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Vertical Integration and Patent Licensing in Upstream and Downstream Markets

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Abstract

The present paper studies and compares different vertical integration structures on consumers and total surplus with licensing by mean of a fixed fee in two successive homogeneous-good Cournot duopolies where one of the firms in each market has a different cost-reducing innovation. The key difference between the present model and models in the existing literature is that here we suppose the existence of two different patents in upstream and downstream markets. In each market we find two firms: the patent holding firm and a non innovative firm. In upstream market, the innovative firm owns an innovation allowing to reduce the input marginal production cost. In downstream market the innovative firm owns an innovation allowing to reduce marginal cost of transforming the input into output. We discuss different structures of vertical integration and we show that consumer surplus and total surplus are depending of cost-reducing innovation in upstream and downstream markets and the structure of vertical integration.

Key words: Cournot successive markets, Fee licensing, Vertical integration, process innovation

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Introduction

According to OCDE report¹, determining whether a merger will be likely to promote or prevent innovation requires a complex, case-specific inquiry. A merger could lead to efficiencies in research and development, yet reduced rivalry and greater market power could slow the post-merger rate of technological change. Once an innovation is patented, the patent holding firm has three different licensing regimes: fixed fee which is not depending of the product quantity, a per-unit royalty and an auction. In order to maximize its profit, an innovative firm in successive markets can not only license its patent but also try to integrate vertically. We can find some examples in the economy like in the air transport industry where we find travel agencies in the downstream market and firms responsible of catering and maintenance in the aircrafts. In petroleum industry we find oil stations in downstream markets and exploring oil firms in the upstream market. Kamien and Tauman (1984,1986), Katz and Shapiro (1986), Kamien and al (1992) and Wang (1998) compared different patent licensing regimes and found many different results depending on the position of the innovative firm : outsider or insider firm. Wang (1998) finds that royalty licensing can be superior to fixed-fee licensing for the patent-holding firm when the cost-reducing innovation is non-drastic. Wang (2002) studied a differentiated Cournot model and finds that fixed fee licensing contract is better for consumer surplus then a royalty. Vishwasrao (2006) assembled a data set of all foreign technology licensing agreements entered into by manufacturing firms in placecountry-regionIndia between 1989 and 1993 and found that licensing contracts are more likely to use royalties when sales are relatively high, while increased volatility of sales and greater profitability favor fixed fee contracts. Sen (2002) considered a Cournot oligopoly with at least three firms, where one of the firms has a cost-reducing innovation. He proposed a general version of royalty contract, and show that this contract enables the innovator firm to earn the monopoly profit with the reduced cost. Sen (2004) finds that for an outsider innovator in a Cournot oligopoly, royalty licensing could be superior to both fixed fee and auction when the number of licenses can take only integer values. Caballero, Moner and Sempere (2002) analyzed a multi-stage non-cooperative game involving an outside patent-holder, who seeks to licence a process innovation, and two price-setting firms located on a circumference. They compared three licensing policies: an auction, a fixed fee and a per unit output royalty. They find, contrary to standard results, that royalties yield higher payoffs to the patent-holder than do an auction policy or a fixed fee policy regardless of the size of the innovation. Sandonis and Fauli (2002) characterized situations where licensing a cost reducing innovation to a rival firm using two-part tariff contracts (a fixed fee plus a linear per unit of output royalty) reduces social welfare. Triest and Vis (2007) consider the relevant cash

¹ OCDE (2007) Competition, Patents and Innovation

flows that result from owning the patent and show that valuation of patents on production process improvements cannot be done without good knowledge of technology, markets and competitors. Encaoua and Lefouili (2006) investigated the choice of an intellectual protection regime for a process innovation. They found that choosing between patent and trade secrecy depends on patent strength, cost of imitating a patented innovation relative to the cost of imitating a secret innovation, and the innovation size. Brocas (2003) analysed a model in which two upstream innovators are investing to improve process innovations used by two downstream producers. She found that when technologies are not costlessly substitutable, the prices of licenses rise with the size of the switching costs. Buehler and Schmutzler (2007) examined the interplay of endogenous vertical integration and cost-reducing downstream investment in successive oligopoly. They found that vertical integration increases own investment and decreases competitor investment (intimidation effect). Matsushima (2004) investigated the location strategies of upstream and downstream firms using a Hotelling-type product differentiation model (linear city model). He showed that more inefficient transport technologies of upstream firms may enhance welfare and showed that vertical mergers occur if the transport costs of upstream firms are large enough. Mukherjee and Zanchettin (2007) studied vertical integration and product innovation as interdependent strategic choices of vertically related firms. They found that, although product differentiation allows to soften product market competition and to avoid market foreclosure, the downstream market may prefer less product differentiation to deter vertical integration. Mukherjee (2003) examined the incentive for licensing in the downstream industry when the firms in the upstream industry have market power. He shows that licensing in the downstream industry is profitable if and only if licensing increases competition in the upstream industry. Lemarie (2005) analysed a situation where a patent holder is considered as an upstream firm who can license its innovation to some downstream companies that compete on a final market with differentiated products. He shows that a license based on a royalty works better with vertical integration, and that consequently, the patent holder have some interest to vertically integrate if it enables him to apply a royalty based license. He finds that the effect of vertical integration on the social surplus can be either positive or negative. Sandonis and Fauli (2005) presented a model in which an independent research laboratory owns a patented process innovation ready to be used by an industry that produces differentiated goods. They show first, that the vertical merger is profitable only in the case of small innovations, whereas a merger increases welfare only for significant innovations; second, all profitable vertical mergers reduce welfare. In this paper we analyse, in successive markets, situations of vertical integration and process innovation in a model where we find licensing contracts in both upstream and downstream markets. The key difference between the present model and models in the existing literature is that here we suppose the existence of two different patents in upstream and downstream markets and each patent holding firm can license its patent only in the market

where it's acting. In each market we find two firms: the patent holding firm and a non innovative firm. In the upstream market, the innovative firm owns an innovation allowing to reduce the input marginal production cost. In the downstream market the innovative firm owns an innovation allowing to reduce marginal cost of transforming the input into output. We suppose that patent holding firms use only fixed fees licensing contracts. We study different forms of vertical integration and we show that consumer surplus and total surplus are depending of cost-reducing innovation in upstream and downstream markets.

We will analyse the case of non integration with no licensing contracts in upstream and downstream markets, and then the cases of only one licensing contract in the downstream or the upstream markets and last the case where the two innovative firms license their two patents. Then we will analyse three cases of vertical integration: vertical integration between only innovative firms in upstream and downstream markets, vertical integration between only non innovative firms in upstream and downstream markets and vertical integration between both innovative firms and non innovative firms. In which case, we will compare consumer surplus and total surplus. We find that when there is no vertical integration, consumer surplus and total surplus are better when non innovative firms in upstream and downstream markets benefit from licensing contracts. When patent holding firms integrate vertically and the non innovative firms integrate vertically too, consumer surplus is better when the two innovations are licensed only when innovations are non drastic. If innovations are drastic, consumer surplus is better when there are no licensing contracts in upstream and downstream markets. When only patent holding firms are vertically integrated, the two other non innovative firms are none integrated, the licensing of the innovations in downstream and upstream markets is better for consumer surplus only when innovations are drastic. When patent holding firms are not integrated and only non innovative firms are vertically integrated, total surplus is better only when the two innovations are licensed independently of their magnitudes and the consumer surplus is better only when the two innovations are non drastic or intermediate

The paper is organised as follow. The model and game stages are presented in section 2. We consider the case where firms in each successive market do not vertically integrate in section 3. Then we discuss three structures of vertical integration in section 4.

1 Model

We consider two downstream firms v_1 and v_2 in a Cournot duopoly producing homogeneous outputs q_1 and q_2 and two upstream firms m_1 and m_2 producing

homogeneous inputs s_1 and s_2 . The (inverse) market demand function is given by $p = a - Q$, where p denotes price and Q represents industry output. With the old technology, both downstream firms produce at constant unit production cost c . The cost-reducing innovation by firm v_1 creates a new technology that lowers its unit cost by the amount of ε ($0 < \varepsilon < c$). With the old technology, both upstream firms produce at constant unit production cost d . The cost-reducing innovation by firm m_1 creates a new technology that lowers its unit cost by the amount of γ ($0 < \gamma < d$).

We suppose:

- Upstream firms sell inputs s_1 and s_2 to downstream firms at the price w
- Each downstream firm needs one unit of input to produce exactly one unit of output
- Input producing cost d is less than its selling price w in the intermediate market ($d < w$) and the price of the input and its transforming cost into an output is less than the selling price of the output in the final market ($w + c < a$). We suppose consequently that $c + d < a$
- Entry of new firms in the market is not profitable

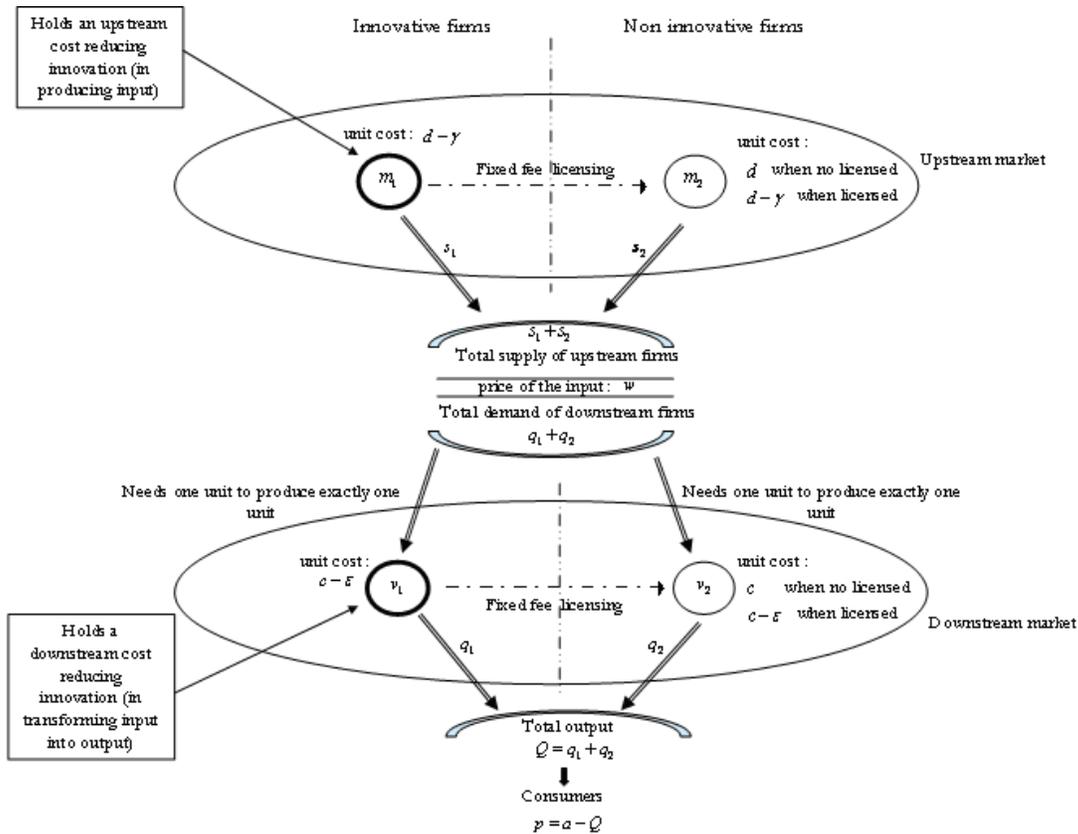


Figure 0 : Model

A licensing game consists of three stages. In the first stage, each patent-holding firm acts as a Stackelberg leader in setting a fixed licensing fee equal to the difference in profits of each non innovative firm without and with the use of the new technology. In the second stage, each non innovative firm (the would-be licensee) acts as a Stackelberg follower in deciding whether to accept the offer from each patent holder (in the same market). In the third stage, the two upstream firms engage in a noncooperative competition in quantities of input and in the fourth and last stage, the two downstream firms engage in a noncooperative competition in quantities of output.

2 Licensing contracts in downstream and upstream markets without vertical integration

We start our analysis by considering the downstream Cournot duopoly where patent holding firm v_1 has an unit output cost production of $c - \varepsilon$, the non innovative firm v_2 has an unit output cost production of c when firm v_2 do not benefit from the innovation of firm v_1 and has an unit output cost production of $c - \varepsilon$ when it benefits from a downstream license. In the upstream Cournot duopoly, patent holding firm m_1 has an unit input cost production of $d - \gamma$, the non innovative firm m_2 has an unit input cost production of d when firm m_2 do not benefit from the innovation of firm m_1 and has an unit output cost production of $d - \gamma$ when it benefits from an upstream license.

Comparing the consumer surplus and the total surplus for the four possible cases of licensing and no licensing in upstream and downstream markets, We find²:

	No upstream licensing
No downstream licensing	$CS_{PL_PL_m1_v1_m2_v2} = -\frac{1}{72}(-28 + 2\varepsilon + \gamma)(8 + 2\varepsilon + \gamma)$
downstream licensing	$CS_{PL_F_m1_v1_m2_v2} = -\frac{1}{18}(-14 + \varepsilon)(4 + \varepsilon)$

	upstream licensing
No downstream licensing	$CS_{F_PL_m1_v1_m2_v2} = -\frac{2}{81}(-19 + \varepsilon + 2\gamma)(8 + \varepsilon + 2\gamma)$
downstream licensing	$CS_{F_F_m1_v1_m2_v2} = -\frac{4}{81}(-19 + 2\varepsilon + 2\gamma)(4 + \varepsilon + \gamma)$

² $a = 6, b = 1, c = 1, d = 1, 0 < \varepsilon < 1, 0 < \gamma < 1$

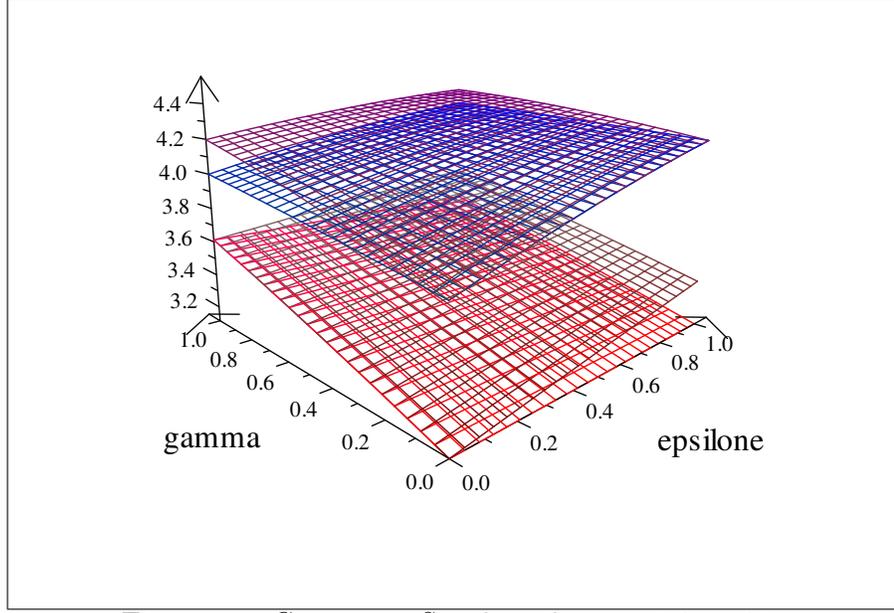


Figure 1: Consumer Surplus when no integration

Result 1

Consumer surplus is better when there's no vertical integration and when the two innovations are licensed in the upstream and downstream markets. The bad situation for consumers is when patent holding firms benefits alone from their innovations.

	No upstream licensing
No downstream licensing	$TS_{PL_PL_m1_v1_m2_v2} = \frac{20}{3} + \frac{7}{3}\varepsilon + \frac{7}{6}\gamma + \frac{1}{6}\varepsilon^2 + \frac{1}{6}\varepsilon\gamma + \frac{7}{24}\gamma^2$
downstream licensing	$TS_{PL_F_m1_v1_m2_v2} = \frac{20}{3} + \frac{7}{3}\varepsilon + \frac{8}{9}\gamma + \frac{1}{6}\varepsilon^2 + \frac{2}{9}\varepsilon\gamma + \frac{4}{9}\gamma^2$

	upstream licensing
No downstream licensing	$TS_{F_PL_m1_v1_m2_v2} = \frac{208}{27} + \frac{34}{27}\varepsilon + \frac{68}{27}\gamma + \frac{29}{54}\varepsilon^2 + \frac{4}{27}\varepsilon\gamma + \frac{4}{27}\gamma^2$
downstream licensing	$TS_{F_F_m1_v1_m2_v2} = \frac{208}{27} + \frac{68}{27}\varepsilon + \frac{28}{9}\gamma + \frac{4}{27}\varepsilon^2 + \frac{4}{9}\varepsilon\gamma + \frac{8}{27}\gamma^2$

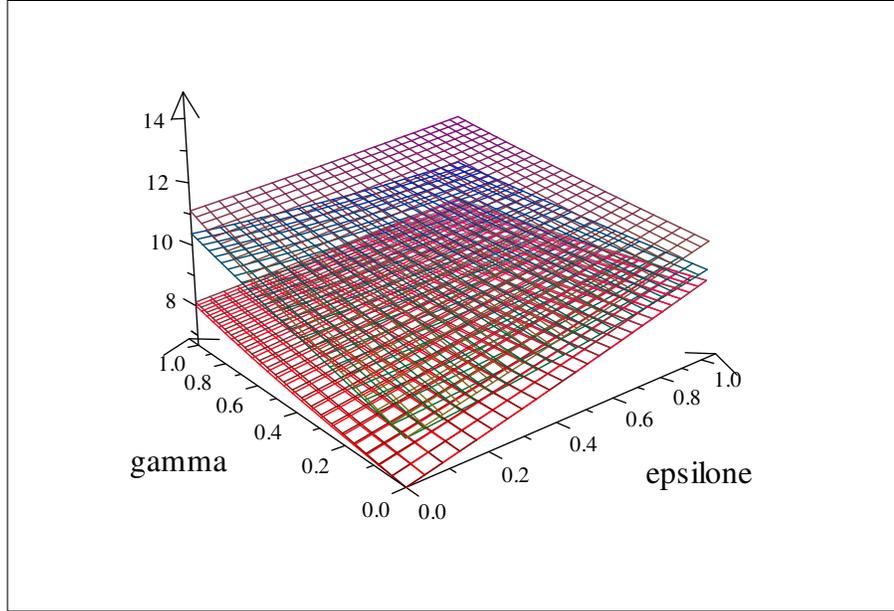


Figure 2: Total Surplus when no integration

Result 2

Total surplus is always better when there is no vertical integration and when both innovations in downstream and upstream markets are licensed.

3 Licensing contracts in downstream and upstream markets with vertical integration

In this section we examine different licensing cases with different structures of vertical integration. We discuss three main structures:

- Vertical integration between innovative firms (m_1, v_1) in one hand and vertical integration between non innovative firms (m_2, v_2) in the other hand
- Vertical integration between only innovative firms (m_1, v_1)
- Vertical integration between only non innovative firms (m_2, v_2)

For each vertical integration structure, we will compare four cases of licensing:

- No license in upstream market and no license in downstream market
- No license in upstream market and license in downstream market
- License in upstream market and no license in downstream market
- License in upstream market and license in downstream market

3.1 Vertical integration between innovative firms and vertical integration between non innovative firms

We find³:

	No upstream licensing
No downstream licensing	$CS_{PL_PL_m_1v_1_m_2v_2} = -\frac{1}{72}(-20 + 2\varepsilon + \gamma)(16 + 2\varepsilon + \gamma)$
downstream licensing	$CS_{PL_F_m_1v_1_m_2v_2} = -\frac{1}{18}(-10 + 2\varepsilon + \gamma)(8 + 2\varepsilon + \gamma)$

	upstream licensing
No downstream licensing	$CS_{F_PL_m_1v_1_m_2v_2} = -\frac{1}{18}(-10 + \varepsilon + 2\gamma)(8 + \varepsilon + 2\gamma)$
downstream licensing	$CS_{F_F_m_1v_1_m_2v_2} = -\frac{2}{9}(-5 + \varepsilon + \gamma)(4 + \varepsilon + \gamma)$

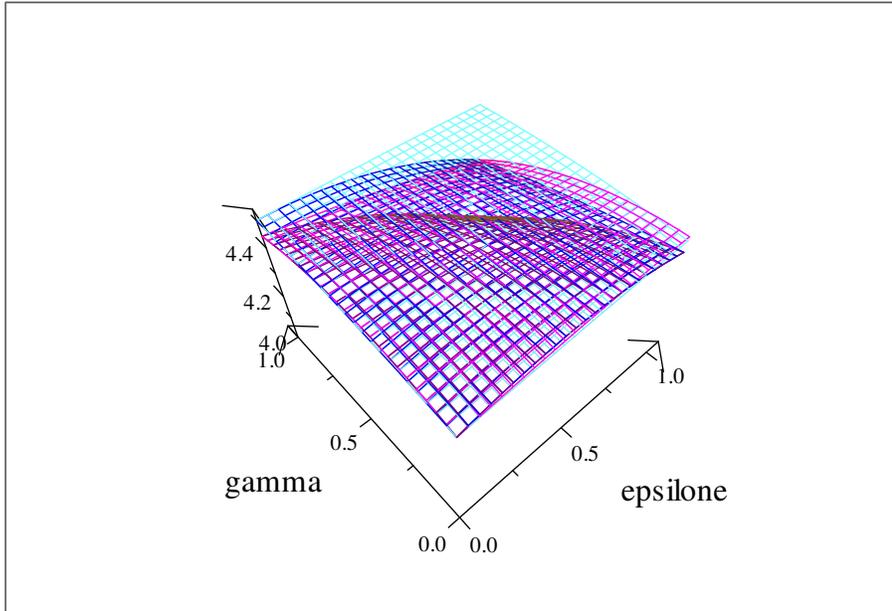


Figure 3: Consumer Surplus for a $m_1v_1 - m_2v_2$ integration

Result 3

When there are vertical integration between innovative firms and vertical integration between non innovative firms, consumer surplus is better if innovations in upstream and downstream markets are not licensed and innovations

³ $a = 6, b = 1, c = 1, d = 1, 0 < \varepsilon < 1, 0 < \gamma < 1$

are drastic. If innovations are non drastic, consumer surplus is better when innovations are licensed. When innovations are intermediate, surplus consumer is better when only one innovation is licensed.

	No upstream licensing
No downstream licensing	$TS_{PL_PL_m_1v_1_m_2v_2} = 8 + \varepsilon + \frac{7}{6}\gamma + \frac{1}{2}\varepsilon^2 + \frac{5}{6}\varepsilon\gamma + \frac{7}{24}\gamma^2$
downstream licensing	$TS_{PL_F_m_1v_1_m_2v_2} = 8 + 2\varepsilon + \gamma + \frac{1}{2}\gamma^2$

	upstream licensing
No downstream licensing	$TS_{F_PL_m_1v_1_m_2v_2} = 8 + \varepsilon + 2\gamma + \frac{1}{2}\varepsilon^2$
downstream licensing	$TS_{F_F_m_1v_1_m_2v_2} = 8 + 2\varepsilon + 2\gamma$

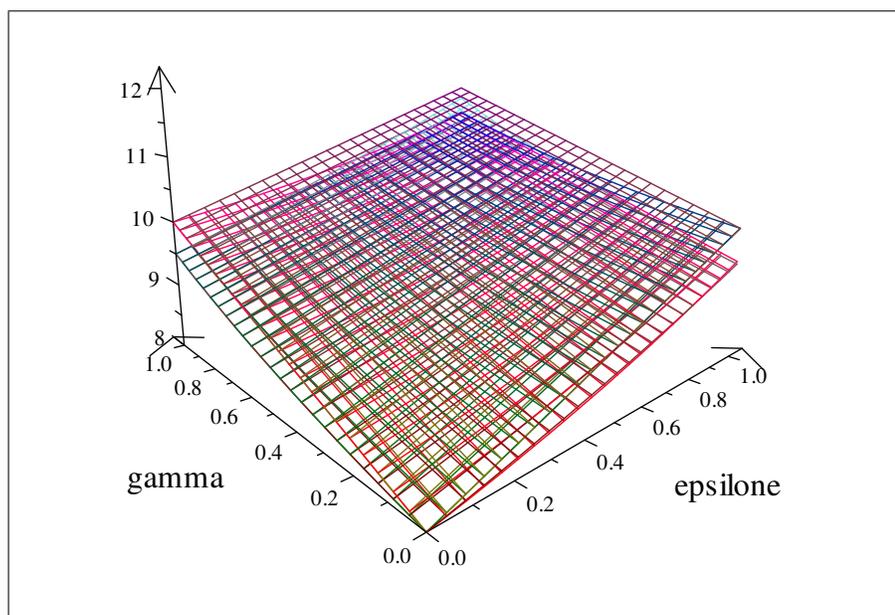


Figure 4: Total Surplus for a $m_1v_1 - m_2v_2$ integration

Result 4

When there are vertical integration between innovative firms and vertical integration between non innovative firms, total surplus is always better when both innovations are licensed

3.2 Vertical integration between only innovative firms

We find⁴:

	No upstream licensing
No downstream licensing	$CS_{PL_PL_m_1v_1_m_2_v_2} = -\frac{1}{18}(-10 + \varepsilon + \gamma)(8 + \varepsilon + \gamma)$
downstream licensing	$CS_{PL_F_m_1v_1_m_2_v_2} = -\frac{1}{288}(-44 + 7\varepsilon + 5\gamma)(28 + 7\varepsilon + 5\gamma)$

	upstream licensing
No downstream licensing	$CS_{F_PL_m_1v_1_m_2_v_2} = -\frac{1}{18}(-10 + \varepsilon + 2\gamma)(8 + \varepsilon + 2\gamma)$
downstream licensing	$CS_{F_F_m_1v_1_m_2_v_2} = -\frac{1}{8}(-8 + \varepsilon + \gamma)(4 + \varepsilon + \gamma)$

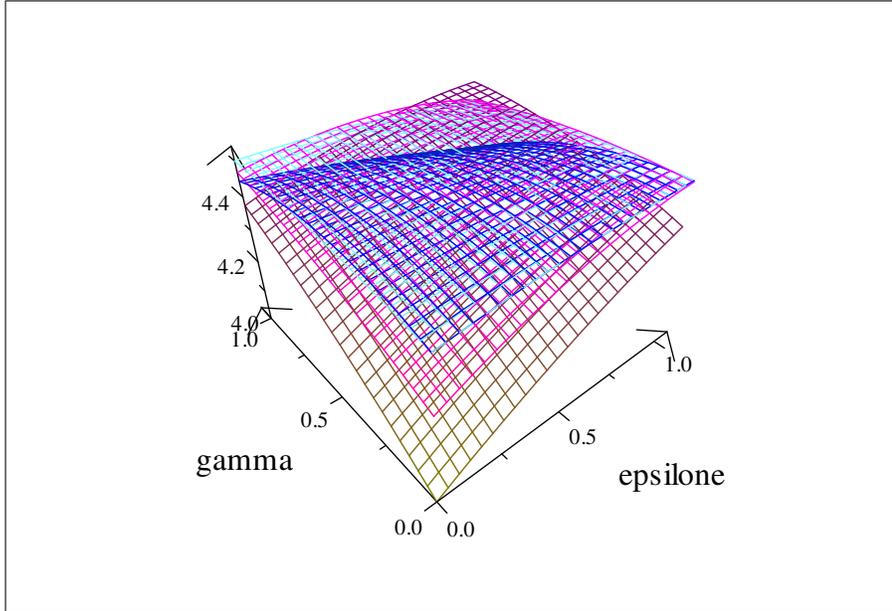


Figure 5: Consumer Surplus for a $m_1v_1 - m_2 - v_2$ integration

Result 5

When innovative firms integrate vertically while non innovative firms are not integrated, consumer surplus is better if both innovations in downstream and upstream markets are licensed and drastic. If innovations are not drastic, consumer surplus is better when only upstream innovation is licensed.

⁴ $a = 6, b = 1, c = 1, d = 1, 0 < \varepsilon < 1, 0 < \gamma < 1$

	No upstream licensing
No downstream licensing	$TS_{PL_PL_m_1v_1_m_2_v_2} = 8 + \frac{4}{3}\varepsilon + \frac{4}{3}\gamma + \frac{5}{12}\varepsilon^2 + \frac{5}{6}\varepsilon\gamma + \frac{5}{12}\gamma^2$
downstream licensing	$TS_{PL_F_m_1v_1_m_2_v_2} = \frac{49}{6} + \frac{7}{3}\varepsilon + \frac{5}{3}\gamma + \frac{7}{96}\varepsilon^2 + \frac{5}{48}\varepsilon\gamma + \frac{31}{96}\gamma^2$

	upstream licensing
No downstream licensing	$TS_{F_PL_m_1v_1_m_2_v_2} = 8 + \frac{4}{3}\varepsilon + 2\gamma + \frac{5}{12}\varepsilon^2 + \frac{1}{12}\varepsilon\gamma$
downstream licensing	$TS_{F_F_m_1v_1_m_2_v_2} = 8 + \frac{5}{2}\varepsilon + \frac{5}{2}\gamma + \frac{1}{8}\varepsilon^2 + \frac{1}{4}\varepsilon\gamma + \frac{1}{8}\gamma^2$

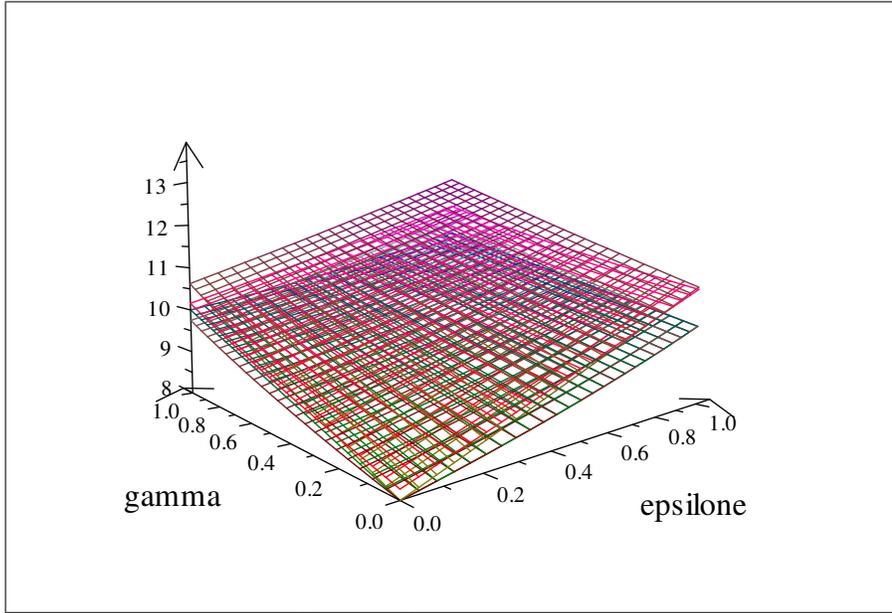


Figure 6: Total Surplus for a $m_1v_1 - m_2 - v_2$ integration

Result 6

When innovative firms integrate vertically while non innovative firms are not integrated, total surplus is better if only one downstream non drastic or intermediate innovation is licensed and if upstream innovation is non drastic. If upstream innovation is intermediate or drastic and downstream innovation is drastic, total surplus is better when both innovations in downstream and upstream are licensed.

3.3 Vertical integration between only non innovative firms

We find⁵:

	No upstream licensing
No downstream licensing	$CS_{PL_PL_m_1_v_1_m_2v_2} = -\frac{1}{72}(-22 + \varepsilon + \gamma)(14 + \varepsilon + \gamma)$
downstream licensing	$CS_{PL_F_m_1_v_1_m_2v_2} = -\frac{1}{288}(-44 + 7\varepsilon + 2\gamma)(28 + 7\varepsilon + 2\gamma)$

	upstream licensing
No downstream licensing	$CS_{F_PL_m_1_v_1_m_2v_2} = -\frac{1}{288}(-44 + 2\varepsilon + 7\gamma)(28 + 2\varepsilon + 7\gamma)$
downstream licensing	$CS_{F_F_m_1_v_1_m_2v_2} = -\frac{1}{288}(-44 + 7\varepsilon + 7\gamma)(4 + \varepsilon + \gamma)$

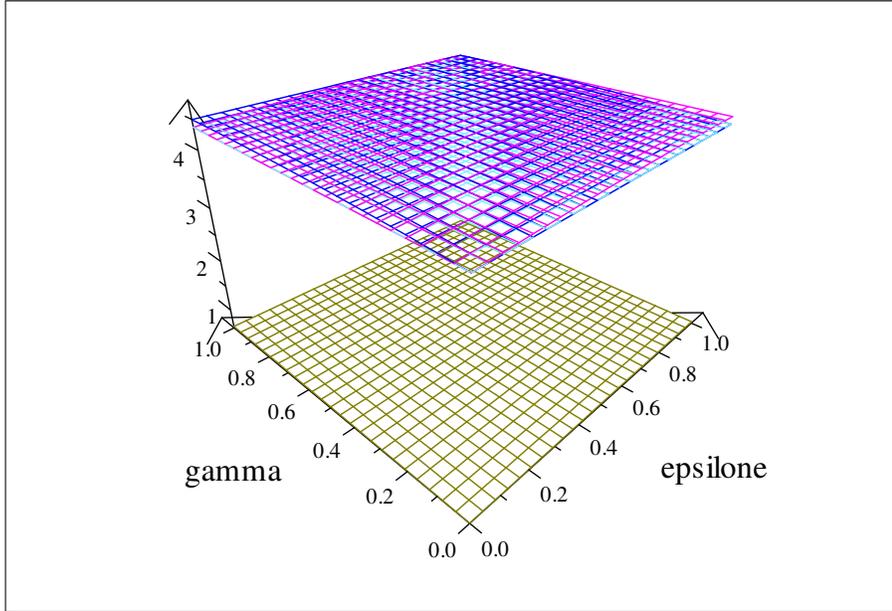


Figure 7: Consumer Surplus for a $m_1 - v_1 - m_2v_2$ integration

Result 7

When non innovative firms are vertically integrated while innovative firms are not integrated, consumer surplus is better if both non drastic innovations are licensed. If only one downstream (upstream) innovation is licensed, consumer

⁵ $a = 6, b = 1, c = 1, d = 1, 0 < \varepsilon < 1, 0 < \gamma < 1$

surplus is better when downstream (upstream) innovation is drastic and upstream (downstream) innovation is intermediate.

	No upstream licensing
No downstream licensing	$TS_{PL_PL_m_1_v_1_m_2v_2} = \frac{49}{6} + \frac{2}{3}\varepsilon + \frac{7}{3}\gamma + \frac{7}{24}\varepsilon^2 + \frac{5}{12}\varepsilon\gamma + \frac{1}{8}\gamma^2$
downstream licensing	$TS_{PL_F_m_1_v_1_m_2v_2} = \frac{49}{6} + \frac{7}{3}\varepsilon + \frac{2}{3}\gamma + \frac{7}{96}\varepsilon^2 + \frac{1}{24}\varepsilon\gamma + \frac{1}{24}\gamma^2$

	upstream licensing
No downstream licensing	$TS_{F_PL_m_1_v_1_m_2v_2} = \frac{49}{6} + \frac{2}{3}\varepsilon + \frac{7}{3}\gamma + \frac{7}{24}\varepsilon^2 + \frac{1}{24}\varepsilon\gamma + \frac{7}{96}\gamma^2$
downstream licensing	$ST_{F_F_m_1_v_1_m_2v_2} = \frac{49}{6} + \frac{7}{3}\varepsilon + \frac{7}{3}\gamma + \frac{7}{96}\varepsilon^2 + \frac{1}{48}\varepsilon\gamma + \frac{7}{96}\gamma^2$

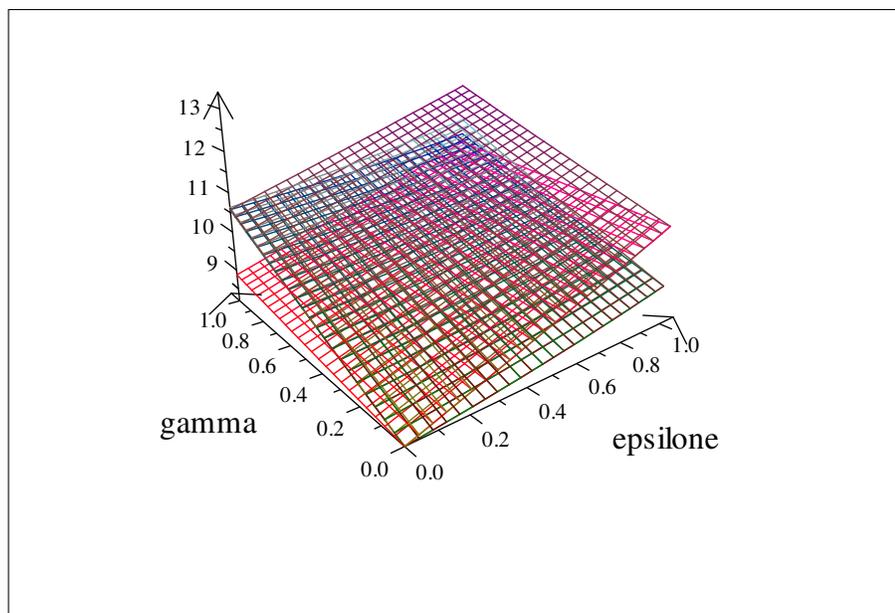


Figure 8: Total Surplus for a $m_1 - v_1 - m_2v_2$ integration

Result 8

When non innovative firms are vertically integrated while innovative firms are not integrated, total surplus is better when both upstream and downstream innovations are licensed.

4 Conclusion

This paper has studied and compared consumer surplus and total surplus in different structures of vertical integration in two successive Cournot duopolies where we find in each duopoly one patent holding firm owning a cost reducing innovation. The innovation of this paper is to treat the case where we find two different innovations in successive markets, as opposed to innovations in only one market in existing literature. We show that the magnitude of innovations in upstream and downstream markets and licensing decision by patent holding firms have direct impact on consumer surplus and total surplus.

Appendix

We have : $c_1 = c - \varepsilon$, $c_2 = c$, $d_1 = d - \gamma$, $d_2 = d$

Profits of downstream firms v_1 and v_2 are :

$$\pi_{v1} = (p - c_1 - w)q_1 \text{ and } \pi_{v2} = (p - c_2 - w)q_2$$

Maximizing downstream profits of respectively firms v_1 and v_2 with respect to q_1 and q_2 we find

$$\begin{cases} \frac{\partial \pi_{v1}}{\partial q_1} = 0 \\ \frac{\partial \pi_{v2}}{\partial q_2} = 0 \end{cases} \text{ and } \begin{cases} \frac{\partial^2 \pi_{v1}}{\partial q_1^2} < 0 \\ \frac{\partial^2 \pi_{v2}}{\partial q_2^2} < 0 \end{cases}$$

$$q_1 = \begin{cases} \frac{a-c+2\varepsilon-w}{3b} & \text{if } \varepsilon < a - c - w \\ \frac{a-c+\varepsilon-w}{2b} & \text{if } \varepsilon \geq a - c - w \end{cases}, q_2 = \begin{cases} \frac{a-c-\varepsilon-w}{3b} & \text{if } \varepsilon < a - c - w \\ 0 & \text{if } \varepsilon \geq a - c - w \end{cases}$$

Total demand of output in downstream market is $z = q_1 + q_2$, trying to write w in function of z , we find:

$$w = \begin{cases} \frac{2(a-c)+\varepsilon-3bz}{2} & \text{if } \varepsilon < bz \\ a - c + \varepsilon - 2bz & \text{if } \varepsilon \geq bz \end{cases}$$

Profits of upstream firms m_1 and m_2 are:

$$\pi_{m_1} = (w - d_1)s_1 \text{ and } \pi_{m_2} = (w - d_2)s_2$$

Maximizing profits of upstream firms m_1 and m_2 with respect to s_1 and s_2 , we find:

$$\text{if } \varepsilon < b(s_1 + s_2)$$

$$s_1 = \begin{cases} \frac{2(a-c)+\varepsilon-2d+4\gamma}{9b} & \text{if } \gamma < \frac{2(a-c)+\varepsilon-2d}{2} \\ \frac{2(a-c)+\varepsilon-2d+2\gamma}{6b} & \text{if } \gamma \geq \frac{2(a-c)+\varepsilon-2d}{2} \end{cases} \text{ and } s_2 = \begin{cases} \frac{2(a-c)+\varepsilon-2d-2\gamma}{9b} & \text{if } \gamma < \frac{2(a-c)+\varepsilon-2d}{2} \\ 0 & \text{if } \gamma \geq \frac{2(a-c)+\varepsilon-2d}{2} \end{cases}$$

$$\text{if } \varepsilon \geq b(s_1 + s_2)$$

$$s_1 = \begin{cases} \frac{a-c+\varepsilon-d+2\gamma}{6b} & \text{if } \gamma < a - c + \varepsilon - d \\ \frac{a-c+\varepsilon-d+\gamma}{4b} & \text{if } \gamma \geq a - c + \varepsilon - d \end{cases} \text{ and } s_2 = \begin{cases} \frac{a-c+\varepsilon-d-\gamma}{6b} & \text{if } \gamma < a - c + \varepsilon - d \\ 0 & \text{if } \gamma \geq a - c + \varepsilon - d \end{cases}$$

Writing the equality between total offer of input by upstream firms and total demand of input by downstream firms $s_1 + s_2 = q_1 + q_2$, we find:

$$w = \begin{cases} \frac{2(a-c)+\varepsilon+4d-2\gamma}{6} & \text{if } \varepsilon < \frac{4(a-c)-4d+2\gamma}{7} \text{ and if } \gamma < \frac{2(a-c)+\varepsilon-2d}{2} \\ \frac{2(a-c)+\varepsilon+2d-2\gamma}{4} & \text{if } \varepsilon < \frac{2(a-c)-2d+2\gamma}{5} \text{ and if } \gamma \geq \frac{2(a-c)+\varepsilon-2d}{2} \\ \frac{(a-c)+\varepsilon+2d-\gamma}{3} & \text{if } \varepsilon < \frac{2(a-c)-2d+\gamma}{4} \text{ and if } \gamma < a - c + \varepsilon - d \\ \frac{(a-c)+\varepsilon+d-\gamma}{2} & \text{if } \varepsilon < \frac{a-c-d+\gamma}{3} \text{ and if } \gamma \geq a - c + \varepsilon - d \end{cases}$$

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