Trade through Endogenous Intermediaries

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Abstract

We apply an intermediation game of Townsend (1983) to analyze trade in an exchange economy through endogenous intermediaries. In this game, each trader has the opportunity to become an intermediary by offering to buy or sell unlimited quantities of the commodities at a certain price vector and for a certain group of customers subject to feasibility constraint. An intermediary will not be active unless some of its customers subsequently choose to trade with it. We introduce an "intermediation core" and show that the subgame-perfect equilibrium allocations of the intermediation game are contained in the intermediation core, similar to the inclusion of competitive equilibrium allocations in the core usually studied. We also identify, in terms of the supporting intermediary structures, intermediation core allocations which are also subgame-perfect equilibrium allocations of the intermediation game. These results provide both a characterization and welfare properties of subgame-perfect equilibrium allocations of the intermediation game.

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1 Introduction

Fundamental to the Walrasian model of exchange is the requirement that trade be governed by a uniform price system. If commodities pass between two consumers in a certain ratio, then they cannot pass between two other consumers in a different ratio. However, prices in the Walrasian model are given *ex machina* and are not responsive to the consumers' buying and selling decisions. A theory is therefore needed to give an account of how prices are formed.

In the literature, several approaches to price formation in general equilibrium settings have been proposed. In this paper, we follow the approach by Townsend (1983). Under this approach, consumers trade through endogenous intermediaries. The approach is described by a two-stage intermediation game. One variant of this game works as follows. In the first stage, each trader individually and simultaneously offers to buy or sell unlimited quantities of the commodities at a certain price vector and for a certain group of other customers subject to feasibility constraint. A trader may be chosen to be customers of more than one intermediaries. However, each trader must subsequently choose to trade with at most one intermediary in the second stage. Furthermore, a trader is obligated to act as an intermediary at the announced terms should some customers of his group choose to trade with him; otherwise, he is free to act as a customer of an intermediary that includes him.[1] The subgame-perfect equilibrium (SPE in short) is chosen as the solution concept for the intermediation game. Notice that a trader's second stage choices may depend on what the other traders choose. For this reason, the *social equilibrium* in Debreu (1952) is applied to the subgames in stage 2.

The subgame-perfect equilibrium of the intermediation game has the following properties. First, each non-intermediating trader maximizes utility by choosing an intermediary with whom to trade and the amount to trade. Second, traders partition themselves into disjoint trading cooperatives, such that there is an active intermediating trader within each cooperative who specifies the terms of trade. Third, there is no incentive for entry of new

[1] This is one of the several variants of the model in Townsend (1983). See Townsend (1978), Boyd and Prescott (1986) and Boyd et al. (1988) for applications of the intermediation games.

intermediaries nor change of strategies by existing intermediaries. It follows that the equilibrium has both cooperative and non-cooperative aspects. However, unlike competitive equilibrium, a SPE allocation is not necessarily contained in the core usually studied. This is largely due to the restricted feasibility of the allocations as imposed by intermediation. The reader is referred to Townsend (1983) for detailed equilibrium analysis of the intermediation game.

The core of an economy is based on coalitional improvements that depend on what each coalition can achieve by its own members. The core usually studied is based on the assumption that any reallocation of a coalition's total endowments among its members is feasible for the coalition. However, it is not clear under this formulation of feasible coalitional allocations how players organize themselves into coalitions and how they carry out the trade.

The purpose of this paper is twofold. First, we apply the idea of intermediation to modify the core of an economy by reformulating feasible allocations for coalitions and for the economy. Specifically, for an allocation to be feasible for a coalition, it must be achievable by having one member behave as an intermediary while the other members act as *price-taking* customers of the intermediary. Thus, at any feasible allocation of a coalition, all members but the intermediating trader maximize utility subject to budget constraint. For an allocation to be feasible for the economy (i.e., for the grand coalition), however, we allow for the possibility that trade is carried out in multiple disjoint intermediaries (see the discussion below Definition 3). Hence, we require that there exist a partition of all traders into disjoint sub-coalitions, such that the restriction of the allocation to any sub-coalition is achievable by having some trader in the sub-coalition act as an intermediating trader for the rest of the sub-coalition. We call the resulting core the *intermediation core*. The intermediation core is explicit about how traders organize themselves into coalitions, and how trade gets carried out.[2]

[2]It follows that feasible coalitional allocations in this paper are different from those in both Mas-Colell (1975) and Qin et al. (2006). In the former, feasible allocations of a coalition are required to be allocations in competitive equilibrium of the sub-economy composed of members of the coalition, while in the latter, no one is required to maximize utility subject to budget constraint.

Second, we compare the SPE allocations of the intermediation game with allocations in the intermediation core. We show that SPE allocations of the intermediation game are contained in the intermediation core under general conditions. Furthermore, we identify, in terms of the supporting intermediary structures, intermediation core allocations which are also SPE allocations of the intermediation game. Our result implies that an intermediation core allocation can be supported as a SPE allocation of the intermediation game whenever all intermediaries in the supporting intermediary structure have at least two customers. This stability of the intermediation core allocations resembles the contestability notion in industrial organization (Baumol et al., 1982). In particular, the two-customers requirement ensures that, for any active intermediary, there is always a contestable intermediary who stands ready to serve the other customers at the same terms. These results help to characterize equilibrium allocations of the intermediation game and to analyze their welfare properties.

The rest of the paper is organized as follows. Section 2 introduces the intermediation game, its subgame-perfect equilibrium, and the intermediation core. Section 3 establishes the main results and Section 4 concludes. The appendix contains an example showing an unequal treatment of at an intermediated core allocation.

2 The Economy

We consider an exchange economy with finite ℓ physical commodities. Let N be the set of traders. A trader $i \in N$ has an initial endowment $\omega^i \in \mathbb{R}^{\ell}_{++}$ and consumption set $X^i \in \mathbb{R}^{\ell}_{+}$. His preferences are represented by a strictly increasing utility function $U^i : X^i \to \mathbb{R}$. The economy is described by the list $\mathcal{E} \equiv \left(X^i, U^i, \omega^i\right)_{i \in N}$.

2.1 A Intermediation Game

Following Townsend (1983), we consider a non-cooperative intermediation game with endogenous intermediaries. A trader can try to gain market power by offering to intermediate for a certain group of customers. However, the degree of market power is weakened by competition between intermediaries. Specifically, the game has the following two stages

Stage-1

Each trader i announces a subset $C^i \subseteq N$ with $i \in C^i$ and a price vector $p^i \in \mathbb{R}^\ell_+$. The pair $s^i = (p^i, C^i)$ represents trader i's offer to buy or sell unlimited quantities at price vector p^i for customers in $C^i_{-i} = C \setminus \{i\}$. We use $s^i = \emptyset$ to denote the announcement with $C^i_{-i} = \emptyset$, in which case, trader i forgoes the opportunity to act as an intermediary.

Stage-2

Given Stage-1 announcements $s = (s^1, \cdots, s^n)$, trader i's feasible choices are as follows.

(i) $s^i = \emptyset$

In this case, trader i can either choose to trade with an intermediary offered by a trader in $\left\{j \in N : i \in C^j\right\}$ or stay autarkic. Denote this choice by $d^i(s)$. Here, $d^i(s) = j$ means that i chooses to trade with j while $d^i(s) = 0$ means that he chooses to stay autarkic. When $d^i(s) = j$, trader i also chooses a net-trade vector $z^i(s)$ such that

$$p^j \cdot z^i(s) = 0 \text{ and } z^i(s) + \omega^i \in X^i, \tag{1}$$

where $j = d^i(s)$. The resulting bundle for trader i is

$$x^i(s) = \begin{cases} z^i(s) + \omega^i & \text{if } d^i(\mathbf{s}) \neq 0, \\ \omega^i & \text{if } d^i(\mathbf{s}) = 0. \end{cases} \tag{2}$$

(ii) $s^i \neq \emptyset$

In this case, if $d^k(s) \neq i$ for all $k \in C^i_{-i}$, trader i can act as if $s^i = \emptyset$. If $d^k(s) = i$ for some $k \in C^i_{-i}$, however, trader i must act as an intermediary characterized by his Stage-1 announcement. The resulting bundle for i is

$$x^i(s) = \omega^i - \sum_{\left\{k : d^k(\mathbf{s}) = i\right\}} z^k_i(s). \tag{3}$$

If the bundle in (3) does not belong to X^i, then it is not feasible for i to act as an intermediary for the group $\left\{k : d^k(\mathbf{s}) = i\right\}$. In that case, trader i will act as if $s^i = \emptyset$.

Observe that traders' feasible choices in the second stage depend on each others' choices within this stage. Because of this, we apply the *social equilibrium* in Debreu (1952) to this stage in our determination of subgame-perfect equilibrium of the two-stage game.

Definition 1. A *subgame-perfect equilibrium* is a strategy profile $(s^{*i}, d^{*i}, z^{*i})_{i \in N}$ such that for each trader i

(i) $(d^{*i}(\mathbf{s}), z^{*i}(\mathbf{s}))_{i \in N}$ is *maximal* for all Stage-1 choices $\mathbf{s}$, given $(d^{*j}, z^{*j})_{j \neq i}$; and

(ii) s^{*i} is *maximal*, given $(s^{*j})_{j \neq i}$ and given $(d^{*j}, z^{*j})_{j \in N}$.

2.2 Intermediation Core

We modify the feasibility of an allocation for a coalition by requiring that the allocation be achieved with one member behaving as an intermediary for the rest of the coalition's members. The intermediating member is the organizer of both the coalition and the trade behind the allocation. Formally,

Definition 2. Given $C \subseteq N$, a C-allocation $\left(x^i\right)_{i \in C}$ is *feasible* for coalition C if

$$\sum_{i \in C} x^i = \sum_{i \in C} \omega^i \tag{4}$$

and there a price schedule p such that, x^i solves

$$\max_{x^i} U^i\left(x^i\right) \text{ subjectto } p \cdot x^i \leq p \cdot \omega^i \tag{5}$$

for all $i \in C$ except for at most one member in C.

The set of all C-feasible allocations is denoted by $F(C)$. The trader whose bundle does not maximize utility subject to budget constraint is the intermediating member. The rest of the members are the customers of the intermediary. In case each member's bundle maximizes utility subject to budget constraint, any one of them can be considered as the intermediating trader.

Definition 3. An allocation $\left(\bar{x}^i\right)_{i \in N}$ is in the intermediation core if there exists a partition $\{\bar{C}^k\}_{k=1}^m$ of N such that $(\bar{x}^i)_{i \in \bar{C}^k} \in F(\bar{C}^k)$, for $k = 1, 2, \cdots, m$, and there does not exist any coalition $C \subseteq N$ such that $U^i(x^i) > U^i(\bar{x}^i),\ \forall i \in C$, for some C-allocation $(x^i)_{i \in C} \in F(C)$.

We call the collection $\{(\bar{p}^1, \bar{C}^1), \cdots, (\bar{p}^m, \bar{C}^m)\}$, with $\bar{p}^k$ a supporting price vector for $\bar{C}^k$ for all k, a *supporting intermediary structure* for the allocation $\bar{x} = (\bar{x}^i)_{i=1}^n$. To be in the intermediation core, we allow an allocation to be achievable through multiple disjoint intermediaries in stead of just one grand intermediary. Notice also that the intermediation core remains the same if for any coalition C, we modify $F(C)$ by allowing trade between members in coalition C be achievable though multiple disjoint intermediaries.

Since coalitional improvements through intermediation are more restrictive and since competitive equilibrium allocations are in the core usually studied, it follows that competitive equilibrium allocations are in the intermediation core. We summarize this result in the following lemma whose proof is omitted. An implication of this lemma is that the intermediation core of an economy is non-empty under general conditions.

Lemma 1. *Competitive equilibrium allocations are intermediation core allocations.*

3 Main Results

We begin with a result that shows that competition between endogenous intermediaries is strong enough to make SPE allocations of the intermediation game intermediation core allocations. We then establish a partial converse of this result; namely, an intermediation core allocation is also a SPE allocation of the intermediation game whenever all intermediaries in the supporting intermediary structure have at least two customers.

Theorem 1. *The SPE allocations of the intermediation game are intermediation core allocations.*

Proof. Let $(\bar{x}^i)_{i \in N}$ be an allocation resulting from a SPE, $(s^{*i}, d^{*i}, z^{*i})_{i \in N}$, of the intermediation game. Suppose it is not in the intermediation core. Then, there exist a coalition C

and $(x_i)_{i\in C} \in F(C)$ such that

$$U^i\left(x^i\right) > U^i\left(x^{*i}\right), \ \forall i \in C. \tag{6}$$

Let p be the price schedule that supports $\left(x^i\right)_{i\in C}$. Choose $j \in C$ whose bundle x^j does not maximize U^j subject to budget constraint.[3] Now, consider $s^j = (p, C)$. By (6), each trader $i \in C$ with $i \neq j$ chooses to trade with j. That is, each $i \in C_{-j}$ chooses

$$d^{*i}\left(s^j, \mathbf{s}^*_{-j}\right) = j, \ \forall i \in C \setminus \{j\}. \tag{7}$$

Since $(x)_{i\in C} \in F(C)$ and since p is the supporting price vector, (7) implies that $x^i(s_j, s^*_{-j}) = x^i$ for all $i \in C_{-j}$; hence, $x^j(s_j, s^*_{-j}) = x^j$. By (6), $(x^{*i})_{i\in N}$ cannot be a SPE allocation of the intermediation game, which is a contradiction. □

We now establish a partial converse of Theorem 1.

Theorem 2. *Let $x^* = (x^{*i})_{i\in N}$ be in the intermediation core with supporting intermediary structure $\{(p^{*1}, C^{*1}), \cdots, (p^{*m}, C^{*m})\}$. If $|C^{*k}| \geq 3$ for all $k = 1, 2, \cdots, m$, then x^* is a SPE allocation.*

Proof. For each $k = 1, 2, \cdots, m$, let $j_1^k \in C^{*k}$ be the intermediating trader for the other members of C^{*k} and $j_2^k, j_3^k \in C^{*k}$ be two other members. Now, consider players' strategies

$$s^{*j_1^k} = \left(p^{*k}, C^{*k}\right), \ k = 1, 2, \cdots, m, \tag{8}$$

$$s^{*i} = \begin{cases} \left(p^{*k}, C^{*k} \setminus \{j_1^k\}\right), & \exists k : i = j_2^k \text{ or } j_3^k; \\ \emptyset, & \text{otherwise}, \end{cases} \tag{9}$$

and

$$d^{*i}(s^*) = \begin{cases} \emptyset, & \exists k : i = j_1^k; \\ j_1^k, & \exists k : i \in C^{*k} \text{ and } i \neq j_1^k. \end{cases} \tag{10}$$

Since $(x^{*i})_{i\in N}$ is an intermediation core allocation, it follows form (8) and (9) that $d^*(s^*)$ in (10) is maximal. In addition, any strategy profile that that has $(s^*, d^*(s^*))$ as the path of play can implement allocation x^*. Thus, it suffices to show that $(s^*, d^*(S^*))$ is a SPE path.

[3] In case everyone's bundle maximizes utility subject to budget constraint, choose j arbitrarily.

To this end, notice that considering $(s^*, d^*(s^*))$ as candidate SPE path, we only need to specify traders' choices at the off-equilibrium path in the events, in which a *single* trader deviates. For any trader j and for any $s^j = (p, C) \neq s^{*j}$, define $d^{*i}(s^j, s^*_{-j})$ for $i \neq j$ by

$$d^{*i}(s^j, s^*_{-j}) = \begin{cases} 0, & d^{*k}(s_j, s^*_{-j}) = i, \\ j, & i \in C, U^i(x^i(p)) > U^i(x^{*i}), d^{*k}(s^j, s^*_{-j}) \neq i, \forall k, \\ j^k \in J^{*k}_{-ij}, & i \in C \cap C^{*k}, U^(x^i(p)) \leq U^i(x^{*i}), d^{*k'}(s^j, s^*_{-j}) \neq i, \ \forall k', \\ j^k \in J^{*k}_{-ij}, & i \in C^{*k} \setminus C, d^{*k'}(s^j, s^*_{-j}) \neq i, \ \forall k', \end{cases} \tag{11}$$

where $J^{*k} = \{j^k_1, j^k_2, j^k_3\}$, $J^{*k}_{-ij} = J^{*k} \setminus \{i, j\}$, and define $d^{*j}(s^j, s^*_{-j})$ by

$$d^{*j}(s^j, s^*_{-j}) = \begin{cases} 0, & \text{if } \exists k : d^{*k}(s^j, s^*_{-j}) = j, \\ d^{*j}(s^*), & \textit{otherwise}. \end{cases} \tag{12}$$

By (11) and (12), $d^{*i}(s^j, s^*_{-j})$ is maximal for all i. Thus, it only remains to show that trader j has no incentive to deviate from $s^*j = (p^{*j}, C^{*j})$ to $s^j = (p, C)$.

Since x^* is in the intermediation core, either j or some member $i \in C_{-j}$ such that $d^{*i}(s_j, s^*_{-j}) = j$ must not be strictly better off. However, by (11), the members in C who choose to transact with j must be strictly better off. It follows that j, the deviator, cannot be strictly better off. This shows that trader j does not have has any incentive to unilaterally deviate in Stage 1 given the State-2 maximal choices in (11) and (12) and the others' Stage-1 choices in (8)-(10). $\square$

The three-members restriction ensures that, for any active intermediary, there is always a contestable intermediary who proposes to serve all customers at the same price. If there are at least two active intermediaries one of which has only one customer, the intermediating trader of the one-customer intermediary could have beneficially proposed a different price vector for perhaps a different set of customers. This is shown in Example 1 below, in which an intermediation core allocation need not be a SPE allocation.

Example 1. Consider an exchange economy with two physical commodities and $N = \{1, 2, 3\}$. Traders' preferences are representable by utility function $U^i(x_1, x_2) = \min\{x_1, x_2\}$ for $i = 1, 2, 3$. Traders' endowments are given by $\omega^1 = (10, 0), \omega^2 = (0, 9), \omega^3 = (1, 0)$. We show that the allocation $\bar{x} = (\bar{x}_1, \bar{x}_2, \bar{x}_3)$ with $\bar{x}^1 = (5.5, 4.5)$, $\bar{x}^2 = (4.5, 4.5)$, and $\bar{x}^3 = (1, 0)$ is in the intermediation core. Notice that $(\bar{x}^1, \bar{x}^2) \in F(\{1, 2\})$ and $\bar{x}^3 \in F(\{3\})$. Notice also that no single trader alone can improve upon the allocation. Since neither trader 1 nor trader 3 is endowed with good 2, they together cannot improve upon the allocation. Trader 2 and trader 3 cannot join together and improve upon the allocation since the maximum amount of good 1 that trader 2 can have is 1 unit. Trader 1 and trader 2 cannot together improve upon the allocation because their bundles consist of a Pareto optimal allocation for them.

Let $(x^1, x^2, x^3) \in F(N)$. Observe that it is infeasible trader 3 to intermediate for the other two because of his endowment. Suppose first that trader 1 intermediates with relative price $\rho = p_2/p_1$. Then,

$$x^2 = (\frac{9\rho}{1+\rho}, \frac{9\rho}{1+\rho}) \text{ and } x^3 = (\frac{1}{1+\rho}, \frac{1}{1+\rho}) \Rightarrow x^1 = \left(\frac{10+2\rho}{1+\rho}, \frac{8}{1+\rho}\right).$$

Thus, for $U^2(x^2) > U^2(\bar{x}^2)$, it must be $\rho > 1$. However, with $\rho > 1$,

$$U^1(x^1) = \frac{8}{1+\rho} < 4 < U^1(\bar{x}^1).$$

Suppose now that trader 2 intermediates. Then,

$$x^1 = (\frac{10}{1+\rho}, \frac{10}{1+\rho}) \text{ and } x^3 = (\frac{1}{1+\rho}, \frac{1}{1+\rho}) \Rightarrow x^2 = \left(\frac{11\rho}{1+\rho}, \frac{9\rho-2}{1+\rho}\right).$$

Thus, for $U^1(x^1) > U^1(\bar{x}^1)$, it must be $\rho < 11/9$. However, with $\rho < 11/9$,

$$U^2(x^2) = \frac{9\rho-2}{1+\rho} < 4.05 < U^2(\bar{x}^2).$$

In summary, the grand coalition cannot improve upon allocation $\bar{x}$.

We now show that $\bar{x}$ cannot be a SPE allocation. To this end, consider any strategy profile $(s^i, d^i, z^i)_{i \in N}$ that results in allocation $\bar{x}$. Note that the prices of the intermediating trader must be strictly positive.[4] Because of this, trader 3 cannot intermediate for both

[4] Suppose first that trader 1 is a customer. Then, the price of good 1 must be strictly positive for him to be able to afford $(5.5, 4.5)$. If the price of good 2 is zero, then trader 1 would demand for bundle $(10, 10)$ which is not feasible. If trader 2 is the customer, however, then the price of good 2 must be positive for him to be able to afford bundle $(4.5, 4.5)$. Hence, if the price of good 1 is zero, then trader 2 would demand for bundle $(9, 9)$, which is not compatible with trader 1 receiving $(5.5, 4.5)$.

trader 1 and trader 2. The reason is as follows. Trader 3 is endowed with 1 unit of good 1 only. Thus, it is not feasible for him to act as an intermediary for trader 1 or trader 2 alone. On the other hand, if both traders 1 and 2 are customers of trader 3, then they will demand at price vector p for a total quantity of $(10p_1 + 9p_2)/(p_1 + p_2) > 9$ units of each good. Since total endowment of good 2 is 9 units, it is not feasible for trader 3 to intermediate for both traders 1 and 2. Notice also that trader 3 cannot be a customer of either trader 1 or trader 2, because otherwise he would have demanded for a bundle different from $\bar{x}^3$.

Consider a deviating strategy $\tilde{s}^1 = (\tilde{p}, \tilde{C})$ with $\tilde{C} = \{1, 2, 3\}$ and $\tilde{\rho} = \tilde{p}_2/\tilde{p}_1 < 1$. Since trader 2 cannot be a customer of trader 3, $d^2(\tilde{s}^1, s_{-1}) = 1$. In that case, trader 1 is bound to intermediate. Thus, $d^3(\tilde{s}^1, s_{-1}) = 1$ and

$$x^1 = (11, 9) - \left(\frac{9\tilde{\rho}}{1+\tilde{\rho}}, \frac{9\tilde{\rho}}{1+\tilde{\rho}}\right) - \left(\frac{1}{1+\tilde{\rho}}, \frac{1}{1+\tilde{\rho}}\right) = \left(\frac{10+2\tilde{\rho}}{1+\tilde{\rho}}, \frac{9}{1+\tilde{\rho}}\right).$$

As a result, trader 1's utility level is

$$U^1\left(x^1\right) = \min\left\{\frac{10+2\tilde{\rho}}{1+\tilde{\rho}}, \frac{9}{1+\tilde{\rho}}\right\} = \frac{9}{1+\tilde{\rho}} > 4.5 = U^1\left(\bar{x}^1\right).$$

□

4 Conclusion

In this paper we followed the approach by Townsend (1983) to consider trading in an exchange economy through endogenous intermediaries. Under this approach, each consumer can form an intermediary by offering to buy and sell unlimited quantities of the commodities at a certain price vector for a group of other consumers. We introduced an intermediation core by reformulating coalitional feasible allocations. Like the inclusion of the competitive equilibrium allocations in the core, we showed that the subgame-perfect equilibrium allocations are contained in the intermediation core. Furthermore, we showed that conditions exist with which the subgame-perfect equilibrium allocations fill up the intermediation core. This paper contributes to the literature on intermediation by providing tools for characterizing the subgame-perfect equilibrium allocation of the intermediation game and for analyzing their welfare properties.

A Unequal Treatment Property of Intermediation Core

Consider an economy in which there are three types of traders, a, b and c, each of which consists of two identical traders. We name them as $a_1, a_2, b_1, b_2, c_1, c_2$. Preferences and endowments of type-b and type-c traders are given by $U^b(x_1, x_2) = \min\left(\frac{1}{2}x_1, x_2\right)$, $U^c(x_1, x_2) = \min\left(x_1, \frac{1}{2}x_2\right)$, $\omega^b = (1, 0)$, $\omega^c = (0, 1)$. The type a's endowment is $\omega^a = (10, 10)$ and its preferences will be precisely described later.

We consider an allocation achievable by having the two type a's traders as the intermediating traders. In particular, we let $\frac{1}{2} \le p \le \frac{2}{3}$ be the price ratio and $\{b_i, c_i\}$ be the set of customers for the intermediary trader a_i is willing to offer. $i = 1, 2$. See Figure 1(a). The dash-curve is the utility frontier of the two customers in each intermediary, which is obtained as the price ratio of the intermediary changes within the range of $\frac{1}{2} \le p \le \frac{2}{3}$. when they are customers in the same intermediary, while the bold-curve is the utility frontier obtainable by themselves.[5]

Notice that each type a trader receives the following bundle from intermediating with price ratio $\frac{1}{2} \le p \le \frac{2}{3}$ and customer set $\{b, c\}$:

$$\left(10 + \frac{p(p-1)}{(1+2p)(2+p)}, 10 + \frac{1-p}{(1+2p)(2+p)}\right) \tag{13}$$

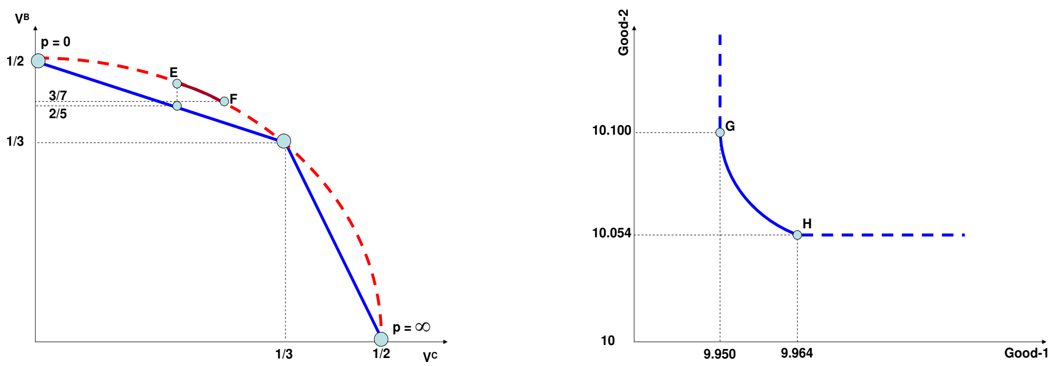
The locus of these bundles is presented by the $G - H$ curve in Figure 1(b).

We now demonstrate that the allocations achieved with the preceding range of price ratios and intermediary structure are intermediation core allocations. Let $C \subseteq N$ be any coalition.

(i) $C = \{b_i, c_j\}$: The utility level of b in this coalition is at most equal to $\frac{2}{5}$, which is lower than $\frac{3}{7}$, the utility level of b at the candidate allocation.

(ii) $C = \{a_i, b_1, b_2, c_j\}$: In order to make both traders of type b strictly better off the price ratio must be smaller than p. This will certainly make an trader type-c worse

[5] This utility frontier when both of them are customers is represented by $V^b(p) = \frac{1-2V^c}{2-3V^c}$, while the frontier when both of them form an intermediary is represented by $V^b = \frac{1}{2} - \frac{V^c}{2}$ when $0 \le p \le 1$, and $V^b = 1 - 2V^c$ otherwise.



(a) (b)

Figure 1: (a) A candidate allocation is on $E-F$ arc, with $\frac{1}{2} \leq p \leq \frac{2}{3}$. (b) An indifference curve of a trader type-a. The portion $G-H$ corresponds to the consumption function of an intermediary type A in (13) given that $\frac{1}{2} \leq p \leq \frac{2}{3}$.

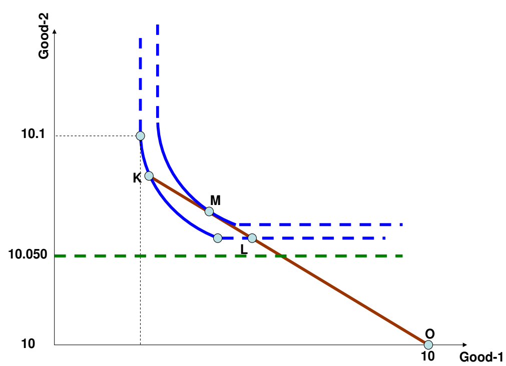
off. Hence, C cannot improve upon the candidate allocation. The similar argument applies to coalitions: $\{a, b, c_1, c_2\}$, $\{a, b_1, b_2, c_1, c_2\}$, $\{a_1, a_2, b_1, b_2, c_1, c_2\}$, $\{b_1, b_2, c\}$ or $\{b, c_1, c_2\}$.

(iii) $C = \{a_i, b_j\}$: As Figure 1(b) shows, trader a_i must receive at least 10.054 units of good-2 (see the indifference curve of an a_i in Figure 1(b)). This will leave b_j with at most $10 - 10.054 = -0.054$, which is not feasible.

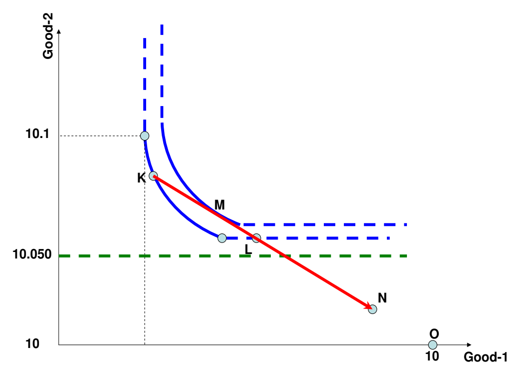
(iv) $C = \{a_1, a_2, b_i\}$: To make b_i better off, the net endowments left after satisfying b_i is at most $\left(20 + \frac{p}{2+p}, 20 - \frac{1}{2+p}\right)$. Hence, the maximum consumption level of good-2 for a type a trader is strictly less than 10.054. This implies that none of the type a can be better off.

(v) $C = \{a_1, a_2, b_i, c_j\}$: Suppose a_1 is the intermediating trader. To induce one trader of type b and one trader of type c, the proposed price must be $\frac{1}{2} \leq p \leq \frac{2}{3}$. See Figure 2(a)-2b. Let point **K** in Figure 2(a) denote the net endowment of type a traders after b_i and c_j have completed their trade, and line **K-O** represents the budget line for a.

The customer type a will choose allocation **M**. To achieve such allocation, her net-trade is given by segment **O**-**M**. Consequently, the opposite trade position relative to an allocation **K** will be the net trade of the intermediating trader (see Figure 2(b)). Using a simple geometric argument, the segment **O**-**M** is always longer than segment **K**-**M**. As a result, the consumption bundle of the intermediating trader (allocation **N**) is always below the original indifference curve of a (shown in Figure 2(b)). This implies that the intermediating trader is worse off. The similar argument applies to cases where b_i or c_j is the intermediating trader.



(a)



(b)

Figure 2: (a) The budget line and optimal consumption allocation for a customer type a. (b) The final allocation of the intermediary type a.

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