Studying the Properties of the Correlation Trades

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Abstract

This thesis tries to explore the profitability of the dispersion trading strategies. We begin examining the different methods proposed to price variance swaps. We have developed a model that explains why the dispersion trading arises and what the main drivers are. After a description of our model, we implement a dispersion trading in the EuroStoxx 50. We analyze the profile of a systematic short strategy of a variance swap on this index while being long the constituents. We show that there is sense in selling correlation on short-term. We also discuss the timing of the strategy and future developments and improvements.

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"He who love touches, walks not in darkness." Plato (427-347BC).

Contents

1 Introduction

Traditionally, investors have tried to maximize returns by taking directional positions[1] of equities, fixed income securities and currencies based on their vision on future return. However the rapid development of derivatives markets brings about new investment opportunities. With the help of these derivatives investors can obtain exposure to Gamma or Vega of underlying security. So instead of (or in addition to) just taking directional positions based on predictions of underlying prices, investor with foresight of volatilities might also add value to their portfolio by engaging in option positions in a delta neutral way. Under this view, the volatility itself, as the correlation, have become in tradable assets in the financial markets.

There are several means of getting exposure to volatility. One way is trading options. However we are interested in volatility (or correlation as we will discuss below) exposure rather than stock price exposure. It is well-known that trading stock options produces exposure to both the volatility movement but also to the price. Different authors has showed that the price of the stock in equities does not follow a lognormal distribution and that the volatility of the stock is not constant. Therefore problems with hedging emerge if we use Black-Scholes market assumptions (volatility is constant, no transaction costs, no jumps in the asset dynamics, no liquidity problems, the stocks can be traded continuously and we have an infinite number of different strikes). Moreover, the high transaction cost due to dynamic hedging is the major shortcoming for such a strategy. New financial products more adapted to the preferences of the market participants have emerged in the last years in order to set up successful strategies to gain pure exposure to volatility: Variance Swap, Variance (Volatility) Forward (Future), Forward Variance Swap and Option on volatility. Among them Variance (Volatility) Swaps are the most heavily traded kind and hence most interesting to study[2].

Variance swaps are forward contracts on the future realized variance. Its payoff is the difference between the subsequent realized variance and the fixed variance swap rate, which can be implied from option prices as the market's risk neutral expectation on future variance. Therefore, by logging in these contracts, investors do not bet on future return, they will benefit if they have a good view or intuition on the difference between the future variance and

[1] A directional position try to take into account the view of the investor on the underlying asset. For example, if the investor thinks the future price of the stock will increase he will buy the stock today. If his view is that the price will decrease, he will sell the stock today. A more sophisticated strategy should be use options with the stock as underlying asset.

[2] Mainly because the pricing and hedging can be done with calls and puts options.

the current variance swap rate.

Some authors have discussed that the spread between the index implied volatility and the realized volatility have been in average positive and it has exceeded the subsequent realized spread for single stocks. Carr and Wu[3] showed that the variance risk premium is much more significant for equity indices than for individual stocks suggesting that correlation risk could be the reason. As a result of this higher variance risk premium for equity indices, it is tempted to systematically short variance swap on equity indices rather than on individual stocks. The way to implement this view is known as dispersion trading strategy (correlation trading or volatility trading are also common names in the market for this type of strategy). The name of the strategy (dispersion trading) emerges as an attempt to capture the essence of the trading: how the constituents of a basket disperse themselves around a level that is set by the investor.

In this paper we implement the dispersion strategy and we show that the systematically short of correlation has positive average returns. However, the distribution function exhibits highly negative skew of return due to large losses happening during crashes environments[4]. Moreover, we show that the weights of the elements in the portfolio created will be crucial in order to get higher returns with the strategy implemented. We also conclude that dispersion trading implemented with variance swaps has sense as a position risk management tool rather than a profit generator in lower volatility environments.

This paper is organized in four parts: Part I is a survey of the volatility models, dispersion trading basis as well as a simple model to explain the drivers and the intuition behind the trading. Part II reviews the different methods proposed to trade volatility as well the pricing and replication strategies for the variance swaps. Part III shows the implementation of a dispersion trading. We analyze the profile of a systematic short strategy of a variance swap on the EuroStoxx 50 being long on the constituents of this index. Part IV contains the conclusions and future improvements and/or advances.

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[3] "A Tale of Two Indices", P Carr, L Wu, February 23, 2004.

[4] This is consistent with the empirical fact that the volatility is negatively correlated with the underlying price process.

Part I
Volatility Models

The aim of this section is to provide a review of the different models proposed for the volatility. We will derive a simple model in order to understand how the correlation arises as the main driver of the profit and loss in the dispersion trading strategies. The volatility of a stock is the simplest measure of its risk or uncertainty. Formally, the volatility σ^R is the annualized standard deviation of the stock's returns during the period of interest[5]. In this paper we will compute the realized volatility as the annualized volatility of the daily underlying returns on the underlying over the contract period assuming zero mean for the returns:

$$\sigma^R = \sqrt{\frac{\sum_i \log(\frac{S_{i+1}}{S_i})^2}{n-1} \times 252} \tag{1}$$

Here S_i is the price of the underlying in day i and the factor 252 is set in order to get the annualized volatility.

The Black-Scholes model assumes that the volatility is constant. This assumption is not always satisfied by real-life options as the probability distribution of the equity has a fatter left tail and a thinner right tail than the lognormal distribution is incompatible with derivatives prices observed in the market. Data show that the volatility is non-constant[6] and can be regarded as an endogenous factor in the sense that it is defined in terms of the past behavior of the stock price. This is done in such a way that the price and volatility form a multidimensional Markov process[7].

Different authors have presented different approaches to explain the skew observed in the market[8]. Merton extended the concept of volatility as a deterministic function of time. This approach is followed by Wilmott (1995). This is a deterministic volatility model and the special case where the volatility

[5] We can distinguish amongst three kinds of values: realized (σ^R), implied (σ^I), and theoretical (σ) volatility. Realized values can be calculated on the basis of historical market data in a specified temporal horizon. Implied values are values implied by the option prices observed on the current day in the market. Theoretical values are values calculated on the basis of some theory.

[6] This effect is known as skew in equities and smile in currencies.

[7] A random process whose future probabilities are determined by its most recent values. A formal definition given by Papoulis (1984, p. 535) is: A stochastic process is called Markov if for every n and $t_1<t_2<\ldots<t_n$ we have that: $p(x(t_n) \leq x_n \forall t \leq t_{n-1}) = p(x(t_n) \leq x_n \mid x(t_{n-1}))$

[8] See Frey (1997) for an excellent survey of stochastic volatility models.

is a constant reduces to the well-known Black-Scholes model, that suggests changes in stock prices are lognormal distributed. But the empirical test by Bollerslev (1986) seems to indicate otherwise. Hull (2000) assumes that the volatility is not only a function of time but also a function of the stock price at time t; where the stock price is driven by an Itô diffusion equation with a Brownian motion. Buff and Heston developed a model in which the time variation of the volatility involves an additional source of randomness besides $\{W_1(t)\}$, represented by $\{W_2(t)\}$:

$$dS(t) = \mu S(t)dt + \sigma(t, S(t))S(t)dW_1(t) \tag{2}$$

$$d\sigma(t) = \alpha(t, \sigma(t))dt + b(t, \sigma(t))dW_2(t) \tag{3}$$

Here both Brownian motions can be correlated processes. Elliot and Swishchuk (2002) consider that the volatility depends on a random parameter x such as $\sigma(t) = \sigma(x(t))$.The value x(t) is is some random process. Buff (2002) develops another approach based on Avellaneda (1995) called uncertainty volatility scenario. In Buff's model a concrete volatility surface is selected among a candidate set of volatility surfaces. This approach addresses the sensitivity question by computing an upper bound for the value of the portfolio under arbitrary candidate volatility, and this is achieved by choosing the local volatility $\sigma(t, S(t))$ among two extreme values $\sigma_{\max}$ and $\sigma_{\min}$. such that the value of the portfolio is maximized locally. Wu (2002) proposed a new approach in which the volatility depends on a stochastic volatility delay $\sigma(t, S(t+\theta))$ where the delay is given by $\theta \in [-\tau, 0]$. The advantage of this model is that replicates better the behaviour of the historical volatility.

The last three approaches or stochastic volatility models introduce a second "non-tradable" source of randomness. These models usually obtain a stochastic volatility model, which is general enough to include the deterministic model as a special case. The concept of stochastic volatility was introduced by Hull and White (1987), and subsequent development includes the work of Wiggins (1987), Johnson and Shanno (1987), Scott (1987), Stein and Stein (1991) and Heston (1993). However, the main drawback of the stochastic volatility models is that appears preferences of the investors in the specification of the models and the calibration is quite more complicated and subjective.

2 The Heston model

The Heston model is one of the most widely used stochastic volatility models today. Its attractiveness lies in the powerful duality of its tractability and robustness relative to other stochastic volatility models. Therefore we will discuss this one in great detail. In the Black-Scholes-Merton model, a derivative is dependent on one or more tradable assets. The randomness in the option value is solely due to the randomness of these underlying assets. Since the assets are tradable, the option can be hedged by continuously trading the underlying. This makes the market complete, i.e.,every derivative can be replicated. In a stochastic volatility model, a derivative is dependent on the randomness of the asset and the randomness associated with the volatility of the asset's return. Heston modelled the asset price and the volatility by use of the following stochastic differential equation:

$$
\begin{aligned}
dS(t) &= \mu S(t)dt + \sqrt{\gamma} S(t) dW_1(t) && (4)\\
d\gamma &= \eta(\theta - \gamma)dt + \sigma\sqrt{\gamma} dW_2(t) && (5)\\
dW_1 dW_2 &= \rho dt && (6)
\end{aligned}
$$

μ is the drift, γ is the volatility that is modelled with a mean reversion process, η is the speed of reversion to the level θ and σ is the volatility of the volatility mean reversion process. ρ is the correlation coefficient between both the stock and the volatility uncertainty source.

There are many economic, empirical, and mathematical reasons for choosing a model with such a form. Empirical studies have shown that an asset's log-return distribution is non-Gaussian. It is characterized by heavy tails and high peaks (leptokurtic). There is also empirical evidence and economic arguments that suggest that equity returns and implied volatility are negatively correlated (also termed 'the leverage effect')[9]. This departure from normality plagues the Black-Scholes-Merton model with many problems. In contrast, Heston's model can imply a number of different distributions. ρ which can be interpreted as the correlation between the log-returns and volatility of the asset, affects the heaviness of the tails. Intuitively, if $\rho > 0$ then volatility will increase as the asset price/return increases. This will spread the right tail and squeeze the left tail of the distribution creating a fat right-tailed distribution. Conversely, if $\rho < 0$ then volatility will increase when the asset price decreases, thus spreading the left tail and squeezing the right tail of the distribution creating a fat left-tailed distribution (emphasizing the fact that equity returns and its related volatility are negatively correlated).

The effect of changing the skewness of the distribution has also an impact on the shape of the implied volatility surface. This fact can be captured by the introduction of the correlation parameter ρ. The model can incorporate a variety of volatility surfaces and hence addresses another shortcoming of

[9] See Cont 2001 for a detailed statistical and empirical analysis.

the Black-Scholes-Merton model, that is, constant volatility across differing strike levels.

σ affects the kurtosis (peak) of the distribution. When σ is zero the volatility is deterministic and hence the log-returns will be normally distributed. Increasing σ will then increase the kurtosis only, creating heavy tails on both sides. Again, the effect of changing the kurtosis of the distribution impacts on the implied volatility. Higher σ makes the skew/smile more prominent. This makes sense relative to the leverage effect. Higher σ means that the volatility is more volatile. This means that the market has a greater chance of extreme movements. So, writers of puts must charge more and those of calls, less, for a given strike.

η is the mean reversion parameter. It can be interpreted as the degree of "volatility clustering". This is something that has been observed in the market, for example, large price variations are more likely to be followed by large price variations[10]. The aforementioned features of this model enable it to produce a big amount of distributions. This makes the model very robust and hence addresses the shortcomings of the Black-Scholes-Merton model. It also provides a framework to price a variety of options incorporating dynamics closer to real ones observed in the market.

3 Why Dispersion Trading?

We will assume a filtered probabilistic space $(\Omega, F, \{F_t\}_{t\in[0,T]}, P)$ where Ω is the sample space, F is a sigma-algebra, $\{F_t\}_{t\in[0,T]}$ is the filtration associates to the sigma-algebra and P is the probability measure.

An index measures the price performance of a portfolio of selected stocks. It allows us to consider an index as a portfolio of stock components. Assume that we have an index composed by two stocks: S_1 and S_2 in proportions w_1 and w_2, where $w_1 + w_2 = 1$. We will model both stock prices with a diffusion process $\{S_i\}_{i=1,2}$. The index is traded as an asset in our market and also follows a diffusion process $\{S_{index}\}$. We want to replicate the index creating a basket B_k with the two assets S_1 and S_2 and in the same proportions w_1 and w_2. If the weights to create the replicated portfolio are the same as the weights in the index replicated, then the σ_{index} should be equal to the σ_{basket}. The replication strategy is just given by finding two processes $\{h(t)\}$ and $\{g(t)\}$ that replicate the payoff function considered:

$$H_T = H_0 + \int_{[t,T]} h(t)dS_t + \int_{[t,T]} g(t)dP_t \tag{7}$$

[10] See Cont (2001) for an excellent study.

The dynamics of the element considered are given by:

$$
\begin{aligned}
dS_i &= \mu_i S_i dt + \sigma_i S_i dW_i, \ i = 1, 2 &(8)\\
dS_{index} &= \mu_{index} S_{index} dt + \sigma_{index} S_{index} dW_{index} &(9)\\
dW_{index} &= \lambda dW_1 + (1-\lambda) dW_2, \qquad \lambda \in [0,1] &(10)\\
S_{index} &= w_1 S_1 + w_2 S_2 &(11)
\end{aligned}
$$

The dynamics of the replicated basket is given by applying Itô's lemma on $(w_1 S_1 + w_2 S_2)$:

$$d(w_1 S_1 + w_2 S_2) = dB_k \tag{12}$$

We will assume that the sources of uncertainty present in both asset prices processes are correlated with parameter ρ. The Brownian motion presents in the index dynamics process will be a linear combination of both uncertainty sources $\{W_1\}$ and $\{W_2\}$. We can review both processes as a function of two independent Brownian motions, that is, $dW_1 = \rho dZ_2 + \sqrt{1-\rho^2} dZ_1$ and $dW_2 = dZ_2$, where $\{Z_1\}$ and $\{Z_2\}$ are uncorrelated Brownian motions. Moreover, the same decomposition can be applied to:

$$dW_{index} = \lambda(\rho dZ_2 + \sqrt{1-\rho^2} dZ_1) + (1-\lambda) dZ_2 \tag{13}$$

Then the dynamics of the basket and the index are given by:

$$
\begin{aligned}
dS_{index} &= S_{index}[\mu_{index} dt + \sigma_{index} dM] &(14)\\
dM &\sim N[0, (\lambda\rho + (1-\lambda))^2 t + \lambda^2(1-\rho^2)t] &(15)\\
dB_k &= B_k[(\mu_1 w_1 S_1 + \mu_2 w_2 S_2) dt + d\hat{A}] &(16)\\
d\hat{A} &\sim N[0, (w_1\sigma_1 S_1 \rho + w_2 \sigma_2 S_2)^2 t + (w_1 S_1 \sigma_1 \sqrt{1-\rho^2})^2 t] &(17)
\end{aligned}
$$

$\{\hat{A}\}$ is a random process with mean zero and the variance of the replicated portfolio. Because the basket is created synthetically and the index is traded itself as an asset, there exists a new component in the volatility of the replicated portfolio: the correlation coefficient (or more generally, the correlation matrix). That is, a prior, the σ_{index} should be equal to the σ_{basket} if we have chosen the components of the basket in the same proportion as in the index; but because the new synthetically created portfolio takes into account the correlation coefficient between assets, the volatility of the basket can differ of the volatility of the index. This is the essence of the dispersion trading and the main driver of the profit and loss in the strategy. This is the reason why dispersion trading is sometimes also known as correlation trading. The profitability of this broadly depends on the realized correlation versus the correlation implied by the original set of prices. Moreover, it shows that the dispersion trading itself is not an arbitrage opportunity. It is straightforward to understand that depending on the values of the correlation coefficient (or matrix in a multidimensional case) the betting in the future evolution of the correlation will allow us to make or lose money. That is, there is risk when we take a position in the strategy. A second order driver is the volatility of the elements in the portfolio: how the volatility of the volatility changes on time[11].

This is consistent with the empirical evidence that shows that index options are more expensive than individual stock option because they can be used to hedge correlation risk[12]. The explanation is that the correlations between stock returns are time varying and will blow up when the market is suffering big losses. During market crisis a higher correlation between components would increase the volatility of the index, leading into higher index volatility than individual stock [13]. From an economic point of view, the premium in the index is the premium that the investors pay in excess in order to reduce the risk of their portfolios taking positions in the index.

Suppose that the correlation is 1, then the theoretical volatility of the index should be given by the volatility of the basket, that is, $\sigma_{index} = (w_1\sigma_1 S_1 + w_2\sigma_2 S_2)^2$. Because the correlation is bounded between $[-1, 1]$, this theoretical value should be the upper bounded of the maximum dispersion in the volatility. In the same way we can get a lower bound to the dispersion given by zero correlation. So the dispersion bounds are given by $0 \leq \sigma_{spread} \leq \sqrt{|\sigma^2_{index} - (\sum_i w_i S_i \sigma_i)^2|}$ and we can take positions in the the spread between implied volatility and realized volatility. Now the spread has become in a tradable asset.

Therefore we have shown that dispersion has two components: volatility and correlation. If we are short in correlation and real correlation goes to one we can still make money if volatilities are high enough. The effect of the two (correlation and volatility) are not really separable. If we implemented a short dispersion trade, we would not make money, unless we sold so much index gamma that our break-even correlation decreased so much, that we would be vulnerable to spikes in correlation.

An extension of the model with three assets is considered. Let us assume

[11] For a detailed explanation see "Realized Volatility and Correlation", Andersen 1999.

[12] "The relation between implied and realized volatility". Christensen, B.J., & Prabhala, N.R. Journal of Financial Economics(1998).

[13] We use the term volatility trading as well as dispersion trading or correlation trading in equivalent way, but the reader do not must to forget that in the end the correlation is the main driver of the dispersion strategy as we aforesaid.

that the index dynamics is described by the diffusion equation aforementioned, and the three assets also follow diffusion processes. The assets are correlated with (known) correlation matrix Σ instead of a number ρ . The uncertainty of the index is given by the linear combination of the assets' uncertainty sources as before. We replicate the index by buying the stocks in the same proportion of this. Note that now the correlation is a $3x3$ matrix. As a previous step, we want to rewrite the correlated Brownian motions as a function of independent Brownian motions. Then the decomposition is given by:

$$W = C^T Z \tag{18}$$

$$\begin{pmatrix} W_1(t) \\ W_2(t) \\ W_3(t) \end{pmatrix} = \begin{pmatrix} c_{11} & c_{21} & c_{31} \\ 0 & c_{22} & c_{32} \\ 0 & 0 & c_{33} \end{pmatrix} \begin{pmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) \end{pmatrix} \tag{19}$$

Here the vector W is composed by the correlated Brownian motions and Z is given by the independent Brownian motions. The matrix C^T is a direct result of the Choleski's decomposition of Σ. Note the correlation matrix is a symmetric and positive definite matrix, it can be efficiently decomposed into a lower and upper triangular matrix following Cholesky's decomposition, that is:

$$\Sigma = C^T.C \tag{20}$$

Then the task is to get the coefficients c_{ij} that will give the expression of the correlated Brownian motions as a function of independent Brownian motions. The coefficients are given by:

$$c_{ii} = \sqrt{\left(a_{ii} - \sum_{k=1}^{i-1} c_{ik}^2\right)} \tag{21}$$

$$c_{ji} = \frac{\left(a_{ji} - \sum_{k=1}^{i-1} c_{jk} c_{ik}\right)}{c_{ii}} \tag{22}$$

The uncertainty source of the index is given by the following linear relationship between the stocks' Brownian motions:

$$dW_{index} = \lambda dW_1 + \theta dW_2 + (1 - \lambda - \theta) dW_3, \qquad \lambda, \theta \in [0,1] \tag{23}$$

Then we can compute the dynamics of the portfolio replicated as:

$$d(w_1 S_1 + w_2 S_2 + w_3 S_3) = dB_k \tag{24}$$

And the dynamics is given by:

$$\begin{aligned} dB_k &= B_k[(\mu_1 w_1 S_1 + \mu_2 w_2 S_2 + \mu_3 w_3 S_3)dt + dG] && (25) \\ dG &\sim N[0, (w_1\sigma_1 S_1 c_{11})^2 t + (w_1\sigma_1 S_1 c_{21} + w_2\sigma_2 S_2 c_{22})^2 t && (26) \\ &\quad +(w_1\sigma_1 S_1 c_{31} + w_2\sigma_2 S_2 c_{32} + w_3\sigma_3 S_3 c_{33})^2 t] \end{aligned}$$

$\{G\}$ is a random process with mean zero and the variance of the replicated portfolio. The coefficients c_{ji} take into account the effect of the correlation between assets. They are expressed as:

$$c_{11} = \sqrt{\rho_{11}} = 1 \tag{27}$$

$$c_{21} = \frac{\rho_{21}}{\sqrt{\rho_{11}}} = \rho_{21} \tag{28}$$

$$c_{22} = \sqrt{\rho_{22} - \left(\frac{\rho_{21}}{\sqrt{\rho_{11}}}\right)^2} = \sqrt{1-\rho_{21}^2} \tag{29}$$

$$c_{31} = \frac{\rho_{31}}{\sqrt{\rho_{11}}} = \rho_{31} \tag{30}$$

$$c_{32} = \frac{\rho_{32} - \frac{\rho_{31}}{\sqrt{\rho_{11}}}\frac{\rho_{21}}{\sqrt{\rho_{11}}}}{\sqrt{\rho_{22} - \left(\frac{\rho_{21}}{\sqrt{\rho_{11}}}\right)^2}} \tag{31}$$

$$c_{33} = \sqrt{\rho_{33} - \left(\frac{\rho_{31}}{\sqrt{\rho_{11}}}\right)^2 - \left(\frac{\rho_{32} - \frac{\rho_{31}}{\sqrt{\rho_{11}}}\frac{\rho_{21}}{\sqrt{\rho_{11}}}}{\sqrt{\rho_{22} - \left(\frac{\rho_{21}}{\sqrt{\rho_{11}}}\right)^2}}\right)^2} \tag{32}$$

This procedure can be used to construct a model for N assets correlated with matrix Σ. The conclusions are, in essence, the same as with three assets. We observe that as the basket´s size increases, the dispersion effect can be higher (or lower). In fact, the dominant term in the volatility of the basket is the squared weighted sum of individual volatilities, where the weights are given by both the product of the correlation between stocks and the weight of the assets in the index. The main driver is the correlation and it is introduced in the basket's volatility by the choice of the weights. Another conclusion is that the bigger is the portfolio the bigger is the dispersion effect. Again a second order effect is the individual stocks' volatility. However, it can play an important role in the dispersion when the volatility spikes suddenly.

Part II
Trading volatility

As we have remarked before, volatility and correlation themselves, have becoming tradable assets the financial markets. There are two main approaches: hedging strategies versus econometric models. In the first approach we distinguish four type of strategies to trade volatility[14]: 1) sell index vanilla options and buy options on some components in the index; 2) sell index straddles or strangles[15] and buy straddles on the components in the index; 3) sell variance swaps on the index and buy variance swaps on the components, and 4) buy or sell correlation swaps[16]. By keeping a delta-neutral position, volatility trades attempt to eliminate the random effect of the underlying asset prices and try to get only exposure to the volatility (or correlation).

Positions in call and put options in order to trade volatility have the disadvantage not only that it is a very expensive trade to execute (delta hedging, roll the strikes...) but it is also difficult to keep track of the sensitivities (Greeks and second order Greeks). The same problem arise when we are dealing with straddles and strangles: if you want to gain exposure to the volatility, you will be long, and if you want to decrease your exposure, you will be short of at-the-money options. Again this trade is very expensive to execute and to keep track of the sensitivities (like calls and puts options,

[14] Other instruments as gamma swaps have appeared in a dispersion trade, because gamma swaps eliminate the natural short "spot cross-gamma" (when spot price goes down or up you end up with a volatility position) you get exposed to when using variance swaps.

The reason for this can be seen in some limiting cases, in which for example a stock drops massively and gets volatile. You need to have a vega position that matches the index weight, but that index weight has just dropped. So you can trade variance swaps to get neutral, but going in and out of variance swaps is not cheap. If you had used a gamma swap, the drop in the stock price effectively decreases the vega exposure, so you do not need to rebalance. However these type of contracts are not widely used.

[15] Investment strategies that involve being long or short in call and put options with same maturity and same strikes (straddles), or with same maturity and different strikes (strangles).

[16] A correlation swap is just a forward contract in the realized correlation. The correlation swap gives direct exposure to the average pairwise correlation of a basket of stocks agreed at entering in the contract. The main disadvantage of this strategy is that the market for these products is less liquid than other (variance and volatility swaps).

these contracts can take on significant price exposure once the underlying moves away from its initial level; an obvious solution to this problem is to delta-hedge with the underlying). The third strategy works but the problem is to choose the right amount of components and the size of the elements to set up the strategy. No delta hedging is neccesary if we weighted the components in the right amount as we discuss later. In fact, this will be the method implemented by us in the empirical study because it relies on the use of the widely traded variance swaps. The fourth strategy has pure correlation exposure, but there is not enough liquidity in this market so it is not very used.

4 Specific contracts for trading volatility

We start our review with instruments that trades correlation and after that, we focus our attention to specific volatility (variance) products because these are the most trade in the market.

Correlation, in the easiest way, can be trade with Correlation Swaps. A correlation swap is just a forward contract in the realized correlation. The correlation swap gives direct exposure to the average pair wise correlation of a basket of stocks agreed in the contract. The main disadvantage of this strategy is that the market for these products is less liquid than others (variance and volatility swaps). The other strategy for trading correlation is implemented with variance swaps: it profits directly from the returns and volatilities of the stocks in the index becoming more dispersed over time.

4.1 Variance swaps

Volatility swaps and variance swaps are relatively new derivative products since 1998. A variance swaps is an Over-The-Counter (OTC) financial derivative which payoff function is given by:

$$H_T = N \times (\sigma_R^2 - K_{var}^2) \tag{33}$$

Here σ_R^2 is the realized variance at expiry and K_{var}^2 is the delivery price agreed at inception. N is the notional amount in money units per unit of volatility. That is, a variance swap is a forward contract in the future variance.

Note that the payoff is convex in volatility. As a consequence, variance swaps strikes trade at a premium compared with a volatility swap. Our goal will be to calculate the fair price of the variance swap, that is, the delivery price K_{var}^2 that makes the contract have zero value. Different approaches have been discussed in the literature, from a replication strategy based in an option portfolio to the expected value of the payoff under the risk neutral measure. We focus our study in the replication strategy with a portfolio of options whose payoff is the same that the variance swap payoff. Other methods proposed will be discussed.

4.1.1 Pricing variance Swaps: Replication Strategy

Following Demeterfi (1999)[17], it can be shown that if one has a portfolio of European option of all strikes, weighted in inverse proportion to the squared of the strike, you will get exposure to the variance that is independent of the price of the underlying asset. This portfolio can be used to hedge a variance swap, and as a consequence, the fair value of the variance swap will be the value of the portfolio. The only assumption that we need in order to derive the price is that the underlying follows a diffusion process without jumps :

$$dS(t) = \mu(t, S(t))dt + \sigma(t, S(t))dW(t) \tag{34}$$

The strategy can be implemented as follows. Consider a portfolio of European options of all strikes k and a single time to expiry $\tau = T - t$, weighted inversely proportional to k^2 :

$$\pi(S, \sigma\sqrt{\tau}, S^*) = \sum_{K \geqslant S^*} \frac{1}{k^2} C(S, k, \sigma\sqrt{\tau}) + \sum_{K < S^*} \frac{1}{k^2} P(S, k, \sigma\sqrt{\tau}) \tag{35}$$

Because out-of-the-money options are more liquid, we use put options $P(S, k, \sigma\sqrt{\tau})$ for strikes k varying continuously from zero up to some arbitrary reference price S^*, and call options $C(S, k, \sigma\sqrt{\tau})$ for strikes k varying continuously from S^* to infinity. Note that the Vega of a European option in the Black-Scholes world is the same for a call as for a put, provided that the strike and other parameters are the same. S^* is chosen to be the approximate at-the-money forward price of the underlying asset that marks the boundary between liquid puts and liquid calls. Under the Black-Scholes assumptions, it can be shown[18] algebraically that the portfolio value (and as a consequence, the fair value of the variance swap) is given by:

$$\pi(S, \sigma\sqrt{\tau}, S^*) = \frac{2}{T}\left(\frac{S - S^*}{S^*}\right) + \frac{1}{2}\sigma^2\tau - \frac{2}{T}\log\left(\frac{S}{S^*}\right) \tag{36}$$

[17] "More Than You Ever Wanted To Know About Volatility Swaps", K Demeterfi, E Derman, M Kamal, J Zou, March 1999.

[18] "More Than You Ever Wanted To Know About Volatility Swaps", K Demeterfi, E Derman, M Kamal, J Zou, March 1999.

Here the first term in our portfolio is a long position in the stock S and a short position in a bond. The second term is a log contract (which is not traded in the market but can be replicated with options[19]).

Note that the variance exposure of the portfolio $\pi(S, \sigma\sqrt{\tau}, S^*)$ is $\frac{\tau}{2}$, and this quantity is independent of the current price S of the underlying asset[20]. This observation gives a basis for the replicating strategy for a variance swap in a Black-Scholes world. The main disadvantage of this approach is that, in order to hedge the portfolio, we need and infinite number of strikes appropriately weighted to replicate a variance swap. It is in practice impossible. Also under volatility skew, Black-Scholes assumptions do not hold and the errors introduced in pricing can be considerable.

4.1.2 Pricing variance Swaps: Expected Value

The second approach is more general. Given the payoff function for a variance swap, the expected present value of the contract under the risk neutral measure is given by the fundamental pricing formula:

$$H_t = E^Q\left[e^{\left(-\int_t^T r(s)ds\right)} \times H_T \mid F_t \right] \tag{37}$$

where $F_t\}_{t\in[0,T]}$ is the filtration generated by the brownian motion $W(t)$. The fair value K_{var}^2 will be the expected value of the future realized variance that makes the contract have zero value:

$$K_{var} = \frac{1}{T} E^Q\left[e^{\left(-\int_t^T r(s)ds\right)} \times \int_t^T \sigma^2(t)dt \right] \tag{38}$$

Thus the pricing problem is reduced to building an algorithm for the estimation of the future value of volatility. It can be shown that the fair

[19] See Neuberger (1994 and 1996).

[20] Peter Carr. "Toward a theory of volatility trading", 2002

price of the variance swap under the assumption of a continuous diffusion process is given by a portfolio of call, put options and a log contract[21]:

$$\begin{aligned} K_{var} &= \frac{2}{T}\left[rT - (\frac{S_0 e^{rT}}{S^*} - 1) - \log\frac{S^*}{S_0}\right] \\ &\quad + \frac{2}{T}\left[e^{rT}\int_0^{S^*}\frac{1}{K^2}P(K)dK + e^{rT}\int_{S^*}^{\infty}\frac{1}{K^2}C(K)dK\right] \end{aligned} \tag{39}$$

The term:

$$e^{rT}\int_0^{S^*}\frac{1}{K^2}P(K)dK + e^{rT}\int_{S^*}^{\infty}\frac{1}{K^2}C(K)dK \tag{40}$$

is a portfolio of options (calls and puts with strike K) with final payoff given by:

$$f(S_T) = \frac{2}{T}\left(\frac{S_T - S^*}{S^*} - \log\frac{S_T}{S^*}\right) \tag{41}$$

This equation reflects that implied volatilities can be regarded as the market's expectation of future volatilities. We can use market prices of the options to obtain an estimation of the future variance[22]. This is the method used by us in this paper in order to calculate the fair price of the variance swap. The election relies in the use of tradable options calls and puts in the pricing strategy and therefore the liquidity of this market. In practice, only a small set of discrete option strikes is available and used to calculate variance swap rates. Demeterfi al showed that with some assumption on the implied volatility smile, variance swap rate can be approximated as a linear function of at-the-money forward implied volatility. If the smile is flat, the variance swap rate will be equal to at-the-money implied volatility. If the smile does exist, then the variance swap rate will be higher than the at-the-money implied volatility[23].

[21] Note that the price is not really the fair variance, because the procedure of calculating the log contract is an approximation. This one always over-estimates the value of the log contract. See Appendix A in "A guide to volatility and variance swaps". The Journal of Derivatives, summer 1999, pp. 9-32.

[22] The implied volatility is an umbiased and efficient forecast on future volatility and subsumes the information content of historical volatility (see Christensen 1998).

[23] Carr and Lee ("Robust replication of volatility derivatives") get the lower bounds for variance swaps. They show that the at-the-money implied volatility is a lower bound for the variance swap rate. Dupire also derived lower bounds for variance claims 'notably for a call on variance ("Model free results on volaitlity derivatives").

4.1.3 Pricing variance Swaps: Other approaches

Other approaches have been developed in different papers from an econometric point of view. Financial return volatility data is influenced by time dependent information flows which result in pronounced temporal volatility clustering. These time series can be parameterized using Generalized Autoregressive Conditional Heteroskedastic (GARCH) models[24].The key element in these approaches is the calibration of the parameters of the model used. Note that GARCH models use data under the real measure (observed data in the market), but the calibration of the models is under the risk neutral world. It has been found that GARCH models can provide good in-sample parameter estimates and, when the appropriate volatility measure is used, reliable out-of-sample volatility forecasts. Javaheri, Wilmott and Haug (2002) discussed the valuation and hedging of a GARCH(1,1) stochastic volatility model. They used a general partial differential approach to determine the first two moments of the realized variance in a continuous or discrete context. Then they approximate the expected realized volatility via a convexity adjustment. Brockhaus and Long (2000) provided an analytical approximation for the valuation of volatility swaps and analyzed other options with volatility exposure. Théoret, Zabré and Rostan (2002) presented an analytical solution for pricing of volatility swaps, proposed by Javaheri, Wilmott and Haug (2002). They priced the volatility swaps within framework of GARCH(1,1) stochastic volatility model. Swishchuk (2004) proposes a new probabilistic approach based on Heston (1993) to price variance swaps.

4.2 Volatility swaps

A volatility swap is a forward contract on the annualized volatility. The payoff function is given by:

$$H_T = N \times (\sigma_R - K_{var}) \tag{42}$$

where σ_R is the realized stock volatility,K_{var} is the annualized volatility delivery price and N is the notional amount of the swap in money per annualized volatility point. That is, the holder is swapping a fixed volatility K_{var} on the realized future volatility .

4.2.1 Pricing Volatility swaps

Valuing a volatility forward contract or swap is no different from valuing a variance swap. The value of a forward contract H_T on future realized variance

[24]For a detailed explanation of GARCH models see Levy, G F. “Implementing and Testing GARCH models”, NAG Ltd Technical Report TR4/ 00, 2000.

with strike price K_{var} is the expected present value of the future payoff under the risk-neutral measure:

$$H_t = E^Q \left[e^{\left(-\int_t^T r(s)ds \right)} \times H_T \mid F_t \right] \tag{43}$$

The fair value K_{var} is not the square root of the variance swap price due to convexity effect[25]. In order to compute it we need to take into account the approximation of Javaheri (2002) for the convexity adjustment:

$$E\left(\sqrt{\sigma_R^2}\right) \simeq \sqrt{E(\sigma_R^2)} - \frac{var(\sigma_R^2)}{8 \times E(\sigma_R^2)^{3/2}} \tag{44}$$

There is no simple replication strategy for volatility swaps. Demertefi imply that "its value depends on the volatility of the underlying variance, that is, on the volatility of volatility ". Despite that it is difficult to hedge (this difficulty is the main reason why volatility swaps are not widely traded), Carr shows that the volatility swap rate is well approximated by the Black-Scholes at-the-money implied volatility of the same maturity [26].

Part III

Empirical Testing

We analyze in this part of the thesis the profile of a systematic short strategy of a variance swap on the index while being long the constituents[27]. We show that there is sense in selling correlation on short-term. Moreover, we show that this is not an arbitrage strategy because there is risk of suffering losses. In order to study the properties of the strategy, we get the daily

[25] Using Jensen inequatility we can bound the problem by:
$E(\sqrt{\sigma_R^2}) < \sqrt{E(\sigma_R^2)} = \sqrt{K_{var}}$
For a more detailed study see "At-the-money Implied as a Robust Approximation of the Volatility Swap Rate", Peter Carr, Roger Lee, working paper.

[26] "At-the-money Implied as a Robust Approximation of the Volatility Swap Rate", Peter Carr, Roger Lee, working paper.

[27] In the period considered (1 January 2002 to 14 March 2007) there are 57 companies and the index. However, only 50 companies compose the EuroStoxx 50, effect collected by the weight matrix.

profit and losses (and the distribution function) for the strategy proposed: sell correlation on the index and buy the constituents of the index. Different weights in the strategy have been used in order to get more volatility exposure or more correlation exposure.

5 Data

The data have been provided by the Merrill Lynch London Equity Derivatives Strategy Team. We have the three months volatility surfaces for both the EuroStoxx 50 and the constituents of the same, from 1 January 2002 to 14 March 2007. The volatility surfaces are given by a set of strikes from 70 to 130 uniformly spaced (five points apart), where 100 is considered at-the-money. As we have mentioned before, the volatility surfaces are implied volatilities calculated from a model. The model considered is Black-Scholes and the market is the internal market at Merrill Lynch. We have the LIBOR (daily) for the same period of time. The daily prices of the stocks and the index, dividend yield for both the index and the stocks, and the weights of the stocks in the index are also provided.

5.1 Cleaning the data

The first step is to compute the three months realized volatility with the daily prices of the stocks. We have applied the formula proposed in Part I to calculate it. Please note that the dividends have been introduced in the calculation of the realized volatility when it has been necessary[28]. Moreover, the computation of the annualized rate of the dividend yield is done at this step. We compute the right weight matrix taking into account that the weights of the stocks in the index have been changing every three months (in March, June, September and December there are companies that enter, go out or change their weight in the index).

To evaluate the properties of the dispersion trades, we will need a clean data set of index and single stock variance swap prices. In order to compute the prices, we use the volatility surfaces of the index and the constituents. The main problem arises because there exist dates in the volatility surfaces database that are without data. The explanation is that the trader have not trade options at those dates for some companies. Therefore, the second step is the volatility surface reconstruction (interpolation) problem.

[28]The return during a time interval including ex-dividend day is given by $\log\left(\frac{S_i+\text{div}}{S_{i-1}}\right)$.

We define the spread as the difference between the implied volatility minus the realized one. Note that the realized volatility is just one value for each date, but the implied -due to the skew- differs for different strikes, so we have 13 implied volatility values (one for each strike) in each date. A prior there is no relationship between both the implied volatility value and the realized volatility (implies are obtained from a model and realized volatilities are a measure of the underlying's price movement over the past history). We will assume in this thesis that the implied volatility at-the-money is the one that we use to compare with the values of the realized volatility.

The daily continuity of the realized volatility is assured because it is calculated with the daily (in a three month horizon) stock prices. We compute the spread between both the implied volatility and the realized volatility for every strike of the volatility surface.

In order to interpolate the volatility surfaces, we need to keep in mind that we must to have continuity in both the volatility surface slope and the spread. The strategy proposed to reconstruct the surfaces has been a linear interpolation of the spread[29]. The value obtained has been added to the realized volatility in order to get the reconstructed implied volatility level. A second refinement have been done. There are periods of time with the same volatility skew for all the dates, changing suddenly in the next date. This is produced because the traders have not updated the skew values until this moment. However, this is clearly wrong and an interpolation in these values has been also considered following the aforementioned procedure.

The third step his to prepare the clean data set to work with. We have 57 matrices for the volatility surfaces data (one for each company) and 1 matrix for the index. We have the LIBOR rate vector, one matrix of weights and one matrix of spot prices. A matrix with the dividend for each company is also computed. The time horizon considered is three months.

6 Computing the variance swap prices

Once the data is ready, we can start to compute the three months variance swaps prices assuming that every day we enter in a new contract. The method used at this stage has been the hedging approach developed by Demertefi and explained in Part II. Because we have a discrete set of strikes, a slight modification have been introduce in the pricing formula to adapt it to the

[29]We have choose linear interpolation because it is the easier one. However, other methods more complex were implemented as splines and parabolic interpolation and the results were quite similar.

discrete framework. Concretely, we use[30]:

$$K_{var} = \frac{2}{T}\left[rT - \left(\frac{S_0 e^{rT}}{S^*} - 1\right)\right] - \log\left(\frac{S^*}{S_0}\right) + e^{rT}\pi_{CP} \tag{45}$$

where π_{CP} is a portfolio of call and put options given by:

$$\pi_{CP} = \sum_i w(K_{ip})P(S, K_{ip}) + \sum_i w(K_{ic})C(S, K_{ic}) \tag{46}$$

where $w(K_i)$ is determined by the slope of the calls and puts options used to replicate the log-payoff contract for the different strikes K_i [31]. Note that if we have not skew (that is, just one value of the implied volatility at-the-money) we can also use this approach. The dominant driver of expected realized variance will be the at-the-money volatility (the skew is important but is of secondary importance). Therefore when we integrate with respect to the strike over the vanilla call and put prices we can just use no strike dependency.

We assume in the implementation of the model that 100 is the at-the-money value. Strikes out-the-money correspond to put options and in-the-money to call options. We obtain in the computation the total cost of the options portfolio π_{CP} and the contribution of each strike value in the volatility surface to the cost. In fact, only the strikes closer to the at-the-money value are the main driver of the portfolio cost. The fair price of the variance swap K_{var} [32] is given by the equation aforementioned. We get 57 prices vectors (one for each constituent of the index) and one variance swaps price vector for the index.

7 Dispersion trading

It is well documented [33]that the implied volatility is generally higher than the subsequent realized volatility, a phenomenon most significant for out-of-the-money options. As for variance swaps levels, it is evidenced [34]that

[30]This model is implemented because it assumes stochastic volatility (but any specific model is necessary). We obtain with this method both the variance swap price and the hedging positions. See "More Than You Ever Wanted To Know About Vitality Swaps", K Demeterfi, E Derman, M Kamal, J Zou, March 1999.

[31]See Appendix for a detail explanation.

[32]Note that the price is not really the fair variance, because the procedure of calculating the log contract is an approximation with puts and calls options. This one always overestimates the value of the log contract.

[33]"Predicting volatility in the foreign exchange market", Jorion, P. Journal of Finance, 1995.

[34]"A Tale of Two Indices", P Carr, L Wu, February 23, 2004

the variance swaps rates generally exceed the subsequent realized variance. This is not surprising because the variance swap rate is a non-linear function of implied volatility across different strikes. In the case of a liquid market, where the volatility smile is pronounced ,the variance swap rate is higher than the at-the-money volatility[35].

An economic explanation of the overestimation of implied volatility is the negative correlation between index returns and volatility. Therefore people tend to long volatility to hedge market turndown. Another source of volatility risk premium is identified in Demertefi as the variance of volatility of the underlying. Carr[36] discovered that the variance risk premium is much more obvious for equity index than for individual stocks. Some authors suggest the reason to be the pricing of correlations between the components. That is to say, index options are more expensive than individual stock option because they can be used to hedge correlation risk. The explanation is that the correlations between stock returns are time varying and will blow up when the market is suffering big losses. In this way during market distress the soar of correlation between components would amplify the volatility of the index, leading to higher volatility of the index than individual stocks. Hence the market is charging a higher premium of volatility risk from indices. Some traders report positive average profit of systematic short strategy of variance swap[37]. On the other hand, a systematic short strategy exhibits highly negative skew[38] of return due to large losses caused by markets in decline. This is consistent with the empirical fact that the volatility is negatively correlated with the underlying price process.

[35] "More Than You Ever Wanted To Know About Volatility Swaps". Demeterfi, Derman, Kamal and Zou. March 1999.

[36] A Tale of Two Indices, P Carr, L Wu, February 23, 2004

[37] "Strategic and Tactical Use of Variance and Volatility". Morgan Stanley's report, 2003

[38] The skew and kurtosis measure how the values are distributed around the mode. A skew value of zero indicates that the values are evenly distributed on both sides of the mode. A negative skew indicates an uneven distribution with a higher than normal distribution of values to the right of the mode, a positive value for the skew indicates a larger than normal distribution of values to the left of the mode. The kurtosis of the distribution indicates how narrow or broad the distribution is. A positive value for the kurtosis indicates a narrower distribution than a normal Gaussian, a negative value indicates a flatter and broader distribution. A normal Gaussian distribution has a kurtosis of zero. The larger the kurtosis of the parameter, the better.

7.1 Setting the dispersion trading weights

In order to implement dispersion trading, the first step is to choose the proportion of variance swaps contracts that we will buy or sell. Two types of weights are proposed in the literature. The weights are chosen in order to get a portfolio Vega neutral (volatility exposure) or Theta neutral (correlation exposure)[39].

7.1.1 Vanilla Dispersion Trade

The weights are chosen in the same proportion of the members in the index. In this case the exposure of the volatility is the main driver of the P&L. A long position in the strategy involves buying 50 variance swaps in the constituents of the index and being short in one variance swap in the index. A vanilla dispersion trade is long in volatility and short in correlation. Let us use the model developed in section 3 to explain this. Assume the case that there exists correlation between the two assets in the index proposed and it is different of zero. We showed that the theoretical index variance is given by:

$$\sigma^2_{index} = (w_1\sigma_1 S_1\rho + w_2\sigma_2 S_2)^2\, t + \left(w_1 S_1\sigma_1\sqrt{1-\rho^2}\right)^2 t \tag{47}$$

A long strategy is profitable for us if the volatilities of the individual stocks increase. The correlation is bounded by one and in this extreme case the index volatility is equal to the replicated basket volatility. Therefore the volatility of the index increases at a lower rate than the one of the stocks because individual volatilities are weighted by the correlation coefficient (that is less than one). Moreover, the larger is the decrease in the correlation the bigger is the P&L. Please note that although the strategy profits directly from changes in volatility of the index versus volatility of the stocks, the driver of this movements is always generated by the correlation matrix.

The natural approach in view of the results of vanilla dispersion trading, that is, the correlation as a main driver, is to set up weights that increase the exposure to the correlation between constituents. This is called correlation-weighted dispersion trading.

7.1.2 Correlation-weighted dispersion trading

The weights may be chosen keeping the portfolio independent of changes in the volatility. However, this strategy would need dynamic hedging to assure

[39] The index Vega exposure is bigger than the component Vega exposure because the correlation is lower than one.

a Vega neutral portfolio after inception. The dispersion implemented in our project is static, that is, after entering in the contract we keep the position in the variance swaps until expiry. This implies that we can get a portfolio Vega neutral at inception. However, when the correlation movements arise the volatility exposure will be developed during the life of the contract[40]. We weight each variance swap in the constituents as follows:

$$\alpha_i = w_i \times \rho \times \left(\frac{\sigma_i^{\mathrm{Im}\,plied}}{\sigma_{index}^{\mathrm{Im}\,plied}} \right) \tag{48}$$

where w_i is the weight of the stock in which we are setting the contract in the index and $\frac{\sigma_i^{\mathrm{Im}\,plied}}{\sigma_{index}^{\mathrm{Im}\,plied}}$ is the ratio of implied volatilities of the stock and the index. The correlation ρ is the implied index correlation. It is a measure derived from the implied volatility of the index and the constituents pair wise correlations. It is computed using[41]:

$$\rho = \frac{\sum_{i<j} w_i w_j \sigma_i \sigma_j \rho_{ij}}{\sum_{i<j} w_i w_j \sigma_i \sigma_j} \tag{49}$$

Here ρ_{ij} is the pair wise correlation between the stock i and j.

According to this weights, the main driver of the strategy would be the correlation as we can see in the expression for the index variance in our model. However, because the trade is not Vega neutral, there exists other second order driver in the P&L due to the stocks variance. That is, if we are long in the trade and a particular stock variance peaks producing more dispersion, then this volatility effect will be added to the P&L generated by the correlation. If the position held is short, it will result in bigger losses.

7.2 Profit and loss

The P&L of the dispersion trading with weights α_i being long in the index and short in the constituents, implemented with variance swaps, is defined by:

$$\begin{aligned} P\&L \;&=\; \left[\frac{N}{2 \times k_{varIndex}} (k_{varIndex}^2 - \sigma_{Index}^2) \right] \times e^{-rT} \\ &- \left[\sum_{i=1}^{50} \frac{N}{2 \times k_{var_i}} \alpha_i \left(k_{var_i}^2 - \sigma_i^2 \right) \right] \times e^{-rT} \end{aligned} \tag{50}$$

[40] Seing the index variance formula developed in our model is straightforward to understand this.

[41] "Variance Swaps Guide". JPMorgan Equitiy Derivative Strategies, 2006.

where $k_{varIndex}$ is the index variance swap fair strike, σ^2_{Index} the index realized variance, $k^2_{var_i}$ the stock i variance swap strike, σ^2_i the stock i realized variance and α_i the weights chosen for the strategy. The discount factor is assumed to be continuous, where r is the LIBOR rate and T is the maturity (3 months). N is the variance notional, and it is weighted by the factor $\frac{1}{2\times k}$ in order to express the amount in Vega notional. The Vega notional is the average profit or loss for a one per cent change in volatility. The reason is that traders use to think in terms of volatility and therefore the P&L is quoted in terms of the volatility.

7.3 Empirical results

We study the performance of systematic short strategy of a variance swap on the index and being long on the constituents. The setup is that every day we enter in a contract with expiry three months that involves selling a variance swap on the EuroStoxx 50 and buying 50 variance swaps on the constituents. The realized variance is as defined as in Part I. The transaction cost is assumed to be zero. The notional of the contracts is one (Euros). We implement the different weights aforementioned for both: getting volatility exposure and getting correlation exposure. We considered the period from 1 January 2005 to 8 December 2006. It is showed that the systematic short strategy does provide positive mean of return (3 months). However, the risk of suffering big losses due to large unexpected volatility in market crashes is inevitable in this strategy. Therefore, the risk of losses exists and the distribution of profits and losses has high skew. We compute the correlation matrix for the P&L obtained with the different weights proposed. We observe that the correlation for both correlation exposure strategies (P&L) is almost identical and very high (0.99). The correlation of the volatility exposure strategy versus the exposure strategy P&L is also very high (0.8). We compute the correlation of the P&L with the index log-returns. The value obtained is very low for both correlation-weighted and vanilla dispersion trading (5%). The average return is positive and higher for both correlation-weigthed schemes than vanilla weighted scheme. Traditionally there was a bias for realized dispersion to be higher than implied, but the market has gotten more and more efficient and it has produced that the profits obtained with the dispersion trading are lower than historically.

Table 1 at Appendix shows the main statistics of the dispersion trading implemented for the different weights chosen. The P&L (in Vegas) for the different weights schemes and the distribution functions are showed in Figures 1, 2, 3 and 4.

7.3.1 Subsamples

The risk premium becomes negative mostly due to major market corrections. We can see a correction in 2005 and 2006 clearly in the P&L. The effect is more important in correlation-weighted schemes because the exposure to the correlation is higher than in vanilla schemes. Remember that the correlation is the main driver as we showed. On the other hand, a suddenly increased of the correlation if we are long in the contract also will produce bigger losses than vanilla.

A clear example is the 2006 correction. It affects to the returns accumulated during one year. However, the accumulated gains continue to be positive while and after the crash event. The effect is showed for the P&L in Figure 5. For some investors, particularly those with short volatility positions through variance swaps and dispersion trades, the increase in volatility produced huge losses as we can see in Figure 5.

The reason of the losses was the increase in the (realized) stocks correlation above the implied level (the "bet" level in the contract). The volatility spiked in May and June a result of a pick-up in inflation, anticipated interest rate hikes and a sell-off in emerging market assets. The effect is also showed by the VIX Index[42]. This index jumped from 13.35 points on May 16 to 18.26 points on May 23. On June 13, it closed at a year high of 23.81 points before falling to 13.03 points on June 29. This is almos perfectly correlated with the P&L implemented (around 99.5%).

The outperforming of the correlation-weighted schemes is clearly higher than the vanilla weights. In almost all the dates the correlation weight scheme makes profit, but between March and June in 2006 the realized volatility of the index was bigger than the implied (larger realized correlation than implied) and the P&L were negative. Think of it in a insurance term: variance swap buyers generally pay premium to protect them against crashes (the reason of our positive average return), when the market crashes actually happen, variance swap seller have to pay for the claims. This also reflects the convexity of the variance swaps payoffs. This point reflects that dispersion trading implemented with variance swaps can has sense as a position risk management tool rather than a profit generator in lower volatility environments. If we want to hedge against P&L losses corresponding to volatility spikes, we could obviously employ a variance swap or maintain an appropriate option position. But if we want to hedge the portfolio against a spike in correlation, an easier, although less than perfect, hedge we would try would be to find an asset that is significantly correlated with the correlation of the index. There

[42]The VIX Index measures the expectation of the market of 30-day volatility on S&P 500 index option prices.

is the risk that correlation would also break down at that time, but we would test prospective candidates with a simulation. Another approach would be to study the principal components analysis in the returns of our portfolio, and use a factor model. Then we could isolate the correlation and cancel with another asset.

Therefore a first approximation in order to prevent the losses should be a variance swap contract with a cap on maximum loss[43]. Since the systematic short strategy is positively correlated with the return of underlying and the losses happen mainly during market crashes, we might be able to use some vanilla instruments that can insure investors against market turndown. We hope that they can also protect the systematic short strategy of variance swap. Keep in mind this is not strange because the variance swap can be replicated by a basket of put and calls with different strikes. Buying back some options can certainly offset some risk of variance swap. If we buy back all the options that construct the variance swap, there will be no risk at all. However, the skew of the P&L can become positive with the help of the puts. This is because the over-the-counter put becomes profitable in down market. But it is not necessarily profitable when the variance swap contract is suffering a big loss.

Another option can be to use barrier options. The idea to buy down-and-in barrier put to cap the loss of variance swap comes from the intuition that, whenever the down-and-in put comes into existence, the underlying had probably been through a down and volatile period. Down-and-in barrier put is much cheaper than vanilla put of the same strike if the barrier is low. Therefore with a low barrier and a relatively high strike, a down-and-in barrier put option is very likely to end in-the-money for either straight downward market or rebound. This captures the characteristic of being volatile.

7.3.2 Timing

As have been show in last section, systematic short strategy exhibits highly negative skew of return due to large losses happening during market crashes. This is consistent with the empirical fact that the volatility is negatively correlated with the underlying price process. Thereafter it is important to determine a timing trading strategy of variance swap. A good time to put on dispersion trade is when correlation is high.

One most straight-forward idea to find a better forecaster of future volatility than the variance swap rate. The idea comes from that if you have perfect foresight of future volatility then you will know perfectly when to short

[43] "Pricing Options on Realized Variance", P Carr, H Geman, D Madan, M Yor, August 2004.

and when to long. So if one comes up with a better predictor of volatility than variance swap rate and trade according to the difference between the two, we would expect higher return than systematic short. However, current literature points out that option implied volatilities produce superior volatility forecasts compared to time series forecasters (for example GARCH)[44]. Although Martens suggests the historical variance calculated by high frequency intra-day data could compete with implied volatility in a time series approach, it is hard to obtain high frequency data with history long enough to back test the strategy.

Another problem is that, because big loss of systematic short strategy unexceptionally happens during market crashes, it is crucial to predict volatility of crash period to avoid loss. However, time-series approach's predictability on volatility during market crashes is seriously questionable. Therefore, rather than trying to predict the volatility in a traditional way, we could try to identify the driving factors for variance risk premiums, which is actually the P&L of dispersion trading. Another idea is to predict the market crashes. It is inspired by the fact that the highly negative skew of P&L of systematic short strategy is mainly due to market crashes. Intuitively, these losses could have been avoided if we can foresee the coming of market crashes. Of course there are other losses that are not caused by market crisis, but those are relatively cheap to afford.

Martens suggested that we short variance swap whenever the risk premium is high or low. Ideally we would like to have positive return across different periods. Keep in mind that in reality it will take investor extra confidence to go long variance swaps if they consider it an profit driver instead of protection of regular investment. With regard to skewness of the P&L profile obtained, it is profitable to short volatility swap when the historical volatility is high.

From a practical point of view, Burgardht and Lane suggest the use of volatility cones[45]. They proposed this method as a solution to determine if the current volatility level is higher or lower than the historical level.

7.3.3 Optimal Contract Duration

We have drawn results for contracts that expiry in 3 months. The dispersion trading P&L is a function of: weights, correlation, volatility, size of the index and time. A natural question arises: what is the optimal contract

[44] "Predicting financial volatility: High-frequency time -series forecasts vis-à-vis implied volatility", Martin Martens, Jason Zein 2003.

[45] "The Sampling Properties of Volatility Cones", Stewart Hodges and Robert Tompkins. Financial Options Research Centre, University of Warwick, Coventry.

duration in order to maximize the P&L? Remember that the main driver is the correlation. Therefore, we are interested in periods of time with the higher difference between implied correlation and the subsequence realized correlation. The volatility also plays an important role. It is well-known that the volatility shows a mean-reversion effect in equities. Therefore I have computed the P&L for different maturities: three months, six months and one year. The P&L obtained with a three month dispersion trading outperformances the other two for both correlation dispersion trading and vanilla dispersion trading.

7.3.4 Optimal weights

The P&L obtained varies depending on the weights chosen. We have showed that the bigger is the exposure to the correlation the larger are the profits (losses) of the strategy. This is consistent with the fact that the correlation is the main driver in dispersion trading strategies. The convexity effect appears due to the variance swaps. However, we can improve the P&L choosing the optimal allocation for each variance swap. We have the P&L as a function of the weights α_i. Therefore, we suggest to maximize the expected return of our strategy constraint to that the volatility is minimized:

$$\max_{\alpha_i} \; E[p\&l(\alpha_i)] - k \times E\left[p\&l^2(\alpha_i)\right] \tag{51}$$

where k is an arbitrary constant.

Under this new allocation the P&L obtained outperformances both dispersion trading implemented in a 2%.

7.3.5 Optimal portfolio size

We showed in section II that dispersion effect is highly correlated with the portfolio's size. However, many constituents in the portfolio can offset the dispersion P&L (or amplify it). We ran different dispersion strategies with sub-baskets of the original portfolio, that is, EuroStoxx 50. The P&L obtained is maximized when the components in the sub-basket have the higher negative correlation. However, the P&L are only 3% bigger than we consider the whole EuroStoxx 50.

Part IV
Conclusions and Future Developments

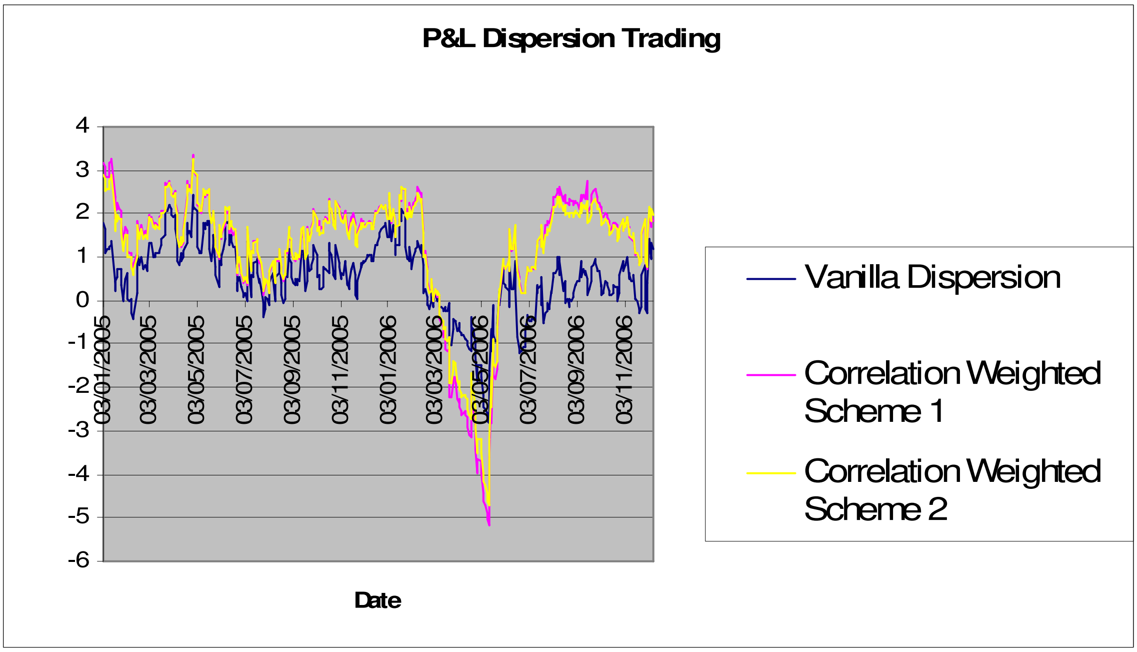
The rapid development of derivatives market has enabled investors to gain exposure to volatility. So instead of just taking directional positions based on predictions of future returns, investor with foresight of volatilities might also make money by engaging in the appropriate volatility product. Variance swap is the most heavily traded among the volatility products. The payoff of this contract is the difference between the future realized return variance and the predetermined variance swap rate.

This thesis tries to explore the profitability of the dispersion trading strategies. We begin examining the different methods proposed to price variance swaps. We have developed a model that explains why the dispersion trading arises and what the main drivers are. We have investigate the P&L obtained with the different strategies proposed in the market. We show that correlation-based dispersion trading produces the bigger P&L. This strategy demonstrates positive mean of return, which is consistent with the fact that the implied volatility is generally higher than the future realized volatility. This strategy has a potential of suffering large loss in bullish market. We show that dispersion trading strategies are not arbitrage strategies. We have computed the distribution of the P&L obtained. We show that the distribution of the P&L shows thick tails. This is consistent with the extreme events that appear in a bullish market (for example, the correction in the market of 2005). We proposed a method to choose the optimal weights that produces outperforming of the strategies used in the market. The timing of the strategy is also studied.

As a further it might be interesting to check the seasonality of the return profile and design one optimal entry time. Furthermore, the mark to market value of variance should also be studied because for long maturity contract one might prefer to close the position before maturity.

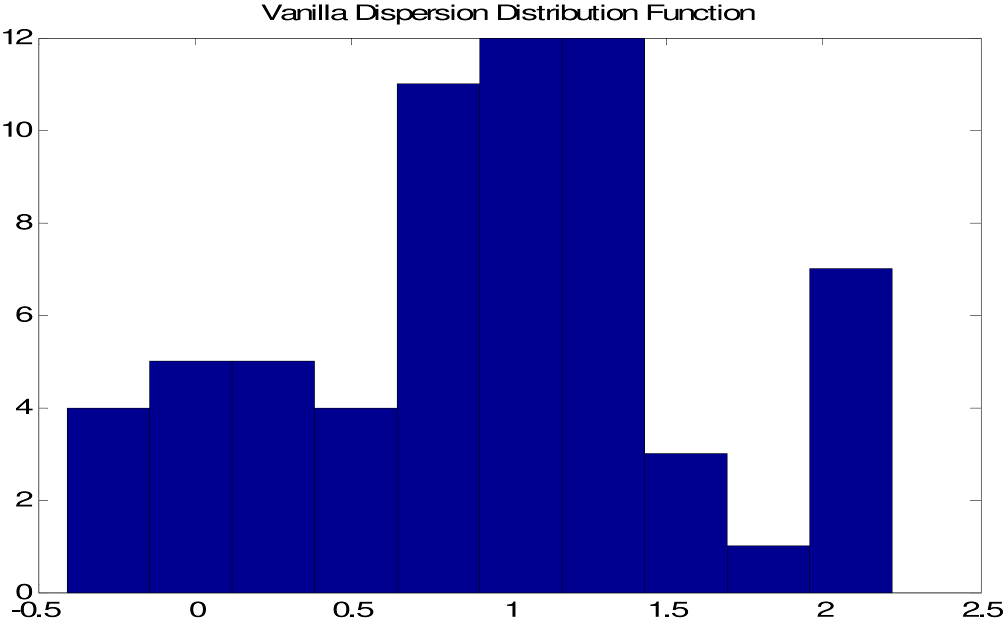
We have studied the dispersion trading in a Equity Index. An interesting exercise could be to implement the strategy in Credit Indices. This approach needs a previous step: the modification of the pricing method proposed by Derman in order to use Credit Options (in both single Credit Default Swaps and Credit Indices).

Figure 1.



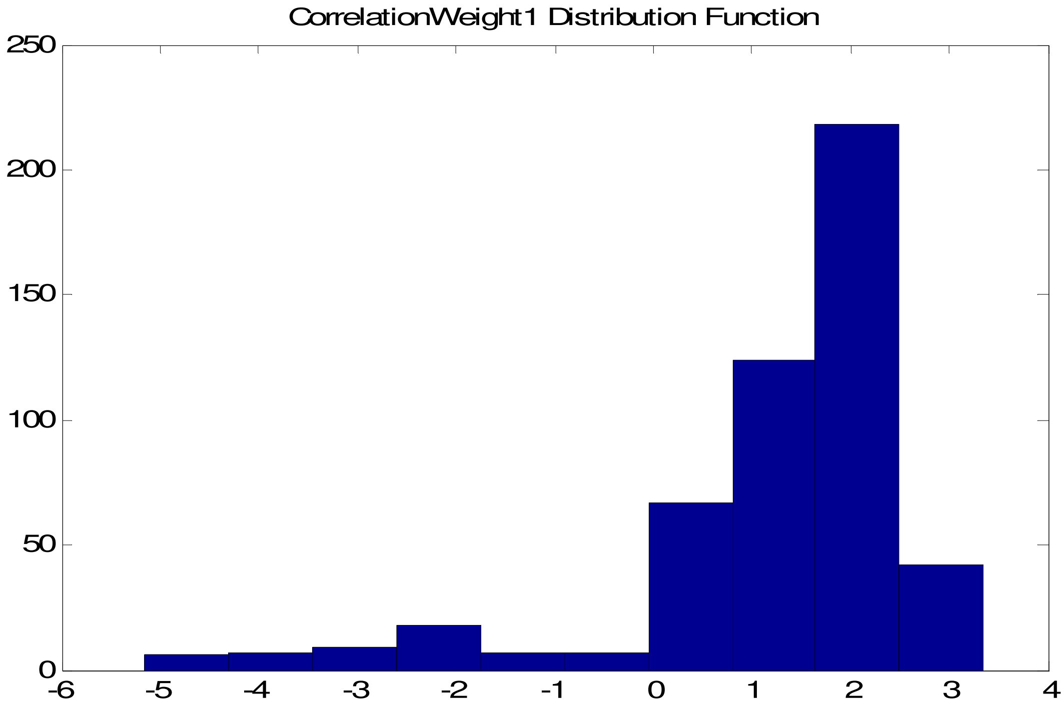
Description Figure 1: P&L (in Vega Notional) obtained selling a variance swap on the EuroStoxx 50 and being long on the constituents. Vanilla dispersion gets exposure to the volatility and correlation-weighted dispersion gets exposure to the correlation. The expiry is three months and the Notional is one Euro.

Figure 2.



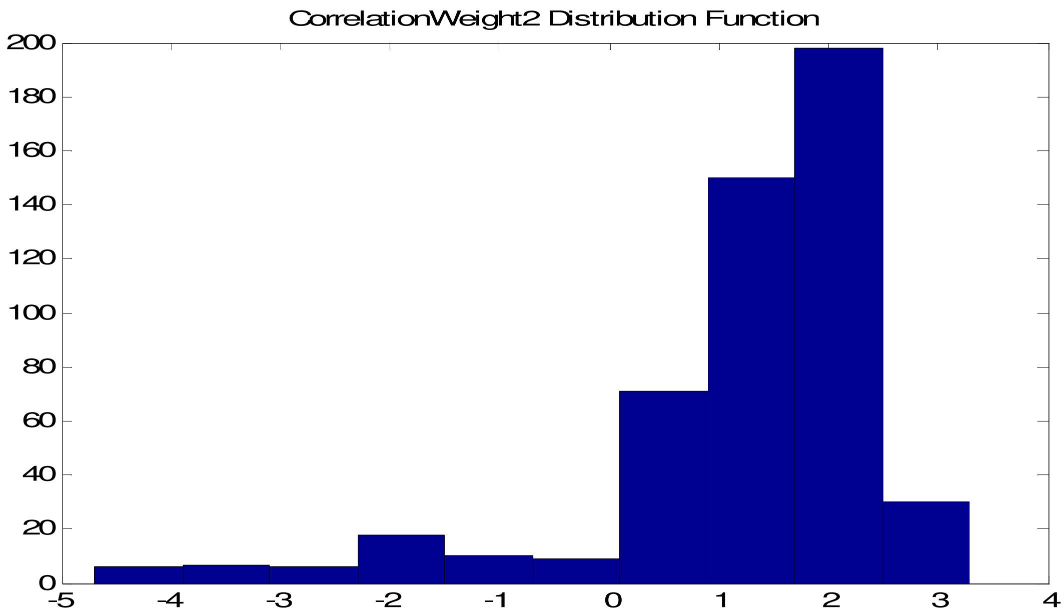
Description Figure 2: Distribution function obtained for the Vanilla Dispersion Trading on the EuroStoxx 50. The weight used is w_i.

Figure 3.



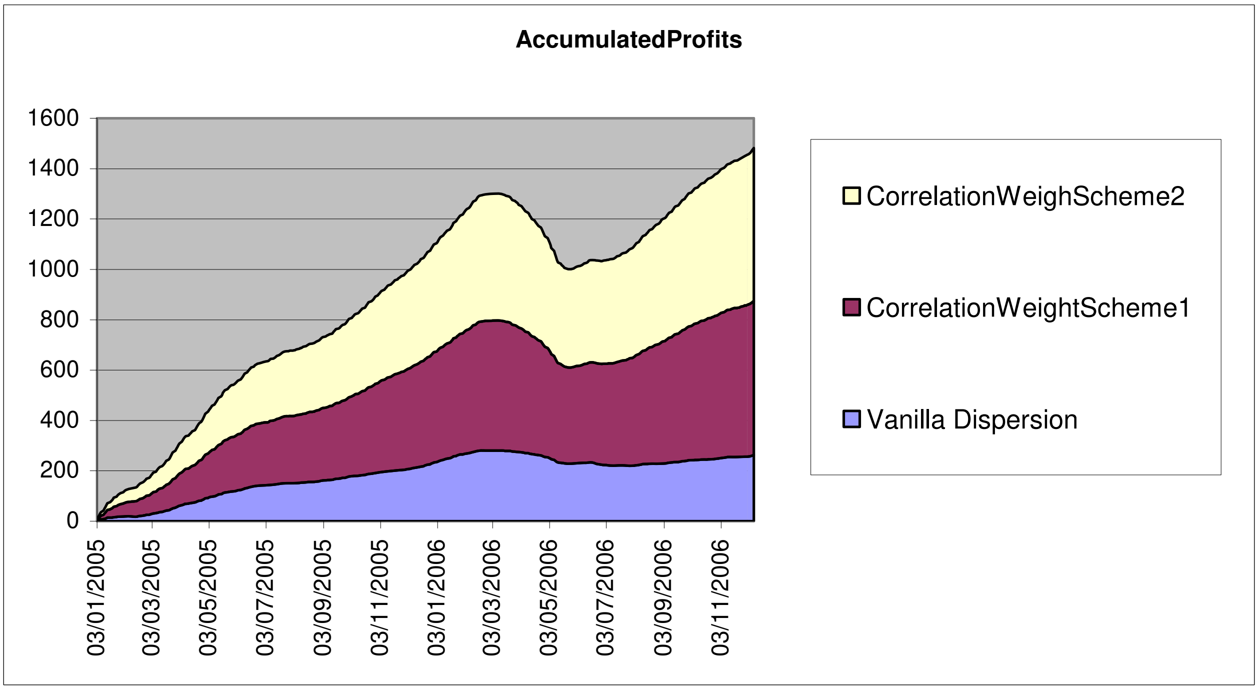
Description Figure 3: Distribution function obtained for the Correlation-Weighted Dispersion Trading (Scheme 1) on the EuroStoxx 50. The weight used is $w_i \times \rho \times \dfrac{\sigma_i^{implied}}{\sigma_{index}^{implied}}$.

Figure 4.



Description Figure 4: Distribution function obtained for the Correlation-Weighted Dispersion Trading (Scheme 2) on the EuroStoxx 50. The weight used is $w_i \times \sqrt{\rho}$.

Figure 5.



Description Figure 5: Accumulated profits (in Euros) from the different strategies implemented. The initial cost assumed in the strategy is zero. We observe that the 2006 correction affected to the P&L but it did not produce a negative balance in the accumulated gains.

Table 1.

Weight αi	**Average Return**	**STD of Return**	**Risk-Return (no annualized)**	**Skew**
w_i	0.52669	0.83866	0.62085	-0.75189
$w_i \times \rho \times \frac{\sigma_i^{implied}}{\sigma_{index}^{implied}}$	1.2111	1.4948	0.8101	-2.0271
$w_i \times \sqrt{\rho}$	1.2023	1.3576	0.88561	-2.0149

Description Table 1: Main Statistics of the P&L for the different strategies implemented.

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