External constraint and financial crises with balance sheet effects

Meixing Dai

BETA, University of Strasbourg, France

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Meixing DAI*

BETA, University of Strasbourg, France

Abstract: This paper investigates the dynamic implications of Krugman’s (1999) model of financial crises with balance-sheet effects, which has a considerable impact on the literature as well as the teaching of international financial crisis. By explicitly taking account of wealth accumulation and external equilibrium condition, it is shown that a financial crisis in emerging market economies, instead of being interpreted as a jump from a good to a bad equilibrium with zero investment and zero foreign debt, could be explained as a jump from an unstable dynamic trajectory to a stable one. The dynamic framework illustrates well the analysis of different factors at the origin of financial vulnerability and crisis. By discriminating the financial crises according to the severity of their negative impacts on the domestic economy, the present study also adds some insights in the analysis of policy implications.

Keywords: Financial crisis, currency crisis, balance sheet effect, external solvency constraint.

JEL Classification: F31, F32, F41.

* BETA, University of Strasbourg, 61, avenue de la Forêt Noire – 67085 Strasbourg Cedex – France. Phone: (+33) 03 68 85 21 31; Fax: (+33) 03 68 85 20 71, e-mail: dai@unistra.fr.

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1. Introduction

In the aftermath of 1997 Asian financial crisis, there was much controversy among economists about its origin and nature. A decade later, economists have learned much about the relation between financial and exchange crises even the underlying dynamics before and after the crises are not enough discussed.

Two generations of currency crisis models have been developed before the 1997 Asian turmoil and are pertinent in explaining other particular crisis in the 1990s. The first focused on budgetary deficits and the effect of its continuing monetary financing (Krugman, 1979; Flood and Garber, 1984). The second (Obstfeld, 1994; Sachs et al., 1996), explained the crisis as the result of a conflict between a nominal exchange rate peg and the desire of pursuing an expansionary monetary policy, leading to the existence of multiple equilibria.

In the major crisis countries of Asia, however, neither of these stories has much relevance. In terms of conventional fiscal measures, the governments of the distressed economies were in quite good shape at the beginning of 1997. While growth had slowed and some signs of excess capacity appeared in 1996, none of them faced the kind of clear trade-off between employment and exchange rate stability.

The third-generation models of currency crisis were then developed to answer the particular questions raised during the Asian crisis. Many of these models have in common the idea that the crisis should be seen as a result of a shock that is amplified by what Bernanke et al. (1999) have called a financial accelerator mechanism. The basic story is similar: Since firms’ assets and liabilities are denominated in domestic and foreign currency respectively, real currency depreciation can have a large effect on output if it affects the credit access of some subset of agents; moreover this effect on output may in turn affect the exchange rate,
further amplifying the shock and causing it to persist. Most have argued that the core of the problem lies in the banking system (Corsetti et al., 1999; Chang and Velasco, 2001).

For Krugman (1999), any useful model of the crisis must involve some mechanism that produces multiple equilibria. He has remarked that the transfer problem, debated by Keynes and Ohlin in the 1920s, was central to what has happened to Asia. This issue has been remarkably absent from many formal models perhaps because the modellers have been mainly concerned with the behaviour of investors in one-good models rather than with the real economy per se. Hence, they don’t pay much attention to the movement in the terms of trade (or the real exchange rate) and the reversal in the current account. In practice, this reversal has been achieved partly through massive real depreciation, partly though severe recession that produces a compression of imports.

Using a model with balance-sheet effect, Krugman (1999) has sketched the transfer problem as another way of explaining the Asian crisis: foreign currency debts and firms’ leveraged financing make the domestic economy fragile and prone to crisis. He provides then a static analysis of his model and has found that there are three equilibria with the crisis brought on by a pure shift in expectations, leading to possible jump from high investment equilibrium to one with zero investment. A large literature of financial crisis that focuses on the story of balance-sheet effects and leverage constraint similar to Krugman (1999) has been developed into different directions (Allen et al., 2002). For example, Mendoza (2002) has shown that a mismatch between the denomination of debt and income exacerbates financial crises in emerging markets. Aghion et al. (2001, 2004a, b) have integrated the credit side with monetary side in order to study explicitly the problem of exchange rate stabilisation and provide optimal monetary policy prescriptions when a crisis occurs. Schneider and Tornell (2004) have developed a model based on sectoral asymmetries in corporate finance with
currency mismatch and borrowing constraints arising endogenously, giving rise to self-fulfilling crises.

Empirically, Upadhyaya and Upadhyaya (1999) have found that, with few exceptions, currency devaluation in Asian countries fails to make any effect on output over any length of time - short run, intermediate run, or long run. Kim and Ying (2007), using the pre-1997 crisis data and the trade-weighted exchange rate, have observed no evidence of contractionary devaluations. In fact, currency devaluation appears strongly expansionary in several countries. But their exercise suggests also that the crisis period was indeed different. However, after currency devaluation, the level of GDP can remain permanently below its initial trend, suggesting that the shocks underlying a currency crisis are persistent (Hong and Tornell, 2005). Taking a long term view and considering a great number of countries borrowing in foreign currencies, Bordo et al. (2010) has shown empirically that financial crises, driven by exposure to foreign currency, resulted in significant permanent output losses.

The theoretical advances in the literature of international financial crisis, achieved after the 1997 Asian financial crisis, have also great pertinence for understanding the repercussions of subprime crisis on Central and Eastern European countries and Iceland (Krugman (2010)). In effect, many of these countries have large amount of foreign debt denominated in foreign currencies (see, for example, Sirtaine and Skamnelos (2007), Von Hagen and Siedschlag (2008), Buiter and Sibert (2009), Stokes (2009) and Danielsson (2009) among others) and they are vulnerable to a devaluation or depreciation of national currency, exactly as in 1997 Asian financial crisis. It is astonishing that the great lessons of the 1997 Asian crisis are not learned.2

1 This can also be explained by the shift, induced by violent shocks, in the behaviours of international lenders as well as that of domestic firms, leading to less lending (or borrowing) and hence less investment and lower growth rate of the GDP.

2 A debate emerged in the late 1990s regarding the causes of the prevalence of foreign currency denominated foreign debt in emerging markets. Some saw it as a consequence of moral hazard. Hausmann (1999), and Eichengreen and Hausmann (1999) advanced the “original sin hypothesis” to describe a situation where “the
In this context, it is particularly interesting to revisit Krugman’s (1999) article which is distinguished by its profound insight within a simple framework and its capability of dealing with a large number of issues on currency and foreign debt crisis. It is among the early articles to draw the attention of economists on issues of debt composition and particularly currency denomination. It has a large impact on the literature on the currency and debt crisis and is largely used in graduate teachings.\(^3\) However, while the model of Krugman (1999) is dynamic, there are no ulterior extensions investigating the dynamic implications of this model. The central objective of this paper is to fill the gap and to supply a graphical analysis of the dynamics of Krugman’s (1999) model by introducing some supplementary assumptions and equations, notably equations of wealth accumulation and external equilibrium condition. This allows us to see from a different angle the mechanism of financial crisis in the model initially developed by Krugman for Asian emerging market countries exposed to foreign currency debt.

The reminder of the paper is structured as follows. In the next section, we present the dynamic model. In the section after, we describe the steady state. In section 4, we examine the dynamic properties of the model and the mechanism of financial crisis. In section 5, we discuss factors at the origin of financial fragility and crisis. The section 6 gives an analysis of policy implications. The last section concludes.

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\(^3\) For example, Krugman’s model (1999) is included in a number of books such as Gandolfo (2002), Isard et al. (2000), Allen (2004), etc… The article of Krugman (1999) is also on the reading lists of some leading Master’s programs across the world, e.g. Open economy macroeconomics (Summer 2004) by Charles Wyplosz at Graduate Institute of International study, Geneva, Switzerland; International Economics II (Fall 2006) by Ricardo Caballero, and Guido Lorenzoni at MIT, USA; Advanced Macroeconomics for the Open Economy (Fall 2009) by Jeffrey A. Frankel, at Kennedy School of Government, Harvard University; Financial crises, bubbles and crashes (BMSS 2008) by Hans-Joachim Voth at Barcelona Macroeconomics Summer School, Spain; International macroeconomics (fall 2008) by Giancarlo Corsetti and Philippe Martin at Sciences-Po, Ecole Polytechnique, ENSAE, France; International Macroeconomics (February, 2010) by Patrick Pintus in the Master Program in Economics and Finance at Université de la Méditerranée & GREQAM-IDEP, France.
2. The model

We extend the small open-economy model of Krugman (1999), which does not include the money, to a multi-period model. The model is described by the following equations:

\[ y_t = G(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1, \]  
\[ y_t = (1-\mu)I_t + (1-\mu)C_t + p_t X = (1-\mu)I_t + (1-\alpha)(1-\mu)y_t + p_t X, \]  
\[ p_t = \frac{y_t[1-(1-\alpha)(1-\mu)]-(1-\mu)I_t}{X}, \]  
\[ I_t \leq (1+\lambda)W_t, \]  
\[ 1+r_t = p_t^{-\mu}G_h(K_t, L_t), \quad \text{with} \quad K_t = I_{t-1}p_t^{-\mu}, \]  
\[ \frac{(1+r_t)p_t}{p_{t+1}} \geq 1 + r_t^*, \]  
\[ I_t \geq 0, \]  
\[ W_t = \alpha y_t - (1+r_{t-1})D_{t-1} - p_t(1+r_{t-1}^*)F_{t-1}, \]  
\[ D_t + p_t F_t = I_t - W_t, \]  
\[ B_{ce} = p_t X - \mu I_t - \mu C_t - r_{t-1} D_t - r_{t-1}^* p_t F_{t-1} + (D_t - D_{t-1}) + p_t(F_t - F_{t-1}) \]  
\[ = p_t X - \mu I_t - (1-\alpha)\mu y_t - r_{t-1} D_t - r_{t-1}^* p_t F_{t-1} + (D_t - D_{t-1}) + p_t(F_t - F_{t-1}). \]

with \( y_t \) denotes the output, \( K_t \) the stock of physical capital, \( L_t \) the labour, \( I_t \) the investment, \( C_t \) the consumption, \( p_t \) the real exchange rate (or the relative price of foreign goods), \( X_t \) the exportations, \( W_t \) the net wealth of domestic entrepreneurs, \( D_t \) the domestic currency denominated debt, \( F_t \) the foreign currency denominated debt, \( r_t \) the domestic real interest rate and \( r_t^* \) the foreign real interest rate. Among these variables, \( y_t, K_t, I_t, C_t, W_t \) and \( D_t \) are measured in terms of domestic goods, and \( X_t \) and \( F_t \), are in terms of foreign goods.
Equation (1) represents the production function, assumed to be Cobb-Douglas, of the small open economy that produces a single good each period using capital and labour. Capital is created through investment and it is assumed that it lasts only one period, so that this period’s capital is equal to last period’s investment. This assumption allows putting aside Diamond-Dybvig-type concerns over maturity mismatch between capital and foreign debt.

Equation (2) describes the market clearing condition for domestic goods. The functions of demand for consumption and investment are derived under the simple assumption that the residents of this economy are divided into two distinct classes. Workers, who receive a share $1 - \alpha$ of domestic income, lack access to the capital market and therefore spend all their income within each period. Entrepreneurs, who are assumed to be single-mindedly engaged in wealth accumulation, saving and investing all their income, create and own domestic capital until they have exhausted domestic investment opportunities. After that, they will spend the surplus of their revenue over investment. The domestic and foreign goods, with a unitary elasticity of substitution, are not perfect substitutes. A share $1 - \mu$ of both consumption and investment is spent on domestic goods, $\mu$ on imports. The rest of the world is large and spends a negligible fraction of its income on domestic goods. The value of domestic exports in terms of foreign goods is assumed to be fixed with $X_t = X$, i.e. the foreign elasticity of substitution is also unitary.

From equation (2), the domestic real exchange rate is expressed as in equation (3). According to (3), the higher is the investment, the lower the real exchange rate.

Inequality (4) specifies that the ability of domestic entrepreneurs to invest is limited by their wealth in the way of Bernanke et al. (1999). Lenders impose a limit on leverage,

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4 We introduce this assumption to close the small open-economy model. Otherwise, these entrepreneurs can accumulate an infinite amount of wealth. This assumption might not be plausible one for describing some emerging economies having sufficiently accumulated capital. As illustrated by recent international developments of South Korean firms, ambitious and innovative entrepreneurs might be tempted to invest on international financial market and in industrial projects in other countries. For other manners of closing the model, see Schmitt-Grohé and Uribe (2003).
justified by motives such as risk of insolvability and asymmetric information. Hence, entrepreneurs can borrow at most \((1 + \lambda)\) times their initial wealth.

Equation (5) shows that the return of domestic investment is determined by the marginal productivity of capital. Because a share \(\mu\) of investment falls on foreign goods, the price index for investment relative to that of domestic output is \(p_t^{-\mu}\). Hence, \(G_k(K_t, L_t)\) is not the return on savings denominated in domestic goods. One unit of domestic good saved in \(t\) produces \(p_t^{-\mu}\) units of capital and therefore the return on domestic bonds (by arbitrage) should be equal to \(p_t^{-\mu}G_k(K_t, L_t) = \alpha_t^{1-\mu} p_t^{-\mu} p_t^{-\mu} = \alpha_t^{1-\mu} p_t^{-\mu}\).

According to inequality (6), entrepreneurs will not borrow beyond the point at which the real return on domestic investment, determined by equation (5), equals that on foreign investment. It is similar to uncovered interest rates parity (UIRP), which compares the return that can be achieved with domestic capital (by converting a unity of foreign goods into domestic goods at the real exchange rate \(p_t\), then converting the next-period return back into foreign goods at \(p_{t+1}\)) and the rate of return of foreign asset \(r_t^*\).

Equation (7) imposes that investment cannot be negative.

Entrepreneurs own a wealth which is defined by equation (8). They hold all domestic capital and receive a share \(\alpha\) of domestic revenue that will not be spent for consumption. The income accruing to capital within the current period \((\alpha \gamma_t)\) also represents the value of domestic capital since it lasts only one period. At aggregate level, they owe debt to international lenders with the “currency composition” of debt taken as given.

Equation (9) describes the evolution of foreign debt contracted by domestic entrepreneurs. All investment not financed by their wealth is financed by foreign debt. In other words, it represents the balance-sheet constraint of these entrepreneurs.
Equation (10) represents the external equilibrium condition or balance of payments. Under flexible exchange rate regime where the balance of payments exhibits neither deficit nor surplus, i.e. $B_{ee} = 0$, equation (10) is equivalent to the combination of (2), (8) and (9).

Equations (9) and (10) represent the logic extension of Krugman’s model. They are important to our understanding of the underlying dynamic features of a crisis prone economy.

In the following, we assume for simplicity that the flexibility on the labour market will ensure full employment and the supply of labour is normalized to unity so that $L_t = 1, \forall t$.

Assume that entrepreneurs risk neutral and maximise their firms’ benefits in period $t$. Therefore, if $\alpha l_{t-1}^{-1} p_{t-1}^{r_{t-1}^{m}} \geq 1 + r_t$, i.e. the marginal revenue is higher than (or equal to) the marginal cost of investment, the leverage constraint (4) is binding, i.e.:

$$p_{t+1}(1 + r_t^*) = (1 + r_t)p_t.$$  \hspace{1cm} (11)

In other words, when the marginal rate of return of domestic investment is higher than the cost of financing additional investment, entrepreneurs find it advantageous to borrow from abroad the maximal amount allowed by the leverage constraint. If $\alpha l_{t-1}^{-1} p_{t-1}^{r_{t-1}^{m}} < 1 + r_t$, the constraint (4) will not be binding and entrepreneurs will borrow less than that is authorised by the leverage multiplier.

The ratio of foreign currency (or goods) denominated debt relative to domestic currency denominated debt is assumed to be $\nu$:

$$\frac{F_t}{D_t} = \frac{F_{t-1}}{D_{t-1}} = \nu.$$  \hspace{1cm} (12)

The ratio $\nu$ can be modified arbitrarily to examine the effect of its variation on economic equilibrium. In the following, $D_t$ is simply named “foreign debt” even though the “total foreign debt” is a multiple of it, i.e. $D_t + p_t F_t = (1 + \nu p_t)D_t$. 

8
Foreign banks would pay attention to the wealth left to entrepreneur after the payment of principal and interests. If this financial indicator is zero or negative, no bank will lend to these entrepreneurs. Using equations (1), (8), (11) and (12), we obtain:

\[
W_t = \alpha l_{t-1} p_t^{-\mu} - \left[ \frac{p_t}{p_{t-1}} + p_t \nu \right] (1 + r_{t-1}^*) D_{t-1}.
\] (13)

If \( W_t < 0 \), international lenders will lose money while reducing their lending to zero. All entrepreneurs will bankrupt and invest zero, i.e. the constraint (7) will be binding.

We define an iso-wealth curve representing the combinations of foreign debt and investment that ensure a constant wealth. For a given wealth \( W_0 \), equation (13) can be rewritten as:

\[
D_{t-1} = \frac{\alpha l_{t-1} p_{t-1}^{\mu} - p_{t-1} W_0}{(1 + \nu p_{t-1}) p_t (1 + r_{t-1}^*)}.
\] (14)

For given \( W_0 \), \( p_{t-1} \) and \( p_t \), there is a positive relation between investment and foreign debt as shown in Fig. 1.

A given combination of investment and foreign debt \((I_0, D_0)\) can correspond to different levels of entrepreneurs’ wealth, depending on the values of exogenous variables and parameters such as \( r_{t-1}^* \), \( X \), \( \mu \) and \( \alpha \).

![Fig. 1: Iso-wealth curve.](image-url)
We notice that a variation of $\lambda$ has no effect on the iso-wealth curve. According to equation (14), the curve Iso-wealth will rotate counter-clockwise around $C$ to a position like the curve Iso-wealth $bis$ corresponding to lower net wealth, $W_0'$ (see Fig. 1), if there is:

- an increase in $r_i^*$ that reduces entrepreneurs’ wealth.

- a decrease in $X$, since this implies according to equation (3) a depreciation of the real exchange rate, decreasing the current value of domestic product but increasing the value of principal and interests on foreign debt.

- an increase in $\mu$ that, in increasing demand for foreign goods, implies a depreciation of the real exchange rate and hence has similar effect as a decrease in $X$.

However, an increase in $\alpha$ has ambiguous effects on entrepreneurs’ wealth. It implies, on the one hand, an increased revenue accrued to capitalists, and on the other hand, a depreciation of the real exchange rate (due to higher productivity of existing capital and hence higher output, and higher part of revenue that is exported) which increases thus the value of foreign debt when measured in domestic currency.

The model has nine endogenous variables, i.e. $y_i$, $C_i$, $I_i$, $K_i$, $r_i$, $p_t$, $W_t$, $D_t$ and $F_t$. It can be resolved in two steps. The first step consists to solve the reduced system of difference equations describing the evolution of $p_t$, $I_t$ and $D_t$. In the second step, we can solve for $F_t$, $K_t$, $y_i$, $C_i$, $r_i$ and $W_t$ given the solutions of $p_t$, $I_t$ and $D_t$.

Substituting $y_i$ defined by equation (1) into equation (3) yields:

$$p_t = \frac{I^{\alpha}_{t-1}p_t^{-\alpha \mu}[1-(1-\alpha)(1-\mu)]-(1-\mu)I_t}{X}. \quad (15)$$

Using equations (1), (8), (11) and (12) to eliminate $F_t$, $W_t$ and $r_i$ leads to the difference equation of $I_t$ which verifies the condition $\alpha I_{t-1}^{\alpha} p_t^{-\alpha \mu} \geq 1 + r_i$:
\[ I_t = (1 + \lambda)\psi t^{-\alpha\mu} - (1 + \lambda) \frac{1 + \psi}{p_{t-1}} p_t (1 + r_{t-1}^*) D_{t-1}. \] (16)

The dynamics of private foreign debt, which is equal to the country’s external debt in the present model, can be described equivalently by equation (9) or equation (10) with \( B_{ee} = 0 \).

Using equation (9) and eliminating \( F_t \), \( W_t \) and \( r_t \) as before lead to:

\[ D_t (1 + p_t) = I_t - \beta t^{-\alpha\mu} + \frac{1 + \psi}{p_{t-1}} p_t (1 + r_{t-1}^*) D_{t-1}. \] (17)

Equations (15) and (16)-(17) allow describing the dynamic behaviours of real exchange rate, investment and foreign debt. They are non-linear difference equations and cannot be solved explicitly. However, the static and dynamic properties of this reduced system can be studied with the help of phase diagram.

3. Steady state

The steady state equilibrium is attained when the dynamic effects of shocks are fully exercised, i.e. \( p_{t+1} = p_t = p_{t-1} = \bar{p} \), \( I_{t-1} = I_t = \bar{I} \), \( D_t = D_{t-1} = \bar{D} \) and \( r_t = r_{t-1} = \bar{r} = r = r_{t-1} \).

Using the condition \( a t^{-\alpha\mu} \geq 1 + r \), we obtain that the steady state level of investment must satisfy the following condition:

\[ \bar{I} \leq \hat{I} = \arg\{a \bar{t}^{-\alpha\mu} = 1 + \bar{r}^*\}. \] (18)

For given international interest rate, the highest level of investment, \( \hat{I} \), that is rational to realise at the steady state, corresponds to the one that equalizes the marginal productivity of capital and the financial opportunity cost of investing.

The steady state counterparts of equations (15), (16) and (17) are given as follows:

\[ \bar{p} = \frac{t^{-\alpha} - (1 - \alpha)(1 - \mu)] - \mu)\bar{I}}{X}. \] (19)
The steady state equilibrium of the reduced dynamic system is defined by the vector \( (\bar{I}, \bar{p}, \bar{D}) \) which checks simultaneously equations (18)-(21). To ensure a positive real exchange rate, we impose that \( \bar{I}^a[1 - (1 - \alpha)(1 - \mu)] - (1 - \mu)\bar{I} > 0 \).

Given the steady state level of investment, equation (19) determines the steady state real exchange rate. Equation (20) corresponds to the combination of investment and foreign debt along the most rapid wealth-accumulation path, limited by condition (18). Equation (21) is the solvency constraint of entrepreneurs, which implies that the surplus of wealth over investment is equal to interest payments on foreign debt. Equation (21) can be substituted by the steady state counterpart of equation (10) meaning that, in the absence of entry and exit of capital, the trade surplus must be equal to interest payments on foreign debt. Since the debt in this model is only foreign and uniquely contracted by entrepreneurs, their solvency constraint is then identical to the external solvency constraint of the economy.

The system constituted of equations (19)-(21) can be decomposed into two sub-systems that can be separately solved. Substituting \( p \) given by equation (19) into equations (20)-(21) and rearranging the terms lead to

\[
\bar{D}_{\text{inv}} = \frac{\alpha(1 + \lambda)\bar{I}^a X^{1+\alpha \mu} \{\bar{I}^a[1 - (1 - \alpha)(1 - \mu)] - (1 - \mu)\bar{I}^{\gamma - \alpha \mu} - X\bar{I}\}}{(1 + \lambda)\{X + \nu\bar{I}^{\alpha}[1 - (1 - \alpha)(1 - \mu)] - \nu(1 - \mu)\bar{I}(1 + \bar{r}^*)\}}, \tag{22}
\]

\[
\bar{D}_{\text{ee}} = \frac{\bar{I}^a X^{1+\alpha \mu} \{\bar{I}^a[1 - (1 - \alpha)(1 - \mu)] - (1 - \mu)\bar{I}^{\gamma - \alpha \mu} - X\bar{I}\}}{\{X + \nu\bar{I}^{\alpha}[1 - (1 - \alpha)(1 - \mu)] - (1 - \mu)\bar{I}\}\bar{r}^*}, \tag{23}
\]

where \( \bar{D}_{\text{inv}} \) and \( \bar{D}_{\text{ee}} \) denote, for all levels of investment, the highest steady state debt compatible with the leverage constraint and the external solvency constraint respectively.
Equations (22)-(23) constitute a sub-system incorporating the effects of the real exchange rate on wealth and can be solved to obtain $\bar{I}$ and $\bar{D}$ taking account of condition (18).

For each value of $\bar{D}_{|_{lnv}}$, we can determine a “financeable” level of investment that would occur if the leverage constraint (4) was binding. It determines, taking account of the effect of investment on the real exchange rate, and hence on balance sheets, how much credit could be extended to domestic firms at maximum. Since risk-neutral entrepreneurs use maximally the leveraged financing, equation (22) represents also the set of combinations of foreign debt and optimal investment for all $\bar{I} < \bar{I}$.

According to equation (22), $\bar{D}_{|_{lnv}}$ is positive for $\bar{I} \in ]0, \bar{I}[$ and it equals zero for $\bar{I} = 0$ and $\bar{I} = \bar{I}$, with:

$$
\bar{I} = \text{arg}\{\alpha(1 + \lambda)\bar{I}^{\alpha-1}X^{\alpha\mu} \{\bar{I}^{\alpha}[1 - (1 - \alpha)(1 - \mu)] - (1 - \mu)\bar{I}^{\alpha-\alpha\mu} = 1\}.
$$

Equation (23) implies that $\bar{D}_{|_{lee}}$ is positive for $\bar{I} \in ]0, \bar{I}_d[$ with

$$
\bar{I}_d = \text{arg}\{ar{I}^{\alpha-1}X^{\alpha\mu} \{\bar{I}^{\alpha}[1 - (1 - \alpha)(1 - \mu)] - (1 - \mu)\bar{I}^{\alpha-\alpha\mu} = 1\} \text{ and it equals zero for } \bar{I} = 0 \text{ and } \bar{I} = \bar{I}_d.
$$

The relative position of $\bar{D}_{|_{lnv}}$ and $\bar{D}_{|_{lee}}$ at the steady state is examined through their difference:

$$
\bar{D}_{|_{lnv}} - \bar{D}_{|_{lee}} = \frac{(1 + \lambda + \lambda\bar{r}^*)X\bar{T} - [1 + (1 - \alpha)\bar{r}^*](1 + \lambda)\bar{T}X^1 + \alpha\mu \{\bar{T}^{\alpha}[1 - (1 - \alpha)(1 - \mu)] - (1 - \mu)\bar{T}^{\alpha-\alpha\mu}\}}{(1 + \lambda)\{\bar{X} + \bar{v}\bar{T}^X[1 - (1 - \alpha)(1 - \mu)] - \bar{v}(1 - \mu)\bar{T}^X(1 + \bar{r}^*)\}.
$$

(25)
As the denominator at the right hand of equation (25) is positive, it is straightforward to show that $\mathcal{D}_{\text{inv}} - \mathcal{D}_{\text{lee}} = 0$ has two solutions. The first solution is zero and the second solution denoted by $\tilde{T}_{I}$ is positive with:

$$
\tilde{T}_{id} = \arg \left\{ 1 + (1 - \alpha)\alpha^* \left( 1 + \lambda \right) \alpha^{-1} X^{\alpha} \left( \alpha \left[ 1 - (1 - \alpha)(1 - \mu) \right] - (1 - \mu) \alpha^{-1} \alpha^{\alpha \mu} \right) = 1 + \lambda + \lambda \alpha^* \right\}.
$$

Thus, we have:

$$
\begin{align*}
\mathcal{D}_{\text{inv}} - \mathcal{D}_{\text{lee}} & \geq 0, \text{if } T \geq \tilde{T}_{id} \\
& < 0, \text{if } T < \tilde{T}_{id}.
\end{align*}
$$

(26)

According to (26), for $T < \tilde{T}_{id}$ (respectively $T \geq \tilde{T}_{id}$), the level of foreign debt compatible with external equilibrium is higher than (respectively equal to or lower than) the maximal foreign debt allowed by the leverage constraint. Rewrite $\hat{T}$ and $\tilde{T}_{id}$ given by (18) and (24) as

$$
\hat{T} = \arg \left( \alpha^{-1} p^{-1} = \frac{1 + \alpha^{*}}{\alpha} \right) \quad \text{and} \quad \tilde{T}_{id} = \arg \left( \alpha^{-1} p^{-1} = \frac{1 + \alpha^{*} + \lambda \alpha^{*}}{(1 + (1 - \alpha)p^{*})(1 + \lambda)} \right),
$$

respectively. Since

$$
\frac{1 + \alpha^{*} + \lambda \alpha^{*}}{(1 + (1 - \alpha)p^{*})(1 + \lambda)} > \frac{1 + \alpha^{*}}{\alpha},
$$

it is easy to show that:

$$
\hat{T} < \tilde{T}_{id}.
$$

(27)

According to (27), the level of investment $\tilde{T}_{id}$ corresponding to the crossing point of these two curves is not a steady state equilibrium.

Taking account of above discussion, the curves $\mathcal{D}_{\text{inv}}$ and $\mathcal{D}_{\text{lee}}$ could be represented as in Fig. 2. At high levels of $I$, the leverage constraint (4) will not bind. Instead, investment is determined by equations (5) and (11) and consequently limited to $\hat{T}$. According to conditions

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5 We have $\mathcal{D}_{\text{inv}} - \mathcal{D}_{\text{lee}} = 0$ if $(1 + \lambda + \lambda \alpha^{*})X = (1 + (1 - \alpha)\alpha^{*})X^\alpha \left( \alpha \left[ 1 - (1 - \alpha)(1 - \mu) \right] - (1 - \mu) \alpha^{-1} \alpha^{\alpha \mu} \right)$. The left hand of the last equation is a line with constant positive slope and the right hand denoted by $\Phi(I)$ is a curve with positive slope: $\Phi = (a \alpha (1 - \mu) + (1 - \alpha) \alpha^{*}) \left( 1 + \lambda \right) \alpha^{-1} X^{\alpha} \left( \alpha \left[ 1 - (1 - \alpha)(1 - \mu) \right] - (1 - \mu) \alpha^{-1} \alpha^{\alpha \mu} \right)$.

The left hand of the last equation is a line with constant positive slope and the right hand denoted by $\Phi(I)$ is a curve with positive slope: $\Phi' = a \alpha (1 - \mu) + (1 - \alpha) \alpha^{*} \left( 1 + \lambda \right) \alpha^{-1} X^{\alpha} \left( \alpha \left[ 1 - (1 - \alpha)(1 - \mu) \right] - (1 - \mu) \alpha^{-1} \alpha^{\alpha \mu} \right)$ with $\lim_{I \to \infty} \Phi' = \infty$, $\Phi' = a \alpha (1 - \mu) + (1 - \alpha) \alpha^{*} \left( 1 + \lambda \right) \alpha^{-1} X^{\alpha} \left( \alpha \left[ 1 - (1 - \alpha)(1 - \mu) \right] - (1 - \mu) \alpha^{-1} \alpha^{\alpha \mu} \right) < 0$. That shows that there is a second solution for $I$ denoted by $\tilde{T}_{I}$ with $\tilde{T}_{I} > 0$. 

---
(18) and (27), the set of feasible combinations of debt and investment is constituted of AB (i.e., \( I = \hat{I} \)) and OB (i.e., the dark part on the curve \( \overline{D}_{\text{inv}} \)). For \( \hat{I} < \tilde{I} \), it is optimal for risk-neutral entrepreneurs to use fully the leverage effect since this is the most rapid path of wealth accumulation. When \( \tilde{I} \) is attained, entrepreneurs will reduce the leverage to a level smaller than \((1+\lambda)\).

![Fig. 2: Steady state equilibria.](image)

The position of the curves \( \overline{D}_{\text{inv}} \) and \( \overline{D}_{\text{ee}} \) depends on the value of \( X, r^*, \mu, \nu \) and \( \alpha \).

Whereas \( \lambda \) is a determinant of \( \overline{D}_{\text{inv}} \), it does not affect \( \overline{D}_{\text{ee}} \). These two curves rotate counterclockwise around \( O \), if:

- \( X \) decreases, implying a depreciation of the real exchange rate. That will increase the value of foreign currency debt when measured in domestic currency.\(^6\)

- \( r^* \) rises, increasing financial burden of foreign debt.

- \( \mu \) increases, leading to similar effect as a decrease in \( X \).

\(^6\) Denote \( \Pi = \tilde{I}^\mu \left[ 1 - (1 - \alpha)(1 - \mu) \right] (1 - \mu) \tilde{I} > 0 \), deriving \( \overline{D}_{\text{inv}} \) and \( \overline{D}_{\text{ee}} \) given by equations (22)-(23) with respect to \( X \) yields:

\[
\frac{\partial \overline{D}_{\text{inv}}}{\partial X} = \frac{\mu \sigma^2 (1 + \lambda) \sigma^2 X^2 + \mu \Pi [1 + \alpha (1 + \lambda) \sigma X + \lambda \Pi] - \lambda \Pi}{(1 + \lambda) (X + \Pi)^2 (1 + \lambda^2)} > 0, \quad \forall \overline{D}_{\text{inv}} > 0; \quad \text{and}
\]

\[
\frac{\partial \overline{D}_{\text{ee}}}{\partial X} = \frac{\alpha \sigma^2 X^2 + \alpha \nu \Pi [1 + \alpha (1 + \lambda) \sigma X + \lambda \Pi] - \lambda \Pi}{(X + \Pi)^2 \nu^2} > 0, \quad \forall \overline{D}_{\text{ee}} > 0.
\]
The above-mentioned changes in exogenous variables and parameters imply that the entrepreneurs’ wealth and the trade account surplus are reduced to levels that can only sustain a lower foreign debt if both leverage and external solvency constraints are respected.

An increase in \( v \) also induces a counter-clock rotation of these curves around \( O \). However, it exercises its effects in a different manner. In effect, it is equivalent to an increase of total foreign debt. To keep foreign debt at a constant level compatible with the leverage and external solvency constraints, domestic currency debt must be reduced. In the case of a decrease in \( \lambda \), it limits the foreign borrowing by entrepreneurs but does not affect the external solvency constraint.

We remark that a variation of \( \alpha \) has contradictory effects on the curves \( \frac{\partial \bar{D}_{lnv}}{\partial \alpha} \) and \( \frac{\partial \bar{D}_{\lambda}}{\partial \alpha} \). It ambiguously affects the entrepreneurs’ wealth and hence the maximal amount of foreign debt which can be borrowed by them. It similarly affects the trade account and therefore the level of foreign debt sustainable by the trade surplus.\(^7\) On the one hand, an increase in \( \alpha \) will raise the total revenue that can be distributed (due to positive effect on the production) and the part of national revenue distributed to entrepreneurs. This increases the surplus of entrepreneurs’ revenue over investment and thus their capability to repay principal and interests on foreign debt. In other words, it decreases workers’ consumption and importations and increases the trade surplus necessary to pay interests on foreign debt. On the other hand, an increase in \( \alpha \) implies a depreciation of the real exchange rate due to a higher productivity of existing capital and hence a larger output as well as larger part of output that can be exported. After the depreciation, the value of foreign currency debt increases in terms of domestic currency.

\(^7\) Deriving \( \bar{D}_{lnv} \) and \( \bar{D}_{\lambda} \) given respectively by equations (22) and (23) with respect to \( \alpha \) leads to:

\[
\frac{\partial \bar{D}_{lnv}}{\partial \alpha} = \left\{ \frac{(1+\lambda)\Gamma\alpha X^{1+\alpha}P_{\alpha}^{-\alpha} - (1+\lambda)^{\alpha+1} (X+\Pi) \ln X + \alpha (1+\lambda) \frac{\Gamma\alpha X^{1+\alpha}P_{\alpha}^{-\alpha} - X T \ln T}{(1+\lambda) (X+\Pi) \ln X - (1-\alpha) (1-\mu) \Gamma\alpha X^{1+\alpha}P_{\alpha}^{-\alpha} - X T \ln T} \right\}
\]

\[
\frac{\partial \bar{D}_{\lambda}}{\partial \alpha} = \left\{ \frac{(1+\lambda)\Gamma\alpha X^{1+\alpha}P_{\alpha}^{-\alpha} + (1+\lambda) \frac{\Gamma\alpha X^{1+\alpha}P_{\alpha}^{-\alpha} - X T \ln T}{(X+\Pi) \ln X - (1-\alpha) (1-\mu) \Gamma\alpha X^{1+\alpha}P_{\alpha}^{-\alpha} - X T \ln T} \right\}
\]

As the numerator of these two derivatives has some positive terms and some negative terms, the net effect is ambiguous.
Hence, total foreign debt becomes too high and must be reduced to respect both leverage and external solvency constraints.

The foreign debt is non-negative, i.e. \( \overline{D} + \overline{F} \geq 0 \), \( \forall \hat{I} \). This condition translates the assumptions given before, i.e. entrepreneurs have objective of attaining wealthy statute in exploiting as quick as possible all domestic investment opportunities; and once this statute is obtained, they will not seek to increase further their wealth by spending the surplus of their revenue over the investment. Under the above condition, there are only two stationary equilibria (Fig. 2). One corresponds to the point \( O \), where investment and foreign debt are both zero. At this equilibrium, lenders do not believe that entrepreneurs have any collateral. Another equilibrium is the point \( A \) where foreign debt is zero and investment equals \( \hat{I} \).

4. Dynamics and mechanism of financial crisis

In a static analysis, Krugman has shown that there are three equilibria where investment equals financeable investment that depends on wealth and leveraged financing. The worst equilibrium is the one with zero investment. According to Krugman, a financial crisis happens due to the interaction between borrowing, real exchange rate and wealth. The total amount that entrepreneurs can borrow from foreign lenders depends on their wealth. The wealth of each individual entrepreneur depends on the total amount of such borrowing. The borrowing of individual entrepreneur has a negative externality on the community, since the volume of capital inflow affects the terms of trade and hence the valuation of foreign-currency-denominated debt. Due to this externality, a decline in capital inflows can adversely affect the balance sheets of domestic entrepreneurs, reducing their ability to borrow and hence further reducing capital inflows.
Sharing the basic story with Krugman, we remark that the three equilibria do not find their equivalent in a dynamic analysis. Analysing the steady state of the economy reveals that there are only two equilibria.

The phase diagram represented in Fig. 3 allows discussing the dynamic stability of the economy. At the right of the curve $\bar{D}_{ee}$, the external debt is too high in the sense that the trade surplus is not sufficient to pay interests on existing foreign debt and the latter keeps increasing due to current account deficit, and *vice versa*. Under the line $\hat{I}$ and at the left of the curve $\bar{D}_{inv}$, as the foreign debt is smaller than the maximum authorised by international lenders, entrepreneurs’ wealth increases and they can invest more and more. Above the line $\hat{I}$ the investment is too high and it has a rate of return smaller than that offered by international financial market. Consequently, investment must be reduced to increase the marginal rate of return of capital. Under the line $\hat{I}$ and at the right of the curve $\bar{D}_{inv}$, the foreign debt is too high to satisfy the leverage constraint and the entrepreneurs’ wealth will be insufficient to sustain the past level of investment. Therefore, investment must be reduced.

The equilibrium point A is stable. Meanwhile, the equilibrium point O is not stable since a small wealth is sufficient to start a wealth accumulation process which leads to higher and
higher investment and foreign debt, leading the domestic economy finally to the equilibrium point A. A more detailed discussion, in distinguishing seven areas in Fig. 3, allows us to understand better the underlying dynamics of investment and foreign debt.

In area 1 (area ABO), bounded at the above by the line $\hat{I}$ and at the right by $\hat{D}_{\text{inv}}$, the foreign debt is smaller than the limit fixed by leverage constraint and is compatible with external solvency constraint. In this area, risk neutral entrepreneurs tend to invest in using at maximum the leveraged financing so that they can enrich themselves as quickly as possible, i.e. along the segment OB of the curve $\hat{D}_{\text{inv}}$. They have the possibility to progressively reduce their foreign debt and invest with their own financial resources once $\hat{I}$ is attained. While the path OB is realisable, it is extremely fragile and can lead to financial crisis when the domestic economy is facing adverse external shocks or international lenders turn pessimistic about the perspectives of the domestic economy. Entrepreneurs with risk aversion will choose a wealth-accumulation path in the interior of the area ABO.

In area 2, delimited by the line $\hat{I}$, the curve $\hat{D}_{\text{ee}}$ and the curve OB, the debt is higher than the limit fixed by the leverage constraint but is compatible with the external solvency constraint. If the combination of investment and foreign debt happens to be in this area, entrepreneurs are constrained to reduce investment in order to reduce foreign debt. The temporary equilibrium that will realise after adjustment of international lending can be more or less favourable, depending on the confidence of international lenders placed on these entrepreneurs and the perspectives of the domestic economy. For a level of wealth $W_0$, instead of reducing their lending to a level corresponding to a point such as C, international lenders with herd behaviour could reduce their lending to zero, diminishing hence the
investment to a level equal to $W_0$. A jump from a point of disequilibrium with high foreign
debt to a temporary equilibrium without it becomes probable when the debt is short run.

In area 3, which is at the right of the curve $D_{le}^*$ and under the line $\hat{I}$, entrepreneurs have
high foreign debt, exceeding the limit imposed by the leverage constraint and that by external
solvency constraint. If the foreign debt is not too high, a quick reaction of international
lenders will allow them to recover totally or most of their lending. If they wait, the situation
could deteriorate and lead to a loss for lenders while all entrepreneurs bankrupt.

In area 4, above the line $\hat{I}$ and bordered at the left by the curve $D_{inv}^*$ for $I > \bar{I}_{id}$ and the
curve $D_{ee}^*$ for $I \in [\hat{I}, \bar{I}_{id}]$, the investment is higher than $\hat{I}$ and the foreign debt is too high to
be compatible with both leverage and external solvency constraints. Rational entrepreneurs
will reduce their investment until $\hat{I}$. The reaction of international lenders will depend on the
initial level of foreign debt as well as that of wealth.

In area 5, between curves $D_{inv}^*$ and $D_{ee}^*$ with $I \in [\hat{I}, \bar{I}_{id}]$, the investment is higher than $\hat{I}$.
The foreign debt, while being compatible with the external solvency constraint, is too high to
be compatible with the leverage constraint. Entrepreneurs will have to reduce their investment
until $\hat{I}$. If they do not make this decision, they will be constrained by foreign lenders to
reduce their investment after all. Since foreign debt is not excessively high and entrepreneurs’
wealth is large, a severe financial crisis is impossible.

In area 6, between curves $D_{inv}^*$ and $D_{ee}^*$ with $I > \bar{I}_{id}$, the investment is higher than $\hat{I}$
and the foreign debt, while being compatible with the leverage constraint, is too high for the
long term external solvency constraint to be respected. Entrepreneurs must reduce their

\footnote{To the difference of the figure given in Krugman (1999), not all firms will bankrupt and investment will not
finally fall to zero in this dynamic analysis.}
investment until $\hat{I}$ and their foreign debt correspondingly. On the contrary, the foreign debt will increase due to unpaid interests and that will not be allowed by rational and well-informed international lenders, who would consider that the increasing current account deficit is not sustainable.

In area 7, delimited at the above by the curve $\overline{D}_{ee}$, $\forall I \in \hat{I}_{id}, \bar{I}_{d}[\]$ and at the right by the curve $\overline{D}_{inv}$ with $I \in \hat{I}, \tilde{I}_{id}[\]$, the investment is higher than $\hat{I}$. The foreign debt is compatible with both constraints. Entrepreneurs will reduce investment and foreign debt until the point A is attained.

In the areas 1 and 7, the behaviour of this model is relatively uninteresting in terms of crisis probability. In the area 1, the economy has a high rate of return on investment and the accumulation of capital stock is delayed by not making leverage constraint binding. In the area 7, reducing orderly both investment and foreign debt is optimal and feasible. An outburst of severe financial crisis is improbable since the wealth is sufficiently high with regard to existing foreign debt. In these two areas, there will be nothing that resembles an Asian-style financial crisis as discussed in the literature.

The financial crises in this dynamic setting are not explained by jumps between the multiple stationary equilibria due to self-fulfilling expectations. In contrast, they can be reinterpreted as jumps from a temporary equilibrium with high foreign debt to another one with less or no foreign debt, situated on two distinct dynamic trajectories. For a financial crisis to take place, it is necessary to consider some factors that make foreign lenders panicking. For example, two channels of contagion, i.e. monsoonal effects and spill-over effects (Masson, 1999), can be used here to explain a financial crisis. The monsoonal effects emanate from the global environment (in particular, from policies in industrial countries), and sweep over all emerging countries to a greater or lesser extent. Thus, a monetary contraction
in the world leading country could considerably raise the financial costs of investment for an indefinite horizon and hence put some emerging market economies under great financial pressures. Due to spill-over effects, crisis in one country may affect other emerging markets through the linkages operating through trade, economic activities or competitiveness. A typical example is a currency devaluation of a rival country in crisis, which reduces potentially the exportations of the domestic economy.

Due to the effects of contagion, an economy initially on a feasible accumulation path such as OB in Fig. 3 could be found to be situated at the right of \( \bar{D}_{lev} \) or in the worst case at the right of \( \bar{D}_{ee} \) as these curves rotate counter-clockwise. For emerging market economies that initially offer high rate of return of capital, this situation could result from previous choices of entrepreneurs who underestimate the effects of exogenous shocks on macroeconomic and financial stability and the repercussion of these shocks on the rate of return of their investment.

We remark that although the leveraged financing is assumed to have a rigid multiplier \( \lambda \), international lenders can choose to reduce drastically the debt to a level that they consider as safe for their principal and interest payments. This is more likely if the debt is short-run. Under a flexible exchange regime, a severe reduction of investment and foreign debt in a short time can take place when some shocks modify drastically and adversely the current and future real exchange rates and hence reduce entrepreneurs’ wealth to a dangerous low level.

Any temporary equilibrium along the line OB is submitted to the possibility that a loss of lenders’ confidence will be validated by financial collapse if these lenders change radically their behaviour. In normal time, domestic entrepreneurs could finance their investment in using at maximum the leveraged financing. But in bad time, panicking international lenders could reduce arbitrarily the leverage multiplier to a level that they consider as safe. They would examine attentively if the exogenous shocks are sufficiently important so that the
economy is going to collapse with investment and entrepreneurs’ wealth falling to zero. This scenario is possible only if investment and foreign debt are both jumping variables. If the investment is only amortised partially in every period and the foreign debt is long term, the immediate collapse will be less severe and these two variables will be on a path corresponding to a rampant crisis, less violent than the one with sudden withdrawal of foreign lending.

Even though the basic part of this model is the same as Krugman, the story talked here is somewhat different from his one. In fact, Krugman worked with three equilibria that are made possible by using a static analysis. A financial crisis corresponds then to a jump from the equilibrium with high investment to the one with zero investment with all entrepreneurs bankrupt, given that the intermediate equilibrium is unstable. By analysing the dynamics of the model, we have shown that there are only two steady state equilibria. A financial crisis is then a dynamic phenomenon of an emerging market economy that uses extensively foreign debt (foreign currency denominated or not) to finance its development. It is a jump from one dynamically unstable trajectory of wealth-accumulation to another one with less investment and lower foreign debt when international lenders become pessimistic. When severe financial crisis takes place, not all entrepreneurs are systematically bankrupt. The collapse (or shift from a dynamic trajectory of high debt to another one with low or no debt) does not imply that the previous investments were unsound. The problem is instead one of financial fragility in a dynamic context.

5. Factors at the origin of financial fragility and crisis

It is largely documented that Asian countries, which have been drawn into 1997 financial crisis, had a balance of payments characterized by large current account deficits compensated by net inflow of foreign capital. In the dynamic setting, this could correspond to a situation
where ambitious entrepreneurs expand their investment using at maximum the leveraged financing along or near the line OB. That implies an increasing inflow of foreign capital. In the following, we consider the dynamic framework how different factors could create financial fragility and cause a financial crisis.

5.1. Factors at the origin of financial fragility

The financial fragility in this model has nothing to do with the mismatch between short-term debt and long-term investments; nor does it appear to depend on foreign exchange reserves. Krugman has highlighted the difference between his story of financial fragility and that told by others (e.g. Chang and Velasco, 2001) by considering the conditions under which this fragility can occur — namely, when financeable investment responds more than one to actual level of investment. That leads him to consider the following factors as being able to cause financial collapse: (i) High leverage; (ii) Low marginal propensity to import; (iii) Large foreign-currency debt relative to exports.

In the dynamic setting, we re-examine below if they make the domestic economy more fragile and more likely to fall into financial crisis in the event of adverse exogenous shocks or unfavourable change in international lenders’ opinion.

![Diagram](image-url)

Fig. 4: High leverage and steady state equilibrium.
**High leverage**

An increase in the leverage, i.e. a higher $\lambda$, will induce the curve $D_{inv}$ to rotate clockwise without modifying the position of the curve $D_{lee}$. Whatever is the value of $\lambda$, $D_{inv}$ is always at the left of $D_{lee}$, $\forall I < \hat{I}$. A temporary equilibrium like B’ in Fig. 4 is less susceptible of keeping the confidence of international lenders than a point like B. In effect, if shocks come to induce $D_{lee}$ to rotate counter-clockwise, it is more probable to find the point B’ at the right of the new curve (not drawn in Fig. 4) representing the external equilibrium condition. That corresponds to a situation where the external debt is not sustainable.

**Low marginal propensity to import**

A decrease in marginal propensity to import will induce the curves $D_{inv}$ and $D_{lee}$ to rotate clockwise with the later being more sensible. Since a low marginal propensity to import leads domestic entrepreneurs to contract higher foreign debt, it will increase the fragility of the domestic economy and reduce the latter’s capability of buffering against adverse shocks.

**Large foreign-currency debt relative to exports**

The ratio of foreign-currency debt relative to exports is a complex indicator which is not clearly discussed by Krugman. A large ratio of foreign-currency debt relative to exports can be due to multiple factors. Using equations (12), (19) and (22), the ratio can be written as:

$$\frac{pF}{X} = \frac{pvD}{X} = \frac{\alpha v(1+\lambda)\hat{T}^\alpha X^{\alpha\mu}[1-(1-\alpha)(1-\mu)]-(1-\mu)\hat{T}^\mu}{(1+\lambda)\hat{X}\{X+v\hat{T}^\alpha[1-(1-\alpha)(1-\mu)]-(1-\mu)\hat{T}^\mu\}-(1-\mu)\hat{T}^\mu(1+\tilde{r}^\mu)}.$$

(29)
An increase in $v$, $\mu$, $\lambda$ as well as a decrease in $r^*$ and $X$ could lead domestic firms to contract a larger foreign currency debt, implying an increase in the ratio $\frac{p^F}{X}$ while an increase in $\alpha$ has ambiguous effect. A larger ratio $\frac{p^F}{X}$ will make these firms more vulnerable to financial crisis. The ratio $\frac{p^F}{X}$ depends also on the investment and hence the stage of economic development. It is to note that some parameters or exogenous variables, such as $\lambda$, $r^*$ and $X$, can vary adversely and brutally, leading to financial crisis as discussed in the following.

5.2. Factors at the origin of a financial crisis

The factors discussed above explain the fragility of emerging market economy using foreign currency debt to finance its development. They matter because they make the circular loop from past investment to real exchange rate to balance sheets to current investment more powerful. But they don’t explain why Asian financial crisis takes place. All afflicted Asian economies were peculiarly vulnerable to financial crisis due to high leverage and unusually high levels of debt denominated in foreign currency. These borrowings have placed them before an increased risk of financial collapse if the real exchange rate depreciated. We show that the factors that were present in Asian crisis can also generate a financial crisis under floating (or quasi floating) exchange rate regime as is observed in Iceland and Central and Eastern European countries.9 Generally, the factors leading to financial fragility can become factors at origins of financial crisis if they come to change adversely. Consider here some others not considered in the above discussion.

Foreign monetary policy

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9 Some Central and Eastern countries are in the Exchange Rate Mechanism II, preparing their entry into the European Economic and Monetary Union, which allows their exchange rate between their national currency and the euro to fluctuate with a standard fluctuation band of 15% above and below the central exchange rate.
One important factor which is present previous to Asian financial crisis is the high 3-month Libor interest rate (Kwack, 2000). In effect, previous to the crisis, the Fed has adopted a restrictive stance for its monetary policy, inducing hence higher interest on international financial markets. This shock can affect considerably the temporary equilibrium based on leveraged financing. According to equations (22) and (23), an increase in $r^*$ shifts the line $\hat{I}$ to $\hat{I}'$, and induces the curves $\hat{D}_{\text{inv}}$ and $\hat{D}_{\text{we}}$ to rotate counter-clockwise (Fig. 5).

An increase in foreign interest rate reduces the foreign funds that can borrow domestic entrepreneurs. Furthermore, they have to pay high interest rate, reducing thus their wealth from $W_0$ to $W'_0$. If the net wealth is sufficiently high at the initial temporary equilibrium, the following adjustment can place the economy on a trajectory such as the curve OB’. On the contrary, the foreign debt can either jump to a low level if it is short term or follow a trajectory characterized by long-lasting financial crisis if it is long term.

Lender’s attitudes towards domestic entrepreneurs
In the present model, the leverage of investment over entrepreneurs’ wealth is considered as fixed. When a financial crisis hits other emerging market economies, lenders may change their mind and hence reduce the leverage multiplier. In this case, the curve $D|_{inv}$ rotates counter-clockwise, while the curves $D|_{lee}$ and iso-wealth stays unaffected.

A reduction in the leverage multiplier may translate an increased risk aversion or loss of confidence of international lenders on the future perspective of the domestic economy or/and world economy. It is not sufficient to lead to the occurrence of a financial crisis except when their pessimistic expectations are realised. If entrepreneurs’ wealth is not influenced, investment and foreign debt will adjust orderly towards lower levels.

**Contagious financial crisis**

A currency devaluation of a neighbour country that is competitor of the domestic county on the international goods market reduces the latter’s exportations.\(^\text{10}\) That induces the curves $D|_{inv}$ and $D|_{lee}$ to rotate counter-clockwise and could make the initial temporary equilibrium with high investment and high foreign debt unsustainable judging by the leverage constraint and/or external equilibrium condition. If the leverage constraint is not respected meanwhile external solvency constraint is respected, the panic of lenders might not be very severe since the reduction of liquidity in the domestic economy will not bankrupt all entrepreneurs. In effect, a reduction of exportations will impact negatively the entrepreneurs’ wealth since it implies a depreciation of the real exchange rate and hence increases the value of foreign currency debt measured in domestic currency. A surprise decrease in exportations would make some domestic entrepreneurs insolvent and could lead to a financial crisis. The severity

\(^{10}\) We neglect the effect of neighbour country’s currency devaluation on the domestic price level. This can be justified by assuming that the domestic country produces similar goods as its neighbour but does not import goods from its neighbour for consumption or investment.
of the crisis depends on initial wealth, leverage multiplier as well as amplitude of the fall in exportations.

However, if the external solvency constraint is also violated, a deep crisis could easily materialize since the high level of debt would be backed by an insufficient level of wealth or even negative wealth. Thus, the liquidation of all firms might not leave enough financial resources to pay all foreign debt and interests.

6. Policy analysis

The dynamic setting can be easily used to analyze three policy issues considered by Krugman (1999). Since the framework is formulated in non-monetary terms, it is more adapted for analysing the situation of an emerging market economy under floating exchange rate regime. This is to some extent the case in Central and Eastern European countries and Iceland. Therefore, we do not consider explicitly the exchange rate peg in the following. Nevertheless, an exchange rate peg can be mimicked by a real exchange rate peg.

Preventive measures against financial crisis

Krugman has argued that the widespread imposition of Chilean-type restrictions on short-term borrowing denominated in foreign currencies, which reduces short-term foreign-currency exposure, could not allow emerging market countries to reduce significantly the risks of being forced into crisis by a loss of confidence. In addition, as long as a country has free convertibility of capital, short-term foreign loans are only one of many different possible sources of capital flight. Our dynamic analysis gives a more nuanced analysis by allowing the case where only a small part of foreign loans is short term, foreign debt will not be a totally free jumping variable and will not fall to zero whenever there is a financial crisis.
Consider a contagious financial crisis in a competing country, which decreases the expected exportations of the domestic country in the future. The curves $\overline{D}_{\text{inv}}$ and $\overline{D}_{\text{le}}$ rotate counter-clockwise (the curves representing $\overline{D}_{\text{le}}$ and $\overline{D}'_{\text{le}}$ are not given in Fig. 6 for simplicity). Such event reduces the sustainable level of foreign loans and makes the iso-wealth curve rotate counter-clockwise around the point C, a temporary equilibrium point attained before the financial crisis but not anymore sustainable. If short-term foreign debt is sufficiently important, the economy will jump along the new iso-wealth curve to the point C’, the crossing point between the Iso-wealth bis and the curve $\overline{D}'_{\text{inv}}$, or to another one down the curve Iso-wealth bis and at the left of the curve $\overline{D}'_{\text{inv}}$. Then, the economy follows a trajectory along $\overline{D}'_{\text{inv}}$ or another trajectory with less leverage leading again to higher investment and higher foreign debt in the future when the dust of financial panic has settled.

A high level of long-term foreign debt implies that the economy is on a trajectory of less violent but long-lasting financial crisis if a temporary equilibrium happens to be at the right of the curve $\overline{D}'_{\text{inv}}$. The fact that the holders of short-term foreign debt will refuse to roll it over could generate an exchange rate depreciation that bankrupts some entrepreneurs until the

![Fig. 6: The effect of contagious crisis with mixture of short and long term debt.](image-url)
foreign debt is sufficiently low (i.e. at left of the curve $\overline{D}_{\text{inv}}^i$). However, higher rate of return of capital will allow entrepreneurs who have survived the crisis to regain the confidence of international lenders.

The nuance that we introduce in the analysis of the impact of the financial crisis does not put into question Krugman’s proposition about the appropriate prophylactic policy; i.e., to avoid any transmission of international crisis to the domestic economy, it is necessary to discourage firms from taking on foreign-currency-denominated debt of any maturity. In effect, the real-exchange-rate impact of adverse shocks could negatively affect domestic investment in the presence of such borrowing. The negative effects are magnified to cause economic distress when such borrowing is high and when capital-market imperfections are important. In the absence of foreign currency debt (i.e. $v = 0$), a shock like a decrease of exportations will not modify $\overline{D}_{\text{inv}}^i$ as well as $\overline{D}_{\text{ex}}^e$ according to equations (27) and (28), but can impact negatively the wealth of the current period according to equation (8). If the shock is small, the current level of entrepreneurs’ wealth can stay above that of last period allowing international lenders to maintain the same level of lending for the following periods. On the contrary, these lenders will reduce the lending to a level compatible with lower level of wealth. The adjustment could then be realised downside along the curve $\overline{D}_{\text{inv}}^i$. However, extreme shocks, such as a sudden decrease of exportations inducing an ample depreciation of the real exchange rate and a brutal decrease in output,\footnote{The current stock of capital depends negatively on current real exchange rate.} and hence a severe reduction of entrepreneurs’ wealth, can lead to a financial panic.

In this framework, we can consider another measure to avoid the contagion by international financial crises, i.e., the government limits the leveraged financing used by domestic entrepreneurs. Limiting the use of leveraged financing to a level (on a path at the left
of OB in Fig. 6) less than the leverage multiplier allowed initially by international lenders will
decrease the probability that these entrepreneurs become insolvent in the event of adverse
shocks.

**Policy during the financial crisis**

In the period before 1997, Asian countries have grown rapidly and liberalised their capital
account as urged by the FMI while keeping nominal exchange rate peg. The peg of Asian
monies to US dollar is an important characteristic of these countries. To avoid the risks of
financial trauma due to foreign currency debt was a major reason why the IMF advised its
Asian client countries to follow the much-criticized “IMF strategy” which consists to defend
their currencies with high interest rates rather than simply letting they devaluate. Even though
this model does not allow a direct analysis of monetary policy, we can get some insight at the
nature and consequences of the IMF strategy by imagining that the effect of that strategy is to
hold the real exchange rate $p$ constant even when the willingness of international lenders to
finance investment declines. Stabilizing the nominal and real exchange rates, while closing
one channel for potential financial collapse, opens another: according to equation (8),
increasing strongly domestic interest rate hence the interest rate paid to international lenders
implies that the sustainable level of foreign debt is reduced drastically, especially when
leverage is high.\(^{12}\)

Following Krugman, the provision of emergency lines of credit must not be used to avoid
the currency crisis, i.e. defending the un-defendable nominal exchange rate peg, but to prevent
the financial crisis that could result from it due to high level of foreign currency debt. These

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\(^{12}\) In the case of Thailand, given the overvaluation, Merton (1998) considers that one mistake of the government
is not to take a once-and-for-all devaluation. Another mistake is to raise local interest rate since the interest rate
parity theorem tells us that those speculators who have already sold short baht in the forward market will be
enriched, not hurt. On the contrary, it can hurt entrepreneurs and hence precipitate a financial crisis.
credit lines would have to do more than just provide balance-of-payments financing or even provide lender-of-last resort facilities to banks.

Assume as before that the domestic economy is submitted to a contagious crisis and the curves $D_{low}$ and $D_{lee}$ rotate counter-clockwise (the representation of $D_{lee}$ is omitted in Fig. 7 for simplicity). The above measure is justified if the level of investment before the crisis is less than $\hat{I}$. However, due to changing international context, the foreign debt could become too high to respect the leverage constraint but is always compatible with external solvency constraint (point C in Fig. 7).

![Fig. 7: The effect of contagious crisis and the IMF credit lines.](image)

Keeping the temporary equilibrium at the point C, while the anticipated depreciation of domestic currency (and hence that of the real exchange rate) indicates a possible temporary equilibrium at the left of the curve $D_{low}$, can consume much financial resources. To keep the level of investment corresponding to the point C, the IMF’s financial aid must be enough to make up the credit being lost by firms so as to allow investment to continue, given that some foreign lenders will reduce their lending. If the IMF funds are used to defend the nominal exchange rate peg, the aid must be large enough to substitute all private foreign lending desiring quitting the country at the official nominal exchange rate.
If the IMF credit lines are not used to defend the nominal exchange rate peg, a temporary equilibrium like the point C is defendable. In the case where the domestic currency is already depreciated, there is less or no incentive for capital flight, particularly when the IMF financial aid can be used, through a mechanism that funnels the funds to troubled entrepreneurs, to limit the bankruptcy due to lack of liquidity.\textsuperscript{13} If the IMF is willing to help the country in crisis but without excess investment, a sufficiently large credit line would never actually have to be used, since its very existence would prevent the financial crisis from ever getting under way. Standstill agreements on foreign-currency debt negotiated with foreign lenders can greatly increase the efficacy of the IMF aid. Nevertheless, the currency crisis is not easily avoidable if the capital convertibility is always in place. In Fig. 7, the economy can pass from point C to point E when entrepreneurs reduce progressively their debt owed to the IMF.

Another measure to rule out the possibility of a downward financial spiral, which has been used by Malaysia during the 1997 crisis, is to impose a curfew on capital flight. In the Fig. 7, it consists to keep the economy at the point C by keeping unchanged the real exchange rate and by not letting the foreign lenders reducing their lending. It can be justified by the fact that standstill agreements on foreign-currency debt are not sufficient to avoid the twin crises. This measure allows avoiding the exchange rate and financial crises if capital-account convertibility is suspended so that any form of capital flight is not anymore possible. This measure is efficient only when the shock at the origin of these crises is transitory and the domestic investment is not too high compared with the level which is rational to realize when international economic and financial environment becomes normal. It is then even in the self-interest of investors to impose emergency capital controls since not all investors can escape from the twin crises undamaged. However, if the adverse shocks are permanent, capital will

\textsuperscript{13} Generally, it is political difficult to justify the rescue of these entrepreneurs who are considered as culpable on the ground that their excesses brought on the crisis in the first place.
find progressively other ways to quit the country and the curfew on capital flight will loss its efficacy.

*Rebuilding the economy after the financial crisis*

Considering the bad equilibrium with zero investment, Krugman suggested that the main problem was that the entrepreneurs who drove investment and growth before the crisis are now effectively bankrupt and unable to raise capital.

In the dynamic analysis, the situation depends on the initial levels of wealth and foreign currency debt of every individual entrepreneur. If the aggregate wealth of entrepreneurs after the devaluation of the domestic currency is high and little affected by the twin crises, many entrepreneurs can survive and reimburse their foreign debt and continue to develop in a context with less competition and higher profitability since the investment level is lower. Then, this will create a new wave of development soon after the twin crises. In this case, the efforts focused on bank restructuring and recapitalization are sufficient.

Consider a second case where the aggregate wealth after the devaluation is near zero or negative, only some prudent entrepreneurs can survive and reimburse their foreign debt and develop more quickly with high profitability and weak competition since a majority of entrepreneurs have bankrupted. Even if a big wave of development can happen soon after the twin crises, the short-run deep economic recession is however too costly. Consequently, the efforts focused on bank restructuring and recapitalization are not sufficient. Solutions proposed by Krugman, such as rescuing bankrupt entrepreneurs through some kind of “private sector Brady Plan”, and/or growing a new set of entrepreneurs and/or welcoming foreign direct investment, are needed to reduce the economic and social consequences of the twin crises and to allow an accelerating recovery of the domestic economy.
7. Conclusion

This paper analyses the dynamic implications of the Krugman’s (1999) model with balance sheet effects by extending it to explicitly take account of wealth accumulation constraint and external equilibrium condition. The revisit to Krugman’s analysis is interesting since it has a considerable impact on the literature of international financial crisis, it is largely used in the teaching of open-economy macroeconomics across the world, and it is still applicable to actual financial crisis affecting some Central and Eastern European countries and Iceland.

Basically, our analysis complements these of Krugman by extending his analysis to a dynamic framework, which allows better illustrating his comments about different factors at the origin of financial fragility and financial crisis in emerging market economies. The present analysis does support most of the discussions intuitively made by Krugman on the policy implications of his model while suggesting some nuanced analysis of crisis scenarios and policy implications thanks to the dynamic framework.

The static analysis of such a model implies the existence of three equilibria. Thus, a financial crisis corresponds to a jump from the equilibrium with high investment to the one with zero investment with all entrepreneurs bankrupt, given that the intermediate equilibrium is unstable. By analysing the dynamics of the model, this study shows that there are only two steady state equilibria. As a dynamic phenomenon of an emerging market economy that uses extensively foreign currency denominated debt to finance its development, a financial crisis can be analysed as a jump from one trajectory of wealth-accumulation with higher foreign currency debt to another one with less investment and lower foreign currency debt when international lenders become pessimistic.

The dynamic setting provides better illustration of the dynamics inherent to the Krugman’s model. In the initial graphic illustration of the analysis made by Krugman, an
crisis will lead to the bad equilibrium with zero investment and zero foreign debt. By introducing wealth accumulation constraint and external equilibrium condition beside the leverage constraint, it is shown in this paper how external shocks can dynamically impact the domestic economy and in which cases, these shocks can generate a financial crisis. Furthermore, the dynamic analysis allows discriminating the case where an economy could collapse to the equilibrium with zero investment and the cases where it returns to a temporary equilibrium with low investment. Hence, the severity of financial and currency crises and their economic consequences would vary with the level of wealth. Even in the event of severe financial crisis, not all entrepreneurs are systematically bankrupt. Therefore, the equilibrium with zero investment is not dynamically stable in the sense that a very small amount of capital owned by the surviving entrepreneurs could jump-start a wealth-accumulation process after the financial crisis.

References: