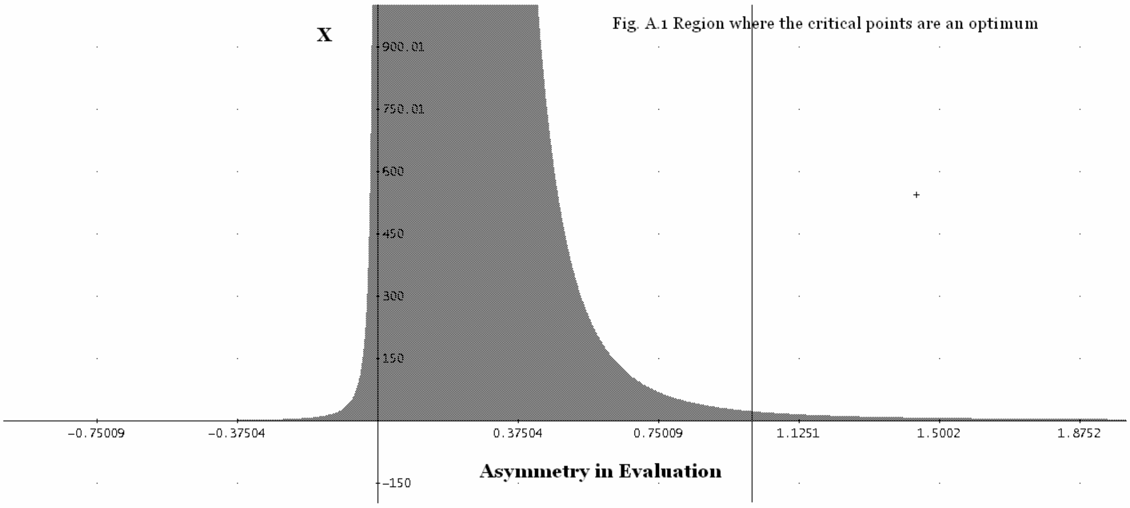


Fig. A.1 Region where the critical points are an optimum

for agent 2 we have:

$$\pi_2\left(z_1^*, h_1^*, z_2, 0\right) = \frac{z_2\left(\delta^4 + 2\delta^2 + 1\right)^2 \delta X}{\delta^4 X^2 + z_2\left(\delta^4 + 2\delta^2 + 1\right)^2} - z_2 \qquad (9.13)$$

$$\frac{\partial \pi_2}{\partial z_2} = \frac{\delta X z_2\left(\delta^4 + 2\delta^2 + 1\right)^2}{\left[\delta^4 X^2 + z_2\left(\delta^4 + 2\delta^2 + 1\right)^2\right]^2} - 1 = 0 \qquad (9.14)$$

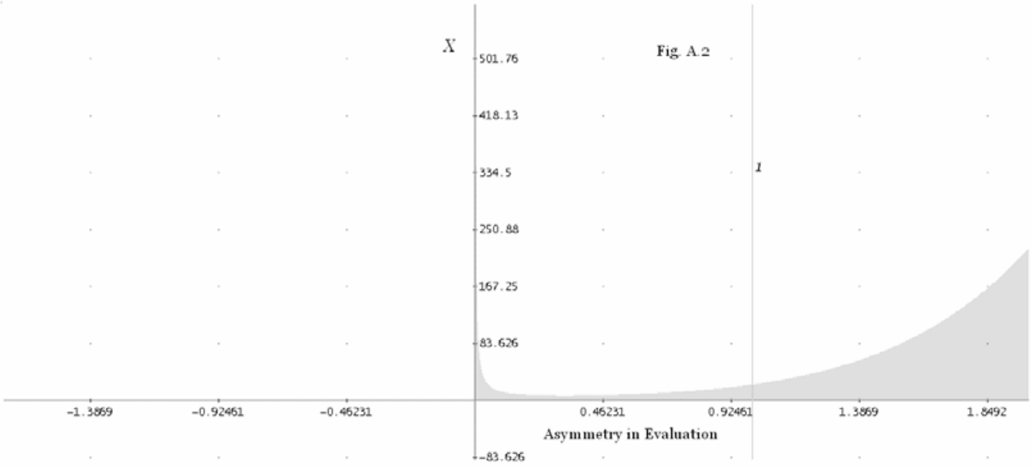
$$z_2 = -\frac{\delta^{5/2} X^{3/2}\left(X^{1/2}\delta^{3/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^4 + 2\delta^2 + 1\right)^2} \qquad (9.15)$$

$$\pi_2 = \frac{\delta X\left(X^{1/2}\delta^{3/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^4 + 2\delta^2 + 1\right)^2} \qquad (9.16)$$

compare (9.16) and (11.2)

$$\frac{\delta^3 X\left(\delta^2 - 1\right) + \left(\delta^2 + 1\right)^2}{\left(\delta^2 + 1\right)^2} > \frac{\delta X\left(X^{1/2}\delta^{3/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^4 + 2\delta^2 + 1\right)^2}$$

also in this case consider the plot below:



Then, it is clear that the critical points $\left(z_1^{T*}, z_2^{T*}, h_1^*, h_2^*\right)$ do not constitute a global maximum, namely a nash equilibrium.

Asymmetry in abilities

To verify whether $\left(z_1^{a*}, z_2^{a*}, h_1^{a*}, h_2^{a*}\right)$ is an optimum also in this case I compute the payoff function for agent 1 $\pi_1\left(z_1, h_1, z_2^*, h_2^*\right)$. The payoff function becomes:

$$\pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\frac{a\delta^6 X^3}{a^4 z_1\left(h_1+1\right)+4a^3\delta^2 z_1\left(h_1+1\right)+6a^2\delta^4 z_1\left(h_1+1\right)+a\delta^6\left(4h_1 z_1+X^2+4z_1\right)+\delta^8 z_1\left(h_1+1\right)} - h_1 + X - z_1$$

(29.1)

The Hessian matrix for agent 1 is:

$$H_1\left(z_1, h_1, z_2^*, h_2^*\right) = \begin{pmatrix} \dfrac{\partial \pi_1}{\partial z_1 z_1} & \dfrac{\partial \pi_1}{\partial h_1 z_1} \\ \dfrac{\partial \pi_1}{\partial z_1 h_1} & \dfrac{\partial \pi_1}{\partial h_1 h_1} \end{pmatrix} =$$

$$= \begin{pmatrix} -\dfrac{2a\delta^6 X^3\left(h_1+1\right)^2\left(a^4+4a^3\delta^2+6a^2\delta^4+4a\delta^6+\delta^8\right)^2}{\left[a^4 z_1\left(h_1+1\right)+4a^3\delta^2 z_1\left(h_1+1\right)+6a^2\delta^4 z_1\left(h_1+1\right)+a\delta^6\left(4h_1 z_1+X^2+4z_1\right)+\delta^8 z_1\left(h_1+1\right)\right]^3} & -\dfrac{a\delta^6 X^3\left(a^4+4a^3\delta^2+6a^2\delta^4+4a\delta^6+\delta^8\right)\left[a^4 z_1\left(h_1+1\right)+4a^3\delta^2 z_1\left(h_1+1\right)+6a^2\delta^4 z_1\left(h_1+1\right)+a\delta^6\left(4h_1 z_1-X^2+4z_1\right)+\delta^8 z_1\left(h_1+1\right)\right]}{\left[a^4 z_1\left(h_1+1\right)+4a^3\delta^2 z_1\left(h_1+1\right)+6a^2\delta^4 z_1\left(h_1+1\right)+a\delta^6\left(4h_1 z_1+X^2+4z_1\right)+\delta^8 z_1\left(h_1+1\right)\right]^3} \\ -\dfrac{a\delta^6 X^3\left(a^4+4a^3\delta^2+6a^2\delta^4+4a\delta^6+\delta^8\right)\left[a^4 z_1\left(h_1+1\right)+4a^3\delta^2 z_1\left(h_1+1\right)+6a^2\delta^4 z_1\left(h_1+1\right)+a\delta^6\left(4h_1 z_1-X^2+4z_1\right)+\delta^8 z_1\left(h_1+1\right)\right]}{\left[a^4 z_1\left(h_1+1\right)+4a^3\delta^2 z_1\left(h_1+1\right)+6a^2\delta^4 z_1\left(h_1+1\right)+a\delta^6\left(4h_1 z_1+X^2+4z_1\right)+\delta^8 z_1\left(h_1+1\right)\right]^3} & -\dfrac{2a\delta^6 X^3 z_1^2\left(a^4+4a^3\delta^2+6a^2\delta^4+4a\delta^6+\delta^8\right)^2}{\left[a^4 z_1\left(h_1+1\right)+4a^3\delta^2 z_1\left(h_1+1\right)+6a^2\delta^4 z_1\left(h_1+1\right)+a\delta^6\left(4h_1 z_1+X^2+4z_1\right)+\delta^8 z_1\left(h_1+1\right)\right]^3} \end{pmatrix}$$

(29.2)

The leading principal minors alternate signs as follows:

$$\left|H_{11}\right| < 0,$$

$$\left|H_{12}\right| > 0 \Leftrightarrow 3a^4 z_1\left(h_1+1\right)+12a^3\delta^2 z_1\left(h_1+1\right)+18a^2\delta^4 z_1\left(h_1+1\right)+a\delta^6\left(12h_1 z_1 - X^2+12z_1\right)+3\delta^8 z_1\left(h_1+1\right) > 0.$$

(29.3)

These conditions say nothing abou whether or not any of these local extrema is a global max. Then I compute the limits of agent 1's payoffs.

$$\lim_{h_1 \to 0} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\frac{a\delta^6 X^3}{a^4 z_1 + 4a^3\delta^2 z_1 + 6a^2\delta^4 z_1 + a\delta^6\left(X^2 + 4z_1\right) + \delta^8 z_1} + X - z_1 \tag{29.4}$$

$$\lim_{h_1 \to \infty} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\infty$$

$$\lim_{z_1 \to 0} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -h_1$$

$$\lim_{z_1 \to \infty} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\infty$$

therefore, it is still necessary to check for $h_i = 0, i = 1, 2$. Consider the payoff function of agent 1:

$$\pi_1\left(z_1, z^*_2, 0, h_2^*\right) = -\frac{a\delta^6 X^3}{a^4 z_1 + 4a^3\delta^2 z_1 + 6a^2\delta^4 z_1 + a\delta^6\left(X^2 + 4z_1\right) + \delta^8 z_1} + X - z_1$$

(29.5)

$$\frac{\partial \pi_1}{\partial z_1} = \frac{a\delta^6 X^3\left(a^4 + 4a^3\delta^2 + 6a^2\delta^4 + 4a\delta^6 + \delta^8\right)}{\left(a^4 z_1 + 4a^3\delta^2 z_1 + 6a^2\delta^4 z_1 + a\delta^6\left(X^2 + 4z_1\right) + \delta^8 z_1\right)^2} - 1 = 0$$

(29.6)

$$z_1 = \frac{X\delta^{3/2}\left[\left(a+\delta^2\right)^2\left(\delta^2\left(a\left(X-2\right)-\delta^2\right)-a^2\right)^{1/2} + \delta^{3/2}\left(a^2 + a\delta^2\left(2-X\right)+\delta^4\right)\right]}{\left(a+\delta^2\right)^4}$$

(29.7)

$$\pi_1 = \frac{X\left[2\delta^{3/2}\left(a+\delta^2\right)^2\left(\delta^2\left(a\left(X-2\right)-\delta^2\right)-a^2\right)^{1/2} - a^4 - \delta^2\left(4a^3 + a^2\delta\left(6\delta-1\right)+a\delta^3\left(4\delta+X-2\right)+\delta^5\left(\delta-1\right)\right)\right]}{\left(a+\delta^2\right)^4}$$

(29.7)

Now compare (31) and (29.7)

$$\frac{a^2\left(X+1\right)+a\delta^2\left(2-X\right)+\delta^4}{\left(a+\delta^2\right)^2}>$$

$$>-\frac{X\left[2\delta^{3/2}\left(a+\delta^2\right)^2\left(\delta^2\left(a\left(X-2\right)-\delta^2\right)-a^2\right)^{1/2}-a^4-\delta^2\left(4a^3+a^2\delta\left(6\delta-1\right)+a\delta^3\left(4\delta+X-2\right)+\delta^5\left(\delta-1\right)\right)\right]}{\left(a+\delta^2\right)^4}$$

(29.8)

First, it is possible to show that this inequality does not hold for $X\rightarrow\infty$. Secondly, the plots below show the region when inequality (34.8) holds once attached different arbitrary values to X, namely when $X=100, X=200, X=1000$.

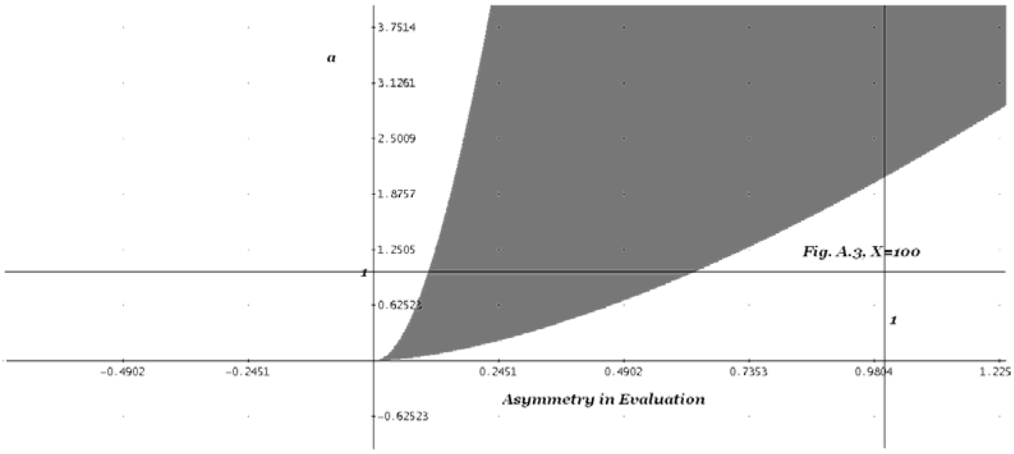


Fig. A.3, X=100

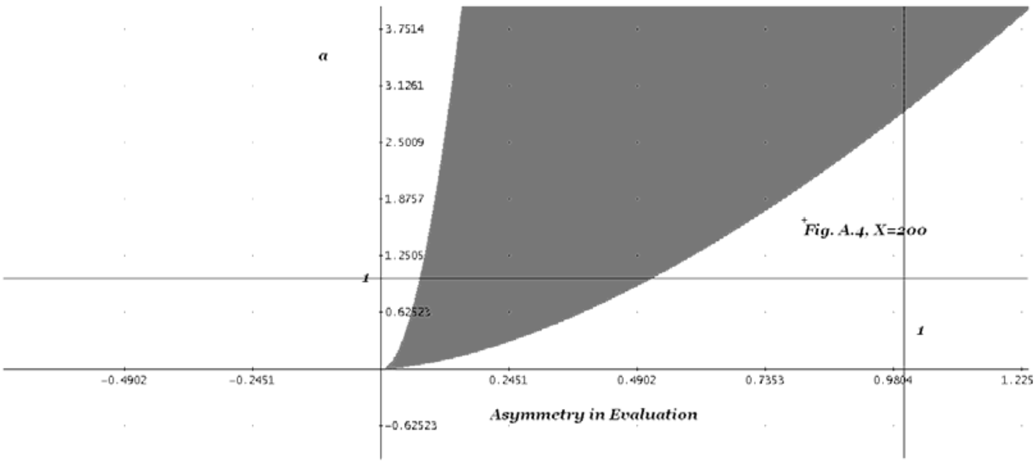
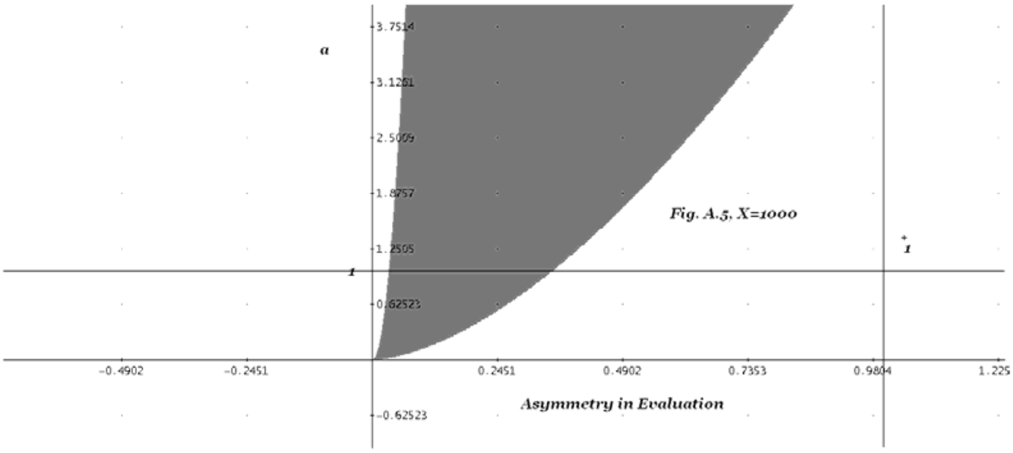


Fig. A.4, X=200



In particular the shaded areas show when the critical points $\left(z_1^{a*}, z_2^{a*}, h_1^{a*}, h_2^{a*}\right)$ represent an optimum for agent 1 in a (δ, a) space.

Destructive Conflict and sharing rule

To verify whether $\left(z_1^{s*}, z_2^{s*}, h_1^{s*}, h_2^{s*}\right)$ is an optimum also in this case I compute the payoff function for agent 1 $\pi_1\left(z_1, h_1, z_2^*, h_2^*\right)$. The payoff function becomes:

$$\pi_1 = \frac{\beta X z_1\left(\delta^2+1\right)^4\left(h_1+1\right)}{\beta^2\delta^6X^2 + z_1\left(\delta^2+1\right)^4\left(h_1+1\right)} + \frac{X(1-\beta)}{2} - h_1 - z_1 \qquad (37.1)$$

The Hessian matrix for agent 1:

$$H_1\left(z_1, h_1, z_2^*, h_2^*\right) = \begin{pmatrix} \dfrac{\partial\pi_1}{\partial z_1 z_1} & \dfrac{\partial\pi_1}{\partial h_1 z_1} \\ \dfrac{\partial\pi_1}{\partial z_1 h_1} & \dfrac{\partial\pi_1}{\partial h_1 h_1} \end{pmatrix} =$$

$$= \begin{pmatrix} -\dfrac{2\beta^3\delta^6X^3\left(\delta^2+1\right)^4\left(h_1+1\right)^2}{\left[\beta^2\delta^6X^2 + z_1\left(\delta^2+1\right)^4\left(h_1+1\right)\right]^3} & \dfrac{\beta^3\delta^6X^3\left(\delta^2+1\right)^4\left[\beta^2\delta^6X^2 - z_1\left(\delta^2+1\right)^4\left(h_1+1\right)\right]}{\left[\beta^2\delta^6X^2 + z_1\left(\delta^2+1\right)^4\left(h_1+1\right)\right]^3} \\ \dfrac{\beta^3\delta^6X^3\left(\delta^2+1\right)^4\left[\beta^2\delta^6X^2 - z_1\left(\delta^2+1\right)^4\left(h_1+1\right)\right]}{\left[\beta^2\delta^6X^2 + z_1\left(\delta^2+1\right)^4\left(h_1+1\right)\right]^3} & -\dfrac{2\beta^3\delta^6X^3\left(\delta^2+1\right)^4 z_1^2}{\left[\beta^2\delta^6X^2 + z_1\left(\delta^2+1\right)^4\left(h_1+1\right)\right]^3} \end{pmatrix}$$

(37.2)

The leading principal minors alternate in sign as follows:

$$|H_{21}| < 0,$$
$$|H_{22}| > 0 \Leftrightarrow \beta^2\delta^6 X^2 - \left[3z_1\left(\delta^2+1\right)^4\left(h_1+1\right)\right] < 0. \qquad (37.3)$$

These conditions say nothing about whether or not any of these local extrema is a global max. Then I compute the limits of agent 1's payoffs.

$$\lim_{h_1 \to 0} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = \frac{\beta z_1 X\left(\delta^2+1\right)^4}{\beta^2\delta^6X^2 + z_1\left(\delta^2+1\right)^4} + \frac{X(1-\beta)}{2} - z_1$$

$$\lim_{h_1 \to \infty} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\infty$$

$$\lim_{z_1 \to 0} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = \frac{X(1-\beta)}{2} - h_1$$

$$\lim_{z_1 \to \infty} \pi_1\left(z_1, z_2^*, h_1, h_2^*\right) = -\infty \qquad (37.4)$$

therefore, it is still necessary to check for $h_1 = 0$ and $z_1 = 0$. Consider the payoff function of agent 1:

$$\pi_1\left(z_1, z_2^*, 0, h_2^*\right) = \frac{\beta X z_1\left(\delta^2+1\right)^4}{\beta^2\delta^6X^2 + z_1\left(\delta^2+1\right)^4} + \frac{X(1-\beta)}{2} - z_1 \qquad (37.5)$$

$$\frac{\partial \pi_1}{\partial z_1} = \frac{\beta^3 \delta^6 X^3 \left(\delta^2+1\right)^4}{\left[\beta^2 \delta^6 X^2 + z_1 \left(\delta^2+1\right)^4\right]^2} - 1 = 0$$

(37.6)

$$z_1 = -\frac{\delta^3 \beta^{3/2} X^{3/2} \left(\beta^{1/2} \delta^3 X^{1/2} - \delta^4 - 2\delta^2 - 1\right)}{\left(\delta^2+1\right)^4}$$

(37.7)

$$\pi_1 = \frac{X\left[2\beta^2 \delta^6 X - 4\delta^3 X^{1/2} \beta^{3/2} \left(\delta^2+1\right)^2 + (\beta+1)\left(\delta^2+1\right)^4\right]}{2\left(\delta^2+1\right)^4}$$

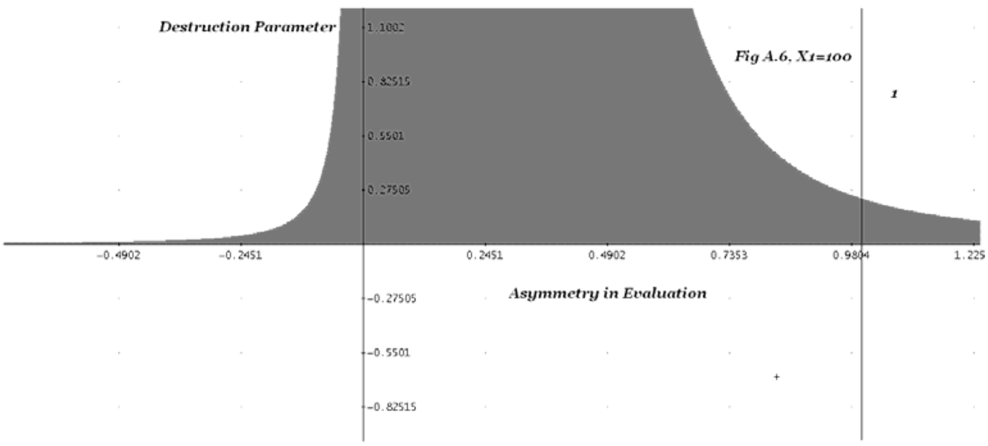
(37.8)

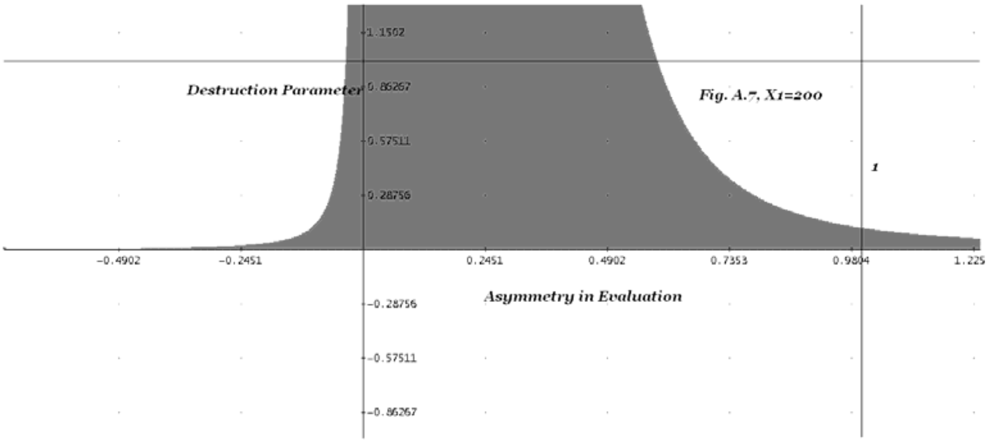
Compare (37.8) and (38)

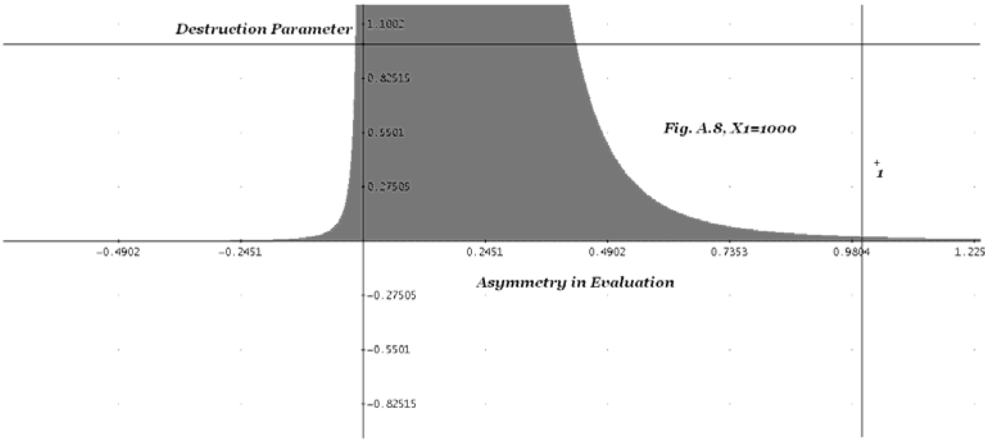
$$\frac{X}{2}\left[1-\beta\frac{\left(\delta^2-1\right)^2}{\left(\delta^2+1\right)^2}\right]+1 > \frac{X\left[2\beta^2 \delta^6 X - 4\delta^3 X^{1/2} \beta^{3/2} \left(\delta^2+1\right)^2 + (\beta+1)\left(\delta^2+1\right)^4\right]}{2\left(\delta^2+1\right)^4}$$

(37.9)

the plots below show the region when inequality (37.9) holds once attached different arbitrary values to X, namely when $X = 100, X = 200, X = 1000$.







In particular the shaded areas show when the critical points $\left(z_1^{s*}, z_2^{s*}, h_1^{s*}, h_2^{s*}\right)$ represent an optimum for agent 1 in a (δ, β) space. It is clear that the critical points $\left(z_1^{s*}, z_2^{s*}, h_1^{*}, h_2^{*}\right)$ do not constitute a global maximum, namely a NE.

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