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September 2009

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March 2010

Abstract

To analyze the welfare gain from allowing for differentiated patent protection across sectors, this study develops a two-sector quality-ladder growth model in which patent breadth is a policy variable and derives the optimal patent breadth under two policy regimes. We show that (i) the optimal *uniform* patent breadth is a weighted average of the optimal *sector-specific* patent breadth, and (ii) the optimal sector-specific patent breadth is larger in the sector that has a larger market size and more technological opportunities. To derive the optimal policy, we allow for an arbitrary path of patent breadth and derive the optimal path by solving a Stackelberg differential game. We find that the optimal path of patent breadth under each regime is stationary, time-consistent and subgame perfect. Finally, we perform a numerical investigation and find that even a moderate degree of asymmetry across sectors can generate a significant welfare cost of uniform patent protection.

***Keywords*: ** economic growth, R&D, uniform patent protection, time-consistent patent policy

***JEL classification*: ** O31, O34

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> "The economic evidence is overwhelming that innovation works differently in different industries, and that the way patents affect innovation also differs enormously by industry. The question for patent policy is how to respond to those differences."
>
> – Burk and Lemley (2009, p. 4-5)

1. Introduction

In a recent book, Burk and Lemley (2009) argue that the courts should tailor the unitary patent rules through interpretations and applications to suit the different needs of diverse industries.[1] An important shortcoming of the patent system is that diverse industries, such as pharmaceuticals, software and semiconductors, are governed by the same set of rules. For example, as a result of the TRIPS agreement,[2] the statutory term of patent in the US is 20 years for inventions across almost all fields of technology, and this one-size-fit-all patent policy is unlikely to provide the appropriate incentives for innovation in every industry. Fortunately, there are other patent-policy instruments that can be adjusted by policymakers. An important example is patent breadth that determines the broadness or scope of a patent. When an inventor applies for a patent, she makes a number of claims about the invention in her application to be reviewed by a patent examiner. The Burk-Lemley proposal implies that the courts should be given the discretion to decide how broadly or narrowly these patent claims are to be interpreted on a case-by-case basis tailoring to the needs of different industries.

To analyze the welfare implications of allowing for sector-specific patent protection, this study develops a two-sector quality-ladder growth model in which patent breadth is a policy

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[1] Burk and Lemley (2009) note that the courts already treat innovation across industries differently, but they also argue that the current degree of differentiation is insufficient.

[2] The World Trade Organization's Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), initiated in the 1986-94 Uruguay Round, establishes a minimum level of intellectual property protection that must be provided by all member countries.

variable. Then, we derive the optimal patent breadth under two policy regimes (i) uniform patent breadth across sectors and (ii) sector-specific patent breadth. Finally, we perform a numerical investigation on the potential welfare gain from differentiated patent breadth across sectors.

We extend the quality-ladder model of Grossman and Helpman (1991) by incorporating two sectors that are differentiated by market size and technological opportunity. Within this framework, we show that (i) the optimal patent breadth is larger in the sector that has a larger market size and more technological opportunities, and (ii) the optimal *uniform* patent breadth is a weighted average of the optimal *sector-specific* patent breadth and the optimal weight is given by each sector's market size. Comparing the differences in economic growth and social welfare under the two policy regimes, we find that although the growth-rate differential is zero in this model, the welfare difference is generally nonzero and determined by the relative technological opportunity and market size across sectors. This finding has an important policy implication that even if empirical studies do not find a significant improvement in growth upon implementing differentiated patent protection across sectors, the welfare gain can still be significant. In the numerical analysis, we find that a moderate degree of asymmetry across sectors can generate a significant welfare gain from allowing for sector-specific patent breadth.

Some interesting recent studies, such as Acemoglu and Akcigit (2009) and Mosel (2009), also analyze the implications of differentiated patent protection across sectors. In addition to some modeling differences in the growth-theoretic framework, the present study differs from the above studies in the following ways. Firstly, these studies model patent protection as a constant parameter and numerically compute the parameter value that maximizes growth or welfare. In contrast, we allow for an arbitrary path of patent breadth and then analytically derive the optimal path by solving a differential game,[3] in which policymakers move first by choosing a time path of patent breadth and households response by choosing a time path of consumption (i.e., a Stackelberg differential game). We find that the optimal path of patent breadth under each policy regime is stationary, time-consistent and subgame perfect. Time consistency and subgame perfectness imply that policymakers have no incentive to deviate from the optimal path of patent breadth under any realization of the state variables along and off the equilibrium path.

In their seminal study, Kydland and Prescott (1977) consider patent protection as an important example of time-inconsistent policies for which they point out the following problem. "Given that resources have been allocated to inventive activity which resulted in a new product or process, the efficient policy is not to permit patent protection." To show that optimal patent policy is not necessarily time inconsistent, this study adopts a differential-game approach and derives time-consistent optimal patent breadth in a modified version of the Grossman-Helpman (1991) model,[4] which is a workhorse model in the R&D growth literature. Time inconsistency does not arise in this model because the equilibrium allocation at any point in time depends only on the current level of patent breadth and is independent of future patent policies.

A second difference with Acemoglu and Akcigit (2009) is that while they consider the level of patent protection to be differentiated by the technological gap between the leader and the follower in an industry, we consider patent breadth to be differentiated by an industry's market size and technological opportunity that drive the observable industry differences in productivity growth and R&D intensity according to Klenow (1996). In other words, we examine a different set of industry-specific characteristics that are also important features of the economy and hence

[3] A differential game is a dynamic game in which the state variables evolve according to differential equations. See, for example, Dockner *et al.* (2000) for a comprehensive textbook treatment on differential games.
[4] It can be shown that the optimal patent breadth is also time consistent in the original Grossman-Helpman model. A proof is available upon request from the author.

complement the analysis in Acemoglu and Akcigit (2009), who also find a significant welfare gain from sector-specific patent protection. Thirdly, while Mosel (2009) considers a related set of industry-specific characteristics in a different model, he focuses on the growth effects of sector-specific patent protection. Given that growth maximization does not necessarily give rise to welfare maximization, it is interesting to consider the welfare effects as well, and the present study fills in this gap in the literature.

The seminal study of the patent-design literature is Nordhaus (1969), who concludes that the optimal level of patent protection should tradeoff the static welfare costs against the dynamic gains from innovation. A comprehensive review of the subsequent developments in this literature can be found in Scotchmer (2004). While most studies in the patent-design literature are based on a qualitative partial-equilibrium setting, the macroeconomic literature plays a complementary role in providing a dynamic general-equilibrium (DGE) analysis on patent policy. For example, Futagami and Iwaisako (2003, 2007) derive the optimal patent length in a version of the Romer model and show that it can be finite. Li (2001) extends the Grossman-Helpman (1991) model to consider patent breadth and finds that it has a positive effect on R&D and growth. As for quantitative DGE analysis, Kwan and Lai (2003) evaluate the quantitative implications of patent length in a version of the Romer model and find that extending the effective lifetime of patent would lead to a substantial increase in R&D and welfare. Chu (2009) builds on the quality-ladder model in O'Donoghue and Zweimuller (2004) to provide a quantitative analysis on the effects of blocking patents and finds that reducing the negative effect of blocking patents on R&D would lead to a significant increase in welfare. All of these studies are based on R&D-driven growth models that have only one R&D sector.[5] The present study complements them by analyzing the welfare implications of patent policy in a growth model that features multiple R&D sectors.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium and analyzes its dynamic properties. Section 4 derives the optimal patent breadth under the two policy regimes. Section 5 provides a quantitative analysis on welfare. The final section concludes, and proofs are relegated to Appendix A.

2. A two-sector quality-ladder growth model with patent breadth

The quality-ladder model is based on Grossman and Helpman (1991).[6] We modify their model by incorporating (i) patent breadth as a policy variable following the formulation in Li (2001) and (ii) two sectors that are differentiated by market size and technological opportunity. Klenow (1996) develops a two-sector Romer model with three industry-specific characteristics that are commonly discussed in the industrial organization literature, and he finds that market size and technological opportunity best explain the empirical differences in productivity growth and R&D intensity across industries. As for the third industry-specific characteristic (i.e., appropriablility), it is captured in the model by the different rates of endogenous creative destruction across sectors.

In the following model, patent breadth is allowed to be a time-varying (but deterministic) variable. Then, in Section 3, we show that the optimal path of patent breadth under each policy regime is stationary, time-consistent and subgame perfect in this modified Grossman-Helpman model. Given that the Grossman-Helpman model has been well-studied, the familiar features will be briefly described to conserve space while the new features will be described in more details.

[5] O'Donoghue and Zweimuller (2004) also analyze the case of two R&D sectors in one of their extensions. However, their focus is on the distortionary effect of patent polices on the allocation of R&D across sectors. Therefore, they only consider exogenous changes in the uniform level of patent protection.

[6] See, also, Aghion and Howitt (1992) and Segerstrom *et al.* (1990) for the other pioneering studies on the quality-ladder growth model.

2.1 Households

There is a unit continuum of identical households, and their lifetime utility is given by

(1) $$U = \int_0^\infty e^{-\rho t} \ln C_t \ .$$

C_t denotes consumption at time *t*, and the parameter $\rho > 0$ is the subjective discount rate. Households maximize utility subject to

(2) $$\dot{V}_t = R_t V_t + W_t - P_t C_t \ .$$

P_t denotes the price of consumption at time *t*. Each household supplies one unit of labor (chosen as the numeraire) to earn a wage income W_t that will be normalized to unity. V_t is the value of assets owned by households, and R_t is the nominal rate of return. The familiar Euler equation is

(3) $$\dot{C}_t / C_t = r_t - \rho \ ,$$

where $r_t \equiv R_t - \dot{P}_t / P_t$ is the real interest rate.

2.2 Consumption

To consider a two-sector R&D-based growth model, consumption is aggregated from two types of final goods $i \in \{1,2\}$. This aggregation process can be done by the households themselves or by competitive firms, and these two formulations are equivalent. We follow Klenow (1996) to consider a Cobb-Douglas aggregator given by

(4) $$C_t = (Y_{1,t})^{\alpha} (Y_{2,t})^{1-\alpha} \ ,$$

where $\alpha \in (0,1)$ is the market-size parameter. We use this Cobb-Douglas aggregator instead of a CES aggregator because we want to allow $Y_{1,t}$ and $Y_{2,t}$ to grow at different rates. In the case of a

CES aggregator, $Y_{1,t}$ and $Y_{2,t}$ growing at different rates is incompatible with a balanced-growth path. The first-order conditions for $Y_{1,t}$ and $Y_{2,t}$ are respectively

(5) $$P_{1,t}Y_{1,t} = \alpha P_t C_t ,$$

(6) $$P_{2,t}Y_{2,t} = (1-\alpha)P_t C_t .$$

Therefore, α determines the output share of the two types of final goods (i.e., the market size).

2.3 Final goods

Final goods $i \in \{1,2\}$ are produced by a standard Cobb-Douglas aggregator over a continuum of differentiated intermediate goods $j \in [0,1]$.

(7) $$Y_{i,t} = \exp\left(\int_0^1 \ln X_{i,t}(j)dj\right).$$

This sector is perfectly competitive, and final-goods firms take both the output and input prices as given. Given (7), the familiar price index for $Y_{i,t}$ is $P_{i,t} = \exp\left(\int_0^1 \ln P_{i,t}(j)dj\right)$.

2.4 Intermediate goods

In each sector $i \in \{1,2\}$, there is a continuum of differentiated intermediate goods indexed by $j \in [0,1]$. Each intermediate goods *j* of sector *i* is produced by a monopolistic leader, who holds a patent on the latest innovation and dominates the market until the next innovation occurs. The production function for the leader of intermediate goods *j* in sector *i* is

(8) $$X_{i,t}(j) = z^{n_{i,t}(j)} L_{i,t}(j) .$$

$L_{i,t}(j)$ denotes the number of workers producing intermediate goods *j* of sector *i*. $z > 1$ is the exogenous step size of productivity improvement from each innovation. $n_{i,t}(j)$ is the number of innovations that have occurred in intermediate goods *j* of sector *i* as of time *t*. The marginal cost of production for the leader of intermediate goods *j* in sector *i* is

(9) $$MC_{i,t}(j) = W_t / z^{n_{i,t}(j)}.$$

As commonly assumed in the literature, the current leader and the former leader engage in Bertrand competition. The profit-maximizing price for the current leader is a constant markup over the marginal cost.

(10) $$P_{i,t}(j) = \mu_{i,t}(W_t / z^{n_{i,t}(j)}),$$

where $\mu_{i,t} = z^{b_{i,t}}$ and $b_{i,t} \in (0,1]$ is the level of patent breadth at time *t*.[7] Grossman and Helpman (1991) assume complete patent protection against imitation (i.e., $b_{i,t} = 1$). Li (2001) generalizes the policy regime to allow for incomplete protection, and we follow Li's (2001) formulation of patent breadth here. Because of incomplete protection, the current leader's invention enables the former leader to increase her productivity by a factor of z^{1-b} without infringing the current leader's patent. Therefore, the limit-pricing markup for the current leader is z^b. A larger patent breadth enables the current leader to charge a higher markup, and the resulting increase in profit improves the incentives for R&D.[8] For the rest of this study, we use $\mu_{i,t} \equiv \mu(z, b_{i,t})$ to denote patent breadth for convenience and consider changes in $\mu_{i,t}$ coming from changes in $b_{i,t}$ only.

[7] When an inventor applies for a patent, she makes a number of claims about the invention to be patented. If these claims are narrowly interpreted, then competitors may be able to imitate around them to avoid infringement.

[8] Li (2001) generalizes (7) to a CES production function. In this case, the markup is given by $\min\{z^b, \varepsilon/(\varepsilon-1)\}$, where $\varepsilon \in (1,\infty)$ is the elasticity of substitution between intermediate goods. Therefore, when $z^b < \varepsilon/(\varepsilon-1)$, the effect of patent breadth on R&D and growth is the same as in the case of a Cobb-Douglas production function.

2.5 R&D

Denote the value of the latest invention in intermediate goods *j* of sector *i* by $V_{i,t}(j)$.[9] Because of the Cobb-Douglas specification in (7), the amount of profit is the same across industries within a sector (i.e., $\pi_{i,t}(j)=\pi_{i,t}$ for $j\in[0,1]$). As a result, $V_{i,t}(j)=V_{i,t}$ for $j\in[0,1]$ in a symmetric equilibrium in which the arrival rate of innovation is equal across industries within a sector.[10] The familiar no-arbitrage condition for $V_{i,t}$ is

$$R_t V_{i,t}=\pi_{i,t}+\dot{V}_{i,t}-\lambda_{i,t}V_{i,t}. \tag{11}$$

The left-hand side of (11) is the nominal return on this asset. The right-hand side of (11) is the sum of (a) the profit $\pi_{i,t}$ generated by this asset, (b) the potential capital gain $\dot{V}_{i,t}$, and (c) the expected capital loss $\lambda_{i,t}V_{i,t}$ due to creative destruction for which $\lambda_{i,t}$ is the *aggregate* Poisson arrival rate of innovation in sector *i*.

There is a unit continuum of R&D entrepreneurs in each sector *i*. They hire R&D workers $H_{i,t}$ to create inventions, and the expected profit for R&D in sector *i* is

$$\Pi_{i,t}=V_{i,t}\tilde{\lambda}_{i,t}-W_t H_{i,t}, \tag{12}$$

where $\tilde{\lambda}_{i,t}=\varphi_i H_{i,t}$ is the *individual* Poisson arrival rate of innovation. Following Klenow (1996), we allow the technological-opportunity parameter φ_i to vary across sectors.[11] Without loss of generality, we assume that $\varphi_1\leq\varphi_2$. The zero-expected-profit condition for R&D in sector *i* is

[9] It will become clear why we use $V_{i,t}$ to denote the market value of inventions and V_t to denote the value of assets owned by households.

[10] We follow the standard approach in the literature to focus on the symmetric equilibrium. See, for example, Cozzi et al. (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the quality-ladder growth model.

[11] In the literature, the parameter φ_i is sometimes referred to as R&D efficiency. Because our study relates to Klenow (1996), we follow his terminology to refer to φ_i as technological opportunity. Intuitively, in a sector that has more technological opportunities, the chance of discovering an invention is higher for a given amount of R&D input.

(13) $$\varphi_i V_{i,t} = W_t = 1,$$[12]

where the second equality of (13) follows from choosing labor as the numeraire.

The Cobb-Douglas specification in (7) implies that each intermediate goods *j* of sector *i* employs an equal number of production workers. Substituting (8) into (7) yields $Y_{i,t} = Z_{i,t}L_{i,t}$, where the level of technology in sector *i* is defined as

(14) $$Z_{i,t} \equiv \exp\left(\int_0^1 n_{i,t}(j)dj \ln z\right) = \exp\left(\int_0^t \lambda_{i,\tau} d\tau \ln z\right),$$

where the second equality of (14) uses the law of large numbers. Differentiating the log of (14) with respect to time yields the growth rate of total factor productivity (TFP) in sector *i* given by

(15) $$g_{i,t} \equiv \dot{Z}_{i,t} / Z_{i,t} = \lambda_{i,t} \ln z,$$

where $\lambda_{i,t} = \widetilde{\lambda}_{i,t} = \varphi_i H_{i,t}$ in equilibrium.

3. Decentralized equilibrium

The equilibrium is a time path of allocations $\{C_t, Y_{i,t}, X_{i,t}(j), L_{i,t}, H_{i,t}\}_{t=0}^{\infty}$, a time path of prices $\{P_t, P_{i,t}, P_{i,t}(j), W_t, R_t, V_t, V_{i,t}\}_{t=0}^{\infty}$, and a time path of polices $\{\mu_{i,t}\}_{t=0}^{\infty}$. Also, at each instant of time,

(a) households choose $\{C_t\}$ to maximize utility taking $\{P_t, W_t, R_t\}$ as given;

(b) competitive firms produce $\{C_t\}$ by using $\{Y_{i,t}\}$ as inputs to maximize profit taking $\{P_t, P_{i,t}\}$ as given;

(c) competitive final-goods firms in sector *i* produce $\{Y_{i,t}\}$ by using $\{X_{i,t}(j)\}$ as inputs to maximize profit taking $\{P_{i,t}, P_{i,t}(j)\}$ as given;

[12] The sector with a larger φ_i attracts more R&D and hence has a higher rate of creative destruction that reduces $V_{i,t}$.

(d) the leader of intermediate goods *j* of sector *i* produces $\{X_{i,t}(j)\}$ and chooses $\{P_{i,t}(j)\}$ to maximize profit taking $\{W_t\}$ as given;

(e) competitive R&D entrepreneurs in sector *i* choose $\{H_{i,t}\}$ to maximize expected profit taking $\{W_t, V_{i,t}\}$ as given;

(f) the labor market clears such that $L_{1,t} + L_{2,t} + H_{1,t} + H_{2,t} = 1$;

(g) the market value of inventions adds up to the value of assets owned by households such that $V_{1,t} + V_{2,t} = V_t$.

3.1 Balanced-growth path

In this section, we firstly derive the equilibrium labor allocations for an arbitrary path of patent breadth $\{\mu_{1,t}, \mu_{2,t}\}_{t=0}^{\infty}$. Then, we show that given a stationary path of patent breadth $\{\mu_1, \mu_2\}_{t=0}^{\infty}$, the economy is always on a unique and stable balanced-growth path.[13]

Lemma 1: *Given an arbitrary path of patent breadth $\{\mu_{1,t}, \mu_{2,t}\}_{t=0}^{\infty}$, the equilibrium labor allocations at time t are*

(16) $$L_{1,t} = \alpha\left(1 + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2}\right)\frac{1}{\mu_{1,t}},$$

(17) $$L_{2,t} = (1-\alpha)\left(1 + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2}\right)\frac{1}{\mu_{2,t}},$$

[13] As in Grossman and Helpman (1991), the implicit assumptions behind this result are (i) at any point in time, each industry has an existing leader with a competitor one step down the quality ladder and (ii) R&D entrepreneurs always implement their inventions immediately (i.e., ruling out endogenous implementation cycles).

(18) $$H_{1,t} = \alpha\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\left(\frac{\mu_{1,t}-1}{\mu_{1,t}}\right)-\frac{\rho}{\varphi_1},$$

(19) $$H_{2,t} = (1-\alpha)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\left(\frac{\mu_{2,t}-1}{\mu_{2,t}}\right)-\frac{\rho}{\varphi_2}.$$

The equilibrium labor allocations at time t only depend on the level of patent breadth at time t. Furthermore, given a stationary path of patent breadth $\{\mu_1, \mu_2\}_{t=0}^{\infty}$*, the economy is always on a unique and stable balanced-growth path.*

Proof: See Appendix A.■

Equations (16) – (19) reveal that in this model, the equilibrium labor allocations are independent of future patent policies. This is a property of the Grossman-Helpman model, in which a higher level of patent breath $\mu_{i,t}$ at any time t is accompanied by an increase in $\lambda_{i,t} = \varphi_i H_{i,t}$ in such a way that $V_{i,t}$ remains unchanged.[14] In the next section, we will use this convenient feature of the model to derive a time-consistent path of optimal patent breadth, which turns out to be stationary.

Given a stationary path of patent breadth, the economy is on a balanced-growth path, and the steady-state equilibrium allocations are quite intuitive. A larger α increases both L_1 and H_1. Intuitively, as the market size of final goods 1 increases, the economy devotes more labor to production and R&D in sector 1. A larger μ_1 decreases L_1 and increases H_1. A larger patent breadth in sector 1 leads to a reallocation of labor from production to R&D within the sector. However, note that the sum of L_1 and H_1 is independent of μ_1. In other words, a change in the relative level of patent breadth does *not* lead to a reallocation of labor across sectors. Similar to an increase in μ_1, a larger φ_1 decreases L_1 and increases H_1. Interestingly, in this case, L_2 and

[14] See the proof of Lemma 1.

H_2 also decrease. In other words, as the technological opportunity of sector 1 improves, the economy not only reallocates labor from production to R&D within the sector but also across sectors. Finally, the consumption growth rate $g_t \equiv \dot{C}_t / C_t$ along the balanced-growth path is

$$g = \alpha g_1 + (1-\alpha) g_2 = (\alpha \varphi_1 H_1 + (1-\alpha)\varphi_2 H_2) \ln z \,. \tag{20}$$

4. Optimal patent breadth

The previous section shows that given a constant level of patent breadth, the economy is always on a balanced-growth path. This section shows that the optimal path of patent breadth under each regime is indeed stationary. We firstly derive the *sector-specific* optimal patent breadth and then the *uniform* optimal patent breadth. Finally, we derive the first-best allocation and compare it with the equilibrium allocations under the two policy regimes.

4.1 Sector-specific optimal patent breadth

This section derives the optimal path of sector-specific patent breadth denoted by $\{\mu_{1,t}^*, \mu_{2,t}^*\}_{t=0}^{\infty}$. Technically, we are solving a Stackelberg differential game,[15] in which policymakers move first by choosing a time path of $\{\mu_{1,t}, \mu_{2,t}\}_{t=0}^{\infty}$ and then households respond by choosing a time path of consumption. It is well known that this Ramsey approach usually gives rise to time-inconsistent policies (i.e., after households make their best response, policymakers have incentives to deviate from their chosen policies *ex post*). Time inconsistency does not arise in this model because the equilibrium allocation at any time *t* depends only on the current level of patent breadth and hence is independent of future patent policies. Therefore, policymakers have no incentive to manipulate future policies for the purposing of influencing current allocations.

[15] See, for example, Xie (1997) and Karp and Lee (2003) for a discussion.

Proposition 1: *The optimal path of sector-specific patent breadth is stationary, time-consistent, subgame perfect and given by*

$$\mu_{1,t}^* = \mu_1^* = \alpha\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\varphi_1 \ln z}{\rho}, \tag{21}$$

$$\mu_{2,t}^* = \mu_2^* = (1-\alpha)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\varphi_2 \ln z}{\rho}. \tag{22}$$

Proof: See Appendix A.■

We impose a parameter restriction $\max\{\mu_1^*, \mu_2^*\} \leq z$ to ensure that the breadth parameter b_i is between zero and one for $i \in \{1,2\}$. Equations (21) and (22) show that a larger discount rate decreases the optimal patent breadth in both sectors. This is because the benefit of a higher growth rate on households' welfare becomes smaller as ρ increases. The quality step size z has a positive externality effect on the growth rate as shown in (15); therefore, a larger z increases optimal patent breadth in both sectors. An improvement in sector 1's technological opportunity φ_1 increases the optimal patent breadth in sector 1 and decreases that of sector 2. Similarly, an increase in α (i.e., sector 1's market size) increases the optimal patent breadth in sector 1 and decreases that of sector 2. Substituting (21) and (22) into (16) – (19) yields

$$L_1(\mu_1^*) = \frac{\rho}{\varphi_1 \ln z}, \tag{23}$$

$$L_2(\mu_2^*) = \frac{\rho}{\varphi_2 \ln z}, \tag{24}$$

$$H_1(\mu_1^*) = \alpha\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)-\left(1+\frac{1}{\ln z}\right)\frac{\rho}{\varphi_1}, \tag{25}$$

(26) $$H_2(\mu_2^*) = (1-\alpha)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)-\left(1+\frac{1}{\ln z}\right)\frac{\rho}{\varphi_2}.$$

We will compare (23) – (26) to the first-best labor allocations in Section 4.3.

4.2 Uniform optimal patent breadth

This section considers the policy regime under uniform patent breadth denoted by $\overline{\mu}_t \equiv \mu_{1,t} = \mu_{2,t}$ and derives the optimal path of uniform patent breadth $\{\overline{\mu}_t^*\}_{t=0}^{\infty}$. As before, we are solving a Stackelberg differential game, in which the policymakers move first by choosing a time path of $\{\overline{\mu}_t\}_{t=0}^{\infty}$ and households respond by choosing a time path of consumption.

Proposition 2: *The optimal path of uniform patent breadth is stationary, time-consistent, subgame perfect and given by*

(27) $$\overline{\mu}_t^* = \overline{\mu}^* = \alpha\,\mu_1^* + (1-\alpha)\mu_2^* = (\alpha^2\varphi_1 + (1-\alpha)^2\varphi_2)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\ln z}{\rho}.$$

Proof: See Appendix A.■

Proposition 2 shows that the optimal uniform patent breadth is a weighted average of the optimal sector-specific patent breadth, and the optimal weight is determined by α. The effects of ρ and z on the optimal patent breadth are the same as before. As for an increase in α, it has a positive (negative) effect on $\overline{\mu}^*$ if $\alpha\varphi_1$ is greater (less) than $(1-\alpha)\varphi_2$. Intuitively, a larger α increases the optimal patent breadth of sector 1 and decreases that of sector 2. Therefore, when the level of patent breadth is constrained to be the same across sectors, whether a larger α increases or decreases $\overline{\mu}^*$ depends on the relative magnitude of the above two forces. At a large (small) α , the effect from sector 1 (sector 2) dominates, so that $\overline{\mu}^*$ is an U-shape function in α . Similarly, φ_i has an U-shape effect on $\overline{\mu}^*$, which is initially decreasing in φ_i and subsequently increasing in φ_i, because an increase in φ_i also leads to opposing effects on the optimal patent breadth in the two sectors. Substituting (27) into (16) – (19) yields

(28) $$L_1(\overline{\mu}^*) = \frac{\alpha}{\alpha^2\varphi_1 + (1-\alpha)^2\varphi_2}\left(\frac{\rho}{\ln z}\right),$$

(29) $$L_2(\overline{\mu}^*) = \frac{1-\alpha}{\alpha^2\varphi_1 + (1-\alpha)^2\varphi_2}\left(\frac{\rho}{\ln z}\right),$$

(30) $$H_1(\overline{\mu}^*) = \alpha\left(1 + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2}\right) - \left(1 + \frac{\alpha\varphi_1}{\alpha^2\varphi_1 + (1-\alpha)^2\varphi_2}\frac{1}{\ln z}\right)\frac{\rho}{\varphi_1},$$

(31) $$H_2(\overline{\mu}^*) = (1-\alpha)\left(1 + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2}\right) - \left(1 + \frac{(1-\alpha)\varphi_2}{\alpha^2\varphi_1 + (1-\alpha)^2\varphi_2}\frac{1}{\ln z}\right)\frac{\rho}{\varphi_2}.$$

4.3 First-best allocation

In this section, we drive the first-best labor allocations by having the social planner chooses a time path of $\{L_{1,t}, L_{2,t}, H_{1,t}, H_{2,t}\}_{t=0}^{\infty}$ to maximize (1). The optimization yields a corner solution in which either $H_{1,t}$ or $H_{2,t}$ is equal to zero for all *t* depending on whether $\alpha\varphi_1$ is greater or less than $(1-\alpha)\varphi_2$. For illustrative purposes, we consider $\alpha\varphi_1 > (1-\alpha)\varphi_2$, so that $H_{2,t} = 0$ for all *t*.

Lemma 2: *The optimal path* $\{L_{1,t}^*, L_{2,t}^*, H_{1,t}^*, H_{2,t}^*\}_{t=0}^{\infty}$ *is stationary and given by*

(32) $$L_{1,t}^* = L_1^* = \frac{\rho}{\varphi_1 \ln z},$$

(33) $$L_{2,t}^* = L_2^* = \left(\frac{1-\alpha}{\alpha}\right)\frac{\rho}{\varphi_1 \ln z},$$

(34) $$H_{1,t}^* = H_1^* = 1 - \frac{\rho}{\alpha\varphi_1 \ln z},$$

(35) $$H_{2,t}^* = H_2^* = 0.$$

Proof: See Appendix A.■

Comparing (23) – (26) and (32) – (35) shows that $L_1(\mu_1^*) = L_1^*$ and $L_2(\mu_2^*) > L_2^*$. In other words, compared to the first-best allocations, the equilibrium under $\{\mu_1^*, \mu_2^*\}$ devotes too much labor to production in sector 2 and too little labor to R&D (i.e., $H_1(\mu_1^*) + H_2(\mu_2^*) < H_1^* + H_2^*$). Also, the first-best allocations (34) and (35) are efficient in terms of allocating R&D labor to the sector that has a larger effect on welfare (recall that $\alpha\varphi_1 > (1-\alpha)\varphi_2$). As for the equilibrium allocations under $\{\mu_1^*, \mu_2^*\}$, we see that $H_1(\mu_1^*) < H_1^*$ and $H_2(\mu_2^*) > H_2^* = 0$. Therefore, the first-best optimal growth rate is strictly higher than the equilibrium growth rate under sector-specific patent breadth unless $\alpha\varphi_1 = (1-\alpha)\varphi_2$, in which case the growth rates are equal.

Comparing (28) – (31) and (32) – (35) shows that $L_1(\overline{\mu}^*) > L_1^*$ and $L_2(\overline{\mu}^*) > L_2^*$. In other words, the equilibrium under uniform patent breadth allocates too much labor to production in both sectors and too little labor to R&D (i.e., $H_1(\overline{\mu}^*) + H_2(\overline{\mu}^*) < H_1^* + H_2^*$). As for the allocation of R&D labor, we see that $H_1(\overline{\mu}^*) < H_1^*$ and $H_2(\overline{\mu}^*) > H_2^* = 0$. Therefore, the first-best growth rate is also strictly higher than the equilibrium growth rate under uniform patent breadth unless $\alpha\varphi_1 = (1-\alpha)\varphi_2$, in which case the growth rates are equal.

5. Growth and welfare differences between policy regimes

In this section, we consider the growth and welfare differences between the two policy regimes. We find that although the growth difference is zero, the welfare difference is a function of α and φ_1/φ_2, and the magnitude is generally non-negligible. To compare the growth difference, we firstly substitute (25) and (26) into (20) to derive the equilibrium growth rate under sector-specific optimal patent breadth $g(\mu_1^*, \mu_2^*)$ and then substitute (30) and (31) into (20) to derive the equilibrium growth rate under uniform optimal patent breadth $g(\bar{\mu}^*)$. In both cases, we find that the equilibrium growth rate is

$$(36) \qquad g(\mu_1^*, \mu_2^*) = g(\bar{\mu}^*) = (\alpha^2\varphi_1 + (1-\alpha)^2\varphi_2)\left(1 + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2}\right)\ln z - \rho(1 + \ln z).$$

Therefore, the growth difference between the optimal sector-specific and uniform patent breadth is zero in this model. However, the following results show that despite this zero growth-rate differential, the welfare difference can be non-negligible. This is because uniform patent breadth achieves the same growth rate as sector-specific patent breadth but with a less efficient allocation of R&D labor (i.e., $H_1(\mu_1^*) + H_2(\mu_2^*) < H_1(\bar{\mu}^*) + H_2(\bar{\mu}^*)$).[16] Therefore, under uniform patent breadth, there is less labor available for production, which decreases the level of consumption and hence social welfare relative to the equilibrium under sector-specific patent breadth.

Given the balanced-growth behavior of the model under a stationary path of patent breadth, the households' lifetime utility in (1) can be re-expressed as

$$(37) \qquad U = \frac{1}{\rho}\left(\ln C_0 + \frac{g}{\rho}\right) = \frac{1}{\rho}\left[\alpha\left(\ln L_1 + \left(\frac{\varphi_1 \ln z}{\rho}\right)H_1\right) + (1-\alpha)\left(\ln L_2 + \left(\frac{\varphi_2 \ln z}{\rho}\right)H_2\right)\right],$$

[16] This inequality can be shown by using (25), (26), (30), (31) and a few steps of mathematical manipulation.

where the second equality is obtained by dropping the exogenous terms $Z_{1,0}$ and $Z_{2,0}$. Given (37) as a measure of social welfare, we substitute (23) – (26) into (37) to compute the level of social welfare under sector-specific patent breadth denoted by $U(\mu_1^*, \mu_2^*)$ and substitute (28) – (31) into (37) to compute the level of social welfare under uniform patent breadth denoted by $U(\bar{\mu}^*)$.

Proposition 3: *The welfare difference* $\Delta U \equiv U(\mu_1^*, \mu_2^*) - U(\bar{\mu}^*)$ *can be expressed as*

$$\Delta U = \frac{1}{\rho}\left[\ln\left(\alpha^2 \frac{\varphi_1}{\varphi_2} + (1-\alpha)^2\right) - \left(\alpha \ln\left(\alpha \frac{\varphi_1}{\varphi_2}\right) + (1-\alpha)\ln(1-\alpha)\right)\right] \geq 0, \tag{38}$$

which becomes a strict inequality if $\alpha\varphi_1 \neq (1-\alpha)\varphi_2$.

Proof: See Appendix A.■

Given that $\rho\Delta U$ depends on only two parameters $\alpha \in (0,1)$ and $\varphi_1 / \varphi_2 \in (0,1]$, we can numerically evaluate (38) to examine the properties of $\rho\Delta U$ without loss of generality. Figure 1 plots the welfare difference against $\alpha \in (0,1)$ and $\varphi_1 / \varphi_2 \in [0.2,1]$.[17] For the ease of interpretation, the welfare difference is re-expressed as δ denoting the equivalent variation in consumption per year defined as $U[C_0(\mu_1^*, \mu_2^*), g(\mu_1^*, \mu_2^*)] = U[(1+\delta)C_0(\bar{\mu}^*), g(\bar{\mu}^*)]$.

[Insert Figure 1 here]

Figure 1 shows that for a given φ_1 / φ_2, the welfare difference δ is an M-shape function in α. Figure 2 plots δ against α for various values of φ_1 / φ_2.

[Insert Figure 2 here]

[17] The welfare difference can be very large when $\varphi_1 / \varphi_2 \in (0,0.2)$; thus, we report the results for $\varphi_1 / \varphi_2 \in [0.2,1]$ only. However, the properties of $\rho\Delta U$ are the same as in the rest of the parameter space.

Suppose $\varphi_1/\varphi_2 = 1$. In this case, the two sectors are symmetric when $\alpha = 0.5$. Under symmetry, the welfare loss from imposing uniform patent breadth is zero. As α deviates from 0.5 in either direction, the welfare loss becomes positive. This explains the U-shape pattern around 0.5 for $\varphi_1/\varphi_2 = 1$. As $\alpha \to 1$, the model becomes a one-sector model in which only sector 1 matters. In this case, $\bar{\mu}^* \to \mu_1^*$; therefore, the welfare loss δ approaches zero. The same is true for $\alpha \to 0$. This explains the M-shape pattern of δ for $\varphi_1/\varphi_2 = 1$. As φ_1/φ_2 decreases, the optimal patent breadth of sector 2 increases while that of sector 1 decreases. Therefore, households benefit from differentiated patent breadth even when $\alpha = 0.5$. When $\varphi_1/\varphi_2 < 1$, a uniform patent breadth is optimal only if α increases above 0.5 to diminish the importance of sector 2. This explains why the interior minimum of δ in Figure 2 shifts to the right as φ_1/φ_2 decreases.

[Insert Figures 3 and 4 here]

Figures 3 and 4 plot δ against φ_1/φ_2 for various values of α. Figure 3 shows that when $\alpha \in (0,0.5]$, the welfare loss δ is always decreasing in φ_1/φ_2. Intuitively, when $\alpha \in (0,0.5]$, the optimal patent breadth of sector 1 is smaller than that of sector 2; however, this gap shrinks as φ_1/φ_2 increases. Consequently, the welfare loss from imposing a uniform level of patent breadth diminishes as φ_1/φ_2 increases when $\alpha \in (0,0.5]$.

Figure 4 shows that when $\alpha \in (0.5,1)$, the welfare loss δ can become non-monotonic in φ_1/φ_2. When $\alpha \in (0.5,1)$, it is not necessarily the case that $\mu_1^* < \mu_2^*$. Only if φ_1/φ_2 is small enough, then $\mu_1^* < \mu_2^*$. In this case, as φ_1/φ_2 increases, the gap between μ_1^* and μ_2^* shrinks, and the welfare loss δ decreases. However, if φ_1/φ_2 becomes sufficiently large, then $\mu_1^* > \mu_2^*$. In this case, any further increase in φ_1/φ_2 would widen the gap between μ_1^* and μ_2^*; as a result, the welfare loss δ becomes increasing in φ_1 / φ_2. This explains the potential U-shape pattern of δ when $\alpha \in (0.5,1)$. In other words, whenever α is sufficiently large (small) such that μ_1^* is above (below) μ_2^* for a given φ_1 / φ_2, an increase (a decrease) in φ_1 / φ_2 would widen the gap between μ_1^* and μ_2^* and hence magnify the welfare loss δ. Finally, as $\alpha \to 1$, the welfare difference δ approaches zero for any φ_1 / φ_2. The same is true for $\alpha \to 0$.

Having understood the qualitative pattern of δ as a function of α and φ_1 / φ_2, we now consider the magnitude of the welfare loss. Figure 1 shows that the welfare loss ranges from zero to as large as 50% of consumption per year. To focus on the effect of asymmetry in technological opportunity across sectors, Table 1 summarizes the welfare costs of uniform patent protection for $\alpha = 0.5$ from Figure 1.

Table 1: Welfare costs of uniform patent breadth for α = 0.5

φ1/φ2	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
δ	34.2%	18.7%	10.7%	6.1%	3.3%	1.6%	0.6%	0.1%	0.0%

The policy implication from this illustrative numerical exercise is that even a moderate degree of asymmetry in technological opportunity across sectors can generate a non-negligible welfare cost of one-size-fit-all patent policy. Also, empirical evidence suggests that φ_i varies significantly across sectors. For example, Klenow (1996) finds that although R&D intensity and TFP growth at the industry level are positively correlated, R&D explains only a small fraction of the variation in industry-level TFP growth implying that technological opportunity differs significantly across industries. Furthermore, (15) implies that the log of TFP (level) in sector *i* can be expressed as $\ln Z_{i,t} = (\varphi_i \ln z) S_{i,t}$, where $S_{i,t} \equiv \int_0^t H_{i,\tau} d\tau$ denotes the stock of R&D in sector *i*. A number of empirical studies, such as Verspagen (1995), Los and Verspagen (2000) and Cameron (2000), have estimated the effect of R&D stock on the level of TFP/output at the industry level, and they also find that the effect of R&D stock varies substantially across industries.

6. Conclusion

This study has developed a two-sector R&D-based growth model and used the growth-theoretic framework to analyze the welfare gain from implementing differentiated patent protection across sectors. We find that the welfare gain can be substantial. However, this analysis is based on the assumption that policymakers (or the courts in the case of the Burk-Lemley proposal) are well-informed about the different characteristics across industries. In reality, it could be quite costly to acquire this kind of information. Therefore, for real-world policy applications, the welfare gain from implementing industry-specific patent protection should be evaluated in conjunction with the information-acquisition costs, and the magnitude of these costs remains as an empirical question. Finally, the issue of scale effects is set aside by normalizing the supply of labor to unity so that it is the *share* of labor devoted to R&D that determines growth as in the second-generation R&D-based growth model.[18]

[18] See Jones (1999) for an excellent discussion on scale effects in R&D-based growth models.

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Appendix A

Proof of Lemma 1: Households' current-value Hamiltonian is

(A1) $$\Omega_t = \ln C_t + \omega_t (R_t V_t + W_t - P_t C_t) .$$

The first-order conditions are

(A2) $$\frac{\partial \Omega_t}{\partial C_t} = \frac{1}{C_t} - \omega_t P_t = 0 ,$$

(A3) $$\frac{\partial \Omega_t}{\partial V_t} = \omega_t R_t = \omega_t \rho - \dot{\omega}_t ,$$

(A4) $$\frac{\partial \Omega_t}{\partial \omega_t} = R_t V_t + W_t - P_t C_t = \dot{V}_t .$$

The transversality condition is $\lim_{t\to\infty} e^{-\rho t} \omega_t V_t = 0$. Combining (A3) and (A4) yields

(A5) $$\dot{\omega}_t V_t + \omega_t \dot{V}_t = \omega_t V_t \rho + \omega_t W_t - \omega_t P_t C_t .$$

Given households' first-order conditions, we use the market equilibrium conditions to solve (A5). From (13), $W_t = \varphi_1 V_{1,t} = \varphi_2 V_{2,t}$. Combining this condition with $V_{1,t} + V_{2,t} = V_t$ yields

(A6) $$V_t = \left(\frac{1}{\varphi_1} + \frac{1}{\varphi_2} \right) W_t .$$

Substituting (A2) and (A6) into (A5) yields

(A7) $$\dot{\omega}_t V_t + \omega_t \dot{V}_t = \omega_t V_t \left(\rho + \frac{\varphi_1 \varphi_2}{\varphi_1 + \varphi_2} \right) - 1 .$$

(A7) is a one-dimensional differential equation in $\omega_t V_t$, and the dynamic system is characterized by saddle-point stability. Therefore, $\omega_t V_t$ must jump to its unique steady state; otherwise, the transversality condition would be violated. To see this result, integrating (A7) with respect to time yields

(A8) $$\omega_t V_t = e^{\rho t}\beta + \left(\rho + \frac{\varphi_1\varphi_2}{\varphi_1+\varphi_2}\right)^{-1},$$

where β is an integration constant. The transversality condition implies that $\beta = 0$. Therefore, $\omega_t V_t = \left(\rho + \frac{\varphi_1\varphi_2}{\varphi_1+\varphi_2}\right)^{-1}$ for all *t*. Substituting (A6) and $W_t = 1$ into this condition yields

(A9) $$\omega_t = \left(1 + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2}\right)^{-1}$$

for all *t*. Because ω_t is stationary, (A3) implies that $R_t = \rho$ for all *t*. Also, (A2) implies that

(A10) $$P_t C_t = \left(1 + \frac{\rho}{\varphi_1} + \frac{\rho}{\varphi_2}\right).$$

In other words, nominal expenditure on consumption and the nominal interest rate are constant regardless of whether the path of patent breadth $\{\mu_{1,t}, \mu_{2,t}\}_{t=0}^{\infty}$ is stationary or not.

The rest of this proof derives the equilibrium labor allocations for an arbitrary path of patent breadth. From (5), (6) and (10), the factor payments to production workers in the two sectors are respectively

(A11) $$W_t L_{1,t} = P_{1,t} Y_{1,t} / \mu_{1,t} = \alpha P_t C_t / \mu_{1,t},$$

(A12) $$W_t L_{2,t} = P_{2,t} Y_{2,t} / \mu_{2,t} = (1-\alpha) P_t C_t / \mu_{2,t}.$$

Combining (A11) and (A12) yields

(A13) $$\frac{L_{1,t}}{L_{2,t}} = \frac{\mu_{2,t}}{\mu_{1,t}}\left(\frac{\alpha}{1-\alpha}\right).$$

The monopolistic profits in the two sectors are respectively

(A14) $$\pi_{1,t} = \left(\frac{\mu_{1,t}-1}{\mu_{1,t}}\right) P_{1,t} Y_{1,t} = \left(\frac{\mu_{1,t}-1}{\mu_{1,t}}\right) \alpha P_t C_t,$$

(A15) $$\pi_{2,t} = \left(\frac{\mu_{2,t}-1}{\mu_{2,t}}\right)P_{1,t}Y_{1,t} = \left(\frac{\mu_{2,t}-1}{\mu_{2,t}}\right)(1-\alpha)P_tC_t \, .$$

(13) implies that $\dot{V}_{i,t} = 0$. Imposing $\dot{V}_{i,t} = 0$ on (11) yields

(A16) $$\pi_{i,t} = (\rho + \lambda_{i,t})V_{i,t} \, ,$$

where we have applied the previously derived result ($R_t = \rho$ for all *t*). Substituting (A11), (A14) and (A16) into (13) yields

(A17) $$H_{1,t} = (\mu_{1,t} - 1)L_{1,t} - \rho / \varphi_1 \, .$$

Similarly, substituting (A12), (A15) and (A16) into (13) yields

(A18) $$H_{2,t} = (\mu_{2,t} - 1)L_{2,t} - \rho / \varphi_2 \, .$$

To close the model, we use the labor-market clearing condition given by

(A19) $$L_{1,t} + L_{2,t} + H_{1,t} + H_{2,t} = 1 \, .$$

Solving the four equations (A13), (A17) – (A19) yields (16) – (19).■

Proof of Proposition 1: In general, the households' Hamiltonian co-state variable ω_t should be treated as a state variable in the policymakers' dynamic optimization problem. However, (A9) shows that ω_t is constant; therefore, we can directly substitute (4) and $Y_{i,t} = Z_{i,t}L_{i,t}$ for $i \in \{1,2\}$ into (1) to derive the policymakers' current-value Hamiltonian given by

(A20) $$\Phi_t(\mu_{1,t}, \mu_{2,t}) = \ln C_t + \phi_{1,t}\dot{Z}_{1,t} + \phi_{2,t}\dot{Z}_{2,t} \, ,$$

where $\ln C_t = \alpha[\ln Z_{1,t} + \ln L_{1,t}(\mu_{1,t})] + (1-\alpha)[\ln Z_{2,t} + \ln L_{2,t}(\mu_{2,t})]$, $\dot{Z}_{1,t} = Z_{1,t}(\varphi_1 \ln z)H_{1,t}(\mu_{1,t})$ and $\dot{Z}_{2,t} = Z_{2,t}(\varphi_2 \ln z)H_{2,t}(\mu_{2,t})$. The labor allocations $L_{1,t}(\mu_{1,t})$, $L_{2,t}(\mu_{2,t})$, $H_{1,t}(\mu_{1,t})$ and $H_{2,t}(\mu_{2,t})$ are given by (16) – (19). The first-order conditions are

(A21) $$\frac{\partial \Phi_t}{\partial \mu_{1,t}} = -\frac{\alpha}{\mu_{1,t}} + \phi_{1,t} Z_{1,t} (\varphi_1 \ln z)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\alpha}{(\mu_{1,t})^2} = 0,$$

(A22) $$\frac{\partial \Phi_t}{\partial \mu_{2,t}} = -\frac{1-\alpha}{\mu_{2,t}} + \phi_{2,t} Z_{2,t} (\varphi_2 \ln z)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{1-\alpha}{(\mu_{2,t})^2} = 0,$$

(A23) $$\frac{\partial \Phi_t}{\partial Z_{1,t}} = \frac{\alpha}{Z_{1,t}} + \phi_{1,t}\frac{\dot{Z}_{1,t}}{Z_{1,t}} = \phi_{1,t}\rho - \dot{\phi}_{1,t},$$

(A24) $$\frac{\partial \Phi_t}{\partial Z_{2,t}} = \frac{1-\alpha}{Z_{2,t}} + \phi_{2,t}\frac{\dot{Z}_{2,t}}{Z_{2,t}} = \phi_{2,t}\rho - \dot{\phi}_{2,t}.$$

Manipulating (A23) yields $\dot{\phi}_{1,t} Z_{1,t} + \phi_{1,t}\dot{Z}_{1,t} = \phi_{1,t} Z_{1,t}\rho - \alpha$. As before, this differential equation is characterized by saddle-point stability, so that $\phi_{1,t} Z_{1,t}$ must jump to its unique steady-state value given by

(A25) $$\phi_{1,t} Z_{1,t} = \alpha/\rho$$

for all t. Substituting (A25) into (A21) yields

(A26) $$\mu_{1,t} = \alpha\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\varphi_1 \ln z}{\rho}$$

for all t. Manipulating (A24) yields $\dot{\phi}_{2,t} Z_{2,t} + \phi_{2,t}\dot{Z}_{2,t} = \phi_{2,t} Z_{2,t}\rho - (1-\alpha)$, and this differential equation is also characterized by saddle-point stability. Therefore, $\phi_{2,t} Z_{2,t}$ must also jump to its unique steady-state value given by

(A27) $$\phi_{2,t} Z_{2,t} = (1-\alpha)/\rho$$

for all t. Substituting (A27) into (A22) yields

(A28) $$\mu_{2,t} = (1-\alpha)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\varphi_2 \ln z}{\rho}$$

for all t. (A26) and (A28) show that the optimal path of sector-specific patent breadth is stationary. Given that the equilibrium labor allocations (16) – (19) are independent of future policies, the policymakers have no incentive to deviate from their chosen path of patent breadth at any point in time along the equilibrium path (i.e., time consistency). Furthermore, given that (A26) and (A28) are stationary and independent of the state variables, they are optimal under any realization of the state variables along and off the equilibrium path (i.e., subgame perfectness).■

Proof of Proposition 2: The policymakers' current-value Hamiltonian in the case of uniform patent breadth is

$$\Phi_t(\overline{\mu}_t) = \ln C_t + \phi_{1,t}\dot{Z}_{1,t} + \phi_{2,t}\dot{Z}_{2,t}\,, \tag{A29}$$

where $\dot{Z}_{1,t}$, $\dot{Z}_{2,t}$ and $\ln C_t$ can be re-expressed as in Proposition 1. The labor allocations $L_{1,t}(\overline{\mu}_t)$, $L_{2,t}(\overline{\mu}_t)$, $H_{1,t}(\overline{\mu}_t)$ and $H_{2,t}(\overline{\mu}_t)$ are given by (16) – (19) as before. The first-order conditions are

$$\frac{\partial \Phi_t}{\partial \overline{\mu}_t} = -\frac{1}{\overline{\mu}_t} + \phi_{1,t}Z_{1,t}(\varphi_1 \ln z)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\alpha}{(\overline{\mu}_t)^2} + \phi_{2,t}Z_{2,t}(\varphi_2 \ln z)\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{1-\alpha}{(\overline{\mu}_t)^2} = 0\,, \tag{A30}$$

$$\frac{\partial \Phi_t}{\partial Z_{1,t}} = \frac{\alpha}{Z_{1,t}} + \phi_{1,t}\frac{\dot{Z}_{1,t}}{Z_{1,t}} = \phi_{1,t}\rho - \dot{\phi}_{1,t}\,, \tag{A31}$$

$$\frac{\partial \Phi_t}{\partial Z_{2,t}} = \frac{1-\alpha}{Z_{2,t}} + \phi_{2,t}\frac{\dot{Z}_{2,t}}{Z_{2,t}} = \phi_{2,t}\rho - \dot{\phi}_{2,t}\,. \tag{A32}$$

Manipulating (A31) and (A32) yields (A25) and (A27). Substituting them into (A30) yields

$$\overline{\mu}_t = \alpha^2\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\varphi_1 \ln z}{\rho} + (1-\alpha)^2\left(1+\frac{\rho}{\varphi_1}+\frac{\rho}{\varphi_2}\right)\frac{\varphi_2 \ln z}{\rho} \tag{A33}$$

for all t. (A33) shows that the optimal path of uniform patent breadth is stationary. (A33) is time-consistent and subgame perfect for the same reasons as in Proposition 1.■

Proof of Lemma 2: The social planner's current-value Hamiltonian is

(A34)
$$\begin{aligned}\Psi_t &= \alpha \ln Z_{1,t} + \alpha \ln L_{1,t} + (1-\alpha)\ln Z_{2,t} + (1-\alpha)\ln L_{2,t} \\ &+ \psi_{1,t} Z_{1,t}(\varphi_1 \ln z) H_{1,t} + \psi_{2,t} Z_{2,t}(\varphi_2 \ln z) H_{2,t} + \psi_{3,t}(1 - L_{1,t} - L_{2,t} - H_{1,t} - H_{2,t})\end{aligned}.$$

The first-order conditions are

(A35)
$$\frac{\partial \Psi_t}{\partial L_{1,t}} = \frac{\alpha}{L_{1,t}} - \psi_{3,t} = 0,$$

(A36)
$$\frac{\partial \Psi_t}{\partial L_{2,t}} = \frac{1-\alpha}{L_{2,t}} - \psi_{3,t} = 0,$$

(A37)
$$\frac{\partial \Psi_t}{\partial H_{1,t}} = \psi_{1,t} Z_{1,t}(\varphi_1 \ln z) - \psi_{3,t} \le 0,$$

(A38)
$$\frac{\partial \Psi_t}{\partial H_{2,t}} = \psi_{2,t} Z_{2,t}(\varphi_2 \ln z) - \psi_{3,t} \le 0,$$

(A39)
$$\frac{\partial \Psi_t}{\partial Z_{1,t}} = \frac{\alpha}{Z_{1,t}} + \psi_{1,t}\frac{\dot{Z}_{1,t}}{Z_{1,t}} = \psi_{1,t}\rho - \dot{\psi}_{1,t},$$

(A40)
$$\frac{\partial \Psi_t}{\partial Z_{2,t}} = \frac{1-\alpha}{Z_{2,t}} + \psi_{2,t}\frac{\dot{Z}_{2,t}}{Z_{2,t}} = \psi_{2,t}\rho - \dot{\psi}_{2,t}.$$

As before, integrating (A39) and (A40) with respect to time and setting the integration constants to zero as implied by the transversality conditions yields $\psi_{1,t} Z_{1,t} = \alpha / \rho$ and $\psi_{2,t} Z_{2,t} = (1-\alpha)/\rho$. Substituting these conditions into (A37) and (A38) yields

(A41)
$$\psi_{3,t} = \frac{\alpha \varphi_1 \ln z}{\rho} > \frac{(1-\alpha)\varphi_2 \ln z}{\rho},$$

which follows from $\alpha\varphi_1 > (1-\alpha)\varphi_2$. Substituting (A41) into (A35) and (A36) yields (32) and (33). Combining (32), (33), $H_{2,t} = 0$ from (A38) and $L_{1,t} + L_{2,t} + H_{1,t} + H_{2,t} = 1$ yields (34).■

Proof of Proposition 3: We already know from (36) that the growth difference is zero across the two regimes. Therefore, the welfare difference is given by the difference in initial consumption.

$$\text{(A42)} \quad \Delta U = \frac{\ln C_0(\mu_1^*, \mu_2^*) - \ln C_0(\bar{\mu}^*)}{\rho} = \frac{\alpha \ln[L_1(\mu_1^*, \mu_2^*)/L_1(\bar{\mu}^*)] + (1-\alpha)\ln[L_2(\mu_1^*, \mu_2^*)/L_2(\bar{\mu}^*)]}{\rho}.$$

Substituting (23), (24), (28) and (29) into (A42) yields

$$\text{(A43)} \qquad \Delta U = \frac{1}{\rho}\left[\alpha \ln\left(\frac{\alpha^2\varphi_1 + (1-\alpha)^2\varphi_2}{\alpha\varphi_1}\right) + (1-\alpha)\ln\left(\frac{\alpha^2\varphi_1 + (1-\alpha)^2\varphi_2}{(1-\alpha)\varphi_2}\right)\right].$$

Applying a few steps of mathematical manipulation to (A43) yields

$$\text{(A44)} \qquad \Delta U = \frac{1}{\rho}\left[\ln\left(\alpha^2\frac{\varphi_1}{\varphi_2} + (1-\alpha)^2\right) - \left(\alpha \ln\left(\alpha\frac{\varphi_1}{\varphi_2}\right) + (1-\alpha)\ln(1-\alpha)\right)\right].$$

Finally, given that $\ln(.)$ is a concave function, Jensen's inequality implies that (A44) is weakly positive, and a strict inequality emerges if $\alpha\varphi_1 \neq (1-\alpha)\varphi_2$.■

Figure 1: Welfare differences between sector-specific and uniform patent breadth

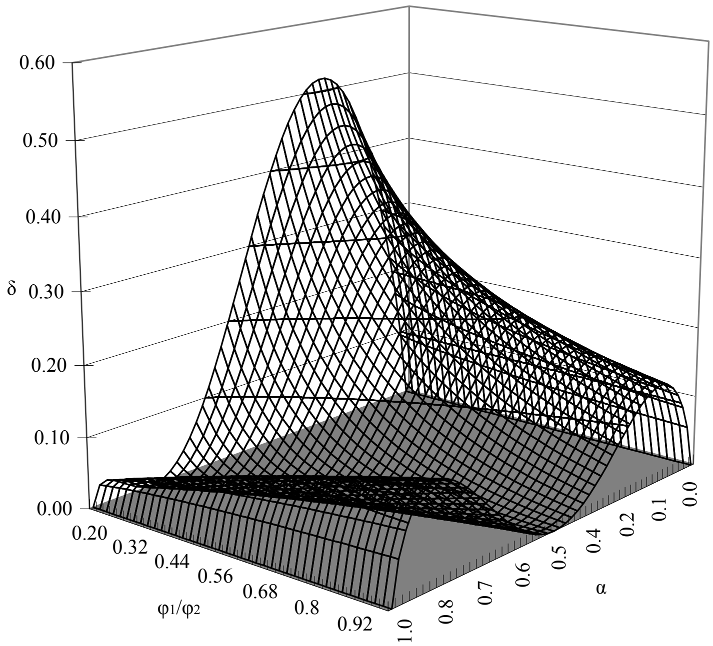

Figure 2: Welfare differences as a function of α

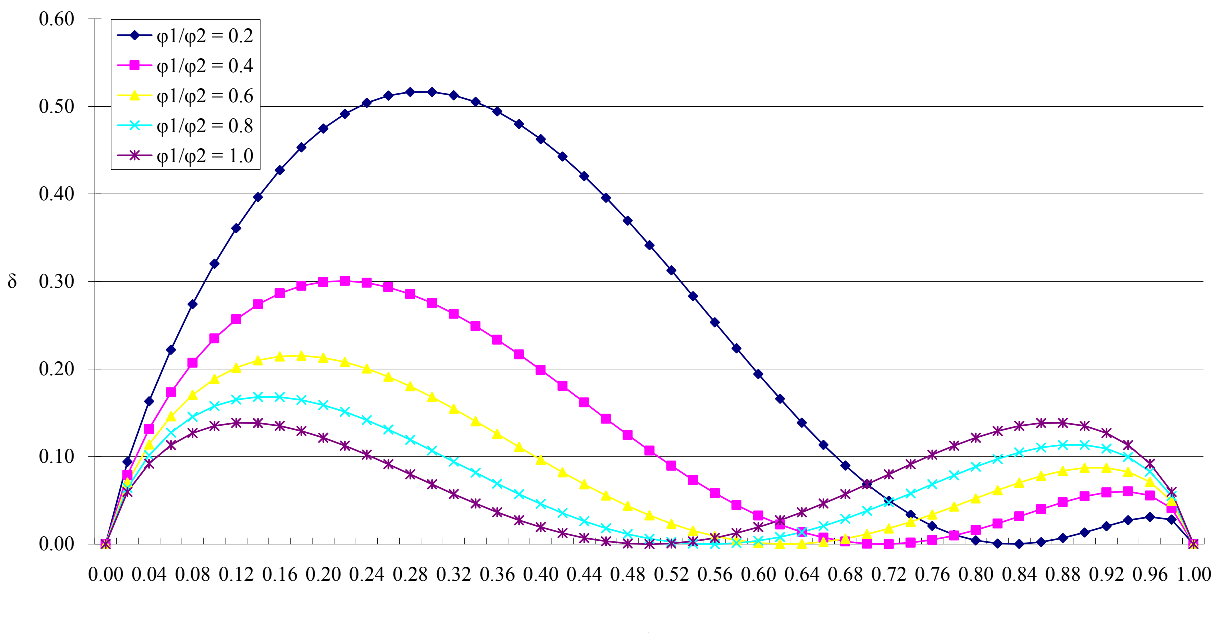

Figure 3: Welfare differences as a function of φ1/φ2

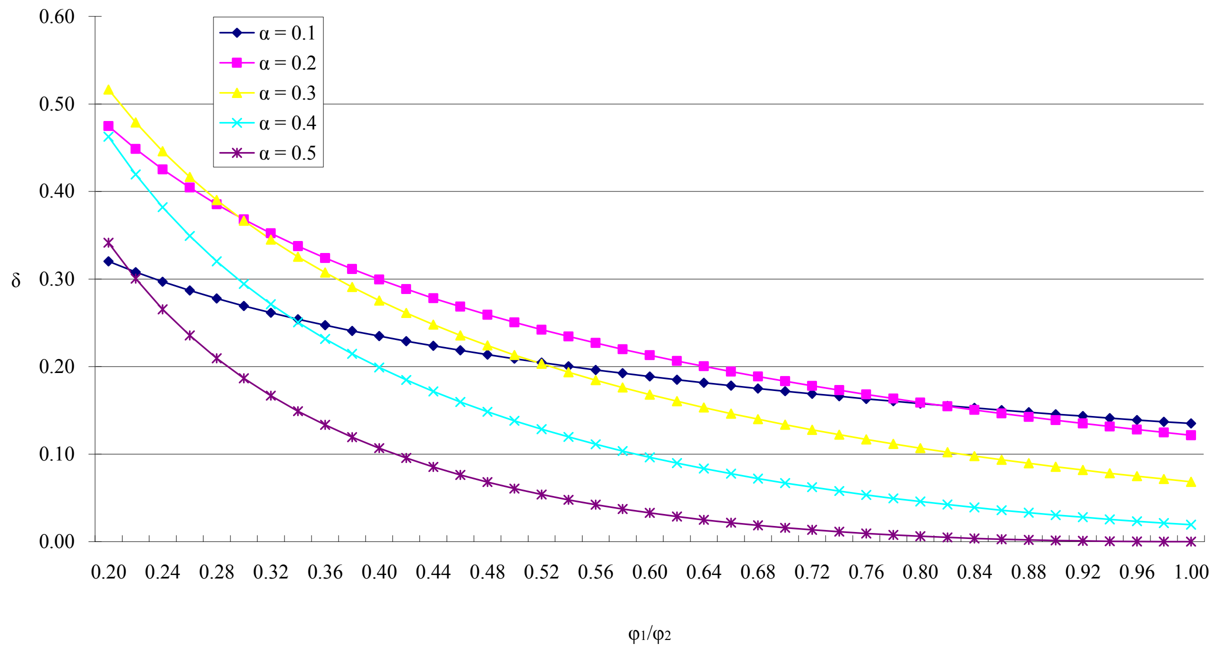

Figure 4: Welfare differences as a function of φ1/φ2