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International Intellectual Property Rights: Effects on Growth, Welfare and Income Inequality

Angus C. Chu\textsuperscript{a} and Shin-Kun Peng\textsuperscript{b}

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Abstract

What are the effects of strengthening developing countries’ protection for intellectual property rights on economic growth, social welfare and income inequality in the global economy? To analyze this question, we develop a two-country R&D-based growth model with wealth heterogeneity. We find that the North experiences higher growth and welfare at the expense of higher income inequality while the South experiences higher growth at the expense of lower welfare and higher income inequality. As for global welfare, there exists a critical degree of cross-country spillovers below (above) which global welfare decreases (increases). In light of these findings, we discuss policy implications on China’s accession to the World Trade Organization in 2001.

Keywords: economic growth, income inequality, intellectual property rights, TRIPS

\textit{JEL classification:} O34, O41, D31, F13

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1. Introduction

The World Trade Organization (WTO) Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) establishes a minimum level of intellectual property rights (IPR) protection that must be provided by all member countries by 2006. Given that developed countries (the North) generally had a much higher level of IPR protection than developing countries (the South) prior to TRIPS, this agreement is likely to have asymmetric effects on the North and the South. As an example of the North (the South), we consider the US (China). Table 1 presents the index of patent rights from Park (2008) for the US and China. It shows that as a result of TRIPS, the level of patent protection in China is converging towards the level in the US. Given the importance of TRIPS, this study analyzes its effects on economic growth, social welfare and income inequality in the global economy.

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Specifically, we develop a two-country R&D-based growth model with wealth heterogeneity among households. In the model, both the North and the South invest in R&D, but the North has a higher degree of innovative capability than the South. Within this framework, we derive the following results. Firstly, strengthening patent protection in either country increases both countries’ (a) economic growth by increasing R&D and (b) income inequality by raising the return on assets. Then, following Grossman and Lai (2004) and Lai and Qiu (2003), we derive the pre-TRIPS Nash equilibrium level of patent protection that is sub-optimally low due to the positive externality of patent policy. Also, we find that the North chooses a higher level of patent protection than the South and imposing the North’s higher level of patent protection on the South as required by TRIPS increases (decreases) welfare in the North (the South).³

¹ The index is a scale of 0 to 5, and a larger number indicates stronger patent protection. See Park (2008) for details.
² As a result of TRIPS, the statutory term of patent in the US was extended from 17 years (counting from the issue date of the patent) to 20 years (counting from the earliest filing date). However, due to the difference in the starting date, the effective extension was minimal. As for China, it extended the patent length from 15 to 20 years in 1992. Prior to joining the WTO, China reformed its patent system again in 2000 in compliance with the TRIPS agreement; see footnote 4 for details of this reform.
³ See, for example, Reichman (1995) for a more detailed discussion on the requirements of TRIPS.
This welfare implication is consistent with Grossman and Lai (2004) and Lai and Qiu (2003). It is perhaps not surprising that the South would be worse off by deviating from its best response. Therefore, the intriguing question is whether TRIPS would improve or reduce global welfare. We find that there exists a critical degree of cross-country spillovers below (above) which global welfare is lower (higher) under TRIPS while Lai and Qiu (2003) find that global welfare always improves as a result of TRIPS. This difference arises because we introduce a structural parameter to allow for varying degrees of cross-country spillovers captured by the importance of foreign goods in domestic consumption. In our model, the degree of the positive externality in the Nash equilibrium is determined by this structural parameter. When the share of foreign goods in domestic consumption is small, cross-country spillovers of innovation are small as well. In this case, imposing the North’s level of patent protection on the South makes the South worse off without making the North much better off. This finding has important policy implications. First, it implies that the North is not always able to compensate the South even in the presence of costless transfers. Secondly, a sufficient degree of global integration is necessary in order for the harmonization of IPR protection to improve global welfare.

Furthermore, our model with heterogeneous households enables us to analyze the effects of TRIPS on income inequality within each country in addition to growth and welfare. Under TRIPS, the North experiences higher growth and higher welfare at the expense of higher income inequality. As for the South, it experiences higher growth at the expense of lower welfare and higher income inequality. Intuitively, a higher growth rate increases the rate of return on assets through the Euler equation, and this higher return on assets increases the income of asset-wealthy households relative to asset-poor households in each country. This result suggests that although the representative-agent welfare analysis of TRIPS in previous studies can be robust to an extension with heterogeneous households, it is useful to also analyze the distributional consequences within a country given that income inequality can be a social concern.
For example, China amended its patent law in 2000 in anticipation of its accession to the WTO in 2001. Since this amendment, the annual growth rate of applications for invention patents in China has increased to 23% (compared to less than 10% before 2000). Hu and Jefferson (2009) provide empirical evidence to show that the patent-law amendment in 2000 is a major factor for China’s recent surge in patenting activities. Also, R&D as a share of GDP in China increases from an average of about 0.7% in the 90’s to 1.49% in 2007. At the same time, the rising income inequality in China poses the country a serious challenge on domestic stability. In 2007, China’s Gini coefficient rises to 0.47 that is above the threshold of 0.45 indicating potential social unrest. Our theoretical analysis suggests that strengthening IPR protection in China as a result of TRIPS worsens its income inequality in addition to potentially reducing its social welfare as also implied by previous studies. Given the current situation in China, the distributional consequence seems to be more alarming. In a panel regression, Adams (2008) finds that strengthening IPR protection indeed has a positive and statistically significant effect on income inequality. His estimates imply that increasing Park’s (2008) index by one (on a scale of zero to five) is associated with an increase in the Gini coefficient of 0.01 to 0.02 (on a scale of zero to one) in developing countries.

We should emphasize that there are also other important factors contributing to the rising income inequality in China, and patent policy is only one of them. Furthermore, China’s accession to the WTO carries other benefits, such as lower trade barriers, which are not considered in this partial analysis of patent policy. In the model, we introduce a parameter to capture trade barriers and find that lower trade barriers improve social welfare. Therefore, a complete welfare analysis on China’s accession to the WTO should trade off the welfare gain from lower trade barriers against the welfare loss from TRIPS.

Our study relates to the literature on IPR protection and North-South trade. Early studies in this literature focus on the effects of IPR in reducing imitation from the South and encouraging technology transfer from the North through licensing or foreign direct investment. In these studies, innovative

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4 The changes include (a) providing patent holders with the right to obtain a preliminary injunction against the infringing party before filing a lawsuit, (b) stipulating standards to compute statutory damages, (c) affirming that state and non-state enterprises enjoy equal patent rights, and (d) simplifying the patent application process, examination and transfer procedures and unifying the appeal system.

5 This data is obtained from China Statistical Yearbook.
activities are usually assumed to take place only in the North. However, two other important reasons for strengthening IPR in the South are (a) to provide incentives for the North to develop technologies that are also used by the South, and (b) to provide incentives for the South to invest in innovative activities. To fill in this gap in the literature, recent theoretical studies, such as Lai and Qiu (2003) and Grossman and Lai (2004), have started to consider the important role of TRIPS in providing sufficient incentives for innovation in both the North and the South. Our paper follows this branch of studies to focus on this aspect of TRIPS.

Lai and Qiu (2003) and Grossman and Lai (2004) derive the Nash equilibrium level of patent protection in an open-economy variety-expanding model, in which both the North and the South invest in R&D, and analyze the welfare effects of harmonizing IPR protection. We complement these interesting studies by analyzing the effects of TRIPS on the growth-inequality tradeoff in a quality-ladder model with endogenous growth and by allowing for varying degrees of cross-country spillovers. To our knowledge, our study is the first to analyze the effects of TRIPS on welfare, growth and inequality simultaneously. Modeling varying degrees of cross-country spillovers also has surprising implications on global welfare.

Since the seminal study of Simon Kuznets (1955), the tradeoff between growth and inequality has been an important issue in economics. On one hand, early theoretical and empirical studies tend to find a negative growth-inequality relationship. On the other hand, the more recent studies tend to find a positive relationship. Forbes (2000) finds a positive empirical growth-inequality relationship and argues that the different results in previous studies are due to omitted-variable bias and measurement error. García-Peñalosa and Turnovsky (2006, 2007) argue that the theoretical growth-inequality relationship should be

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7 See, for example, Diwan and Rodrik (1991).
8 For example, in a panel regression, Chen and Puttitanun (2005) find that strengthening IPR in developing countries indeed has a positive and significant effect on their innovations.
ambiguous and depends on the underlying structural changes. Incorporating wealth heterogeneity into an AK growth model, they show that a positive relationship is more likely to emerge.

Although the capital-accumulation-driven growth models are useful frameworks for analyzing many macroeconomic issues, they are not suitable for evaluating innovation policies. Therefore, this study incorporates wealth heterogeneity into an open-economy R&D-based growth model to analyze the effects of TRIPS. In a related study, Chu (2010) analyzes the effects of patent policy on the growth-inequality tradeoff in the US using a closed-economy quality-ladder model with wealth heterogeneity.\footnote{See also Chou and Talmain (1996), Li (1998), Zweimuller (2000), Foellmi and Zweimuller (2006) and Bertola et al. (2006). These studies focus on the effects of inequality on growth but do not consider the effects of patent policy on income inequality.}
The present study differs from Chu (2010) by (a) developing an open-economy model, (b) deriving the level of patent protection as the outcome of a policy game between countries, and (c) considering the effects of patent policy on welfare in addition to growth and income inequality. As argued by Chen and Turnovsky (2010), “…virtually the entire growth-inequality literature is restricted to a closed economy, which is a severe shortcoming given the increasing openness characterizing most economies.”

This paper also relates to the literature on R&D underinvestment, patent policy and economic growth. Griliches (1992) provides a survey on empirical studies that find the social return to R&D to be much higher than the private return. Jones and Williams (1998, 2000) use these empirical estimates to show that in an R&D-based growth model, the socially optimal level of R&D is at least two to four times higher than the market level. A number of studies, such as Li (2001), O’Donoghue and Zweimuller (2004) and Chu (2009), analyze how patent breadth increases R&D and economic growth in R&D-based growth models that feature a representative household. Given that patent policy may affect the distribution of income, the present study contributes to this literature by providing a growth-theoretic framework that highlights the distributional consequences of patent policy in an open economy.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium and analyzes its properties. Section 4 considers the effects of TRIPS on growth, welfare and income inequality. Section 5 concludes with some suggestions for future research.
2. The model

The underlying quality-ladder model is based on Grossman and Helpman (1991a). We modify the model by (a) extending it to a two-country setting with trade in intermediate inputs similar to Peng et al. (2006), (b) allowing for wealth heterogeneity among households, and (c) considering incomplete patent breadth (i.e., patent protection against imitation) as in Li (2001). There are two countries denoted by the North (n) and the South (s). As in Lai and Qiu (2003) and Grossman and Lai (2004), both countries invest in R&D, but the North has a higher degree of innovative capability than the South. Also, trade is balanced as commonly assumed in the literature. Given that the quality-ladder growth model has been well-studied, the familiar components of the model will be briefly described in Sections 2.1 to 2.4. To conserve space, we only present the equations for the North. However, the readers are advised to keep in mind that for each equation that we present, there is an analogous equation for the South.

2.1. Households

There is a continuum of identical households (except for the initial holding of wealth) on the unit interval $h \in [0,1]$ in each of the two countries indexed by a superscript $\in \{n, s\}$. Households are immobile across countries. In country $n$, household $h$’s utility function is given by

$$U^n(h) = \int_0^\infty e^{-\rho t} \ln C^n_t(h) dt.$$

$C^n_t(h)$ denotes the consumption of household $h$ in country $n$ at time $t$. $\rho > 0$ is the exogenous discount rate. Each household maximizes utility subject to the following law of motion for asset accumulation.

$$\dot{V}^n_t(h) = R^n_tV^n_t(h) + W^n_t - P^n_tC^n_t(h).$$

12 Lai and Qiu (2003) and Grossman and Lai (2004) consider patent protection in the form of patent length in their variety-expanding models. Given that we have a quality-ladder model, we consider patent protection in the form of patent breadth, which is an equally important patent-policy instrument commonly discussed in the patent-design literature. Using China as an example, its statutory length of patent has been 20 years since 1993.

13 In a similar (closed-economy) model, Chu (2010) considers a more general iso-elastic utility function and shows that the positive relationship between patent protection and income inequality is robust to this specification change. To simplify the analytical derivation, we focus on the log utility function in this study.
\( V_t^n(h) \) is the value of financial assets owned by household \( h \) in country \( n \) at time \( t \). Household \( h \)’s share of financial assets at time 0 is exogenously given by \( s_{t,0}^n(h) \equiv V_0^n(h) / V_0^n \) that has a general distribution function with a mean of one and a standard deviation of \( \sigma^v_n \) (i.e., the coefficient of variation of wealth).

\( R_t^n \) is the nominal rate of return on assets in country \( n \). We assume home bias in asset holding such that the shares of monopolistic firms in each country are solely owned by domestic households.\(^\text{14}\) Household \( h \) inelastically supplies one unit of labor to earn a wage income \( W_t^n \). \( P_t^n \) is the price of consumption in country \( n \). From household \( h \)’s intertemporal optimization, the familiar Euler equation is given by

\[
\frac{\dot{C}_t^n(h)}{C_t^n(h)} = \frac{\dot{C}_t^n}{C_t^n} = r_t^n - \rho,
\]

where \( r_t^n \equiv R_t^n - P_t^n / P_t^n \) is the real interest rate in country \( n \). Equation (3) shows that consumption of households within a country grows at the same rate.

### 2.2. Final goods

Consumption in country \( n \) is a Cobb-Douglas aggregate of two types of final goods.\(^\text{15}\)

\[
C_t^n = \frac{(C_t^{n,n})^{1-\alpha} (C_t^{n,s})^{\alpha}}{(1-\alpha)^{1-\alpha} \alpha^\alpha},
\]

where \( C_t^{n,s} \) refers to final goods (in country \( n \)) that are produced with intermediate inputs imported from country \( s \). The parameter \( \alpha \in [0,0.5] \) captures the importance of foreign goods in domestic consumption.

Later on, we will show that this parameter also determines the degree of cross-country spillovers. There is

\(^{14}\text{It is useful to note that home bias does not eliminate the positive externality of IPR protection in generating profits to be earned by foreign households. When a country strengthens IPR protection, foreign firms owned by foreign households still earn a larger amount of profits. What home bias does is to naturally link the degree of this positive externality to the share of goods traded, which is determined by the domestic importance of foreign goods.}\)

\(^{15}\text{This type of Armington aggregator is commonly used in open-economy macroeconomic models for aggregating tradable goods across countries. A more general form of Armington aggregator is of the CES form, which we do not consider because we want to allow } C_t^{n,n} \text{ and } C_t^{n,s} \text{ to grow at different rates.}\)
a large number of competitive firms producing final goods with a standard Cobb-Douglas aggregator over a continuum of differentiated intermediates goods \( i \in [0,1] \).

\[
C_i^{n,n} = \exp \left( \int_0^1 \ln C_i^{n,n} (i) \, di \right),
\]

(5)

\[
C_i^{n,s} = \exp \left( \int_0^1 \ln C_i^{n,s} (i) \, di \right).
\]

(6)

\( C_i^{n,s} (i) \) refers to intermediate goods \( i \) (in country \( n \)) that are produced by inputs from country \( s \).

2.3. Intermediate goods

In country \( n \), there is a continuum of industries indexed by \( i \in [0,1] \). In each industry, an industry leader produces \( X_i^{n,n} (i) \) and \( X_i^{s,n} (i) \) (which are the necessary inputs for \( C_i^{n,n} (i) \) and \( C_i^{n,s} (i) \) respectively) and dominates the market until the arrival of the next innovation.\(^{16}\) The leader holds a patent in each country on the industry’s latest technology. Using the leader’s input \( X_i^{n,n} (i) \), the level of output for \( C_i^{n,n} (i) \) is

\[
C_i^{n,n} (i) = z^{N_i^{n} (i)} X_i^{n,n} (i).
\]

(7)

\( z > 1 \) is the exogenous step size of quality improvement from each innovation, and \( N_i^{n} (i) \) is the number of innovations that have occurred in industry \( i \) of country \( n \) as of time \( t \). In other words, \( z^{N_i^{n} (i)} \) represents the quality of each unit of input produced by the leader while \( X_i^{n,n} (i) \) is the quantity of input produced.

Similarly, using the leader’s input \( X_i^{s,n} (i) \), the level of output for \( C_i^{s,n} (i) \) is

\[
C_i^{s,n} (i) = (1 - \tau) z^{N_i^{s} (i)} X_i^{s,n} (i),
\]

(8)

where \( \tau \in [0,1] \) denotes an iceberg transportation cost that captures trade barriers.

\(^{16}\) Grossman and Helpman (1991a) show that the next innovation comes from another innovator due to the Arrow displacement effect.
To produce one unit of $X_{i}^{n,n}(i)$ or $X_{i}^{s,n}(i)$, the industry leader needs to employ one unit of workers. Therefore, the production function is

$$X_{i}^{n,n}(i) + X_{i}^{s,n}(i) = L_{i,i}^{n,n}(i) + L_{s,i}^{s,n}(i) = L_{i,i}^{n}(i).$$

$L_{i,i}^{n}(i)$ is the total number of workers employed in industry $i$ of country $n$. The leader’s marginal cost of producing one unit of $X_{i}^{n,n}(i)$ or $X_{i}^{s,n}(i)$ is

$$MC_{i}^{n}(i) = W_{i}^{n}.$$

Implicitly, we have assumed that the industry leader must employ domestic workers to produce for both domestic and foreign markets and abstracted from the issues of foreign direct investment, licensing and overseas imitation in order to keep the model tractable.\(^{17}\)

As commonly assumed in quality-ladder models, the current and former industry leaders engage in Bertrand competition, and the familiar profit-maximizing price for the current industry leader is a constant markup over the marginal cost. The prices for $X_{i}^{n,n}(i)$ and $X_{i}^{s,n}(i)$ are respectively

$$P_{i}^{n,n}(i) = \mu(z_{i}, b_{n})MC_{i}^{n}(i),$$

$$P_{i}^{s,n}(i) = \mu(z_{i}, b^{i})MC_{i}^{n}(i),$$

where $\mu(z, b) = z^{b}$ for $b \in (0,1]$. $b^{n}$ ($b^{i}$) captures the level of patent breadth in country $n$ ($s$). In Grossman and Helpman (1991a), there is complete patent protection against imitation (i.e., $b = 1$). Li (2001) generalizes the policy environment to capture incomplete patent protection against imitation (i.e., $b \in (0,1]$).\(^{18}\) Due to incomplete patent breadth, the former leader can partly imitate the current leader’s invention in order to increase the quality of her product by a factor of $z^{1-b^{s}}$ ($z^{1-b^{i}}$) in country $n$ ($s$) without infringing the current leader’s patents. As a result, the limit-pricing markups for the current leader

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\(^{17}\) These interesting issues have been studied in a related literature. See Grossman and Helpman (1991b), Helpman (1993), Lai (1998), Yang and Maskus (2001), Glass and Saggi (2002a, b) and Dinopoulos and Segerstrom (2010).

\(^{18}\) This is known as lagging patent breadth in the literature. See, for example, O’Donoghue and Zweimuller (2004) for an analysis of leading patent breadth in dynamic general-equilibrium models. For the purpose of the current study, the consideration of lagging patent breadth is more relevant for developing countries.
are $z^b_n$ in country $n$ and $z^s_n$ in country $s$ respectively. An increase in $b$ in either country enables the current leader to charge a higher markup in that country. The resulting increases in profits and the value of inventions improve the incentives for R&D. From the rest of this study, we denote patent protection as $\mu'' \equiv \mu(z, b'')$ for convenience and consider changes in $\mu''$ coming from changes in $b''$ only.

### 2.4. R&D

Denote the expected value of an innovation in industry $i$ of country $n$ as $\bar{V}_t^n(i)$. Due to the Cobb-Douglas specification in (5) and (6), the amount of profits is the same across industries within a country (i.e., $\pi_i^n(i) = \pi_i^s(i)$ and $\pi_i^s(i) = \pi_i^t(i)$ for $i \in [0,1]$). As a result, $\bar{V}_t^n(i) = \bar{V}_t^n$ in a symmetric equilibrium that features an equal innovation arrival rate across industries within a country.\(^{19}\) We denote the sum of profits generated by an innovation in country $n$ as $\pi_t^n \equiv \pi_t^n + \pi_t^s$. Because of home bias in asset holding, the market value of inventions in country $n$ equals the total value of assets owned by domestic households (i.e., $\bar{V}_t^n = V_t^n$). The familiar no-arbitrage condition for $V_t^n$ as an asset is

\begin{equation}
R_t^n V_t^n = \pi_t^n + \bar{V}_t^n - \lambda_t^n V_t^n,
\end{equation}

which equates the interest rate to the asset return per unit of assets. The right-hand side of (13) consists of the sum of (a) the monopolistic profit $\pi_t^n$ generated by this asset, (b) the potential capital gain $\bar{V}_t^n$, and (c) the expected capital loss $\lambda_t^n V_t^n$ due to creative destruction for which $\lambda_t^n$ is the Poisson arrival rate of innovation in country $n$.

There is a continuum of R&D entrepreneurs indexed by $j \in [0,1]$ in each country, and they hire workers for R&D. The expected profit for entrepreneur $j$ in country $n$ is

\begin{equation}
\pi_{t,j}^n (j) = V_t^n \lambda_t^n (j) - W_t^n L_{t,j}^n (j).
\end{equation}

\(^{19}\) We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi et al. (2007) for a theoretical justification for the symmetric equilibrium in the quality-ladder growth model.
The Poisson arrival rate of innovation for entrepreneur $j$ in country $n$ is $\lambda^j_n(t) = \varphi^n L^j_n(t)$, where $\varphi^n$ is the productivity of R&D workers (i.e., innovative capability) in country $n$. Without loss of generality, we assume $\varphi^n \geq \varphi^i$. Because of free entry, R&D entrepreneurs make zero expected profit such that

$$\text{(15)} \quad V^n_t \varphi^n = W^n_t.$$

This condition determines the allocation of labor between production and R&D within each country.

3. Decentralized equilibrium

In this section, we define the equilibrium and show that the aggregate economy is always on a unique and stable balanced-growth path. In Section 3.1, we show that the wealth distribution is stationary. In Section 3.2, we consider the income distribution. In Section 3.3, we derive a welfare function for policymakers and characterize the Nash equilibrium as well as the globally optimal patent protection.

Equilibrium is a time path of prices $\{R^n_t, W^n_t, P^n_t, P_{i,n}^n(t), P_{r,n}^n(i), V^n_t(h), V^n_t\}_{t=0}^\infty$ and allocations $\{C^n_t(i), C^n_t(i), X^{n,n}_t(i), X^{r,n}_t(i), L_{r,i}^n(i), L_{r,j}^n(j), C^n_t(h), C^n_t(i), C^n_t, C^n_t\}_{t=0}^\infty$. Also, at each instant of time,

a. household $h$ chooses $\{C^n_t(h)\}$ to maximize (1) subject to (2) taking $\{R^n_t, W^n_t, P^n_t\}$ as given;

b. perfectly competitive final-goods firms maximize profit taking prices as given;

c. the leader in industry $i$ produces $\{X^{n,n}_t(i), X^{r,n}_t(i)\}$ and chooses $\{P_{n}^{n,n}(i), P_{r}^{n,n}(i), L_{r,i}^n(i)\}$ to maximize profit according to the Bertrand competition and taking $\{W^n_t\}$ as given;

d. R&D entrepreneur $j$ chooses $\{L_{r,j}^n(j)\}$ to maximize profit taking $\{W^n_t, V^n_t\}$ as given;

e. the market for consumption clears such that $\int_0^1 C^n_t(h)dh = C^n_t = \frac{(C^n_t)^{1-\alpha} (C^n_t)^\alpha}{(1-\alpha)^{1-\alpha}}$;

f. the market for domestic final goods clears such that $C^n_t = \exp\left(\int_0^1 \ln C^{n,n}_t(i)di\right)$. 

g. the market for foreign final goods clears such that 
\[ C_{t}^{n,s} = \exp\left(\int_{0}^{1} \ln C_{i}^{n,t} (i) di\right) \].\(^{20}\)

h. the domestic market for intermediate goods \( i \) clears, i.e., 
\[ C_{i}^{n,a} (i) = z^{N_{t}^{i}} X_{i}^{n,n} (i) \]

i. the overseas market for intermediate goods \( i \) clears, i.e., 
\[ C_{i}^{n,s} (i) = (1 - \tau) z^{N_{t}^{i}} X_{i}^{n,n} (i) \]

j. the labor market clears such that 
\[ \int_{0}^{1} L_{x,i}^{n} (i) di + \int_{0}^{1} L_{x,j}^{n} (j) dj = 1 \]; and

k. the value of trade in intermediate goods is balanced such that 
\[ P_{i}^{n,s} C_{i}^{n,s} = P_{i}^{n,n} C_{i}^{n,n} \].\(^{21}\)

**Lemma 1:** The aggregate economy is always on a unique and stable balanced-growth path, in which the equilibrium allocation of labor in country \( n \) is given by

\[ L_{x}^{n} (\mu^{n}, \phi^{n}) = \left(1 - \frac{1}{\mu^{n}} + \frac{\rho}{\phi^{n}}\right) \]
\[ L_{x}^{i} (\mu^{i}, \phi^{n}) = \left(\frac{1}{\mu^{i}} + \frac{\rho}{\phi^{n}}\right) \]
\[ L_{x}^{r} (\mu^{n}, \mu^{i}, \phi^{n}) = 1 - \left(\frac{1}{\mu^{n}} + \frac{1}{\mu^{i}} + \frac{\rho}{\phi^{n}}\right) \]

**Proof:** See Appendix A.\( \blacksquare \)

Lemma 1 shows that the aggregate economy always jumps immediately to a unique and stable balanced-growth path. Furthermore, the properties of the equilibrium labor allocation are quite intuitive. An increase in \( \mu^{i}, \mu^{r} \) or \( \phi^{n} \) improves the incentives for R&D. As a result, labor is reallocated away from the production sector to the R&D sector. To ensure that \( L_{x}^{r} > 0 \), we impose a lower bound on R&D productivity.

---

\(^{20}\) To be more precise, we are referring to final goods produced using foreign intermediate inputs.

\(^{21}\) These price indices will be defined in the proof of Lemma 1.
Condition R (R&D productivity): \[ \phi^n > \rho / (\Gamma^n - 1), \]

where \[ \Gamma^n = \left( \frac{1 - \alpha}{\mu^n} + \frac{\alpha}{\mu^i} \right)^{-1}. \]

Given the equilibrium allocation of labor, the next lemma characterizes the equilibrium outcomes for other aggregate variables. In (19), the arrival rate of innovation is increasing in domestic R&D. In (20), the growth rate of consumption in country \( n \) is increasing in the arrival rate of innovation in each country. Therefore, an increase in \( \mu^n, \mu^i, \phi^n \) or \( \phi^i \) increases domestic R&D and/or foreign R&D as well as the consumption growth rate in each country. As for the level of consumption, it is derived in (21).

**Lemma 2:** On the balanced-growth path, the other aggregate variables are given by

\[ \lambda^n(\mu^n, \mu^i, \phi^n) = \phi^n L^n, \]  

\[ \frac{\dot{C}^n}{C^n} \equiv g^n(\mu^n, \mu^i, \phi^n, \phi^i) = [(1 - \alpha)\lambda^n + \alpha \lambda^i] \ln z, \]

\[ C^n_t = \left( 1 + \frac{\rho}{\phi^n} \right) \frac{W^n_t}{P^n_t}. \]

**Proof:** See Appendix A. ■

### 3.1. Wealth distribution

I adopt a similar approach as in García-Peñalosa and Turnovsky (2006, 2007) to show that the distribution of wealth is stationary on the balanced growth path. The value of wealth in country \( n \) evolves according to

\[ V^n_t = R^n V^n_t + W^n_t - P^n C^n_t. \]

Combining (2) and (22), the law of motion for \( s_{r,n} (h) \equiv V_t (h) / V^n_t \) is given by

\[ \frac{\dot{s}^n_{r,n} (h)}{s^n_{r,n} (h)} = \frac{W^n_t - P^n C^n_t (h)}{V^n_t (h)} - \frac{W^n_t - P^n C^n_t}{V^n_t}. \]
From (15) and (21), \( s_{r,t}^n(h) \) evolves according to a simple linear differential equation given by

\[
\dot{s}_{r,t}^n(h) = \rho s_{r,t}^n(h) + \left(1 - s_{r,t}^n(h) \left(1 + \frac{\rho}{\varphi^n}\right)\right)\varphi^n.
\]

(24) describes the potential evolution of \( s_{r,t}^n(h) \) given an initial value of \( s_{r,0}^n(h) \cdot s_{r}^n(h) \equiv C_r^n(h) / C_t^n \) is a stationary variable from (3), so that the last term in (24) is constant. The coefficient on \( s_{r,t}^n(h) \) given by \( \rho \) is constant and positive. Therefore, the only solution consistent with long-run stability is \( \dot{s}_{r,t}^n(h) = 0 \) for all \( t \). From (24), \( \dot{s}_{r,t}^n(h) = 0 \) for all \( t \) implies that \( s_{r,t}^n(h) = s_{r,0}^n(h) \) and

\[
C_r^n(h) = \left(1 + \frac{\rho s_{r,0}^n(h)}{\varphi^n}\right)\frac{W_t^n}{P_t^n}
\]

for all \( t \). Lemma 3 summarizes the stationarity of the wealth distribution in country \( n \).

**Lemma 3:** For every household \( h \) in country \( n \), \( s_{r,t}^n(h) = s_{r,0}^n(h) \) for all \( t \).

**Proof:** Proven in the text.\( \blacksquare \)

### 3.2. Income distribution

In this section, we derive a measure of income inequality. We consider inequality in real income, which is the appropriate measure because it is invariant to the unit of denomination. Household \( h \)’s real income \( Y_t^n(h) \) is the sum of the real return on financial assets and the real wage rate given by

\[
Y_t^n(h) = \tau_t^n Y_t^n(h) / P_t^n + W_t^n / P_t^n.
\]

From (3), (15) and Lemma 3, the share of real income earned by household \( h \) simplifies to

\[
s_{y,t}^n(h) \equiv \frac{Y_t^n(h)}{Y_t^n} = \frac{(\rho + g^n)s_{r,0}^n(h) + \varphi^n}{\rho + g^n + \varphi^n}
\]

for all \( t \). The standard deviation of income share (i.e., the coefficient of variation of income) is
Proposition 1: Holding $\rho$ and $\varphi^n$ constant, income inequality $\sigma_y^n$ is increasing in the growth rate $g^n$.

Proof: See (28).

Intuitively, a higher growth rate drives up the real interest rate through the Euler equation, and the resulting higher rate of asset return increases the income share $s_y^n(h)$ of asset-wealthy households (i.e., $s_y^n(h) > 1$) while it decreases that of asset-poor households (i.e., $s_y^n(h) < 1$). This positive relationship between growth and inequality is consistent with recent empirical studies, such as Li and Zou (1998) and Forbes (2000). Next, we consider the effects of an exogenous increase in patent protection on growth and income inequality. Corollary 1 shows that a higher level of patent protection in either country increases R&D, economic growth and income inequality in both countries.

Corollary 1: An increase in $\mu^n$ or $\mu^t$ increases growth and income inequality in both countries.

Proof: See (20) and (28).

Equation (28) indicates an interesting difference between the AK model and the quality-ladder model. The AK model in García-Peñalosa and Turnovsky (2006, 2007) requires elastic labor supply to generate an endogenous income distribution while the quality-ladder model generates an endogenous income distribution even with inelastic labor supply. See Chu (2010) for a quality-ladder model with heterogeneous households and elastic labor supply.


3.3. Social welfare

Due to the balanced-growth behavior of the model, the utility of household $h$ in country $n$ simplifies to

$$U^n(h) = \frac{\ln C^n_0(h)}{\rho} + \frac{g^n}{\rho^2},$$

(29)

Substituting (25) into (29) yields

$$U^n(h) = \frac{1}{\rho} \left( \ln \left( 1 + \frac{\rho s^n_0(h)}{\varphi^n} \right) + \ln \left( \frac{W^n_0}{P^n_0} \right) + \frac{g^n}{\rho} \right).$$

(30)

The lifetime utility of a household depends on the growth rate and the initial level of consumption, which in turn depends on the initial real wage rate and the share of assets owned by the household. Although the ownership of assets varies across households, (30) shows that this household-specific term is independent of patent protection. This property is a result of the log utility function, and this convenient feature allows us to abstract from choosing a social welfare function for the government.

**Lemma 4:** After dropping the exogenous terms, the initial real wage in country $n$ can be decomposed into

$$\ln(W^n_0 / P^n_0) = -\ln \mu^n + \alpha \ln(1 - \tau) + \alpha \ln(W^n_0 / W^1_0).$$

(31)

**Proof:** See Appendix A. ■

Lemma 4 shows that the initial real wage in country $n$ has three components (a) the negative effect of markup pricing from patent protection, (b) the negative effect of trade barriers, and (c) the relative wage rate across the two countries. An expression for the relative wage can be derived from the balanced-trade condition $P^{*,n} C^{*,n} = P^{*,s} C^{*,s}$, which simplifies to

$$\frac{W^n_t}{W^s_t} \equiv \omega^n(\varphi^n, \varphi^s) = \frac{\mu^n}{\mu^s} \left( \frac{L^n_{i,s}}{L^s_{i,s}} \right) = \left( 1 + \frac{\rho}{\varphi} \right) \left( 1 + \frac{\rho}{\varphi^n} \right) \geq 1$$

(32)

$^{23}$ For example, the benefit of China’s accession to the WTO may be captured by a reduction in trade barriers that increases social welfare in China.
for all $t$. (32) shows that the relative wage is independent of patent protection and depends on the relative R&D productivity between the North and the South. Substituting (31) and (32) into (30) and dropping the terms that are independent of patent protection yield the welfare of any household $h$ in country $n$ as a function of $\mu^n$ and $\mu'$ given by

$$\Omega^e(\mu^n, \mu') \equiv -\ln \mu^n + \frac{g^n(\mu^n, \mu')}{\rho}.$$  

Equation (33) has three interesting properties. Firstly, the welfare component that depends on patent protection is the same across households. Secondly, (33) captures the tradeoff between the static cost $-\ln \mu^n$ and the dynamic benefit $g^n/\rho$ of patent protection as in the seminal study of Nordhaus (1969). Thirdly, (33) and the analogous condition for the South show that while the welfare cost of raising $\mu^n$ falls entirely on the North, the welfare gain is shared with the South because $g^s$ is increasing in $\mu^n$. Due to this positive externality, the Nash equilibrium level of patent protection is suboptimal. To see this result, $g^s = [(1-\alpha)\lambda^s + \alpha \lambda^n] \ln z$ is increasing in $\mu^n$ via two channels of cross-country spillovers: (a) $\mu^n$ increases $g^s$ via $\lambda^n$, and (b) $\mu^n$ increases $g^s$ via $\lambda'$. Channel (a) captures technology spillovers across countries. Channel (b) captures the positive effect of domestic patent protection on foreign return to R&D. The degree of these cross-country spillover effects is determined by the structural parameter $\alpha$.

Upon deriving the welfare function, we firstly characterize the Nash equilibrium level of patent protection in the two countries denoted by $(\mu^e_{NE}, \mu^e_{NE})$. As in Grossman and Lai (2004), the policymaker in each country chooses the domestic level of patent protection once and for all at time 0 to maximize domestic households’ welfare in (33) taking the foreign level of patent protection as given. In other words, the policymakers in the two countries play a one-shot game at time 0. Also, we assume an interior solution for the equilibrium level of patent protection such that $\mu < z$ (i.e., $b < 1$) in each country.
Proposition 2: The Nash equilibrium level of patent protection is given by

\[
\mu^*_{NE}(\phi^n, \phi^s) = \left( 1 - \alpha \right) \left( \frac{\phi^n}{\rho} + 1 \right) + \alpha \left( \frac{\phi^s}{\rho} + 1 \right) \ln z, 
\]

\[
\mu^*_{NE}(\phi^n, \phi^s) = \left( 1 - \alpha \right) \left( \frac{\phi^n}{\rho} + 1 \right) + \alpha \left( \frac{\phi^s}{\rho} + 1 \right) \ln z. 
\]

Proof: See Appendix A.■

As in Lai and Qiu (2003) and Grossman and Lai (2004), we find that the Nash equilibrium level of patent protection is stronger in the North than in the South unless either (a) \( \alpha = 0.5 \) or (b) \( \phi^n = \phi^s \).

We assume that neither (a) nor (b) hold such that \( \mu^*_{NE} > \mu^*_{NE} \). In Proposition 3, we derive the globally optimal patent protection denoted by \( (\mu^n_{GO}, \mu^i_{GO}) \equiv \arg \max(\Omega^n + \Omega^i) \). If cross-country spillovers are absent (i.e., \( \alpha = 0 \)), then \( \mu^*_{NE} = \mu^n_{GO} \) and \( \mu^i_{NE} = \mu^i_{GO} \). Otherwise, \( \mu^*_{NE} < \mu^n_{GO} \) and \( \mu^i_{NE} < \mu^i_{GO} \) implying suboptimal patent protection in the Nash equilibrium. For the rest of the analysis, we assume that \( \alpha > 0 \).

Proposition 3: The globally optimal level of patent protection is given by

\[
\mu^n_{GO}(\phi^n, \phi^s) = \left( 1 - \alpha \right) \left( \frac{\phi^n}{\rho} + 1 \right) + \alpha \left( \frac{\phi^s}{\rho} + 1 \right) \ln z > \mu^*_{NE}, 
\]

\[
\mu^i_{GO}(\phi^n, \phi^s) = \left( 1 - \alpha \right) \left( \frac{\phi^n}{\rho} + 1 \right) + \alpha \left( \frac{\phi^s}{\rho} + 1 \right) \ln z > \mu^*_{NE}. 
\]

Proof: See Appendix A.■

Corollary 2: An increase in \( \alpha \) increases \( \mu^n_{GO} - \mu^*_{NE} \) and \( \mu^i_{GO} - \mu^*_{NE} \).

Proof: See Appendix A.■

\[24\] To be consistent with previous studies, we use this utilitarian approach to define global welfare.
Corollary 2 shows that the Nash equilibrium level of patent protection deviates from the globally optimal level as $\alpha$ increases because the positive externality in the Nash equilibrium is increasing in $\alpha$. Intuitively, a larger degree of cross-country spillovers raises the degree of positive externality and hence worsens the sub-optimality of the Nash equilibrium.

4. Effects of TRIPS

In this section, we analyze the effects of TRIPS on growth, welfare and income inequality. Following Lai and Qiu (2003), we define the policy regime under TRIPS as $\mu_{\text{TRIPS}} = \mu_{\text{TRIPS}} = \mu_{\text{NE}}$. In summary, we find that the North experiences higher growth, higher welfare and higher income inequality. As for the South, it experiences higher growth, lower welfare and higher income inequality. Under TRIPS, the South’s level of patent protection increases from $\mu_{\text{NE}}$ to $\mu_{\text{TRIPS}}$. This higher level of patent protection increases economic growth in both countries (i.e., $g_{\text{TRIPS}}^{n} > g_{\text{NE}}^{n}$ and $g_{\text{TRIPS}}^{s} > g_{\text{NE}}^{s}$). However, the higher growth also raises inequality in both countries (i.e., $\sigma_{y,\text{TRIPS}}^{n} > \sigma_{y,\text{NE}}^{n}$ and $\sigma_{y,\text{TRIPS}}^{s} > \sigma_{y,\text{NE}}^{s}$). As for welfare, (33) shows that the higher growth in the North unambiguously increases its welfare (i.e., $\Omega_{\text{TRIPS}}^{n} > \Omega_{\text{NE}}^{n}$). As for the South, the increase in $\mu^{s}$ causes opposing effects on its welfare. One is the positive growth effect. The other is the negative welfare effect of markup pricing that reduces consumption. However, from the definition of the Nash equilibrium, a unilateral deviation from the best response must render the South worse off (i.e., $\Omega_{\text{TRIPS}}^{s} < \Omega_{\text{NE}}^{s}$). Proposition 4 summarizes these findings.

**Proposition 4:** In the North, the effects of TRIPS on growth, welfare and income inequality are (a) $g_{\text{TRIPS}}^{n} > g_{\text{NE}}^{n}$, (b) $\Omega_{\text{TRIPS}}^{n} > \Omega_{\text{NE}}^{n}$, and (c) $\sigma_{y,\text{TRIPS}}^{n} > \sigma_{y,\text{NE}}^{n}$. In the South, the effects of TRIPS on growth, welfare and income inequality are (a) $g_{\text{TRIPS}}^{s} > g_{\text{NE}}^{s}$, (b) $\Omega_{\text{TRIPS}}^{s} < \Omega_{\text{NE}}^{s}$, and (c) $\sigma_{y,\text{TRIPS}}^{s} > \sigma_{y,\text{NE}}^{s}$.

**Proof:** Proven in the text. ■
The above welfare implication is perhaps not surprising given the definition of Nash equilibrium. Therefore, the intriguing question is whether global welfare increases or decreases as a result of TRIPS, and we compare the level of global welfare between the Nash equilibrium and the policy regime under TRIPS. We find that there exists a critical degree of cross-country spillovers captured by \( \alpha \) below (above) which global welfare is lower (higher) under TRIPS. Proposition 5 summarizes this result, and Figure 1 plots \( \Delta \Omega \equiv (\Omega_{TRIPS}^n + \Omega_{TRIPS}^s) - (\Omega_{NE}^n + \Omega_{NE}^s) \) against \( \alpha \).

**Proposition 5:** There exists a cutoff value \( \bar{\alpha} \in (0,0.5) \) such that (a) \( \Omega_{TRIPS}^n + \Omega_{TRIPS}^s < \Omega_{NE}^n + \Omega_{NE}^s \) if \( \alpha \in (0,\bar{\alpha}) \), and (b) \( \Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s \) if \( \alpha \in (\bar{\alpha},0.5) \).

**Proof:** See Appendix A. ■

![Figure 1: Changes in global welfare under TRIPS](image)

In Figure 1, we see that as \( \alpha \) approaches zero, \( \Delta \Omega < 0 \) because the two countries are almost in autarky and the South’s optimal patent protection is weaker than that of the North. Forcing the South to adopt the North’s level of patent protection causes the South to experience a welfare loss while the welfare gain for the North is negligible. As \( \alpha \) rises above 0, \( \Delta \Omega \) increases in \( \alpha \) because the positive externality in the Nash equilibrium reduces the welfare loss in the South and increases the welfare gain in the North. As \( \alpha \to 0.5 \), \( \Delta \Omega \) becomes zero because the Nash equilibrium is the same as the policy regime under TRIPS, such that \( \mu_{NE}^s = \mu_{TRIPS}^s \). When \( \alpha \) is slightly below 0.5, \( \Delta \Omega \) becomes positive because \( \mu_{NE}^s < \mu_{TRIPS}^s < \mu_{GO}^s \) implying that the South’s level of patent protection under TRIPS is moving towards
the globally optimal level. For intermediate values of $\alpha$, there exists a critical degree $\overline{\alpha}$ below (above) which global welfare under TRIPS is lower (higher) than in the Nash equilibrium. In other words, there must be a sufficient degree of global integration in order for a harmonization of IPR protection to improve global welfare.

5. Conclusion

This paper analyzes the effects of TRIPS on growth, welfare and income inequality simultaneously. In summary, strengthening patent protection in developing countries as a result of TRIPS increases global economic growth but also worsens global income inequality. Whether it increases global welfare depends on the degree of cross-country spillovers. To derive these results, we incorporate wealth heterogeneity among households into an open-economy quality-ladder model. Our model belongs to the class of first-generation R&D-based growth models that exhibit scale effects (i.e., a larger economy experiences faster growth). We eliminate scale effects by normalizing each country’s population size to unity.\(^{25}\)

In our model, we have abstracted from some interesting issues, such as licensing, foreign direct investment, and North-South product cycles. In reality, both of (a) technology transfer from the North to the South and (b) providing sufficient incentives for the South to innovate are important reasons for strengthening IPR in the South. For analytical tractability and the relative lack of attention to the latter issue in the literature, we follow Lai and Qiu (2003) and Grossman and Lai (2004) to focus on (b) only. Therefore, one direction for future research is to account for these issues in a model with heterogeneous households. Furthermore, given that the enforcement of IPR is as important as the statutory law in reality, it would be interesting for future studies to consider IPR enforcement as well.

Although our model is designed to analyze the positive externality associated with IPR protection in developed and developing countries, the two countries in the model can easily be relabeled as two

\(^{25}\) The literature has two other ways of dealing with scale effects (a) the semi-endogenous growth model and (b) the second-generation model that combines quality improvement and variety expansion. Our model’s implication that devoting a larger share of labor to R&D would increase growth is consistent with the second-generation models. See, for example, Jones (1999) for a discussion on scale effects in R&D-based growth models.
developed countries by assuming that they have similar levels of R&D productivity. In this case, the Nash equilibrium level of patent protection continues to be lower than the globally optimal level as long as \( \alpha \) is greater than zero. In other words, a coordination failure of patent policy can exist even among developed countries suggesting the importance of also evaluating whether the level of IPR protection chosen by developed countries is indeed optimal from the perspective of global welfare.

Finally, in our model, income inequality is generated by an unequal distribution of (financial) capital income, and patent policy affects income inequality through the rate of return on assets. Therefore, even if inventions do not represent a significant share of assets in reality,\(^{26}\) the effect of patent policy on income inequality can still be significant in the presence of other capital incomes that depend on the real interest rate. Although the prevailing wisdom is that income inequality is mainly caused by an increase in the skill premium (i.e., the relative wage between skilled and unskilled workers), some studies, such as Atkinson (2000, 2003), argue that inequality in capital income is also playing an increasingly important role. For example, Reed and Cancian (2001) show that capital income contributes to one quarter of the increase in income inequality in the US in the 90’s while it accounts for less than one-tenth of the increase in the 70’s. Therefore, the current study also serves the purpose of providing an open-economy R&D-based growth model that highlights the increasing importance of capital income on income inequality.

\(^{26}\) Nakamura (2003) calculates that the market value of intangible assets in the US is at least $5 trillion in 2000 (i.e., about 50% of US GDP). Although intangible assets include patents and copyrights that are innovation-related, they also include trademarks and goodwill that may be unrelated to innovation.
References


Appendix A

**Proof of Lemma 1:** In this proof, we first show that aggregate expenditure on consumption \( E_t^n \equiv P_t^n C_t^n \) in country \( n \) always jumps immediately to a unique and stable steady-state value. Then, we show that this steady-state value determines a unique and stationary equilibrium allocation of labor in country \( n \).

Choosing labor as the numeraire in country \( n \) (i.e., \( W_t^n = 1 \) for all \( t \)) implies that \( V_t^n \varphi^n = 1 \) for all \( t \) from (15). Given that \( \varphi^n \) is constant, \( \dot{V}_t^n = 0 \). Integrating (2) over \( h \in [0,1] \) and then setting \( \dot{V}_t^n \) to zero yield

\[
(A1) \quad E_t^n = W_t^n + R_t^n V_t^n = 1 + R_t^n / \varphi^n .
\]

Using its definition, the law of motion for aggregate expenditure on consumption is given by

\[
(A2) \quad \frac{\dot{E}_t^n}{E_t^n} = \frac{\dot{P}_t^n}{P_t^n} + \frac{\dot{C}_t^n}{C_t^n} = R_t^n - \rho
\]

from (3) because \( \dot{C}_t^n / C_t^n = \dot{C}_t^n(h) / C_t^n(h) \) for all \( h \in [0,1] \). Substituting (A1) into (A2) yields

\[
(A3) \quad \frac{\dot{E}_t^n}{E_t^n} = \varphi^n (E_t^n - 1) - \rho ,
\]

which is plotted in Figure 2.

![Figure 2 Phase Diagram](image)

For any initial value of \( E_t^n \) below \( 1 + \rho / \varphi^n \), \( E_t^n \) eventually converges to zero violating the households’ utility maximization. For any initial value of \( E_t^n \) above \( 1 + \rho / \varphi^n \), \( E_t^n \) eventually increases to a point in which all the workers are allocated to production. A zero allocation of R&D workers violates the R&D
entrepreneurs’ profit maximization. Therefore, to be consistent with long-run stability, \( E^n_t \) must always jump to its unique non-zero steady state given by

\[
E^n_t = 1 + \rho / \varphi^n .
\]

From (A2), \( \dot{E}_t^n = 0 \) implies that \( R^n_t = \rho \) for all \( t \).

Next, we derive the equilibrium allocation of labor. The price index for \( C_i^n = (C_i^n)^{1-\alpha} (C_i^n)^{\alpha} \) is \( P_t^n \equiv (P_t^n)^{1-\alpha} (P_t^n)^{\alpha} \). The price index for \( C_i^{n,s} \) is \( P_t^{n,s} \equiv \exp \left( \int_0^1 \ln \left( Z_i^n \right) \right) = \frac{\mu^n W_t^n}{Z_t^n} \), where

\[
Z_t^n \equiv \exp \left( \int_0^1 N_t^n(i) \ln z \right). \]

Similarly, the price index for \( C_i^{s,s} \) is \( P_t^{s,s} = \frac{\mu^n W_t^n}{(1-\tau)Z_t^n} \). From (5), (7) and (9), the aggregate production function for \( C_i^n = Z_t^n L_i^{n};n \). Similarly, from (6), (8) and (9), the aggregate production function for \( C_i^{s,s} = (1-\tau)Z_t^n L_i^{s,s} \). For country \( n \), the value of export is \( P_t^{s,n} C_i^{s,n} \) while the value of import is \( P_t^{n,n} C_i^{n,n} \). The balanced-trade condition is

\[
P_t^{s,n} C_i^{s,n} = P_t^{n,n} C_i^{n,n} \Leftrightarrow L_{s,i}^{n,n} = \left( \frac{\mu^n}{\mu^s \omega_t^n} \right) L_{s,i}^{s,n} ,
\]

where \( \omega_t^n \equiv W_t^n / W_t^s \) denotes the relative wage rate. The conditional demand functions in country \( n \) for domestic and foreign final goods are \( P_t^{n,n} C_i^{n,n} = (1-\alpha)P_t^{n,n} C_i^{n,n} \) and \( P_t^{n,n} C_i^{n,n} = \alpha P_t^{n,n} C_i^{n,n} \). Combining these two conditions yields

\[
P_t^{n,n} C_i^{n,n} = \frac{P_t^{n,n} C_i^{n,n}}{1-\alpha} = \frac{P_t^{n,n} C_i^{n,n}}{\alpha} \Leftrightarrow L_{s,i}^{n,n} = \left( \frac{\alpha}{1-\alpha} \right) \omega_t^n L_{s,i}^{s,n} .
\]

Substituting (A6) into (A5) yields

\[
L_{s,i}^{n,n} = \left( \frac{\mu^n}{\mu^s} \right) \left( \frac{\alpha}{1-\alpha} \right) L_{s,i}^{s,n} .
\]
Substituting $E^n_t = P^n_t C^n_t / (1 - \alpha) = \mu^n \Lambda^n_t / (1 - \alpha)$ into (A4) yields (16). Then, substituting (16) into (A7) yields (17). Finally, substituting (16) and (17) into the labor-market clearing condition yields (18). A similar exercise yields the unique, stable and stationary equilibrium allocation of labor in country $s$. ■

**Proof of Lemma 2:** The arrival rate of innovation in country $n$ is

(A8) \[ \lambda^n_t = \phi^n L^n_t. \]

The growth rate of $Z^n_t = \exp \left( \int_0^1 N^n_i (i) d \ln z \right) = \exp \left( \int_0^1 \lambda^n_t d \tau \ln z \right)$ is given by

(A9) \[ \frac{\dot{Z}^n_t}{Z^n_t} = \lambda^n_t \ln z. \]

The balanced-growth rate of consumption in country $n$ is

(A10) \[ \frac{\dot{C}^n_t}{C^n_t} = (1 - \alpha) \frac{\dot{Z}^n_t}{Z^n_t} + \alpha \frac{\dot{Z}^s_t}{Z^s_t} = [(1 - \alpha) \lambda^n_t + \alpha \lambda^s_t] \ln z. \]

Finally, aggregating (2) over $h \in [0,1]$ yields the level of consumption in country $n$ given by

(A11) \[ C^n_t = \frac{W^n_t + R^n_t V^n_t}{P^n_t} = \left( 1 + \frac{P_t}{\phi^n} \right) \frac{W^n_t}{P^n_t} \]

because $\dot{V}^n_t = 0$, $R^n_t = \rho$ and $V^n_t \phi^n = W^n_t$. ■

**Proof of Lemma 4:** The price index for consumption at time 0 is $P^n_0 \equiv (P^n_0, \ldots, (P^n_s)^\alpha, \ldots)$, where $P^n_0 = \mu^n W^n_0 / Z^n_0$ and $P^n_s = \mu^n W^s_0 / [(1 - \tau) Z^s_0]$ from the proof of Lemma 1. The initial levels of technology $Z^n_0 \equiv \exp \left( \int_0^1 N^n_i (i) d \ln z \right)$ and $Z^s_0 \equiv \exp \left( \int_0^1 N^s_i (i) d \ln z \right)$ are exogenous. After dropping these exogenous terms, $\ln (W^n_0 / P^n_0)$ simplifies to (31). ■
Proof of Proposition 2: After dropping the terms that are independent of patent protection, the welfare of any household $h$ in country $n$ is

\[(A12)\quad \Omega^n = -\ln \mu^n + \frac{g^n}{\rho} .\]

The arrival rates of inventions in the two countries are

\[(A13)\quad \lambda^n = \varphi^n - \left(1 - \frac{\alpha}{\mu^n} + \frac{\alpha}{\mu^s}\right)(\varphi^n + \rho) .\]

\[(A14)\quad \lambda^s = \varphi^s - \left(1 - \frac{\alpha}{\mu^s} + \frac{\alpha}{\mu^n}\right)(\varphi^s + \rho) .\]

Substituting (A13) and (A14) into (A10) yields

\[(A15)\quad g^n = \left(1 - \alpha\right)\left(\varphi^n - \left(1 - \frac{\alpha}{\mu^n} + \frac{\alpha}{\mu^s}\right)(\varphi^n + \rho) + \alpha\left(\varphi^s - \left(1 - \frac{\alpha}{\mu^s} + \frac{\alpha}{\mu^n}\right)(\varphi^s + \rho)\right)\right)\ln z .\]

Substituting (A15) into (A12) and then dropping the exogenous terms yield

\[(A16)\quad \Omega^n = -\ln \mu^n - \left(1 - \alpha\right)\left(1 - \frac{\alpha}{\mu^n} + \frac{\alpha}{\mu^s}\right)(\varphi^n + \rho) + \alpha\left(1 - \frac{\alpha}{\mu^s} + \frac{\alpha}{\mu^n}\right)(\varphi^s + \rho)\frac{\ln z}{\rho} .\]

Differentiating (A16) with respect to $\mu^n$ yields

\[(A17)\quad \frac{\partial \Omega^n}{\partial \mu^n} = -\frac{1}{\mu^n} + \left(1 - \alpha\right)\left(\frac{1}{\mu^n} - \left(\frac{1}{\mu^n} + \frac{\alpha}{\mu^s}\right)^2(\varphi^n + \rho) + \left(\frac{\alpha}{\mu^n}\right)^2(\varphi^s + \rho)\right)\ln z = 0 .\]

Solving (A17) yields (34), and (35) can be obtained by a similar derivation.

Proof of Proposition 3: Combining (A16) and the analogous condition for country $s$ yields

\[(A18)\quad \Omega^n + \Omega^s = -\ln \mu^n - \ln \mu^s - \left(1 - \frac{\alpha}{\mu^n} + \frac{\alpha}{\mu^s}\right)(\varphi^n + \rho) + \left(1 - \frac{\alpha}{\mu^s} + \frac{\alpha}{\mu^n}\right)(\varphi^s + \rho)\frac{\ln z}{\rho} .\]

Differentiating (A18) with respect to $\mu^n$ yields
\[
\frac{\partial (\Omega^p + \Omega^s)}{\partial \mu^n} = - \frac{1}{\mu^n} + \left( \frac{1 - \alpha}{(\mu^s)^2} \right) (\varphi^n + \rho) + \left( \frac{\alpha}{(\mu^s)^2} \right) (\varphi^s + \rho) \ln \frac{z}{\rho} = 0.
\]

Solving (A19) yields (36), and (37) can be obtained by a similar derivation. ■

**Proof of Corollary 2:** Subtracting (34) from (36) and differentiating \( \mu_{GO}^n - \mu_{NE}^n \) with respect to \( \alpha \) show that the sign of \( \partial (\mu_{GO}^n - \mu_{NE}^n) / \partial \alpha \) is given by the sign of \( (1 - 2\alpha) > 0 \) for \( \alpha < 0.5 \). Similarly, from (35) and (37), differentiating \( \mu_{GO}^i - \mu_{NE}^i \) with \( \alpha \) shows that the sign of \( \partial (\mu_{GO}^i - \mu_{NE}^i) / \partial \alpha \) is also given by \( 1 - 2\alpha \). ■

**Proof of Proposition 5:** As \( \alpha \to 0 \), \( \Omega_{TRIPS}^n + \Omega_{TRIPS}^s < \Omega_{NE}^n + \Omega_{NE}^s \) because the countries are almost in autarky so that \( \mu_{GO}^i < \mu_{TRIPS}^i \). As \( \alpha \to 0.5 \), \( \Omega_{TRIPS}^n + \Omega_{TRIPS}^s = \Omega_{NE}^n + \Omega_{NE}^s \) because the Nash equilibrium is the same as the policy regime under TRIPS such that \( \mu_{NE}^i = \mu_{TRIPS}^i \). The rest of the proof shows that there exists an intermediate range of \( \alpha \) for which \( \Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s \). From (34) and (37), \( \mu_{GO}^i - \mu_{TRIPS}^i \) is an increasing function in \( \alpha \). As \( \alpha \to 0.5 \), \( \mu_{GO}^i > \mu_{TRIPS}^i \). Therefore, there must exist a threshold denoted by \( \tilde{\alpha} \in (0,0.5) \) above which \( \mu_{NE}^i < \mu_{TRIPS}^i < \mu_{GO}^i \). When \( \alpha \in [\tilde{\alpha},0.5) \), it is sufficient for \( \Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s \) to hold, and there exists a lower critical value \( \bar{\alpha} \in (0,\tilde{\alpha}) \) above which \( \Omega_{TRIPS}^n + \Omega_{TRIPS}^s > \Omega_{NE}^n + \Omega_{NE}^s \) still holds. In this case, the South’s level of patent protection moves from one suboptimal level to another suboptimal level (i.e., \( \mu_{NE}^i < \mu_{GO}^i < \mu_{TRIPS}^i \)). In summary, when \( \alpha < \bar{\alpha} \), \( \Omega_{TRIPS}^n + \Omega_{TRIPS}^s < \Omega_{NE}^n + \Omega_{NE}^s \). As \( \alpha \) increases above \( \bar{\alpha} \), the opposite is true. ■