Measuring inflation through stochastic approach to index numbers

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Abstract

This study attempts to estimate the rate of inflation in Pakistan by a stochastic approach to index numbers which provides not only point estimate but also confidence interval for inflation estimate. There are two approaches to index number theory namely: the functional economic approach and the stochastic approach. The attraction of stochastic approach is that it estimates the rate of inflation in which uncertainty and statistical ideas play a major roll of screening index numbers. We have used extended stochastic approach to index numbers for measuring the Pakistan inflation by allowing for the systematic changes in the relative prices. We use CPI data covering the period July 2001--march 2008.

1. Introduction

Inflation is a rise in the price and wages caused by an increase in the money supply and demand for goods, resulting in a fall in the value of money. Wilson (1982) said "inflation is sharp increase in the rate of change of some price index above a previous normal level." Inflation has a continuous propensity to rise for general level of price. Inflation is measured by using consumer price index number. Consumer price index is directly related to people's purchasing power of fixed basket of goods and services. When inflation rises, people can buy only small number of things and vice versa. A sharp rise in inflation hits the poor people more severely than rich because former is more vulnerable. Inflation is also used in negotiating wages, providing social security benefits, reviewing contracts etc.

The objective of this study is to estimate the rate of inflation in Pakistan. So far the Laspeyres index based on fixed basket approach to index numbers is used to estimate the rate of inflation for Pakistan. In this study we used the stochastic approach to index numbers to estimate the rate of inflation which has not been used for Pakistan before. The advantage of the stochastic approach is that it not only provides a point estimate of the rate of inflation but also provides complete probability distribution of inflation where as other approaches provide only point estimate of rate of inflation.

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To estimate the rate of inflation there are two main approaches namely functional approach to index numbers of Diewert (1981), Balk (1995) which dates back to Fisher (1922), and second is stochastic approach to index numbers theory reintroduced by Frisch (1936), Theil (1975), Clements (1981), Clements (1987), Selvanathan (1989), Diewert (1995), Crompton (2000), Selvanathan (2003), Selvanathan (2004), Selvanathan (2006), Clements (2006), Clements (2007).

The functional approach utility based which according to Diewert (1981) takes maximum utility with optimum expenditures. This approach is difficult to apply in real world because perhaps it is impossible to know the income of every individual and utility of things for every one of them. Cost of living index vary with income significantly. Diewert (1995) said that flexible index number formula for economic approach is 'superlative'. However with homothetic type restrictive assumptions this approach can be applied to estimate the rate of inflation.

In a fixed basket approach index is function of prices and quantities. The very problem to apply this approach is that in this approach commodities included in the basket are constant. These commodities included in the basket can only be changed after a period of ten years. As the time passes people substitute old things with new ones. So this does not include exactly the same things which are in use of people in reality. Its modified form is test or axiomatic approach to index numbers. This approach is also used for the same purpose. Balk (1995) provided a complete survey of this approach. Balk (1995) said that in fixed basket approach prices and quantities of goods and servises are two separate variables which do not depend on each other.

Any index number with this approach has to satisfy certain set of axioms or tests. Still there is no index number which is universally accepted to satisfy all the axioms. Only Fisher ideal index number satisfy all the axioms about which Diewert (2007) said

> "However it could be argued that the list of tests or axioms that was used to establish the superiority of the Fisher Ideal index number might have been chosen to favor this index."(Diewert (2007))

For example, Laspeyres index and Paasche satisfy 17 tests among the list of 20 tests. The Tornqvist index satisfies 11 tests among 20 tests. One index number satisfies some axioms and the other index satisfies other set of axioms. When comparing two index numbers it is difficult to decide which one is better because they satisfy different axioms among all.

Alternative to these approaches, for the measurement of inflation one good approach is stochastic approach to index numbers, which was reintroduced by Frisch (1936). Stochastic approach to index numbers used the same information to measure the rate of inflation as those of other approaches. The information given is the prices of n (any constant) commodities in two time periods say t and t-1. According to Clements (1987), the difference between the prices of the period's t and t-1 contains two portions one is trend component (rate of inflation) and the other is random component. The trend component will move in a systematic way. Selvanathan (2006) said that the sum of price change would contain only the pure trend component because taking sum removed the random component. So the average change of the price measures the rate of inflation.

> "If we have n prices, then the rate of inflation can be estimated by taking some form of average of the n price change. The stochastic approach to index numbers can be viewed as a signal extraction problem. To illustrate consider the simplest case whereby each of the n proportionate price change is the sum of the underlying rate of inflation and an independent random component. Here each observed price change is the reading on the rate of inflation 'contaminated' by the random term. The averaging of the price change serves to eliminate as much as possible of the contamination and leaves an estimate of the underlying signal, the rate of inflation." (Selvanathan (2006))

Frisch (1936) expressed his point of view about the rate of inflation as follow:

> "Here the assumption is made that any change that takes place in the 'price level' ought, so to speak, to manifest itself as a proportional change of all prices. Whatever, deviation there is from this strict proportionality must be looked upon as due to other causes than those we think of when we speak of the price level change." (Frisch (1936))

Firstly, like other traditional approaches stochastic approach to index numbers provides point estimate of inflation. It has been found that the estimate through this stochastic approach is approximately close to that of official inflation.

Secondly, the stochastic approach to index numbers provides the whole distribution of the rate of inflation. Stochastic approach gives standard error for the rate of inflation where as other approaches only give point estimate rate of inflation. The standard errors obtained from this approach can be used to get an interval estimate of rate of inflation. The main attraction of the stochastic approach over other approach is that the stochastic approach to index numbers could be used to express the inflation target in a range. For example if the interval estimator X percent ± 1.96 standard errors is taken for the parameter implies we are 95% confident that this range will contain true parameter of inflation. Some level of confidence is gained that estimated range will contain the true inflation but in point estimation we are not sure that the estimated point will approach to true inflation or not. According to Casella (2002) though we loss some precision in estimating confidence interval but at the same time we gain some confidence coefficient about the parameter. The advantage of the confidence interval is

> "The purpose of using an interval estimator rather than a point estimator is to have some guarantee of capturing the parameter of interest."Casella (2002)

Thirdly, according to Clements (1987) the stochastic approach to index numbers is closely related to regression theory. All the estimators which are used in the stochastic approach to index numbers are derived by ordinary least square method. So this derivation of estimators develops a link between the index number theory and the least square theory. The merit of these estimators, derived through this link is that they will adopt the properties of the least square theory that is they are BLUE (best linear unbiased estimators). Their variance will be constant and they are independent.

The weighted stochastic approach to index numbers, an extension of the simple stochastic approach to index numbers gives weights to all the commodities according to their importance in consumer's budget and hence it use weighted average for the point estimation.

After the Frisch (1936), no remarkable work done on the stochastic approach to index number for a long time perhaps due to Keynes (1930) criticism. The main criticism of Keynes (1930) was that the portion of change other than trend component in prices could never be pure random. This change can be systematic due to importance of commodities. Later on Clements (1987) extended the stochastic approach to index number by allowing systematic changes in prices of commodities. He introduced a commodity specific component in the model. He found that results were not significantly different after allowing systematic changes in the commodities. Clements (1987) is also extension of Clements (1981) where he focused on Divisia index numbers. Clements (1981) argues that The Divisia index numbers can be interpreted and estimated as regression coefficients under plausible error specification. So when Divisia index numbers are applied then rate of inflation is the parameter to be estimated from the individual price changes. Selvanathan (1989) extended the approach by allowing within group changes. His results were unbiased for the parameters as was proved with simulation.

Clements (1987) imposed an assumption on the variance of errors that when commodities have larger budget shares then the variance will be smaller but when commodities had smaller budget shares then the commodities had larger variation. He derived estimators for this particular type of hetroscadsticity. So results will only be unbiased when such type of hetroscadsticity does not exists in the data. To overcome this problem Crompton (2000) formulated a generalized estimator of variance which did not concern with any type of hetroscadsticity and provided unbiased results. He applied his estimator on Australia data from 1973 December to 1996 June and provided this results

A brief discussion of some recent work on stochastic approach to index numbers is as follows. Selvanathan (2003) noted thatt the Crompton's approach makes a reasonable reduction in variance than that of Clements(1987). After correction, it was seen that 75% values of standard error with Crompton's approach were smaller than that of Clements (1987) approach. Selvanathan (2003) also wrote a book on international consumption comparisons. He used the data of OECD (organization of economic cooperation development) and LD (Least Developing) countries and applies the stochastic approach to index numbers. Anderle (2002) estimated rate of inflation through stochastic approach to index numbers. He provided the attractive features to adopt the stochastic approach to index numbers. He said its first attraction is its link with probability theory and the other is that observations belonged to a complete distribution. Selvanathan (2004) estimated rate of inflation through stochastic approach to index numbers using the data of 23 OECD countries. Clements (2006) reviewed the stochastic approach to index numbers and highlights all the work done on this approach till 2006. According to Clements (2007) stochastic index numbers was equivalent to the familiar optimal combination of forecasts with the individual prices playing the role of n forecasts of the overall rate of inflation. This leads to new analytical results on the impact of adding additional information within the stochastic approach framework.

Rest of the paper is organized as follows. Section 2 gives review of simple stochastic approach to index numbers, and then weighted average of stochastic approach to index is reviewed and is used to match with Divisia index number. It is also used to calculate Laspeyres index. Section 3 gives the empirical study of the approach for Pakistan and finally we conclude.

2. Stochastic Approach to Index Number Theory

2.1 Simple Average of Prices:

If prices of n commodities in two time periods say t and t-1 are given, subtraction of prices of period t-1 from period t will give change between prices. According to Selvanathan (2006) each price change is equal to the rate of inflation plus other components. Average of these changes will give the rate of inflation because averaging will remove the other components (errors) if other components are random. Let suppose p_{it} be the price of i^{th} commodity where i =1, 2,..., n during t^{th} time, where t= 1, 2 , . . . , T, also let $\mathrm{Dp}_{\mathrm{it}} = \log p_{it} - \log p_{i,t-1}$ be the log-change of prices. Let this log-change be made up of a trend component α_t (rate of inflation) and a random component ε_{it}, since random component will be zero on average:

$$\mathrm{Dp}_{\mathrm{it}} = \alpha_t + \varepsilon_{it} \text{ where i = 1, 2,..., n, t= 1, 2, ... , T} \tag{2.1}$$

Precision of the results with this model will increase when the relative prices are not too high. Where $E(\mathrm{D}p_{it}) = \alpha_t$ because $E(\varepsilon_{it}) = 0$. Here α_t is constant for all commodities and $E(\varepsilon_{it}) = 0$ means that relative prices are zero on average. Assume that the error term ε_{it}'s are independent over commodities i.e. $\operatorname{cov}(\varepsilon_{it}, \varepsilon_{jt}) = 0$ where i≠j and $\operatorname{var}(\varepsilon_{it}) = \sigma_t^{\,2}$. Under these assumptions, the BLUE (Best Linear Unbiased Estimator) of α_t can be derived from Eq.(2.1) as

$$\hat{\alpha}_t = \frac{\sum_{i=1}^{n} Dp_{it}}{n} \quad \text{and} \quad \operatorname{var}(\hat{\alpha}_t) = \frac{\sigma_t^{\,2}}{n} \tag{2.2}$$

Error variance can be estimated by unbiased estimator , $\hat{\sigma}_t^{\,2} = \dfrac{\sum_{i=1}^{n} (Dp_{it} - \hat{\alpha}_t)^2}{n-1}$ (2.3)

From this equation it is observed that when relative prices vary substantially then random errors in (Eq.2.1) will be high and so will be Eq.(2.2) also high. Hence the sampling variance of estimated rate of inflation will be higher which implies that the rate of inflation is less precisely estimated. That is over all rate of inflation becomes less well defined and relative price changes take substantial part of the over all changes. In other words we can say that the model Eq.(2.1) is more appropriate when variation in relative prices did not take substantial part. Inflation is less precisely estimated having large sampling variance. The assumptions imposed on the model are obviously very strict.

2.2 **Budget Share Weighted Average of Prices and Divisia Index**

If the rate of inflation is calculated by giving relative importance to all the commodities then perhaps it will give more reliable results than that of giving equal importance to all the commodities. For example, food will always be more important in the consumer's budget than education. So in such case weighted average is more appropriate than simple average.

Let w_{it} be budget shares of ith commodity at time t, then the arithmetic average of budget shares of ith commodity is given by $\overline{w_{it}} = \frac{1}{2}$ ($w_{it}+w_{i,t-1}$) at time t and t-1. As in simple average method assume that relative price changes ε_{it} are independent. Their expected value is zero. But the assumption that their variance is constant over commodities is relaxed, so Eq. (2.2) is replaced with the following equation.

$$\text{cov}(\varepsilon_{it}, \varepsilon_{jt}) = 0 \text{ where } i \neq j \text{ and } \text{var}(\varepsilon_{it}) = \frac{\lambda_t^2}{\overline{w_{it}}} \tag{2.4}$$

This is variance of the changes in the relative prices of commodities, and is inversely proportional to the weights $\overline{w_{it}}$. As the weights of commodities rises the variance of the relative prices falls. As food is very important in the consumer's budget, so scope for relative prices to change is less. But if we consider Tobacco which is less important in the consumer's budget so change in its relative prices is considerable. According to Clements (1987) the specification (2.4) is plausible. The vector form of the Eq. (2.1) is

$$Dp_t = \alpha_t \tau + \varepsilon_t \text{ where } t=1, 2, \ldots, T \tag{2.5}$$

Where $Dp_t = [Dp_{it}]$, $\tau = [1, \ldots, 1]$ and $\varepsilon_t = [\varepsilon_{it}]$.

The variance-covariance matrix of the relative price changes using Eq. (2.4) and taking the diagonal matrix of weights $\overline{W_t} = diag[\overline{w_{1t}}, \ldots, \overline{w_{nt}}]$

$$\text{var} - \text{cov}(\varepsilon_t) = \lambda_t^2 \overline{W_t}^{-1}$$

(2.6)

Due to hetroscadsticity Clements (1987) applied GLS (Generalized Least Square) to Eq.(2.5) under Eq.(2.6) to obtain estimator of the rate of inflation $\hat{\alpha}_t = (\tau' \overline{w_t} \tau)^{-1} (\tau' \overline{w_t} Dp_t)$ after simplification it becomes

$$\hat{\alpha}_t = \sum_{i=1}^{n} \overline{w_{it}} Dp_{it} \tag{2.7}$$

This is BLUE. Its interpretation is the same as that of budget share weighted average of the price log changes. It also matches with the first moment of the Divisia index where Divisia index is defined as expenditure share weighted average of the price log changes. Thus Divisia index comes out as the GLS estimator of rate of inflation. Now variance of $\hat{\alpha}_t$ is

$$\mathrm{var}(\hat{\alpha}_t) = \lambda^2{}_t = \frac{(Dp_t - \alpha_t \tau)' \overline{w_t} (Dp_t - \alpha_t \tau)}{n-1} = \frac{\sum_{i=1}^{n} \overline{w_{it}} (Dp_{it} - \alpha_t)^2}{n-1} \tag{2.8}$$

Now let us check its reliability by its comparison with a standard measure. The Divisia second order moment will be

$$\varphi_t = \sum_{i=1}^{n} \overline{w_{it}} (Dp_{it} - Dp_t)^2 \tag{2.9}$$

Where $Dp_t = \hat{\alpha}_t$. This measures how much apart relative prices are from their mean. If φ_t =0 then it means that there is no change in relative prices. Comparing (2.8) and Eq.(2.9) it can be observed that variance of Divisia index is the same as Divisia second order moment multiplied by 1/(n-1), which confirms the reliability of the derived Divisia variance..

2.3 **Laspeyres Price Index Method**

Let P_{i0} be the price of the ith commodity in the base period 0 and q_{i0} denote the quantity of the ith commodity in the base period 0. Let $p_{i0}q_{i0}$ be the expenditure on the ith commodity in base period. Let $p_{i0}q_{it}$ be the consumption of the ith commodity in the current period. Then assume the regression of $p_{i1}q_{it}$ on $p_{i0}q_{it}$.

$$p_{it}q_{i0} = \gamma_t p_{i0}q_{i0} + \varepsilon_{it}, \text{ where i=1, 2, 3,…, n} \tag{2.10}$$

Here the constant γ_t is a constant over the commodities and it measures the rate of inflation over all commodities. It varies with time but is constant with commodities. Let impose the assumptions on Eq.(2.10) E (ε_{it}) =0 and $\mathrm{cov}(\varepsilon_{it}, \varepsilon_{jt}) = \sigma_t^2 p_{i0} q_{i0} \delta_{ij}$, where δ_{ij} **denote the kronecker delta** and defined as a function of two variables that is equal to zero when variables have different values and equal to one when variables have the same value.

The variability of disturbances falls as expenditures on commodities in the base period falls and vice versa. For the solution heteroscedasticity and to find estimator of γ_t Selvanathan (2006) applied GLS to Eq.(2.10).

$$\hat{\gamma}_t = \frac{\sum_{i=1}^{n} p_{it} q_{i0}}{\sum_{i=1}^{n} p_{i0} q_{i0}} \quad \text{or} \quad \hat{\gamma}_t = \sum_{i=1}^{n} w_{i0} \frac{p_{it}}{p_{i0}} \tag{2.11}$$

Here $w_{i0} = \frac{p_{i0} q_{i0}}{\sum_{i=1}^{n} p_{i0} q_{i0}}$ are the budget shares of ith commodity in the base period and $\sum_{i=1}^{n} p_{i0} q_{i0}$ are the total expenditures in the period 0. Eq. (2.11) is a Laspeyres ndex which the index represents the relative prices of various commodities of buying the base period quantities in the current period. The advantage of Laspeyres index formula is that the budge share weights remain unchanged and only information on prices need to be obtained. However, it has some limitation that the index obtained by this formula gets inflated as the distance between base period and current period is large.

If we subtract one from Laspeyres index, it becomes equal to Divisia index. That is $\hat{\gamma}_t - 1 \approx \sum_{i=1}^{n} \overline{w}_{i0} D p^0{}_{it} = \hat{\alpha}_t$. The Divisia index and the Laspeyres index are approximately equal when base period and current period are close to each other. The variance of $\hat{\gamma}_t$ according to Selvanathan (2006) is Variance of estimated rate of inflation $\operatorname{var}(\hat{\gamma}_t) = \frac{\sigma_t{}^2}{\sum_{i=1}^{n} x_{it}{}^2}$ where $\sigma_t{}^2$'s unbiased estimator is

$$\hat{\sigma}_t^2 = \frac{\sum_{i=1}^{n} (y_{it} - \hat{\gamma}_t x_{i0})^2}{n-1} \qquad \text{which implies} \qquad \operatorname{var}(\hat{\gamma}_t) = \frac{1}{n-1} \sum_{i=1}^{n} w_{i0} \left(\frac{p_{it}}{p_{i0}} - \hat{\gamma}_t\right)^2 \tag{2.12}$$

This variance of Laspeyres index increases as the relative prices of the commodities increase. It becomes difficult to estimate rate of inflation precisely as the relative prices take much part in the change in prices. When the prices of individual commodities move away the over all index (rate of inflation) is not precise. In that case width of the confidence band will be wider. Also when the distance between the current period and the base period becomes larger the Laspeyres index is less well defined. Now Eq. (2.12) can be written as

$$\operatorname{var}(\hat{\gamma}_t) = \frac{1}{n-1}\sum_{i=1}^{n} w_{i0}\left(\frac{p_{it}}{p_{i0}} - 1 - \hat{\gamma}_t + 1\right)^2 \text{ or } \operatorname{var}(\hat{\gamma}_t) = \frac{1}{n-1}\psi_t \text{ Where } \psi_t = \sum_{i=1}^{n} w_{i0}(Dp^0{}_{it} - \hat{\alpha}_t)^2 \qquad (2.13)$$

This ψ_t is equal to φ_t which is Divisia second order weighted moment about mean. So variance of rate of inflation is similar to Divisia second order weighted moment about mean multiplied by 1/ (n-1). Eq. (2.13) measures the degree of dispersion of relative prices & by Divisia index we also get the same measure.

2.4. **Allowing for the Systematic Changes in the Relative Prices**

Consider the model with an additional term to allow the some sustained change in the relative prices of commodities.

$$Dp_{it} = \alpha_t + \beta_i + \mu_{it} \quad \text{Where i=1, 2, . . ., n and t=1, 2, . . ., T} \qquad (2.14)$$

Where α_t denotes the trend in the prices and β_i denotes the part of change in the relative prices which is not random. The error term μ_{it} is assumed to be random error, so its expected value is zero and independent over time as well as commodities. Hence, $\operatorname{cov}(\mu_{it}, \mu_{js}) = 0$ where i $\neq$ j and t $\neq$ s. Let w_{it} denote the budget share of ith commodity at time t. It is also assumed that the variance of this error is not stable over time and commodities; some form of hetroscadsticity is present in the model, the variance of errors is assumed to decrease as the commodity becomes more important in the consumers budget. The error variance grows as the commodity takes less weight in consumer's budget.

$$\operatorname{var}(\mu_{it}) = \frac{v_t^2}{w_{it}} \qquad (2.15)$$

The data is pooled against time as well as commodity also, so it becomes difficult to estimate Eq. (2.15) with both variances. Let us first slightly relax the assumption Eq.(2.15) and assume that it is only commodity dependent and not that of time. And after first round of derivation we shall again assume that is time dependent and we shall work with Eq.(2.15)

$$\operatorname{var}(\mu_{it}) = \frac{v^2}{w_i} \qquad (2.16)$$

Since it varies only with commodities where $\overline{w_i}$ are arithmetic averages of $\overline{w_{it}}$. Practically, the difference between $\overline{w_i}$ and $\overline{w_{it}}$ is of no importance as budget shares tend to change slowly over time. So $\overline{w_i} \approx \overline{w_{it}}$. Another issue with Eq. (14) is that it is under identified. Hence to overcome this problem, Clements (1987) impose the constrained $\sum_{i=1}^{n} \overline{w_i} \beta_i = 0$ which is budget share weighted average of the systematic part of the changes in relative prices. Now Clements applied GLS to the model Eq.(2.14) under Eq. (2.16) to estimate α_t & β_i.

$$\hat{\alpha}_t = \sum_{i=1}^{n} \overline{w_i} Dp_{it} \, , \hat{\beta}_i = \frac{\sum_{t=1}^{T} (Dp_{it} - \alpha_t)}{T} \tag{2.17}$$

Both of which are BLUE. The above estimators are founded by replacing Eq. (2.15) with Eq. (2.16) which varies only with respect to commodities and not with time. Now let us find estimators by taking assumption Eq. (2.15) in which variance is time dependent and replace $\overline{w_{it}}$ with $\overline{w_i}$. Clements (1987) substitute Eqn. (2.17) in (2.14) and obtain SSE (sum of squares of errors) provided below

$$\delta_t^2 = \sum_{i=1}^{n} \overline{w_i} (Dp_{it} - \tilde{\alpha}_t)^2 + \sum_{i=1}^{n} \overline{w_i} (\overline{Dp_i} - \overline{\tilde{\alpha}})^2 - 2 \sum_{i=1}^{n} \overline{w_i} (Dp_{it} - \tilde{\alpha}_t)(\overline{Dp_i} - \overline{\tilde{\alpha}}) \tag{2.18}$$

Where $\sum_{i=1}^{n} \zeta_{it}{}^2 = \delta_t^2$ and on the right hand side, the very first term is similar to the 2nd order weighted moment about mean of Divisia index. The only difference is that in Divisia index technique weights $\overline{w_{it}}$ are used while here in stochastic approach we use its average $\overline{w_i}$. The second term measures the variation which comes in relative price of commodities on average but is not independent of time. When trend in price change is small and other components (errors) become large then inflation measurement will be poor. The third term covariance) on the right hand side of Eq. (2.18) is positive when on average the relative prices of those goods that increase during t th time also increase and vice versa. The GLS estimators of α_t & β_i after substituting (2.17) in (2.14) are

$$\hat{\alpha}_t = \sum_{i=1}^{n} \overline{w_i} Dp_{it} \, , \hat{\beta}_i = \frac{\sum_{t=1}^{T} \frac{(Dp_{it} - \alpha_t)}{\delta_t^2}}{\sum_{t=1}^{T} \frac{1}{\delta_t^2}} \tag{2.19}$$

As can be seen, the estimator $\tilde{\alpha}_t$ of rate of inflation is identical to that estimated previously, the reason is that the new weighting factor $\frac{1}{\delta_t^2}$ is same for all commodities with in the period t and we measure inflation during t by using only $\overline{w_1}\,\overline{w_2}...\overline{w_n}$ and $Dp_{1t}, Dp_{2t}...Dp_{nt}$ the estimator $\tilde{\beta}_i$ of the systematic component of the ith relative price change is now a weighted average of this price change over all t periods. In contrast, $\hat{\beta}_i$, defined in Eq. (2.17), is an un weighted average. The weights in new approach $\varphi_1, \varphi_2, ...\varphi_T$ are inversely proportional to $\delta_t^{\,2}$ which in turn is proportional to the error variance in time period t; thus smaller weight is allocated to those observations which have higher variance.

The Clements (1987) variance of the estimated rate of inflation can be estimated by the formula

$$\text{var}(\hat{\alpha}_t) = \frac{\delta_t^{\,2}}{n-1} \quad \text{or} \quad \text{var}(\overline{\hat{\alpha}_t}) = \frac{1}{T^2}\sum_{t=1}^{T}\text{var}(\hat{\alpha}_t) \tag{2.20}$$

The corresponding Crompton (2000) variance formula is

$$Var(\hat{\alpha}_t) = \sum_{i=1}^{n}\overline{w_i}\,\hat{\zeta}_{it}^{\,2} \tag{2.21}$$

This is weighted average of the squared random residuals. If the rate of inflation is constant then the Eq. (2.20) or Eq. (2.21) gives answer zero.

The Clements (1987) variance of systematic changes which occur in the relative prices is

$$Var(\hat{\beta}_t) = \frac{i}{n-1}\sum_{t=1}^{T}\frac{1}{\delta_t^2}\left(\frac{1}{\overline{w_i}} - 1\right) \tag{2.22}$$

This variance will get increased as the commodity become less important in the consumer’s budget.

3. An empirical illustration

We have used CPI monthly data covering the period July 2001 to March 2008 for Pakistan obtained from Federal Bureau of Statistics monthly Bulletin of Statistics. The details of the data are as follows. The number of commodity classifications is 10 namely: Food and Beverages, Apparel Textile and Foot Wear, House Rent, Fuel and Lighting, House Hold Furniture and Equipment, Transport and

Communication, Recreation and Entertainment, Education, Cleaning Laundry and Personal appearance, and Medicare.

Interpretation of results[†] is given as; the point estimate for mean monthly rate of inflation comes out to be 0.640% by Divisia index number using Eq. (2.19). The standard error of monthly rate of inflation is estimated to be 0.049% for the Divisia index number using Eq.(2.20). The point estimate 0.640% is close to the mean CPI log change of 0.597% which is computed using Eq. (2.2), which is expected. Laspeyres index number estimate by using Eq. (2.11) comes out to be 0.68% using which is close to Divisia index number approximately. The State Bank of Pakistan also uses Laspeyres index to find the point estimate of rate of inflation. All three point estimates of rate of inflation are not very much different from each other. The standard error for the rate of inflation using Eq. (2.20) is 0.0499% and using Eq. (2.21) is 0.052%, the difference may be to structure of heteroscedasticity. Heteroscedasticity present in our data is not the same structure which was assumed by Clements (1987) because when we computed the residual variance; it did not come out inversely proportional to the corresponding weights. We have also computed Crompton's[‡] variance for the rate of inflation in this study. Almost 59% of the values of variance computed by using Crompton's formula are smaller than those computed using Clements's formula, which shows that structure of heteroscedasticity in our data, may be different than that of Clements (1987). That is variances of inflation computed by Eq. (2.20) are biased.

In figure 1, scatter diagram of the estimate of rate of inflation is plotted against its corresponding standard error (Using Eq. (2.20)), solid line is the least square regression line. Graph shows that there is a positive relationship between the rate of inflation and its standard error. So when rate of inflation is higher then it is difficult to estimate it precisely in absolute sense i.e standard error of rate of inflation tends to rise. However, the ratio of increase in rate of inflation is higher than the increase in standard error. Figure 2, in which scatter diagram of estimate of inflation is plotted against its standard error (using Eq. (2.21) plotted, confirms that precise measure of inflation is not possible when its standard error is higher.

In Figure 3, 95% confidence band is constructed assuming the normal distribution $\hat{\alpha}_t$ ±1.96 (var $\hat{\alpha}_t$)$^{1/2}$ for the estimated rate of inflation, using Clements formula. It is assumed that prices are log

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† All the results are multiplied by 100.

‡ It is robust to unknown forms of heteroscedasticity

normally distributed. According to Clements (1987; p345) "it is reasonable". The jump in inflation and increase in the width of confidence band is noted in June 2006. This may be due to the earthquake shock in 2005 in Pakistan. Again in figure 4, 95% confidence band for rate of inflation is plotted using Crompton's formula, all the points lie in 95% confidence interval.

Table 3. Estimates of Relative Price Changes: Pakistan, Jul 2001-Mar 2008

Commodity Groups	$\hat{\beta}_i$	S.E of $\hat{\beta}_i$	T-Value
Food & Beverages	0.19273	0.02941	13.25218
Apparel, Textile & Footwear	-0.2287	0.0949	-0.33417
House Rent Index	-0.11	0.04372	1.990688
Fuel and Lighting	0.17681	0.08623	4.334916
House hold Furniture & Equipment	-0.5316	0.13119	-2.55043
Transport & Communication	-0.521	0.08603	-3.76636
Recreation & Entertainment	-0.6145	0.26499	-1.57543
Education	-0.6381	0.12783	-3.45101
Cleaning Laundry & Personal Appearance	0.67028	0.09676	8.963107
Medicare	-0.3668	0.16621	-1.02151

Table 1 shows that the relative price of food beverages is 0.19273, an increase that is highly significant even at one percent level of significance with the t-value 13.25; the point estimate 0.19273 is more than six times its standard error. The estimated relative price of Apparel, Textile and Footwear is constant, with the t-value -.33. The estimated relative price of house rent index decline with t-value 1.99. Next the estimated relative price of fuel and lighting increase by 0.1768 percent. This value is highly significant even at one percent. The estimated relative price of Household Furniture and Equipment declines with the t-value -2.55, which is significant at one percent. The estimated relative price of Transport and communication also declines with t-value -3.76636, which is significant at one percent. The estimated relative price of recreation and Entertainment is almost constant. The estimated relative price of education is declined by -0.6381% which is significant at 5%. At the end, the estimated relative price of cleaning laundry and personal Appearance is increased in real terms by 0.67, which is not ignorable. Medicare does not considerable change, as its estimated relative price is not significant even at 5%.Our model pass all diagnostic tests. Autocorrelations up to lag 10 are also computed and no high correlation was found except food has 0.2 and Cleaning 0.21.

We suggest that if prices of individual commodities are widely dispersed then level of uncertainty for measuring the rate of inflation will be very high. Otherwise this stochastic approach for measuring inflation is highly effective and more flexible in setting the inflation target in a range with certain level of confidence. Therefore, we suggest agencies that publish point estimate of rate of inflation should also use this approach to estimate the rate of inflation.

Tables of results can be provided upon request.

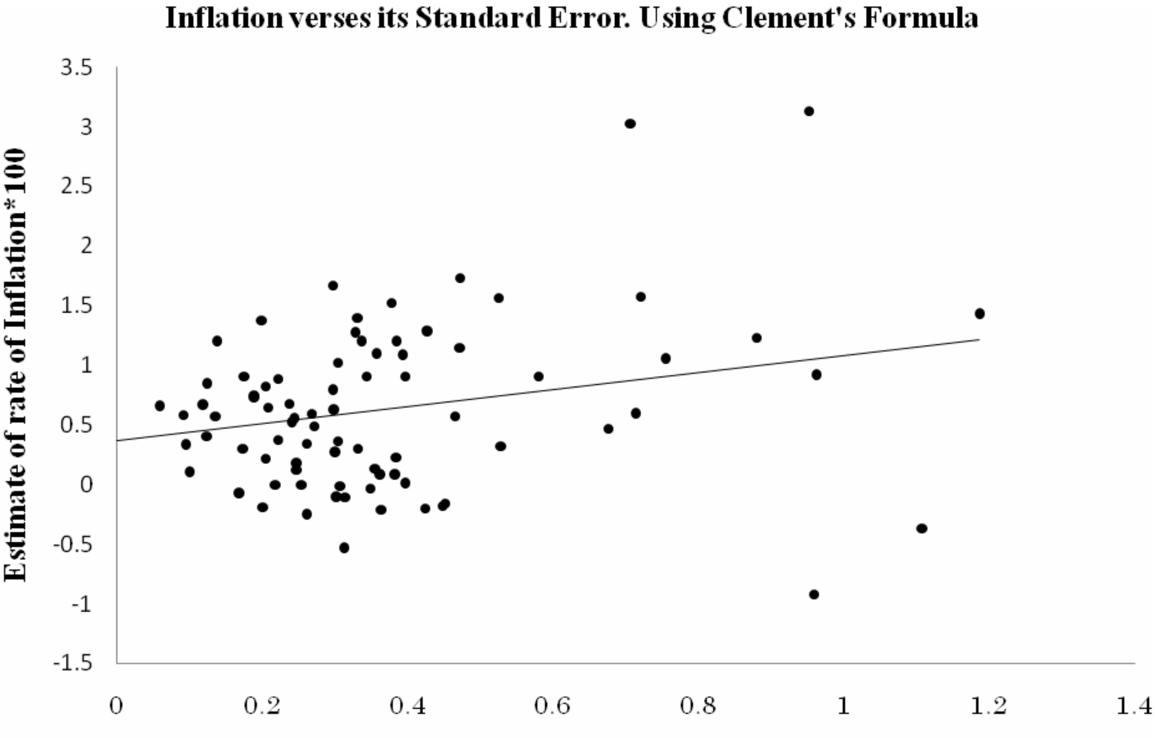


Figure 1.

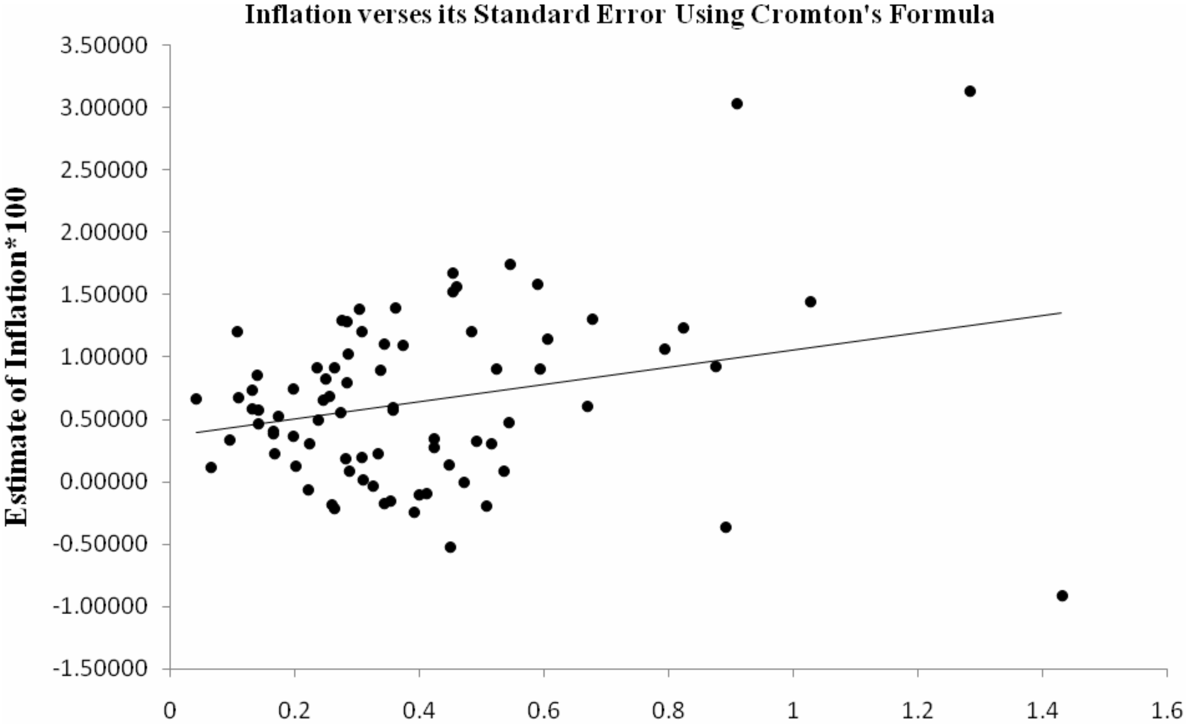


Figure 2.

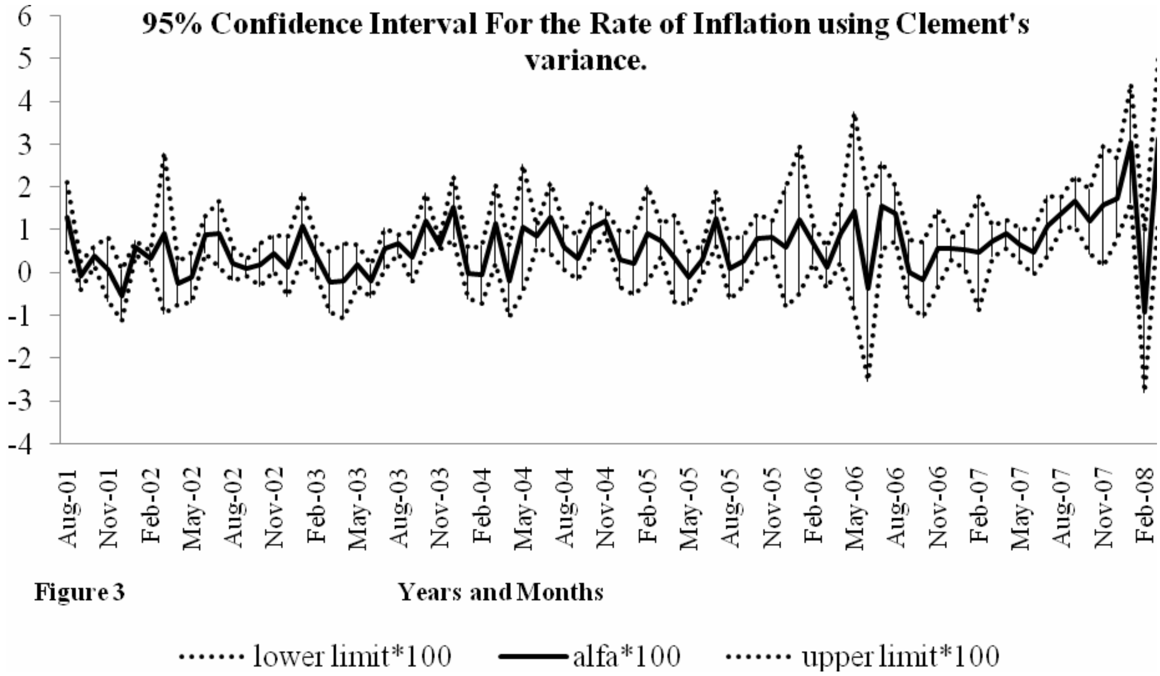
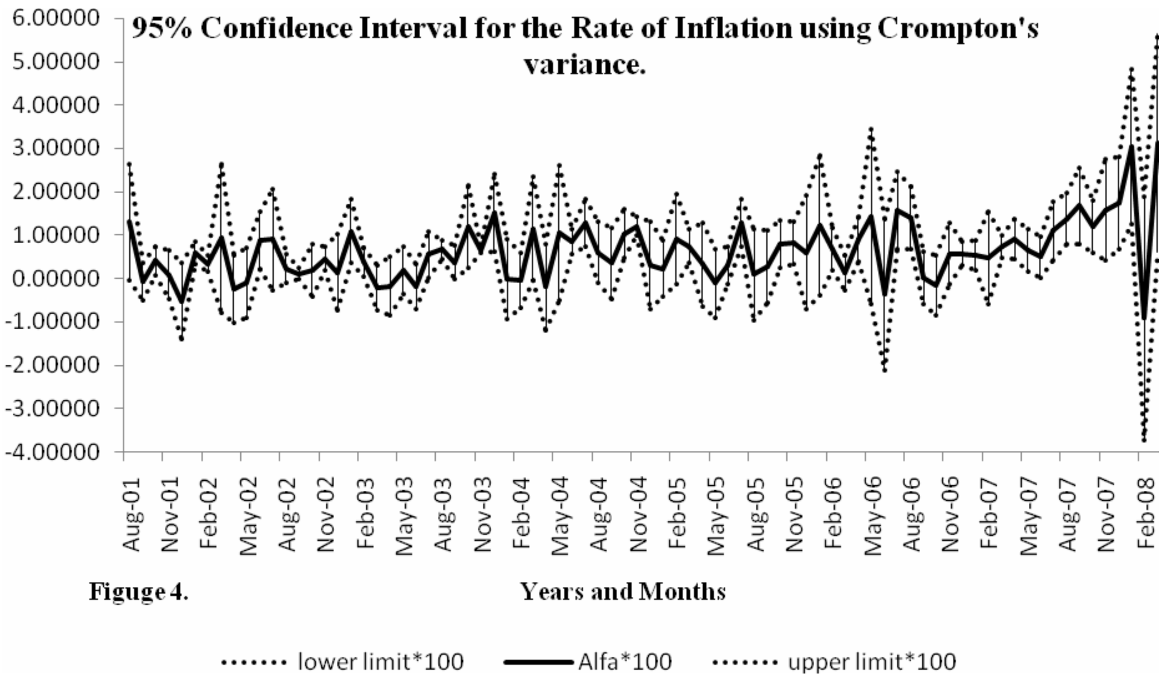


Figure 3



Figuge 4.

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