Measuring the Welfare Cost of Inflation in Zimbabwe

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Title: Measuring the Welfare Cost of Inflation in Zimbabwe

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Abstract
The study estimates the long-run equilibrium relationship between money balance as a ratio of income and treasury bill rate for the Zimbabwean economy. These estimates are done for two periods, the entire period (using quarterly data) of 1980:01 to 2005:04 and the hyperinflationary period (using monthly data) of 1999:01 to 2005:12. These estimates are in turn used to obtain estimates for the welfare cost of inflation. Using the Johansen technique, the research estimates a log-log specification and a semi-log model of the above relationship for the two periods. Estimates suggest that the welfare cost of inflation for Zimbabwe ranges between 0.9% and 23.4% of GDP for a band of 10 to 300% of inflation in the case of estimations done for the entire period. Welfare cost estimates for the hyperinflationary period are 0.4% and 27.6% of GDP, respectively.

JEL classification: E31; E41; E52

Keywords: Cointegration; Interest elasticity; Money Demand; Welfare Cost of Inflation
1. Introduction

Although “global disinflation” (a term inverted by Kenneth Rogoff) saw the average inflation rates in developing countries falling from 31 percent in the first half of the 1980s to less than 6 percent since 2000 (Craig and Rocheteau, 2005), Zimbabwe’s inflation trend has been on the opposite path. Other world evidence also shows that average inflation has fallen from 9 percent in the first half of the 1980s to less around 2 percent in 2000 in advanced economies, while the figure for Latin America and transition economies dropped from more than 100 percent to about 10 percent since 1993 to date.

In the first decade after independence (in 1980), Zimbabwe was seen as a relatively moderate inflation country, with average annual inflation rates hovering below 15 percent (Chhibber et al, 1989). Nevertheless, Makochekanwa (2007) traces how the country’s inflation trend skyrocketed especially since 1999, turning the country into a hyper-inflationary economy. Currently, the country is grappling with monthly inflation rates of above 100 000 percent, the latest being 100 580.2 percent for the month of January 2008. Thus, the “global disinflation” apparatus employed in other parts of the world (referred above) which includes institutional changes such as greater central banks independence, improved monetary regimes and better macroeconomic policies have not been implemented in Zimbabwe (at least to a magnitude which is sufficient to ensure low levels of inflation). To the contrary, inflation financing in the form of money printing spearheaded by the Reserve Bank of Zimbabwe (RBZ) has been the norm, especially in the past five or so years.

In general, the reasons why inflation finance is not an advisable option of government finance can be categorized into two: the redistributive and disruptive (welfare costs) aspects, with the former having been received much attention in literature than the latter\(^1\). Redistributive effects include the hardship involved by people whose income and wealth are fixed in monetary terms, redistribution from lender to borrower, and redistribution from private sector to government.

On the other hand, the disruptive or welfare cost of inflation involves the misallocations of resources that may result from the heightened uncertainties concerning future relative and absolute prices. Bailey (1956: 93) defines welfare cost as:

“\textit{...a tax on the holding of cash balances, a cost which is fully analogous to the welfare cost (or excess burden) of an excise tax on a commodity or productive service}”

Graphically, welfare cost of inflation is therefore defined as the area under the inverse demand function – the consumer surplus – that can be gained by reducing the nominal interest rate from a positive level of \(i\) to the lowest possible level (perhaps to zero rate) (Lucas, R.E., jr, 2000).

\(^1\) For instance, more (if not all) emphasis on the cost of inflation in undergraduate university courses are on the redistributive effects.
Inflation, more so high inflation, has a cost that it inflicts on the society. Evidence point out that even moderate inflation rates of 10 percent generates substantial costs to the society. Literature is abounding with evidence of the welfare cost of inflation even for as low rates as below 10 percent. Table 1 provides evidence of the welfare cost of inflation in a number of countries as estimated from the studies conducted in those countries.

Table 1: Empirical results on the welfare cost of inflation

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Country studied</th>
<th>Inflation rate (%)</th>
<th>Welfare cost (% of GDP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gupta, R &amp; J. Uwilingiye</td>
<td>2008</td>
<td>South Africa</td>
<td>3</td>
<td>0.34 (0.34)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>0.67 (0.76)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>1.08 (1.43)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15</td>
<td>1.56 (2.41)</td>
</tr>
<tr>
<td>Ireland, P. N</td>
<td>2007</td>
<td>USA</td>
<td>3</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>0.22</td>
</tr>
<tr>
<td>Lagos, R &amp; R. Wright</td>
<td>2005</td>
<td>USA</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Serletis et al</td>
<td>2004</td>
<td>Canada and USA</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Lucas, R. E, jr</td>
<td>2000</td>
<td>USA</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>Lucas, R. E, jr</td>
<td>1981</td>
<td>USA</td>
<td>10</td>
<td>0.45</td>
</tr>
<tr>
<td>Fisher, S</td>
<td>1981</td>
<td>USA</td>
<td>10</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Source: author compilation
Note: * means results from log-log estimations;
      + means results from semi-log estimations.

Table 1 testifies that inflation (and most importantly high inflation) results in welfare cost to any society. On the other hand, figures shows that Zimbabwe’s inflation has been on an upward trend for close to two decades. Despite these two fact (the welfare cost of inflation and Zimbabwe’s high inflation trend), no study (to the best knowledge of the author) has been done to attempt to measure the welfare cost of such high inflation for the Zimbabwean economy. To this end, this paper aims to close this gap in literature about the welfare cost of inflation in the country. To try and capture the welfare costs of higher inflation, the study estimated models for two periods: the entire period (1980 to 2005) using quarterly data and the higher or hyperinflation (hyperinflation as defined by Cagan, 1956) period from 1999 to 2005 using monthly data.

The remainder of the paper is structured as follows: Section 2 presents the theoretical underpinnings to the estimation of the welfare cost of inflation, with Section 3 discussing data sources, stationarity and cointegration tests. Section 4 presents estimations from both the log-log and semi-log money demand specifications, as well as calculating the welfare cost estimates for the Zimbabwean economy for the two periods. Conclusion is presented in section 5.
2. Theoretical framework

A point of departure is the fact that money provides some services to society by facilitating exchange of commodities. Thus, the cost of inflation corresponds to a reduction in these services provided by money. Since inflation by its nature erodes the purchasing power of money balances, economic agents tend to conduct their transactions with fewer money balances as the inflation rate increases. For instance, they resort to alternative payment arrangements, such as credit or barter, which can be less efficient or more costly. In some cases they also buy the services of financial intermediaries to help manage their cash balances.

In order to capture and measure the cost of inflation, there is need to get a (an imaginary) sense of the non-monetary benefits generated by a stock of money and assign them some equivalent value in dollars. Typically, economists measure the convenience that one enjoys by holding cash with the nominal interest rate (which is approximately the sum of a real interest rate and the anticipated inflation rate). The idea behind the use of nominal interest rate is that holding cash is convenient because it can facilitate exchange, and people are willing to give up something for that convenience. What they are willing to give up is what they could have earned had they put the cash to some non-risky money-making use, that is, if they had invested it in the safest interest bearing asset available (Craig and Rochetau, 2005). As an example, if the interest rate on government securities is 8 percent, then the services provided by holding an additional dollar should be worth 8 cents a year.

Among the methods used to capture and measure the welfare cost of inflation are the “welfare triangle” which gave rise to the consumer surplus approach which was developed by Bailey (1956) and also employed by Lucas (2000), the Lucas’ (2000) compensating variation approach and the search model of monetary exchange which was formulated by Lagos and Wright (2005). This paper employs the Bailey (1956) consumer surplus approach (as employed by Lucas 2000). This approach utilizes techniques from both public finance and applied microeconomics.

To simplify the analytical discussion, one strong assumption is made. The study assumes that the real interest rate is close to zero so that terms “inflation rate” and “interest rate” can be used interchangeably (and this does not affect the conclusions of the study).

If the nominal interest rate measures the nonpecuniary benefit that money gives people, one can approximately calculate the cost of inflation by estimating the proportion of the benefit which is lost when inflation rises. As pointed by Craig and Rochetau (2005). The benefit lost is a function of the fact that people hold less money (in real terms) as the rate rises; less money held equals less of its benefit obtained. In Figure 1, we represent the relationship between the interest rate and the stock of real money balances in the economy. (This relationship is referred to as the money demand function). To facilitate the calculation of welfare cost of inflation, we compare resource allocations when nominal interest rate is greater than zero \((r > 0)\) to a benchmark case of zero nominal interest rate \((r = 0)\). When the stock of real balances is 50 (just an arbitrary point along
the possible values of money balances), the benefit that one enjoys by holding an additional dollar is measured by the interest rate that corresponds to 50 real balances on the money demand curve (in this case, 10 percent). Equivalently, it is the length of the segment between points X and Y. The total benefit provided by real balances can then be identified as the area under the money demand curve (that is, the sum of all segments under the curve). The maximum total productivity of real balances occurs when the interest rate is zero, because at this point, one loses nothing by holding money (the level of real balances in Figure 1 corresponding to this point is 100). Thus the welfare cost of inflation is minimized when the nominal interest rate is zero (a famous result known as the *Friedman rule*).

If the interest rate increases from 0 to 10 percent, then individuals economize on their use of real money balances. In Figure 1, real balances fall from 100 to 50. The area under the money demand relationship, the “triangle” \( \text{XYZ} \) in Figure 1, measures the welfare cost of having a positive interest rate of 10 percent relative to zero. Equivalently, it captures the loss to society in terms of lost production and wasted resources due to the fact that people reduce their real money balances from 100 to 50.

To measure the welfare triangle \( \text{XYZ} \), one therefore needs to estimate the money demand in Figure 1 and then compute the area under this curve. This is what Fischer (1981) and Lucas (2000), among others did to come up with their estimates recorded in Table 1. Thus, to this end, the paper will follow Lucas’ (2000) theoretical model and will briefly present it below.

**Figure 1: The Welfare Triangle**

![Figure 1: The Welfare Triangle](image)

*Source: Bailey (1956: 95)*
Thus, the analysis so far indicates that money demand specification is paramount in investigating the appropriate magnitude of the welfare cost of inflation in any economy. Lucas (2000) analyzed two competing demand for money specifications. One, motivated by Meltzer (1963), relates the natural logarithm of \( m \), the ratio of nominal money balances to nominal income, to the natural logarithm of \( r \), the short-term nominal interest rate. This specification is represented as follows:

\[
\ln(m) = \ln(A) - \eta \ln(r)
\]

where \( A > 0 \) is a constant and \( \eta > 0 \) measures the absolute value of the interest elasticity of money demand. The other money demand specification, adapted from Cagan (1956), links the log of \( m \) instead to the level of \( r \) via

\[
\ln(m) = \ln(B) - \zeta r,
\]

where \( B > 0 \) is a constant and \( \zeta > 0 \) measures the absolute value of the interest semi-elasticity of money demand for money with respect to the interest rate.

Lucas (2000), after applying the outline of Bailey (1956), translated the evidence on money demand into a welfare cost estimate. As before, Bailey (1956) defined the welfare cost of inflation as the area under the inverse demand function – or the consumer surplus that could be gained by reducing the interest rate from \( r \) to zero. Thus, if \( m(r) \) is the estimated function and \( \psi(m) \) is the inverse function, the welfare cost can algebraically be defined as:

\[
w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_{0}^{r} m(x)dx - rm(r)
\]

Given that the function \( m \) has the dimensions has the dimensions of a ratio to income, so does the function \( w \). The value of \( w(r) \) shows the fraction of income people would require as compensation in order to make them indifferent between living in a steady-state with an interest rate constant at \( r \) and an otherwise identical steady state with an interest rate of (or near) zero. Lucas (2000) also shows that the welfare cost of inflation, \( w(r) \), as a percentage of GDP is represented by:

\[
w(r) = A \left( \frac{\eta}{1 - \eta} \right) r^{1 - \eta}
\]

when money demand takes the log-log form and

\[
w(r) = B \left[ 1 - (1 + \zeta r)e^{-\zeta r} \right]
\]

when money demand takes the semi-log form.

An important observation from (4) and (5) is that an estimate of the interest elasticity of money demand is central in evaluating the welfare cost of inflation. Thus, the first step is
to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate (Gupta, R and Uwlingwe, J, 2008). The coefficients obtained in the long-run relationships are then inserted into the relevant places in equations (4) and (5) to get the welfare cost of inflation.

3.1 Data Analysis

3.2 Data Sources

The study employed both monthly and quarterly time series data, with the former covering the first month of 1999 to the last month of 2005. On the other hand, the quarterly series was from the first quarter of 1980 to the fourth quarter of 2005. The two series were obtained from Reserve Bank of Zimbabwe (RBZ). The two variables used in this research are the real money balances ratio (rm3) generated by dividing the broad money supply (m3) with the nominal income (nominal GDP), and short term 91 days Treasury bill rate (tbr). Also necessary transformations are done, especially when estimating the logarithmic values.

3.2 Stationarity and Stability tests

Given the nature of time series variables, this section presents the univariate characteristics of the data. Stationarity tests are done on the three variables, \( lrm3 \), \( ltb\)r and \( tbr \) using the Augmented – Dickey – Fuller (ADF), Kwiatkowski-Phillips-Schmidt-Shin (KPSS) and Phillips Peron (PP). These tests were performed for all the three series for both the quarterly and monthly data sets. As reported in Appendix Tables A1 and A2, all the three variables follow autoregressive process with unit root. In both the ADF and PP, the null hypothesis of a unit root could not be rejected for the variables expressed in level form, while for the KPSS; the null of stationarity was rejected for all the variables in level form. Thus, since the variables were non-stationary, the Johansen test for cointegration between \( lrm3 \) and \( ltb\)r (1) and \( lrm3 \) and \( tbr \) (2) was employed for both the entire and the hyperinflation periods.

Stability tests of the VAR models (log-log, and semi-log specifications for both the entire and hyperinflation periods), including a constant as an exogenous variable were also performed. The results are presented in Tables 3 through to 6 of the Appendix. Since no roots were found to lie outside the unit circle for the estimated VARs based on 4 lags under the log-log and the semi-log specifications for the entire; and based on 2 lags for the hyperinflationary period, the study conclude that all the four VARs are stable and suitable for further analysis.
3.3 Cointegration Test

The study tested for the cointegrating relationship based on the Johansen (1991, 1995) approach. For this purpose, we included four lags in the quarterly (entire period) VARs, and 2 lags for the monthly (hyperinflationary) VARs. In both the quarterly and monthly VARs, we allowed the level data to have linear trends, but the cointegrating equations to have only intercepts. Based on the Pantula Principle, both the Trace and the Maximum Eigen Value tests, showed that there is one stationary relationship in the data \( r = 1 \) at 5 percent level of significance for both the log-log and the semi-log specifications, in both the quarterly and monthly estimations. The results have been reported in Table 7 through to Table 10 of the Appendix.

4. Empirical Results

The long run estimates for the log-log and semi-log specifications for entire period, that is, 1980 (first quarter) to 2005 (fourth quarter), are given as follows:

\[
lnm3 = -1.158424 - 0.247493tbr \quad \text{(log – log specification)} \\
\text{(6)} \\
(-3.42012)
\]

\[
lnm3 = -0.524234 - 2.01980tbr \quad \text{(semi-log specification)} \\
\text{(7)} \\
(-2.94003)
\]

While the corresponding estimates for the hyperinflationary period estimated using monthly data are as follows:

\[
lnm3 = -1.391661 - 0.480984tbr \quad \text{(log – log specification)} \\
\text{(8)} \\
(2.43290)
\]

\[
lnm3 = -0.20827 - 0.053159tbr \quad \text{(semi-log specification)} \\
\text{(9)} \\
(-3.12943)
\]

The interest elasticities for both log-log specifications, in absolute term are 0.24749 and 0.480984, respectively for the entire period and the hyperinflation period. The corresponding interest semi elasticities also in absolute terms are 2.01980 and 0.053159, respectively. As can be seen from equations (6) through (9), the signs of the interest rate variable in all the four equations have both correct signs and are significant, thus adhering to economic theory. Thus, based on these four equations (two for entire period and two for hyperinflation period), the author is now ready to calculate the welfare cost of inflation for Zimbabwe following Lucas (2000), and as presented earlier in equations (4) and (5).

Calculation for the intercept and slope coefficients reported for the entire period under the log – log specification results in the values of \( A = 0.313980625 \) and that of \( \eta = \)
0.247493, while for the semi-log specification the values of $B = 0.592009$ and that of $\xi = 2.0198$. The respective values for the hyperinflation estimations are $A = 0.248662$, $\eta = 0.480984$, $B = 0.811988$ and $\xi = 0.053159$.

Given the above values, the next step will be inserting these values into the corresponding formula for the welfare cost measures, given by equation (4) for log-log specification and equation (5) for semi-log specification. It is also assumed that real interest rate over the entire period (1980 to 2005) was equal to 39.91, while for the hyperinflation period was 97.09. These respective real interest will imply a nominal rate of interest equal to 39.91 (entire period) and 97.09 (hyperinflation period). These real and nominal interest rates will allow the calculation of the benchmark values of the welfare cost of inflation ($w$) under price stability in the respective periods\(^2\). Reading from real interest baselines stated above, a value of $r = 49.91$ corresponds to a ten percent rate of inflation, while, when $r = 89.91$ will imply a 50 percent inflation and so on for the entire period calculations. During the hyperinflation period, values of $r = 107.09$ and $r = 147.09$ will mean inflation rates of 10 and 50 percent, respectively. The actual welfare cost of inflation are then evaluated by subtracting the value of $w$ at an inflation equal to zero from the value of the same at a positive rate of inflation.

Table 2 therefore present the measures of the welfare costs of inflation, under the log-log and the semi-log specifications for two periods under consideration and for the inflation rates of 10, 50, 100, 150, 200 and 300 percent, for both the two periods\(^3\).

<table>
<thead>
<tr>
<th>Inflation rate</th>
<th>Entire Period</th>
<th>Hyperinflation Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Log-Log</td>
<td>Semi - Log</td>
</tr>
<tr>
<td>10</td>
<td>0.9</td>
<td>2.2</td>
</tr>
<tr>
<td>50</td>
<td>4.4</td>
<td>10.2</td>
</tr>
<tr>
<td>100</td>
<td>8.1</td>
<td>17</td>
</tr>
<tr>
<td>150</td>
<td>11.6</td>
<td>20.6</td>
</tr>
<tr>
<td>200</td>
<td>14.8</td>
<td>22.3</td>
</tr>
<tr>
<td>300</td>
<td>20.8</td>
<td>23.4</td>
</tr>
</tbody>
</table>

As can be seen from Table 2, an inflation rate of 10 percent results in welfare costs to the magnitude of 0.9 percent of GDP for log-log specification and 2.2 percent of GDP for semi-log specification, when one considers the entire period. The corresponding welfare

\(^2\) Note, as in Ireland (2007), we define the real rate of return to be equal to the difference between the nominal interest rate and the inflation rate, where the inflation rate is obtained as the percentage change in the seasonally adjusted series of the CPI.

\(^3\) Whilst most studies limit their analysis for inflation below 20 percent, the fact that the Zimbabwean economy has generally been experiencing relatively higher inflation (above 20 percent) for a long time necessitates the study to go up to 300 percent inflation rates in the analysis.
costs for the hyperinflation period are 1.2 percent and 0.4 percent of GDP, respectively. On the other hand, a 300 inflation rate is more damaging during the hyperinflationary period as it results in a welfare cost of 24.4 and 27.6 percent of GDP for the log-log and semi-log, respectively; while the same inflation rate results in welfare cost of 20.8 and 23.4 percent of GDP, respectively for the entire period. The other inflation figures are interpreted the same.

5. Conclusion

This research uses the Johansen (1991, 1995) cointegration technique to first obtain an appropriate long-run money demand relationship for the Zimbabwean economy and then, in turn, construct welfare cost estimates based on the money demand function, as outlined in Lucas (2000). This exercise was done for both the quarterly data over the period of 1980:01 to 2005:04 as well as for the monthly data for the period 1999:01 to 2005:12. In both cases, estimates were done for log – log function and semi – log specifications. The research’s findings indicates that the welfare cost of inflation for Zimbabwe ranges between 0.9 percent and 23.4 percent of GDP for a band of 10 to 300 percent of inflation in the case of estimations done for the entire period. The corresponding welfare cost of inflation estimates for the hyperinflationary period are 0.4 percent and 27.6 percent of GDP, respectively.

Given these estimates, one can conclude that the high inflationary environment which has characterized the Zimbabwean economy for a relatively long time has hurt the country’s long run economic growth. Thus, the authorities need to implement serious anti-inflationary policies and measures.
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## APPENDIX

Table A1: Univariate characteristics of all the variables (Quarterly series)

<table>
<thead>
<tr>
<th>Series</th>
<th>Model</th>
<th>ADF</th>
<th>KPSS</th>
<th>PP</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>τ, τ_μ, τ_t</td>
<td>φ2, φ1</td>
<td>τ, τ_μ, τ_t</td>
<td>τ, τ_μ, τ_t</td>
</tr>
<tr>
<td>LRM3</td>
<td></td>
<td>1.868, 0.983, -0.911</td>
<td>3.96, 0.966, -----</td>
<td>0.187**, 0.330, -----</td>
<td>2.033, 0.733, -0.933</td>
</tr>
<tr>
<td>D(LRM3)</td>
<td></td>
<td>-8.364***, -8.103***, -8.02***</td>
<td>35.36***, 65.66***, -----</td>
<td>0.246***, 0.533**, -8.127***</td>
<td>-8.363***, -8.049***</td>
</tr>
<tr>
<td>LTBR</td>
<td></td>
<td>-4.022**, -1.279, 0.918</td>
<td>3.45, 10.522***, -----</td>
<td>0.050, 1.213***, 0.330</td>
<td>-2.316, -1.165, -0.933</td>
</tr>
<tr>
<td>D(LTBR)</td>
<td></td>
<td>-7.587***, -7.615***, -7.467***</td>
<td>20.456***, 30.829***, -----</td>
<td>0.044, 0.052, 1.80</td>
<td>-6.540***, -6.573***, 1.80</td>
</tr>
<tr>
<td>TBR</td>
<td></td>
<td>-0.613, 0.766, 1.470</td>
<td>7.613**, 9.144, -----</td>
<td>0.1024, 1.031***, -7.096***</td>
<td>-0.970, 1.670, 1.80</td>
</tr>
<tr>
<td>D(TBR)</td>
<td></td>
<td>-8.236***, -8.034***, -7.917***</td>
<td>25.227***, 35.834***, -----</td>
<td>0.097, 0.294, -7.170***</td>
<td>-7.20***, -7.096***</td>
</tr>
</tbody>
</table>

*(*[*][**]) Statistically significant at a 10(5)[1] percent level

**Key:** τ_t: Means Trend and Intercept

τ_μ: Means intercept

τ: Means None

The Augmented Dickey- Fuller, Kwiatkowski-Phillips-Schmidt-Shin and Phillips Peron results tests for non-stationarity shows that all the variables appear to be integrated of order one that is stationary after first differencing.
The Augmented Dickey - Fuller, Kwiatkowski-Phillips-Schmidt-Shin and Phillips Peron results tests for non-stationarity shows that all the variables appear to be integrated of order one that is stationary after first differencing.

### Stability Tests: Quarterly Models

#### Table 3: Diagnostic Statistics of the Reduced-Form Var Model (Log_Log)

Roots of Characteristic Polynomial  
Endogenous variables: LRM3  LTBR  
Exogenous variables: C  
Lag specification: 1 4

<table>
<thead>
<tr>
<th>Root</th>
<th>Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.997210</td>
<td>0.997210</td>
</tr>
<tr>
<td>0.922806</td>
<td>0.922806</td>
</tr>
</tbody>
</table>

No root lies outside the unit circle.  
VAR satisfies the stability condition.
### Table 4: Diagnostic Statistics of the Reduced-Form Var Model (Semi_log)

| Roots of Characteristic Polynomial
| Endogenous variables: LRM3 TBR
| Exogenous variables: C
| Lag specification: 1 4

<table>
<thead>
<tr>
<th>Root</th>
<th>Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.991343</td>
<td>0.991343</td>
</tr>
<tr>
<td>0.808561</td>
<td>0.808561</td>
</tr>
<tr>
<td>0.384523</td>
<td>0.384523</td>
</tr>
<tr>
<td>-0.057201</td>
<td>0.057201</td>
</tr>
</tbody>
</table>

No root lies outside the unit circle.
VAR satisfies the stability condition.

### Monthly Models

### Table 5: Diagnostic Statistics of the reduced-From VAR Model (Log-log)

| Roots of Characteristic Polynomial
| Endogenous variables: LRM3 LTBR
| Exogenous variables: C
| Lag specification: 1 2

<table>
<thead>
<tr>
<th>Root</th>
<th>Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.969076</td>
<td>0.969076</td>
</tr>
<tr>
<td>0.891598</td>
<td>0.891598</td>
</tr>
</tbody>
</table>

No root lies outside the unit circle.
VAR satisfies the stability condition.

### Table 6: Diagnostic Statistics of the reduced-From VAR Model (Semi-log)

| Roots of Characteristic Polynomial
| Endogenous variables: LRM3 TBR
| Exogenous variables: C
| Lag specification: 1 2

<table>
<thead>
<tr>
<th>Root</th>
<th>Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.977088</td>
<td>0.977088</td>
</tr>
<tr>
<td>0.889408</td>
<td>0.889408</td>
</tr>
</tbody>
</table>

No root lies outside the unit circle.
VAR satisfies the stability condition.
Quarterly Series

Table 7: Estimation and Determination of Rank (Log-log)
Sample (adjusted): 1976Q2 2005Q4
Included observations: 119 after adjustments
Trend assumption: No deterministic trend
Series: LRM3 TBR
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized</th>
<th>No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>At most 1</td>
<td>0.104229</td>
<td>13.53667</td>
<td>12.32090</td>
<td>0.0311</td>
</tr>
</tbody>
</table>

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized</th>
<th>No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>At most 1</td>
<td>0.104229</td>
<td>12.87831</td>
<td>11.22480</td>
<td>0.0254</td>
</tr>
</tbody>
</table>

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Table 8: Estimation and Determination of Rank (Semi_log)
Sample (adjusted): 1976Q2 2005Q4
Included observations: 119 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: LRM3 TBR
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized</th>
<th>No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>At most 1</td>
<td>0.132090</td>
<td>20.58034</td>
<td>20.26184</td>
<td>0.0452</td>
</tr>
<tr>
<td>Hypothesised No. of CE(s)</td>
<td>Eigenvalue</td>
<td>Max-Eigen Statistic</td>
<td>0.05 Critical Value</td>
<td>Prob.**</td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>-------------</td>
<td>---------------------</td>
<td>---------------------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>None *</td>
<td>0.132090</td>
<td>16.85841</td>
<td>15.89210</td>
<td>0.0352</td>
<td></td>
</tr>
<tr>
<td>At most 1</td>
<td>0.030793</td>
<td>3.721937</td>
<td>9.164546</td>
<td>0.4551</td>
<td></td>
</tr>
</tbody>
</table>

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

### Monthly Series

**Table 9: Estimation and Determination of Rank (Log-Log)**

Sample (adjusted): 1999M03 2005M12
Included observations: 82 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: LRM3 LTBR
Lags interval (in first differences): 1 to 2

#### Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.438647</td>
<td>50.96867</td>
<td>20.26184</td>
<td>0.0000</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.043203</td>
<td>3.621423</td>
<td>9.164546</td>
<td>0.4714</td>
</tr>
</tbody>
</table>

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

#### Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesised No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max-Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.438647</td>
<td>47.34725</td>
<td>15.89210</td>
<td>0.0000</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.043203</td>
<td>3.621423</td>
<td>9.164546</td>
<td>0.4714</td>
</tr>
</tbody>
</table>
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Table 10: Estimation and Determination of Rank (Semi-Log)
Sample (adjusted): 1999M03 2005M12
Included observations: 82 after adjustments
Trend assumption: No deterministic trend (restricted constant)
Series: LRM3 TBR
Lags interval (in first differences): 1 to 2
Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.432788</td>
<td>49.48730</td>
<td>20.26184</td>
<td>0.0000</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.035825</td>
<td>2.991535</td>
<td>9.164546</td>
<td>0.5823</td>
</tr>
</tbody>
</table>

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values