Time, the value of money and the quantification of value

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TIME, THE VALUE OF MONEY AND THE QUANTIFICATION OF VALUE

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ABSTRACT
This paper establishes, and illustrates for the case of the UK, a temporal method for calculating the labour values of outputs from any process or sector of a market economy.

It exhibits the temporal calculation of the Monetary Equivalent of Labour Time (MELT), the general ratio between monetary and labour time magnitudes, for a single national economy, but does not correct for the consequences of value transfers within the world economy as a whole. The method is nevertheless generalisable, provided international value transfers are properly accounted.
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Note: Though this study was conducted using standard published data it has been supplemented with pre-publication data kindly supplied by the UK Office of National Statistics. As is normal in preparing long data series, it is necessary to splice series from different runs, prepared to different accounting standards at different times, and to interpolate between I/O series before 1989, when yearly I/O balances became the new basis of UK National Accounts preparation. In preparing this first draft I have not had the time to validate or cross-check the resultant raw data to standards I consider adequate for publication. In addition two further methodological flaws in the National Income Accounts data require correction to present an accurate picture of the real value relations governing a Northern Industrial and Financial power, namely, the measurement of depreciation and capital stock, and the accounting for value transfers from the rest of the world. These corrections will be the subject of future work. These limitations make it inadvisable to draw policy conclusions from the data and this is not the purpose of this paper. The main purpose of the paper is to illustrate differences in methodology and demonstrate that qualitatively and quantitatively different results flow from these differences. The results should therefore not be cited. A disseminable version will follow.

SUMMARY

This paper operationalises a radical new approach to the measurement of value in terms of labour hours, which has become known as the Temporal Single System (TSS) approach (See for example the collection of papers in Freeman and Carchedi 1995). This approach accords with the work of Shaikh and Tonak, and also Moseley, insofar as it begins by transforming the National Income Accounts to distinguish properly between productive and unproductive labour. It differs, however, in the way it transforms the resultant transformed money magnitudes into labour-time equivalents, yielding substantially different results — for example, indicating a rise in the organic composition of capital where the traditional approach finds this is static or falling, and thus confirming Marx’s treatment of accumulation.

The principal existing tradition is an equilibrium or static tradition, in that it estimates values as Vertically-Integrated Labour-Embodied magnitudes, inverting a Leontieff Matrix to determine the amounts of labour time that would hypothetically be necessary to reproduce the economy using current technology, if this economy were static (which it is not). In consequence, the value transferred by inputs does not correspond to the actual labour time required to produce them. The new approach estimates values temporally, estimating the value transferred by inputs as the labour time actually used to produce them.

The existing tradition is also dualist, in that it ignores the role of money and the purchases of inputs for money, which transfers magnitudes of labour time from one purchaser to another. The new approach recognises the role of money, but without (as in the Value-Form approach) losing the essential distinction either between price and value, or between labour magnitudes and money magnitudes.

The paper calculates the monetary expression of labour time (MELT) in the framework of the temporal single-system (TSS) approach, for Britain between 1965 and 1992. This decisive, economy-wide coefficient permits us to convert any money magnitude to an equivalent labour-time magnitude, and vice versa. On this basis we calculate the following magnitudes:

(i) Alternative measures of the following standard neoclassical magnitudes: real output, the price level, the capital-output ratio, the profit rate and value added.

(ii) sectoral value outputs and price-value ratios for the principal UK National Accounts standard sectors at an 11-sector disaggregation level.

This operational procedure produces quantitatively different results from the two main alternative operational procedures, namely:

(i) measures of real output derived from money prices adjusted using the neoclassical price-index

(ii) vertically-integrated value

We argue that these differences show how Marx’s value categories may be given a meaningful empirical content in way that distinguishes his results not merely qualitatively but quantitatively from the alternative interpretations of Marx extant in the Twentieth Century.

INTRODUCTION

Our starting point is the idea that National Accounts data, using the techniques adopted by Shaikh and Tonak(1990), Moseley(1982), also employed by Freeman (1991), Fazeli (1996) and other of Shaikh’s co-workers, can and should be transformed to yield quantitative measures of variable capital, constant capital, surplus value, gross output and hence the rate of exploitation both in money terms and in terms of socially-necessary abstract labour time.
The principal steps of this transformation are common ground between a wide variety of authors. The central distinction between productive and unproductive labour, the consequent distinctions between produced and derived incomes, and their centrality as the foundation of a critique of neoclassical national income accounting is restated.

We diverge from Shaikh neither on the use of National Accounts data nor his critique of Input-Output methodology but on the definition of labour values derived from them and in particular the assumption that vertical integration produces labour values. We believe vertical integration supplies instead an alternative use-value measure of value to untransformed price data.

We make a critical assessment of the use made of Input-Output matrices by Shaikh, Ochoa, Petrovic and Cockshott-Cottrell, of their empirical finding that aggregate price magnitudes correlate closely with vertically-integrated labour coefficients, and of their interpretations of this finding.
THE RESULTS

Output: Neoclassical 'real' net and circulating capital-based gross value

Figure 1

Output: Stock-Based and Circulating Capital-based gross value

Figure 2
Figure 3

Different Measures of the Monetary expression of Labour-Time

Figure 4
Price levels: labour value (Stock-based MEL) and NIA measures

![Graph showing Price levels: labour value (Stock-based MEL) and NIA measures](image)

**Figure 5**

Capital-Output ratio, two measures

![Graph showing Capital-Output ratio, two measures](image)

**Figure 6**
A tale of two Organic Compositions

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Figure 7
Price and value sectoral comparisons

Price/Value ratios (Pounds per hour)

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Figure 8
DESCRIPTION AND PROOF OF THE METHOD USED

The unproductive/productive distinction

The earliest use of National Accounting Statistics for the purposes of obtaining quantitative estimates of Marxian quantities, according to Shaikh and Tonak (1994) [henceforth ST] was by Terashima in 1924. To the thorough bibliography in this work we would add only the compilation edited by Paul Dunne (1991) summarising outcomes of over ten years of European discussion and the recent compilation by Davis (1992) which in addition to reprinting some works cited by ST, adds some further work in the Economic Surplus tradition of Baran and Sweezy. For reasons of space I shall not repeat or summarise this here.

Shaikh and Tonak’s work represents the most authoritative statement of work in this tradition and the starting point for almost all subsequent work. They introduced, in particular, the decisive distinction between productive and unproductive labour which is critical to all further analysis. This distinction makes it impossible to take National Accounts or input-output data and from them directly calculate Marxian magnitudes such as the rate of surplus value, variable capital, hence the rate of profit, whatever view is taken of the relation between price and value.

ST instituted the first universal procedure for, so to speak, untransforming national accounts data to recover basic Marxian categories: for passing backwards from the outward form of appearance of the data to its underlying origin. The fundamental approach is best described by the authors themselves but I think it is reasonable to describe it as rooted in a methodological critique of the national account categories. Thus, like all genuine empirical work, this tradition begins by a systematic enquiry into what is that is being measured:

The measurement of national product lies at the core of all systems of national accounts (Carson and Honsa 1990:28-9). In this regard, it is interesting to note that most critics of official accounts accept the basic definitions of production embodied in the national accounts, and seek instead to extend and improve their coverage. Issues of coverage are evidently important. But the definition of production is clearly prior, and this is precisely where we differ from orthodox economists. Thus, while our own criticism is part of the general chorus, it is quite different in character rom most of the others, and has different implications.

As Maniatis (1996) notes, every single study since this time that has applied the productive/unproductive distinction has revealed essentially the same qualitative phenomena, and moreover the quantitative story that is told, when this distinction is applied, is different from the neoclassical story. If one considers that the
financial sector is productive of value as opposed to merely parasitical, one gets a completely different picture of the economy. If one considers that mere trading in goods without modifying them in any way except to change the formal owner, is capable of adding to the ‘real’ wealth of an economy, one ends up with a completely different picture of what will alter the well-being of its inhabitants for the better.

The separation of the activities of the economy into those which genuinely produce new value, and those which merely circulate it, is fundamental to Marx’s approach in almost any reading of what he was trying to do.

What this work established is that this difference, makes a difference. What makes their method impossible to ignore is that these conceptual distinctions are the foundation of a quite distinct procedure which yields quite distinct quantitative results. These conceptual distinctions are not only prior to all quantitative measurement but distinguish this method of enquiry sharply from orthodox economics.

The measurement of the value of constant and variable capital

We take the whole of this body of work as read, in what we present here, for the simple reason that we do not have the time to repeat it. We do not repeat it also because we have no essential difference from it and we regard it as the starting point of all serious subsequent work. We have applied the methodology developed to pre-transform the data by reallocating categories of expenditure wrongly recorded in the national accounts as costs of production, to their correct position as unproductive expenditures out of revenue. These same corrected raw figures are used in all subsequent calculations and comparisons except for the neoclassical measures of real output, the price level which obviously cannot be thus transformed since the neoclassicals must be left free to calculate their own measures of output and price level in their own chosen manner, or the comparison is an unfair one.

Problems arise with measures which, we assert, have been calculated in the neoclassical manner but ascribed to Marx or use as the basis of comparisons with Marx. In particular this applies to the derived measures of the capital-output ratio and organic composition. In later work we hope to correct this to make the two measures directly comparable, or rather to ensure that the only distinction rests in the method used to measure the value of constant and variable capital.

This latter is our principal difference with preceding work as it so far stands. Our differences is in the measurement of the categories of price and value themselves, that is, the procedure to be adopted in untransforming money prices back into labour values. The conceptual distinctions between the various interpretations of Marx, all of which relate to the meaning of the concept of value, are not mere refinements of language but make an enormous practical difference in the interpretation of virtually all published data.

The meaning assigned to the concept of labour-value by the great majority of those working with input-output data begins from two ideas, both introduced by Bortkiewicz, and both challenged by recent work:

1) values are defined in the Bortkiewicz tradition as the solution to a set of simultaneous equations which represent rates of exchange in a hypothetical economy, which would permit this economy to reproduce itself if net value added in each sector were directly proportional to labour-hours worked.¹

2) the value transmitted to constant capital (constant capital) by consumed means of production is given by the vertically-integrated (henceforth VI) values so calculated, and the value appropriated by the workers (variable capital) is given likewise by these vertically-integrated values.

The first, seminal, alternative to this interpretation is the view associated with Duncan Foley, Gérard Duménil, and others generally known as the ‘New Interpretation’, who argued that the value of variable capital at least is represented in effect, not by vertically-integrated values of consumed wage goods but by the value of the money that was spent on them.

A second alternative now known as the single-system approach was first published by Wolff, Roberts and Callari in 1982 and argued that not only variable, but also constant capital, is represented not by VI values of consumed means of production but by the value of the money spent on them.

An alternative expression that brings both these procedures within a common linguistic framework is the term suggested by Ramos and Rodríguez, the ‘monetary expression of value’, which in the New

¹ in some interpretations this is asserted directly to mean that value added is proportional to wages. Other mathematical formulations exist but are indistinguishable in the practical outcome that results.
Interpretation framework would be the inverse of the value of money, and in the single-system framework is the amount of labour represented in exchange by a unit of currency. In this paper we shall use the term ‘monetary expression of labour-time’ (MELT) to make clear that what is involved is a relation between two measures, one in terms of money and the other in terms of (socially necessary abstract) labour-time. I do not regard the terminological question as particularly decisive except insofar as the pre-New Interpretation usage of the ‘value of money’ took it to mean the labour embodied in the money commodity, and it is therefore useful to avoid this confusion.

Clearly the implications of the single-system are more far-reaching. Whereas the New Interpretation definition impinges on at most the definition of variable capital and hence surplus value, it need not in any way modify the definition of value itself since, in common with the dualist or vertically-integrated approach, the value contribution of consumed constant capital is still given directly without the intermediary of money. The New Interpretation has been characterised by a co-existence between the more radical account of Foley and those of writers such as Glick and Ehrbar, probably Duménil and Levy also, and of Mohun, who tend to see it merely as a different way of reading the same equations – hence the name New Interpretation.

This cannot be the case with the single-system interpretation, even in its simultaneous version, which modifies the definition of value itself since in this interpretation, constant capital is itself a different magnitude and thus, so is value.

Although the WCR calculation, since adopted by Fred Moseley, has not been applied to our knowledge to quantitative measurement, a paper by Anwar to the Bergamo conference produced a number of investigations of the correlation between aggregate value outputs calculated in this manner. The method implies that the value of inputs is given not by the value of the money actually paid for them but, as with the vertically-integrated system, the money which would be paid (that is, the prices which would obtain) in a hypothetical economy which would reproduce at these prices on the assumption that value added were proportional to advanced (or in some versions, circulating) capital.

A criticism of this approach which we develop in our main paper to this conference is that in consequence none of the magnitudes referred to in this interpretation correspond to what actually exists in the economy. In common with all general equilibrium methodology the concepts are therefore defined, not with reference to the actual economy but to a hypothetical state which it does not and cannot attain. The justification offered in the past is that this hypothetical state is a first approximation or centre of gravity of the actual state of the economy. The problem is, as our own work has shown, it is neither a first approximation nor a centre of gravity if productivity is rising systematically, as is the case under capitalism. If a justification for this procedure is to be produced, it must in our view stand on its own two feet without reference to this refuted claim to serve as an approximation. Indeed, the quantitatively different results produced here add further weight to this assertion.

The temporal approach

A further and distinct development of the single-system approach, which we term the temporal single-system approach, and which is applied here, takes an entire different view in the following essential respect: we deal, neither with hypothetical values nor with hypothetical prices but with the actual prices at which goods are traded, that is, with the directly observable magnitudes of the economy. This is the evident consequence of our critique of Bortkiewicz whose ‘correction’ to Marx is rooted in the claim that inputs to production should be valued, not at the prices for which these inputs are purchased, but at the prices realised by the outputs resulting from them.

In fact, once it is accepted that such a thing exists as the ‘value of money’ in the sense discussed either by the New Interpretation or by Single-System interpretations of any stripe, it is evident that this cannot refer to the value of money in the normally-accepted sense of the value embodied in the money-commodity. For a fuller discussion of this we refer the reader elsewhere, particularly to Rodriguez. The ‘value of money’ in this sense refers to the purchasing power of money over the labour of society and must necessarily do so. Differences arise, therefore, from the meaning which is to be assigned to the term ‘the labour of society’. The approach to be taken here is that money, in exchange, stands as the representative of pre-existing value arising from production, that is, the dead labour of society. In this sense the term ‘value of money’ is quite distinct from, for example, Smith’s concept of labour commanded which refers to living labour.

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2 In this paper Shaikh termed these ‘Marxian’ prices of production.
What does it mean, to say that the goods consumed as constant capital transfer social labour to the outputs and this is measured by the money paid for them? Essentially, it means that in exchange these goods represent a definite proportion of the value in circulation, a portion of what Marx terms ‘the totality of commodities and money in circulation’. If we consider the immediate outcome of production, it is evident that there is a definite mass of such commodities and they have at a given point in time a definite money price. But they also have a definite value, measured in hours. Any purchase of any portion of these commodities therefore represents a certain definite aliquot portion of this total value. If, for example, the money expression of goods currently in circulation is £100 million and their total value is 2,000 worker-years, then each £1 million represents 1/100th of these 2,000 worker-years, namely 20 worker-years. Consequently, we may calculate the dead labour represented by any sum of money, including clearly the dead labour entering into any part of the labour process, or the whole of the labour process. In consequence we may measure the value resulting from this labour process, since we have now reduced both dead and living labour to the same homogenous magnitude, abstract labour-time. The value arising from the process is then the sum of the abstract dead and living labour embodied in it, calculated in this manner.

This value is quite distinct from the price, and the difference is a quantitative one. In exchange the values that have arisen in the manner we have just described will be exchanged against money and this money, in exactly the same way but with new proportions, will represent value, measurable in hours. Some will realise more hours than their value, and some will realise less. Conversely, since every amount of value may equally well be represented in money, we may assign a money magnitude to their value and we will find that their value, measured in money, is in general systematically different from their price.

Table 1 illustrates this for the UK economy in 1992, using the Monetary Expression of Labour Time which we calculate using the method explained below and is equal in this year to £45,540 per worker-year. That is, in circulation, each worker-year exchanges for (realises) £45,540.

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Table 1: prices and values in 1992

(Legend: 1 Agriculture, hunting, forestry & fishing; 2 Mining & quarrying; 3 Manufacturing; 4 Electricity, gas & water supply; 5 Construction; 7 Transport, storage & communication; 9 Public admin., national defence & compulsory social security; 10 Education, health & social work; 11 Other services including sewage & refuse disposal)

The underlying principle is thus as follows: at any given moment in time, a definite magnitude of money represents, in exchange, a definite amount of previously-created and accumulated dead labour. On this basis, any money magnitude may be converted into an amount of labour. In this form it may be added to living labour, with which it is directly comparable.

In each period, new prices are established in circulation on the basis of the values arising thus from the immediately preceding phase of production. In consequence a new monetary expression of labour time, is established and in the next phase of production, the dead labour represented by the goods entering production are correspondingly altered.

The two questions that arise are therefore:
may this critical parameter, the monetary expression of labour time, be calculated sequentially from
the data?

since the estimate of this parameter depends on the initial condition, how likely is it that this initial
estimate will introduce errors into the procedure?

We seek to show that it can indeed be calculated, and that the error term will rapidly die away

The procedure

This account proceeds for pedagogical reasons in five stages

In the first stage I assume all constant capital is circulating and that the MELT at time t(0) is accurately
known. In the second stage I enquire into the effects of an error in the original estimate of the MELT and
prove that the error term in subsequent estimates of the MELT must decay rapidly.

In the third stage I make a comparison with other measures of the MELT and also of the price level, and
explain why I think the measure I propose is a useful generalisation and in particular, why it differs from
neoclassical price indices, and how it expresses Marx’s concept that value may only be created by living
labour.

Fourth, I extend the analysis to fixed capital and finally, I look at practical problems, in particular the
measurement of depreciation, obtaining comparable measures of the consumption of circulating constant
capital, and a solution to the question of unproductive labour.

Notation

(a) I find time-subscripts surrounded by brackets confusing in E-Mail so I’d like to adopt a simple
convention: I use + and - for lags and leads. Thus if C means constant capital at time t=0 then C+ means
constant capital at time t=1, C– means Constant capital at time t=−1, and so on. I use X+ to refer to the
output of period t=0.

(b) To distinguish the value of a commodity in hours from the same thing measured in money, I’ll write C
and £C respectively.

(c) £C and C are not vectors but scalar sums. (£C is the money value of all consumed inputs in the whole
economy, C is the same thing in hours). This is not therefore a single-sector illustration but applies to
any arbitrary number of sectors.

(d) A second source of confusion, the quantitative divergence of price from value, need not intrude as long
as we are dealing with the economy as a whole. Total price is equal to total value, so X£ is both the total
price and the total value of outputs measured in money; likewise X is both the total price and the total
value of outputs measured in hours. Since we are using the single-system interpretation, £C is both
the total price of inputs and the total value they transmit to the outputs, again measured in money; likewise C
is both the total price of inputs and the total value they transmit to the outputs, measured in hours.

General assumptions about data

The data required from public records is:

(a) the price of total output excluding transfers during each period: £X+, £X++, £X+++ etc
(b) the price of total inputs to productive sectors of the economy during each period £C, £C+, £C++ etc
(c) the total productive hours worked in the economy during each period L, L+, L++ etc. Take this as directly
equal to total abstract labour added in production, that is to total new value in hours.
(d) the initial value of the inputs at time t=0, namely C.

This is not a model: no necessary mathematical relations are assumed between any datum from this list and
any other. The conclusions are valid for all models and any assumptions, or if there is no model at all. My
concern is to provide the general theoretical background to all models by furnishing the terms, definitions
and magnitudes in which any more concrete theory needs to be expressed, if it is to make the distinctions
necessary to understand the workings of a market economy. We are concerned only to operationalise the
calculation of the key value parameters of a market economy from observed data.
The proposal

Suppose initially that over the whole of the economy £C is entirely consumed in a period, with the addition of L hours of abstract labour, and that the resultant produce sells for £X'. By X' I intend what Marx designates as C'. £X' corresponds more loosely to what Marx designates as M', the monetary measure of C'. I mention this only because it may help fix ideas; establishing the precise correspondence with Marx's notation is a more complex issue.

It should be noted that in common with Marx, we are concerned with the gross output of all commodities arising from production, namely C+L, not with net output (L = V+S, or value added as it is normally known).

Suppose as indicated that at time t=0, C is known. The MELT at this time, though not yet needed given our knowledge of C, can be calculated from C and £C. Call it n. Its units are £/hr. Then

\[ n = \frac{\text{£C}}{C} \]

Adding dead labour C to living labour L gives the value of output in hours

\[ X' = C + L \]

X' is thus derived from the data of the period independent of the price for which output actually sells. So far, we have not used the quantity £X'. It is in this sense, I would argue, that value is objectively given independent of prices, in each period. What is past is objective in all possible senses of the word: it is independent of the thinker, it is not mutable and it does not depend on our desires. It is interesting to compare this idea, which I think is Marx's, with what I see as a quite similar position expressed by a number of Post-Keynesian thinkers eg Arestis (1988:42):

Post-Keynesian analysis is thus crucially concerned with historical time which is sharply distinguished from logical time. The essence of recognising the importance of historical time in an economic system is that 'its past is given and cannot be changed, and that its future is uncertain and cannot be known' (Moore 1979a:121).

From the data we do however know £X'. This is required not for the calculation of X' but of n', the MELT governing the exchange of money for dead labour in the next period.

\[ n' = \frac{\text{£X'}}{X'} \]

n' is thus not (for me) a ratio between two flow quantities but between two stock quantities. I think this distinction is the source of much confusion and I think is probably the most controversial issue to be discussed. To my way of thinking the MELT is the ratio between the money value of a definite ensemble of goods at a definite point in time and the labour value of the same ensemble of goods at the same point in time.

Hence n is at any given moment a universal parameter of the economy. It expresses the number of hours of dead labour which a given magnitude of money can purchase, at a specific point in time. That is, it expresses the purchasing power of a unit of money, measured in hours.

Hence it converts any money magnitude into a number of hours, the labour value which that money magnitude represents, considered as an aliquot share of the total produce of society. [Conversely of course it furnishes the monetary measure of the value-creating capacity of new living labour]. It thus serves, for example, as the basis for calculating the value output by any individual capital, since it will allow us to convert the money expenditures of the capitalists into hours of abstract labour and hence to add up dead and living labour. It serves as an index of the general purchasing power of money, by reducing all money sums to shares in the total labour output of society.

It is not the same as labour commanded and does not suffer the Smithian confusion between the cost of labour-power and its value-creating capacity; the labour in question is dead, the result of production and not its precondition. Nor is it the same as neoclassical purchasing power; it is independent of the amount of use value in which this labour is embodied. It is a quite distinct quantity with no counterpart either in pre-Marxian classical theory nor in subsequent neoclassical developments.

Because it is a universal parameter of the economy it can be used to calculate C', as follows: £C' is known and so now is n'. Hence

\[ C' = \frac{\text{£C'}}{n} \]
We have hence re-established the inductive conditions necessary to repeat the calculation, namely $C^+$ is known. We can therefore repeat the iteration to establish $X^{++}$, hence $n^{++}$, hence $C^{++}$ and so on as long as there is data.

It is instructive and illustrative to enquire on which parameters of the data this depends, because it helps to establish the status of the MELT. Observe that at the end of each period a definite portion of output £X re-enters production in the next period. We have designated its monetary magnitude (ascertainable from the data) by £$C^+$. Since not all of the use-values represented in X re-enter production we can be certain that in general

\[ £C^+ < £X \]

Let us use the letter $c$, appropriately time-subscripted, for the ratio of £$C^+$ to £X, that is, for the proportion of output which re-enters production in the next period:

\[ c^+ = \frac{£C^+}{£X} \]

the proportion of the previous period's outputs which re-enter production in the next period as constant capital, and note that

\[ c < 1 \]

Now, since

\[ £C^+ = n^+ \times C^+ \]

and

\[ £X^+ = n^+ \times X^+ \]

it follows that $c$ is also equal to the ratio between the labour value $C$ and the labour value $X$

\[ c = \frac{C}{X} \]

It is thus a dimensionless measure of the share of the value total output from one period which re-enters production in the next period. Moreover, it turns out that on this definition, the MELT whilst an important and useful parameter, is not required for the calculation of labour values. What is required, is knowledge of the ratio of £C to £X, and knowledge of the initial value $C$ at time $t=0$.

It is therefore I think false to argue, as some have, that the temporal definition of value depends on the temporal definition of the MELT. The reverse is the case. This can be seen as follows: we can re-express the iterative formula for $X^+$ as follows:

\[ X^+ = C + L \]

\[ X^{++} = (C^+) + L^+ \]

Re-expressing $C^+$ using the calculation then yields

\[ X^{++} = (C^+) + L^+ \]
\[ = c^+£X + L^+ \]
\[ = c^+(C + L) + L^+ \]

so that we have now determined the value output of the second period solely with respect to the parameter $c$, measurable from money data. In general, reverting to (t) notation:

\[ X(T) = C.\{c(1).c(2).c(3)...c(T)\} \]
\[ + L.\{c(1).c(2).c(3)...c(T)\} \]
\[ + L(1)\{c(2).c(3).c(4)...c(T)\} \]
\[ + L(2)\{c(3).c(4).c(5)...c(T)\} \]
\[ + L(T) \]

That is, successive layers of dead labour incorporated in the product from previous time periods are steadily reduced by repeated multiplication by the coefficient $c$, being the proportion of the value of the product of each period which re-enters the next period of production as constant capital.
On the propagation of errors

This leads to the next question: what if there is an error in our initial estimate of C? Let us suppose that the initial estimate of C is in error by a factor u, so that instead of the true estimate C we employ a false estimate

\[ C^* = (1 + u)C \]

so that u, which may of course be positive or negative, represents the proportionate error in our estimate of C. It is known that c(t) is always less than 1. Hence over the given period it must have an upper bound also less than 1, which we will designate d.

Now consider the relation between the true measure of X(T) and the estimate X(T)*. In the expression above, all the quantities c(t) are known precisely. The only difference between X(T) and X(T)* is thus the first term. Hence the proportionate error in X(T)*, defined by

\[ u(T)^* = \frac{X^*(T) - X(T)}{X(T)} \]

is given by

\[ C \cdot \left( c(1) \cdot c(2) \cdot c(3) \cdots \cdot c(T) \right) / X(T) \]

Because all c(t) are bounded above by d, this is strictly less than

\[ u(T)^* = \frac{C \cdot d^T}{X(T)} \]

But X(T) is always at least equal to L(t). Hence the proportionate error in our estimate decays exponentially, as the proportion of the value of the product that is represented by the past dead labour of time t=0, likewise decays as fresh layers of living labour are added to the social product. Moreover empirically this is a robust estimate since c is in fact equal more or less to 1-(the share of consumption in output) which is around 70% for the UK economy. Within 10 time periods any error would thus decay to around 3% of its initial magnitude.

Differences with other conceptions

The most important feature of the MELT as thus proposed is that it permits a clear distinction between changes in price levels brought about by rising or falling productivity and changes brought about by a nominal change in the unit of account. This is best illustrated by contrasting this measure with neoclassical price indices, which attempt to operationalise, albeit in a confused and contradictory manner, the concept of subjective value or value in use.

With a constant MELT, if the elements of \( X^* \) are produced with less labour than the elements of C due to rising productivity, the price level in the neoclassical sense will be recorded as having fallen. This is because their labour content is smaller or, to put it the other way around, the same amount of labour produces more of them. If, therefore, the same amount of labour is represented in the same amount of money, then this same amount of money will purchase more use-value. A price-index will show that the purchasing power of money has increased.

Thus the MELT is not the same as the neoclassical price level and our proposal cannot be identified with price indexation in the neoclassical sense. The MELT is to my thinking a true measure of the extent of nominal change in the price level, and the neoclassical measure is a false one, because it confuses changes in productivity with changes in the unit of account.

The immediate conclusion is that neoclassical price indexes understate inflation because they mistake improvements in productivity for a fall in the price level.

It is therefore, I think, quite dangerous to assume in the discussion of Marx's value theory that economics already possesses a valid concept of price level, through the prism of which Marx's value categories are to be studied. On the contrary, I view Marx's value categories as the principal instrument for the critique of the neoclassical categories, and the above argument suggests this is an extremely practical matter which will yield very different empirical results from all neoclassical results and therefore lead to very different conclusions.

Second, it is extremely instructive to compare the MELT thus calculated with the measure of value generally attributed to Marx, namely the ratio between the use-value of a definite magnitude of the money commodity and the labour required to produce it.
If a particular commodity such as gold is employed as unit of account, then the relation between the price level measured in gold will depend on the relative rate of growth of productivity in the gold industry and in the rest of the economy. With a uniform rate of productivity growth, or indeed provided merely that productivity in the gold industry keeps pace with the average rise in productivity elsewhere, the change in the gold price level will be precisely equal to the change in the MELT. On average over a medium timescale this seems quite likely, since a large part of gold extraction costs consist of dead labour and dead labour enters production at the average for society as a whole.

In my view this is the real underlying rationale behind Marx’s references to gold as the monetary measure of value. Thus it turns out to be far less inconsistent than proposed, to interpret Marx’s monetary theory as not primarily a metallic money theory and not dependent on the assumption of a metallic or otherwise intrinsically valuable money commodity. In my view what Marx wished to do, quite properly, is distinguish changes in price that result from changes in the productivity of labour, from changes in price resulting from a nominal rise in all prices, that is a general fall in the purchasing power of the money commodity.

Third, we should compare this measure of the MELT with the measure first proposed by Duncan under the name of the value of money, which I think of as expressing a very similar idea to the inverse of the MELT. Under what circumstances would these two measures differ and what would this difference represent?

If n remains constant then the New Interpretation measure (of the MELT) will remain constant and they will be identical. To show this consider the quantity d, the inverse of the value of money as defined by Duncan. This is given by the money value added divided by L:

$$d = \frac{\text{\£X} - \text{\£C}}{\text{L}}$$

or, which is the same thing,

$$d = \frac{\text{\£X} - \text{£C}}{(\text{X} - \text{C})}$$

The familiar MVA/L, money-value-added over new living labour.

If n remains constant (n=n') then in the expression for d' we can write

$${\£X}' = n{X}'$$

$${\£C} = n\text{C}$$

and it quickly follows that $${d}' = n$$ and is hence constant.

If on the other hand d is constant, the two measures may or may not be equal depending on past history. Consider for example the case where £C=10 = constant, L=10 = constant, d=£2/hr=constant. In this case

$${\£X}' = \text{\£C} + d\text{L} = 10 + 2.10 = 30;$$

$${X}' = \text{\£C} + \text{L} = 10 + 10 = 20;$$

and n = 3/2.

However, it seems to me that the problem with d is that in a regime of falling prices it makes it appear that value has been created out of nothing, or to put it another way, value-creation is not proportional to labour-time. Since it has been argued that d expresses the idea that only living labour can create value, I think this needs to be studied carefully. An illustration follows.

Suppose first:

$$\text{\£C}=10; \text{ L} = 10; \text{\£X}' = \text{\£20}; \text{ C} = 10$$

that is, the money value added to the product is £10 for the application of 10 hours. In this case the n and the d measure agree. The fact that C=10 is not needed for the calculation of d; it is simply (£20-£10)/10 = £1/hr. For the calculation of n we observe that X = 10 + 10 = 20 so n = £20/20 = £1/hr likewise.

Suppose at the beginning of the next period the data shows £C' = £10 again. Both measures would, I think, then agree that C' = 10 hours, being the money value of C' times the MELT. Suppose that L' =10 also in this period and that, however, due to a fall in the price level, £X'' = £15, that is, the output is sold for £5 less than before. I would argue as follows: the value of the output is given by the value of constant capital plus the value added by living labour, namely

$$X'' = 10 + 10 = 20$$
Thus, very straightforwardly, the value of the product has been augmented, due to the application of living labour, by the time worked. I would have thought that the use of $d$ leads to the following conclusion:

**Step 1:** 10 hours of living labour are expressed in £5 money-value-added to the product. Therefore the value of money is $\frac{1}{2}$ hr/£ and the MELT is £2/hr.

**Step 2:** what is the value of the product? Since it cost £15 and each of the £15 represents 2 hours, it seems to me that the value of the product is 30, that is

$$X'' = 30$$

But in this case, 10 hours of living labour have added 20 hours of value. It is for this among other reasons that I am more inclined to the idea that $n$ better represents the monetary expression of labour.

**Stocks**

Considering all such commodities, they have a total value, inherited from history. This value, a definite magnitude in hours, is expressed in another definite magnitude in money, which we can observe, namely the total money price of all these commodities (Marx's 'total price'). From this we can calculate the new monetary expression of labour-time (inverse of the 'value of money' in the above sense) which in general changes from one period to the next. It is simply the ratio of one measure to the other. It represents the purchasing power of money over the accumulated dead labour of society or, as Marx puts it, the monetary expression of this labour.

With this calculated, we may determine values in the next period in a modified manner. Given the initial condition this calculation is determinate. Suppose for example that the commodities in society contain initially 10,000 hours of labour which is expressed in (priced at) £20,000 and suppose the capitalists then consume goods worth £10,000 in constant capital.

Then we know that this constant capital represents one-half of the aggregate value in circulation, and hence its value is 5,000 hours. If we know how many hours the workers work - let us say 5000 - then we know that in the next period the gross value output will be

$$5,000 + 5,000 = 10,000$$

Now suppose the capitalists spend £5,000 on wages and £2000 in frivolities. We know how many hours society consumed in this period: it is the sum of the C, the V and frivolities:

$$5,000 + 2,500 + 1,000 = 8,500$$

The new total value of commodities in society is then the old 10,000, less the 8,500 consumed, plus the 10,000 newly-created goods, that is 11,500 hours:

$$10,000 - 8,500 + 10,000 = 11,500$$

Note that the increase in value of the commodities in society is exactly equal to the surplus value of 2,500, less the frivolities. This represents accumulation. The calculation can now be repeated for the next period; once we observe the new prices we can calculate a new monetary expression of labour-time.

At this point since in circulation new relative prices are determined, some will lose value while others gain. However this is a pure redistribution. Since circulation may neither destroy nor create value, which (I agree) arises entirely in production, it follows that in fixing relative prices, the market only redistributes value which already exists as a result of the prior phase of production.
Accounting for the difference

The main difference which is evident from the results, is that the results we obtain for ‘output’ (gross value output) is simply different from the neoclassical result. Without entering into the reasons this already establishes the principal point to which we want to draw attention, namely that it is not sufficient, in attempting to understand such concepts in Marx as the organic composition of capital (which for our estimates rises for an entire historical period, the very period during which the possibility of a rising organic composition as the basis for the crisis of UK profitability was simply ruled out by the ‘intelligent’ literature in favour of a profit-squeeze analysis) or the price level, simply to replace Marx’s concepts by the ‘equivalent’ neoclassical concept. It must least be considered – and we believe there are good grounds for asserting so – that Marx had in mind something very different, which permits us to explain differences that the conventional measures cannot express. Since this is dealt with at length in another paper to this conference, I shall leave it at there. In conclusion however I do want to deal briefly with what is probably the single most important difference between this work and the alternative proposal for measuring labour values advanced by ST, namely the idea of using vertically-integrated labour values. The general basis for our critique of this has been advanced elsewhere. Here I want to deal with an obvious and important contradiction between our results for sectoral values, and the findings of many researchers that values and prices are highly correlated.

The problem can be seen with reference to figures 8 and 9. As figure 8 shows, there is a clear difference between the value output of each sector and the price output of each sector. Each hour of value in the output of the mining and quarrying sector, for example, sells for twice as much as an hour of the output of the manufacturing sector, almost certainly reflecting the high rents obtained in the oil sector. Yet according to studies using the vertically-integrated method, price and value correlate almost perfectly. How is this extremely significant difference to be accounted for?

In my view this difference, unlike the proceeding differences, does not arise from a difference in the measurement technique and indeed, when I calculate using vertically-integrated labour values on the full 132-sector I/O tables, I obtain the same contradiction. That is, if using vertically-integrated values, one calculates the price/value ratio in output, one finds that the data is just as widely distributed as the data given above.

The difference does not arise, therefore, from the method of calculating labour-values in the main. Rather, it arises from the conception of the term ‘correlation’ that is applied. As figure 9 shows, a very significant correlation is observed between the aggregate value output of the various sectors of the economy and their aggregate price output. It is this correlation between aggregate value and aggregate price output that is taken to prove a high level of correlation between price and values. This is explicit. Ochoa writes the following:

> In connection with cross-sectional series, the error of ‘spurious correlation’ is known to be a problem. Clearly, if we are trying to establish a relationship between a and b, and if we define \( x = az \) and \( y = bz \), the correlation coefficient between \( x \) and \( y \) will overestimate the correlation between \( a \) and \( b \)… I am clear that we can increase the extent of common variation of \( P \) and \( M \) by judicious manipulation of the physical units [Ochoa 1985: 129]

The problem that then presents itself is as follows: actually, input-output matrices contain no physical units. As Leontief himself noted, the unit of measurement in input-output data is not tons, numbers of machines or bushels but dollars. What the I/O tables measure are commodity flows that are already in money. They cannot be identified with physical matrices, even supposing a physical matrix could be constructed, much less use-values. There is actually no means, given the I/O data, of constructing any equivalent of the normal concept of ‘price’ or of ‘value’, that is, the money price paid for a unit of something or the number of hours in a unit of something. It simply does not contain this information. Therefore, correlation studies based on I/O data do not actually compare ‘price’ with ‘value’ in the normal accepted sense of this word. Some stand-in, some proxy for this must be adopted. The proxy adopted by all workers in the field is the aggregate output of a sector, where ‘sector’ is defined by the standard industrial classification. Ochoa explains this as follows:

> The question reduces itself to which is the appropriate population: unit prices or sectoral outputs. The only unambiguously defined elements with common characteristics are sectoral outputs, so their two properties (market price and computed price) can legitimately be compared [Ochoa 1985:130]

We would not seek here to quarrel with the reasons given but we would argue that it does place important limitations on the conclusions that can justifiably be drawn. What is it that is being compared?
The problem is that the sectors being compared are all of different sizes, and moreover this variation in size arises from nothing intrinsic to the data but from the choices made by the accountants when they define which business is to be in which sector. Therefore there are not one, but two sources of variation in the magnitude of the aggregate output, namely the price-value deviations on the one hand, and the variations in the size of the sector on the other, this latter obviously affecting both aggregate price and aggregate value equally.

In honour to a tradition first established by Anwar himself, I thought the best way to test the hypothesis of a correlation between price and value was to apply it to randomly-generated data. I therefore constructed a 66-sector economy by assuming a random distribution of price-value ratios within a range of \( \pm 40\% \), a random distribution of sector sizes, and a random distribution of organic compositions also in the range \( \pm 40\% \). As can be seen from the last two figures, it generates a very good correlation.

If, of course, all sectors were the same size, then the comparison would correlate the output in price terms of each sector with a value that did not change. Since, what meaning can be attached to the notion that all sectors are the same size? We do not know their physical size independent of the money units used to measure them. ‘Same size’ could only mean that either the output of every sector sold for the same aggregate price or that every sector, using the calculated values, had the same value output. But in this case there would be no correlation in the normal sense, since there would be no variation in one of the variables that entered into the correlation. A true test of price-value identity in this case would be that both variables showed no variation. Another way of thinking about this is to enquire into the price-value ratio. What would happen if goods actually sold at their values? In that case, goods whose value was 10 hours would always sell for the same amount of money, no matter which sector they came from. To put it another way, goods in which one hour was embodied would always sell for the same uniform price. The best test of price-value equality, no matter what system of measurement is used, is therefore in my opinion to compare price-value ratios by sector. In the even of complete equality of prices and values, these would be the same.

It is for those who apply this methodology to test it by this criterion. We do not propose to do this for them. We would observe, however, that it is a universal criterion for price-value equality that is independent of the method of measurement. If unit price really correlate with unit values then all price-value ratios should be the same.

It is my view that the observed correlations are very significant and do tell us something about the economy. I do not seek to dismiss the important body of work in this field. Cockshott and Cottrell have also introduced an important new dimension in this work by comparing the correlations that arise from vertically-integrated labour values with similar correlations from other vertical integrations, and show that the correlations that result from labour integration are more significant. I have no doubt that this proves that important structural information about an economy is proved by these techniques.

However, what I dispute is that it proves no error will result, if the prices of outputs are taken as directly representative of values.

In a nutshell, the practical difference between us is this: the proposal that arises from the use of vertically-integrated labour values is to use the prices of outputs as directly representatives of value magnitudes. This, I would dispute. On the contrary, the dispersion of output prices and the consequent dispersion of profit rates in the economy is precisely the motor force that drives the movement of capital from one sector to another, which provides the mechanism of interaction between supply and demand and constitutes the actual law of motion of capital. On the contrary, by recognising that the prices of inputs, when adjusted by the method that I propose for a rise in nominal prices (a rise in the MELT) actually represent the value transferred to the product into which these inputs enter, we are able to distinguish precisely those variations in price that arise from the conditions of production (variations in the value of outputs as the sum of the price of inputs and direct labour), and those that arise from circulation (the subsequent sale of these values for different money sums per unit of value).

It it is a very peculiar outcome of a doctrine which places such emphasis on the direct embodiment of labour at the point of production, and turns its back on any recognition of the intermediation of circulation at this point, turns out when it comes to the sale of the outputs to make no practical distinction at all between value and price, and thereby ends up in practice doing what it so energetically campaigns against, that is, directly taking money as the representative of value.
## Randomly-generated value-price deviations

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### Figure 10

Value price correlations for randomly-generated data

### Figure 11

References (incomplete, see main text)