Perfect Competition

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ABSTRACT

In his 1987 entry on ‘Perfect Competition’ in *The New Palgrave*, the author reviewed the question of the perfectness of perfect competition, and gave four alternative formalisations rooted in the so-called Arrow-Debreu-Mckenzie model. That entry is now updated for the second edition to include work done on the subject during the last twenty years. A fresh assessment of this literature is offered, one that emphasises the *independence* assumption whereby individual agents are not related except through the price system. And it highlights a ‘linguistic turn’ whereby Hayek’s two fundamental papers on ‘division of knowledge’ are seen to have devastating consequences for this research programme.

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An allocation of resources generated under perfect competition is an allocation of resources generated by the pursuit of individual self-interest and one which is insensitive to the actions of any single agent. Self-interest is formalised as the maximisation of profits over production sets by producers and the maximisation of preferences over budget sets by consumers, both sets of actions being taken at a price system which cannot be manipulated by any single agent, producer or consumer. An essential ingredient then in the concept of perfect competition, that which gives the adjective *perfect* its thrust, is the idea of *economic negligibility* and, in a set of traders with *many* equally powerful economic agents, the related notion of *numerical negligibility*. Perfect competition is thus an idealised construct akin (say) to the mechanical idealisation of a frictionless system or to the geometric idealisation of a straight line.

Following the lead of Wald, a mathematical formalisation of perfect competition in a setting with an exogenously-given finite set of commodities and of agents was developed in the early fifties in the pioneering papers of Arrow, Debreu and McKenzie. It was shown that convexity and independence assumptions on tastes and technologies guarantee that a competitive equilibrium exists, and that a Pareto-optimal allocation can be sustained as a competitive equilibrium under appropriate redistribution of resources. It was also shown, drawing on the tacit assumption that markets are universal but by avoiding any convexity assumptions, that with local non-satiation, every competitive allocation is Pareto-optimal. Relegating precise definitions to the sequel, we refer the reader to Koopmans (1961) for a succinct statement of the theory; Debreu (1959) and McKenzie (2002) remain its standard references, Fenchel (1951) and Rockafellar (1960) its mathematical subtexts, and Weintraub (1985) and Ingrao-Israel (1987) its sources of historical appraisal.

However, in its exclusive focus on drawing out the implications of convexity and agent-independence for a formalisation of perfect competition, the theory remained silent about environments with increasing marginal rates, in production and in consumption, as well as those where private and social costs and benefits do not coincide, to phrase this silence in Pigou's (1932) vocabulary of a preceding period. In particular, the notion of perfect competition that was fashioned by the initial theoretical development had no room for economic phenomena emphasised, for example, in the papers of Hotelling (1938), Hicks

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Author's Note: A version of the above text will appear in a new, forthcoming edition (2007) of *The New Palgrave: A Dictionary of Economics*.

(1939) and Samuelson (1954). It took around two decades to show that at least as far as collective consumption and public goods were concerned, the theory had within it all the resources for an elegant incorporation, but of course within the confines and limitations of its purview; see Foley (1970) and his followers. Non-convexities in production and consumption were a different matter entirely; they required mathematical tools that went beyond convexity, and further development had to await the invention of non-smooth calculus of Clarke and his followers; see Rockafellar-Wets (1998) and Mordukhovich (2006) for a comprehensive treatment.

A robust formalisation of the idea of perfect competition for non-convex technological environments in the specific form of marginal cost-pricing equilibria, with the regulation of the increasing returns to scale producer(s) given an explicit emphasis, can be outlined under each of the three headings of the theory identified by Koopmans: existence and the two welfare theorems. Marginal cost pricing equilibria exist under suitable survival and loss assumptions, but are not globally Pareto optimal even under the assumption of universality of markets. Finally, Pareto optimal allocations can be sustained as marginal cost-pricing equilibria under appropriate redistribution of resources. Moreover, under the terminology of Lidahl-Hotelling equilibria, Khan-Vohra (1987) provide the existence of an equilibrium concept that incorporates both public goods and increasing returns to scale in one sweep. This work on perfect competition in the presence of individualised prices stemming from collective consumption and a regulated production sector (or sectors) merits an entry in its own right, and rather than a detailed listing of the references, we refer the reader to Vohra (1992) and Mordukhovich (2006; Chapter 8) for details and references.

Three observations in connection with this recent, but already substantial, literature are worth making. First, in the attempts to generalise the second theorem, one can discern a linguistic turn whereby both the Arrow-Debreu emphasis on decentralisation and the Hicks-Lange-Bergson-Samuelson-Allais equality of marginal rates are seen as special cases within a synthetic treatment emphasising the intersection of the cones formalising marginal rates; Khan's (1988) introduction is a forceful articulation of this point of view. Second, a canonical formulation of the notion of marginal rates, despite fits and starts, now seems within reach, though a notion that works well for the necessary conditions may not be the one suited for the question of existence; see Hamano (1989) and Khan (1999). Finally, conceptual clarity requires an understanding of circumstances when this type of non-convex theory bears a strong imprint of its finite-dimensional, convex counterpart, as detailed in Khan (1993), and when its higher reaches require a functional-analytic direction totally different from that charted out in the pioneering papers of the fifties; see Bonnisseau-Cornet (2006) for references to recent work.

With price-taking assumed rather than proved, there is no over-riding reason why a formalisation of perfect competition must limit itself to a setting with a finite, as opposed to an infinite number of (perfectly-divisible) commodities. Indeed, another set of pioneering papers of Debreu, Hurwicz and Malinvaud, written in the fifties with an eye to a theory of intertemporal allocation but over a time horizon that is not itself arbitrarily given, fixed and finite so to speak, did consider the decentralisation of efficient production plans as profit maximising ones. But again, it was only two decades later that the work of Bewley, Peleg-Yaari, Gabszewicz and Mertens inaugurated sustained attempts to provide a general formalisation of perfect competition over infinite-dimensional commodity spaces, see Khan-Yannelis (1991). The work can again be categorised under Koopmans' three headings of the theory, but relative to its finite-dimensional counterpart, it emphasised that the separation of disjoint convex sets, and the use of aggregate resources to furnish a bound on the consumption sets to ensure compactness, prove to be matters of somewhat greater subtlety. In short, a compact set of an infinite-dimensional commodity space is "rather large" and its cone of non-negative elements "rather small". Indeed, as Negishi's method of proof attained dominance, the imbrication of the convexity assumption in a clear demarcation of fixed-point theorems for issues of existence and separating hyperplane theorems for those of decentralisation, no longer obtains. The subject is surveyed in Mas-Colell-Zame (1991), but another survey is perhaps overdue as exploration of individual mathematical structures, ordered structures in particular, reveal hitherto unforeseen essentials, and increasing returns to scale and other non-classical phenomena are inevitably accommodated; see the references of Aliprantis, *et al.* (2002, 2006), on the one hand, and those of Shannon (1999) and Bonnisseau (2002) on the other.

However, the question persists as to what meaning can be given to the study of perfect competition in a setting with an exogenously-given infinite-dimensional commodity space where markets open only once and there is no room for the correction of mistakes and unfulfilled plans? If the extension of the theory requires additional technical assumptions, how do they translate into *desiderata* that are of relevance for the formalisation of the coherence of decentralised, self-interested decision-making of independent agents acting independently of each other? Even if, for example, the *uniform properness* assumption of Mas-Colell (1986) and his followers could be pinned down as a formalisation of bounded marginal rates of substitution [see the notion of a Fatou cone in Araujo, *et al.* (2004) and one failed attempt in Khan-Peck (1989)], what does it say about the set-up of the model itself that lifts this up to be a limitation as fundamental as that of convexity or independence? If the underlying motivation for the extension to infinite-dimensional commodity spaces is time, risk, quality, information or location, how do these considerations manifest themselves in the infinite-dimensionality of the commodity space, in a situation that necessities (or precludes) one commodity in an economy being numerically negligible relative to the entire set? More sharply, why the resulting problems ought not to be more squarely faced in simpler partial equilibrium models, rather than studied under the limitation of a construction whose primary emphasis is the viability and desirability of static interaction? We defer these issues to turn to our principal theme; namely, how to formalise the perfectness of perfect competition?

The point is that the assumption of a finite number of agents embodied in all of this work is an explicit admission of the fact that the economic non-negligibility of each agent, at least in principle, and therefore her non-manipulation of, and corresponding submission to, the price-system furnishes a somewhat muted maximisation of her self-interest. In terms of the emphasis on *negligibility* as a pre-requisite for a rigorous formalisation of perfect competition, as is being emphasised in this entry, the postulated behaviour of individual agents in the so-called Arrow-Debreu-McKenzie model of perfect competition, with or without infinite commodities, externalities and increasing returns to scale, leads to the rather natural puzzlement as to what is it precisely that guarantees an agent's passive acceptance of the price system, leave alone individualised pricing rules, and that too in a construction whose primary motivation is consistency and generality. In the vernacular due to Hurwicz (1972), one that has gained increasing currency since the eighties, what is it that makes this model of the economic system incentive-compatible? how is its gloss of the intuitive notions of *negligibility, large* and *many* to be made precise?

Five conceptually separate attempts to answer this question are distinguished here; these alternative but inter-related formalisations of perfect competition draw their meaning from two early conjectures: (i) Edgeworth's (1881) conjecture on the shrinking of the *core* to its set of competitive allocations (again, precise definitions to follow), and (ii) Farrell's (1959) conjecture on the existence of competitive equilibrium in a environment that is not necessarily convex. Interpreted literally, both conjectures are clearly false for a given finite economy, but the first can be distinguished from the second in not being simply a case of dispensing with an assumption in a result whose basic contours are wellestablished, but rather in going beyond Koopmans' categorisation of perfect competition to include a solution concept other than that of Pareto optimality. It is in the reliance of the core notion as a test for the perfectness of competition, in working with a third fundamental theorem of welfare economics, so to speak, and in giving precision to the ambiguity inherent in the term *shrinking*, that allows an entry into the formalisation of the negligibility of individual agents. However, at this point, the discussion demands the rigor of notation and definitions; and since the essence of the ideas can be adequately communicated in the context of an economy without producers, that is, in an *exchange economy*, we confine ourselves to this case.

An *exchange economy* consists of a commodity space L, a set of traders T, a space of trader characteristics ρ defined on the commodity space, and a mapping ε from T into ρ with the triple $\varepsilon(t) \equiv (X(t), \geq t, e(t))$ specifying the characteristics of agent t in T. The space of characteristics is thus a product space constituted by consumption sets $X(t) \subseteq L$, by binary relations $\geq t$ over $X(t) \times X(t)$, preferences over the consumption set read "preferred or indifferent to", and by initial endowments $e(t) \in X(t)$. An *allocation* $x : T \to L$ is an assignment of commodity bundles such that $x(t) \in X(t)$ for all t in T and such that the summation, suitably formalised, of $(x(t) - e(t))$ over T is zero, or, in the case of *free-disposal*, less than or equal to zero. In any case, the fundamental economic problem facing a particular exchange economy, as discussed above and being given symbolic formulation here, is the choice of an allocation.

An allocation $x : T \to L$ is said to be in the *core* if there does not exist any other allocation y and a coalition $S \subseteq T$, suitably formalised, such that $y(t) \geq t\, x(t)$ and not $x(t) \geq y(t)$ for all $t \in S$, and that the summation of $(y(t) - e(t))$ over S is zero, or again with free disposal, less than or equal to zero. A perfectly competitive allocation of resources is a price-based allocation where a *price system* is a non-zero, continuous linear function on the commodity space L. A *competitive equilibrium* is a pair (p, x) where p is a price system and x an allocation such that for all t in T, $x(t)$ is a maximal element for $\geq t$ in the *budget set* $\{y \in X(t)) : (y, p) \leq (e(t), p)\}$. Here (y, p) denotes the valuation of the commodity bundle y by the function p and, in case L is the Euclidean space IR^{ℓ}, the Reisz representation theorem allows it to be given a simple accounting interpretation of an inner product $(y, p) = \sum_{i=1}^{\ell} p_i y_i$; see Rudin (1974) for this theorem and, if unspecified, for other terminology. For any *competitive equilibrium* (p, x), x will be referred to as a *competitive allocation*. In terms of the earlier discussion of infinite-dimensional commodity spaces, the commodity space L has presumed on it enough mathematical structure so as to give meaning to the ordering "less than or equal to", to the summation operator in the notion of an allocation and of a blocking coalition, and to linearity and continuity in the notion of a price-system. Conceptually, what is of consequence here is that competitive allocations can be viewed as making precise the idea of some sort of individual rationality, and core allocations as making precise the idea of some sort of group rationality.

In Aumann's (1964) formulation of perfect competition, the set of traders is the Lebesgue unit interval, the commodity space is the Euclidean non-negative orthant IR^{ℓ}_{+}, the set of admissible coalitions the Borel σ-algebra on the unit interval, and summation, Lebesgue integration. Under the assumption of Lebesgue measurability of preferences $\geq$, and of Lebesgue integrability of the initial endowments $e(\cdot)$, he proved that the set of competitive allocations of such an economy coincides with its set of core allocations and, in Aumann (1966), that neither set is empty. These precise and elegant affirmations of the conjectures of Edgeworth and Farell did not require any convexity hypotheses on preferences, and what is perhaps of equal significance, they furnished a precise formulation of an idealised limit economy in which price-taking is rendered theoretically reputable: every agent is numerically and economically negligible in that his or her effect, not only on the price system, but also on the equilibrium allocation, is precisely zero. An agent has a negligible weight very much akin (say) to the probability of a particular point on a dart-board being hit by a dart.

The seminal nature of Aumann's conception was quickly realised and incorporated into the mainstream: the metaphor of a continuum of agents is appealed to validate aggregation in a removal of idiosyncratic uncertainty even in models of a representative agent in theoretical work in macroeconomics and other, so-called applied, fields, work that is nothing if not an investigation of competition, perfect or otherwise. Two observations are worth making. First, whereas Aumann's assumption of a Lebesgue unit interval was only a simplifying one, and that the results hold for any arbitrary *atomless* and finite measure space, Lebesgue (rather than Riemann-Stieltjes) integration is essential to a theory based on T as the set of agent-names, and therefore free of any topological considerations; see Khan-Sun (2002; Introduction) for a detailed exposition of this point. Indeed, Shapley has even questioned the postulate of measurability, leave alone continuity, for a notion of an allocation whose very *raison d'etre* is a formalisation of independent individual self-interest. Second, since the theory is based on a neglect of sets of measure zero, it is the conception of an allocation as an equivalence classes of functions, rather than of functions themselves, that is identified by the theory. Put more sharply, Pareto-optimal allocations in an economy with a continuum of agents do not exist if their definition is taken verbatim from that of a finite economy, and not recast in terms of coalition of positive measure. In any case, the theory of an economy ε conceived as a measurable map, at least in its finite-dimensional embodiment, is a testimony to the power of Lyapunov theorem on the range of an atomless vector measure and to a powerful mathematical theory of the integration of correspondences that emerges as its corollary; Hildenbrand (1974) is the relevant reference.

A contemporaneous formulation of Vind (1964) short-circuits some of these issues concerning sets of zero measure by ignoring agents altogether, and focussing instead on coalitions, each with their own preferences and endowments, as the primitive data of the economy. Allocations then are measures on a non-atomic measure-space, and the notions of core and competitive allocations, correspondingly defined, can be shown to be identical solution concepts. This is a formulation of perfect competition that is also measure-theoretic, but one, alternative to that of Aumann, that explicitly does away with mathematical integration as its necessary micro-foundation. However, by assuming countable additivity, Vind enabled Debreu (1967) to draw on Radon-Nikodym differentiation to effect a reconciliation. It took subsequent work of Armstrong and Richter to give fuller autonomy to this alternative point of view by first eliminating countable additivity, and then in setting the discussion in the framework of non-atomic Boolean algebras; see Armstrong-Richter (1986) and their references. Whereas the technical underpinning of this approach is now clearly seen to be the Armstrong-Prikry extension of the Lyapunov theorem, it is perhaps fair to say that the conceptual ramifications of this alternative (perhaps syndicalist) vision have yet to be fully explored and understood; see Avallone-Basile (1998) and Basile-Graziano (2001) for references to current research.

The Brown-Robinson (1975) formulation of perfect competition, the third to be discussed here, returns to the methodological individualism of Aumann, and requires the set of agent-names T to be an internal star-finite set, the commodity space to be $^*\ IR_+^\ell$, the nonstandard extension of IR_+^ℓ based on manipulable infinitely large and infinitesimally small numbers, the summation in the definitions of allocations and core to be summation over internal sets, the set of admissible coalitions to be the set of all internal subsets of T and ε to be an internal map from T to $^*\rho$, the set of agent characteristics modelled on $^*IR_+^\ell$. Such a formulation utilises methods of nonstandard analysis, a specialisation in mathematical logic due to A. Robinson; see Loeb-Wolff (2000) for details and references. On replacing equality by equality modulo infinitesimals in the definitions of allocation and the core, Brown-Robinson (1975), and without their ad-hoc standardly bounded assumption on allocations, Brown-Khan (1980), showed the equivalence (and Brown (1976) the existence) of core and competitive allocations of a non-standard economy without any convexity assumptions on preferences. Loeb's (1963) combinatorial analogue of Lyapunov's theorem provided the mathematical underpinning of the theory. This alternative affirmation of the conjectures of Edgeworth and Farell is another way of making precise the concepts of *many* agents and of their individual *negligibility*: meaning can be given to an individual trader's actions having a positive, but infinitesimal, effect on the price system and on an allocation. Even though an initial motivation of this work was to explore a formulation of perfect competition and of a *large* economy in a vernacular alternative to that of measure theory, it was heavily influenced by measure-theoretic formulations, but with an added emphasis on asymptotic implementation, something clear even in the earliest papers of Brown-Robinson and Khan; see Rashid (1987) and Anderson (1991) for details and references.

Strange as it may seem in retrospect, the idealisations of Aumann and Brown-Robinson were criticised on grounds of realism, on the observation that there do not exist economies with uncountably many agents; see Koopmans (1974) and the Georgescu-Roegen-Rashid exchange discussed in Khan (1998). The work categorised here as a fourth formalisation of perfect competition was motivated, in part, by this criticism (ironically also used by Armstrong-Richter as their stated motivation for finitely-additive measures), and, in part, by a methodological curiosity as to whether the results established for non-standard and measure-theoretic economies are artifacts of the way negligibility and large economies were being modelled. Taking its point of departure from the replicated sequences of Debreu-Scarf (1963), the response is to consider a sequence $\mathcal{G} = \{\varepsilon_k\}_{k=1}^{\infty}$ of finite economies based on the commodity space IR_+^{ℓ}, where ε_k is an economy with a set of agents T_k of cardinality k. For each finite economy ε_k, competitive and core allocations can be defined in the conventional way without encountering any technical difficulties in the formalisation of summation or of a coalition. It is clear that agents in ε_k get increasingly numerically negligible with an increase in k, and given a uniformly bounded assumption on initial endowments, also get increasingly economically negligible. For this *perfectly competitive sequence* of economies, one can ask: for any " > 0, however small, does there exist an integer ko such that core allocations of all $\varepsilon_k \in \mathcal{G}$, $k \geq k_o$, can be sustained as approximate competitive equilibria, and whether such equilibria exist, with" indicating in either instance, the degree of approximation? In short, are the formulations of perfect competition in idealised limit economies capable of an asymptotic implementation, with an arbitrarily fine degree of approximation, in economies of arbitrarily large but finite cardinality?

Asymptotic equivalence and existence theorems under varying degrees of generality followed quickly once the problem was posed. We shall not touch upon the various elaborations and refinements except to note that they have been obtained under two disparate techniques, both drawing on the results for an idealised limit economy. The first approach, associated especially with Hildenbrand, is to conceive of an economy as a measure on the space of characteristics and to utilise Skorokhod's theorem and the theory of weak convergence of measures on a topological space (typically metrisable) of characteristics ρ. Under Debreu's rather vivid terminology of "neighbouring economic agents", such topologies were formulated by Debreu, Kannai, Hildenbrand-Mertens, Grodal and others, and surely have independent interest; see Hildenbrand (1974). The second approach is based on the observation that "any sentence which is true in the standard universe is true for internal entities in the non-standard universe", and as such results pertaining to a non-standard exchange economy 'flipped over', as it were, to a corresponding result for a large but finite economy. The differences between the two approaches are interesting from a methodological point of view: the fact that one approach is, in principle, not inherently dependent on any topology on the space of preference relations or on their continuity [as in Khan-Rashid (1976, 1982)] and applies as readily to core as to competitive allocations [as in Khan (1974)], suggests a further look as to how the other may be extended; see Anderson (1992) for a comprehensive treatment. In any case, we have two mutually supporting ways of extracting information for large but finite economies from idealised limit economies, even of the mixed type with atoms that generated the skepticism in the first place; [see Gabszewicz-Shitovitz (1992) and their references. This claim is further underscored by a development due to Loeb (1975)], but before turning to it, we discuss what may be seen as fifth and final formulation of negligibility and thereby of perfect competition.

The asymptotic interpretation of the perfectness of perfect competition concerns sequences of economies, and a question arises as to whether, given an arbitrary economy rather than an arbitrary degree of approximation, one can find the error, independent of the number of agents, with which the equivalence and existence theorems hold? Thus, rather than ask how large is large enough, one asks how small is small for the assumption of price-taking behaviour to be unjustified. For the question posed in this way, initially by Starr (1969), it was the definitive result of Anderson (1978) that capped initial explorations of Arrow-Hahn, Henry, Shaked and others. With the shedding of compactness and continuity assumptions under the nonstandard approach, Anderson observed that the argument in Khan-Rashid (1976) could be based on the Shapley-Folkman theorem instead of that of Loeb (1973) (itself based on Steinitz' theorem), and carried out entirely in standard terms to obtain an elementary equivalence theorem. This yields the asymptotic results as corollaries, and they also come equipped with a rate of convergence, a consideration emphasised by Shapley (1975). The same observation applied to Khan-Rashid (1982) led to an elementary existence theorem; [see Geller's (1986) extension of Anderson-Khan-Rashid (1982)].

In the prominence that it gives to a fixed finite economy, this fifth formulation of perfect competition connects directly to the results whose introduction began this entry; it emphasises that the equalities in the results surveyed by Koopmans, and the counterexamples implicitly underlying them, perhaps ought not be taken completely literally, but rather given a probabilistic cast. In his alternative proof of the Shapley-Folkman theorem, Cassels (1975) had already emphasised this connection, and Mas-Colell deepened it further by appealing to results of especial sophistication concerning the law of large numbers and the central limit theorem. He noted that his refinement of the equivalence theorem has "no analogue in Aumann's continuum of traders model", and that the precise probabilistic estimates that this approach offers have no counterpart in the continuum framework; see Anderson (1992; Sections 8 and 9). However, it is undeniable that it is the exact results for the idealised limit economies that generally indicate the directions of pursuit of the approximations for a finite economy: approximations and numerical algorithms come into play once the exact has been exactly identified. Thus, from a substantive point of view, modulo fine technicalities, how a particular issue pertaining to perfect competition is set, measure-theoretic or non-standard or asymptotic, is largely a contextual matter of analytical convenience and preference.

This conclusion is further sharpened by the methodological unification offered in Loeb (1975); see Khan-Sun (1997b) for exposition. It is the central claim of this entry that Loeb probability spaces go a long way towards settling the question of how the perfectness of perfect competition is to be given a precise mathematical formulation. It is already clear in Aumann's pioneering papers that perfect competition draws from the atomlessness rather than any other particularities of the measure space of agents: the metric on the unit interval, or the topology of any topological measure space, is not, indeed cannot be, of any direct relevance. What is presumably of the essence is that the space of agents' names be hospitable to measurability as well as to independence (the latter term now being used in its precise probabilistic sense rather than as a reference to an absence of externalities), that it generate results capable of straightforward asymptotic implementation, and that, for concepts that revolve only on distributions of the allocations as in Hart-Kohlberg, it yields solutions that are insensitive to a permutation of agent names. In the context of *large* games (discussed below), Khan-Sun (1996, 1999) make the case for Loeb spaces on the basis of these *desiderata* and emphasise their dual identity in the "pushing down" and "lifting up" theorems: being standard, measure spaces, any result on an abstract measure-space (Aumann) economy applies to them, and thereby, to an internal non-standard (Brown-Robinson) economy and hence can be asymptotically interpreted; or alternatively, any approximate result can be translated, as indicated above, to a non-standard economy, and thereby pushed down to its standard Loeb measure-theoretic counterpart. As such, Loeb spaces go a considerable way in obliterating the five-fold categorisation of perfect competition that marks this entry.

Going beyond method to mathematical substance, atomless Loeb spaces are ideally suited for operations ensuring that aggregation removes the irregularities that arise from non-convexities as well as from idiosyncratic uncertainty. In a systematic and far-reaching development, Sun established that the integrals and distributions of correspondences defined on Loeb spaces and taking values in a separable infinite-dimensional Banach space, in the first instance, and into Polish spaces (separable and completely metrisable) in the second, have all the properties that the theory of perfect competition requires of them. Moreover, a perfectly satisfactory law of large numbers for a continuum of random variables is obtained, and for a such a set, that the notions of *independence* and of *exchangeability* are dual in a very elegant sense, and that it yields, as in Duffie-Sun (2006), the existence of an independent random matching. Supplementing the notion of an economy as a random variable, the measurability of the map ε noted above, a stochastic economy can now be formalised as a stochastic process on a product space, the space of agent-names T and an atomless Loeb space of states of nature, Ω, to reveal circumstances under which the distributions of core and competitive allocations of a sampled economy coincide, or approximately coincide in the case of a large economy, with those of the deterministic (population) economy; see Sun (1999). Further application of this substantial theory is noted below; here the reader is referred to Sun's chapters in Loeb-Wolff (2000; Chapters 7 and 8) for exposition and full mathematical references.

In taking stock at this stage, we underscore the fact that even though five robust and logically related methods of studying perfect competition have been illustrated through the conjectures of Edgeworth and Farrell, the discussion could, in principle, equally well have been conducted through alternative tests based on alternative solution concepts: the value [Hart (2002) and his references], or the bargaining set [Anderson (1998)] and his references), or Cournot's conjecture [Mas-Colell (1983), Novshek-Sonnenschein (1983) and their references], all now conceived in a setting where individual agents are negligible. Alternatively, we could discuss applications, particularly in mathematical finance where Arrow markets and ideas of negligibility find concrete expression in derivative financial instruments and in *well-diversified* portfolios [see Anderson-Raimondo (2006) and Khan-Sun (1997a) respectively for references]. However, rather than turn to them and make this entry unmanageable, we draw on the rich and diverse formulation of perfect competition at our disposal to consider the substantive issues broached earlier: public goods, externalities, increasing returns to scale and infinite commodities, all under the rubric of static interaction. Ironically, non-convexities in idealised limit economies have concerned consumptions sets and survival assumptions rather than increasing returns to scale technologies [see Trockel (1984), Hammond (1993) and their references]; research efforts have been most active in the study of public goods and externalities, and here the theory dovetails, from a technical point of view, into work on infinite-dimensional commodity spaces.

The formalisation and defense of perfect competition has, from the very earliest, proceeded on the independence assumption: the fact that individual agents are not related other than through the price system. Thus Hayek (1948; pp. 96-97) quotes Stigler in emphasising the "explicit and complete exclusion from the theory ... of all personal relationships existing between the parties."

Such relationships are *external* to the perfected concept, and to the extent that positive and normative content can be cleanly distinguished, externalities and the Pigovian private/social divergences that they entail, have strong and negative implications for its normative content. If the non-convexities identified by Starrett (1972) are ignored [but also see Otani-Sicilian (1977)], Arrow's universality requirement for the first fundamental theorem of welfare economics can always be met by the creation of markets, fictitious or otherwise, but it leans on a particularly acute form of myopia. Arrow securities and Lindahl prices for public goods, and more generally, prices for contingent commodities and personalised prices for more pervasive externalities bring out an obvious tension between incentive compatibility and efficiency. As brought out in Starrett (1971), if there is a commodity that reflects a particular agent's dependence on my consumption, why should she or I, leave alone the others, take the price of that commodity to be given and non-manipulable? take myself to be economically negligible?

There is also a technical problem in the consideration of pervasive externalities in an idealised limit economy. Since the individualistic, as opposed to the coalitionally-based, approach to perfect competition works with an equivalence class of functions from the space of agent-names to agent-actions rather than the function itself, it is difficult to give meaning to one agent's dependence on the actions of another. In a context of a Lindahl equilibrium of an idealised limit economy, even one with a finite number of commodities and a single public good, one has to reckon with the fact that public goods enjoin equality instead of aggregation, and thereby force the analysis out of a finite-dimensional Euclidean space, as in the Aumann-Brown-Robinson limit theory, to a search for a suitably tractable space of equivalence classes of functions of individualised prices. It is these attendant functional-analytic difficulties, perhaps as much as the fact that the incentive-compatibility problems are most acute in this setting, that has discouraged the initial exploratory attempts of Roberts, Emmons and Khan-Vohra from being followed up; see Khan-Vohra (1985) for references. And it is precisely difficulties of this kind that also prevent a successful theory for idealised limit economies with non-ordered preferences; see Balder's (2000) use of the argument in Khan-Papageorgiou (1985), originally due to Grodal, to turn a positive proof into a negative claim of inconsistency, a claim that derails the initial exploration of Khan-Vohra (1984) and their followers. Externalities, rather than being widespread, need to be controlled and confined in an idealised limit economy. The theory is under active development, and it is too early to say that a formulation sufficiently robust as to be deemed canonical has been achieved; see Balder (2004, 2005), Balder, *et al.* (2006), Cornet-Topuzo (2005), Hammond (1995), Kaneko-Wooders (1994), Noguchi (2005), Noguchi-Zame (2006) and their references.

In its dissociation of the study of perfect competition from its roots in welfare theory, this work makes explicit its connection to game theory. Competitive equilibria with externalities take their place next to marginal-cost pricing and Cournot-Nash equilibria in violating Pareto optimality, but does allow one to ask whether decentralised self-interested decision-making is consistent in the aggregate if it is taken with respect to certain measurable indecis of societal responses rather than solely with respect to a price system. Such a formulation of perfect competition goes back to the early fifties in the papers of McKenzie and Debreu, and to the seventies in Chipman's formulations of Marshallian parametric externalities. Indeed, Arrow-Debreu's original proof of the existence of competitive equilibrium revolved around viewing the economy as a game in which the only "personal relationship" between the parties relates to that with a fictitious auctioneer, a point of view that finds fuller expression in the Shafer-Sonnenschein notion of an *abstract economy*. In more recent investigations of a *large game*, the literature takes another turn towards probability theory, and conceives of an agent's actions as resulting from maximisation but one that takes as given the distribution, or individual moments, of the random variable summarising societal responses. The question then reduces to the existence of such equilibrium distributions, but with social interaction, however limited, recourse has to be made to assumptions on ideal types, and on the *conditional* or *mutual independence* of these types; see Hayek's (1948; p. 47) prescient remarks. This is a theory of competition in which Loeb spaces, and the Dvoretsky-Wald-Wolfowitz extension of the Lyapunov theorem play a dominant role; see Khan-Sun (1999, 2002), Khan-Rath-Sun (2006), Loeb-Sun (2006) and their references to the work of Schmeidler, Radner-Rosenthal, Milgrom-Weber and Mas-Colell.

The technical machinery forged through the study of large games enables a broadened notion of economic negligibility, one that includes *informational negligibility* in an environment with asymmetric information. In a 1936 article on "economics and knowledge," Hayek (1948; pp. 43-44) had already supplemented Adam Smith's emphasis on the *division of labour* by the principle of the *division of knowledge* and asked "whether, in order that we can speak of equilibrium, every single individual must be right, or whether it would not be sufficient if, in consequence of a compensation of errors in different directions, quantities of the different commodities coming on the market were the same as if every individual had been right. A fuller discussion of this problem would have to consider the whole question of the significance which some economists (including Pareto) attach to the law of great numbers in this connection". The issue is "right" about what? The problem devolves on anticipations and expectations, beliefs about beliefs regarding each other and the price system, and it does not require more than a mild degree of scepticism to abandon fictional markets responding to predetermined and universally agreed-upon states of nature. There is need for viable notions of independence and aggregation to eliminate idiosyncratic risk and nullify "combination of fragments of knowledge existing in different minds". Sun (2006) and Sun-Yannelis (2006a, 2006b) give pride of place to the Fubini property in idealised limit economies, and consolidate earlier applications of Loeb spaces for a successful resolution of Malinvaud's work on insurance markets, and that of Gul, McLean and Postlewaite on the compatibility of efficiency and incentive compatibility; also see Jackson-Manelli (1997). Sun also presents compelling arguments why finitely additive measures and the conventional product measure cannot respond to the technical difficulties.

The problems arising from asymmetric information are, at their root, problems of agent interdependence that cannot be internalised through markets, and as such, represent particularly recalcitrant externalities; the assumptions that Sun-Yannelis impose on their signal process can be seen as one successful attempt to subdue them. And in an idealised limit economy with *many* commodities, seen on its own rather than through the externalities' lens, one has to cope with the fact that Lyapunov's theorem is false for an infinite-dimensional vector measure, in addition to all of the problems discussed earlier. It is the *thinness* of its target space, as proposed by Kingman-Robertson in the late sixties, that allows an atomless probability space of agents to work its magic in the form of the existence and equivalence theorems; see Kluvanek-Knowles (1976) and Diestel-Uhl (1977) for necessary and sufficient conditions for the validity of the Lyapunov theorem. There is a hidden assumption, to adopt the postmodern flourish of Tourky-Yannelis (2001), in the Aumann-Brown-Robinson formulations of perfect competition, and the equivalence theorem can fail when the qualitative relationship between the cardinalities of agents and commodities fails; in addition to Muench's example, see Forges, *et al.* (2001) and Serrano, *et al.* (2002). More generally, if the intricacies of reaching binding agreements in coalition-formation cannot be bracketed away, how can a concept embodying group rationality coincide with one hinging on individual rationality? An option, but one that goes against the very grain of this entry, is to dissociate competition from price-taking entirely and derive it as a consequence, as in the no-surplus characterisations of Makowski-Ostroy (2001; Section 9) and Serrano-Volij (2000). The field is under active development; in addition to the papers of Sun, Tourky and Yannelis, see Forges, *et al*. (2002), Herves-Beloso, *et al.* (2005), Martins-da-Rocha (2003, 2004), Podczek (1997, 2003) and their references.

In his classic 1936 *tour de force*, Hayek deconstructed the Arrow-Debreu-McKenzie construction before it was constructed, so to speak, by distinguishing between an *a priori* "pure logic of choice" and an empirical science. In so far as this entry, in its focus on existence and core-equivalence, has concentrated on the adjective *perfect*, and avoided questions of cardinality, computability, learning and stability of a perfectly allocation of resources, it has neglected the noun competition as being outside its scope. For this, the reader could perhaps begin with Morgan (1993), and move from there to Arrow (1986), Buchanan (1987) and Radner (1991), and from there, if she is still so inclined, to the entire gamut of economic theory.

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