Demographics and the Anatomy of International Capital Flows

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The Effect of Ageing

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ABSTRACT

This thesis is built upon two core arguments. The first is the notion that the demographic transition should be narrated through the perspective of ageing rather than population growth and the second is that ageing on a macroeconomic level represents a strong driver of international capital flows. These two arguments are used to discuss the standard prediction in a life cycle framework that ageing leads to dissaving in the aggregate and thus how old economies should tend towards running current account deficits. Using Japan and Germany as the subjects of analysis, this thesis develops the idea that rapidly ageing societies are not, in the main, characterized by dissaving but rather by the fight against it. Finally, a small empirical exercise acts as a perspectivation on the results to suggest why ageing might lead to a reliance on exports and foreign asset income to achieve growth and what this means in a global context.

“I Can’t Understand Why People are Afraid of New Ideas, I am Afraid of the Old Ones”

John Cage

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“The Conventional View Serves to Protect Us from the Painful Job of Thinking”

John Kenneth Galbraith

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1.0 INTRODUCTION

The primary manifestation of the demographic transition in a modern economic context is through ageing and the primary transmission from ageing to the macro economy is through its effect on saving and investment behavior. These two effects taken together suggest a strong impact from the continuing process of ageing on international capital flows. It is in this context that this thesis constructs its main argument. Specifically, it postulates that international capital flows can be seen as the result of interaction between countries that are subject to asymmetric demographic shocks and essentially countries that are at different points in the demographic transition.

The main empirical observation to underpin the investigation is the fact that the world is ageing. Between 1950 and 2010 the developed world saw an increase in its population's median age of 31% (from 29 years to 39.7 years) and this pace is expected to continue. Moreover, ageing is also fast approaching the shores of developing economies and thus the main demographic characteristic of the global economy is rapid ageing. On the macroeconomic level, and as driver of the demographic transition, ageing is a function of two processes in the form of declining fertility and a decline in mortality (increase in life expectancy) and especially, the decade long presence of low fertility means that the entire OECD edifice will be struggling with the effects and consequences of ageing.

The point of departure for this study is the macroeconomic effects of ageing with a focus on open economies where mutual interaction transmits shocks both to the level of individual economies as well as to the global macroeconomic system. This is perhaps the most enduring complicating factor of modern macroeconomics and the analysis presented here is built on the strong notion that whatever perspective we take on macroeconomic inquiries, a strong understanding of demographic processes is central. Within this field a big literature has looked into the connection between demographics and macroeconomics where the main findings support the notion of a strong link between ageing and international capital flows. Especially the idea of intertemporal consumption smoothing[1] following Fischer (1931) represent a key intuition used to think about the connection between demographics and capital flows in relation to the idea of life cycle consumption.

More generally, some of the original contributions that have sought to theorize on the macroeconomic effects from changing demographics were published already in the 1950s and 1960s in the context of

[1] See Appendix 1 for an indicative example of the problems used in the analysis.

Samuelson (1958) and Diamond (1965) on the overlapping generation (OLG) models as well as Modigliani's key contribution in the form of the life cycle hypothesis (LCH) Modigliani and Brumberg (1954) and Ando and Modigliani (1963). In a more recent context, the general intuition derived in the context of intertemporal consumption has been used to formulate the idea of the *intertemporal current account* (Obstfeld and Rogoff (1996)) which has become one of the main work-horse models in open economy macroeconomics. This model and its OLG representation will form the backbone of the theoretical framework in the analysis of capital flows and ageing.

Contemporary studies have provided extensive evidence of the effect on ageing and capital flows. On the idea of asymmetric demographic shocks Bryant (2004, 2006) concretely links capital flows to the presence of trading economies that are moving through the demographic transition in different tempi. In Higgins (1998) empirical evidence is presented to suggest that ageing affect capital flows both in relation to the individual economy (time series) perspective and a global setting (cross-sectional). These results are qualified in Malmberg and Lindh (1999) and Lürhmann (2003) where especially the latter contribution is interesting as it includes effects from future demographic shifts to predict current capital flows as well as it models capital flows in relative terms.

Another type of study uses simulations based on growth theory models. In Henriksen (2002) a two country OLG model is calibrated successfully to the bilateral current account balances of Japan and the US in a historic context. Extending the setting to a multi-country OLG model Feroli (2003) calibrates a model for the G7 economies that shows how demographic differences are good proxies for the drivers of international capital flows and Börsch-Supan et al (2007) sets out to model capital flows in an intra-OECD context and finds equally strong effects from demographically induced capital flows in a global context.

If it is easy to confirm the idea that ageing can act as a strong driver of international capital flows it does not mean that all questions have been answered. One important theoretical and empirical challenge is cast in the context of the later stages of the demographic transition (the phase of rapid ageing) that many OECD economies are now moving towards. Consequently, standard OLG models predict that rapid ageing will lead to dissaving in the aggregate as domestic savings decline further than domestic investment demand. This is not only difficult to verify in an empirical sense but also appears difficult to consolidate theoretically. In the context of the standard macroeconomic framework, we might postulate that the degree of dissaving predicted in this setting is difficult to reconcile with the complex degree of uncertainty facing economic individuals as well as potential bequest motives. More fundamentally however, it is worthwhile to ask whether the extent to which a model with micro foundations shows that some form of dissaving is optimal also means that aggregate dissaving may be optimal from the point of view of an economic entity. This may not be the case and this critical view on dissaving is a key intuitive result for this thesis and essentially guides the choice of Japan and Germany as the main subjects of analysis in the sense that they are the oldest economies in the world measured on median age. It is thus the hypothesis that by studying these two economies in detail, we may learn something about the effect of ageing that can be used to predict what may happen to economies that will succeed them as well as we may learn how ageing will manifest itself in relation to global capital flows.

The thoughts above motivate the main research question for this thesis:

What is the demographic transition and how can it be operationalized in a macroeconomic context to explain international capital flows with specific focus on Japan and Germany.

This problem statement is primarily identified and delimited by the choice of ageing and its effect on international capital flows with the former understood as ageing as a change in an economy's population age structure and the latter as the flow of goods, services and income between open economies. Secondarily the choice of Germany and Japan as subjects of analysis is simply motivated by the fact that they are the oldest economies in the world and thus obvious candidates for investigating the effect of ageing. In order to answer the research question, this thesis mainly employs economics and quantitative analysis as the methodological framework although it is important to emphasize that the analysis and discussion of the demographic transition (DT) will deploy a more interdisciplinary perspective. Most importantly, the central methodological issue for this thesis is to make its main argument easily amendable to the criteria of falsification. The rationale here is essentially a normative one which states that any good scientific argument must first and foremost be judged on its amenability to falsifiable tests.

With regards structure this thesis is divided into five chapters of which sections two and three in particular are important with respect to the answer of the problem formulation. In chapter two, the first part will be answered in the form an analytical piece about the demographic transition, why we must understand it through the perspective of ageing, what ageing is and how ageing is understood in a macroeconomic context. Chapter three will take up the baton from chapter two and answer the second part of the problem statement. This chapter presents the main theoretical model used to explain how demographics drive international capital flows and engages in a concrete analysis on Japan and Germany where the main focus is on the notion of dissaving. Chapter four presents a perspectivation on the analysis and investigates the idea that ageing may lead to a reliance on exports to achieve economic growth and what this might mean in a global context. Chapter five concludes.

2.0 THE DEMOGRAPHIC TRANSITION – TOWARDS UNCHARTED WATERS

In this chapter an ambitious stab is delivered at untangling and connecting the strings which make up the DT. In relation to the overall problem statement, this chapter specifically seeks to answer the following question;

What is the demographic transition, and how can it be operationalized in a macroeconomic context?

In order to provide an answer to this chapter is anchored in two points; one is that the DT is not yet over and the second is that the proper understanding of demographic processes requires a focus on ageing. In this way, three main inquiries are made in order to answer the question above. Firstly, it is elaborated *why* it is important to look at ageing in the context of the demographic transition, secondly it is explored *what* ageing actually is in the form of declining fertility and increasing life expectancy, and finally the chapter finishes with a description of *how* ageing can be understood in a macroeconomic context.

2.1 A TRANSITION TO WHAT AND WHERE – WHY AGEING?

In the traditional account of the DT, changes in mortality and fertility drive a process of four phases, the first of which is really not a phase but more so a state of high mortality and fertility observed in pre-industrial society (pre 1800) Lee (2003). What subsequently sets the transition in motion at the end of the 19th century in Western Europe was a decline in mortality (especially infant mortality) followed later by declining fertility (1890-1920 in Europe) (second and third stage) and was predominantly a result of increased knowledge that helped reduce mortality as a result of contagious and infectious diseases Lee (2003). It is important to note that this simple mechanism did not occur simultaneously in all of today's developed economies, but it remains a defining characteristic of the economic development observed in the 19th and 20th century economic development Lee (2003). Finally, there is last stage which Lee (2003) narrates as a stage of ageing (1950+) even if the baby boom in e.g. the US from 1946 to 1964 represents a an important qualifier to the general discourse of post war ageing. This last phase is given a thorough treatment in the following.

A key question that has attracted lavish attention in the context of the theory of the demographic transition is the extent to which fertility can be seen (or expected) to stabilize at replacement levels, as it was originally envisioned in the classic transition theory. As it turns out, this idea is very difficult to sustain first and foremost in an empirical, but also in a theoretical, context Demeney (1998)[2], Bongaarts (2001) and Caldwell (2003)[3]. Still, old habits linger and following the projections by the United Nations (see data CD) the transition is meant to be finished in 2100 for the entire world. By this time, the world is thus projected to have a fairly stable growth rate in its population due, primarily, to a process of convergence in global fertility levels around replacement levels. In this way, the idea of homeostasis and thus a kind of steady state equilibrium of demographic variables still prevail in official demographic forecasts.

Unfortunately, such projections are ripe with uncertainty and for all intent and purposes quite useless in the context of scientific research Bongaarts (2001). This point needs strong emphasis, since the idea that you can map the transition from here to 2100 is not only problematic it is a categorical fallacy. The point

[2] In Jones et al. (1998)

[3] In Jones et al. (1998)

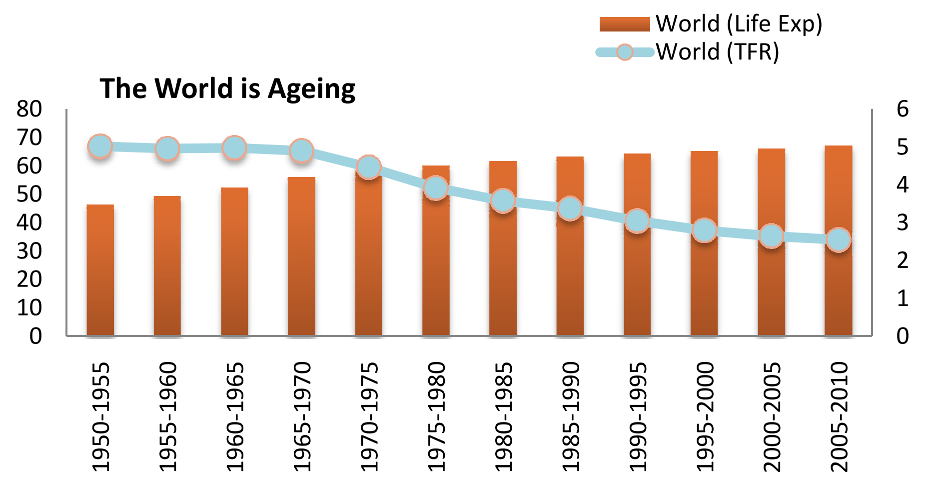
here is that while this may certainly be a potential outcome, it assumes a degree of determinism which is unsatisfactory from the point of view of the dynamism of demographic processes as they evolve.

Rather, it would be interesting to construct contemporary theories to explain what exactly has happened in a post war and essentially post baby boom perspective. This brings us into the idea of a second demographic transition (SDT) Van de Kaa (2002) and the empirical fact that the demographic transition did not end in the 1960s and 1970s with the onset of replacement fertility in a large number of developed and some developing economies. Basically, the SDT is tied to the fact that fertility has continued to decline beyond levels which were originally envisioned by the idea of homoeostasis in the traditional transition. Specifically, and according to Van de Kaa (2002) the difference between the first and second transition is that while the first operated on the basis on a long term decline in mortality the second operates along the lines of fertility and the fact that couples and women now exert full control over their fertility decision Van de Kaa (2002, p. 2). The most interesting aspect of the SDT is thus the unprecedented process of ageing that began in the latter part of the 20th century.

2.1.1 1950-2010 – AGEING BECOMES PERVASIVE

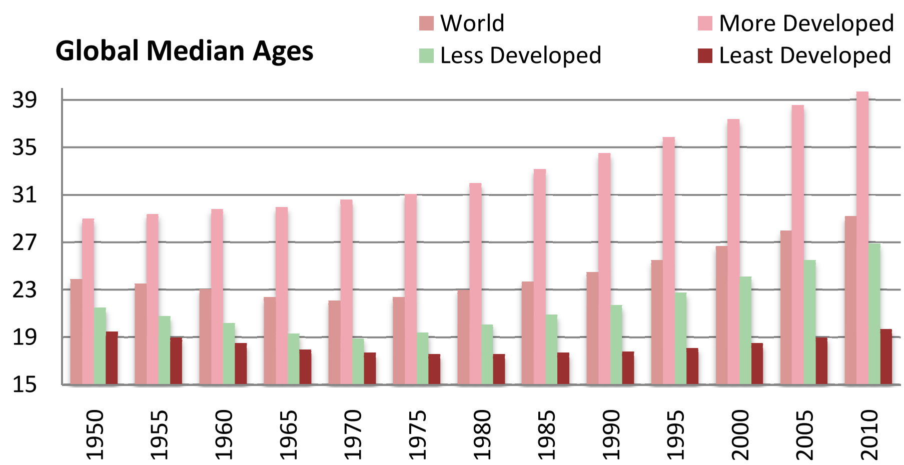
A quick glance of the two figures below (see appendix 2 for more graphs and details on methodology) shows more than anything else why ageing is important to understand demographic change Van de Kaa (2002, p. 11).

FIGURE 2.1



Source: UN, 2010 is an estimate

FIGURE 2.2



Source: UN, 2010 is an estimate

Between 1950 and today the world has experienced a process of unprecedented change in its population structure and more importantly the fundamentals upon which the future population dynamics will occur. In an immediate post-war context world TFR stood at a little over 5 children per women; in 2010 UN estimates this number to be 2.5. In the context of mortality; life expectancy has risen from under 50 to about 68 years on average for both sexes in the same period. Not surprisingly, these trends in mortality and fertility have led to a marked process of ageing proxied by a steady increase in global median ages.

A central argument of this study is that there are considerable advantages to be gained from looking at the DT as a transition in ageing relative to one in population size and growth. This generalization however is too simplistic and looking at the median age on the previous page, it is obvious that the global perspective does not tell the whole story.

In this way, one important observation is that although the entire world is ageing, it is doing so from drastically different initial positions across regions and it is useful to divide this heterogeneity into two. One kind of heterogeneity is observed across regions and countries which are at different stages of their DT. This manifests itself in the fact that the decline in fertility and mortality has moved farther in the developed world than in the developing world. The former has, in some sense, moved further in the DT than the latter. The second kind of heterogeneity is seen through the fact that developing economies are moving through the DT *much faster* than their developed counterparts. Taken together, these two aspects of heterogeneity produce a process of *convergence* whereby the developing world is catching up rapidly with the developed world (OECD) in terms of demographic structure. This process of convergence is clearly present in the data for anything but the poorest countries still stuck in a kind of Malthusian trap[4]; this point is illustrated from the scrutiny of the graphs and data in appendix 2. Take for example the following table which is constructed to show the process of catch-up from the point of view of the developing world with respect to the developed world.

FIGURE 2.3

Ratio of developing world to developed world parameters

Total Fertility Rate	**1950-1955**	**2005-2010**
Less Developed	2,165	1,145
Least Developed	2,349	1,397

Infant Mortality	**1950-1955**	**2005-2010**
Less Developed	2,961	7,620
Least Developed	3,338	12,380

Life Expectancy	**1950-1955**	**2005-2010**
Less Developed	0,617	0,855
Least Developed	0,548	0,714

The numbers show the ratio of the developing world parameters to the developed world parameter and for example the number in the upper right corner (2.165) indicates that in the period 1950-1955 the less developed world had a TFR which was 2.165 times higher than in the developed world.

The numbers are suggestive in terms of demonstrating the process of catch-up; especially in relation to fertility where the global dispersion of fertility has decreased markedly since the 1950s. In the period 1950-1955, fertility in the less and least developed world was consequently more than twice as

[4] In which all output is consumed and none is left for capital accumulation and thus, strictly speaking, growth.

big compared to the developed world. Today, the distance has decreased markedly and strikingly given the fact that almost *all* countries, during this period, have undergone a fertility transition towards lower fertility. In the context of mortality the picture is mixed even though the trend in life expectancy at birth is the same with the less developed[5] and least developed world respectively now being able to muster life expectancies at 86% and 71% of the more developed worlds. The picture on infant mortality is misleading in that it reflects the fact that infant mortality has largely been eradicated in an OECD context. Thus, infant mortality has equally declined in great quantity for the least and less developed world alike. In general, there is of course a zero-bound effect here which tends to blur the conclusion somewhat as fertility and infant mortality cannot, by definition, fall below 0.[6]

This focus on ageing also means that we are able to take a considerable step forward in terms of looking into the future as demonstrated by Lutz et al. (2008) who use probabilistic population forecasting techniques to map ageing into the 21st century. Obviously, such empirical methods represent no miracle tool and the main sources of uncertainty remain. This is especially evident in the context of assigning probabilities to the outcome of fertility, mortality, and migration patterns where the challenges are massive. However, at the current juncture Lutz et al. (2008) is the best piece of research we have on this topic and it is worthwhile to emphasize two points from the study. Firstly, the analysis suggest that the convergence suggested above is set to continue which means that as OECD economies will continue to evolve demographically (i.e. age) emerging economies will, in all likelihood, catch up. Secondly Lutz et al. (2008) also indicate that the proportion of people aged 60+ is set to increase substantially. The numbers here are really extraordinary even if marked by uncertainty; Lutz et al (2008) estimates that, by 2040, the probability of the entire of Europe, Japan/Oceania as well as China having more than a third of its population aged 60+ is over 80%. This, more than anything merits a shift in focus from population growth to ageing.

If the analysis above suggests that ageing represents the main trend of modern demographic processes Malmberg and Sommestad (2000) takes the argument one step further and develop an argument that casts the entire DT in the context of an economy's age structure. In essence, Malmberg

[5] Which comprises all regions of Africa, Asia (excluding Japan), Latin America and the Caribbean plus Melanesia, Micronesia and Polynesia; conversely, the least developed region is a collection of 50 countries specifically defined by the UN. See appendix 1.

[6] Some of the convergence will be a purely mathematical phenomenon.

and Sommestad (2000) develop a framework of the demographic transition that takes its point of departure in the concept of an age transition rather than a transition in population growth. Using Sweden as an example the age transition is argued to consist of four phases; a child phase, a young adult phase, a phase of population maturity and finally a phase of ageing Malmberg and Sommestad (2000, p. 3). It is important to realize that Malmberg and Sommestad (2000) essentially create a new *typology* of the demographic transition and crucially so, one that has the ability to contain both the traditional DT and the SDT. Malmberg and Sommestad (2000) argues that population dynamism stands at the heart of historical macroeconomic analysis, and that one need to introduce ageing and age structure as the main analytical category Malmberg and Sommestad (2000, p. 23).

This final point is the main *raison d'être* for the analyses that follow. It shows that thinking about the DT as a finite process is not reasonable. If instead we use ageing and age structure as the primary analytical vessel through which to analyze population dynamics we will not only be able to better explain the DT; but also, as it will be argued below, it will serve as a entry point into operationalizing demographic dynamics in a macroeconomic context.

2.2 FERTILITY AND LIFE EXPECTANCY – WHAT IS AGEING?

Ageing is driven by two processes in the form of declining fertility and increasing life expectancy and this section attempts to give an analytical account of these two processes, what lies behind them and how economists look at them.

Firstly, and as interesting as life expectancy and longevity is, it will not be subject to a major analysis here. There are two reasons for this. Firstly, a proper analysis of life expectancy would take us far away into biology and medicine sciences which are thoroughly outside the scope of this thesis. Secondly, we are in the fortunate situation that life expectancy, without loss of analytical complexity, can be black boxed as a monotonically increasing function of increasing living standards and economic development. In this way, the contribution of life expectancy to ageing can be seen, in the context of the OECD, as a steady and incrementally increasing factor.

This rather brutal simplification and essentially elimination of life expectancy as a dynamic variable underpins the way ageing is understood in this present inquiry. Consequently, ageing is not cast in the grand scale of evolutionary theory but rather in the form of what concretely provokes the observed process of ageing amongst global economies and how economic theory explains this. It is in this context that life expectancy can be simplified to the extent that is the case here.

Having dispensed swiftly with life expectancy the attention now turns to an analysis of fertility. Contrary to life expectancy the impact from changing fertility on the age structure of society lends itself more readily to the application of economic theory. In the following, the decline in fertility is discussed as the product of two interconnected factors in the form of the quantum effect (Becker and Lewis (1973) and Becker (1977)) and the tempo effect of fertility decline Sobotka (2004)[7].

2.2.1 THE QUANTUM EFFECT – A TRADE-OFF BETWEEN QUALITY AND QUANTITY

In essence, the quantum effect of fertility describes the tendency of women to have fewer children over their entire fertility career. Consequently, the quantum effect says something about the tendency of completed cohort fertility rates[8] to decline over time and what drives this decline. In the context of economic theory, Becker's quantity-quality model of fertility is the main model used to explain families' fertility decision although it is important to note that anthropological theories have been used to derive similar results.

Becker's fundamental insight is both intuitive and strong; not only do families choose how many children to have but also, crucially, how much to spend on them Becker (1977). The interaction between these two decisions are what concretely drives the model and what is now termed the quantity-quality trade off through which a decline in fertility can be explained as the decision, by parts of parents, to substitute quantity of children for their quality. The framework is motivated by the fact that while in Malthus's age fertility could be explained by the age at marriage and the frequency of intercourse during companionship, the advent of contraceptive technology and the change in societal settings gave couples significantly more control over fertility decisions Becker (1977). From the perspective of an economist there thus seemed to be a need to *endogenise* the family's choice of fertility.

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[7] Thomas Sobotka's PHD thesis is really the most comprehensive account here alongside Wolfgang Lutz' work at the Vienna Institute of Demography. More specific references shall be used when deemed relevant.

[8] As oppose to the total fertility rate which is a snapshot of fertility in a given period.

The crucial assumption made by Becker is to let children become consumption goods which provide utility Becker (1977). Following this logic one might infer that an increase in income should lead to an increase in the amount spent on children and thus, as Malthus predicted a large increase in family size. However, this is not necessarily so (Becker 1977). Thus enters Becker most important assumption, namely that the income elasticity of quality is higher than the income elasticity on quantity Becker (1977).[9] In this framework, an increase in income will thus lead to a much larger increase in quality (i.e. investment in a given offspring) than quantity.

Analytically, the quantity-quality tradeoff can be expressed as follows (using Becker and Lewis (1973)), see appendix 3 for derivations:

Assume the following utility function maximized by a representative agent (the consumer (parents))

EQ. 2.1

$$U = (n, q, c)$$

Subject to a simple budget constraint:

EQ. 2.2

$$I = nq\pi_{nq} + c\pi_{c}$$

Where (n) is the number of children, (q) is the quality of children, (c) is consumption of all other goods. Π_{nq} is the price of (nq) where as Π_c is the price of consumption and "I" is a measure of income. Through Becker and Lewis (1973) we can define the following expressions for the marginal utility of (q) and (n) respectively.

[9] Becker even hypothesized that the income elasticity with respect to quantity might be negative which does seem to be resemble the reality in some cases although, by far, not in all contexts; see Becker and Tomes (1976).

EQ. 2.3

$$U_n = \lambda p_n$$

$$U_q = \lambda p_q$$

With $p_n = \pi q$ and $p_q = \pi n$. Becker and Lewis (1973) call these shadow prices and as is readily clear the shadow price of *n* depends positively on *q* whereas the shadow price of *q* depends positively on *n* Becker and Lewis (1973, p 82). The way to understand this is that an increase in quality will be a positive function of the number of children and conversely that an increase in *n* is a positive function of *q*. The economic intuition here is exactly one of a quantity-quality tradeoff as it will be more expensive to increase *n* if *q* is high and conversely more expensive to raise *q* if *n* is high. Thus, if parents wish to increase the amount invested in each child, *n* will tend to be lower than if this preference was not there Becker and Lewis (1973, p 82). As noted above, Becker went a step further and examined the effect of price and income elasticities to illuminate the trade-off. In this respect, it is fruitful to look at two instances. [10] Consider first the case where the income elasticity of *n* is 0 which means that a rise in income will have no effect on the number of children in equilibrium. The first obvious effect from an increase in income is to raise *c* and *q* while keeping *n* unchanged. However, there is another effect too. Thus, recall that the shadow price of quantity ($p_n = \pi q$) is a positive function of *q* and will consequently go up while the shadow prices of quality and consumption will remain the same. This means, quite naturally, that q and c will be substituted for n which will decline Becker and Lewis (1973, p. 84). The second example in the context of price effects (i.e. price elasticities) comes from letting the price of *q* fall due e.g. to an increase in the level of education of the parents. Following the idea above, a decrease in the price of quality will naturally increase *q*, but through the definition of the shadow price of quantity (which depends positively on *q*) also induce a decrease in quantity Becker and Lewis (1973, p. 85).

[10] See appendix 3 for mathematical elaborations on this.

With the model described above,[11] it would be interesting have a brief look at whether it can be verified in reality where it is important to note the two distinct strategies that can be taken. One question would consequently be to investigate whether and how much the quantum effect explains the overall path of declining and lingering low fertility observed across the entire demographic transition. Specifically, and since a steady increase in growth and income goes hand in hand with the DT it would be interesting to test Becker's hypotheses directly. This would then be a perspective *in time* to investigate the extent to which the demographic transition has been driven by a substitution between quantity and quality of children. The second perspective would be independent of time and rather take the form of cross-sectional analysis to investigate whether, in the context of the present, there seems to be a measurable trade-off. Here, we will focus on the latter approach.

In a much cited study from the beginning of the 1990s Erik A. Hanushek finds a large influence of family size on the academic performance of children using data from 1971 to 1975 and thus evidence of the quantum effect. In a more recent study however, Black et al. (2004) qualifies the basic proposition of the quantity/quality trade-off. Consequently, and while the authors do indeed find a negative correlation between family size and the level of the children's education, this is largely found to be explained by *intra-family* differences. Specifically, the study finds that when controlling for birth order the effect of family size on children's education become significantly smaller Black et al (2004). Conversely, the effect of being born later than your siblings of children significantly decreases your educational attainment which suggests that differences within families are more important than family size per se[12].

In summary, the idea of a quantity-quality trade-off seeks to articulate and explain the steady decline in fertility as a function of parents' choice to substitute quantity for quality in the decision to have children. While it is certainly difficult to map the drivers of the quantum effect across the entire decade-long period of the demographic transition (i.e. in a time series perspective) and while the strength and nature of the quantum effect is contested in a cross-sectional perspective, it remains a key theoretical contribution in terms of thinking about the drivers of fertility.

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[11] It should be noted that considerable derivatives exists of this model; for example Lundholm and Ohlsson (2002) argues that the Becker Lewis result only holds if you add a binding child care constraint and thus a framework in which parents will have to purchase, at least, part of the care for their child.

[12] Apart of course from the fact that being born as number 2, 3, … n steadily increases the probability that you will be into a large family. I.e. there is a convergence in perspective here.

2.2.2 THE TEMPO EFFECT OF FERTILITY – POSTPONEMENT OF PARENTHOOD

If the quantum effect potentially says something about the big sweep of fertility decline throughout the DT, the tempo effect represents a much narrower concept in the context of the decline in fertility observed since the 1970s. It thus relates to the observation that a large part of the decline in fertility below and beyond replacement levels is driven by the postponement of first child birth and this trend in fertility creates a set of unique problems both analytically and conceptually in the context of charting the process of ageing. Consequently, it is very easy to imagine how a quantum effect can operate independently of a tempo effect with the latter correcting itself over time. This point is an important one since it highlights the difference between total fertility rates (TFR) and cohort fertility rates (CFR) Sobtoka (2004) where the former is a snapshot of fertility in a given period and the latter a measure of completed fertility for a given cohort. Ideally, we would like to observe cohort fertility rates but it is also quite logical that we can only do this with great difficulty since these figures are only available once fertility has been completed and thus long after one would have had an interest in fertility trends of a given cohort.

If the distinction between TFR and CFR represents a conceptual problem, a far greater issue is of analytical nature. Consequently, if the narration of birth postponement is one of a temporary effect, at what level will fertility rates stabilize and when? More specifically, if we assume that the entire drop in fertility below replacement levels across a wide batch of economies is due to the tempo effect does this mean that fertility levels will see a *full* recovery? This seems dubious and thus opens the door for a much more detailed analysis of the lagged effect from the tempo effect on the quantum effect and the question of couples' ability to manage their fertility decisions over time both in relation to how long the woman can wait (in a biological sense) before having her first child as well as in terms of whether families actually arrive at completing their planned fertility. This thesis will not go into all of these questions, but it is important to keep these issues in mind when progressing further into the analysis of birth postponement.

The tempo effect is essentially an empirical phenomenon observed through birth postponement e.g. in the form of age specific fertility rates for women. Unfortunately, the UN population data base only produces data on age specific birth rates from 1995 and onwards and as such we need to turn to country specific databases in order to visualize and essentially separate the tempo effect from the quantum effect. To that end, appendix 4 presents a number of graphs that attempt to capture the tempo effect in the context of the global economy. The direct comparability of the data is not perfect since we are working with many different national sources but the data tells an unequivocal story. Scrutinizing the graphs for the US, the tempo effect is visualized through the increase in childless women aged 30-34 years old and 35-39 since 1976. Apart from the increase in the percentages that seem to have peaked at around 2000 it is interesting to observe the different levels of the two data columns where the percentage of childless women is much higher for women aged 30-34 years old than for their counterparts aged 35-39. This is the tempo effect in action as it shows how many women enter their thirties without children, a number however which decreases rapidly as they age through their thirties. This conclusion is supported by the fact that the number of live births among women aged more than 30 years has increased steadily since 1976 which suggests a postponement effect (assuming a constant overall fertility rate which is possible in the context of the US).

Turning to Japan, the picture is initially similar with live births sorted by age group showing a strong tempo effect. Between 1970 and 2004 the number of annual live births by woman aged 20-30 fell by an astounding 65% whereas the number of live births by women aged 30+ increased 30%. There is however a very important difference between Japan and the US and it is the quantum effect. In the US the number of total births has actually increased since the 1970s whereas in Japan it has dropped sharply. This is indicative of very strong quantum effect of Japanese fertility alongside the tempo effect; an effect which is largely absent in the US.

Sobotka (2007) presents a multitude of data (see appendix 4) which helps pinpoint the tempo effect in Europe. A good starting point is then to realize that there is considerable heterogeneity in relation to fertility in European countries. The key here is that while all European countries have seen their fertility decline since the 1970s there is a considerable difference with respect to whether fertility has declined below replacement levels (and stayed there). Among the countries with lowest fertility we find Southern Europe, Germany and Eastern Europe whereas the Scandinavian countries (save Sweden), the UK, and France are situated higher levels (although still somewhat below replacement levels). This suggests a significant degree of divergence in the context of how the tempo effect manifests itself in Europe.

If the account above shows the tempo effect associated with a decline of fertility rates below replacement levels, what about in a theoretical light? In this context, the discussion of the postponement effect of fertility decline is closely related to the idea of a second demographic transition Van de Kaa (2002). Essentially, and as the idea of the second demographic transition is specifically linked to the emergence of a new demographic regime where fertility rates drop persistently below replacement levels, a theory of the tempo effect should naturally attempt to explain this decline in a modern context. According to Sobotka (2004) the theoretical explanation for the emergence of fertility postponement is to be found within the realms of socioeconomic theories with a particular emphasis on life course theory. In essence, there are five aspects of interest to Sobotka (2004): Women's entry into the labour force and their subsequent educational attainment, the conflict between employment and parenthood, influence of different forms of uncertainty, changing nature of partnership relations, and finally contraceptive technology (the pill) which gives couples increased control over their fertility Sobotka (2004, p. 9).

It is essentially difficult to rank the importance of these five explanations since they are, by nature, interdependent but it is also clear that particularly one seems to be rise above the others. Consequently, the rise in educational attainment as a function of the importance of the *quality* of human capital seems to be a very important factor explaining the postponement of first births. This effect is especially pronounced in the context of women who have undergone a substantial process of social change in relation to labor market participation and level of education in a post war perspective. For Sobotka (2004) this is one of the principal reasons for birth postponement as he points out that:

In present-day Western societies, the period spent in education is universally perceived as incompatible with family formation. Sobotka (2004) p. 12

Sobotka (2004) presents numerous references to substantiate this point and essentially shows how being enrolled into higher education greatly affects the timing of the first child Sobotka (2004, p. 12). In terms of empirical evidence the list of contributions is vast, but Sobotka (2004) notes two in particular. There is Beets et al. (2001) who shows that an increase in the level of education can explain as much as half the increase in mean age at birth in the context of Dutch women in the period 1931-40 and 1961-65 Sobotka (2004, p. 13); and, there is Meron and Widmer (2003) who show, in a French context, how engaging in post-secondary education may increase the age at first birth with as much as 6 years Sobotka (2004, p. 13).

In general, Sobotka presents a way to think about fertility postponement as he concludes, with great conviction, that looking at the way people organize their lives and make decisions on key events (such as marriage, child birth, education etc) in the form of a life course perspective is chief if you want to track the determinants of fertility postponement.

In summary, the tempo effect constitutes the effect on fertility from birth postponement. Unlike the quantum effect, the tempo effect is strictly related to the idea of the second demographic transition Van de Kaa (2002) and thus the rapid process of ageing we have observed at the onset of the demographic developments observed in a post 1950s perspective. Contrary to the quantity-quality tradeoff, the tempo effect is easy to observe in the data and seems to be a phenomenon which repeats itself, albeit in different forms, in one country after another. One overall encompassing question surrounding the tempo effect is whether period fertility will recover and in turn stabilize cohort fertility. Mixed practical as well as theoretical evidence exist on this question. However, it is very important to realize that the tempo effect, be it temporary or not, *will* have a lasting effect on a country's population pyramid and consequently its demographic path.

2.3 AGEING AND MACROECONOMICS – HOW TO OPERATIONALIZE AGEING?

This section identifies the life cycle hypothesis (LCH) postulated by Modigliani and Brumberg as the core theoretical framework to make ageing operational in a macroeconomic context. This is done in two sub-sections; the first which discusses the theory and the second which takes a look at a few important contributions that make use of the life cycle hypothesis in an empirical context.

The LCH is a classic theory in the context of macroeconomics. Alongside Friedman's permanent income hypothesis (PIH) (Friedman (1957)) it is a core contribution to the theory of consumption and thus a qualifier to the Keynesian result that consumers spend out of current income. The theory, which earned its founder Franco Modigliani a Nobel Prize in 1985, essentially connects consumption to age by linking consumption and saving to the life cycle of the consumer. Like the PIH, the theory builds on the fundamental premise that agents smooth consumption and thus that consumers are forward looking which again means that consumption is intertemporal Deaton (2005). The key point is that the mechanism by which consumers smooth consumption is linked to their age. The LCH first appeared in 1954 in the form of two papers written by Modigliani and graduate student Richard Brumberg. After the death of Brumberg, the papers were left unrevised and were only to appear later in 1980 in the second volume of Modigliani's collected papers Deaton (2005).[13] In this light one often cited contribution is Modigliani and Ando (1963) where the LCH is given a time series treatment where the particularly interesting aspect is that it constitutes an attempt to move the LHC towards the macro level where consumption (and thus savings) becomes a function of the aggregate age structure of society (see appendix 5 for a detailed derivation).

2.3.1 AN OLG REPRESENTATION

Most studies however that study the link between ageing and the macroeconomy inspired by the LHC deploy an overlapping generation framework (OLG) which follows originally Samuelson (1958) and Diamond (1965). Since the focus in the analysis below is on capital flows the OLG framework will be described using the same model as is used in Rogoff and Obstfeld (1996, ch 3.) as well as Börsch-Supan et al. (2007)[14]. In the following, superscript (y) will signify "young" and superscript (o) will signify "old". In short, in this model we assume that our representative individual lives for two periods and crucially; that she only earns income during her "young" years. Following Rogoff and Obstfeld (1996), I also include a government that levies a lump sum tax on both labor income (in the case of the young) and asset income (in the case of the old). The formal problem reads as follows for the representative individual:

EQ. 2.4

$$\underset{C}{MAX}\left[\ln\left(C_t^Y\right)+\beta\ln\left(C_{t+1}^O\right)\right]$$

$$s.t.$$

$$c_t^Y+\frac{c_{t+1}^O}{(1+r)}=y_t^Y-\tau_t^Y+\frac{y_{t+1}^O-\tau_{t+1}^O}{1+r}$$

[13] Much to the chagrin of your humble scribe, these two sources have been absolutely untraceable and thus I am forced to follow the lead of Deaton (2005) as well as of course Ando and Modigliani (1963). Modigliani's own account can be found in his *Collected Paper Edition, vol 2*, but this volume has been very difficult to trace.

[14] Which again is similar to the model in Diamond (1965).

With (c) equal to consumption, (y) equal to income, and (tau) equal to a lump sum tax. Maximizing the utility function subject to the constraint yields the following Euler equation to describe the representative individual's behavior in optimum (see appendix 5).

EQ. 2.5

$$C_t^Y(1+r)\beta = C_{t+1}^O$$

This equation tells us that consumption in old age (t+1) is the value of consumption during the young years (t) discounted by the market as well as subjective discount rate. It is quite clear from this expression that whatever aggregate consumption function derived from this expression it has to be dependent on age. In fact, at all points in time we will have *two* representative individuals in the economy and thus aggregated consumption must be made up of consumption by the young and the old;

EQ. 2.6

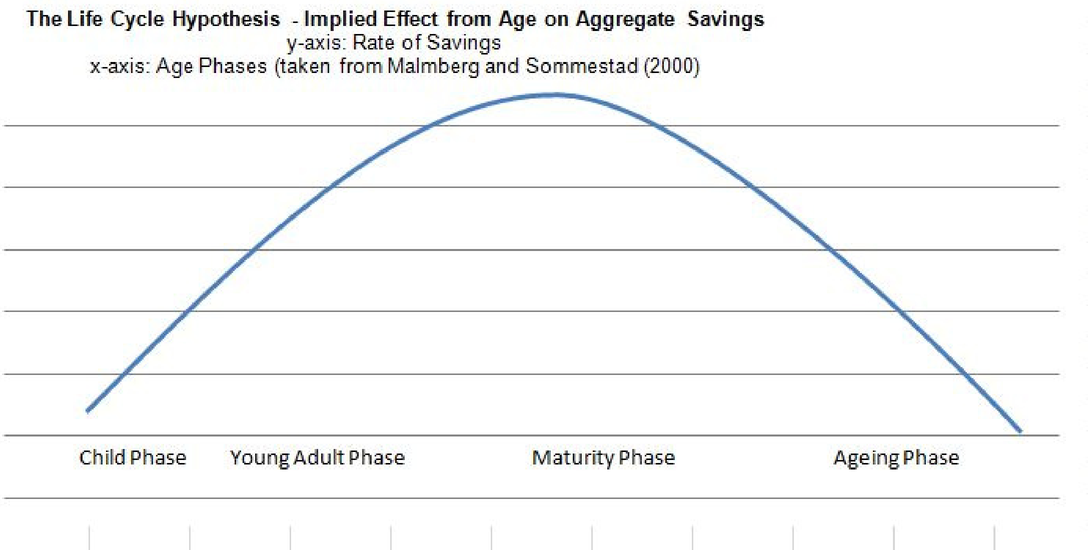
$$C_t = c_t^Y + c_t^O$$

In this way, aggregate consumption is entirely a function of the interplay between the consumption decisions between young and old consumers and essentially a function of consumption smoothing on a macroeconomic level. As we shall see in chapter 4 in the context of the intertemporal current account there is also, naturally, an aggregate savings function.

2.3.2 TAKING THE LIFE CYCLE HYPOTHESIS TO THE DATA

The most basic intuition derived from life cycle theory when applied to the aggregate economy is captured in the graph below which shows the well known hump-shaped relationship between ageing and the supply of savings in the economy.

FIGURE 2.4



One important addition to note in this context is the x-axis which has been parametrisized using the age phases postulated by Malmberg and Sommestad (2000). This does not alter the basic idea that savings is a function of the age structure and more importantly; it conveys the crucial idea that savings of an economy may be a function of its path through the demographic transition. Yet, this representation remains theoretical in its essence and as a first step towards seeing whether this relationship holds in an empirical context, we could hardly do worse than revisiting some of the original work by Modigliani himself.

Consequently, one of the main predictions derived from the life cycle hypothesis is that output growth and savings be positively correlated and Modigliani (1970)[15] provides some interim evidence to this claim using data from the 1950s as he finds a positive relationship between savings and output growth in a

[15] Surveyed in Rogoff and Obstfeld (1996) pp. 152-156.

panel data sample constructed from 36 economies (both developed and developing) Rogoff and Obstfeld (1996, p. 152). However, the relationship is not particularly strong and in this way the life cycle seems to be able to account for only a small fraction of savings Rogoff and Obstfeld (1996, p. 154).

Other prominent studies on the LCH include Mason (1988) and Collins (1991) where the perspective in both studies mainly lies on developing economies and thus whether the decline in the youth dependency ratio (share of population aged <15) across the DT is a driving force of economic growth. Both studies come out strongly in favor of a positive effect on savings from a decline in fertility and thus the existence of a demographic dividend. Collins (1991) uses a sample of 10 Asian economies in the period 1960-1980 to show that those countries who saw a meaningful decline in their youth dependency ratio also experienced a strong increase in gross savings and economic growth. Mason (1988) is a comprehensive literature review where a multitude of evidence is presented to suggest that reducing childrearing in a developing economy context is strongly associated with an increase in aggregate savings.

Despite this evidence, the idea of a single unified life cycle model remains fickle and it is worthwhile to adhere to the notion developed in Browing and Crossley (2000), not of a single life cycle model, but rather of a *life cycle framework* in which economic agents are forward looking with respect to their choice between consumption and saving Browing and Crossley (2000, p. 2). The specific life cycle model I am interested in here is one in which consumers smooth consumption over their entire life time and thus one in which consumption and saving decisions become a function of the agent's age and thus, in the aggregate, an economy's age structure (see figure above). In order to set the stage for the analysis that follows on capital flows, it would be interesting first to have a look at a number of studies that talk about the link between economic growth/performance and ageing.

One important study is Bloom et al (2007) which asks the simple question of whether age structure can help forecast economic growth. Clearly, this would seem a pertinent question since the extent to which this is case it would go a long way into justifying the perspective on ageing as a proxy for demographic processes. The argument is built on the idea that it is the *labour supply as proportion of total population* that matters rather than the mere size of the population and the framework deployed in Bloom et al (2007) consists of an estimation in which economic growth measured as the convergence to the steady state is modeled as a linear function of the determinants of steady state income, working age share of the population, and the lagged value of the convergence towards steady state growth Bloom et al (2007, p. 8). The findings suggest that in-sample forecasts of economic growth (1980-2000) using data from 1960-1980 are significantly improved by the inclusion of age structure as a determining variable and that these results are particularly strong when economic growth (i.e. the dependent variable) is measured as growth relative to world average. This is interesting because it suggests that ageing is an important parameter for an economy's *relative* economic performance and not just its absolute growth rate. This conclusion is supported by Gómez & Cos (2006) which presents evidence to suggest how maturity measured as the share of working age population (prime age workers[16]) represent a good proxy for the link between economic growth and ageing. In quantitative terms, Gómez and Cos (2006) estimate that between 1960 and 2000 the shifts in global working age structure are able to explain roughly 60% of the rise in global GDP from 1960 and 2000.

2.4 THE WHY, WHAT AND HOW OF AGEING AND MACROECONOMICS

If the reader is left a little bit overwhelmed by the amount of information parsed above, it is with the author's full understanding and sympathies. In order to tie the threads together, let us go back to the initial statement of intent in the form of the response to the following question:

What is the demographic transition, and how can it be operationalized in a macroeconomic context?

In order to answer this question, chapter 3 has emphasized the *why*, the *what* and the *how* of ageing. Firstly, the focus was put on the DT, how it has manifested itself and *why* it is important to look at demographic processes through the perspective of ageing. In the second part, the attention was turned to a specific analysis of *what* ageing is in the form of a process determined by declining fertility and increasing life expectancy. Special focus was directed to ageing as a result of lingering below-replacement fertility in the context of the SDT. Finally, the analysis finished off with a look at how we can transfer the need to look at ageing on to macroeconomic analysis. The main point emphasized was that this is best done by adopting the life cycle framework developed by Modigliani and then ultimately how this could be used, in a macroeconomic context, to make inferences on economic growth.

[16] 34-54 age cohort

This is a crucial point for this thesis and underpins the idea that the DT, through ageing, is best operationalized in the context of macroeconomics by anchoring the analysis solidly in life cycle theory.

3.0 AGEING AND CAPITAL FLOWS

In this chapter the attention is directed to the final part of the problem statement. Specifically, it seeks to answer the following question;

How can demographics explain international capital flows in the context of Japan and Germany?

This chapter is divided into four parts. The first part presents the theoretical framework in both a general as well as an OLG context. The second part motivates theoretically how ageing may be expected to affect international capital flows. This part also briefly discusses the empirical validity of dissaving. The third part directs the analysis at Germany and Japan and discusses the idea of dissaving in the context of these two economies. The fourth part briefly summarizes with an answer to the question stated above.

3.1 THEORETICAL FRAMEWORK – THE INTERTEMPORAL CURRENT ACCOUNT

In order to credibly discuss a theoretical framework to handle the question above there are two important aspects to consider initially.

Firstly it is worth separating past contributions on the topic of international capital flows and ageing into two genres. On the one hand there are studies which directly set out to empirically test the relationship between age structure and macroeconomic variables such as the current account /GDP, investment/GDP and savings/GDP (e.g. Higgins (1998), Malmberg and Lindh (1999) and Lürhmann (2003), and on the other hand there are studies that use neo-classical growth theory in the context of OLG models to simulate global capital flows (e.g. Börsch-Supan et al (2007), Domeij and Floden (2004), Feroli (2003) and Henriksen (2002)).

The most important distinction here is that the pure empirical studies may produce predictions that are non-robust because they do not impose the condition of general equilibrium[17]. Consequently and while it is all well and good to argue that an economy may come to run an external surplus (or deficit) as a function of its age, it will not be able to do so without another economy willing (or able) to run a corresponding deficit. In formal terms and in the entire global economy;

EQ. 3.1

$$\sum_{i=1}^{i} \theta_i = 0$$

where θ_i is the net external position of economy i. It follows naturally from this equation that the extent to which one economy may have an intertemporal preference to run an external surplus, there must be an economy with a mirror image preference in order for consumption smoothing through exports and imports to take place. Naturally, in studies where the model takes the form of a growth theoretical model this condition is imposed by definition in the steady state where the factor returns (capital returns in particular) are equated across economies.

Within the context of these two genres the analysis that follows essentially combines the two perspectives, with the important qualifier that it rejects the approach of general equilibrium. Hence, the analysis assumes a small open economy which takes the world interest rate as given as in Rogoff and Obstfeld (1996), but this will *not* be a binding feature in the present context. In this way, the treatment will be cast in the context of partial equilibrium and in this way the model(s) will *not* be closed to develop a steady state result where one would impose, ex post, the world interest rate as a condition for general equilibrium. This choice of methodology is important as it fundamentally relates to the intuition from chapter 2. Consequently, the development of a steady state requires parameter stability or more specifically, it requires that we impose ex-post conditions on a difference equation (or a differential equation) so that it converges to a specified constant (equilibrium). In light of the treatment of

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[17] Well, Lürhmann (2003) uses relative changes in age structure as explanatory variable to explain international capital flows which may be interpreted as a correction for this issue.

demographics in chapter 2 and particularly the idea that the demographic transition is not over, it becomes difficult to sustain the idea of a steady state and parameter stability. In this sense, the models developed below will shy away from *growth theoretical* analysis although the predictions derived from studies using this method will be used to qualify the results.

Secondly, it is important to treat the issue of international factor mobility. Specifically, capital mobility must be near perfect for economies to be able to optimally smooth consumption through time and their external balance. This brings us into one of the most widely investigated results of international macroeconomics in the form of the so-called *Feldstein-Horioka puzzle* which centers on the fact that despite theoretical foundation for the contrary, international differences in investment rates coincide with international differences in saving rates Feldstein and Horioka (1979). Since this study was published a big literature has tried to test this notion and this thesis shall not dig into these contributions but merely note that, in the analysis which follows, capital is assumed to flow freely between economies and thus the Feldstein-Horioka result is not considered binding.

With these considerations pinned down, we can move on to the formal model in the form of the intertemporal current account based on Obstfeld and Rogoff (1996) (and Imbs (2009)[18]). The intuition behind the theory is simple, but powerful. It states that an economy can intertemporally smooth its consumption and investment decisions through its relation, as an open economy, with the rest of the world. For example, an economy can borrow today to finance (excess) consumption or investment by running a current account deficit or it can run a surplus and thereby lend its savings to another country whose intertemporal preference is different from its own. In this sense, the impetus for an economy to run external deficits or surpluses becomes a function of its intertemporal preference for consumption relative to saving. This idea is given a thorough theoretical treatment in the following where I first look at what happens when one transists from a closed economy to an open economy before moving on to a formal look at the intertemporal approach to the current account (see appendix 6).

[18] Imbs (2009) is solely used to aid in solving and representing the model(s).

3.1.1 THE BENCHMARK MODEL

Consider first a standard representation of a closed economy where output is made up by consumption, investment, and government spending.

EQ. 3.2

$$Y_t = C_t + I_t + G_t$$

By definition every unit of output has to equal a unit of income, and national income in any given period can either be saved or consumed. This means that national income can either be put aside for saving or consumed through government (G) or private consumption (C). In this way, we define national saving in any given period as;

EQ. 3.3

$$\begin{aligned} &S_t = Y_t - (C_t + G_t) \\ &\Leftrightarrow \\ &S_t = I_t \end{aligned}$$

Recognize that the right hand side is equal to investment and you get the fundamental identity that savings must equal investment which is a key feature of a closed economy. In an open economy this condition is relaxed and savings do not have to equal investment which, crucially, becomes the way in which an economy can smooth consumption and investment through the current account. Consider then the national identity for an open economy where (X) equals exports and (M) equals imports.

EQ. 3.4

$$Y_t = C_t + I_t + G_t + (X_t - M_t)$$

Now, perform the same steps as above to reach an expression for national savings.

EQ. 3.5

$$S_t = Y_t - (C_t + G_t)$$
$$\Leftrightarrow$$
$$S_t = I_t + (X - M)$$ [19]
$$\Leftrightarrow$$
$$S_t - I_t = (X - M)$$

Thus, if savings in a given period exceed domestic investments the current account will be in a surplus and conversely if savings are less than domestic investment, the economy has to borrow "savings" from abroad, through a current account deficit, in order to finance this investment.

If this description of savings and open economy dynamics represent the stylized facts, the following links open economy dynamics to the principle of intertemporal consumption and thus how equilibrium will be governed by the preference to substitute consumption today for consumption tomorrow. The derivation of the intertemporal current account takes two forms in this thesis. First, a benchmark model is derived and discussed and then a model with overlapping generations is presented in order to capture the intuition of the LCH. A detailed derivation is presented in appendix 6 and the reader is suggested to consult this for mathematical specificities. Throughout, the derivation is inspired by Obstfeld and Rogoff (1996) and Imbs (2009). The benchmark model is motivated through a representative agent framework and the following problem;

---

[19] Assuming that the government budget deficit is balanced.

EQ. 3.6

$$\underset{(c,b,k)}{MAX}(C) = \sum_{s=t}^{\infty} \beta^{s-t} U(c_s)$$

$$s.t.$$

$$C_t + I_t + G_t + B_{t+1} = (1+r)B_t + A_t F(K_t)$$

$$K_{t+1} = K_t + I_t$$

With "C" equal to consumption, "I" equal to investment, "G" equal to government spending, $A_t F(K_t) = Y_t$ (national income), "K" is equal to the capital stock for a given t and finally "B" is equal to net foreign assets at time (t). In order to find an expression for the period value of the current account we manipulate the budget constraint to obtain the following expression (appendix 6).

EQ. 3.7

$$CA_t = (X - M)_t + RB_t$$

This gives us the current account as the sum of net exports + the return on (net) foreign assets from where it is possible to find the same expression as above (appendix 6).

EQ. 3.8

$$\begin{aligned}
&CA_t + I_t \equiv Y_t + rB_t - C_t - G_t \\
&\Leftrightarrow \\
&S_t \equiv Y_t + rB_t - C_t - G_t \\
&\Leftrightarrow \\
&CA_t = S_t - I_t
\end{aligned}$$

If we proceed to solve the problem above by maximizing the utility function subject to the budget constraint[20], by choosing "C", "B", and "K", we get the following expressions (Euler equations) for equilibrium consumption between two periods (see appendix 6);

EQ. 3.9

$$\frac{\beta U'(C_{t+1})\left[A_{t+1}F'(K_{t+1})+1\right]}{U'(C_t)}=1$$

$$\frac{\beta U'(C_{t+1})(1+R)}{U'(C_t)}=1$$

These two equations yield two well known results. Firstly the marginal rate of capital is equal to the prevailing (and constant) interest rate which can be seen as a no arbitrage condition and secondly, we get the famous result that the intertemporal consumption decision is governed by the subjective and market interest rate. In order to move from here and on to an expression for the current account we need some quite elaborate mathematical steps as well as we need to make some assumptions along the way. The first thing we do is to follow convention and assume that $\beta(1+R)=1$ which, following the first order condition, yields;

EQ. 3.10

$$\frac{U'(C_{t+1})}{U'(C_t)}=1 \Leftrightarrow U'(C_{t+1})=U'(C_t) \Leftrightarrow C_{t+1}=C_t=\bar{C}$$

By solving the budget constraint recursively and substituting the constant value for C, it is possible to derive the following expression for the current account (see appendix 6 for more details);

---

[20] Where I substitute in the capital accumulation equation to end up with only one constraint.

EQ. 3.11

$$CA_t = (Y_t - \tilde{Y}_t) - (G_t - \tilde{G}_t) - (I_t - \tilde{I}_t)$$

where a tilde over a variable indicates that it is the permanent (natural) level. It is interesting here to note that that this permanent value is directly associated with Milton Friedman's PIH[21] Obstfeld and Rogoff (1996), Friedman (1957) and in this way the benchmark model is not directly applicable in a life cycle framework although as Deaton (2005) argues these two theories are, in fact, complementary rather than substitutes[22].

In essence, this expression is used to understand the effect on the current account from a one-time shock or unanticipated event Obstfeld and Rogoff (1996, p. 75). It shows that if current output is temporarily higher than permanent output it will affect the current account positively. This follows intuitively from the idea of consumption smoothing on the representative agent level. If output today is higher than a perceived permanent level this excess income should be saved in order to smooth consumption onto the permanent level path. On government spending the expression indicates that when it is higher than its natural level the current account will trend towards deficit all things equal. This prediction is more ambiguous in the sense that it is not certain why a level of government spending higher than the natural level would be financed on international capital markets but the underlying intuition is still a sound one in the sense that such a level of spending would need to be financed through borrowing[23]. Finally, if the demand for investment increases beyond its permanent level, such a need would be met by foreign borrowing through a current account deficit.

The concrete drivers of global capital flows thus take the form of the *intertemporal preference* and smoothing factor of consumption and saving. In this way, and although one could possibly justify the discussion of age within the confines of this model, we are still missing a critical part. Following the

---

[21] Permanent Income Hypothesis

[22] See also Browning and Crossley (2001).

[23] Moreover, considerable ink has been devoted to the question of whether budget deficits lead or cause current account deficits. In short; this result is highly contestable.

intuition of the life cycle theory the representative individual's consumption decision is, in part, a direct function of her age. This aspect is consequently missing, as a direct parameter, in the model derived above.

In principle, there are three ways in which this can be amended.

One is to follow convention of the literature and apply an OLG framework as is done in Obstfeld and Rogoff (1996, ch. 3). A second way is to modify the utility function and follow the ideas e.g. expressed in a recent article by economist César Molinas in which he presents the idea that the subjective discount rate changes with age Molinas (2009). Finally, the third possibility is to modify the budget constraint directly with the consumption function derived in the context of Modigliani and Ando (1963). In the following, the focus will be on options one and two with specific focus on the former and thus the OLG representation.

3.1.2 THE INTERTEMPORAL DISCOUNT RATE

Firstly, we consequently consider the exact same setup as above, but this time with the following expression for utility (so-called isoelastic utility).

Eq. 3.12

$$U(C_s) = \frac{C^{1-1/\sigma}}{1-1/\sigma}$$

Using this functional form for utility and otherwise repeating all steps as above yields the following Euler equation (see appendix 6) for consumption and otherwise the same results;

Eq. 3.13

$$C_{t+1} = \beta^{\sigma}(1+R)^{\sigma} C_t$$

The key difference between what follows and the derivations above is that the assumption of $\beta(1+R)=1$ is relaxed. Performing the same mathematical steps as above (see appendix 6) the following expression of the current account can be derived Rogoff and Obstfeld (1996, p. 75).

EQ. 3.14

$$CA_t = (Y_t - \tilde{Y}_t) - (G_t - \tilde{G}_t) - (I_t - \tilde{I}_t) - \vartheta \frac{W_t}{1+r}$$

Where W(t) is a measure of wealth that includes assets accumulated up until time t as well as future expected income Rogoff and Obstfeld (1996, p. 75) and $\vartheta = 1-(1+r)^{\sigma}\beta^{\sigma}$. The key here is the inclusion of the last term that modifies the preceding expression of the current account. Consequently, the current account is now driven by two factors; one is the smoothing factor which follows from the benchmark case and another is a tilting factor. This tilting factor basically means that for $\vartheta > 0$ the country is relatively impatient and thus the current account is reduced (all things equal) whereas when $\vartheta < 0$ the country is relatively patient and the current account is increased (all things equal) Rogoff and Obstfeld (1996, p. 76). One way to incorporate ageing and life cycle dynamics into this model would then be to assume that the subjective discount rate (and thus in some sense the parameter (ϑ)) be a direct function of age.

EQ. 3.15

$$\vartheta_i = f\left(age_i\right)$$

$$\frac{\partial \vartheta_i}{\partial age_i} < 0$$

Where ϑ_i, the subjective discount rate of individual "i", is a function of age and where the first derivative is negative because we assume that the parameter ϑ_i declines with age (the agent becomes more patient) and thus ageing will tend to increase the current account all things equal. Now, the astute economist will immediately notice the obvious here; namely that the assumption above is *not* possible in the current model setup. Consequently (ϑ)) is a constant and essentially exogenous and may thus not be given a subscript as implied above. However, in an intuitive sense the proposition should not be rejected on this count alone and as it turns out, it may not be so outlandish even if it would be difficult to endogenize these preferences directly. In a recent article[24] Spanish economist César Molinas specifically links ageing and demographics to the notion of an intertemporal discount rate (my emphasis);

> The intertemporal discount rate (IDR) is "the" deep interest rate of an economy. It is like the black hole around which the whole galaxy of other interest rates gravitates. Different societies may have very different IDRs, most likely due to differing demographic profiles. **An aging society is likely to have a preference for lower present consumption relative to future consumption than a younger society**.
>
> César Molinas (2009)

In the present context one could then interpret ϑ as a proxy for the IDR which would suggest a steadily decreasing value of ϑ and thus an increased patience from the point of view of a rapidly ageing economy. This framework would seem to lend evidence to the idea that the propensity (or desire) to run an external surplus increases with age or at least; we can say that this framework stands readily available as a potential explanation to the observed fact that ageing economies are observed empirically to run external surpluses. Essentially, the idea to let the intertemporal (subjective) discount rate govern the way ageing affects the current account is potentially a very fruitful way to model the link between ageing and capital flows (using for example the OLG model developed in Buiter (1981)).

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[24] http://www.voxeu.org/index.php?q=node/3376

3.1.3 THE CURRENT ACCOUNT AND OLG

Perhaps the most widely used framework with which to operationalize the effect of demographics (ageing) on international capital flows is the OLG framework. In fact, most recent studies seek to link international capital flows to demographics deploy a variant of the OLG framework (e.g. Börsch-Supan et al. (2007), Feroli (2003), and Henriksen (2002)) and in this way it is important to see how an expression for the current account might look like in the context of overlapping generations.

Firstly, it is important to note that the general expression for the current account is exactly the same as in the benchmark model. Consequently, the current account can be expressed as the difference (in two periods) between net foreign assets;

EQ. 3.16

$$CA_t = B_{t+1} - B_t$$

Because the model used in Rogoff and Obstfeld (1996, ch. 3) also has an active government that levies a lump sum tax this expression can be written as follows;

EQ. 3.17

$$CA_t = B_{t+1} - B_t = (B_{t+1}^P - B_t^P) + (B_{t+1}^G - B_t^G)$$

Where superscript (P) indicates private and superscript (G) indicates governmental. In order to derive the current account in this context (appendix 6) it is necessary to go back to the model derived in the context of the description of the LHC (appendix 5). Recall consequently, the aggregate consumption function as;

EQ. 3.18

$$C_t = c_t^Y + c_t^O$$

As a natural function of this expression, there must also be a corresponding function for (net) aggregate savings. Following Rogoff and Obstfeld (1996, Ch. 3 p. 137) and appendix 6 the *net* aggregate savings function is given by;

EQ. 3.19

$$S_t = S_t^Y + S_t^O = B_{t+1}^P - B_t^P$$

Using this result in the expression for the current account yields:

EQ. 3.20

$$CA_t = B_{t+1} - B_t = (S_t^Y + S_t^O) + (B_{t+1}^G - B_t^G)$$

This expression is very important. If we abstract for a minute from the asset accumulation of the government we now have an expression in which the current account is a *direct* function of ageing in the form of net saving of the consumers in the economy. The following segment discusses a specific result in this model in the form of the idea of dissaving as a function of old age, whether it is plausible or even optimal.

3.2 RAPID DISSAVING, AN INEVITABLE CONSEQUENCE OF OLD AGE?

One of the logical consequences of the life cycle hypothesis is the prediction that rapidly ageing societies, at some point, will enter a stage of dissaving as the assets of consumers are run down into old age and that this ultimately would entail *old countries running external deficits*. This can be shown in the model above by assuming a process by which the old contiuously, because of lingering sub replacement fertility, will outnumber the young. This means that the dissaving of the old will be larger than the saving of the young; in absolute values and assuming that the government's foreign asset position is neutral $\left[CA_t = \left(S_t^Y + S_{t+1}^O\right)\right]$, we get:

EQ. 3.21

$$\begin{aligned} &\left|S_{t+1}^O\right| > \left|S_t^Y\right| \\ &\Leftrightarrow \\ &\left|B_t^P\right| > \left|B_{t+1}^P\right| \\ &\Leftrightarrow \\ &CA_t < 0 \end{aligned}$$

In order for this to produce a current account deficit, the central assumption is that savings decline faster than the decline in domestic investment demand Higgins (1998)[25], but also that the *stock* of savings is steadily eroded as consumers, in the aggregate, run down their assets.

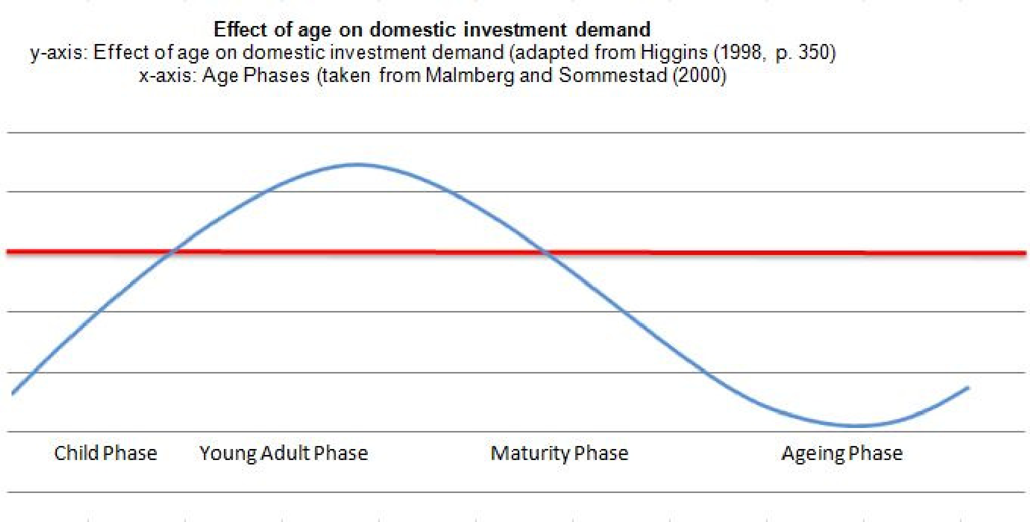
In a theoretical sense, this is a key result in a standard OLG model as it is assumed that old consumers de-cumulate their entire asset base up until their demise. In a representative agent framework with no uncertainty, everything else would clearly be suboptimal.

With this dynamic in mind, one of the best empirical conceptualizations of the way ageing may generate capital flows on the basis of life cycle dynamics is found in Higgins (1998) that uses panel data

[25] See also Lürhmann (2003) and Malmberg and Lindh (1999).

estimations with savings (investment) as well as the current account balance as dependent variables to be modeled as a function of age structure. One of the most important results from this study is the distinction between what is coined as the center of gravity of investment demand and savings supply in and how these two do not occur simultaneously. Consequently, consider the traditional hump shaped savings function which follows naturally from a standard life cycle framework (see section 2.3) and compare it with the graph below that plots domestic investment demand as a function of age as estimated by Higgins (1998);

FIGURE 3.1



Again, I have chosen an aggregate representation of this relationship with the phases of ageing from Malmberg and Sommestad (2000) on the x-axis to denote an aggregate time series perspective from the point of view of an individual economy. In this sense, the function above depicts the theoretical relationship between the evolution of the demographic transition and domestic investment demand. The thing to notice is how the peak in investment demand occurs earlier than does the supply of savings when both are taken as a function of ageing Higgins (1998) and Feroli (2003). Such a mismatch between in an open economy has to materialize itself in the form of either an external surplus or deficit. The best way to think about this is to imagine that savings and investment are in a race governed and controlled, as it were, by the transition in age structure that occurs as a result of the demographic transition. Initially, as the transition sets in with a decline in mortality and where fertility only follows with a lag, investment demand outruns the supply of savings and the economy is running an external deficit.

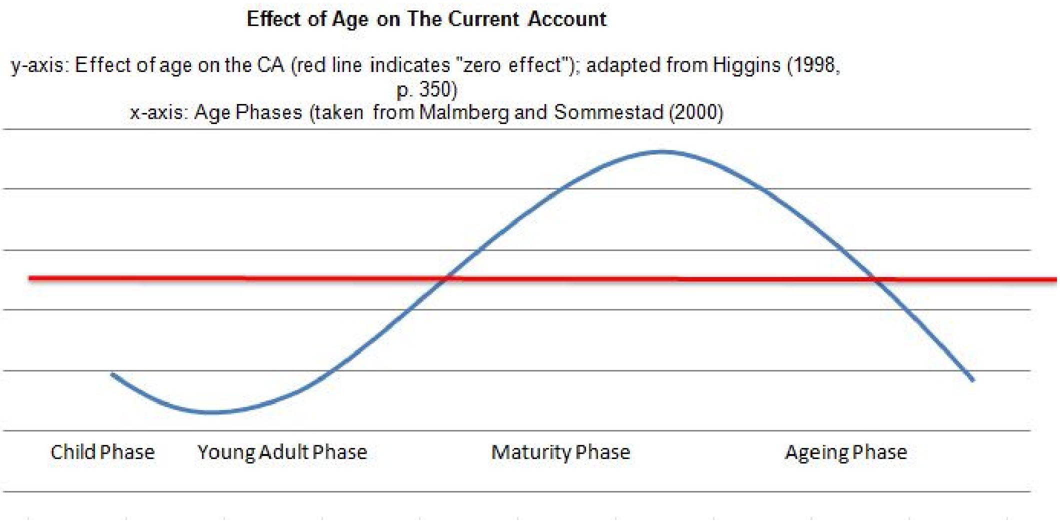
Steadily however, the supply of saving catches up with investment demand which itself begins to decline and thus the external balance moves into a surplus. Finally, the pace of savings accumulation is replaced by outright decumulation (dissaving) and the external balance moves into deficit as savings decline faster than domestic investment demand.

This remains a stylized account however, and it is important to make two important qualifiers. Firstly it is obvious that it cannot be held up as a determinist account for the evolution in all economies. This goes back to the point of convergence in global ageing (chapter 2) and the fact that the sum of all external positions must equal 0. Theoretically however, it may be motivated through the idea that it may *only* materialize itself to the extent that there are two or more economies subject to asymmetric demographic shocks Bryant (2004, 2006) as a function of the fact that they are in different stages of their demographic transitions. Here, the veil of general equilibrium is then needed in order to qualify and, essentially, quantify the exact nature of international capital flows at a given point in time.

Secondly, there is the robustness of the results. Higgins (1998) is a core citation when it comes to the effect of ageing on saving and investment dynamics, but for example Malmberg and Lindh (1999) specifically sets out to match the results from Higgins (1998) and finds results that are qualitatively different. Concretely, they find that while there is indeed a measurable difference from ageing on savings and investment dynamics, the peak effect from ageing on investment saving are in, some of their estimations, directly opposite to those of Higgins (1998) where the reason cited is that they are restricting their analysis to an OECD context (with Higgins (1998) using a world sample). Lürhmann (2003) on the other hand finds the same results as Higgins (1998) and specifically that future trends of demographic change (which are known today) affect present day trends in capital flows.

These qualifiers notwithstanding, the representation above leads us to the hypothetical relationship between the demographic transition and the sign of the current account.

FIGURE 3.2



This graph is important as it captures the essence of applying the LCH to international capital flows. Following this representation, an economy moving through the DT will be characterized first by a current account deficit which moves steadily over to an external surplus as the share of prime savers is maximized in proportion of total population over to a final stage of external deficit as the amount of dissavers steadily outnumber the amount of savers. In this model, the initial phase of external deficit that occurs in the early stages of the transition is often termed the demographic dividend Bloom and Williamson (1998) and is associated with a period in which investment demand rises much faster than the savings supply as a function of the rapid increase in the share of the labor force to total population. The second stage which brings the economy into a stage where domestic savings exceed domestic investment demand is driven by two phases. One is naturally the increase in savings as the number of prime workers[26] is maximized as proportion to the total population, but it is also important to note that an equally large part of this process is driven by a decline in domestic investment demand as the age structure steadily tilts towards old age and grinds down domestic capacity.

It is within this framework that the assumption of aggregate dissaving as a function ageing can be challenged. In particular, much uncertainty arises in relation to when an economy may seen to be old in an absolute sense and thus when this stage of dissaving is expected to occur. In this context, the

[26] E.g. 35-45 years or 35-54 years following Gomez and Pablo (2006)

question that needs to be addressed is not whether dissaving won't occur at some point as an inevitable result of ageing that simply moves forward into the distant horizon, but whether it is in fact an optimal response and if it is not; what economies can and will do to postpone or evade the stage of dissaving. Most crucially, what happens once an economy moves into the ageing phase (following Malmberg and Sommestad (2000)) and stays there? This is an important question simply because it is the phase where many OECD economies will move within the next decade (even if we could argue that Germany and Japan are already there).

In order to get to grips with this argument, consider first a closed economy with a rapidly ageing population as a result of lingering below replacement fertility. Clearly, in such an economy there is no escape from dis-saving since there are no external leakages through which excess savings can exit the economy. In the jargon used above, the race between savings and investment is off as they are tied together by laws of gravity that govern a closed economic system. Savings will have to equal investment in all periods and as the population ages, investment demand will decline and consumers will dissave (in the aggregate) and domestic companies will respond by investing less. In the alternative event that dissaving does not occur the return on capital would drift ever closer to the zero bound as a function of a continuous and increasing excess supply of savings. Taking this example to the extreme where fertility never recovers to above replacement levels[27] such an economy would face tremendous challenges in terms of maintaining its social institutions intact, particularly in the case of pension and healthcare systems.

Although this example is outlandish in its extreme version it may be closer than we would like to admit especially in relation to economies such as Japan and Germany and indeed the entire OECD edifice. However, there is an important aspect missing in this account. Japan, Germany are open economies.

Consider consequently next an open economy with a rapidly ageing population and, for the sake of argument, an expensive welfare system to finance. What would happen in the context of rapid dissaving in which the external balance would trend towards a negative value? How would such an economy ever be able to finance a consistent external deficit? What would the return of any foreign borrowing be in such an economy, and how would the economy ever build up its capital accumulation potential again? Surely, these are relevant questions which are hardly addressed within conventional OLG models. In fact,

[27] Or where it takes very long time for it to do so.

it is very difficult to see how a rapidly ageing economy would ever be able to finance a persistent external deficit. Specifically, it is likely that the evolution of such a process would be a steady stream of sovereign defaults as the economy cannot pay its foreign creditors each time pushing the economy down the ladder as the future stream of revenues from its domestic activities shrink thereby allowing less leeway towards international borrowing than before. Ultimately of course this economy would have to leave international capital markets all together and realize that savings, after all, have to equal investment as in a closed economy with a dramatic drop in investment rates (and thus growth) to follow. As such, the hypothesis of the effect from dissaving in the context of an open economy is very difficult to reconcile with economic common sense and intuition.

This would indicate that ageing, contrary to what the OLG and standard life cycle theory predict, is associated with a higher propensity to run an external surplus or at least, that dissaving becomes a *sub-optimal* response to ageing. In order to tie this in with the present theoretical framework, we might narrate it in the context of an intertemporal preference for savings relative to consumption that would have the effect to keep the external balance in surplus.

Notwithstanding the effect on savings and investment of companies[28] where one would expect capital expenditure decisions to increasingly react to foreign instead of domestic demand, this intertemporal preference would transmit itself through two additional channels in our economy; one would be for the old to dissave less drastically than assumed by the standard life cycle framework and the other would be for the young generation to save more to compensate for the dissaving of the old. If these two effects were to materialize as a function of age, we would be in the opposite situation than postulated by the initial model above;

[28] Note that in the standard OLG model, the definition of private savings do not discriminate between corporate and household savings.

EQ. 3.22

$$
\begin{aligned}
&CA_t = S_t - I_t \\
&\Leftrightarrow \\
&\left|\left(S_t^Y + S_t^O\right)\right| > \left|I_t\right| \\
&\Leftrightarrow \\
&CA_t > 0
\end{aligned}
$$

Whether this is a more accurate description of the dynamics of ageing and international capital flows is ultimately an empirical question, but on the basis of the discussions above the following two theoretical propositions are still ventured in connection to the notion of dissaving;

Proposition 1 – Although an ongoing process of ageing on a macroeconomic level will hypothetically, and in the limit, lead to aggregate dissaving, this may not be an optimal response. Rather, it seems plausible that an economy will attempt to maintain savings on a relatively higher level by exploiting the ability to intertemporally substitute consumption for savings by running an external surplus which materializes as the domestic supply of savings exceed domestic investment demand. Once again, it must be emphasized that this is only possible if there is another economy willing (able) to run a corresponding deficit.

Proposition 2 (corollary to proposition 1) – Domestic demand and thus the ability to generate economic growth measured as e.g. headline GDP declines with age as a natural function of declining domestic investment demand and consumption.

On the first proposition it is important to note that it is not entirely outside the grasp of an OLG model imbued with a more complex structure than the one above (see e.g. Collins (1991)). For example one could introduce uncertainty which would be bound to generate an increase in savings of the older generations. Another route would be to include bequest motives (i.e. the desire to leave inheritances) as well as one could introduce measures to correct for the nature of the pension system where one would assume that the presence of a generous PAYGO system would lead to a higher degree of dissaving than if agents cannot expect to receive transfers in kind Börsch-Supan (2004). Finally, and something which would be very complicated in a representative agent framework, one would also want to correct for the effect of ageing on the saving of the young and working cohorts. In this sense, the OLG model remains a more versatile tool than it perhaps appears here, but the analysis will continue assuming that whatever model we might conjure, some form of macroeconomic dissaving is a natural consequence of aggregating from a representative agent framework and it is this assumption which is challenged here.

Proposition 2 may of course seem pulled rather out of the blue here, but is not unreasonable and quite intuitive if we think about the studies (e.g. Bloom et al. (2007)) that show how ageing has a non-neglieble effect on the buoyancy of economic performance.

After such theoretical postulates, it would seem appropriate to move into a more concrete and empirical discussion of the life cycle theory in the context of capital flows with a strong eye to the notion of dissaving.

3.2.1 EVIDENCE OF AGGREGATE DISSAVING?

It is important to emphasize at the offset that when you set out to test the literature against this question the answer you will get in return is neither clear nor decisive. In this way, the notion of dissaving as a function of ageing remains covered in uncertainty and notable divergence across economies.

In Börsh-Supan (2004) a general argument is presented to suggest that although it is difficult to imagine a completely disjoint relationship between dissaving and ageing, the pace and nature of such a process not to mention its specific trajectory are highly uncertain and essentially represent an aspect of theory that is ill understood by economists Börsch-Supan (2004). Specifically, much uncertainty arises in relation to the existing pension system where a generous PAYGO[29] pension system would lead to an overall flatter savings schedule and thus lower dissaving into old age as economic agents can expect a generous life time annuity from the state Börsch-Supan (2004). On the other hand, the absence of a public pension would lead to a situation in which the aggregate savings schedule is more in tune with the life cycle model in so far as goes the increased need to save for retirement during working years that would naturally translate into marked dissaving into old age Börsch-Supan (2004). This point should then translate into two important observations. One is the presence of cross country heterogeneity as a

[29] Pay-as-you-go; essentially this system is “unfunded” and benefits are paid directly from current workers' contributions and taxes.

function of different pension systems (see e.g. figure 17, Börsch-Supan (2004, p. 30) and the other would be the trickier issue of the effect from pension reform on the savings of the working cohorts as well as the effect from aggregate ageing itself. The big issue here is naturally that assuming forward looking behavior by part of economic agents, the effect of any pension reform not to mention the effect from ageing itself on expectations formation are bound to be a complicating factor. Börsch-Supan (2004) argues that the extent to which pension systems are reformed away from PAYGO due to the obvious cost levied on public debt dynamics, one would expect savings to increasingly resemble a standard life cycle framework in which economic agents save more when they are young to dissave into old age.

Moreover and in relation to direct empirical applications the estimations in Higgins (1998) are not robust in the context of savings behavior into old age. Consider then the result that the negative coefficients which relate ageing to dissaving (negative saving) are significant only for the age groups 60+ which implies that the trajectory of dissaving is very difficult to capture as an empirical regularity Higgins (1998, p. 349). Secondly, applying a 90% confidence interval (see Higgins (1998, p. 350) the trajectory of savings at old age becomes subject to market fluctuation around the zero effect bound which exactly represents the threshold when the age structure would provoke dissaving in the aggregate Higgins (1998, p. 349).

Moving on to a general equilibrium treatment of international capital flows Börsch-Supan et al. (2007) applies the intuition from a simple life cycle model to a closed economy in which the real rate of return will decline with ageing as the stock of capital will grow relative to the amount of projects in which it can be invested. This has two main effects in a closed economy. One is that it will engender an intergenerational transfer from the old to the young in the sense that asset holders (the old) will lose out relatively to income earners (the young). The second effect is exactly the notion of dissaving as the older generations start to run down their asset base. The main prediction in an open economy context is then a direct result of the closed economy dynamics. The simulations of Börsch-Supan et al. (2007) consequently suggest how capital initially (up until 2030; Börsch-Supan et al. (2007, p. 23)) will flow from the relatively old to the relatively young economies but that this will reverse once we get into a period of rapid dissaving. Specifically, the model predicts that the US will be running a persistent current account deficit (as a young economy) relative to the rest of OECD and especially Europe; by 2040 and 2050 and once dissaving kicks in, the regions should settle with a balanced external account.

Although entirely plausible, the analysis in Börsch-Supan et al. (2007) directs our attention to one immediate problem in the context of dissaving and its transmission to global capital flows. Quite simply the extent to which this might occure seems, at this point in time, to be a question of predicting what might happen in 2030 and beyond. Needless to say, this is anything but satisfactory from the point of view of the empirically minded economist not least because predicting what might happen in 2030 is not only difficult but also, in the opinion of your humble scribe, quite futile. In order however to derive some interim conclusions, two points should be emphasized. Firstly, considerable uncertainty and heterogeneity between countries suggest that aggregation of life cycle behavior to infer macroeconomic dissaving is very difficult. Secondly, it is equally important to note that given the knowledge we have about the future trajectory of ageing in an OECD context the prospect of aggregate dissaving is very real even if we cannot say when exactly it will occur and with what force.

In the following, the analysis will be augmented with an empirical account on Japan and German where the main idea is to discuss the two propositions stated above in relation to these two economies.

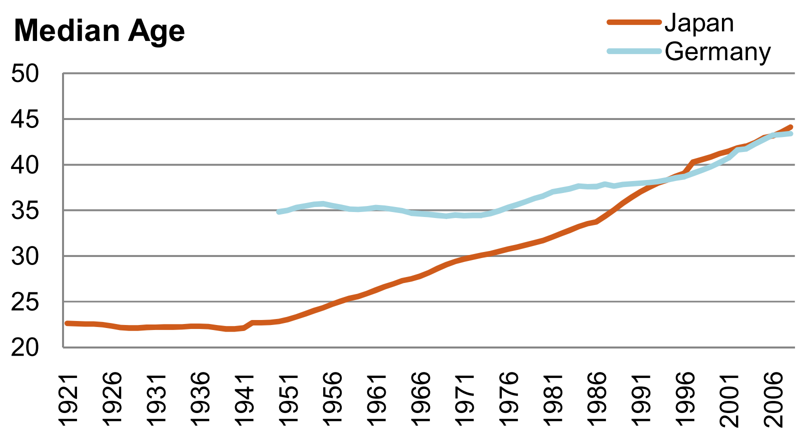
3.3 AN APPLICATION ON GERMANY AND JAPAN

In so far as ageing and its effect on savings behavior represent the main focus of the present analysis it provides a simple answer to the question of why Japan and Germany are interesting. Consequently, Japan and Germany are the two oldest countries in the world measured by median age[30]and thus also two economies where we should expect the effects from ageing to manifest themselves most forcefully.

Although the demographic evolution of Germany and Japan has been characterized by notable difference if we peer across the entire 20th century, the two economies' experience conform to the framework depicted in chapter 2 by showing a remarkable degree of convergence in the latter phases of the age transition.

[30] Well, Monaco is older at 45.2. Germany had a median age of 43.4 in 2008 and Japan one of 44.1 in 2008.

FIGURE 3.3

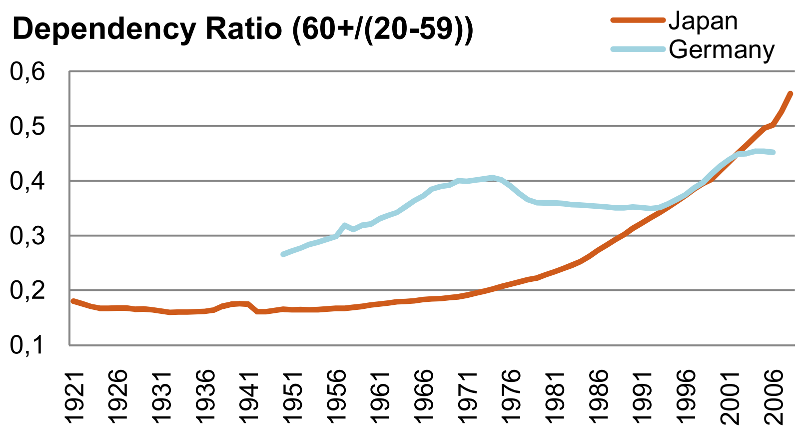


Source: National Statistical Offices, Nationmaster, own calculations

Germany was considerably older than Japan in an immediate post war context and in this regard it is fascinating to look at the remarkable process of catch-up from the part of Japan as life expectancy has increased at the same time as fertility has plunged (see appendix 7). Especially the demographic development from 1950-1970 differed considerably between the two economies but, from 1970 and onwards, the pace of fertility decline and increase in life expectancy increased in the context of Japan to catch up and essentially surpass that of Germany.

As a result Germany and Japan are not only already old, they are set to age rapidly which can be shown by the steady increase of the old age dependency ratios in the two economies.

FIGURE 3.4



Source: National Statistical Offices, Nationmaster, own calculations

The picture here is very clear and especially so for Japan where the increase in the old age dependency ratio measured as the proportion of +60 to 20-60 is used. The picture in Germany appears different and unfortunately, the data set I have is not as detailed as the one I have from Japan. In Börsch-Supan (2004) where the dependency ratio is defined as (65+/15-64) the projected dependency ratio for both Germany and Japan is set to increase markedly over the next decades.

3.3.1 (DIS)SAVING INTO OLD AGE?

In terms of empirical data both the OECD and IMF database have been used to extract data on national accounts. The OEDC database was used to get quarterly national accounts for the period Q2-1960 to Q3 2008[31] and in order to get comparative data for the two countries and to get as long a sample period as possible, OECD's CARSA methodology[32] was chosen. One important obstacle to the data retrieval process was the fact that the full sample time series only incorporates the difference between imports and exports to proxy the external balance. This is a problem since it omits the importance of the income balance that measures the difference between factor income earned abroad (by domestic investors) and factor income earned at home (by foreign investors). In Japan's case this has been mitigated by pulling data for the income balance using the same methodology and then constructing a new times series called GNI (gross national income) which, by definition, is exactly GDP plus the income balance. In this way, the current account balance is defined as net exports + the income balance. Unfortunately, the data for international factor income is only available from Q1-1980 and onwards. In the case of Germany the problem was more profound in the sense that it was not possible to draw data for the income balance using the CARSA methodology. In this way, the analysis on Germany will only include net exports as a proxy for the external balance. The IMF database was used to get annual figures on CA/GDP, I/GDP, and

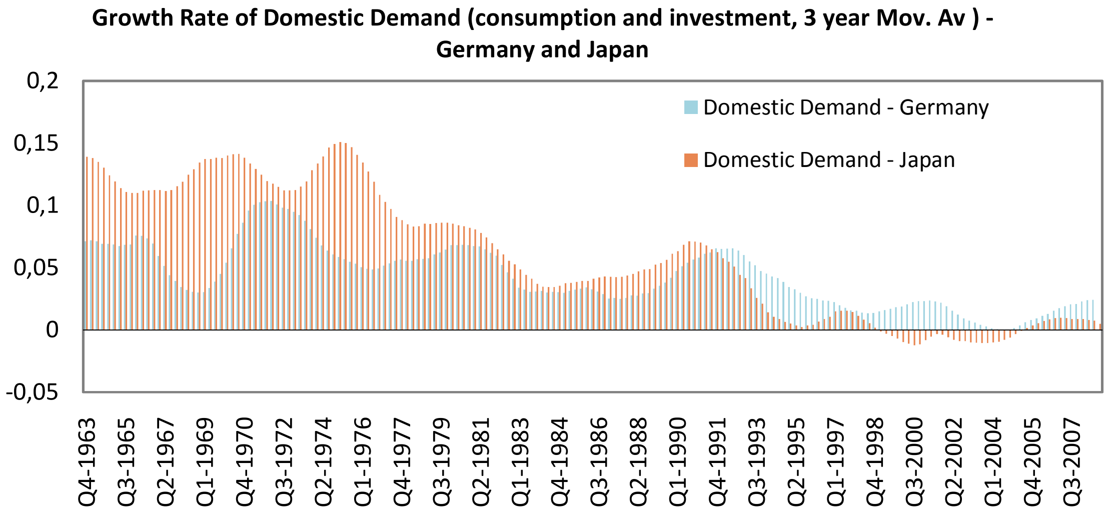
[31] Q4 in the case of Japan. See Data CD for a copy of raw data.

[32] Seasonally adjusted, current prices, annual frequencies

S/GDP[33] in order to perform regressions similar to the ones found in Higgins (1998) and Malmberg and Lindh (1999)[34].

Moving on to the treatment of two propositions above and if we begin with the second, the following graph is important.

FIGURE 3.5 (GROWTH RATE IS YEAR-ON-YEAR)



Source: OECD, own calculations

The graph above shows the growth rate in domestic demand calculated as the annual percentage change of the change in the sum of investment and consumption and both Germany and Japan have clearly observed a decline in growth since the 1960s. Yet, it is important to point out that this is not in itself evidence of proposition 2. In this way, the graphs are bound to include a large *diminishing returns* effect which follows naturally from the fact that as an economy builds up its capital structure from a low base it will see high growth rates that will diminish as the economy evolves and develops. However, even if the chart above is not a smoking gun in terms of linking ageing to a decline in economic growth it is quite easy to verify that such a connection indeed exist. Consequently, we need only go back to Bloom et al

[33] Gross savings.

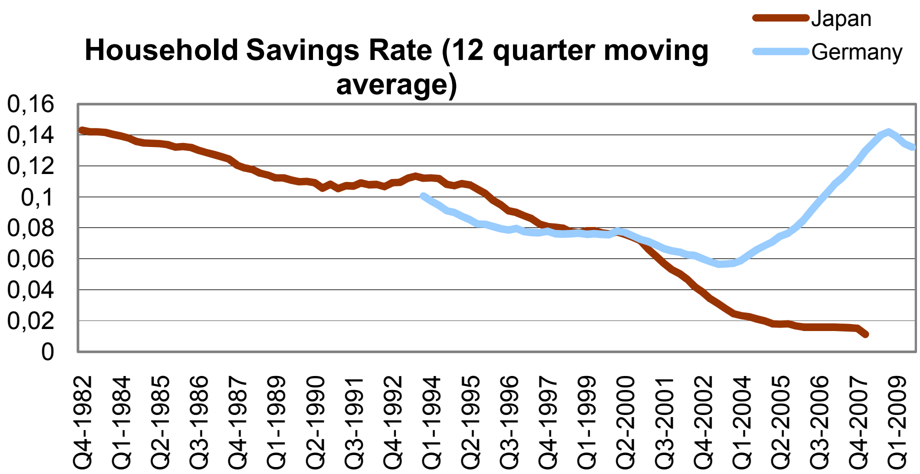
[34] Summary statistics on the data can be found in Appendix 8 where a number of visualizations are also presented.

(2007) who finds a strong and positive correlation between working-age share of the population and economic growth. Depending on the definition, Japan and Germany have seen the share of working-age population decline since the middle of the 1990s in the case of Germany and the end of the 1980s in the case of Japan (see Data CD); and more importantly, we know that this trend is accelerating.

Most studies however that concretely seek to establish the link between growth and ageing tend to focus on the fiscal and public debt burden of an ageing population and how this, in itself, will engender significantly lower growth. For example on Japan, Dekle (2002) is an influential contribution and the estimated time path of Japan's fiscal position and public debt as a result of ageing makes it obviously clear that economic growth will be significantly impaired (even with labour market reforms) as a result of ageing. The obvious question which then arises is whether the growth rate that Japan and Germany may be able to generate on the basis of domestic demand may be enough to fulfill the classic macroeconomic objectives of higher welfare, low unemployment, not to mention maintaining the pension and health care system intact faced with a large burden from the old age dependency ratio.

Concerning the more substantial proposition in the form of the notion of dissaving, it is decidedly more difficult to get a hold on. Firstly and keeping in mind that the saving rate may be defined in many different ways, it is interesting to note the following chart which plots the household savings rate for Japan and Germany respectively;

FIGURE 3.6



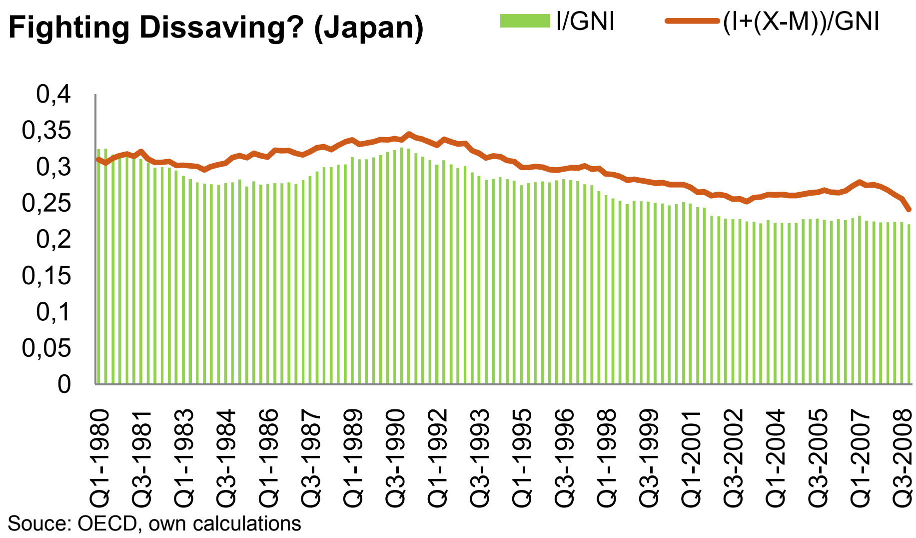
Source: National Statistical Offices, own calculations [35]

[35] Data for Germany only available from 1991 and onwards.

In this way and if we apply a rather naïve application of the lifecycle theory we could say that Japan seems to conform well with the notion of dissaving whereas the spike in the German savings rate from 2003 and onwards seems difficult to reconcile with life cycle theory. However, it is important to be very careful in the interpretation here and especially it is important to distinguish between *stocks* and *flows*. Consider then the fact that the correlation between the change in disposable income and the change in savings is very high for both Japan and Germany (virtually 1 for Japan)[36]. This is consequently a representation of *flows* and indicates that the change in the household savings rate follows very closely the rate of change in income and thus, by derivative, the evolution of the flow (growth) of domestic demand. Especially, in Japan's case it is worth pointing out that in very large parts of the 1990s and the 2000s the annual change in nominal income flow to Japanese households was *negative* (see data CD for specifics) and thus a sharply declining savings rate is not that odd. In this sense the decline in savings (and subsequent bounce in the case of Germany) is bound to track the already stated connection between a decline in the growth rate of domestic demand as a function of ageing. Finally, it is worth re-emphasizing that a decline in savings would only lead to an external deficit in the event that savings declined faster than domestic investment demand.

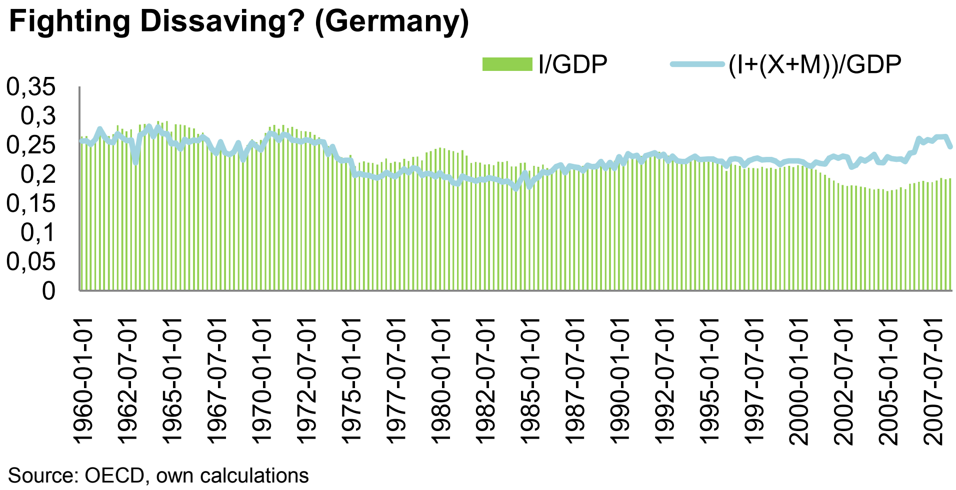
In this light, it would be more interesting then to have a look at total national savings and in particular at the contribution of external savings.

FIGURE 3.7



[36] See Data CD, the spread sheet is named; *"Household Saving Rates - Japan and Germany"*

FIGURE 3.8

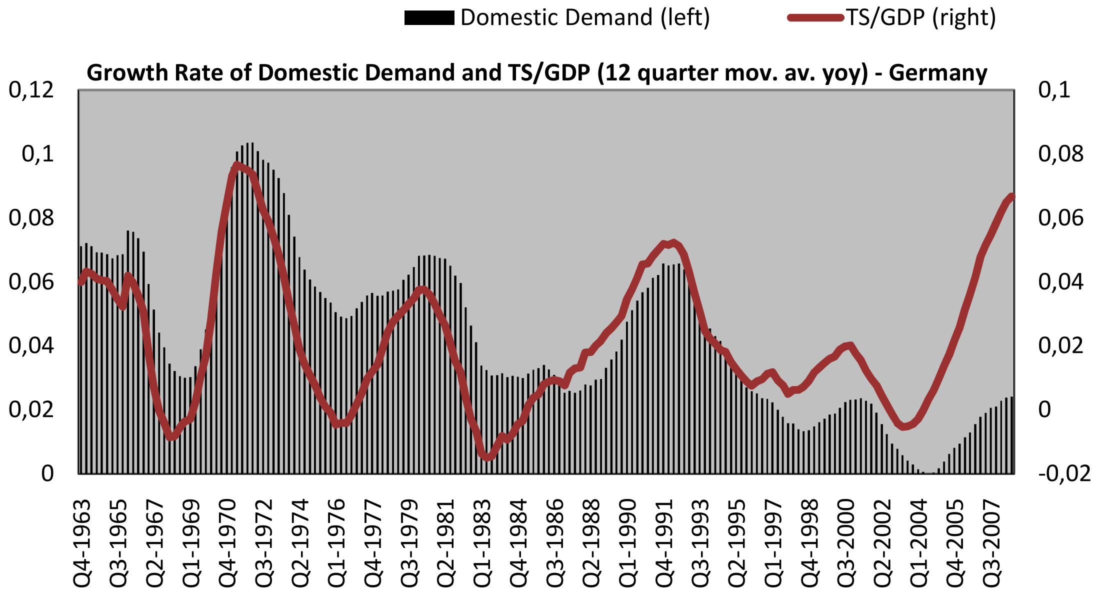


First off, it is clear that investment demand has indeed fallen as a share of national output. The fall is especially pronounced from 1990 and onwards in the context of Japan (the infamous lost decade) and from 1999 and onwards in relation to Germany. The key trend to look for here is the extent to which domestic savings show a tendency to decline faster than domestic investment demand which would translate into a current account deficit. Clearly, this is *not* the case. On the contrary, it appears that total domestic savings defined as the sum of the domestic investment demand plus the external balance has been almost consistently above domestic investment demand. This is especially the case for Japan and in the German case, a structural break appears to be present from 1999-2000 an onwards.[37]

Finally, it is worth combining the intuition from figure 3.5 above into two separate figures for Japan and Germany.

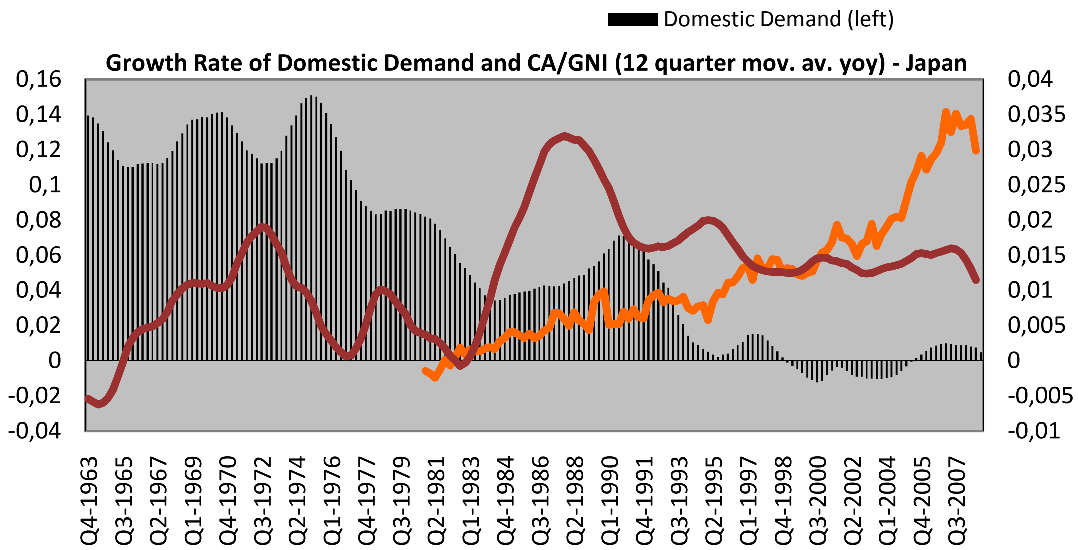
[37] Although one other source of endogeneity here is almost certainly going to be the initiation of the third phase of the EMU in 1999.

FIGURE 3.9



Source: OECD, own calculations [38]

FIGURE 3.10



Source: OECD, own calculations

[38] Where TS = trade surplus and CA = current account (trade surplus + income balance); in Japan CA/GNI is only available from 1980 and onwards.

These two charts depict the growth rate in domestic demand overlaid with the share of the external balance as a share of national income or output. What is interesting to observe is that as the growth rate of domestic demand has steadily declined in the two economies the external balance as a share of national output/income has increased. This is interesting since it suggest that while domestic demand and thus in some way domestic investment demand has declined domestic savings have not declined in the sense that Japan and Germany have both developed a savings surplus to match the borrowing of foreign economies. More specifically, it suggests that domestic investment demand has declined faster than domestic savings thus prompting the development of an external surplus.

In order to give this an empirical application and as a final contribution to the analysis it would be interesting to attempt a replication of the results found in Higgins (1998), Malmberg and Lindh (1999) and Lürhmann (2003). This leads to the estimation of the following fixed effect panel data model;

EQ. 3.23

$$Y_{it} = \alpha_t + \beta_1 \mathbf{X_{it}} + \beta_2 GDPgrowth_{it} + \beta_3 D_i + \beta_4 Y_{it-1} + u_{it}$$

Where "Yit" is either CA/GDP, I/GDP or S/GDP, "**Xit**" is the matrix of age shares of which there are 3 in this estimation (20-39, 40-59, and 60+)[39], "GDPgrowth" is a control variable following Malmberg and Lindh (1999), "Di" is a dummy to allow for country specific effects (1=JPN, 0= GER) and finally "Yit-1" is a control variable to correct for the fact that the dependent variables are likely to be integrated of order 1. The data is taken from the IMF WEO database[40]and spans the years 1980 to 2006 with annual values. In the full stacked model (with i=2 and t=26) this gives 52 observations as the model loses two degrees of freedom from the inclusion of the lagged dependent variables. The model is estimated as a one-way fixed effect model which makes it similar to estimating the model above through simple OLS with the inclusion of the country dummy variably to allow for cross-sectional effects. The results are reported in the table below.

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[39] The age share 0-19 has been left out to avoid severe multicollinearity problems.

[40] See full output on the data CD and the excel sheet named "*Redoing Higgins (1998) and Malmberg and Lindh (1999)*".

FIGURE 3.11

Dependent Variable	CA/GDP	I/GDP	S/GDP
No of Obs.	52	52	52
Adj R-sq	0.80	0,97	0,97
F	35,4***	309,9***	250,4***
Durbin Watson	2,1	1,5	1,6
Estimates *[t-stat] (Standard Error)*			
Intercept	22,5 *[4,0] (5,62)****	-9,87 *[-2,18] (4,52)***	4,68 *[0,96] (4,85)*
20-39	-39,4 *[-3,16] (12,45)****	14,1 *[1,73] (8,12)**	-13,83 *[-1,5] (9,2)*
40-59	-36,2 *[-3,67] (9,86)****	35 *[4,05] (8,64)****	27,9 *[2,01] (13,86)***
60+	-1,98 *[-0,81] (2,45)*	-5,04 *[-1,15] (4,4)*	-11,2 *[-1,93] (5,77)**
GDP Growth	-0,22 *[-3,21] (0,07)****	0,43 *[6,93] (0,06)****	0,33 *[6,15] (0,055)****
Dummy (1=jpn, 0=ger)	1,21 *[2,49] (0,49)***	-0,23 *[-0,55] (0,46)*	1,35 *[1,93] (0,7)**
lag(dep.var)	0,97 *[14,40](0,07)****	0,83 *[17,9] (0,046)****	0,71 *[8,13] (0,088)****[41]

The estimated R-sq values must be taken with a pinch salt as they are significantly biased by the presence of autocorrelation signified by the strong significance of the lagged dependent variable. The Durbin-Watson test statistics however indicate that once first order autocorrelation has been controlled for, the error term are considerably more well behaved which suggest that the estimates are indeed robust.

In terms of the estimated coefficients the interpretation is a bit counterintuitive due to the format of the WEO data. Looking for example at the estimated coefficient for the cohort aged 40-59 at -36.2 in the first estimation, it indicates that a 1% increase in the share of the total population aged 40-59 will lead to a decline in the CA/GDP ratio of 0.362%. This interpretation can be applied to the other results as well.

The results in general indicate a strong effect on macroeconomic variables from the so-called mature part of the population (those aged 40-59). Consequently, this age group is found to be significantly

[41] Where ***, **, * means significance at the 1%, 5% and 10% level respectively; standard errors are robust and thus corrected for heteroscedasticity.

related to both a rise in investment demand and supply of savings. It is interesting here to see though that the model also indicates a strong net effect in the form of this age group's significant negative influence on the current account. This is interesting because other studies tend to find conflicting evidence here most likely because maturity is defined in a narrower age range. In this way, it is interesting that by defining maturity in the broadest way possible, we find a strong negative influence on the current account in relation to Japan and Germany. Concerning, the young age group (aged 20-39) the results by and large confirm the findings of other contributions (e.g. Mason (1988), Collins (1991) and Higgins (1998)), namely that a relatively young population will be associated with a current account deficit (all things equal) as such a population will tend to be characterized by a strong investment demand without a corresponding flow of domestic savings to finance this. "Young" may of course mean many things, but in relation to Japan and Germany (and a developed economy in general) the cohorts aged 20-39 may reasonably be defined as a young trending naturally to mature as they reach an age of 35+.

Turning to the results for the old age group (60+) and thus the results of real interest here is also where the problems arise. The results consequently mirror those of Higgins (1998) and Lürhmann (2003) as the estimates for the oldest age cohorts largely remain outside the grasp of this model framework to capture[42]. Thus and although the model indicate some evidence of dissaving, it is equally important to notice that the positive effect on the current account also seems to decline significantly with age. In this way, we may conclude that while it appears that gross domestic savings seem to be a negative function of old age in Japan and Germany, it is not possible to capture the net effect on the current account which would exactly require that we had a strong result on investment demand (which evidently we have not).

Based on these charts and the connecting analysis the following conclusions can be merited in the context of propositions 1 and 2 stated above.

- On the first proposition it is fairly easy both theoretically and empirically to show that persistent ageing over a prolonged period is likely to lead to significantly lower growth rates. The conflict here especially arises in the context of what we could call a modern capitalist welfare state which simply needs headline economic growth to function properly. Nowhere is this more present than in Japan

[42] This is interesting since for example Lürhmann (2003) uses a considerably more advanced framework than the one above.

and Germany who are already fighting the effects of ageing and the subsequent negative trend in the economic growth rate.

- On the second proposition, it was shown how domestic investment demand and thus savings have indeed declined in Japan and Germany which could be seen as evidence of dissaving provoked by ageing. This would then materialize as a slow, but visible, negative trend in the share of investment in national output. However, it was also shown, in line with proposition 2, that a consistently positive external surplus has compensated both Germany and Japan for the loss of domestic savings. So far then, there does not seem to be any evidence of dissaving in Japan and Germany (in the aggregate!) as the flow and stock of domestic savings is higher than domestic investment demand which produces a current account surplus. This result derived from a mixture of qualitative and quantitative analysis was qualified by an empirical analysis à la Higgins (1998) and Malmberg and Lindh (1999) where evidence of dissaving as a function of old age was indeed found. However it was not possible to discern a net effect on saving and investment dynamics which would formally translate into an effect on the external balance.

In this way, the analysis on Germany and Japan seem to conform well with the first proposition whereas the second proposition has been more difficult to verify. It is important to note however that the empirical observation of a strong propensity for Japan and Germany to run external surpluses indicates that whatever kind of dissaving we may observe from ageing it is either not optimal in the sense of the aggregate economy or has simply not yet materialized.

Consequently, the analysis presented above is bound to suffer from the same degree of uncertainty as do most other empirical studies on this topic. Quite simply, we don't know whether Germany and Japan are sufficiently old to discuss the notion of of dissaving on the aggregate level and since these two economies are the oldest societies we have, we find ourselves standing on the edge glancing into the distant horizon with no possibility to tell what comes next. This is a problem all social scientists face on regular basis and should not deter us from pushing forward, but in the current context it represents a particularly interesting obstacle.

Essentially, the issue can be reduced to the simple question of how old Germany and Japan are in an absolute sense. As it turns out, this is a dreadfully difficult question to answer. If we look to the typology presented in Malmbeg and Sommestad (2000) Japan and Germany are obviously in the ageing phase and thus in a phase where one would expect dissaving to be, if not already occurring, then imminent. Yet, unfortunately it is not so simple. Many studies Henriksen (2002), Börsch-Supan (2007) and Feroli (2003) tend to find that although Japan and Germany are certainly old, they are also expected to run external surpluses well into 21st century and in the case of some studies all the way up to 2040-50. These results are consequently the function of a general equilibrium treatment where dissaving emerges as a hypothetical future driving force of savings and investment dynamics (e.g.. as in the asset meltdown hypothesis) not set to kick in before after 2020 at the earliest. However, as it was demonstrated above, the uncertainty attached to concrete evidence on dissaving on the microeconomic level means that such predictions extrapolated on to the *macroeconomic level* 20-30 years forward in time are bound to be very fuzzy and thus any result in general equilibrium are bound to be even more uncertain. This highlights the delicate sensitivity of any analysis to how one chooses to place the goalposts, as it were, for the transition from a mature economy into an ageing economy.

In fact, I would go one step further and note that although such predictions might be intuitively interesting, they are not particularly useful for the purpose of discussing the current dynamics of demographics and capital flows in a general as well as a specific context of Japan and Germany. Rather, the main point to take home from the analysis above is that an ageing economy may be able to benefit from maintaining domestic savings at a higher level than domestic investment opportunities and thus to run an external surplus. The main goal here is to postpone and ward off the effects of whatever form or variant of dissaving that waits 10, 20 or perhaps even 30 years from now. What is crucial to understand here is then that the focus must be turned to the *current* characteristics of a rapidly ageing economy such as Japan and Germany, why and how these may be a function of ageing, and most importantly; why dissaving ultimately is something decidedly unattractive from the point of view of macroeconomic stability and growth and thus how economies might react and behave in order to prevent it.

3.4 THE DEMOGRAPHIC TRANSITION AND INTERNATIONAL CAPITAL FLOWS

The analysis above has covered a lot of ground both theoretically and empirically and in order to tie this together into a coherent argument it is necessary back to the problem stated in the beginning of this chapter;

How can demographics explain international capital flows in the context of Japan and Germany?

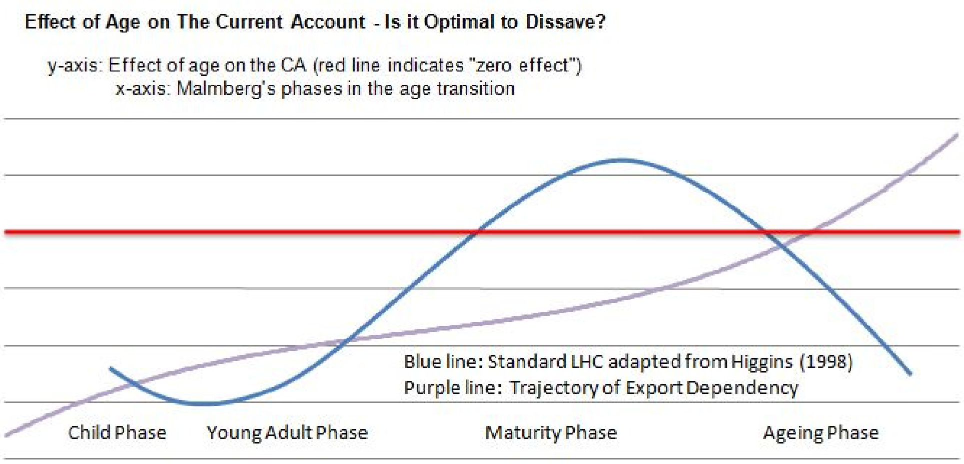
Both theoretical and empirical evidence strongly motivate why and how demographics may drive international capital flows. Concretely, the notion of how demographics drive savings and investment dynamics of an open economy represents a strong way to think about demographics and international capital flows. More specifically, the idea that asymmetric demographic shocks may drive capital flows between economies is an important intuition when thinking about general equilibrium results. Turning the focus towards Germany and Japan and thus to the dynamics of relatively old and rapidly ageing societies, the attention was directed to the notion of dissaving as a key theoretical postulate when applying life cycle analysis. It was shown how this idea was difficult to rationalize and observe theoretically as well as empirically, and how; as a consequence, it may not be the proper lense through which to look at the immediate effect of ageing.

This brings us to the main point of this section and the formal answer to the problem above. Consequently, it has been argued how demographics may explain international capital flows in relation to Germany and Japan through the notion that ageing may lead these economies (and others who will succeed them) towards the propensity to run an external surplus. Concretely, this surplus serves two purposes in the form of providing growth on the one hand and providing a de-facto boost to national savings at a, presumably, higher foreign return thus ultimately also warding off the hypothetical future state of dissaving.

4.0 DISCUSSION AND PERSPECTIVE – EXPORT DEPENDENCY AND GLOBAL IMBALANCES

Can a country become dependent or over reliant on external demand and foreign asset income to achieve economic growth? This may seem an unusual proposition at first, but following the analysis above that stipulates how ageing, at one and the same time, may be associated with a secular decline in the growth rate of domestic demand and a propensity to run an external surplus, it may not be so peculiar after all. In fact, if dissaving is, at best, a fickle consequence of ageing it is possible to take the argument one step further and ask whether dissaving is optimal at all.

FIGURE 4.1



This graph is the same as the ones shown in the preceding analysis, but here it is amended with a second schedule which attempts to capture the notion of export dependency[43]. The important thing to notice is what happens in the ageing phase and in particular when the purple line crosses the "zero effect line". It is here that export dependency might arise as the trend growth of domestic demand becomes so low that it cannot fulfill the objectives of the economy (i.e. high growth in GDP, ongoing finance of welfare

[43] This curve is *pure* speculation and has no direct empirical foundation. Essentially, it is drawn free-hand to illustrate the hypothetical trajectory of export dependency.

services, and low unemployment as the most important ones). In such a situation, net exports and net foreign asset income may become the potential refuge and although all ageing economies, naturally, cannot run surpluses at the same time, the de-facto presence of export dependency may be an important aspect of the description of an ageing economy once they reach the same stage of ageing as Germany and Japan.

In order to make this discussion operational, it is important to note how the notion of Germany and Japan being dependent on exports to grow is not entirely a figment of your author's imagination. Consequently, it is by now an integral part of the market discourse that Germany and Japan are dependent or reliant on exports to grow and how this represents a distinct feature of these economies[44].

In order to give this argument a simple empirical foundation, consider the following estimation;

EQ. 4.1

$$\Delta Y_t = \alpha_0 + \beta \Delta CA_t + u_t$$

Where the left hand side is the change in national output or income and the right hand side is the change in the external balance[45]. On the basis of this simple framework recall the following results in the context of OLS estimation Gujarati (2003).

[44] See e.g. the Absolute Return Partners Newsletter (October 2009), A Country for Old Men and a Bit of Samba, Frances Robinson, Bloomberg (2009) as well as Yoshiaki Nohara, Bloomberg (2009).

[45] Where the external balance will be proxied by net exports in Germany's context.

EQ. 4.2

$$\hat{\beta} = \frac{\text{cov}(x_t, y_t)}{\text{var}(x_t)}$$

$$R^2 = \hat{\beta}^2 \left(\frac{\hat{\sigma}_x^2}{\hat{\sigma}_y^2} \right)$$

$$F = \frac{R^2 / (k-1)}{1 - R^2 / (n-k)}$$

In applying this to the framework above, the idea is to compute the value of these three parameters as a 10 year moving average (41 quarters/observations). This gives us 75 regressions for Japan using the data from Q1-1980 to Q4-2008 and 155 regressions for Germany where the sample size runs from Q1-1960 to Q3-2008. In appendix 9 a series of graphs show the results of these estimations[46] where particularly the following two representations are important[47].

[46] See also Data-CD

[47] In all the regressions where the overall goodness of fits passes a 5% test of significance the estimated beta coefficients for both Japan and Germany are positive.

FIGURE 4.2

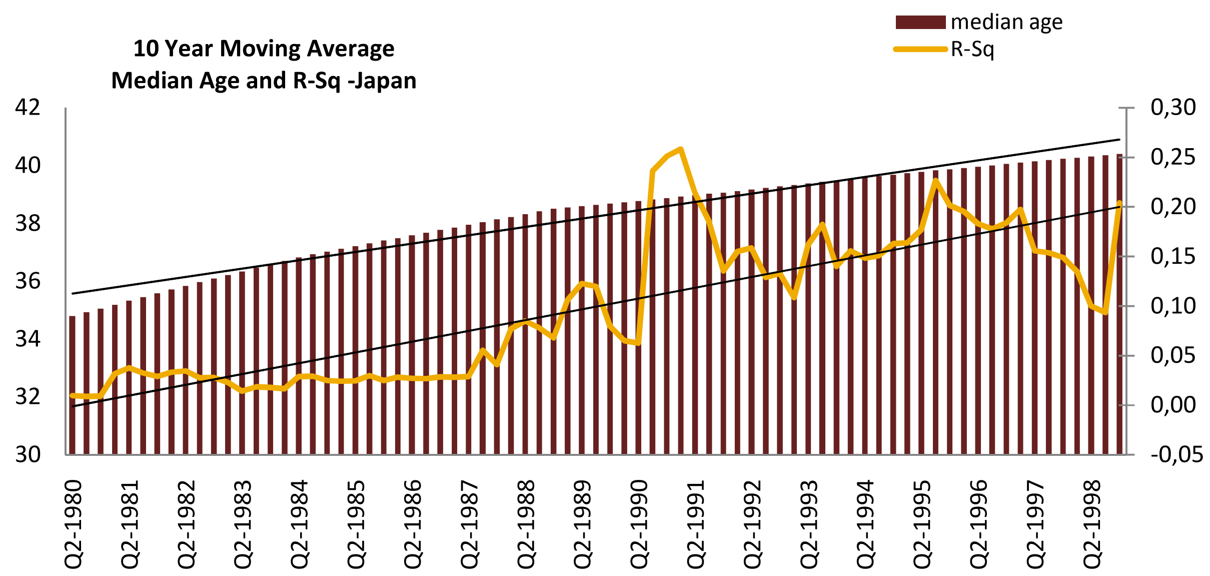
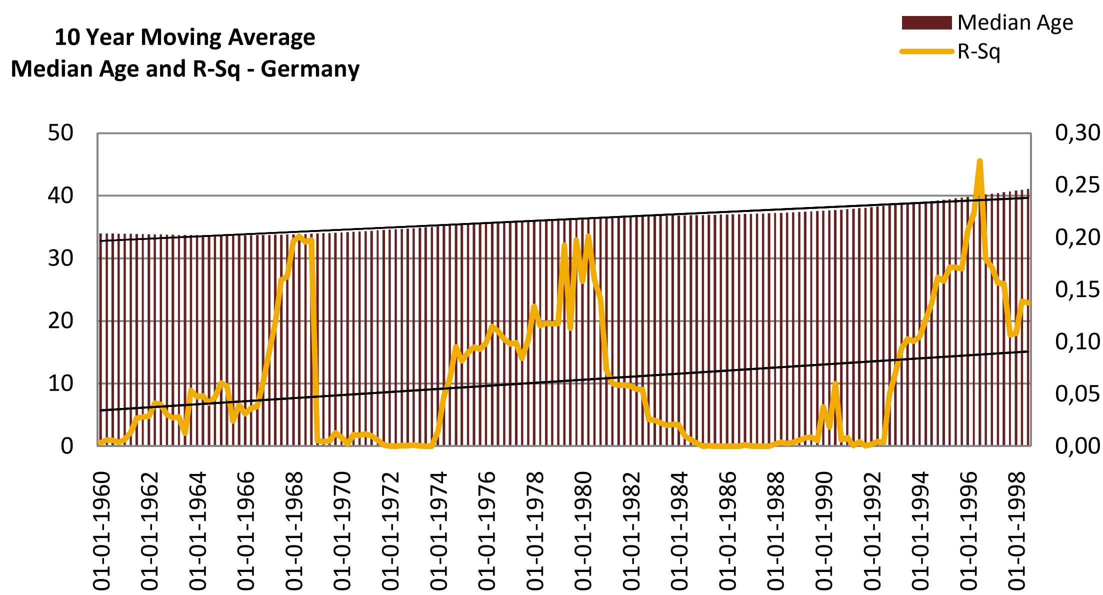


FIGURE 4.3



Here, the evolution of the computed goodness of fit (R-sq) is overlaid with the quarterly median age[48]and finally, Excel has been used to fit a linear trend to the two series.

Beginning with Japan, the results indicate a steady increase in the model's ability to explain the relationship between the variation in the current account and the variation in national income. It is particularly interesting to observe how the period around 1989-1990 exhibits a clear structural break in this relationship. It is important to note then that although Japan has been running a net surplus on its external books throughout the estimation period it is *only* in the latter parts of the sample (1990 and onwards) that the relationship between the current account and national income is statistically significant. Especially the fitted trend lines strongly suggest a growing connection between national income growth and external demand as a function of time and thus in some sense ageing.

Turning to Germany the results are trickier to interpret most likely due to the longer sample period which begins in 1960[49]. Consequently the graphs indicate that that there has been three periods[50] in which the variation in exports has had a strong impact on the variation in GDP. However, the graphs also indicate that the period in question here marked by the transition of German's median age beyond 40 appear to represent the strongest relationship. Yet, this result is also potentially a problem for the argument that ageing is related to dependence on foreign demand to achieve growth. Consequently, it is difficult to argue how the strong relationship in the current period (i.e. apprx. 1998-2008) between external demand and output growth is related to ageing if there has been earlier periods in which Germany was significantly *younger* measured on median age. There is one striking difference though between t Germany's external balance in the three periods in question. Consider then that in the two first periods, Germany was running an external deficit (on a moving average basis) whereas in the third period the external books have been significantly in surplus (appendix 8). This fact restores the idea of ageing and a connection with export dependency in relation to Germany.

In order to give the idea of export dependency a further quantitative perspective it would be interesting to test for cointegration between the estimated R-Sq time series and the quarterly median age, with the

[48] Calculated by assuming that the change over one year is evenly spread out over the year's four quarters.

[49] Of course, the results for Japan could very well have been equally tricky if it had included data from 1960.

[50] The first period is mid 1960s to end 1970s, the second period is mid 1980s to the beginning of 1990s, and the third period is 1998 until now.

latter calculated as a 10 year moving average to match it with the R-Sq vector. The intuition here would be that if the steady increase in median age was found to share a common trend with the R-Sq time series (i.e. long run equilibrium), it would further underpin the notion of a strengthening relationship between exports and economic growth as a function of ageing. The test for cointegration will consequently be based on the following regression;

EQ. 4.3

$$RSq_t = \beta_0 + \beta_1 MedianAge_t + u_t$$

where the subscript "t" signifies a 10 year moving average per period. The results from the test for cointegration can be scrutinized in Appendix 10 and seem to reject the hypothesis of cointegration for both Japan and Germany. However, the estimated t-stats for the Dickey-Fuller test on the residuals are sufficiently large to claim that the results are border line (3.02 and 2.75 for Japan and Germany respectively) since if evaluated against a standard t-distribution, they would be clearly significant at the 1% level.

Since the analysis above provides both empirical and theoretical evidence for the idea that an ageing economy may become dependent on exports and foreign asset income to achieve growth the natural next step would be to ask how this might be transmitted in a global context. Consequently, we know for certain that other economies are moving in the same direction as Germany and Japan in terms of ageing and while export dependency may very well be a natural feature of an ongoing process of ageing, there is a limit to how many economies that can leverage external demand at the same time since they need the importers to make up for their surplus savings. In order to frame this dilemma it is natural to take the point of departure in the discourse on global macroeconomic imbalances.

In this sense and not unlike many other things in the economic and financial sphere, the financial crisis has strongly affected this aspect of the global economy[51] but it is still worthwhile to ponder the nature of the global financial system characterized by these imbalances. Consequently, the global macroeconomic

[51] Although a recent VOX.EU piece by Richard Baldwin and Daria Taglioni (http://www.voxeu.org/index.php?q=node/4209) provides a timely qualifier to this claim.

landscape has long been characterized by what has been coined as Bretton Woods II in which a large batch of especially Asian and oil exporting economies have been pegging their currencies to the US dollar which has manifested itself in the US economy running a large current account deficit to match the savings surplus of emerging markets such as China, South Korea, the Petroexporters, Brazil and Russia. In fact, if we cut a lateral line through this argument we could say that the world has hitherto been characterized by the Anglo-Saxon economies running external deficits to match surpluses in big emerging markets as well as Japan and Germany.[52] That however changed abruptly with the advent of the financial crisis and it is interesting to note the initial response by market participants and many scholars in their interpretation. Consequently, as it became clear that the US economy had been mortally wounded on the back of the subprime mortgage debacle the US Fed slashed nominal interest rates significantly. As a result the USD plummeted which led many commentators to hail the US economy's fall from grace and specifically coined the notion of *decoupling* in which the Eurozone economy and Japan were pinned as the ones taking up the slack in steering forward global demand. Initial versions of the decoupling thesis centered on the shift in emerging market exports from the US to Japan and, especially, the Eurozone and thus in the process also a shift from the US dollar to the Euro as a global reserve currency. As it turned out this was nothing but a mirage masked by the fact that US policy makers essentially acted preemptively to a crisis which turned global during the summer 2007 and now many central banks in the OECD have slowly bitten the bullet and followed Bernanke into quantitative easing to combat the risk of deflation which would be devastating in the context of the debt overhang faced by many economies.

Within this framework and as a general point I find it important to emphasize that the global economy already decoupled from the US, and indeed OECD, economy some time ago. Consequently, it is an irrefutable fact that the global economy is currently undergoing fundamental change in which emerging economies such as India, Turkey, Brazil, China, Chile etc will ascend to account for an ever larger share of global GDP and growth. The crucial question is then; how will this process and the process of global ageing be transmitted to the global economy through capital flows?

It is well beyond the scope of this thesis to dig into a thorough answer of this question, but in order to provide a perspective, I would like to draw the attention to comments made by two of the most

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[52] With the German surplus mainly materializing itself in an intra-European imbalance.

prominent members of the global financial punditry in the form of US economist Paul Krugman (PK) and the Financial Times' chief economics commentator Martin Wolf (MW).

Starting with the former[53] he recently pointed to the fact that external demand was instrumental in ending the slump in Japan that provided a relative economic bounce between 2003 and 2007. PK further goes on to argue how this may present a rather ominous outlook since the extent to which we are all, in the OECD, currently stuck in a "Japan-style" liquidity trap the way out may constitute a rather crowded route. As PK poignantly points out;

> (…) needless to say, we can't all export ourselves out of a global slump. So, how does this end?
>
> Krugman 2009

This is indeed a good question and MW makes a similar argument in a recent column[54] where he points towards the fact that the global imbalances themselves may prove to be an impediment to a swift global recovery.

> In short, if the world economy is to get through this crisis in reasonable shape, creditworthy surplus countries must expand domestic demand relative to potential output. How they achieve this outcome is up to them. But only in this way can the deficit countries realistically hope to avoid spending themselves into bankruptcy.
>
> Martin Wolf (2008)

This is of course a very appealing proposition and also goes to the heart of the idea that, at least, one part of the solution to the current global crisis lies in the resolution of global macroeconomic imbalances. But prey tell; how are these surplus countries going to revert towards a growth path characterized by a more balanced external account and perhaps even an external deficit?

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[53] Paul Krugman (2009) – The Eschatology of Lost Decades, NYT blog post

[54] Martin Wolf (2008) – Global Imbalances Threatens the Survival of Free Trade

It is in this context that the argument presented in this thesis becomes important. Consequently, there is a big risk that these surplus economies (e.g. Japan, Germany and others to follow) simply will not be able to heed the call of MW. The main reason for this inability is then, in part, exactly to be found in the economic profile of a rapidly ageing economy with a median age pushing 40 year mark and beyond. Japan and Germany as well as the economies next in line to reach their age bracket cannot achieve growth based on domestic demand in a way which would allow them to suck up excess global capacity through an external deficit.

This point is very important note and it is my strong belief that the proper understanding of the nature and cause of global macroeconomic imbalances require a solid anchoring in the study of the effect of ageing where particularly the case of Japan and Germany are important test cases.

5.0 CONCLUSION

The main contribution of this thesis is to suggest how rapidly ageing societies will *not*, in the main, be characterised by aggregate dissaving but rather by the fight *against* it. In the context of international capital flows this means that rapidly ageing economies will be characterized by an external surplus and not, as theory would predict an external deficit. According to life cycle theory, savings by consumers should not affect total aggregate savings in the long run because savings today, by definition and through the idea of consumption smoothing, turns into consumption tomorrow. Yet, the analysis and discussion above suggests that this might be the wrong way to read the life cycle hypothesis. Consequently, this thesis proposes that ageing may be associated with the propensity to run an external surplus and how this may, in the limit, lead to a state of export dependency. The first part in particular is easily falsifiable. Consequently and while we should always remember that an economy cannot run an external surplus without another economy running a corresponding deficit, the analysis above is underpinned by the idea that Japan and Germany will not be able to run a consistent external deficit and that this will be the case for other economies who will inevitably follow in their path with respect to ageing.

Going back to the main research question sought illuminated by this thesis the answer was provided in two parts.

Firstly, chapter 2 discussed the concept of the demographic transition in the context of macroeconomic dynamics and presented, as its main argument, the need to focus on a change in age structure when talking about the DT. In this sense, it is important to realize that the DT is not over. Demographic processes are highly dynamic and it is unreasonable to assume that economies will settle in some form or variant of balanced state where demographic variables are constant. This exactly means that the DT is best narrated as a process of ageing and not one of population growth as it allows us to capture the complexity of the DT within a more dynamic theoretical framework than in the traditional account of the DT. Finally, this provides a strong foundation for macroeconomic analysis through the application of life cycle theory.

Secondly, chapter 3 attempted to form a theoretical and empirical framework to suggest how ageing may drive international capital flows with a particular focus on Japan and Germany as the oldest economies in the world. In particular, focus was put on the notion of dissaving as a function of old age and with this, the analysis became increasingly more complex. I believe it is important to emphasize two points.

One is the simple fact that the trajectory, pace, and nature of dissaving, although strongly anchored in basic intuition from life cycle theory, is very uncertain. Second, dissaving should not be seen as a deterministic outcome of ageing but rather it should be discussed not whether it may ultimately occur, but whether it is optimal from the point of view of the macroeconomy. The argument presented above suggests that it isn't. Specifically it outlines that while dissaving may indeed be a defining characteristic of a rapidly ageing economy in the limit, this will first and foremost manifest itself in ageing economies trying to fight dissaving. This suggests then that ageing economies may be expected to keep domestic savings above domestic investments which will result, all things equal, in an external surplus. This proposition was discussed in relation to Germany and Japan where it was shown to provide an important and relevant perspective on these two economies' growth path.

Finally, the findings of chapter 4 were used to form a perspective on the idea that as an ageing economy may be characterized by declining domestic demand at the same time as it develops a propensity to run an external surplus it may be driven towards a state of export dependency. This hypothesis was given an empirical treatment in the context of Japan and Germany where indicative evidence was presented to suggest that these two economies are indeed currently in a state of export dependency.

As a final note it is important to emphasize that there are many factors beyond demographics which may potentially drive and provoke international capital flows. Different market structures, different tax regimes, and difference in technology are but some of these and a full account of international capital flows would require a strong eye to such measures. This qualifier notwithstanding the evidence and analysis presented in this thesis suggest a strong and significant link between demographics and international capital flows. In particular, the notion of how demographics drive savings and investment dynamics in an open economy represents a powerful way to think about demographics and international capital flows.

Further studies on this topic should emphasize two inquiries. One is the study of the notion of export dependency and whether this may really be argued to be a function of ageing and secondly, there is a need to develop models to explain what the apparent propensity of ageing economies to run external surpluses mean in a general equilibrium context as well as, crucially, link this to the notion of global macroeconomic imbalances.

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APPENDICES

APPENDIX 1 – INTERTEMPORAL CONSUMPTION

In this appendix an indicative solution to an intertemporal consumption problem is presented. This is done mainly in order to clearly identify the goal posts with respect to the economic (and mathematical) methodology used in this thesis. Essentially and if we take a very crude overview, there are three solution methods most often deployed by economists in the context of utility maximization under constraint and an intertemporal perspective; one based on a Lagrangean, one based on the Bellman equation (Dynamic Programming) and finally, one set in continuous time using a Hamiltonian.

In the present context, the Lagrangean will be the weapon of choice for the optimization problems which is because this is the method used by the sources (papers and textbooks) used to develop the models.

The Lagrangean and the Intertemporal Dimension

The Lagrangean is a fundamental tool from calculus used to compute constrained optimization problems Sydsæter et al. (2008, pp. 115-124). In the context of neo-classical economics, the Lagrangean is widely used because we often find ourselves in a situation where we want to imbue our economic agents with a constraint in the form of a budget, or other forms of constraints. The key consideration in this case is naturally the presence of dynamics and thus the emergence of the famous intertemporal *Euler equation* as the optimality condition which links consumption in two periods together. In terms of the Euler equation and indeed the theorem which shows why the Euler equation is necessary to solve a dynamic optimization problem it involves quite elaborate mathematics (see e.g. Sydsæter et al. (2008, pp. 290-298)). The main reason is that the Euler condition for optimality is often cast in the context of continuous time optimization which means that we need to handle differential equations and calculus. Following Romer (2006, pp. 55 and 78) we can also cast the Euler equation in a discrete time version which makes the mathematics much easier without losing much in terms of complexity.

The solution method below will be general and closely resemble the method found in academic articles and advanced macroeconomics textbooks. However, I should pay tribute to economics professor Jean Imbs from HEC Lausanne for supplying many of the tips and tricks on how to represent these solutions in a concise way; especially the teaching slides for the Adv. Macro fall 09 and a Doctoral Seminar spring 09 were very helpful. Formally, problems can be found in Romer (2006, ch. 7, ch. 2-3) where similar solutions techniques are used freely to handle a wide range of analytical problems.

Consider consequently the following problem faced by the representative consumer (where we have already defined the value function). Here, the consumer maximizes the sum of discounted utilities by choosing consumption in every period.

A1- EQ. 1

$$V(t)\colon MAX_{Ct}\sum_{T=0}^{\infty} B^{s}(U_t)$$

$$S.T: a_{t+1} = (1+r)a_t + Y_t - C_t$$

$$u'(c_t) > 0$$
$$u''(c_t) < 0$$

Where (a) is equal to assets, (r) is the interest rate assumed constant in this case, (c) is consumption and finally (y) is income (also assumed exogenously given). The value of beta (β) is given by 0< β<1. This variable is a measure of our representative agent's patience. If beta is equal to 1 consumption tomorrow is of equal value to consumption today and if it is zero our agent only cares about consumption today.

The budget constraint states that our assets in time (t+1) is equal to the accrued return on our assets in time (t=0) + income minus consumption. Solving this equation for consumption yields the formal constraint

A1- EQ. 2

$$C_t = (1+r)a_t + Y_t - a_{t+1}$$

This equation simply states that that consumption in time (t=0) cannot exceed income (y) plus the return on our asset (a, t=0) minus whatever assets we *will choose to have* in time (t+1).

Since we are concerned with dynamics and thus the intertemporality of our consumer's decision making it might be a good idea to scrutinize the budget constraint. Note the following three distinct budget constraints which follow naturally from discrete iteration of the first constraint.

A1- EQ. 3

$$a_{t+1} = (1+r_t)a_t + y_t - c_t$$

$$a_{t+2} = (1+r_{t+1})a_{t+1} + y_{t+1} - c_{t+1}$$

$$a_{t+3} = (1+r_{t+2})a_{t+2} + y_{t+2} - c_{t+2}$$

Clearly, with an infinite horizon and assuming a problem formulated in discrete time, our consumer will face one of these identical constraints in each period. Through the use of *recursive substitution* we can express this infinite series of budget constraints and derive what is known as the intertemporal budget constraint. This can be shown by substituting (t+1) into (t+2) and finally this expression into (t+3):

A1- EQ. 4

$$a_{t+1} = (1+r_t)a_t + y_t - c_t$$

$$a_{t+2} = (1+r_{t+1})\left[(1+r_t)a_t + y_t - c_t\right] + y_{t+1} - c_{t+1}$$

$$a_{t+3} = (1+r_{t+2})\left[(1+r_{t+1})\left[(1+r_t)a_t + y_t - c_t\right] + y_{t+1} - c_{t+1}\right] + y_{t+2} - c_{t+2}$$

It is easy to see the recursive element here and realizing the product term originating from the interest rate (discount rate) and the sum (and product term from the interest rate) of income less consumption we get the following expression for the intertemporal budget constraint.

A1- EQ. 5

$$a_{t+n} = \prod_{j=1}^{n-1}(1+r_{t+j})(1+r_t)a_t + \sum_{s=0}^{n-1}\prod_{j=s+1}^{n-1}(1+r_{t+j})(y_{t+s} - c_{t+s})$$ [56]

Another way to see this relationship is of course to solve the budget constraint as a simple first order difference equation where y(t) minus c(t) equal savings at time t.

[56] Here the before mentioned Jean Imbs' lecture notes were very helpful.

A1- EQ. 6

$$a_t = (1+r_t)a_{t-1} + y_t - c_t$$
$$\Leftrightarrow$$
$$a_t = (1+r_t)^t a_{t-1} + \frac{y_t - c_t}{r_t}$$
$$\Leftrightarrow$$
$$a_t = (1+r_t)^t a_{t-1} + \frac{s_t}{r_t}$$

This equation simply states that assets at time t are equal to the return on accumulated assets + the flow of discounted savings.

Finally our consumer is subject to the transversality condition which states that the discounted value of assets (a) is equal to 0 in the limit.

A1- EQ. 7

$$\lim_{t\to\infty} \frac{a_t}{(1+r)^t} = 0$$

Basically, the transversality condition is needed to limit the solutions for infinite period dynamic optimization problems. Finally, we have the value of beta (β) which is 0< β<1. This variable is a measure of our representative agent's patience.

With these considerations out of the way, we can set up the following constrained maximization problem;

A1- EQ. 8

$$L = \sum_{s=0}^{\infty} \left(\beta^s U(c_{t+s}) + \lambda_{t+s} [y_{t+s} + (1+r_{t+s})a_{t+s} - c_{t+s} - a_{t+s+1}] \right)$$

The crucial thing to understand in these kinds of problems is the distinction between state and choice variables. Following the description of the problem above we choose assets and consumption. This leads to three first order conditions; (1), (2) and (3).[57]

A1- EQ. 9

$$\frac{\partial L}{\partial c_{t+s}} = \beta^s U'(c_{t+s}) - \lambda_{t+s} = 0$$
$$\frac{\partial L}{\partial a_{t+s}} = (1 + r_{t+s})\lambda_{t+s} - \lambda_{t+s-1} = 0$$
$$\frac{\partial L}{\partial \lambda_{t+s}} = y_{t+s} + (1 + r_{t+s})a_{t+s} - c_{t+s} - a_{t+s+1} = 0$$

The key to solve this problem is first of all to recognize that as always in such an optimization problem we would first of all like to solve out the Lagrange multipliers in order to get a closed form solution. Moreover, we would like to follow in the mental food steps of Irvin Fischer and Euler and express this solution as a dynamic relationship between consumption in period 0 and period t+1. Fortunately this is quite easy in the present case.

From (1) we get;

A1- EQ. 10

$$\beta^s U'(c_{t+s}) = \lambda_{t+s}$$

Substituting into (2);

$$(1 + r_{t+s})\beta^s U'(c_{t+s}) = \beta^{s-1} U'(c_{t+s-1})$$

[57] The third being the, somewhat trivial, budget constraint in the form of the partial derivative with respect to the lagrangean.

Setting s=1 we get;

$$(1+r_{t+1})\beta U'(c_{t+1}) = U'(c_t)$$

And thus the classic representation of the intertemporal Euler equation;

A1- EQ. 11

$$\frac{(1+r_{t+1})\beta U'(c_{t+1})}{U'(c_t)} = 1$$

This Euler equation shows exactly the relationship we wantes to establish, namely that the consumer's marginal utility in period (t+1) is equal to the marginal utility in period (t) discounted by the market discount rate as well as the subjective discount rate (beta).

This completes the solution of the Lagrangean in terms of defining an optimality condition. In order to proceed from here we would have to impose some ex-post conditions on the marginal utility and substitute back into the budget constraint to get a closed solution (this is naturally done in the problems/models used in the analysis).

APPENDIX 2 – DEMOGRAPHIC DATA

This appendix essentially presents graphs to accompany section 2.1 in the main text which discusses and presents selected demographic data. The raw data (and graphs) can also be scrutinized on the data CD that accompanies the print version of the thesis. First off, there is the methodology used by UN and in particular the fact that it operates on a low, medium and high interval. Since this thesis largely shies away from considering population projections it is of little importance, but for the data 2005-2010 the medium forecast projection is used. In terms of the definition of least, less and more developed the definitions are presented below.

More developed regions: *They comprise all regions of Europe plus Northern America, Australia/New Zealand and Japan (see definition of regions).*

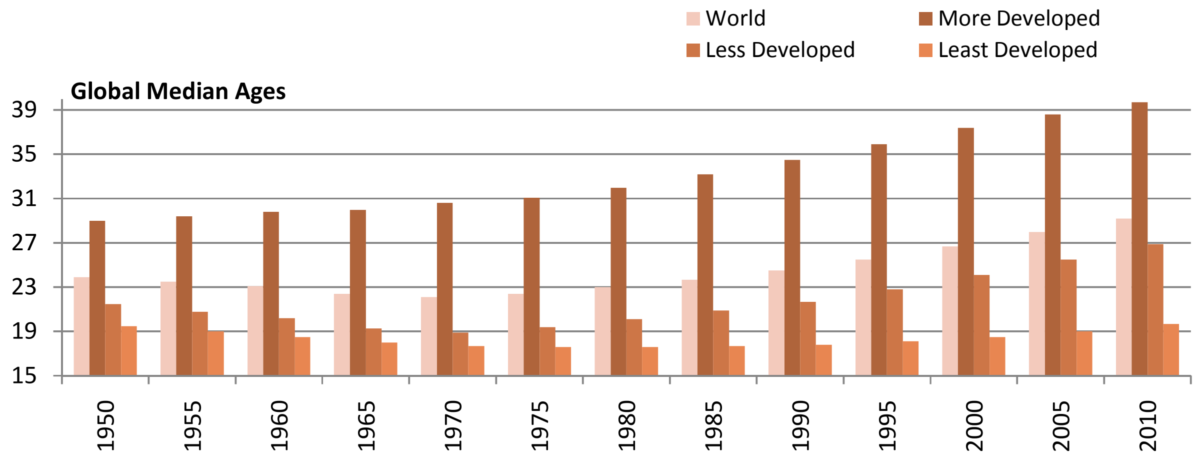
Less developed regions: *They comprise all regions of Africa, Asia (excluding Japan), Latin America and the Caribbean plus Melanesia, Micronesia and Polynesia (see definition of regions).*

Least developed countries: *The group of least developed countries, as defined by the United Nations General Assembly in 2003, comprises 50 countries, of which 34 are in Africa, 10 in Asia, 1 in Latin America and the Caribbean, and 5 in Oceania. The group includes 50 countries - Afghanistan, Angola, Bangladesh, Benin, Bhutan, Burkina Faso, Burundi, Cambodia, Cape Verde, Central African Republic, Chad, Comoros, Democratic Republic of the Congo, Djibouti, Equatorial Guinea, Eritrea, Ethiopia, Gambia, Guinea, Guinea-Bissau, Haiti, Kiribati, Lao People's Democratic Republic, Lesotho, Liberia, Madagascar, Malawi, Maldives, Mali, Mauritania, Mozambique, Myanmar, Nepal, Niger, Rwanda, Samoa, São Tomé and Príncipe, Senegal, Sierra Leone, Solomon Islands, Somalia, Sudan, Timor-Leste, Togo, Tuvalu, Uganda, United Republic of Tanzania, Vanuatu, Yemen and Zambia. These countries are also included in the less developed regions.*

In terms of the graphs, the first section will show the global data from the UN and the second section will plot the OECD data on fertility.

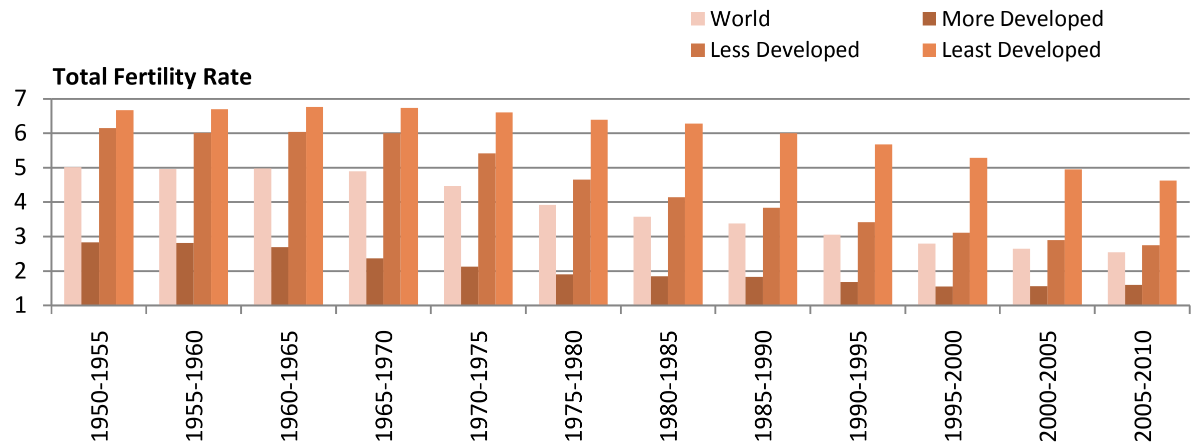
UN Data

A1- FIGURE 1 – GLOBAL MEDIAN AGES



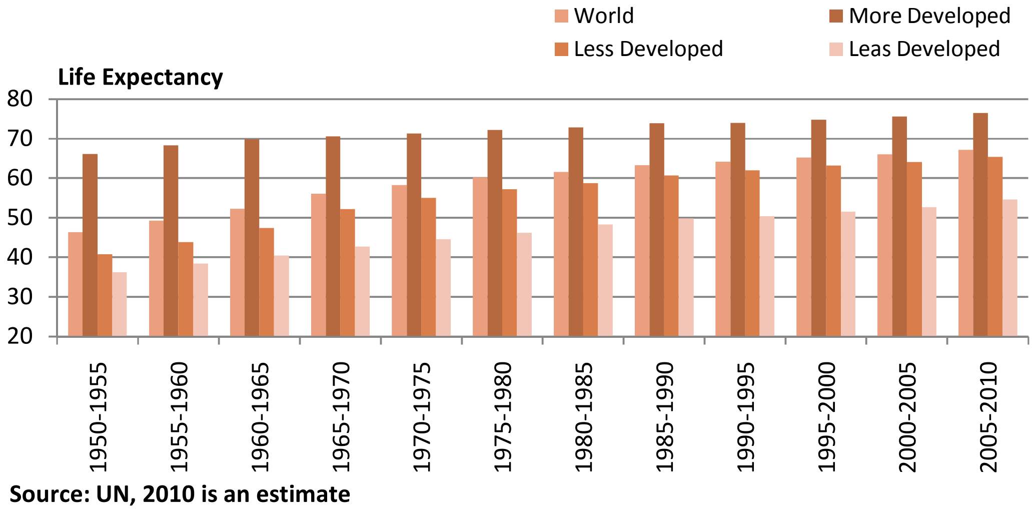
Source: UN, 2010 is an estimate

A1- FIGURE 2 – GLOBAL FERTILITY RATES

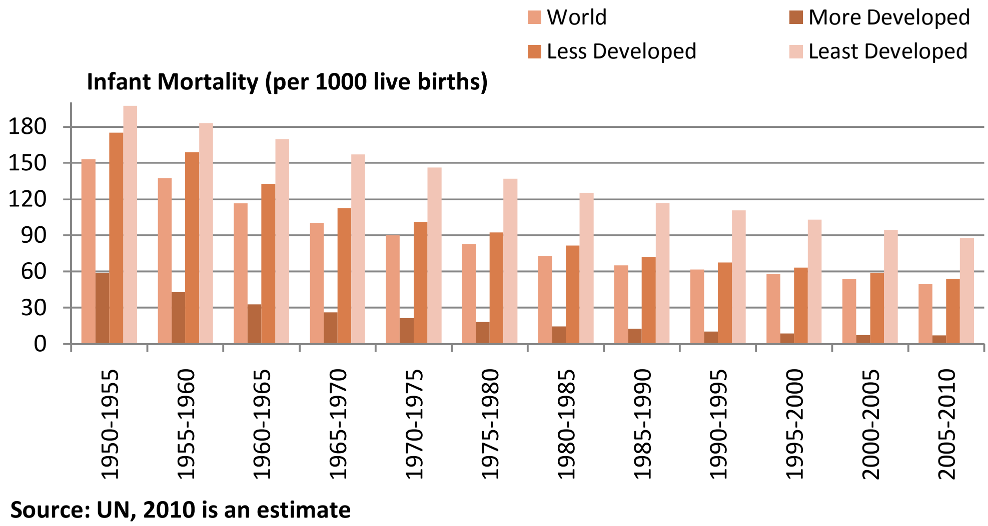


Source: UN, 2010 is an estimate

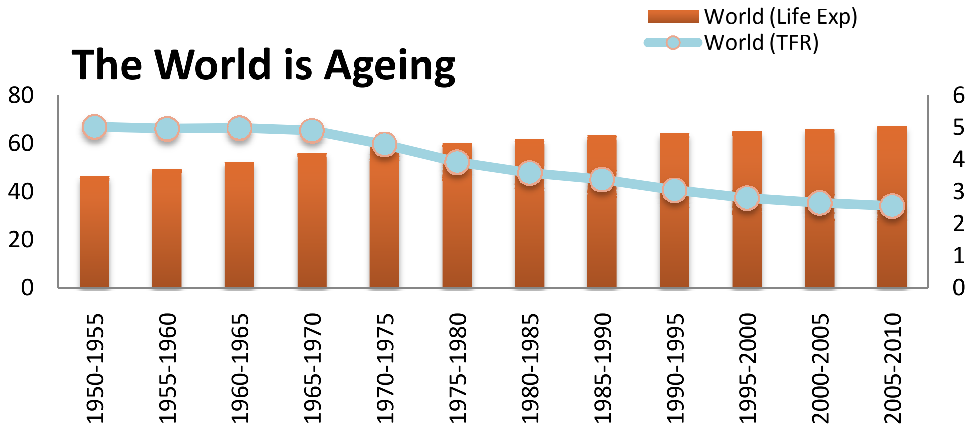
A1- FIGURE 3 – GLOBAL LIFE EXPECTANCY



A1- FIGURE 4 – GLOBAL INFANT MORTALITY



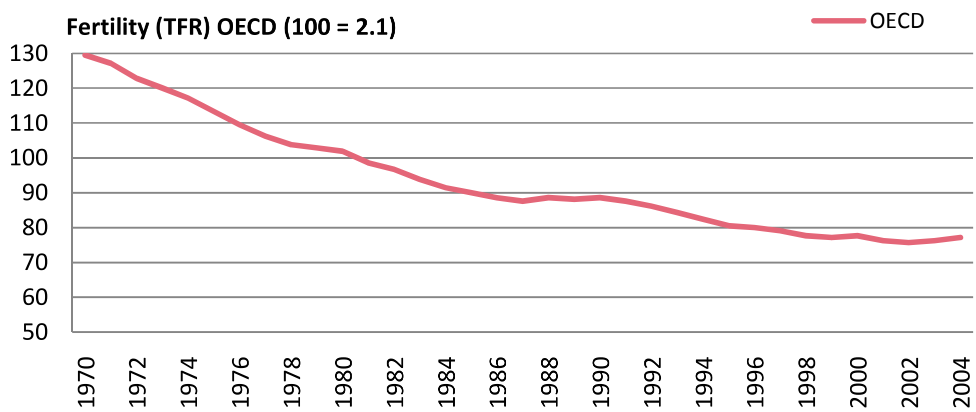
A1- FIGURE 5 – THE WORLD IS AGEING



Source: UN

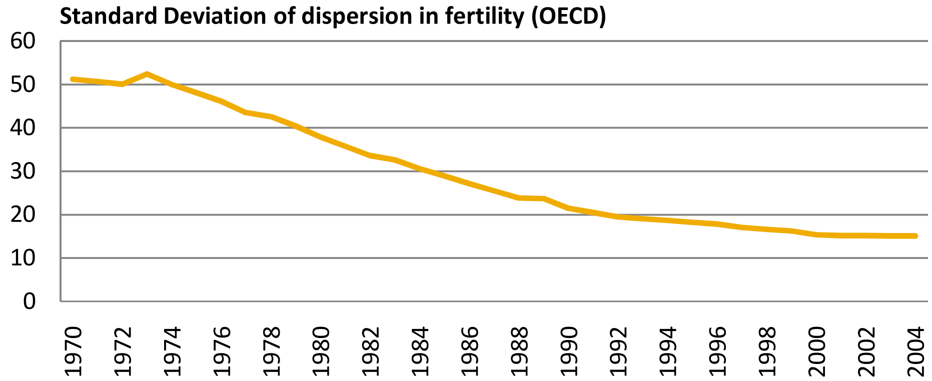
OECD Data

A1- Figure 6 – OECD Fertility Rate



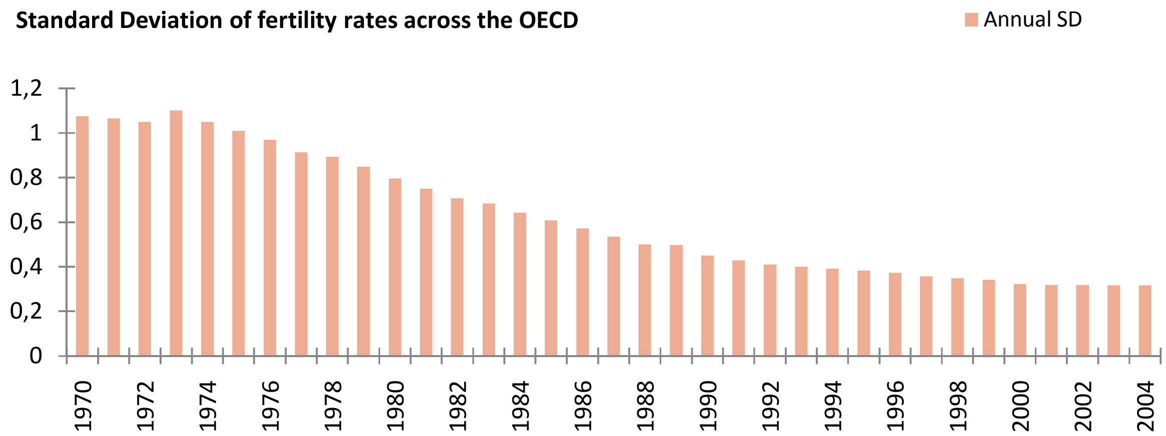
Source: OECD, index based on own calculations

A1- FIGURE 7 – STANDARD DEVIATION OF THE DISPERSION OF OECD FERTILITY RATES



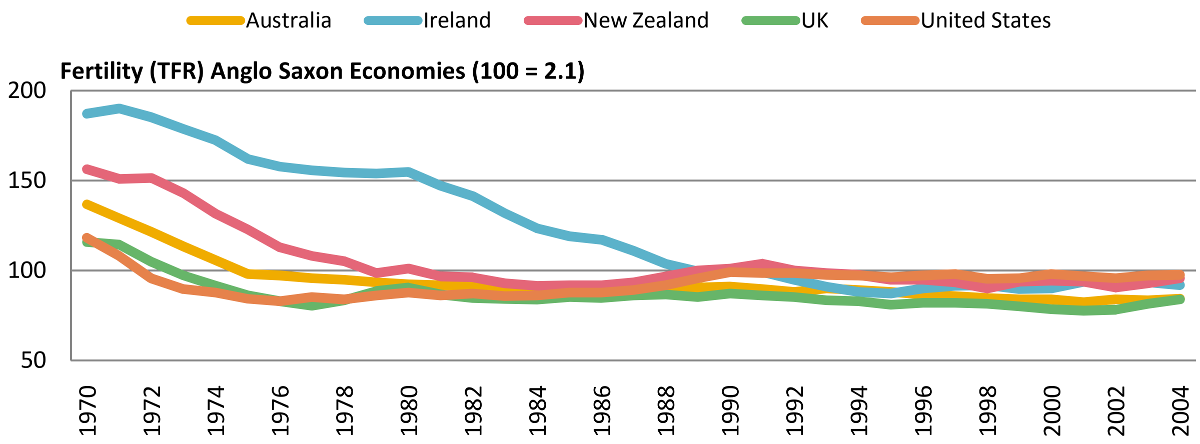
Source: OECD, own calculations. Measured in index points where 100=2.1.

A1- FIGURE 8 – STANDARD DEVIATION OF OECD FERTILITY RATES



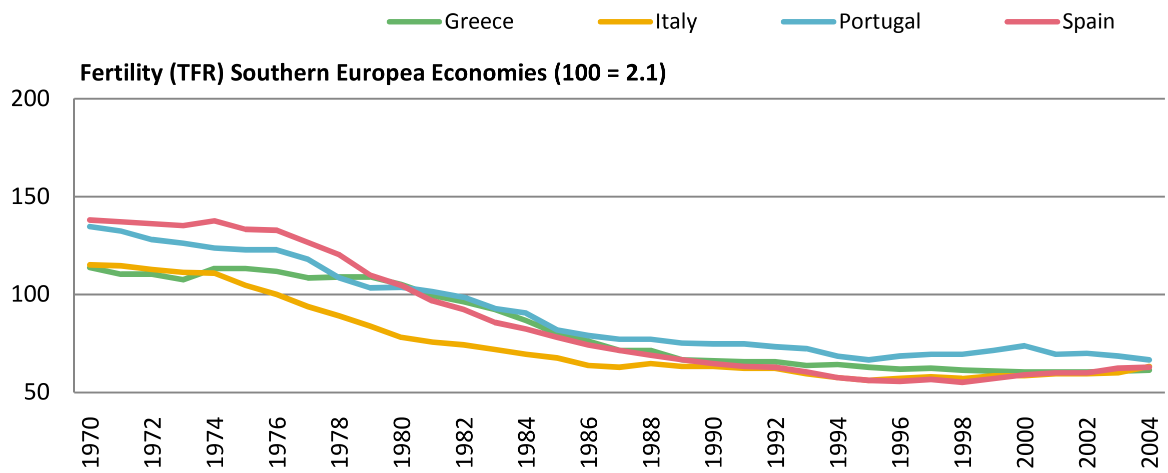
Source: OECD, own calculations

A1- FIGURE 9 – FERTILITY RATES IN THE ANGLO SAXON ECONOMIES



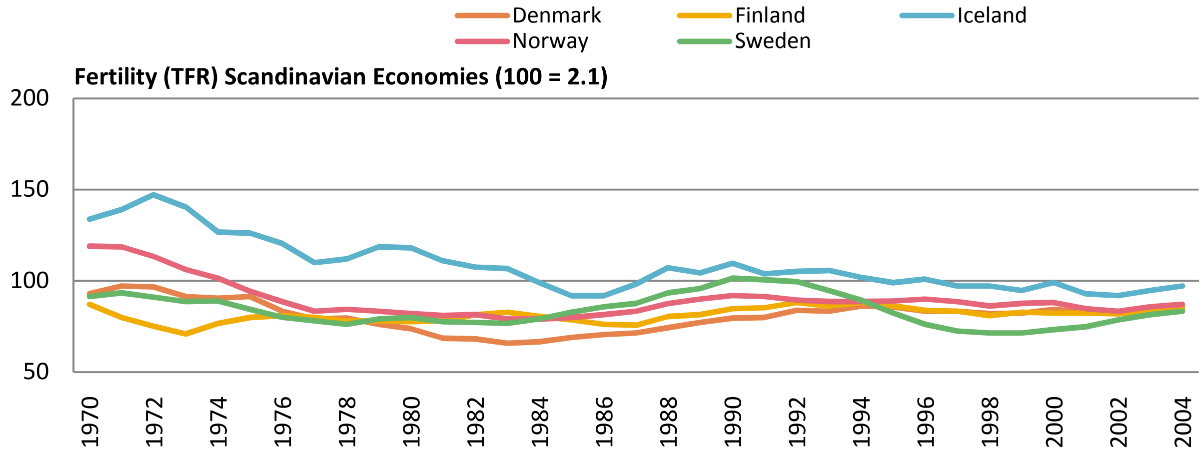
Source: OECD, index based on own calculations

A1- FIGURE 10 – FERTILITY RATES IN THE SOUTHERN EUROPEAN ECONOMIES



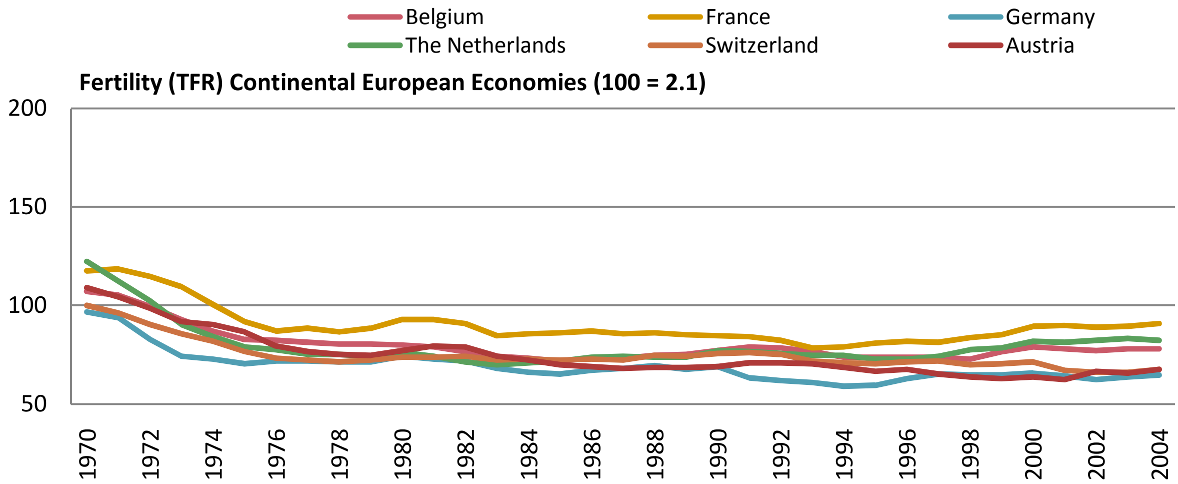
Source: OECD, index based on own calculations

A1- FIGURE 11 – FERTILITY RATES IN THE SCANDINAVIAN ECONOMIES



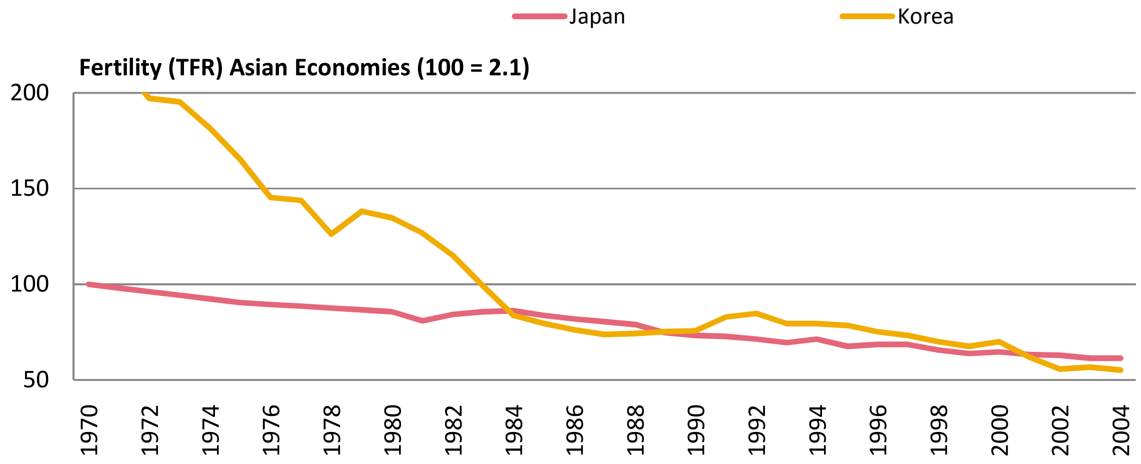
Source: OECD, index based on own calculations

A1- FIGURE 12 – FERTILITY RATES IN CONTINENTAL EUROPEAN ECONOMIES



Source: OECD, index based on own calculations

A1- FIGURE 13 – FERTILITY RATES IN SELECTED ASIAN ECONOMIES



Source: OECD, index based on own calculations

APPENDIX 3 – THE QUANTITY-QUALITY TRADE OFF

This appendix relates to the section concerning the quantum effect of fertility. Specifically, this appendix presents analytical derivations to the model presented in Becker (1960), Becker and Lewis (1973) and Becker and Tomes (1976). The derivations shown below serve two purposes; one is to show analytically the results presented in the main text and the second is to present the model in more rigorous mathematical detail than has been deemed possible in the text. The solution will be based on the methods and equations of Becker and Lewis (1973) and Becker and Tomes (1976);

Consider consequently the representative agent faced with the following problem;

A3-EQ. 1

$$\underset{(q,n,c)}{MAX}\left\{U=(n,q,c)\right\}$$

Subject to the classic budget constraint;

A3-EQ. 2

$$I = nq\pi + c\pi_c$$

Where (n) is the number of children, (q) is the quality of children, (c) is consumption of all other goods. Π_{nq} is the price of (nq) where as Π_c is the price of consumption and I is a measure of income. Setting up the problem gives us the following Lagrangean.

A3-EQ. 3

$$L = U(n,q,c) + \lambda[I - nq\pi_{nq} - c\pi_c]$$

The first order condition looks as follows where we choose both quantity (n), quality (q) and consumption of all other goods (c).

A3-EQ. 4

$$\frac{\partial U(n,q,c)}{\partial n} = U_n - \lambda q \pi_{nq} = 0$$

$$\frac{\partial U(n,q,c)}{\partial q} = U_q - \lambda n \pi_{nq} = 0$$

$$\frac{\partial U(n,q,c)}{\partial c} = U_c - \lambda \pi_c = 0$$

$$\frac{\partial U(n,q,c)}{\partial \lambda} = I - nq\pi - c\pi_c = 0$$

Now, in order to derive the required results we focus the attention on the first order conditions for quantity (n) and quality (q). Now, define the shadow prices of quantity and quality respectively as; $q\pi_{qn} = p_n$ as well as $n\pi_{qn} = p_q$ and you get the result used to explain the quantum effect in the text.

A3-EQ. 5

$$U_n = \lambda p_n$$

$$U_q = \lambda p_q$$

To derive the fundamental results from Becker and Lewis (1973) with respect to the income elasticity of quantity and quality we will work with Eq. 5. First however let us solve the problem by substituting out the Lagrange multiplier from the first order conditions to get a closed form solution for the demand for quantity and quality. Start by simply substituting out the Lagrange multipliers in the original first order conditions for quantity and quality. Focus e.g the attention on the equation for marginal utility with respect to quality.

A3-EQ. 6

$$\frac{\partial U(n,q,c)}{\partial q} = U_q - \lambda n \pi_{nq} = 0$$

And manipulate it as follows;

A3-EQ. 7

$$U_q - \lambda n \pi_{nq} = 0$$
$$U_q = \lambda n \pi_{nq}$$
$$\frac{U_q}{n\pi_{nq}} = \lambda$$

Substitute this into the equation for marginal utility of quantity;

A3-EQ. 8

$$U_n = \frac{U_q}{n\pi_{nq}} q\pi_{nq}$$
$$\Leftrightarrow$$
$$U_n = \frac{U_q}{n} q$$

Now we can express the demand for quality with respect to quantity (and vice versa) as well as quantity and quality as a function of the closed form solution variables. We get two symmetric results for both quantity and quality.

A3-EQ. 9

$$\frac{U_n}{U_q} = \frac{q}{n} \Leftrightarrow \frac{U_q}{U_n} = \frac{n}{q}$$

$$q = \frac{U_n}{U_q} n \Leftrightarrow n = \frac{U_q}{U_n} q$$

Define now the ratio of the marginal utilities as γ . If γ = 1 the demand for quality will be equal to the demand for quantity (for a given level of income). If γ < 1 the demand for quality will be less than the demand for quantity (for a given level of income) and finally if γ > 1 the demand for quality will be greater than the demand for quantity (for a given level of income). It is interesting in this regard to see that this standard solution makes it very difficult to derive the result Becker (presumably) wanted to achieve. For starters, it is dubious to assume that (q) and (n) can be measured in the same quantity which makes the demand function rather innocuous. Secondly, and in strict mathematical terms the term coined above as γ is by definition 1 when we present the problem as is the case here where the Lagrangean has been solved out[58]. This could be amended by discriminating between the price of quantity and quality in the budget constraint or by expressing the problem with separable utility functions for quality and quantity. A more direct way of course would be to allow the marginal utilities to change over time (rather difficult to pull off directly in the current mathematical context) or more specifically to allow their change with respect to an exogenously growing variable (income or prices) to be different. As it turns out, the latter approach is exactly the one taken by Becker in showing the quantity-quality trade off.

In the following I am going to show the income effect which is particularly important since it will show us how to use Becker's ideas in a general perspective of the demographic transition which is marked by a steady process of growth in income.

Consequently, we can go back to Eq. 5 and define the (true) income elasticities of (c), (n), and (q) as [$\eta_{c,}\eta_{n,}\eta_{q,}$]; which is found by changing income while holding prices constant. Now, as Becker and Lewis (1973) pp. 82-83 points out, the *appropriate* prices with which to measure this effect are the marginal costs:

A3-EQ. 10

$$
\begin{aligned}
q\pi_{nq} &= p_n \\
n\pi_{nq} &= p_q \\
c\pi_c &= p_c
\end{aligned}
$$

Whereas the appropriate income measure is total amount spent on (n), (q) and (c) Becker and Lewis (1973) p. 83. Define thus R in the same way as Becker and Lewis (1973) equation (4) p. 83.

[58] This is to say that in the simplest form and without assuming making explicit assumptions about the functional form of the utility function (or price and income effects) or discriminating between the price of quality and quantity we have to conclude that q=n.

A3-EQ. 11

$$R = nq\pi_{nq} + qn\pi_{nq} + c\pi_c = I + nq\pi_{nq}$$
$$\Leftrightarrow$$
$$R = np_n + qp_q + cp_c = I + nq\pi_{nq}$$

Where we have made use of the expression of the budget constraint given by (2). Introducing the income elasticities "defined" above we can write the expression for income as follows[59];

A3-EQ. 12[60]

$$(\Delta)R = (np_n)\eta_n + (qp_q)\eta_q + (cp_c)\eta_c$$

Now, let us follow the example in Becker and Lewis (1973) p. 84 and let η_n be equal to 0. Clearly, we now have the effect described in the main text. An increase in income will increase (q) and (c) and hold n constant due to the assumption that the income elasticity of quantity is 0. The crucial point is that while one would expect quantity to remain constant this is not the case due to the definition of the shadow prices with respect to quantity and quality. Thus the shadow price of quantity with quality ($p_n = q\pi_{qn}$) is a positive function of (q) and as (q) will rise in the context of an increase in income the price of quantity will increase and thus (n) must fall (assuming of course the standard behavior with respect to price elasticity of demand). This is exactly the quantity-quality trade off.

As a final note it might be interesting to look at how plausible the income elasticity assumption with respect to quantity really is. As it turns out, it may be a bit more complicated than that. For example Becker and Tomes (1976) suggests that quantity may exhibit a non-linear relationship with respect to an exogenously given increase in income. The main point here is that while the initial response to an increase in income may be to lower quantity it may later turn into a positive relationship as the increased cost of producing children of very high quality declines due to a very well endowed family (perhaps a dynastic motive is inherent in this).

[59] Where we are looking past the discussion about true versus observed income elasticities.

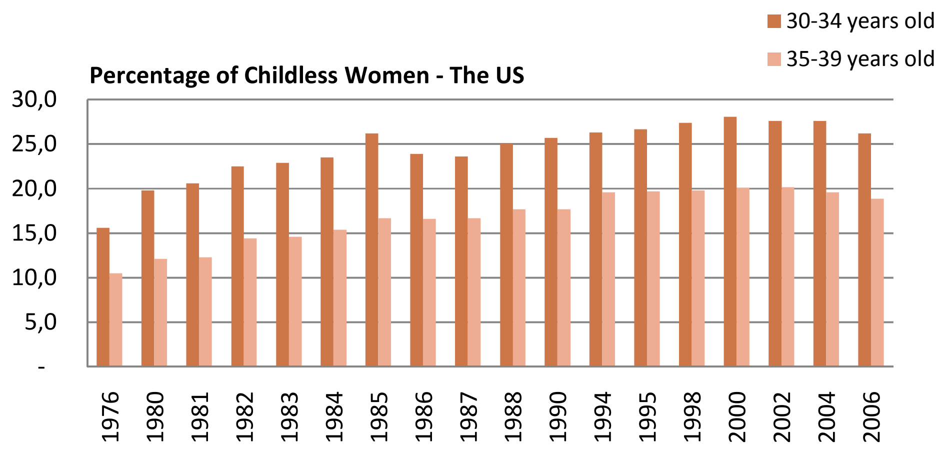
[60] Adopted from equation (5) in Becker and Lewis (1973) p. 83.

APPENDIX 4 – THE TEMPO EFFECT OF FERTILITY

This appendix presents a number of graphs to illustrate the tempo effect in a European, American, as well as a Japanese context. Essentially, it is a bit of a hodge podge since it is not possible to retrieve perfectly symmetric data from the respective statistical offices and since the UN population database only has data on age specific fertility from 1995 an onwards this source has been deemed largely irrelevant to use here. The main messages of the graphs are conveyed in the main text and thus there will be no further explanation in this appendix. As for data sources local statistical offices, Sobotka (2007) as well as the UN population database have been used.

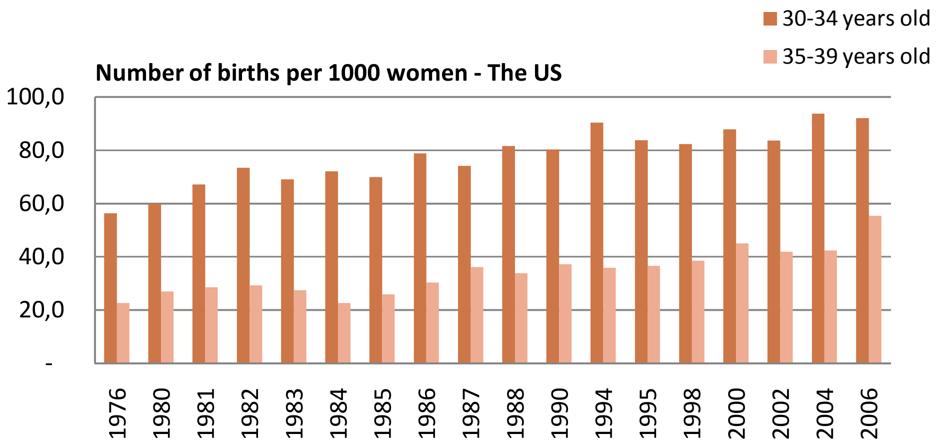
The US

A4-FIGURE 1 – PERCENTAGE OF CHILDLESS WOMEN IN THE US



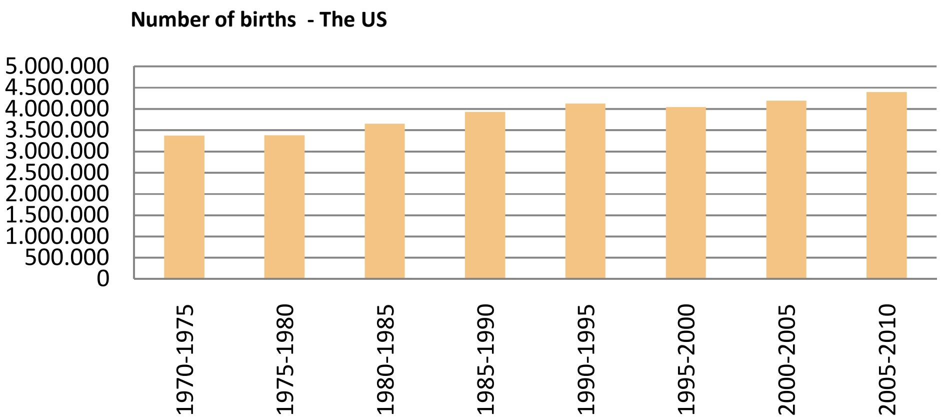
Source: US Census

A4-FIGURE 2 – NUMBER OF BIRTHS PER 100 WOMEN IN THE US



Source: US Census

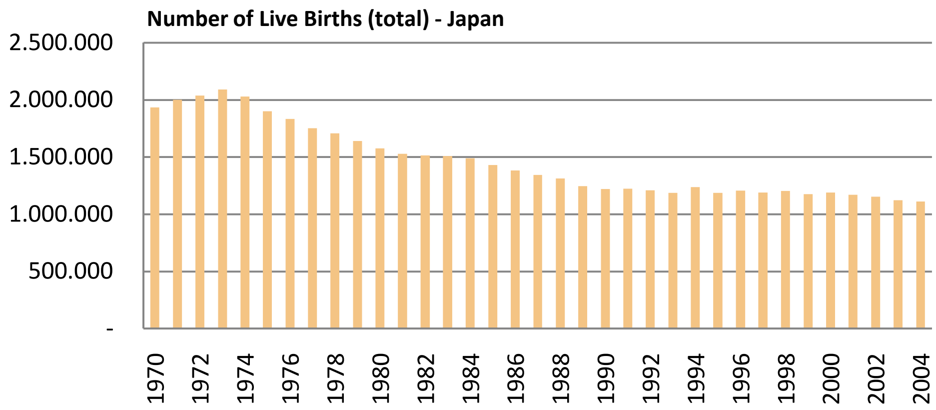
A4-FIGURE 3 – NUMBER OF BIRTHS IN THE US



Source: UN; 2010 is an estimate

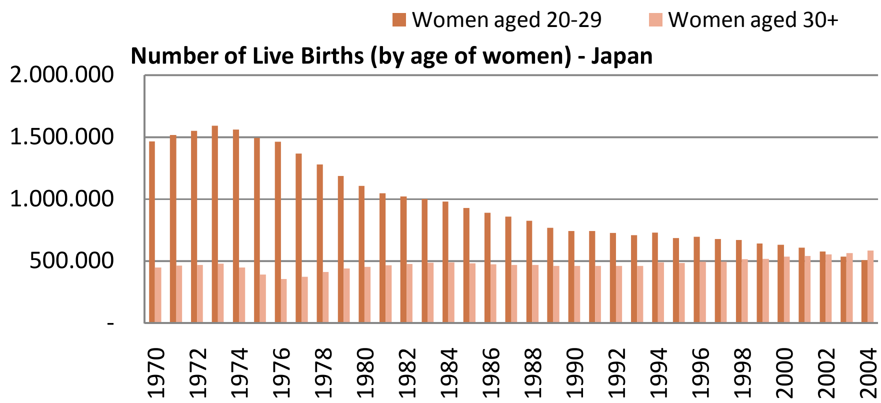
Japan

A4-FIGURE 4– NUMBER OF LIVE BIRTHS IN JAPAN



Source: Japan Statistical Office

A4-FIGURE 5– NUMBER OF LIVE BIRTHS (BY AGE AND WOMEN) IN JAPAN

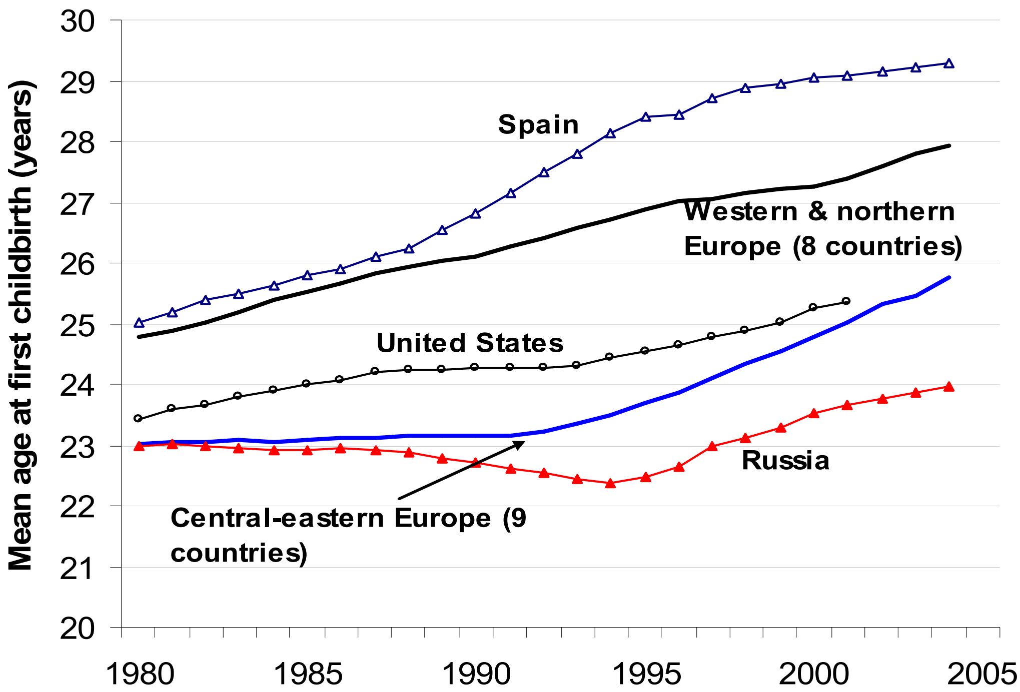


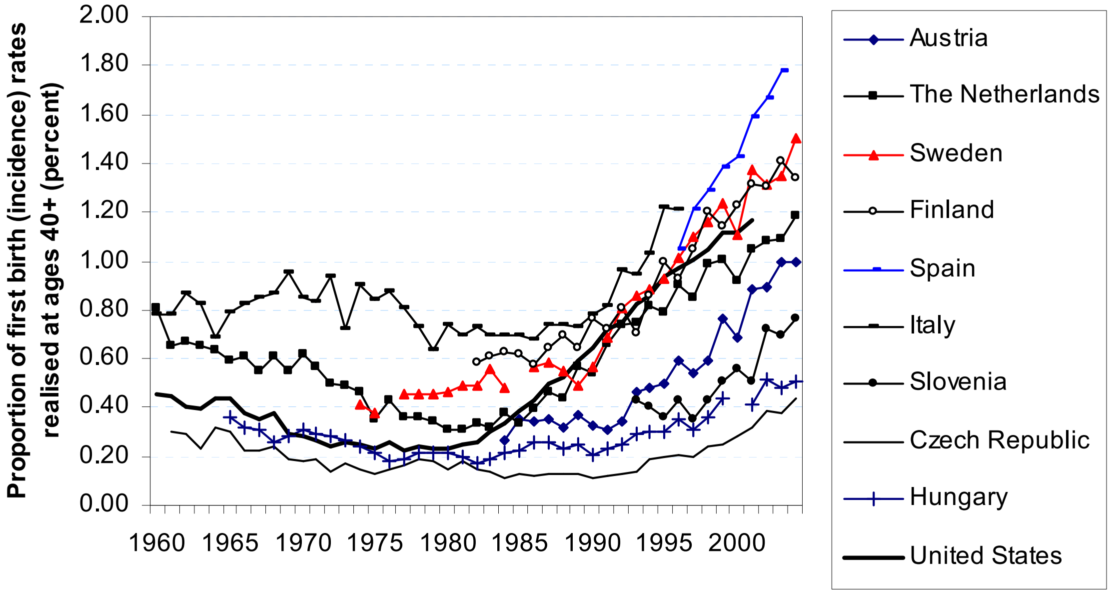
Source: Japan Statistical Office

Europe

In terms of Europe it is difficult to separate EU-15 from EU-12(3) and thus to exclude Eastern Europe. In the following graphs which are taken from Sobotka (2007) the data will consequently be mixed. Note in particular that Sobotka (2007) also includes the United States in the graphs below.

A4 -FIGURE 5 – TEMPO EFFECT GRAPHS FROM SOBOTKA (2007)





Proportion of first birth (incidence) rates realised at ages 40+ (percent)
2.00
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
1960
1965
1970
1975
1980
1985
1990
1995
2000
Austria
The Netherlands
Sweden
Finland
Spain
Italy
Slovenia
Czech Republic
Hungary
United States

APPENDIX 5 – THE LIFE CYCLE MODEL [ANDO AND MODIGLIANI (1963) AND OLG]

This analytical derivation of the LHC is first based on the assumptions and equations in

Modigliani and Ando (1963). It is worthwhile initially to note the underlying assumptions of the model put forward by Modigliani and Ando (1963 pp. 57-59).

- *Assumption I* states that the utility function is homogenous in relation to consumption at different point in times.
- *Assumption II* simply states that the individual does not expect to receive or leave any inheritance.
- *Assumption III* is important in that it formally lays out the idea of consumption smoothing in the sense that it states how the consumer at any age plans to spend her resources evenly over the remainder of her life span.
- *Assumption IV* states that expected earnings and life spans are equal across consumers in the aggregate across all ages.
- *Assumption V* states that the rate of return on assets is constant and expected to be constant.

Although these assumptions are restrictive (especially VI and V) in terms of the model's usefulness to predict and essentially explain the empirical reality, the intuition derived is very strong.

The life cycle model begins with the following simple expression for consumption in year (t) which shows how current consumption is a function of total resources.

A5-EQ. 2

$$c_t = \Omega_t v_t$$

Where v_t is an expression for total resources and Ω_t is a proportionality factor. This factor depends on the functional form of the utility function, the age of the person, and the rate of return on assets. The expression for v_t is given by;

A5-EQ. 2

$$v_t = a_{t-1} + y_t + \sum_{r=t+1}^{N} \frac{E(y_t)}{(1+r_t)^{r-t}}$$

Where a_{t-1} represent the assets a person carry with her from the previous period and is basically in this case a state variable. y_t is current income[61] and $E(y_t)$ is expected income in the nth year of his life which spans N. This equation is a classic economic expression for an agent's total resources as the sum of net worth of assets carried over from the previous period plus current income in this period plus the present value of income the agent expects to earn over the remainder of his life span.

In order to proceed it is convenient to define $E(y_t)$ as average annual expected income as follows;

A5-EQ. 3

$$E(y_t) = \frac{1}{N-T} \sum_{r=T+1}^{N} \frac{E(y_t)}{(1+r_t)^{r-T}}$$

Combining (Eq. 2) with (Eq. 3) gives:

A5-EQ. 4

$$v_t = a_{t-1} + y_t + \sum_{r=t+1}^{N} \frac{\frac{1}{N-T} \sum_{r=T+1}^{N} \frac{E(y_t)}{(1+r_t)^{r-T}}}{(1+r_t)^{r-T}}$$

Substituting into (Eq. 1) yields:

A5-EQ. 5

$$c_t = \Omega_t y_t + \Omega_t (N-T) E(y_t) + \Omega_t a_{t-1}$$

[61] Nonproperty

Which in its aggregate form is expressed as follows

A5-EQ. 6

$$C_t = \Omega_t Y_t + \Omega_t (N - T) E(Y_t) + \Omega_t A_{t-1}$$

What follows now is a rather big leap of faith I think, but in the context of the analytical framework it is very important. Consider consequently equation (1.6) in Modigliani and Ando (1963, p. 58) where the aggregate consumption function is defined.

A5-EQ. 7

$$C_t = \alpha_1 Y_t + \alpha_2 E(Y_t) + \alpha_3 A_{t-1}$$

Although the step from (Eq. 6) to (Eq. 7) may seem innocuous it is quite significant since what we have now is an estimable equation and one that relates Ω_t to coefficients that can be estimated.

The only thing we need now to complete the model is to sort out an expression for $E(y_t)$. In this thesis, I follow the naïve proposition by Modigliani and Ando (1963, pp. 60-61) and assume that expected income is given by current income times a scaling factor. Thus;

A5-EQ. 8

$$E(Y_t) = \gamma Y_t$$

Where 0 < γ < 1. Substituting (8) into (7), we get:

A5-EQ. 9

$$C_t = \alpha_1 Y_t + \alpha_2 \gamma Y_t + \alpha_3 A_{t-1} \Leftrightarrow C_t = (\alpha_1 + \alpha_2 \gamma) Y_t + \alpha_3 A_{t-1}$$

This completes the derivation of the model. Equation (Eq. 9) is quite important in the sense that it defines aggregate consumption in the life cycle model.

The Overlapping Generations Framework

The OLG framework clearly represents the road most often chosen when researchers want to use and operationalize the life cycle hypothesis. As discussed in the main text its origins are the seminal contributions by Peter A. Diamond and Paul Samuelson (Diamond (1965) and Samuelson (1958)).Before, we begin it is important to remember that, in the following, superscript (y) will signify "young" and superscript (o) will signify "old". In short, in this model we assume that our representative individual lives for two periods and crucially; that she only earns income during her "young" years. Following the intuition from Rogoff and Obstfeld (1996), we also include a government that levies a lump sum tax on both labor income (in the case of the young) and asset income (in the case of the old)

Consider consequently the following problem facing the representative individual at the time of entry into the labor force.[62]

A5-EQ. 10

$$\underset{C}{MAX}\left[U(C_t^Y, C_{t+1}^O) = \ln\left(C_t^Y\right) + \beta \ln\left(C_{t+1}^O\right)\right]$$

$$s.t.$$

$$c_t^Y + \frac{c_{t+1}^O}{(1+r)} = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r}$$

Set up the constrained maximization problem as follows;

[62] We are abstracting here from birth and adolescence which would require the utility function to be parameterized with three periods in stead of two.

A5-EQ. 11

$$L = \ln\left(C_t^Y\right) + \beta \ln\left(C_{t+1}^O\right) + \lambda_t \left[y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} - c_t^Y + \frac{c_{t+1}^O}{(1+r)} \right]$$

$$\frac{\partial L}{\partial C_t^Y} = \frac{1}{C_t^Y} - \lambda_t = 0 \Leftrightarrow \frac{1}{C_t^Y} = \lambda_t$$

$$\frac{\partial L}{\partial C_t^O} = \frac{\beta}{C_{t+1}^O} - \frac{\lambda_t}{(1+r)} = 0 \Leftrightarrow \frac{\beta}{C_{t+1}^O} = \frac{\lambda_t}{(1+r)}$$

Combining these first order conditions yields;

A5-EQ. 12

$$\frac{(1+r)\beta}{C_{t+1}^O} = \frac{1}{C_t^Y} \Leftrightarrow \frac{C_{t+1}^O}{(1+r)\beta} = C_t^Y \Leftrightarrow C_t^Y (1+r)\beta = C_{t+1}^O$$

To derive the equations for consumption demand of the “young” and “old” (equations (12) and (13) in Rogoff and Obstfeld (1996, p. 134)), we can combine the budget constraint with the first order condition above.

For consumption by the young we have …

A5-EQ. 13

$$
\begin{aligned}
& c_t^Y + \frac{\beta(1+r)c_t^Y}{(1+r)} = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \\
& \Leftrightarrow \\
& c_t^Y + \beta c_t^Y = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \\
& \Leftrightarrow \\
& (1+\beta)c_t^Y = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \\
& \Leftrightarrow \\
& c_t^Y = \frac{1}{(1+\beta)}\left[y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \right]
\end{aligned}
$$

For consumption by the old we have …

A5-EQ. 14

$$
\begin{aligned}
& \frac{c_{t+1}^O}{(1+r)\beta} + \frac{c_{t+1}^O}{(1+r)} = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \\
& \Leftrightarrow \\
& \frac{c_{t+1}^O(1+r) + \beta(1+r)c_{t+1}^O}{\beta \cdot (1+r)^2} = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \\
& \Leftrightarrow \\
& \frac{c_{t+1}^O\left[(1+r) + \beta(1+r)\right]}{\beta \cdot (1+r)^2} = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \\
& \Leftrightarrow \\
& \frac{c_{t+1}^O\left[1+\beta\right]}{\beta(1+r)} = y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \\
& \Leftrightarrow \\
& c_{t+1}^O = \frac{\beta(1+r)}{1+\beta}\left[y_t^Y - \tau_t^Y + \frac{y_{t+1}^O - \tau_{t+1}^O}{1+r} \right]
\end{aligned}
$$

The crucial point to note in this with this model is the aggregate consumption function which is implied by the setup. Consequently, at all points in time we will have *two* representative individuals in the economy[63] and thus aggregated consumption must be made up of consumption by the young and the old;

A5-EQ. 15

$$C_t = c_t^Y + c_t^O$$

This is a very important conceptualization and represents perhaps the most important way economic scholars are use to represent the life cycle hypothesis as originally proposed by Modigliani and Brumberg.

[63] Hence the idea of *overlapping generations*

APPENDIX 6 – THE INTERTEMPORAL CURRENT ACCOUNT

In this appendix, detailed mathematical derivations will be presented of the intertemporal approach to the current account. The derivations are motivated by the solutions presented in Imbs (2009) and Obstfeld and Rogoff (1996).

The Benchmark Model

Consider first the model presented in Imbs (2009). It is essentially the same as the one in Obstfeld and Rogoff (1996) but since Imbs (2009) uses techniques similar to the ones presented in Appendix 1 this will be the main reference. As in the initial chapters of Obstfeld and Rogoff (1996) we have a small open economy that takes the world interest rate as given and following Imbs (2008)'s solutions we have an infinite horizon treatment which is also shown in Obstfeld and Rogoff (1996, pp. 63-78) . Obstfeld and Rogoff (1996) incorporate government and, more importantly, investment gradually; Imbs (2009) begins with a full representation with both government and investment. Consequently we have the following representative consumer maximization problem;

A6-EQ. 1

$$\underset{(c,b,k)}{MAX}(C) = \sum_{s=t}^{\infty} \beta^{s-t} U(c_s)$$

$$s.t.$$

$$C_t + I_t + G_t + B_{t+1} = (1+r)B_t + A_t F(K_t)$$

$$K_{t+1} = K_t + I_t$$

With "C" equal to consumption, "I" equal to investment, "G" equal to government spending, $A_t F(K_t) = Y_t$ (national income), "K" is equal to the capital stock for a given t and finally "B" is equal to net foreign assets at time (t). It is worth noting that we have no depreciation of the capital stock which implies that the capital stock grows without depreciation and thus that we have no replacement investments. Consequently, forward iteration of the capital accumulation rule yields.

A6-EQ. 2

$$K_{t+2} = K_t + I_t + I_{t+1}$$
$$K_{t+3} = K_t + I_t + I_{t+1} + I_{t+2}$$

$$K_{t+n} = K_t + \sum_{t=1}^{\infty} I_t$$

Before we solve the model it is very important to follow the initial remarks by Imbs (2009) and Obstfeld and Rogoff (1996) and the formal definition of the current account. In the following, we define $A_t F(K_t) = Y_t$. Consequently, start by solving the budget constraint for national output (income).

A6-EQ. 3

$$Y_t = C_t + (K_{t+1} - K_t) + G_t + B_{t+1} - B_t - RB_t$$

And solve for national savings as the sum of the net foreign asset position and investment;

A6-EQ. 4

$$B_{t+1} + K_{t+1} - (B_t + K_t) = Y_t + RB_t - C_t - G_t$$

Now, rewrite this as Imbs (2009):

A6-EQ. 5

$$B_{t+1} - B_t + K_{t+1} - K_t = Y_t + RB_t - C_t - G_t$$

$$\Leftrightarrow$$

$$CA_t + I_t = Y_t + RB_t - C_t - G_t$$

$$\Leftrightarrow$$

$$CA_t = Y_t + RB_t - C_t - G_t - I_t$$

$$\Leftrightarrow$$

$$CA_t = C_t + G_t + I_t + (X - M) + RB_t - C_t - G_t - I_t$$

$$\Leftrightarrow$$

$$CA_t = (X - M)_t + RB_t$$

This gives us the well known expression for the current account in any given period as the difference between imports and exports (goods surplus) plus the return on foreign assets which can be seen as net income in this case since we assume that foreigners also own some of our domestic assets. Thus RB(t) becomes the income balance. The final important result we need to establish is that savings do not necessarily have to equal investment in an open economy framework. This is important since this is what de-facto leads to intertemporal smoothing of consumption and investment on an aggregate level. Define consequently national savings as the sum of the current account and investment (as above). We get the following identity.

A6-EQ. 6

$$CA_t + I_t \equiv Y_t + rB_t - C_t - G_t$$

$$\Leftrightarrow$$

$$S_t \equiv Y_t + rB_t - C_t - G_t$$

$$\Leftrightarrow$$

$$CA_t \equiv S_t - I_t$$

This expression is crucial since it shows how us the current account exactly matches the difference between domestic savings and investment. If domestic savings exceed domestic investment the country is running a current account surplus and conversely, if domestic investment exceeds domestic savings the country is running a current account deficit.

The only thing we need before stating and solving the problem is to find an expression for the budget constraint in the context of infinite horizons. In this way, solve the budget constraint recursively for assets (bonds) in period (t).

A6-EQ. 7

$$(1+R)B_t = C_t + I_t + G_t - Y_t + B_{t+1}$$

$$\Leftrightarrow$$

$$(1+R)B_{t+1} = C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1} + B_{t+2}$$

$$\Leftrightarrow$$

$$B_{t+1} = \frac{C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1} + B_{t+2}}{(1+R)}$$

$$\Leftrightarrow$$

$$B_{t+1} = \frac{C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1}}{(1+R)} + \frac{\frac{C_{t+2} + I_{t+2} + G_{t+2} - Y_{t+2}}{(1+R)} + \frac{B_{t+3}}{(1+R)}}{(1+R)}$$

$$\Leftrightarrow$$

$$(1+R)B_t = C_t + I_t + G_t - Y_t + \frac{C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1}}{(1+R)} + \frac{\frac{C_{t+2} + I_{t+2} + G_{t+2} - Y_{t+2}}{(1+R)} + \frac{B_{t+3}}{(1+R)}}{(1+R)}$$

$$\Leftrightarrow$$

$$(1+R)B_t = \sum_{s=0}^{\infty} \frac{C_{t+s} + I_{t+s} + G_{t+s} - Y_{t+s}}{(1+R)^s} + \lim_{s\to\infty} \frac{B_{t+s}}{(1+R)^s}$$

Where the last expression is our intertemporal budget constraint. Moving on to the solution of the problem, the Lagrangean and its first order conditions write:

A6-EQ. 8

$$L = \sum_{s=0}^{\infty} \left[\beta^s U(C_{t+s}) + \lambda_{t+s} \left((1+R)B_{t+s} + A_{t+s}F(K_{t+s}) - C_{t+s} - K_{t+s-1} + K_{t+s} - G_{t+s} - B_{t+s} \right) \right]$$

$$\frac{\partial L}{\partial C_{t+s}} = \beta^s U'(C_{t+s}) - \lambda_{t+s} = 0$$

$$\frac{\partial L}{\partial B_t} = \lambda_{t+s}(1+R) - \lambda_{t+s-1} = 0$$

$$\frac{\partial L}{\partial B_t} = \lambda_{t+s} A_{t+s} F'(K_{t+s}) - \lambda_{t+s-1} + \lambda_{t+s} = 0 \Leftrightarrow \lambda_{t+s}[A_{t+s}F'(K_{t+s}) + 1] - \lambda_{t+s-1} = 0$$

In order to derive the Euler equations, we set (s=1) and use the first FOC for the marginal utility of consumption which equals the Lagrange multiplier.

A6-EQ. 9

$$\beta U'(C_{t+1})\left[A_{t+1}F'(K_{t+1}) + 1\right] = U'(C_t) \Leftrightarrow \frac{\beta U'(C_{t+1})\left[A_{t+1}F'(K_{t+1}) + 1\right]}{U'(C_t)} = 1$$

$$\beta U'(C_{t+1})(1+R) = U'(C_t) \Leftrightarrow \frac{\beta U'(C_{t+1})(1+R)}{U'(C_t)} = 1$$

This concludes the solution to the representative agent's maximization problem. The final thing before our benchmark model is concluded is to derive an expression for the dynamics of the current account over time. This follows from Imbs (2009) and Obstfeld and Rogoff (1996, pp. 70-76).

In order to proceed from the Euler equations above, we assume that $\beta(1+R) = 1$. This gives the following result from the first order condition.

A6-EQ. 10

$$\frac{U'(C_{t+1})}{U'(C_t)} = 1 \Leftrightarrow U'(C_{t+1}) = U'(C_t) \Leftrightarrow C_{t+1} = C_t = \overline{C}$$

Go back to the intertemporal budget constraint and rewrite it as follows;

A6-EQ. 11

$$(1+R)B_t = \sum_{s=0}^{\infty} \frac{C_{t+s} + I_{t+s} + G_{t+s} - Y_{t+s}}{(1+R)^s} + \lim_{s\to\infty} \frac{B_{t+s}}{(1+R)^s}$$

$$\Leftrightarrow$$

$$(1+R)B_t + \sum_{s=0}^{\infty} \frac{Y_{t+s} - G_{t+s}}{(1+R)^s} = \sum_{s=0}^{\infty} \frac{C_{t+s} + I_{t+s}}{(1+R)^s} + \lim_{s\to\infty} \frac{B_{t+s}}{(1+R)^s}$$

Substitute the constant value for consumption into the intertemporal budget constraint and apply the transversality condition that $\lim_{s\to\infty} \frac{B_{t+s}}{(1+R)^s} = 0$ as well as solve for the constant value of consumption Imbs (2009) and Obstfeld and Rogoff (1996, pp. 62-63 & pp. 70-71).

A6-EQ. 12

$$(1+R)B_t + \sum_{s=0}^{\infty} \frac{Y_{t+s} - G_{t+s}}{(1+R)^s} = \sum_{s=0}^{\infty} \frac{\overline{C}}{(1+R)^s} + \sum_{s=0}^{\infty} \frac{I_{t+s}}{(1+R)^s}$$

$$\Leftrightarrow$$

$$\overline{C} = \frac{R}{1+R}\left[(1+R)B_t + \sum_{s=0}^{\infty} \frac{Y_{t+s} - G_{t+s} - I_{t+s}}{(1+R)^s}\right]$$

This yields consumption as the annuity value of its discounted wealth Obstfeld and Rogoff (1996, p. 62) and Imbs (2009) and thus consistent with Friedman’s permanent income hypothesis. Now, recall one representation of the current account as;

A6-EQ. 13

$$CA_t = Y_t + RB_t - G_t - I_t - C_t$$
$$\Leftrightarrow$$
$$CA_t = Y_t + RB_t - G_t - I_t - \overline{C}$$

Substitute in the value derived above for the constant level of consumption.

A6-EQ. 14

$$CA_t = Y_t + RB_t - G_t - I_t - \frac{R}{1+R}\left[(1+R)B_t + \sum_{s=0}^{\infty}\frac{Y_{t+s} - G_{t+s} - I_{t+s}}{(1+R)^s}\right]$$
$$\Leftrightarrow$$
$$CA_t = Y_t + RB_t - G_t - I_t - \frac{(1+R)R}{1+R}B_t + \frac{R}{1+R}\left[\sum_{s=0}^{\infty}\frac{Y_{t+s} - G_{t+s} - I_{t+s}}{(1+R)^s}\right]$$
$$\Leftrightarrow$$
$$CA_t = Y_t + RB_t - G_t - I_t - RB_t + \frac{R}{1+R}\left[\sum_{s=0}^{\infty}\frac{Y_{t+s} - G_{t+s} - I_{t+s}}{(1+R)^s}\right]$$
$$\Leftrightarrow$$
$$CA_t = Y_t - G_t - I_t + \frac{R}{1+R}\left[\sum_{s=0}^{\infty}\frac{Y_{t+s} - G_{t+s} - I_{t+s}}{(1+R)^s}\right]$$

Now, to move forward from here we define the permanent level of a given variable (X) as its annuity value at the prevailing interest rate Obstfeld and Rogoff (1996, p. 74) and Imbs (2009).

A6-EQ. 15

$$\tilde{X} \equiv \frac{R}{(1+R)}\sum_{s=t}^{\infty}\frac{X_{t+s}}{(1+R)^s}$$

Using this result in the expression above for the current account gives a fundamental equation for the current account Obstfeld and Rogoff (1996, p. 74) and Imbs (2009).

A6-EQ. 16

$$CA_t = Y_t - G_t - I_t + \frac{R}{1+R}\left[\sum_{s=0}^{\infty}\frac{Y_{t+s} - G_{t+s} - I_{t+s}}{(1+R)^s}\right]$$

$$\Leftrightarrow$$

$$CA_t = Y_t - \frac{R}{1+R}\left[\sum_{s=0}^{\infty}\frac{Y_{t+s}}{(1+R)^s}\right] - G_t - \frac{R}{1+R}\left[\sum_{s=0}^{\infty}\frac{G_{t+s}}{(1+R)^s}\right] - I_t - \frac{R}{1+R}\left[\sum_{s=0}^{\infty}\frac{I_{t+s}}{(1+R)^s}\right]$$

$$\Leftrightarrow$$

$$CA_t = (Y_t - \tilde{Y}_t) - (G_t - \tilde{G}_t) - (I_t - \tilde{I}_t)$$

This equation gives a fundamental way to think about the current account. Whenever output is above its permanent level, the CA will tend towards surplus. Whenever government spending is above its permanent level the CA will tend towards deficit and crucially, whenever investment is above its permanent level the CA will tend towards deficit Obstfeld and Rogoff (1996, pp. 74-75) and Imbs (2009).

A Model with Life Cycle characteristics

There are essentially three ways in which to incorporate life cycle characteristics into the framework above. One is to follow, as it were, the *convention* of the literature and apply an overlapping-generations-framework (OLG) as is done in Obstfeld and Rogoff (1996, ch. 3). This approach follows the seminal work from Diamond (1965) and Samuelson (1958). A second option is to follow the ideas expressed in a recent article by economist César Molinas in which he uses the idea that the subjective discount rate changes with age Molinas (2009). As we shall see below, by using a different utility function you can apply this intuition. Finally, there is a third possibility which is to modify the budget constraint with the consumption function derived in the context of Modigliani and Ando (1963). In the following, options one and two will be discussed and derived.

The Isoelastic Utility Function

Ler us begin with the same setup as above but this time with a less general utility function. Specifically, let us assume that the utility function is isoelastic and thus that it takes the following form;

A6-EQ. 17

$$U(C_s)=\frac{C^{1-1/\sigma}}{1-1/\sigma}$$

With this utility function we get the following problem;

A6-EQ. 18

$$\underset{(c,b,k)}{MAX}(C)=\sum_{s=t}^{\infty}\beta^{s-t}\frac{C^{1-1/\sigma}}{1-1/\sigma}$$

$$s.t.$$

$$C_t+I_t+G_t+B_{t+1}=(1+r)B_t+A_tF(K_t)$$

$$K_{t+1}=K_t+I_t$$

Set up the constrained maximization problem the same way as before;

A6-EQ. 19

$$L=\sum_{s=0}^{\infty}\left[\beta^s\frac{C_{t+s}{}^{1-1/\sigma}}{1-1/\sigma}+\lambda_{t+s}\left((1+R)B_{t+s}+A_{t+s}F(K_{t+s})-C_{t+s}-K_{t+s-1}+K_{t+s}-G_{t+s}-B_{t+s}\right)\right]$$

$$\frac{\partial L}{\partial C_{t+s}}=\frac{\beta^s 1-1/\sigma C_{t+s}{}^{-1/\sigma}}{1-1/\sigma}-\lambda_{t+s}=0\Leftrightarrow\beta^s C_{t+s}{}^{-1/\sigma}=\lambda_{t+s}$$

$$\frac{\partial L}{\partial B_t}=\lambda_{t+s}(1+R)-\lambda_{t+s-1}=0$$

$$\frac{\partial L}{\partial B_t}=\lambda_{t+s}A_{t+s}F'(K_{t+s})-\lambda_{t+s-1}+\lambda_{t+s}=0\Leftrightarrow\lambda_{t+s}[A_{t+s}F'(K_{t+s})+1]-\lambda_{t+s-1}=0$$

Since it is only the utility function that changes, we can focus our attention on the Euler equation for consumption. As per usual, we use the first order condition with respect to (Bt). Setting s=1, this gives us;

A6-EQ. 20

$$
\begin{aligned}
&\beta^1 C_{t+1}{}^{-1/\sigma}(1+R) - \beta^0 C_t{}^{-1/\sigma} = 0 \\
&\Leftrightarrow \\
&\beta^1 C_{t+1}{}^{-1/\sigma}(1+R) = C_t{}^{-1/\sigma} \\
&\Leftrightarrow \\
&C_{t+1}{}^{-1/\sigma} = \beta^{-1} \cdot (1+R)^{-1} \cdot C_t{}^{-1/\sigma} \\
&\Leftrightarrow \\
&C_{t+1}{}^{\frac{-1/\sigma}{-1/\sigma}} = \left(\beta^{-1} \cdot (1+R)^{-1} \cdot C_t{}^{-1/\sigma}\right)^{\frac{1}{-1/\sigma}} \\
&\Leftrightarrow \\
&C_{t+1} = \beta^{\frac{-1}{-1/\sigma}} \cdot (1+R)^{\frac{-1}{-1/\sigma}} \cdot C_t^{\frac{-1/\sigma}{-1/\sigma}} \\
&\Leftrightarrow \\
&C_{t+1} = \beta^{\sigma}(1+R)^{\sigma} C_t
\end{aligned}
$$

Where the last term appears as equation (15) in Rogoff and Obstfeld (1996, p. 70). In order to reach an expression for the intertemporal current account, we must first solve for consumption in the budget constraint.

Recall the intertemporal budget constraint as;

A6-EQ. 21

$$
(1+R)B_t = \sum_{s=0}^{\infty} \frac{C_{t+s} + I_{t+s} + G_{t+s} - Y_{t+s}}{(1+R)^s} + \lim_{s\to\infty} \frac{B_{t+s}}{(1+R)^s}
$$

Use this to derive equation (16) in Rogoff and Obstfeld (1996, p. 71);

A6-EQ. 22

$$C_t = \frac{r+\vartheta}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]$$

Where $\vartheta = 1-(1+r)^{\sigma}\beta^{\sigma}$ and from here it is possible to derive a new expression of the current account;

A6-EQ. 23

$$CA_t = (Y_t - \tilde{Y}_t) - (G_t - \tilde{G}_t) - (I_t - \tilde{I}_t) - \frac{\vartheta}{1+r}W_t$$

With W(t) equal to $\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]$.

The key difference between this new expression for the current account and the benchmark case is the operationalization of the subjective discount rate β . Consequently, the current account is now driven by two factors; one is the smoothing factor which follows from the benchmark case and another is a tilting factor Rogoff and Obstfeld (1996, p. 76). This tilting factor basically means that for a $\vartheta > 0$ the country is relatively impatient and thus the current account is reduced (all things equal) whereas when $\vartheta < 0$ the country is relatively patient and the current account is increased (all things equal). One way to incorporate ageing and life cycle dynamics into this model would then be to let the subjective discount rate (and thus the parameter (ϑ)) be a direct function of age. This is, in some sense, what Molinas (2009) argues in the context of debt-deflation dynamics.

Overlapping Generations Framework and the Current Account

In order to derive an expression for the current account with overlapping generations, we follow the lead of Rogoff and Obstfeld (1996, ch. 3) and use the same model was derived earlier in the context of the description of the life cycle model (see appendix 5 for derivation)

The formal expression for the current account is similar, in intuition, to the one noted above. Consequently, we can express the current account as the difference between net assets in two subsequent periods. Specifically, the current account is equal to the sum of net private savings and net public savings.

A6-EQ. 24

$$CA_t = B_{t+1} - B_t = (B_{t+1}^P - B_t^P) + (B_{t+1}^G - B_t^G)$$

In appendix 5 it is shown how aggregate consumption in this setup is made up of the sum of consumption of the young and old;

A6-EQ. 25

$$C_t = c_t^Y + c_t^O$$

As a natural consequence of this aggregate consumption function, there must also exist an aggregate savings function which is essentially symmetric Rogoff and Obstfeld (1996, p. 136-137). Since the young starts out with no assets their savings are essentially equal to their accumulated assets after one period.

A6-EQ. 26

$$S_t^Y = B_{t+1}^P$$

Conversely, we assume (and perhaps erroneously so) that the old decumulate the assets which were accumulated while they were young. This leads to;

A6-EQ. 27

$$S_t^O = -S_t^Y = -B_t^P$$

Using these two results in the aforementioned expression for the current account naturally leads to;

A6-EQ. 28

$$CA_t = B_{t+1} - B_t = (S_t^Y + S_t^O) + (B_{t+1}^G - B_t^G)$$

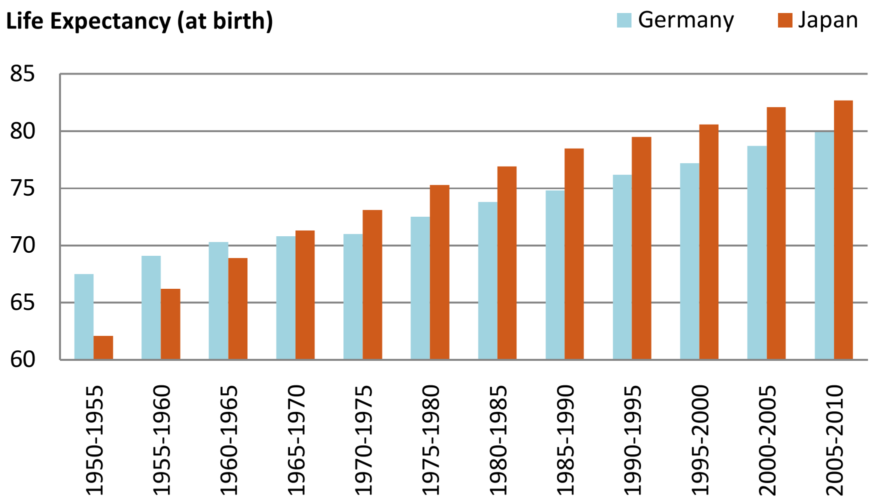
Thus and on an intuitive level, we have accomplished our mission and found and expression for the current account which directly includes ageing as an endogenous variable. The key of course then becomes how to operationalize the savings decision of the young and the old.

APPENDIX 7 – DEMOGRAPHIC DATA, GERMANY AND JAPAN

This appendix contains the graphs used in the description of the demographic transitions of Germany and Japan presented in section 4.1.4 in the main text.

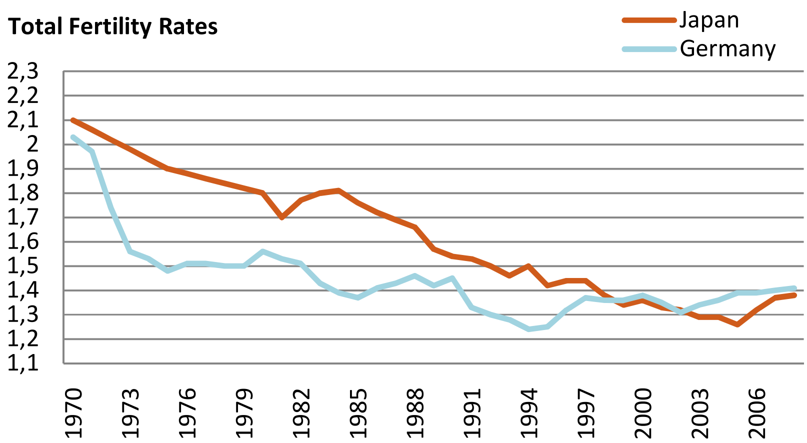
Fertility and Life Expectancy

A7-FIGURE 2 – LIFE EXPECTANCY IN GERMANY AND JAPAN



Source: UN

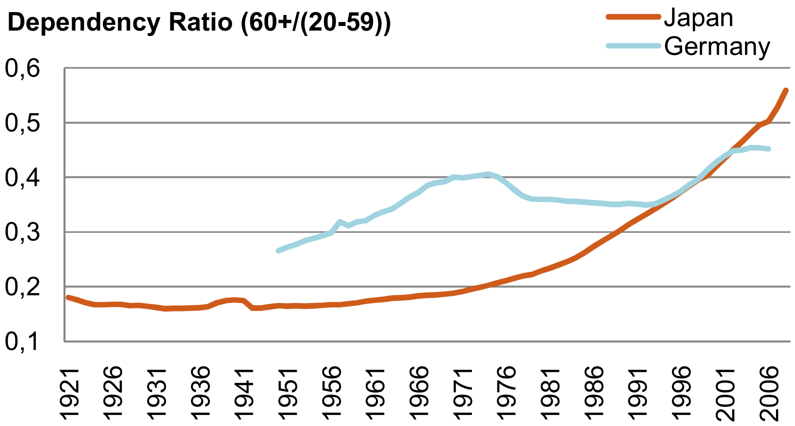
A7-FIGURE 2 – TOTAL FERTILITY RATES IN GERMANY AND JAPAN



Source: OECD and Nationmaster

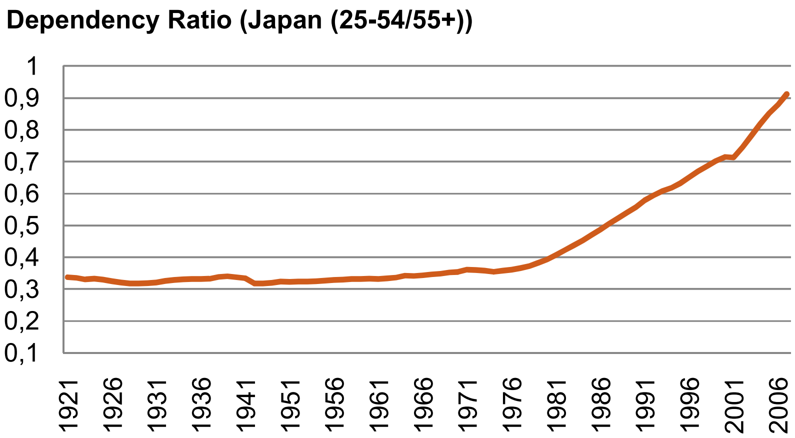
Dependency Ratios

A7-FIGURE 3 – DEPENDENCY RATIOS IN GERMANY AND JAPAN



Source: National Statistical Offices, Nationmaster, own calculations

A7-FIGURE 4 – DEPENDENCY RATIO IN JAPAN



Source: National Statistical Offices, Nationmaster, own calculations

APPENDIX 8 – DISSAVING IN JAPAN AND GERMANY

This appendix presents the summary stats, output and graphs used in Chapter 3. There will be more material in the appendix than in the main text. As a rule of thumb, only the most relevant and topical results in the form of graphs will feature in the main text. As always, the reader is advised to consult the Data CD for specifics.

A8 -FIGURE 1 – SUMMARY STATS (NATIONAL ACCOUNTS, QUARTERLY, OECD)

Germany - Summary Stats	GDP	C	G	I#
Average Growth Rate (yoy)	5,55%	5,44%	6,09%	4,90%
Standard Deviation	3,14%	3,05%	4,76%	6,69%
Maximum	13,32%	12,13%	20,20%	25,74%
Minimun	-0,08%	-0,01%	-2,01%	-13,95%
Skewness	0,376	0,267	0,807	0,127
Kurtosis	-0,707	-0,982	-0,125	0,483

Japan - Summary Stats	GDP (1960-2008)	GNI (1981-2008)	C	G	I
Average Growth Rate (yoy)	7,20%	2,78%	7,32%	8,29%	6,70%
Standard Deviation	6,44%	3,20%	5,99%	6,65%	8,99%
Maximum	20,85%	9,38%	21,64%	35,44%	29,56%
Minimun	-3,91%	-4,43%	-2,15%	-2,31%	-10,20%
Skewness	0,373	1,046	0,325	1,127	0,551
Kurtosis	-0,969	0,226	-1,102	1,522	-0,192

A8-FIGURE 2 – SUMMARY STATS (NATIONAL ACCOUNTS, YEARLY, IMF)

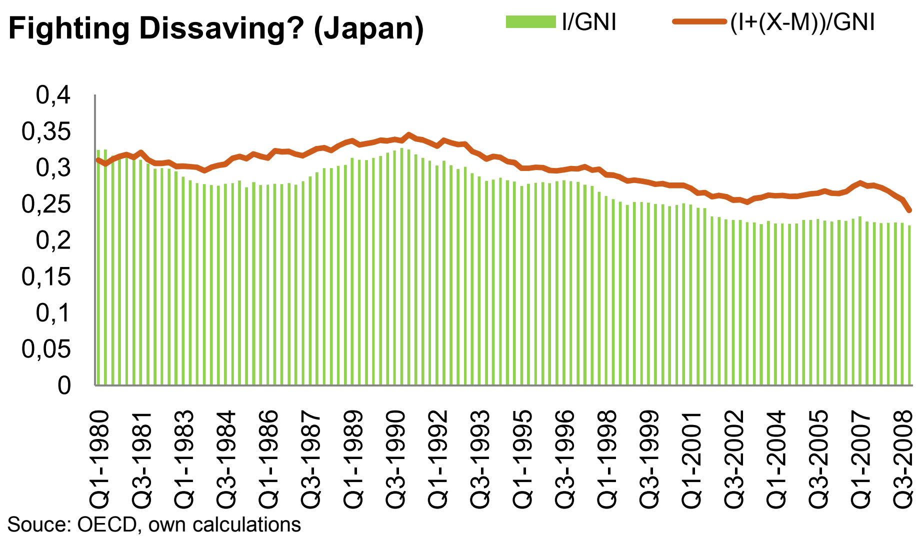
Germany - Summary Stats	GDP	I/GDP	S/GDP	CA/GDP
Average Growth Rate/Value	1,90	22,15	21,91	1,18
Standard Deviation	1,60	2,93	1,83	2,49
Maximum	5,72	28,16	26,19	6,13
Minimum	-0,79	16,87	19,28	-1,73
Skewness	0,423	-0,402	0,651	0,513
Kurtosis	0,238	-0,301	-0,176	-1,193

Japan - Summary Stats	GDP	S/GDP	I/GDP	CA/GDP
Average Growth Rate/Value	2,60	27,93	30,36	2,46
Standard Deviation	2,13	3,12	2,61	1,17
Maximum	7,15	32,72	34,34	4,26
Minimum	-2,05	22,85	25,93	-1,00
Skewness	0,141	-0,199	-0,186	-1,095
Kurtosis	0,013	-1,050	-1,072	1,723

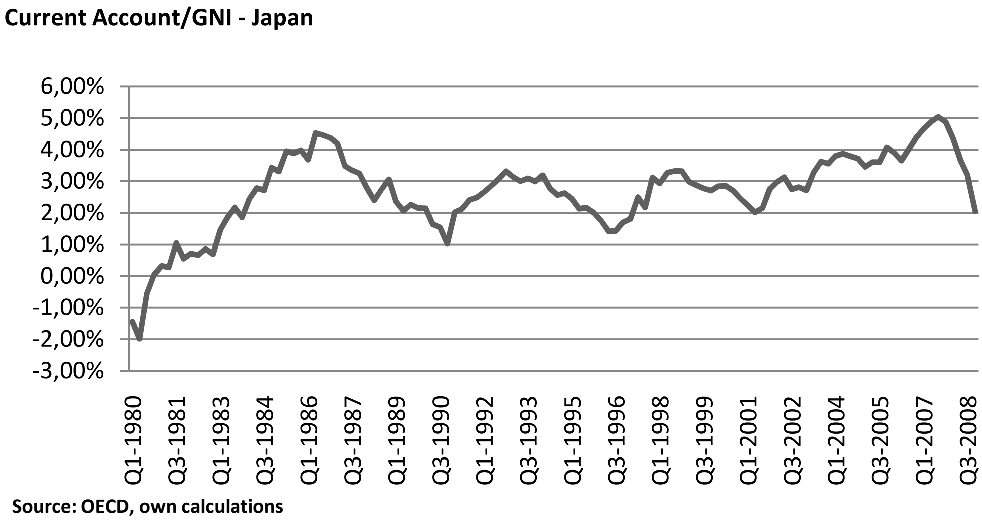
Descriptive Graphs on Investment and the External Balance

Japan

A8-FIGURE 3 – FIGHTING DISSAVING (JAPAN)

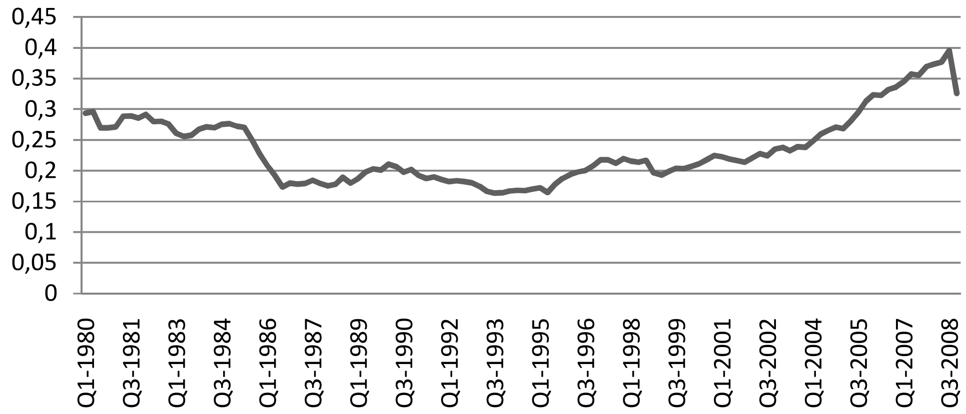


A8-FIGURE 4 – CURRENT ACCOUNT/GNI (JAPAN)



A8-FIGURE 5 – OPENNESS (JAPAN)

Openness [(I+M+income balance)/GNI] - Japan

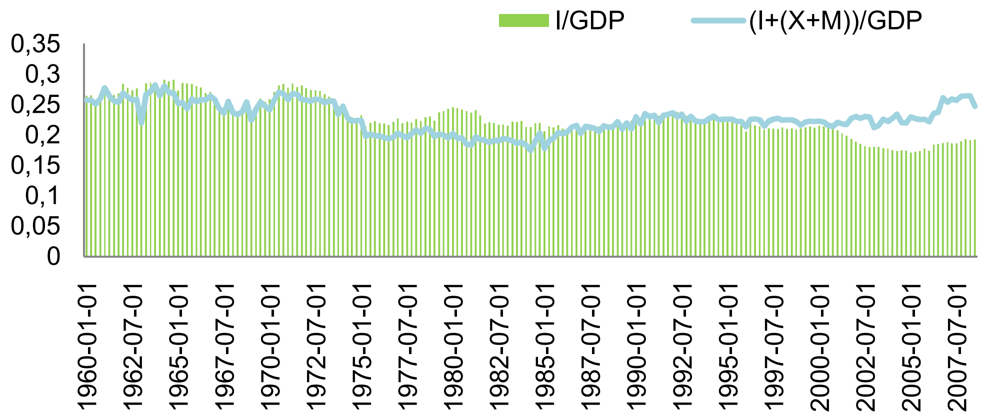


Source: OECD, own calculations

Germany

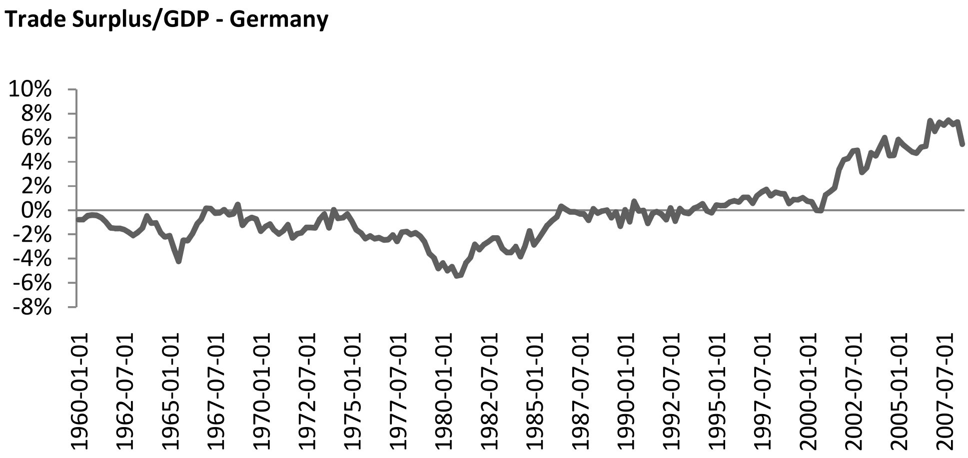
A8-FIGURE 6 – FIGHTING DISSAVING (GERMANY)

Fighting Dissaving? (Germany)



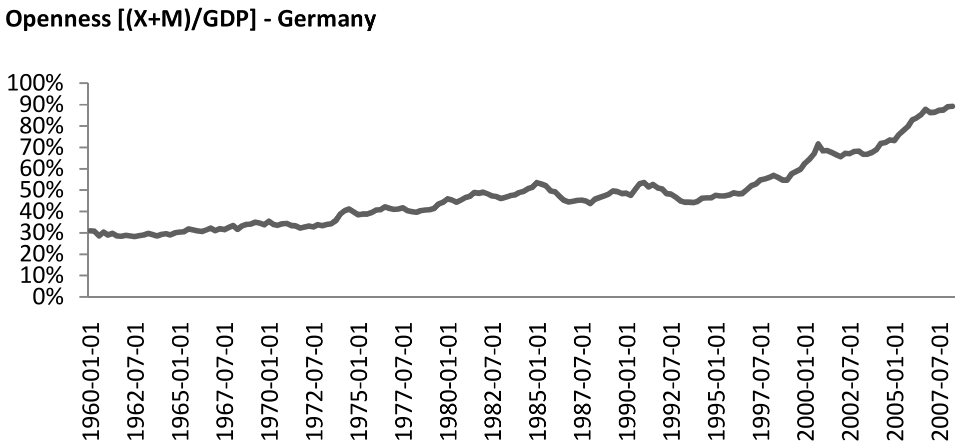
Source: OECD, own calculations

A8-FIGURE 7 – TRADE SURPLUS/GDP (GERMANY)



Source: OECD, own calculations

A8-FIGURE 8 – OPENNESS (GERMANY)



Source: OECD, own calculations

APPRNDIX 9 – ESTIMATION RESULTS CHAPTER 4

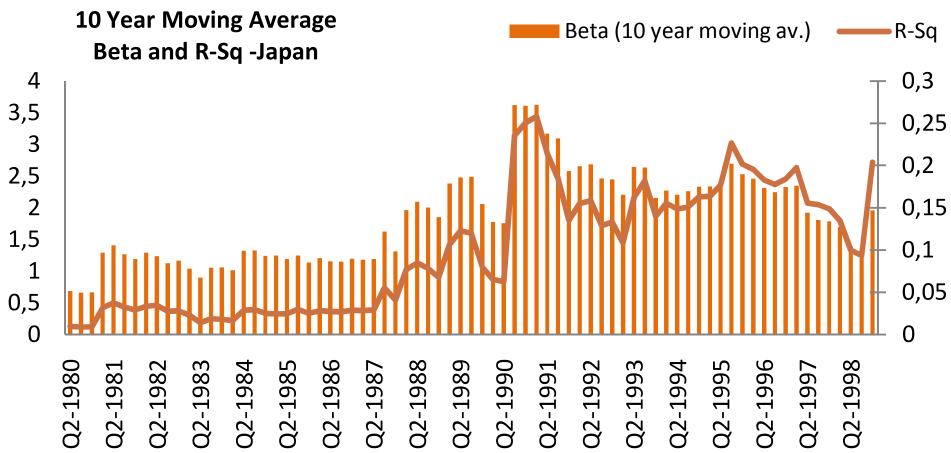
In order to understand the numbers behind these graphs I refer to the Data CD and the Excel sheet with national account data (specifically the tab named "estimations ch. 4") which will give you the details on how the numbers are constructed. Essentially, these values are moving average representations (10 years) of regressions and thus attempt to catch the development of the relationship between net exports (the current account) and output growth. The regression used to do the simulation is the same univariate regression as reported in the text; i.e.

A9-EQ. 1

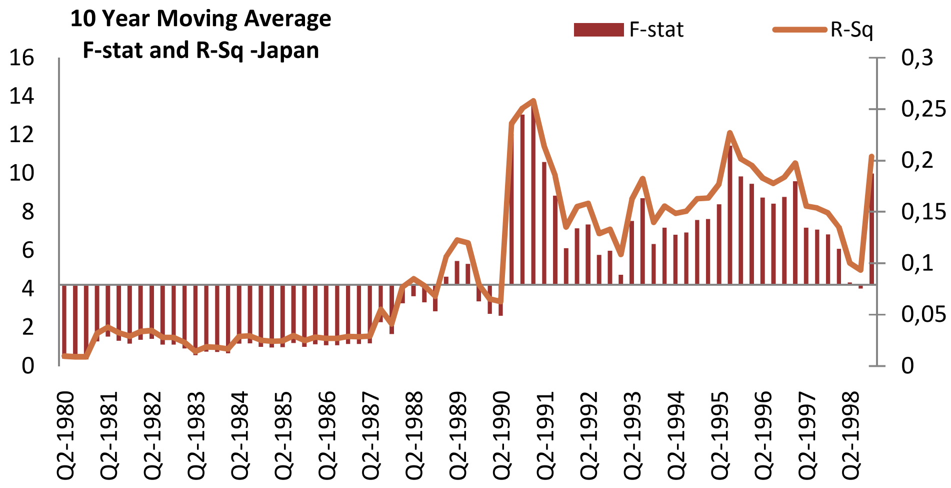
$$\Delta Y_t = \alpha_0 + \beta \Delta CA_t + u_t$$

Japan

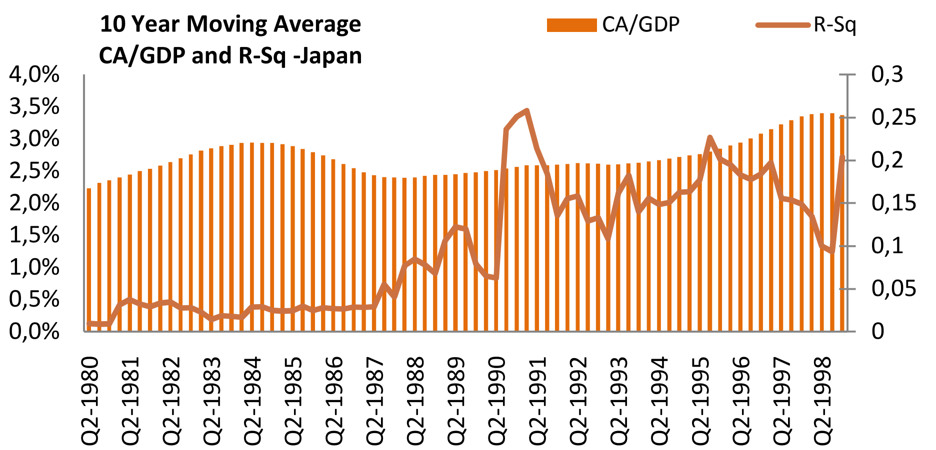
A9-FIGURE 1 – BETA AND R-SQ (JAPAN)



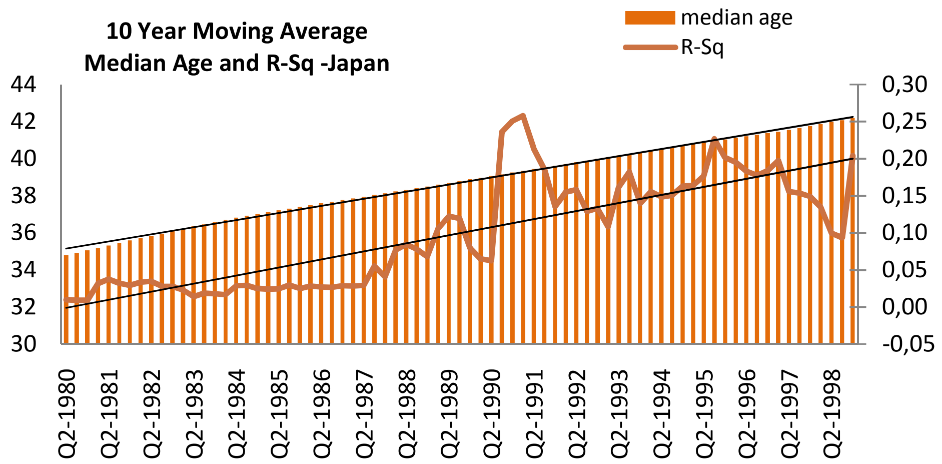
A9-FIGURE 2 – F-STAT AND R-SQ (JAPAN)



A9-FIGURE 3 – CA/GDP AND R-SQ (JAPAN)

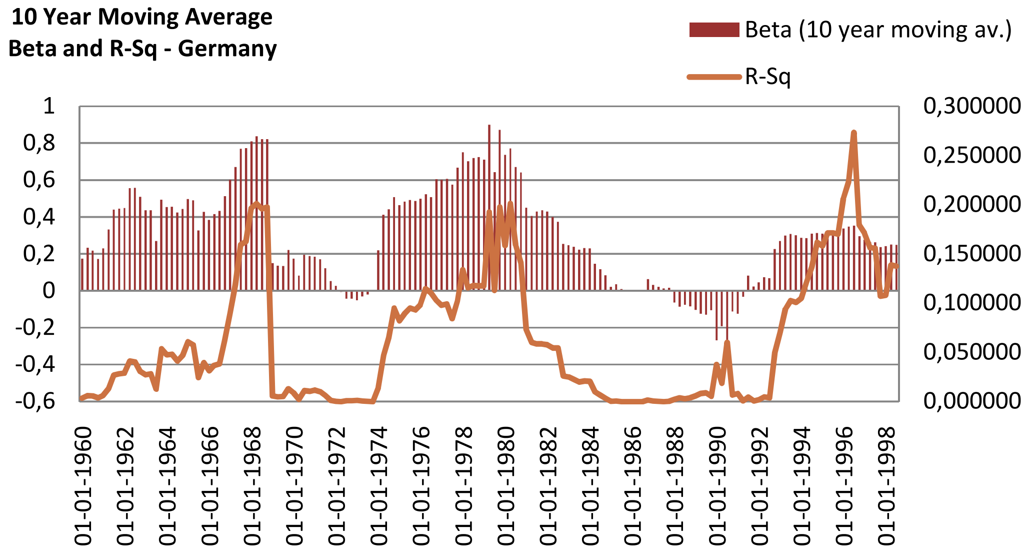


A9-FIGURE 4 – MEDIAN AGE AND R-SQ (JAPAN)

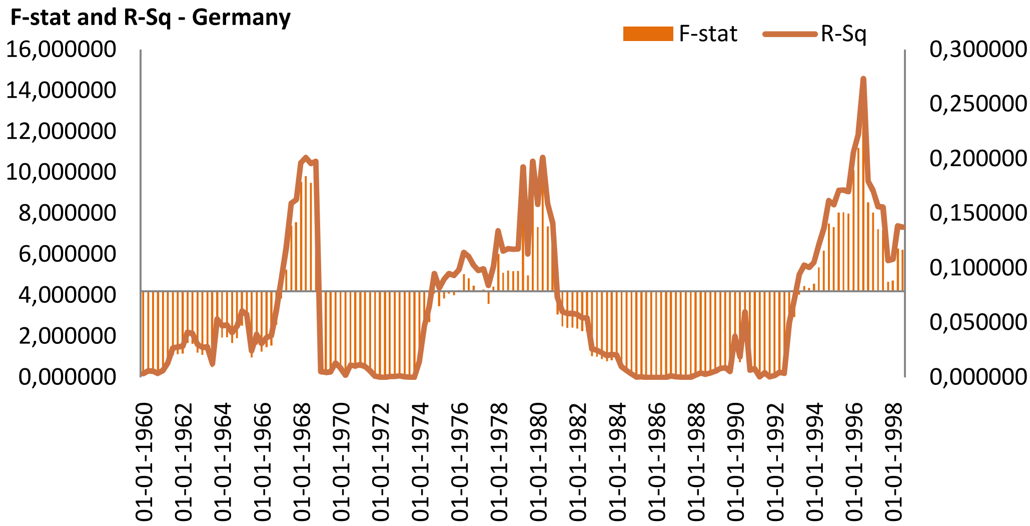


Germany

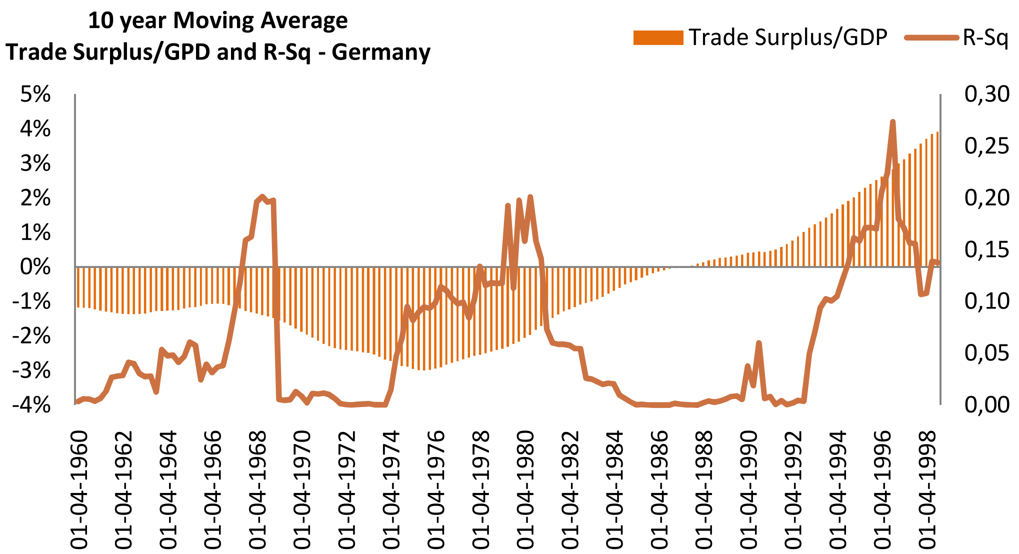
A9-FIGURE 5 – BETA AND R-SQ (GERMANY)



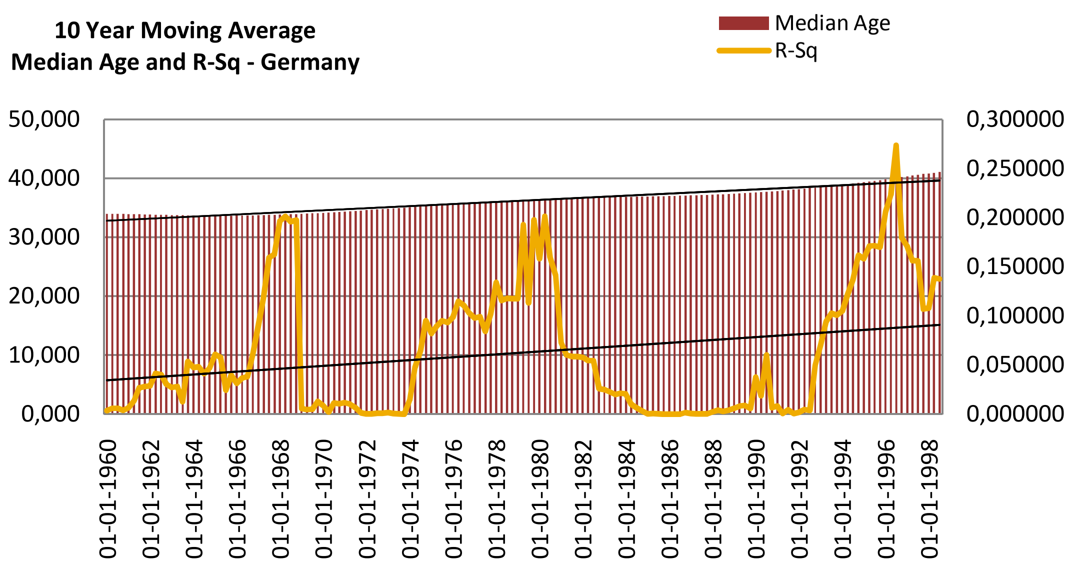
A9-FIGURE 6 – F-STAT AND R-SQ (GERMANY)



A9-FIGURE 7 – TRADE SURPLUS/GDP AND R-SQ (GERMANY)



A9-FIGURE 8 – MEDIAN AGE AND R-SQ (GERMANY)



APPENDIX 10 – R-SQ AND MEDIAN AGE, COINTEGRATED?

The test for cointegration involves three simple steps. The first step involves estimating the level form regression of the estimated R-Sq as a function of median age;

A10-EQ. 1

$$RSq_t = \beta_0 + \beta_1 MedianAge + u_t$$

which returns the following result for Japan and Germany respectively.

A10-TABLE 1 – RESULTS FROM A9-EQ.1

Japan				#
R-sq	0,763			
F	235,171			
N	75			
Variables	*Coefficients*	*SE*	*t-stat*	*p-val*
Intercept	-18,009	1,001	-17,988	0,000
Median Age	0,396	0,026	15,335	0,000
Germany				#
R-sq	0,524			
F	195,755			
N	155			
Variables	*Coefficients*	*SE*	*t-stat*	*p-val*
Intercept	-0,329	0,088	-3,730	0,000
Median Age	0,011	0,002	4,446	0,000

The second step is performed in conjunction with the first step and simply involves obtaining the residuals from the regression estimated above;

A10-EQ. 2

$$\hat{u}_t = RSq_t - \beta_0 - \beta_1 MedianAge_t$$

finally, we perform a Dickey-Fuller test on the estimated residuals. The decision rule is as follows; if the residuals are stationary we conclude that the estimated RSq time series and the evolution of median age are cointegrated and if not; the estimated RSq time series is not likely to cointegrated with median age.

Formally, the Dickey Fuller test starts from the following estimation equation;

A10-EQ. 3

$$\hat{u}_t = \alpha_0 + \alpha_1 \hat{u}_{t-1} + v_t$$

in order to derive the estimation equation substract $\hat{u}_{t-1}$ from both sides of the equation and factor the right hand side.

A10-EQ. 4

$$\begin{aligned}
&\hat{u}_t - \hat{u}_{t-1} = \rho \hat{u}_{t-1} - \hat{u}_{t-1} + v_t \\
&\Leftrightarrow \\
&\Delta \hat{u}_t = (\rho - 1)\hat{u}_{t-1} + v_t \\
&\Leftrightarrow \\
&\Delta \hat{u}_t = \psi \hat{u}_{t-1} + v_t
\end{aligned}$$

This regression will now be estimated using OLS. The null hypothesis is that $\psi = 0$ and thus that there is a unit root (non-stationarity) and conversely if the null can be rejected we can conclude that there is no unit root in the time series. It is important to note two things when using this test. Firstly, we cannot perform hypothesis testing based on standard t-test tables. Consequently, the test statistic derived from the estimation above does not follow a t-distribution. Rather, we must compare them to the following values originally derived by Clive and Granger who devised the test for cointegration used here Greene (2003).

A10-TABLE 2 – CRITICAL VALUES FOR THE COINTEGRATION TEST

No time trend:

Significance level	**1%**	**2.5%**	**5%**	**10%**
Critical value	-3.90	-3.59	-3.34	-3.04

With linear time trend:

Significance level	**1%**	**2.5%**	**5%**	**10%**
Critical value	-4.32	-4.03	-3.78	-3.50

Source: Davidson and MacKinnon (1993), “Estimation and inference in Econometrics”, Table 20.2

The estimation of the Dickey-Fuller test returns the following results;

A9-TABLE 2 – COINTEGRATION RESULTS

Japan				#
R-sq	0,112			
F	9,097			
N	75			
Variables	*Coefficients*	*SE*	*t-stat*	*p-val*
Intercept	0,002	0,034	0,072	0,943
Lag(ut)	-0,222	0,074	-3,016	0,004
Germany				#
R-sq	0,047			
F	7,554			
N	155			
Variables	*Coefficients*	*SE*	*t-stat*	*p-val*
Intercept	0,000	0,002	0,171	0,865
Lag(ut)	-0,094	0,034	-2,749	0,007

Comparing these t-stats to the critical values derived by Granger and Engle we are close (but not close enough) to being able to reject the null which would indicate cointegration[64]. The results however seem borderline and thus if we evaluate against a standard t-distribution we would conclude that the two time series were indeed cointegrated. However, adhering to convention and using the Granger-Engle values, we conclude that these two time series are not cointegrated.

[64] Note in this respect that the p-values reported above correspond to a standard t-test!