Preferences estimation without approximation

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PREFERENCES ESTIMATION WITHOUT APPROXIMATION

ABSTRACT. We devise an estimation methodology which allows preferences estimation and comparative statics analysis without a reliance on Taylor's approximations and the indirect utility function.

1. Introduction

The previous empirical literature that dealt with uncertain utility functions relied on a second-order Taylor's series approximation of the indirect utility function in order to provide some comparatives statics estimates under uncertainty. Examples include Kumbhakar and Tsionas (2008), Kumbhakar and Tsionas (2005), Alghalith (2007), and Appelbaum and Ullah (1997). Evidently, the use of second-order Taylor's series approximation yields inaccurate estimating equations. More importantly, though these studies acknowledged the presence of a utility function, they did not attempt to estimate the value or the functional form of the utility. None of the previous studies treated utility estimation as a goal of the study. Assigning numerical values to preferences (utility) was considered even a more cumbersome endeavor.

In this paper, without relying on Taylor's approximations, we develop a simple econometric methodology that enables the empirical researcher to directly estimate preferences in terms of both the value and the functional form. That is, we will be able to assign numerical values to the agent's preferences. In addition, the functional form of the utility can be estimated with a high level of accuracy. Furthermore, we devise a simpler method of estimating comparative statics under uncertainty. Finally, we deal with the *direct* utility function, as opposed to the indirect utility.

2. Methodology

As an example, we use a standard portfolio model. However, the method is applicable to all standard uncertainty models. The net wealth is specified by $w = (\mathbf{r} - \mathbf{p})\,\mathbf{x}$, where $\mathbf{x}$ is the risky asset (portfolio) vector, $\mathbf{r}$ is the random asset price vector, p is the current (risk-free) price vector. Also, the random price of asset i is given by $r_i = \bar{r}_i + \sigma_i \xi_i$, where σ_i is the standard deviation, $\bar{r}_i \equiv Er_i$ and ξ_i is random. The investor maximizes the expected utility of wealth $Eu\,(w)$ with respect to the asset quantities

$$\max_{\mathbf{x}} Eu\,(w)\,,$$

where u is the utility function.

The solution yields

$$Eu'\,(w^*)\,(r_i^* - p_i) = 0 = (\bar{r}_i - p_i)\,Eu'\,(w^*) + \sigma_i Eu'\,(w^*)\,\xi_i. \tag{1}$$

Clearly,

$$\bar{r}_i = p_i - \frac{\sigma_i Eu'\,(w^*)\,\xi_i}{Eu'\,(w^*)}. \tag{2}$$

We can rewrite (2) as

$$\frac{1}{\bar{r}_i} = \frac{Eu'(w^*)}{p_i Eu'(w^*) - \sigma_i Eu'(w^*)\,\xi_i}. \tag{3}$$

It is established in the empirical literature that the firm/agent has data series for w^*, x_i^*, $\bar{r}_i$, p_i and σ_i (see, for example, Alghalith (2007) and Chavas and Holt (1996) for the methods of obtaining data series for the mean and standard deviation). Thus (3) can be estimated using the following non-linear regression equation

$$\frac{1}{\bar{r}_i} = \frac{\beta_2}{\beta_2 p_i - \beta_1 \sigma_i}, \tag{4}$$

where β_1 and β_2 are the parameters to be estimated; whereas $\bar{r}_i$, p_i and σ_i are observed data. Evidently, β_1 is an estimate of the average value of $Eu'(w^*)\,\xi_i$; likewise, β_2 is an estimate of the average value of $Eu'(w^*)$. If $\beta_1 \lesseqqgtr 0$, then $u'' \lesseqqgtr 0$ (risk-averse, risk-neutral and risk-loving agent, respectively). $\beta_2 > 0$ implies a positive marginal utility of wealth.

2.1 Comparative statics

A standard comparative statics procedure for a single portfolio (it can be easily extended to multiple portfolios) yields

$$dx_i^* = -\frac{x_i^* Eu''(w^*)(r_i - p_i)}{|H|} d\bar{r}_i, \tag{5}$$

$$dx_i^* = -\frac{x_i^* Eu''(w^*)(r_i - p_i)\,\xi_i}{|H|} d\sigma_i, \tag{6}$$

where $|H|$ is the determinant of the Hessian. Equation (5)can be estimated using the following non-linear regression equation

$$dx_i^* = -\beta_3 x_i^* d\bar{r}_i, \tag{7}$$

where β_3 is the parameter to be estimated; dx_i^* and $d\bar{r}_i$ are observed data, which can be obtained by differencing the data series for x_i^* and $\bar{r}_i$. Clearly, β_3 is an estimate of the average value of $Eu''(w^*)(r_i - p_i) / |H|$. Therefore the value of β_3 determines the comparative statics results. Similarly, Equation (6) can be estimated using the same procedure.

2.2 Estimating higher-order derivatives

To estimate the utility functional form with accuracy, we need to estimate the higher-order derivatives of the utility function. For example, the sign of u''' determines prudence and whether the agent has increasing/decreasing/constant absolute risk-aversion. The sign of $u'''' \equiv u^{(4)}$ determines temperance.

The approach we introduce is a simple non-parametric procedure. First, the estimates from (4) can be used to generate data series for $Eu'(w^*)$; that is, to obtain a value for $Eu'(w^*)$ that corresponds to each observation in the data series. This can be achieved by a direct calculation as the following

$$\mu \equiv Eu'(w^*) = -\beta_1 \sigma_i / (\bar{r}_i - p_i). \tag{8}$$

Second, the second-difference of the data series μ and w will be used to obtain estimates for u''' as follows

$$Eu'''(w^*) = \Delta^2 \mu / \Delta^2 w. \tag{9}$$

An average value of $Eu'''(w^*)$ can also be obtained. Similarly, higher-order derivatives can be obtained by

$$Eu^{(n)}(w^*) = \Delta^{(n-1)} \mu / \Delta^{(n-1)} w. \tag{10}$$

Consequently, the functional form of u can be estimated with a high level of precision.

References

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