Carry Trade, Forward Premium Puzzle and Currency Crisis

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Abstract

Recent many empirical studies have argued that currency carry trade have been a driving force behind exchange rate movements, and have explained the latest financial crisis of 2007-2009 in terms of a sudden, massive reversal of carry trade positions.

The aim of this paper is to provide one potential theoretical explanation for questions why currency carry trade becomes profitable, and why a sudden unwinding of carry trade is caused. We propose a new behavioral model of currency bubbles and crashes. We consider that investors trade two currencies: the domestic currency, and the foreign currency. Investors are divided into two groups, the rational investors and the carry traders. The rational investors maximize their expected utility of their wealth in the next period. Carry traders maximize their random utility of binary choice: investing the domestic currency or investing the foreign currency.

We demonstrate that carry-traders' herd behavior, which follows the behavior getting a majority, gives cause to a currency bubble, and their carry trading prolongs bubble. However, depreciation of funding currency slows down as the carry-trader's behavior approaches to a stationary state, so that the return on carry trade predicted by carry traders begins to decrease in the second half of bubble. We demonstrate that decreasing the return on carry trade predicted by carry traders lead to currency crash. Our model also gives a plausible explanation on the forward premium puzzle.

JEL Classification Codes: F31, G01

1. Introduction

Many researchers have focused on the role of carry trade on currency bubble and the possibility that a sudden unwinding of carry trade might give cause to currency crisis. Recent many empirical studies have argued that currency carry trade have been a driving force behind exchange rate movements in the latest financial crisis of 2007-2009 and have explained the large currency crashes in terms of a sudden, massive reversal of carry trade positions. (See for instances, Gagnon, and Chaboud (2007), Cairns, Ho and McCauley (2007), Galati, Heath and McGuire (2007), Brunnermeier, Nagel, and Pedersen (2008), Melvin and Taylor (2009), and Kohler (2010)).

A carry trade is a popular strategy for currency investors which consists of selling low interest-rate currencies and investing in high interest-rate currency. Currency carry trade is profitable if the interest differential is not completely offset by an appreciation of the low interest-rate currency. An increase in carry trade positions generally tends to weaken the low interest-rate currencies and strengthen the high interest-rate currencies. This makes profitability self-fulfilling and attracting further carry trades. As a result of this feedback loop, carry trades tend to be associated with a gradual appreciation of the high interest-rate currency and a depreciation of the low interest-rate currency for a while. However, as some reasons such as changes in interest rate expectations lead carry traders to a sudden unwinding of carry trades, there is a tendency for the high interest-rate currencies to depreciate and the low interest-rate currencies to appreciate sharply.

The aim of this paper is to provide one potential theoretical explanation for questions how currency bubbles and crashes which describes above. We propose a new model of currency bubbles and crashes which are caused by carry trade. The high average payoffs to the carry trade means the violation of uncovered interest parity (UIP) which has been termed the forward premium puzzle: currencies with high interest rates tend to appreciate. Our model also proposes a solution for a forward premium puzzle[1].

We consider that investors trade two currencies: the domestic currency, and the foreign currency. Investors are divided into two groups, rational investors and carry traders. The rational investors chooses that the portfolio of currencies which will maximize his/her expected utility of end-of-period wealth using asset-pricing models such as the CAPM (see e.g., Mossin (1966) and Lintner (1969)). On the other hand, carry

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[1] Empirical studies consistently reject the UIP (see, for a survey on this topic, Engel (1996),.

traders maximize their random utility of alternatives[2], that is, selling foreign currency and buying domestic currency, or selling domestic currency and buying foreign currency. We assume that a carry trader's decision-making is influenced by (i) the decisions of the other carry-traders, and (ii) the carry traders' anticipation of the payoffs to carry trade which is defined as the exponential moving average of the payoffs to carry trade. The carry-trader's utility function of an alternative is composed of the above attributes, and random variable. In our model, we show that as the interaction among carry traders, that is, the extent, that each carry-trader is influenced by the decisions of other carry-traders, is reinforced, carry traders begin to *follow the herd*. In the attribute (ii), we also assume that the carry-traders' expectation of the return on carry trade is adaptive. Our model indicates a mechanism that that carry-traders' herd behavior, which follows the behavior getting a majority, and their attempt to surf currency bubbles, gives cause to a bubble ended up with a crash. This also gives a plausible explanation on the forward premium puzzle.

The paper proceeds as follows. The model is described in Section 2. In Section 3, and in Section 4 we give a theoretical explanation on a mechanism of currency bubble and crash. We give concluding remarks in Section 5.

2. Model

Consider the exchange market in which two currencies are traded. S_t denotes the spot

[2] The qualitative choice models based on maximization of the agent's random utility has been developed by McFadden (1974) .

exchange rate defined as unit of foreign currency per domestic currency. The variables B_t and B_t^* denote holdings of foreign currency and domestic currency at time t, respectively. Currencies are purchased at time t yield interest rates of R_t in foreign currency and R_t^* in domestic currency, respectively. We divide into two groups of investors with different decision making. The first group of investors is a group of rational investors who maximize their expected utility of wealth in the next period using asset-pricing models such as the CAPM (see e.g., Mossin (1966) and Lintner (1969)). The second group of investors is the group of the carry traders who maximize the random utility[3] of the binary choice: investing in a long position in the domestic currency financed by borrowing in the foreign currency, and investing in a long position in the domestic currency financed by borrowing in the foreign currency.

2.1 Rational investors

Let us consider the behavior of rational-investors. We shall assume that there is a number M of rational investors. Their object is to maximize the expected utility $EU(W_{t+1})$ of wealth W_{t+1} in the next period, t+1 by selecting a portfolio mix of the domestic currency B_t^* and the foreign currency B_t. We assume that rational investor's preferences are characterized by the constant-absolute risk aversion (CARA) utility with the coefficient of risk aversion, γ. The rational investors are assumed to be identical. We consider the behavior of the representative-rational investor hereafter. The maximization problem which the rational investors solve is equivalent to the

3 See McFadden (1974).

mean-variance model[4]. That is, in his choice among all the possible portfolios, the rational investor is satisfied to be guided by its expected yields $E(W_t)$ and its variance $V(W_t)$.

$$\underset{B_t, B_t^*}{Max}\ EU(W_{t+1}) = Max\left\{E(W_t) - \frac{\gamma}{2}V(W_t)\right\} \qquad (1)$$

$$s.t. \quad S_t(B_t^* - B_{t-1}^*(1+R_{t-1}^*)) + (B_t - B_{t-1}(1+R_{t-1})) = 0$$

where an investor's wealth is written as

$$W_{t+1} = S_{t+1}^* B_t^*(1+R_t^*) + B_t(1+R_t)$$

The expected value of the wealth $E(W_{t+1})$, and the variance of the wealth $V(W_{t+1})$ is defined as

$$\begin{aligned}
&E(W_{t+1}) = B_t^*(1+R_t^*)E(S_{t+1}) + B_t(1+R_t),\\
&V(W_{t+1}) = E[(W_{t+1} - E[W_{t+1}])^2]\\
&= E[\{B_t^*(1+R_t^*)S_{t+1} + B_t(1+R_t) - (B_t^*(1+R_t^*)E(S_{t+1}) + B_t(1+R_t))\}^2]\\
&= E[\{B_t^*(1+R_t^*)(S_{t+1} - E(S_{t+1}))\}^2]\\
&= \sigma_1^2(B_t^*)^2(1+R_t^*)^2
\end{aligned}$$

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[4] The Arrow-Pratt measure of risk aversion is defined as $\rho \equiv -U''W_{it}/U'$, where $U(W)$ is the utility function, the expectation of which is to be maximized. One can take a Tayler-series approximation to $EU(W)$ and differentiate it with respect to $E(W)$ and $V(W)$ to show that the two definitions of ρ are equivalent.

The utility function will have a constant coefficient of relative risk-aversion if it is exponential in form:

$$U(W) = \frac{1}{\gamma}W^{\gamma},$$

where $\rho = 1-\gamma$. The solution to the one-period maximization problem considered here will be the correct solution to the general intertemporal maximization problem,

if the utility function is further restricted to the logarithmic form, the limiting case as γ goes to zero, which implies $\rho = 1$, or if events occurring during the period are independent of the expected returns that prevail in the following period.

where $E(S_{t+1})$ is the expected value of S_{t+1}, σ_j^2 the variance of S_{t+1}.

The corresponding first-order conditions are:

$$\frac{\partial EU}{\partial B_t^*} = (1+R_t^*)E(S_{t+1}) - \frac{\gamma}{2}[2\sigma^2 B_t^*(1+R_t^*)^2] + \mu S_t^* = 0$$
$$\frac{\partial EU}{\partial B_{0t}} = 1 + R_t + \mu = 0 \qquad (2)$$
$$\frac{\partial EU}{\partial \mu} = S_t(B_t^* - B_{t-1}^*(1+R_{t-1}^*)) + (B_t - B_{t-1}(1+R_t)) = 0$$

where μ denotes the Lagrangian. Holdings of domestic and foreign currencies of the optimal portfolio are:

$$E(S_{t+1}) - \gamma\sigma^2 B_t^*(1+R_t^*) - \frac{(1+R_t)}{(1+R_t^*)}S_t = 0$$

$$B_t^* = \frac{1}{\gamma\sigma^2(1+R_t^*)}\left(E(S_{t+1}) - \frac{(1+R_t)}{(1+R_t^*)}S_t\right)$$

$$B_t = B_{t-1} + (1+R_t)S_t(B_t^* - (1+R_{t-1}^*)B_{t-1}^*) \qquad (3)$$

The well-known uncovered interest-rate parity (UCIP) is written as

$$(1+R_t) = (1+R_t^*)\left(\frac{E(S_{t+1})}{S_t}\right). \qquad (4)$$

Using the UCIP (4), the equation (3) can be rewritten as follows:

$$B_t^* = \frac{S_t}{\gamma\sigma^2(1+R_t^*)^2}\left((1+R_t^*)\frac{E(S_{t+1})}{S_t} - (1+R_t)\right). \qquad (5)$$

Therefore, the rational investors' transaction depends on the expected risk premiums of risky assets. The excess demand (or excess supply) for two currencies by rational investors is calculated by subtracting S_t from S_{t+1}

$$B_t^* - B_{t-1}^* = \frac{1}{\gamma\sigma^2(1+R_t^*)}\left(E(S_{t+1}) - \frac{(1+R_t)}{(1+R_t^*)} S_t \right) - \frac{1}{\gamma\sigma^2(1+R_{t-1}^*)}\left(E(S_t) - \frac{(1+R_{t-1})}{(1+R_{t-1}^*)} S_{t-1} \right)$$

$$B_t - B_{t-1} = (1+R_t)S_t(B_t - (1+R_{t-1}^*)B_{t-1}^*) \tag{6}$$

2.2. Carry traders

Let us assume that there is a large number N of carry traders. Each carry trader is assumed to decide the funding currency and investment currency for each period. That is, each of them is selling the foreign currency and buying the domestic currency or selling the domestic currency and buying the foreign currency. We consider that the individual carry trader maximizes his/her random utility of the alternatives. That is, he/she chooses an alternative with the highest utility[5]. The carry-trader's random utility function is composed of the deterministic part which is assumed to represent average behavior, and a nondeterministic part to represent random deviations from this average. The random utility of alternatives is given as:

$$\begin{cases} U_+ = \bar{U}_+ + \varepsilon_+ \\ U_- = \bar{U}_- + \varepsilon_- \end{cases} \tag{7}$$

where ε_i is a random variable. The carry trader attaches a value, U_i to each of two alternatives, that is, investing in domestic currency financed by selling in the foreign currency (labeled +), and investing in foreign currency financed by selling in the domestic currency (labeled−).

5 The random utility function of discrete choice developed by McFadden (1974) who has developed qualitative choice models based on maximization of the agent's random utility.

A common procedure used in both economics and finance is to assume the existence of a "representative" or "average" individual who is assumed to have tastes equal to the average over all decision makers. Two possible explanations for the stochastic term are given by Hausman and Wise (1978). The first is that a carry trader behaves randomly, perhaps due to random firing of neurons; so that faced repeatedly with the same alternative set. The same individual makes different choices. Second is that there are unobserved characteristics of the individual and unobserved attributes of the alternatives. Given the specification of the utility function, each carry trader is assumed to choose the alternative that maximizes his utility. The maximization of the random utility gives the probability with which each alternative is chosen. The probability that he chooses each alternative is given as :

$$\begin{cases} P_{+} = \Pr[U_{+} > U_{-}] = \Pr[\bar{U}_{+} - \bar{U}_{-} \geq \varepsilon_{-} - \varepsilon_{+}] \\ P_{-} = \Pr[U_{-} > U_{+}] = \Pr[\bar{U}_{-} - \bar{U}_{+} \geq \varepsilon_{+} - \varepsilon_{-}] \end{cases} \tag{8}$$

where $P_{+} + P_{-} = 1$.

We assume that individual carry trader's decision-making is influenced by (i) the other carry-traders and (ii) the return on carry trade H_t anticipated by the carry trader. To describe a carry-trader's utility function which is composed of those determinants, let us introduce a new variable e_t that denotes the normalized excess of the carry traders who sell the foreign currency and buy the domestic currency over the carry traders who sell the domestic currency and buy the foreign currency which is defined as $e_t = (n_t^{+} - n_t^{-})/N$ where n_t^{+} is the number of carry traders who sell the foreign currency and buy the domestic currency, and n_t^{-} is the number of carry traders who sell the domestic currency and buy the foreign currency in the period *t*.

Obviously $n_{+t} + n_{-t} \equiv N$. Using the variable e_t , we rewrite the equation (8),

$$\begin{cases} \bar{U}_+ = \lambda e_t + H_t \\ \bar{U}_- = -\left(\lambda e_t + H_t\right) \end{cases} \tag{9}$$

where the parameter λ is assumed to be positive and constant. Given λ is a positive, an increase in e_t raises the utility of selling the foreign currency and buying the domestic currency, and reduces the utility of selling the domestic currency and buying foreign currency in the direction of the minority decision. This means that the carry trader has a tendency to be in favor of the majority decision. H_t denotes the excess return on carry trade expected by carry traders which is defined as,

$$\begin{aligned} &H_t = (R_t^* - R_t) + Y_{t+1} \\ &Y_{t+1} = \theta \sum_{i=0}^{\infty} (1-\theta)^i \Delta s_{t-i}, \quad \Delta s_{t-i} = \left(\frac{S_{t-i}}{S_{t-i-1}} - 1 \right) \end{aligned} \tag{10}$$

where the parameter θ is constant($0 < \theta < 1$). The carry traders' anticipation Y_t of the relative change of the exchange rate S_t is described as the exponential moving average of the past relative change Δs_t of the exchange rate. Y_t is equivalent to

$$Y_t = (1-\theta) Y_{t-1} + \theta \Delta s_{t-1}, \quad H_{t=0} = H_0 \tag{11}$$

where $0 < \theta < 1$. It means that the carry-trader's expectation on the risk premium is *adaptive*.

As the expected return H_t to carry trade is higher, the utility of selling the foreign currency and buying the domestic currency raises, and the utility of selling the domestic currency and buying foreign currency is reduced.

The equations (10), and (11) means that the carry traders adapt the carry strategies which are a strategy that buys a currency with high return and sells currencies with poor returns over the past periods with a high probability[6].

McFadden (1974) has shown that if the random variable ε_i are independently and identically distributed with the *Gumbell distribution*

$$F(\varepsilon) \equiv \Pr[\varepsilon_i \leq \varepsilon] = \exp[-\exp[-\varepsilon]] \tag{12}$$

The probability that a utility-maximizing carry trader will choose each alternative, is expressed as:

$$\begin{cases} P_+ = \dfrac{\exp[\bar{U}_+]}{\exp[\bar{U}_+] + \exp[\bar{U}_-]} \\ P_- = \dfrac{\exp[\bar{U}_-]}{\exp[\bar{U}_+] + \exp[\bar{U}_-]} \end{cases} \tag{13}$$

Now we introduce a variable ν which is the probability that a transition is attempted by one of the carry-traders, and follows a uniform distribution over carry-traders. We assume that one carry-trader attempts a trade in one time unit. The individual transition probabilities per an unit time period is described as

$$\begin{cases} p_{\downarrow}(e_t) = \nu \cdot \dfrac{\exp[-(\lambda e_t + H_t)]}{\exp[\lambda e_t + H_t] + \exp[-(\lambda e_t + H_t)]} \\ p_{\uparrow}(e_t) = \nu \cdot \dfrac{\exp[\lambda e_t + H_t]}{\exp[\lambda e_t + H_t] + \exp[-(\lambda e_t + H_t)]} \end{cases} \tag{14}$$

The random variable ν determines the time scale in which a transition which is attempted by a carry trader occurs.

$p_{\downarrow}(e_t)$ is the transition probability that one of the carry traders who sell the foreign currency and buy the domestic currency. Inversely $p_{\uparrow}(e_t)$ the transition probability that one of the carry traders who sell the domestic currency and buy the foreign

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6 The fact that momentum strategies yield significant profits, have been well investigated. Jegadeesh and Titman (1993) examine a variety of momentum strategies and document that strategies earn profits.

currency.

The effects of λ and H_t on the transition probabilities can be described as follows:

i) A positive λ enlarges the transition probability in favor of the majority choice and reduces the transition probability in the direction of the minority choice. This positive feedback effect grows for a growing imbalance of choices.

ii) A positive expected return on carry trade H_t increases the probability that a carry trader changes the investment currency from the foreign currency to the domestic currency, and reduces the probability of changing the investment currency from the domestic currency to the foreign currency, and vice versa for negative H_t.

Using the transition probabilities, the equation for the distribution $p(s_t)$ of stochastic process of s is described by

$$\begin{aligned} p(e_{t+1}) - p(e_t) = [\, & w_{\downarrow}(e_t + \Delta e) p(s_t + \Delta e) - w_{\downarrow}(e_t) p(e_t)] \\ & + [w_{\uparrow}(e_t - \Delta e) p(e_t - \Delta e) - w_{\uparrow}(e_t) p(e_t)] \end{aligned} \tag{15}$$

where

$$\begin{cases} w[(e_t - \Delta e) \leftarrow e_t] \equiv w_{\downarrow}(e_t) = n_t^- p_{\downarrow}(e_t) = \dfrac{N}{2}(1 + e_t)\, p_{\downarrow}(e_t) \\ w_{\uparrow}(e_t) \equiv w((e_t + \Delta e) \leftarrow e_t) = n_t^+ p_{\uparrow}(e_t) = \dfrac{N}{2}(1 - e_t)\, p_{\uparrow}(e_t) \\ w_{\downarrow}(e_t + \Delta e) \equiv w(e_t \leftarrow (e_t + \Delta e)) = n_t^- p_{\downarrow}(e_t + \Delta e) = \dfrac{N}{2}(1 + (e_t + \Delta e))\, p_{\downarrow}(e_t + \Delta e) \\ w_{\uparrow}(e_t - \Delta e) \equiv w(e_t \leftarrow (e_t - \Delta e)) = n_t^+ p_{\uparrow}(e_t - \Delta e) = \dfrac{N}{2}(1 - (e_t - \Delta e))\, p_{\uparrow}(e_t - \Delta e) \\ w(e_t' \leftarrow e_t) = 0 \quad for \quad e_t' \neq e_t \pm \Delta e \end{cases} \tag{16}$$

where $\Delta e = N/2$.

The equation (15) is called as the master equation (See Gardiner (1985)). When the number of carry traders is large, the equation (15) is equivalent to the dynamic equation of the mean X_t of $p(e_t)$ (see Weidlich and Haag (1983)),

$$\Delta X_{t+1} \equiv X_{t+1} - X_t = \nu[\tanh(\lambda X_t + H_t) - X_t] \tag{17}$$

where $X_t = (\langle n \rangle_t^+ - \langle n \rangle_t^-)/N$. $\langle n \rangle_t^+$ denotes the number of carry traders who the carry traders who sell the foreign currency and buy the domestic currency at the period *t*, and $\langle n \rangle_t^-$ denotes the number of carry traders who sell the domestic currency and buy the foreign currency at the period *t*. The solution of (17) corresponds to the maximum of the stationary distribution $p_{st}(e_t)$ of the master equation (15), and the equation of (17) describes *the collective behavior of the representative carry-trader.*

Using the difference of $\hat{s}_t$ from period *t* to period *t+1*, the aggregate excess demand for the domestic currency by all carry traders is defined as

$$Q\left[\langle n \rangle_t^+ - \langle n \rangle_{t-1}^+\right] = \frac{QN}{2}\left(X_t - X_{t-1}\right) \tag{18}$$

where Q denotes the amount of the domestic currency which is exchanged in any transaction by a carry trader, and is assumed to be constant. The equation (18) will be utilized when the market exchange rates are calculated under the market clearing conditions in section 2.4.

2.3. Collective behavior of the representative carry traders

Before we discuss about currency bubble and crash, we describe the collective behavior of the representative carry traders using equation (17). As the above conditions i) and ii) indicate, the carry trader's transition probability depends on the variables λ and H_t. For convenience of analysis, let us assume that the return on carry trade H_t is a parameter H. The solutions of the mean equation (17) can be summarized with respect to λ and H as follows:

i) The case of $0 < \lambda < 1$ and arbitrary H:

There is only one possible solution X^{**}. The solution corresponds to the maximum of the stationary distribution $p_{st}(e_t)$. For $H_t = 0$ and $0 < \lambda < 1$, the only one possible solution is zero.

Although in this case which the relatively small value of λ which indicates the strength of interaction among carry traders, the driving force of the collective behavior of carry traders mainly the expected return on carry trade H, a strong herding among carry traders dose not function. In Figure 1 the graphical solution to (17) is plotted for $\lambda < 1$ and the different values of H. When the expected return on carry trade H is positive (negative), the solution moves from zero to a positive value (a negative value).

ii) The case of $\lambda > 1$ and $|H| < \overline{H}$:

$\overline{H}$ is determined by the equation $\cosh^2[\overline{H} - \sqrt{\lambda(\lambda - 1)}] = \lambda$. There are three possible solutions $X^* < X^{**} < X^{***}$. The solution X^* and X^{***} are called the bear-market equilibrium and the bull-market equilibrium respectively.

Therefore, as the interaction among carry traders λ increases, the solution X^{**} is unstable, and appears the bear-market equilibrium $X^{*}(<0)$ and bull-market equilibrium $X^{***}(>0)$. As λ exceeds unity, the stationary distribution $p_{st}(e_t)$ is from unimodal to multimodal. This bifurcation is called as the second-order phase transition. In Figure 2 the graphical solution to (17) is plotted for $\lambda > 1$.

iii) The case of $\lambda > 1$ and $|H| = \bar{H}$:

Two of the tree solutions $X^{*} < X^{**} < X^{***}$ coincide at $X_c = \pm\sqrt{(\lambda - 1)/\lambda}$.

An increase (a decrease) in the expected return on carry trade H causes the curve which indicates the transcendental equation (17) to shift up (down), so that the solutions rise (fall). Figure 3 shows the states that two of the tree solutions coincide.

iv) There case of $\lambda > 1$ and $|H| > \bar{H}$:

There is one solution again. When $\lambda > 1$, and H is negative and decreasing continuously, the stationary distribution $p_{st}(e_t)$ is from multimodal to unimodal. The solution jumps down from X^{***} to X^{*} at the moment that the expected return on carry trade H falls below $(-\bar{H})$. Inversely, the solution jumps up from X^{*} to X^{***} at the moment that or that the expected return on carry trade H exceeds $(+\bar{H})$. This bifurcation is called as the first-order phase transition.

2.4. Market-clearing exchange rates

The market clearing condition requires that the aggregated excess demand (supply) for each currency by rational investors is equal to the aggregated excess demand (supply) by currency traders from the period t-1 to the period t. That is, if one carry-trader changes from an investment in the foreign currency to an investment in the domestic currency, then the exchange rate S_t is adjusted such that rational investors supply the corresponding domestic currency and demand the corresponding foreign currency. That is,

$$M(B_t^* - B_{t-1}^*) = \frac{QN}{2}(X_t - X_{t-1}) . \tag{19}$$

For simplicity of analysis we assume that the interest rate of each currency is fixed over time. Then from equation (6) the aggregated excess demands for currency by rational investors are obtained by multiplying the number M of rational investors:

$$M(B_t^* - B_{t-1}^*) = \frac{M}{\gamma\sigma^2(1+R^*)}\left\{\left(E(S_{t+1}) - E(S_t)\right) - \frac{(1+R)}{(1+R^*)}\left(S_t - S_{t-1}\right)\right\} \tag{20}$$

$$M(B_t - B_{t-1}) = M(1+R)S_t(B_t^* - (1+R^*)B_{t-1}^*) \tag{21}$$

Then, the market clearing conditions are described as

$$\begin{aligned} &M(B_t^* - B_{t-1}^*) - N(X_t - X_{t-1}) \\ &= \frac{M}{\gamma\sigma^2(1+R^*)}\left\{\left(E(S_{t+1}) - E(S_t)\right) - \frac{(1+R)}{(1+R^*)}\left(S_t - S_{t-1}\right)\right\} + \frac{QN}{2}(X_t - X_{t-1}) = 0 \end{aligned} \tag{22}$$

Solving the equations (19) with respect to changes on the exchange rate $\Delta S_t = (S_t - S_{t-1})$ we can obtain changes on the exchange rate which satisfy the market-clearing conditions. In summary, the dynamics of exchange rate can be described as:

$$\begin{cases} S_t - S_{t-1} = \alpha(E(S_{t+1}) - E(S_t)) + \alpha\beta(X_t - X_{t-1}) \\ X_t - X_{t-1} = \nu[\tanh(\lambda X_{t-1} + H_{t-1}) - X_{t-1}] \\ H_t = (R^* - R) + Y_{t+1} \\ Y_t - Y_{t-1} = \theta(\Delta s_{t-1} - Y_{t-1}) \end{cases} \tag{23}$$

where $\alpha = \dfrac{\gamma\sigma^2(1+R_1)^2}{M(1+R_0)}$, $\beta = \dfrac{QN}{2}$ and $\Delta s_{t-i} = \left(\dfrac{S_{t-i}}{S_{t-i-1}} - 1\right)$.

We assume that the term $E(S_{t+1})$, which describes the *fundamental value* of the exchange rate which is renewed by the fundamental news in the period *t*. The terms are often considered as a random variable which fluctuates. To demonstrate the occurrence of currency crisis without the fundamental news, we assume the term $E(S_{t+1})$ are constant over time, that is, $E(S_{t+1}) - E(S_t) = 0$. Then, the changes of the exchange rate $\Delta S_t \equiv (S_t - S_{t-1})$ depend completely on the carry traders' excess demand for the domestic currency, $QN(X_t - X_{t-1})/2$. Since $\alpha > 0$, the change ΔS_t increases (decreases) proportionally with respect to the carry traders' excess demand for the domestic currency, $QN(X_t - X_{t-1})/2$.

3. Currency bubble caused by carry trade

As discussed in subsection 2.3., when the parameter λ, which indicates the degree of intensity of the interaction among carry traders, exceeds unity, the unique solution $X^{**}(=0)$ is unstable, and appears newly two solutions, the bear-market solution $X^*(<0)$ and bull-market solution $X^{***}(>0)$ both of which are stable, under $H_t = 0$. This gives cause to the carry traders' herd behavior. Let us consider the motion of the exchange rate starting from the unstable solution $X^{**}(=0)$. Depending on the value of X_0 at the initial time $t = 0$, the exchange rate S_t can either enter a bull market or a bear market. That is, when the initial value of X_0 is positive (negative), the exchange rate S_t raises, and enter a bubble phase (non-bubble phase).

We now assume that the interest rate R^* of the domestic currency is greater than the interest rate R of the foreign currency. That is, the interest rate differential is positive, $(R^* - R) > 0$. This positively raises the expected return on carry trade H_t. Then, the appreciation of the domestic currency against the foreign currency is due to increases in the carry-traders' excess demand $(QN(X_t - X_{t-1})/2)$ for domestic currency, and the aggregate demand for the domestic currency – investment currency - by carry-traders is increased further due to the carry traders' herding, and it raises further the expected return on carry trade H_t. The increases in the expected return H_t and the carry traders' herd behavior next pull up toward the bull-market solution $X^{***}(> 0)$. As the carry-traders' excess demand for the domestic currency is increased, the domestic currency against the foreign currency appreciates, (that is, the exchange rate S_t rises), and is over-evaluated. This inflationary spiral gives cause to the currency bubble.

For $\lambda > 1$ and $H_t > \bar{H}$ the bear-market equilibrium disappears, and the bull-market equilibrium is unique and stable. Thus, the currency bubble persists until the imbalance of buyers and sellers over the carry traders, e_t approaches to the bull-market equilibrium $X^{***}(> 0)$.

3.1 Forward Premium Puzzle

In the period of bubbles, the actual high return on the investment currency is earned as the result of low expected future returns. That is, rational investors sell the domestic currency because they believe that the carry gains due to the interest-rate differential are offset by a commensurate depreciation of the investment currency. In the opposite direction, more carry traders buy the domestic currency more. The change ΔS_t on the exchange rate is contrary to the rational investors' expectation in the period of bubbles. As a result, the carry traders get a capital gain from the appreciation of the exchange rate S_t in the period of bubbles.

Uncovered interest parity (UIP) predicts that if the low interest rate currency were to appreciate relative to the high interest rate, then any gain on the interest rate differential from the carry trade will be exactly offset by the capital loss resulting from the exchange rate movement, leaving the carry trader no better. However, uncovered interest parity has been studied by many researchers in international economics, and has been widely rejected. High-interest-rate currencies tend to appreciate relative to low-interest-rate currencies. This violation of the UIP is often referred to as the forward premium puzzle. In our model, the carry traders' belief in the success of carry trades can thus become self-fulfilling, and the failure of uncovered interest parity becomes the consequence of carry trades in the period of currency bubble. Thus, our model gives a persuasive explanation on the forward premium puzzle.

4. Currency crash caused by unwinding of carry trade

In the first half of bubbles, the carry-traders' excess demand for the investment currency is sharply increasing, so that the exchange rate S_t is also sharply appreciating, but in the second half of bubbles, as the carry-traders' imbalance X_t is approaching the bull market equilibrium X^{***}, the carry-traders' excess demand $QN(X_t - X_{t-1})/2$ for the investment currency is approaching zero, and so a rise in the exchange rate slows down.

Decreasing the expected return on carry trade H_t changes the bull-market equilibrium X^{***} downward, so that the carry traders' excess demand $QN(X_t - X_{t-1})/2$ for the investment currency declines, and the exchange rate S_t starts to decrease. As the expected return on carry trade H_t decreases, the bear-market equilibrium X^* appears again. This deflationary spiral continues to decrease and H_t becomes negative in its final stage of bubbles even if the interest rate differential $(R^* - R)$ is positive. A sudden currency crash moves unrelated to fundamental news can be due to the unwinding of carry trades. In the end stage of bubbles the expected return H_t declines until $H_t = -\overline{H}$. In an instant when H_t falls below $-\overline{H}$, the probability of the carry trader's selling the domestic currency is higher than the probability of the carry trader's buying the domestic currency and a large-scale unwinding of carry trade is caused. In our model the currency crash is considered as the so called first-order phase transition. See Figure 4. The carry traders' selling on domestic currency in the period of a crash depends on the parameter λ.

After a crash, the rational investors buy the domestic currency, which they sell in the period of bubbles, back when they predict the appreciation of the domestic currency from UIP. After all, the rational investors can make a profit from a long-term investment, while the carry traders lose money.

5. Concluding Remarks

This paper provides one potential theoretical explanation for currency bubble and crash. A merit of this paper is to propose that a model describing the rationality of the carry trader's behavior, and a mechanism of currency bubble and crash which is caused by the carry trader's behavior. This also shed new light on the forward premium puzzle.

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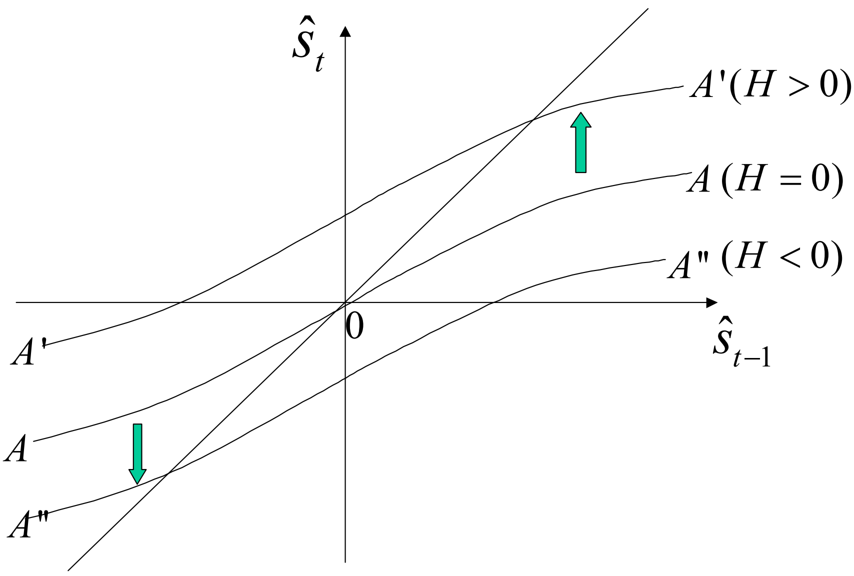

Figure 1: The solutions of the equation (17) for $\lambda < 1$ and the three values of H.

The straight line is 45 degree line.

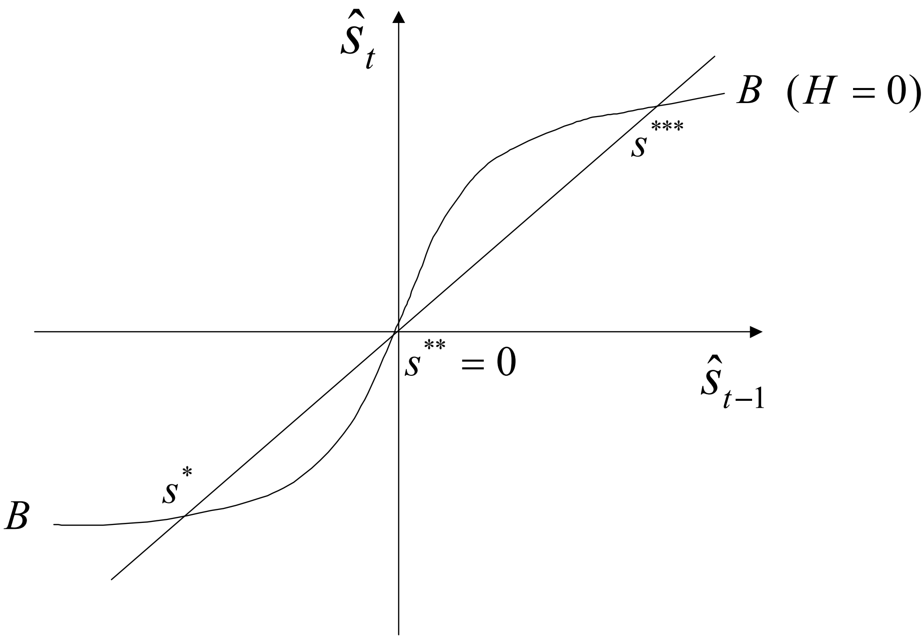

Figure 2: The solutions of the equation (17) for $\lambda > 1$ and $H = 0$. The straight line is 45 degree line.

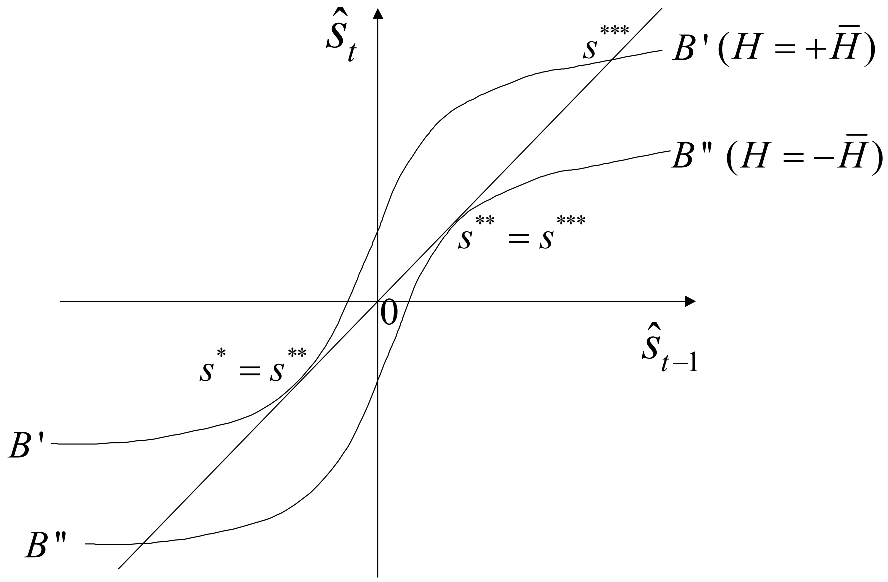

Figure 3: The solutions of the equation (17) for $\lambda > 1$ and $H = \pm\bar{H}$. The straight line is 45 degree line.

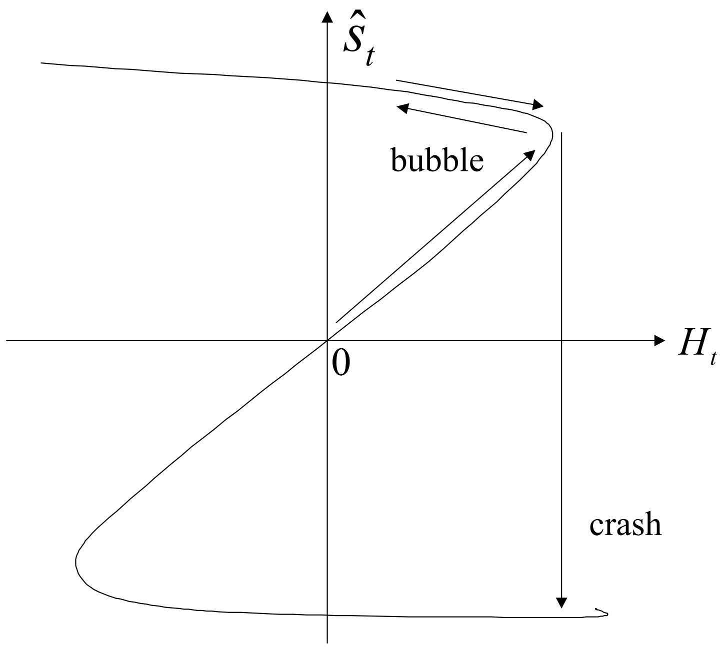

Figure 4: The dynamics of (20) for $\lambda > 1$ and $E(\Delta p_{jt+1}) = 0$. Bubble and Crash.