The compromise efficiency vs. egalitarianism among generations with an infinite horizon

Alcantud, José Carlos R.

Universidad de Salamanca

3 May 2010

The compromise efficiency vs. egalitarianism among generations with an infinite horizon

José Carlos R. Alcantud [1]

Facultad de Economía y Empresa, Universidad de Salamanca, E 37008 Salamanca, Spain

Abstract

This paper concerns ethical aggregation of infinite utility streams. Position i is typically interpreted as the endowment of generation i. We analyze the broad question: In order for the social welfare to increase, the interest of how many generations can be respected if we intend to be "ethical"? Here "ethical" refers to verifying adequate equity axioms, and case-studies cover: extensions of restricted non-substitution; or Hammond Equity-related principles; together with the usual Anonymity axiom.

Key words: Social welfare function, Equity, Pareto axiom, Intergenerational justice

JEL classification

D63, D71, D90.

Email address: jcr@usal.es (José Carlos R. Alcantud).
URL: http://web.usal.es/jcr (José Carlos R. Alcantud).
[1] I acknowledge financial support by the Spanish Ministerio de Ciencia e Innovación under Project ECO2009-07682, and by Junta de Castilla y León under Project SA024A08 and GR-99 Funding.

1 Introduction and motivation

This work is concerned with ethical aggregation of infinite utility streams, thus position i is typically interpreted as the endowment of generation i. We analyze the broad question: In order for the social welfare to increase, the interest of how many generations can be respected if we intend to be egalitarian in the appropriate sense?

With regard to a given infinite utility streams, various efficiency axioms capture the general principle that adequate changes must improve the social welfare. The *Weak Dominance* axiom (WD) captures the following spirit: improving the welfare of *one* single generation suffices to improve the social welfare. In turn, the *Weak Pareto* axiom (WP) requests that *all* generations increase their utility for the social welfare to improve, and the *Strong Pareto* axiom requests that *at least one* generation increases its utility for the social welfare to improve. Intermediate positions have been studied, like the *Strong Monotonicity for Infinite Generations* axiom in Sakai (2006). Under this postulate an increase in the utility of any *infinite* number of generations conveys a higher social welfare.

How far can we go in this efficiency scale under different egalitarian assumptions? Let us focus on the case of evaluations of the streams by utilities. In this context, Basu and Mitra (2003) prove that Strong Pareto is incompatible with the equal treatment of all generations for any non-trivial setting, though Weak Pareto is compatible with that equity postulate in l_∞, the set of bounded real-valued sequences (Basu and Mitra, 2007). Therefore, assuming the equal treatment of all generations imposes a restriction as to the number of generations that must be benefitted for the social evaluation to increase: it is sensible to assume that it is finite, but it is not to claim that it can be arbitrary (because no such numerical evaluation exists). In a related line, Banerjee (2006) yields the incompatibility of Weak Dominance with the egalitarian axiom called HEF in the $\mathbf{X} = [0,1]^{\mathbb{N}}$ context. This imposes a bound to the compromise efficiency vs. consequentialist egalitarianism as expressed by HEF: it is impossible that improving the welfare of any single generation conveys a higher social evaluation as a general efficiency principle. But, suppose that we define axiom WD^2 by asking that improving the welfare of *two* generations must improve the social welfare, which supposes a lesser degree of efficiency than WD. Then there is a strikingly simple explicit evaluation that verifies WD^2 and a reinforced version of HEF called 1RNS: we just take $\sum_{i=2}^{+\infty} \frac{1}{2^i}$. In fact this claim holds in l_∞ and a much stronger version of Pareto-efficiency is obtained. Properties resembling 1RNS are studied in Alcantud and García-Sanz (2010b). Axiom kRNS means that when comparing streams that are constant for the future of generation k (i.e., from generation $k+1$ onwards) then the extended present from the first to the k-th generation does not matter: the higher the "long-run" endowment the better. Peculiarly, if we want to keep the degree of efficiency given by WD^2 and extend the degree of egalitarianism from 1RNS to just 2RNS then we can mimick Banerjee's argument and deduce that impossibility returns. The need for a compromise between degrees of efficiency and of egalitarianism becomes apparent here. We proceed to give some general results about that broad issue.

This work is organized as follows. We set our notation and axioms in Section 2. Assuming that all generations must be treated equally, we disclose the trade-off between the number of generations that must be benefitted for the society to be strictly better, and the number of generations that can be discarded when comparing streams that are constant in the long run. This constitutes Section 3. We perform an analogous analysis under variants of the Hammond Equity principle that capture inequality aversion in Section 4. Section 5 summarizes and gives some concluding remarks.

2 Notation and statement of the axioms

Let $\mathbf{X}$ denote a subset of $\mathbb{R}^{\mathbb{N}}$, that represents a domain of utility sequences or infinite-horizon utility streams. We adopt the usual notation for such utility streams: $\mathbf{x} = (x_1, ..., x_n, ...) \in \mathbf{X}$. By $(y)_{con}$ we mean the constant sequence $(y, y, ...)$, $(x, (y)_{con})$ holds for $(x, y, y, y, ...)$, and $(x_1, ..., x_k, (y)_{con}) = (x_1, ..., x_k, y, y, ...)$ denotes an eventually constant sequence. We write $\mathbf{x} \geqslant \mathbf{y}$ if $x_i \geqslant y_i$ for each $i = 1, 2, ...$, and $\mathbf{x} \gg \mathbf{y}$ if $x_i > y_i$ for each $i = 1, 2, ...$ Also, $\mathbf{x} > \mathbf{y}$ means $\mathbf{x} \geqslant \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$. We use the notation $\mathbb{N}^* = \mathbb{N} \cup \{0\}$

A *social welfare function* (SWF) is a function $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$. In this paper we are concerned with two sets of axioms of different nature on SWFs. They can be rephrased for social welfare relations (SWRs, i.e., binary relations on $\mathbf{X}$) in a direct manner.

Firstly we introduce some equity axioms of two different classes. Axiom AN below is the usual "equal treatment of all generations" postulate. Then we list some consequentialist equity axioms, that implement preference for egalitarian allocations of utilities among generations in various senses.

Axiom AN (*Anonymity*). Any finite permutation of a utility stream produces a utility stream with the same social utility

The next axiom was introduced in Asheim and Tungodden (2004a).

Axiom HEF (*Hammond Equity for the Future*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $\mathbf{x} = (x_1, (x)_{con})$ and $\mathbf{y} = (y_1, (y)_{con})$ $(x_1 > y_1 > y > x)$, then $\mathbf{W}(\mathbf{y}) \geqslant \mathbf{W}(\mathbf{x})$.

When $\mathbf{W}(\mathbf{y}) > \mathbf{W}(\mathbf{x})$ is requested in place of $\mathbf{W}(\mathbf{y}) \geqslant \mathbf{W}(\mathbf{x})$ we refer to HEF$^+$.

HEF states the following ethical restriction on the ranking of streams where the level of utility is constant from the second period on and the present generation is better-off than the future: if the sacrifice by the present generation conveys a higher utility for all future generations, then such trade off is weakly preferred. Asheim and Tungodden (2004a) and Asheim et al. (2007), Section 4.3, explain that it is a very weak equity condition –under certain consistency requirements on the social preferences "condition HEF is much weaker and more compelling than the standard 'Hammond Equity' condition"– that can be endorsed both from an egalitarian and utilitarian point of view.

The following equity axiom is in line with the spirit of HEF$^+$. It captures a very demanding ethical principle: a large improvement in a finite number of generations can never compensate a sustained improvement for all remaining generations.

Axiom RNS (*Restricted Non-Substitution*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $\mathbf{x} = (x_1, ..., x_l, (x)_{con})$ and $\mathbf{y} = (y_1, ..., y_m, (y)_{con})$ with $y > x$, then $\mathbf{W}(\mathbf{y}) > \mathbf{W}(\mathbf{x})$.

A weaker version states that a large improvement in a *fixed* finite number of generations never compensates a sustained improvement for all remaining generations. Formally:

Axiom m**RNS** (*m-Restricted Non-Substitution*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $\mathbf{x} = (x_1, ..., x_m, (x)_{con})$ and $\mathbf{y} = (y_1, ..., y_m, (y)_{con})$ for some generation m, and $y > x$, then $\mathbf{W}(\mathbf{y}) > \mathbf{W}(\mathbf{x})$.

As applied to social welfare relations, 1RNS implies Weak Non-Substitution in Asheim et al. (2008), which implies HEF. Besides, mRNS implies $(m-1)$RNS for $m = 2, 3, ...$

Axioms HE below is another consequentialist equity principle stating that in case of a conflict between two generations, every other generation being as well off, the stream where the least favoured generation is better off must be weakly preferred.

Axiom HE (*Hammond Equity*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $x_j > y_j > y_k > x_k$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$, then $\mathbf{W}(\mathbf{y}) \geqslant \mathbf{W}(\mathbf{x})$.

A variant of this principle is the following postulate of aversion to inequality.

Axiom VWIA (*Very Weak Inequality Aversion*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $x_j > y_j = y_k > x_k$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$, then $\mathbf{W}(\mathbf{y}) \geqslant \mathbf{W}(\mathbf{x})$.

VWIA advocates for a weak preference for streams that do not produce inequality between two generations when such unequal endowments are conflicting, every other generation being as well off. VWIA is called HE(b) in Alcantud and García-Sanz (2010b).

We intend to account for some kind of efficiency too. In this sense the stronger property we deal with is the following Axiom SP, which is weakened to SP^n below.

Axiom SP (*Strong Pareto*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $\mathbf{x} > \mathbf{y}$ then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

Axiom SP^n. If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $\mathbf{x} \geqslant \mathbf{y}$, and there is a finite $I \subseteq \mathbb{N}$ with cardinality at least n such that $x_i > y_i$ when $i \in I$, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

The interpretation of SP^n is clear: if at least n generations improve their welfare, the remaining generations being as well off, then the social evaluation must be higher.

Other axioms that relax Strong Pareto follow.

Axiom WP (*Weak Pareto*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $\mathbf{x} \gg \mathbf{y}$, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

Axiom WD (*Weak Dominance*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and there is $j \in \mathbb{N}$ such that $x_j > y_j$, and $x_i = y_i$ for all $i \neq j$, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

Because even Weak Dominance is incompatible with interesting equity axioms we consider the next relaxed versions.

Axiom WD^n. If there is a finite $I \subseteq \mathbb{N}$ with cardinality at least n, such that $x_i > y_i$ when $i \in I$ and $x_j = y_j$ when $j \notin I$, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

Axiom WD$_n$. If there is I finite with $\{1, 2, ..., n\} \subseteq I \subseteq \mathbb{N}$, such that $x_i > y_i$ when $i \in I$ and $x_j = y_j$ when $j \notin I$, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

The interpretation of these axioms is straightforward too. WD^n claims that if a finite number of at least n generations improve their welfare, the other generations' utilities being unchanged, then the social evaluation must be higher. WD$_n$ is a relaxed version where the first n generations must improve their welfare in order for the consequence to follow.

Our last axiom is implied by SP and implies WD (in fact, it is the conjunction of WD and WP).

Axiom 6 (*Partial Pareto, also PP*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and either $\mathbf{x} \gg \mathbf{y}$ or there is $j \in \mathbb{N}$ such that $x_j > y_j$ and $x_i = y_i$ for all $i \neq j$, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$.

We can list some trivial relationships between concepts. Clearly, WD implies WD^n for each n, and WD^1 is just WD. For a fixed n, SP $\Rightarrow$ SP^n $\Rightarrow$ WD^n $\Rightarrow$ WD$_n$. In the presence of AN, WD^n and WD$_n$ are equivalent. For any index $n = 2, 3, ...$, WD^{n-1} $\Rightarrow$ WD^n and WD$_{n-1}$ $\Rightarrow$ WD$_n$. We refer to axioms of *myopic* Pareto-efficiency to refer to versions SP^n, WD^n and WD$_n$ of Pareto-efficiency, since these axioms are insensitive to positive increases in the endowments of "small" numbers of generations.

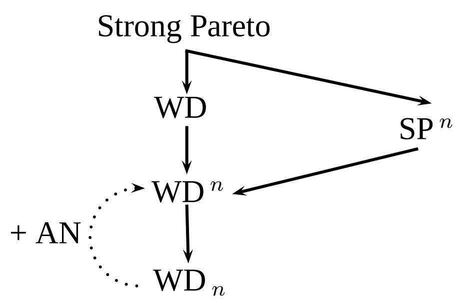

Figure 1. Relationships between efficiency axioms for a fixed n.

3 Egalitarianism as Restricted Non-Substitution

In this Section we investigate to what extent myopic Pareto-efficiency and mRNS can be assured simultaneously. This permits to complement the conclusion that the interest of a single generation as captured by WD is incompatible with HEF (Banerjee, 2006), which is implied by any mRNS egalitarian postulate. Consider the following Proposition 1:

Proposition 1 *Let* $m, n \in \{1, 2, ...\}$.

(a) If $n \geqslant m + 1$, $\sum_{i=n}^{+\infty} \frac{x_i}{2^i}$ *is an explicit SWF on* $\boldsymbol{X} = l_\infty$ *that satisfies SP*^{n} *and mRNS.*

(b) If $n \leqslant m$, *there are not SWFs on* $[0, 1]^{\mathbb{N}}$ *that satisfy WD*$_n$ *and mRNS.*

Proof: Part (a) is immediate. To prove (b) we closely follow Banerjee's (2006) argument thus we proceed by contradiction. For each $0 < x < \frac{1}{2}$ we let

$$L(x) := \mathbf{W}\left(x, \overset{n}{\ldots}, x, \left(\frac{x}{2}\right)_{con}\right) \text{ and } R(x) := \mathbf{W}\left(2x, \overset{n}{\ldots}, 2x, \left(\frac{x}{2}\right)_{con}\right)$$

Then $I(x) := (L(x), R(x))$ is non-empty by WD$_n$. Furthermore $\frac{1}{2} > y > x > 0$ implies $I(x) \cap I(y) = \varnothing$, since $n+1 \leqslant m+1$ and mRNS entail

$$L(y) = \mathbf{W}\left(y, \overset{n}{\ldots}, y, \left(\frac{y}{2}\right)_{con}\right) > \mathbf{W}\left(2x, \overset{n}{\ldots}, 2x, \left(\frac{x}{2}\right)_{con}\right) = R(x)$$

This is impossible because an uncountable number of different rational numbers can be assigned. $\triangleleft$

Proposition 1 concerns how far we can extend the interest of the present. It clarifies the trade-off between the interest of the extended present with respect to both efficiency and egalitarianism as Restricted Non-Substitution. Part (b) assures that if improving the utilities of the first n generations suffices to improve the social welfare, then improving the utilities of the first n generations must compensate sustained improvements for all remaining generations for some particular endowments.

If further we are interested in discussing *how many* generations matter –and not only *which ones* matter–, anonymity must be introduced in the argument. Thus we now focus on elucidating if there exist SWFs that satisfy AN, mRNS and SP^n (or suitable weakenings) in the cases that are not discarded by Proposition 1 (b).

Regarding SP^n the next Proposition settles the matter irrespective of the feasible set of utilities. It discloses an incompatibility between AN and any SP^n by extending the argument in Basu an Mitra (2003) which shows that AN and SP can not be combined in any non-trivial setting.

Proposition 2 *For each $n = 1, 2, ...$, there are not SWFs on $Y^{\mathbb{N}}$ that satisfy AN and SP^n when Y contains more than one element.*

Proof: We use a standard construction (cf., Basu and Mitra, 2003) to produce an uncountable collection $\{E_i\}_{i\in I}$ of infinite proper subsets of $\mathbb{N}$ with the property $i < j \Rightarrow E_i \subsetneq E_j$ and $E_j - E_i$ is infinite, $\forall i, j \in I$. In order to do that, let $\{r_1, r_2, ...\}$ be an enumeration of the rational numbers in $(0, 1)$ and set $E(i) = \{n \in \mathbb{N} : \, r_n < i\}$ for each $i \in I = (0, 1)$.

Suppose by way of contradiction, that $\mathbf{W}$ is a real-valued function on $Y^{\mathbb{N}}$ that verifies AN and SP^n for some $n \in \{1, 2, ...\}$. We assume without loss of generality that $\{0, 1\} \subseteq Y$. With each $i \in I$ we associate the following two utility streams:

$$L(i) \text{ such that } L(i)_p = \begin{cases} 1 & \text{if} \quad p \in E_i \\ 0 & \quad \text{otherwise} \end{cases}$$

$R(i)$ obtained from $L(i)$ by replacing the first n appearences of 0 with a 1

By SP ^{n}, the open interval $(\mathbf{W}(L(i)),\ \mathbf{W}(R(i)))$ is not empty. A routine argument permits to check that $i < j \Rightarrow \mathbf{W}(L(j)) > \mathbf{W}(R(i))$, because $E_j - E_i$ infinite and $E_i \subseteq E_j$ imply that a finite permutation of $L(j)$ can be compared to $R(i)$ via the SP ^{n} assumption. This completes the argument. $\lhd$

Remark 1 *We can now reach a conclusion as to the analysis of SWFs on the ground set* $Y^{\mathbb{N}}$ *when* $Y \subseteq \mathbb{N}^*$*. In this case there are SWFs that satisfy AN, RNS and PP, thus mRNS and WD*^{n} *for all* m*,* n *(cf., Theorem 1 in Alcantud and García-Sanz, 2010a, also 2010b). But even if we are not bound by any mRNS ethical restriction, AN and SP*^{n} *are incompatible irrespective of* n*.*

In order to complete the analysis of SWFs on $[0,1]^{\mathbb{N}}$ under the mRNS egalitarian postulate, we only need to elucidate if AN, mRNS and WD ^{n} (or equivalently, WD $_n$) can be respected simultaneously. The next result solves this question in the negative.

Proposition 3 *There are not SWFs on* $\boldsymbol{X} = [0,1]^{\mathbb{N}}$ *that satisfy AN, mRNS and WD*$_n$ *when* $n \geqslant m+1$ $(n, m \in \mathbb{N})$*.*

Proof: Suppose that $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$ satisfies AN, mRNS and WD $_{m+1}$ (or equivalently, WD $^{m+1}$). If we deduce a contradiction we are done.

We first check that

$$x > y > z \text{ implies } \mathbf{W}(x, \overset{m}{\cdots}, x, (y)_{con}) > \mathbf{W}(z, \overset{m}{\cdots}, z, (y)_{con})$$

because $\mathbf{W}(x, \overset{m}{\cdots}, x, (y)_{con}) > \mathbf{W}(y, \overset{m}{\cdots}, y, z, \overset{m}{\cdots}, z, (y)_{con})$ by WD ^{2m} (which is implied by WD $^{m+1}$), and also $\mathbf{W}(y, \overset{m}{\cdots}, y, z, \overset{m}{\cdots}, z, (y)_{con}) = \mathbf{W}(z, \overset{m}{\cdots}, z, (y)_{con})$ by AN.

For each $0 < x < \frac{1}{2}$ we let

$$L(x) := \mathbf{W}\left(\frac{x}{2}, \overset{m}{\cdots}, \frac{x}{2}, (x)_{con}\right) \text{ and } R(x) := \mathbf{W}(2x, \overset{m}{\cdots}, 2x, (x)_{con})$$

Then $I(x) := (L(x), R(x))$ is non-empty as shown above. Besides, $\frac{1}{2} > y > x > 0$ implies $I(x) \cap I(y) = \varnothing$ due to mRNS:

$$L(y) = \mathbf{W}\left(\frac{y}{2}, \overset{m}{\cdots}, \frac{y}{2}, (y)_{con}\right) > \mathbf{W}(2x, \overset{m}{\cdots}, 2x, (x)_{con}) = R(x)$$

This is impossible because an uncountable number of different rational numbers can be assigned. $\triangleleft$

4 Egalitarianism via inequality aversion

The setting $[0,1]^{\mathbb{N}}$ does not permit to evaluate the streams by utilities that agree with WD and HE/VWIA: cf., Subsection 5.1 in Alcantud and García-Sanz (2010b). This means that under either of the mentioned forms of aversion to inequality, it is not possible to attach numerical evaluations that are positively sensitive to a gain in utility by a single generation. In this Section we verify the tightness of such assertion in two related lines. In the first place, Subsection 4.1 proves that even if we relax the efficiency requirement to any arbitrarily weak WD_n then impossibility is preserved. In the second, Subsection 4.2 proves that if we replace the egalitarian postulate in order to adopt a related axiom (namely, the Pigou-Dalton transfer principle) then the situation is quite different: The latter principle and anonymity can be combined under representability of the social evaluation in the presence of WD. With respect to this existence result we analyze if adopting WP –responsiveness to universal gains of utility– instead of WD –responsiveness to individual gains of utility– preserves the compatibility. We conclude in the negative even if we forgo anonymity. Incompatibility with a representable social ordering arises when VWIA and WP are implemented too. These additional impossibilities stress the peculiar significance of our existence result.

Some preliminary remarks are in order. It is routine to check that when $\mathbf{X} = [0,1]^{\mathbb{N}}$ axioms HE and VWIA are equivalent in the presence of WD (cf., Section 3 in Alcantud and García-Sanz, 2010b). This is not true under weaker assumptions thus in order to check for compatibility with our relaxed versions of WD we need to argue separately for the two axioms. In addition, under WD both egalitarian axioms are equivalent to the following slightly strengthened form of their conjuntion when $\mathbf{X} = [0,1]^{\mathbb{N}}$:

Axiom HE(a)$^+$. If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $x_j > y_j \geqslant y_k > x_k$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$, then $\mathbf{W}(\mathbf{y}) > \mathbf{W}(\mathbf{x})$.

4.1 Hammond Equity and myopic weak dominance

In this Subsection we analyze the interest of how many generations can be respected as to efficiency, when the Hammond Equity spirit is assumed. The following Proposition proves that both HE and VWIA end up in impossibility with every arbitrarily weak WD_n axiom.

Proposition 4 *For any $n \in \mathbb{N}$, there are not SWFs on $\boldsymbol{X} = [0,1]^{\mathbb{N}}$ that verify HE –resp., VWIA– and WD_n.*

Proof: Assume that $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$ satisfies VWIA and WD$_n$. For each $0 < x < 1$ we let

$$L(x) := \mathbf{W}(x, \overset{2n}{\ldots}, x, 0, 0, ...) \text{ and } R(x) := \mathbf{W}\left(\frac{1+x}{2}, \overset{n}{\ldots}, \frac{1+x}{2}, x, \overset{n}{\ldots}, x, 0, 0, ...\right)$$

thus $I(x) := (L(x), R(x)) \neq \varnothing$ due to WD$_n$.[2] A sequential application of VWIA proves that $I(x) \cap I(y) = \varnothing$ for every $\frac{1}{2} > y > x > 0$ (which entails $\frac{1+x}{2} > y > x$) because

$$L(y) = \mathbf{W}(y, \overset{2n}{\ldots}, y, 0, 0, ...) > \mathbf{W}\left(\frac{1+x}{2}, y, \overset{n-1}{\ldots}, y, x, y, \overset{n-1}{\ldots}, y, 0, 0, ...\right) >$$
$$> \mathbf{W}\left(\frac{1+x}{2}, \frac{1+x}{2}, y, \overset{n-2}{\ldots}, y, x, x, y, \overset{n-2}{\ldots}, y, 0, 0, ...\right) > .... >$$
$$> \mathbf{W}\left(\frac{1+x}{2}, \overset{n}{\ldots}, \frac{1+x}{2}, x, \overset{n}{\ldots}, x, 0, 0, ...\right) = R(x)$$

This produces an uncountable number of different rational numbers, a contradiction.

Suppose now that $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$ satisfies HE and WD$_n$. We proceed to assign an uncountable number of different rational numbers, which yields the contradiction. For each $0 < x < 1$ we now let

$$l(x) := \mathbf{W}\left(\frac{1+3x}{6}, \overset{n}{\ldots}, \frac{1+3x}{6}, x, \overset{n}{\ldots}, x, 0, 0, ...\right) \quad \text{and}$$

$$r(x) := \mathbf{W}\left(\frac{1+x}{2}, \overset{n}{\ldots}, \frac{1+x}{2}, x, \overset{n}{\ldots}, x, 0, 0, ...\right)$$

thus $J(x) := (l(x), r(x)) \neq \varnothing$ due to WD$_n$. A sequential application of HE proves that $J(x) \cap J(y) = \varnothing$ for every $\frac{1}{3} > y > x > 0$ (which entails $\frac{1+x}{2} > \frac{1+3y}{6} > y > x$) because

$$l(y) = \mathbf{W}\left(\frac{1+3y}{6}, \overset{n}{\ldots}, \frac{1+3y}{6}, y, \overset{n}{\ldots}, y, 0, 0, ...\right) >$$
$$> \mathbf{W}\left(\frac{1+x}{2}, \frac{1+3y}{6}, \overset{n-1}{\ldots}, \frac{1+3y}{6}, x, y, \overset{n-1}{\ldots}, y, 0, 0, ...\right) >$$
$$> \mathbf{W}\left(\frac{1+x}{2}, \frac{1+x}{2}, \frac{1+3y}{6}, \overset{n-2}{\ldots}, \frac{1+3y}{6}, x, x, y, \overset{n-2}{\ldots}, y, 0, 0, ...\right) > .... >$$
$$> \mathbf{W}\left(\frac{1+x}{2}, \overset{n}{\ldots}, \frac{1+x}{2}, x, \overset{n}{\ldots}, x, 0, 0, ...\right) = r(x)$$

$\lhd$

[2] This argument is inspired in the proof of the incompatibility between WD and HE/VWIA in Alcantud and García-Sanz (2010b).

As is apparent from Proposition 4, if we intend to implement the spirit of the Hammond Equity principle then we must discard any myopic version of the Weak Dominance efficiency postulate. Alternatively, if we admit that strict improvements of the endowments of a finite and higher than n number of generations must convey a strictly better social evaluation, then the spirit of the Hammond Equity principle must be contradicted, large as n may seem. We conclude that a compromise between these two spirits (HE and myopic WD) can not be reached by relaxing the efficiency postulate sufficiently.

4.2 Inequality aversion and responsiveness to gains by one or all generations

As was mentioned, when $\mathbf{X} = [0,1]^{\mathbb{N}}$ no evaluation by a SWF that is positively sensitive to a Paretian improvement by one single generation verifies the equivalent HE, VWIA or HE$(a)^+$. In Subsection 4.1 we have checked that adopting a myopic position as to the efficiency postulate does not reverse the conclusion. Nonetheless, the appeal to the following Pigou-Dalton transfer principle permits to resolve this impossibility.

Axiom PDT (*Pigou-Dalton transfer principle*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that there is $\varepsilon > 0$ with $y_j = x_j - \varepsilon \geqslant y_k = x_k + \varepsilon$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$, then $\mathbf{W}(\mathbf{y}) > \mathbf{W}(\mathbf{x})$.

PDT is a notion of inequality aversion in a cardinal vein that has been introduced in this literature by Bossert et al. (2007) –under the name *strict transfer principle*– and Sakai (2006). [3] We proceed to show that the Pigou-Dalton transfer principle and anonymity can be combined with weak dominance under representability of the social evaluation. This is quite remarkable by contrast with the fact that the PDT postulate is in conflict with upper or lower semicontinuity with respect to the sup topology when we use acyclic SWRs (cf., Hara et al. (2008), Theorem 1 and comment in pp. 185). By incorporating the arguments in Sakai (2006) these authors claim that their "impossibility theorems are robust with respect to the choice of continuity axioms" (a fact that is further enhanced by their Section 4, which shows that their impossibility results are valid for a wide class of topologies). Proposition 5 below contributes to qualifying the robustness of this argument as to the rationality assumptions on the evaluation.

Proposition 5 *There are SWFs on* $\boldsymbol{X} = [0,1]^{\infty}$ *that verify PDT, AN and*

[3] The formulation in Bossert et al. (2007) is different but equivalent: if $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $x_j > y_j \geqslant y_k > x_k$ and $x_j + x_k = y_j + y_k$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$, then $\mathbf{W}(\mathbf{y}) > \mathbf{W}(\mathbf{x})$. This version is parallel in structure to the HE-related axioms.

WD.

Proof: The binary relation on $\mathbf{X}$ given by $\mathbf{x} \sim \mathbf{y}$ if and only if $x_i = y_i$ eventually is an equivalence relation. From each equivalence class $[\mathbf{x}]_\sim$ in the quotient set $\frac{\mathbf{X}}{\sim}$ we select a representative $g^{\mathbf{x}}$. We decompose $g^{\mathbf{x}} = (g_1^{\mathbf{x}}, g_2^{\mathbf{x}}, ...)$ as is standard. Thus when $\mathbf{x} = (x_1, ..., x_n, ...), \mathbf{y} = (y_1, ..., y_n, ...)$ are eventually coincident (i.e., there is k such that $x_i = y_i$ for all $i \geqslant k$) one has $g^{\mathbf{x}} = g^{\mathbf{y}}$.

For each $N \in \mathbb{N}$ and $\mathbf{x} \in \mathbf{X}$ we denote

$$A_N(\mathbf{x}) = (g_1^{\mathbf{x}})^2 + ... + (g_N^{\mathbf{x}})^2 - (x_1^2 + ... + x_N^2)$$

$$B_N(\mathbf{x}) = 2(x_1 + ... + x_N - g_1^{\mathbf{x}} - ... - g_N^{\mathbf{x}})$$

For any fixed $\mathbf{x}$ both sequences $\{A_N(\mathbf{x})\}_N$ and $\{B_N(\mathbf{x})\}_N$ are eventually constant thus it is trivial that the function

$$\mathbf{W}_{PD}(\mathbf{x}) = lim_{N\to\infty}(A_N(\mathbf{x})) + lim_{N\to\infty}(B_N(\mathbf{x})) = lim_{N\to\infty}(A_N(\mathbf{x}) + B_N(\mathbf{x}))$$

is well defined and AN. We proceed to check that it verifies PDT and WD.

The Pigou-Dalton transfer principle derives from the following arithmetic property: for every $m_1, m_2 \in \mathbb{R}$ and $\varepsilon > 0$ such that $m_1 - \varepsilon \geqslant m_2 + \varepsilon$ one has

$$(m_1)^2 + (m_2)^2 > (m_1 - \varepsilon)^2 + (m_2 + \varepsilon)^2 \quad (1)$$

Some simple manipulations transform this claim into $2\varepsilon(m_1 - m_2 - \varepsilon) > 0$, which holds true because $m_1 - m_2 \geqslant 2\varepsilon > \varepsilon > 0$. Now select $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ such that there is $\varepsilon > 0$ with $y_j = x_j - \varepsilon \geqslant y_k = x_k + \varepsilon$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$. Then $g^{\mathbf{x}} = g^{\mathbf{y}}$, and $B_N(\mathbf{x}) = B_N(\mathbf{y})$ eventually. Thus the fact that $\mathbf{W}_{PD}(\mathbf{y}) > \mathbf{W}_{PD}(\mathbf{x})$ is equivalent to proving that $A_N(\mathbf{y}) > A_N(\mathbf{x})$ eventually, which holds if we check that for $N > j, k$ the following holds:

$$(g_1^{\mathbf{x}})^2 + ... + (g_N^{\mathbf{x}})^2 - (y_1^2 + ... + y_N^2) > (g_1^{\mathbf{x}})^2 + ... + (g_N^{\mathbf{x}})^2 - (x_1^2 + ... + x_N^2)$$

This trivially amounts to

$$x_1^2 + ... + x_N^2 > y_1^2 + ... + y_N^2$$

and therefore to

$$x_j^2 + x_k^2 > y_j^2 + y_k^2$$

which holds by Equation (1) as applied to $m_1 = x_j$ and $m_2 = x_k$.

To end the proof we check for WD. Because AN has been ensured, it suffices to prove that if $\mathbf{x}$, $\mathbf{y}$ are such that $x_1 > y_1$ and $x_j = y_j$ when $j > 1$, then $\mathbf{W}_{PD}(\mathbf{x}) > \mathbf{W}_{PD}(\mathbf{y})$. Observe that $g^{\mathbf{x}} = g^{\mathbf{y}}$ and thus

$$\mathbf{W}_{PD}(\mathbf{x}) - \mathbf{W}_{PD}(\mathbf{y}) = -x_1^2 + y_1^2 + 2(x_1 - y_1) =$$

$$= 2(x_1 - y_1) - (x_1 - y_1)(x_1 + y_1) = (x_1 - y_1)(2 - (x_1 + y_1)) > 0$$

This completes the proof. $\lhd$

Remark 2 *Proposition 5 assures the existence of representable social welfare relations with PDT and AN –two different ehical considerations that incorporate intergenerational equity in terms of distributive fairness and equal treatment of the generations– plus WD. We are not aware of any prior construction of a representable social welfare relation that verifies the Pigou-Dalton transfer principle.*

Now it is apparent that in the context of PDT social welfare evaluations, imposing continuity restrictions with respect to a wide class of topologies as specified in Hara et al. (2008), Section 4, is much stronger than imposing anonymity, weak dominance, and representability.

Because WP and WD are unrelated, Proposition 5 leaves the following question open: Can we incorporate positive responsiveness to an increase in the endowment of *every* generation under PDT? Proposition 6 answers this question in the negative.

Proposition 6 *There are not SWFs on $\boldsymbol{X} = [0,1]^{\mathbb{N}}$ that verify PDT and WP.* [4]

Proof: If $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$ satisfies PDT and WP then we can assign an uncountable number of different rational numbers, which is impossible, in the following manner. For each $0 < x < \frac{1}{2}$ we fix $0 < \varepsilon_x$ such that $x + \varepsilon_x < \frac{1}{2}$, and then let

$$L(x) := \mathbf{W}\left(1 - x, x, x, ...\right) \quad \text{and } R(x) := \mathbf{W}\left(1 - \frac{x}{2}, x + \varepsilon_x, x + (\varepsilon_x)^2, ...\right)$$

thus $I(x) := (L(x), R(x)) \neq \varnothing$ due to WP. We proceed to prove that $I(x) \cap I(y) = \varnothing$, i.e., $L(y) \geqslant R(x)$, for every $\frac{1}{2} > y > x > 0$.

Associated with x and y we select n_0, the minimum natural number with the property that $n > n_0$ implies $y > x + (\varepsilon_x)^{n-1}$. We also select $y - x - (\varepsilon_x)^{n_0} > \varepsilon > 0$ sufficiently small to allow for the existence of k_1 and k_2, natural numbers with the properties

$$1 - \frac{x}{2} < 1 - y + k_1\varepsilon \leqslant 1 \quad \text{and} \quad x + \varepsilon_x < y + k_2\varepsilon \leqslant 1$$

[4] Observe that the full force of PDT and WP is not used along the proof. Our argument ensures incompatibility of a weaker version of PDT with a weaker version of WP.

A sequential application of (a relaxed version of) PDT proves that

$$L(y) = \mathbf{W}(1-y, y, y, ...) \geqslant$$

$$\geqslant \mathbf{W}(1-y+\varepsilon, y, \overset{n_0-1}{\cdots\cdots}, y, y-\varepsilon, y, y, ...) \geqslant ... \geqslant$$

$$\geqslant \mathbf{W}\left(1-y+k_1\varepsilon, y, \overset{n_0-1}{\cdots\cdots}, y, y-\varepsilon, \overset{k_1}{\cdots\cdots}, y-\varepsilon, y, y, ...\right) \geqslant ... \geqslant$$

$$\geqslant \mathbf{W}\left(1-y+k_1\varepsilon, y+k_2\varepsilon, \overset{n_0-1}{\cdots\cdots}, y+k_2\varepsilon, y-\varepsilon, \overset{k_1+k_2\cdot(n_0-1)}{\cdots\cdots\cdots\cdots}, y-\varepsilon, y, y, ...\right)$$

(Intuitively: we compare streams where a positive amount ε of utility is exchanged between a generation beyond the n_0 threshold and another before it, which preserves their relative ranking. And we do this $k_1 + k_2 \cdot (n_0 - 1)$ many times). Now (a relaxed version of) WP assures that

$$\mathbf{W}\left(1-y+k_1\varepsilon, y+k_2\varepsilon, \overset{n_0-1}{\cdots\cdots}, y+k_2\varepsilon, y-\varepsilon, \overset{k_2\cdot n_0}{\cdots\cdots\cdots}, y-\varepsilon, y, y, ...\right) \geqslant$$

$$\mathbf{W}\left(1-\frac{x}{2}, x+\varepsilon_x, x+(\varepsilon_x)^2, ...\right) = R(x)$$

because $1 - y + k_1\varepsilon > 1 - \frac{x}{2}$, $y + k_2\varepsilon > x + \varepsilon_x > x + (\varepsilon_x)^2 > ....$, and $y > y - \varepsilon > x + (\varepsilon_x)^{n_0} > x + (\varepsilon_x)^{n_0+1} > ....$ This completes the argument. $\triangleleft$

Proposition 5 yields the significant conclusion that representability of the social ordering is compatible with the respect of some inequality aversion and responsiveness to individual increases of utility. We have evidences that this result is not robust with respect to the choice of the efficiency axiom, v.g., Proposition 6. It is interesting to analyze if this situation is robust with respect to the particular form of inequality aversion. This is not the case when we replace PDT with either VWIA or HE, as we proceed to prove in the next Propositions 7 and 8.

Proposition 7 *There are not SWFs on* $\boldsymbol{X} = [0,1]^{\mathbb{N}}$ *that verify VWIA and WP.* [5]

Proof: If $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$ verifies VWIA and WP then we can assign an uncountable number of different rational numbers, which is impossible, in the following manner. Let $\varepsilon = 0.1$, $\delta = 0.2$. For each $0 < x < \frac{1}{2}$ we let

$$L(x) := \mathbf{W}((x)_{con}) \quad \text{and } R(x) := \mathbf{W}\left(x+\varepsilon, x+\varepsilon^2, x+\varepsilon^3, ...\right)$$

thus $I(x) := (L(x), R(x)) \neq \varnothing$ due to WP. We proceed to prove that $I(x) \cap I(y) = \varnothing$ for every $\frac{1}{2} > y > x > 0$.

Case 1: $y > x + \varepsilon$. Then the conclusion follows from WP since $y > x + \varepsilon^n$ for each n thus $L(y) = \mathbf{W}((y)_{con}) \geqslant R(x)$.

[5] Again, our argument ensures incompatibility of VWIA with a weaker version of WP.

Case 2: $x+\varepsilon \geqslant y$. Now we select z, a number strictly between x and y, such that $x+\varepsilon > x+\varepsilon^2 > ... > x+\varepsilon^n > z > x+\varepsilon^{n+1} > ...$ for some n (that is: a number $y > z > x$ that does not coincide with any $x+\varepsilon^n$). We use the trivial consequence $z > \frac{z+x+\varepsilon^{m+1}}{2} > x+\varepsilon^{m+1}$ for each $m = n, n+1, ...$. A sequential application of VWIA proves that

$$L(y) \geqslant \mathbf{W}\left((z)_{con}\right) \overset{(\dagger)}{\geqslant} \mathbf{W}\left(x+\delta, z, \overset{n-1}{.........}, z, \frac{z+x+\varepsilon^{n+1}}{2}, z, ...\right) \overset{(\ddagger)}{\geqslant}$$

$$\geqslant \mathbf{W}\left(x+\delta, x+\varepsilon, z, \overset{n-2}{.........}, z, \frac{z+x+\varepsilon^{n+1}}{2}, \frac{z+x+\varepsilon^{n+2}}{2}, z, ...\right) \geqslant ... \geqslant$$

$$\geqslant \mathbf{W}\left(x+\delta, x+\varepsilon, ..., x+\varepsilon^{n-1}, \frac{z+x+\varepsilon^{n+1}}{2}, ..., \frac{z+x+\varepsilon^{2n}}{2}, z, z, ...\right)$$

To be precise, inequality (†) derives from $x+\delta > x+\varepsilon > z > \frac{z+x+\varepsilon^{n+1}}{2}$, (‡) derives from $x+\varepsilon > z > \frac{z+x+\varepsilon^{n+2}}{2}$, and so forth. Now WP assures that

$$\mathbf{W}\left(x+\delta, x+\varepsilon, ..., x+\varepsilon^{n-1}, \frac{z+x+\varepsilon^{n+1}}{2}, ..., \frac{z+x+\varepsilon^{2n}}{2}, z, z, ...\right) \geqslant$$

$$\mathbf{W}\left(x+\varepsilon, x+\varepsilon^2, x+\varepsilon^3, ...\right) = R(x)$$

This completes the argument. ◁

Proposition 8 *There are not SWFs on* $\boldsymbol{X} = [0,1]^{\mathbb{N}}$ *that verify HE and WP.* [6]

Proof: If $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$ verifies HE and WP then we can assign an uncountable number of different rational numbers, which is impossible, in the following manner. Let $\varepsilon = 0.1$, $\delta = 0.2$. For each $0 < x < \frac{1}{2}$ we let

$$L(x) := \mathbf{W}\left(x+\varepsilon^2, x+\varepsilon^3, x+\varepsilon^4, ...\right), \quad R(x) := \mathbf{W}\left(x+\varepsilon, x+\varepsilon^2, x+\varepsilon^3, ...\right)$$

thus $I(x) := (L(x), R(x)) \neq \varnothing$ due to WP. We proceed to prove that $I(x) \cap I(y) = \varnothing$, or $\mathbf{W}(y+\varepsilon^2, y+\varepsilon^3, y+\varepsilon^4, ...) \geqslant \mathbf{W}(x+\varepsilon, x+\varepsilon^2, x+\varepsilon^3, ...)$, for every $\frac{1}{2} > y > x > 0$.

Case 1: $y+\varepsilon^2 > x+\varepsilon$, i.e., $y-x > \varepsilon-\varepsilon^2$. Then $y+\varepsilon^{n+1} > x+\varepsilon^n$ follows from trivial computations for each $n = 1, 2, ...$, thus WP yields the thesis.

Case 2: $y+\varepsilon^2 \leqslant x+\varepsilon$. Let $m > 1$ denote the first index for which $y+\varepsilon^{m+1} > x+\varepsilon^m$. This number is well defined because $\lim_k(y+\varepsilon^{k+1}) = y > x = \lim_k(x+\varepsilon^k)$. Observe that $y+\varepsilon^{n+1} > x+\varepsilon^n$ for each $n > m$ too, because $y-x > \varepsilon^m(1-\varepsilon) > \varepsilon^n(1-\varepsilon)$ for all $n > m$. We use the trivial consequence $y+\varepsilon^{n+1} > \frac{1}{2}(y+\varepsilon^{n+1}+x+\varepsilon^n) = \frac{x+y+\varepsilon^n(1+\varepsilon)}{2} > x+\varepsilon^n$ for each $n \geqslant m$.

[6] Again, our argument ensures incompatibility of HE with a weaker version of WP.

A sequential application of HE proves that

$$L(y) \overset{(\dagger)}{\geqslant} \mathbf{W}\left(x+\delta, y+\varepsilon^3, y+\varepsilon^4, ..., y+\varepsilon^m, \frac{x+y+\varepsilon^m(1+\varepsilon)}{2}, y+\varepsilon^{m+2}, ...\right) \overset{(\ddagger)}{\geqslant}$$

$$\mathbf{W}\left(x+\delta, x+\varepsilon, y+\varepsilon^4, ..., y+\varepsilon^m, \frac{x+y+\varepsilon^m(1+\varepsilon)}{2}, \frac{x+y+\varepsilon^{m+1}(1+\varepsilon)}{2}, y+\varepsilon^{m+3}, ...\right)$$

$$\geqslant ... \geqslant$$

$$\geqslant \mathbf{W}\left(x+\delta, x+\varepsilon, ..., x+\varepsilon^{m-2}, \frac{x+y+\varepsilon^m(1+\varepsilon)}{2}, ..., \frac{x+y+\varepsilon^{2m-2}(1+\varepsilon)}{2}, y+\varepsilon^{2m}, ...\right)$$

To be precise, inequality (†) derives from $x+\delta > x+\varepsilon \geqslant y+\varepsilon^2 > y+\varepsilon^{m+1} > \frac{x+y+\varepsilon^m(1+\varepsilon)}{2}$, (‡) derives from $x+\varepsilon \geqslant y+\varepsilon^2 > y+\varepsilon^3 > y+\varepsilon^{m+2} > \frac{x+y+\varepsilon^{m+1}(1+\varepsilon)}{2}$, and so forth. Now WP assures that

$$\mathbf{W}\left(x+\delta, x+\varepsilon, ..., x+\varepsilon^{m-2}, \frac{x+y+\varepsilon^m(1+\varepsilon)}{2}, ..., \frac{x+y+\varepsilon^{2m-2}(1+\varepsilon)}{2}, y+\varepsilon^{2m}, ...\right) \geqslant$$

$$\mathbf{W}\left(x+\varepsilon, x+\varepsilon^2, x+\varepsilon^3, ...\right) = R(x)$$

because $x+\delta > x+\varepsilon$, $x+\varepsilon > x+\varepsilon^2$, ... , and $\frac{x+y+\varepsilon^m(1+\varepsilon)}{2} > x+\varepsilon^m$, ..., $\frac{x+y+\varepsilon^{2m-2}(1+\varepsilon)}{2} > x+\varepsilon^{2m-2}$, $y+\varepsilon^{2m} > x+\varepsilon^{2m-1}$, ... This completes the argument. ◁

5 Concluding remarks

We have studied SWFs on $\mathbf{X} = [0,1]^{\mathbb{N}}$ in order to establish trade-offs between the number of generations that suffice to improve the social welfare, and two general classes of consequentialist equity axioms. The first class admits a gradation in terms of how large is the extended present (in order to compare its interest and the interest of its future). The second class includes variations of the Hammond Equity principle that express aversion to inequality. Our findings for social welfare functions are summarized as follows:

- Proposition 1 concerns how far we can extend the interest of the present. It puts bounds to the compromise between the interest of the extended present with respect to both efficiency (as myopic versions of strong Pareto or weak dominance) and egalitarianism as Restricted Non-Substitution. Together with Proposition 3, it establishes that if anonymity is imposed then we have total incompatibility independently of which compromise we propose. Proposition 2 discards any conjunction of anonymity and myopic versions of the strong Pareto principle in any non-trivial setting, which complements the celebrated result by Basu and Mitra (2003) that anonymity and strong Pareto are incompatible.

- Neither Hammond Equity nor a variation of it called VWIA, which are equivalent under weak dominance, are compatible with any arbitrarily weak version of WD_n. Both are incompatible with WP too (cf., Propositions 7 and 8). Thus the distributive fairness that they capture is not compatible with positive responsiveness to increases of the utilities of any fixed finite number of generations, or of all generations.
- We are not aware of any direct proof that SWFs can implement the Pigou-Dalton transfer principle. Our analysis presents a construction of a social welfare function that verifies such axiom, plus anonymity and weak dominance (cf., Proposition 5). This is in striking contrast with the discussion above. However we have proved that no SWF can verify the Pigou-Dalton transfer principle and Weak Pareto simultaneously.

It has been argued that "continuity and representability can be considered rather demanding in infinite-horizon settings" and that "impossibility results [....] can be avoided if continuity or representability assumptions are dispensed with" (Bossert et al., 2007, p. 588). It is nonetheless true that other routes of escape have been brought to light. Alcantud and García-Sanz (2010a, b) advocate for the use of appropriate domain restriction in order to keep representability when $[0,1]^{\mathbb{N}}$ excludes it, in line with prior studies like Basu and Mitra (2007). Besides, our Propositions 1 and 5 add to other evidences (e.g., Basu and Mitra, 2007, Theorem 5) to conclude that incompatibility of representability and attractive sets of axioms must not be universally presumed.

Hara et al. (2008) have proposed a taxonomy for relevant results on the evaluation of infinite utility streams. By invoking it they have characterized various results by suitable combinations of three crucial criteria: (1) rationality properties of the social evaluation, (2) robustness of the ranking to "small" perturbations of the infinite streams, and (3) sensitivity conditions, in the form of (i) sensitivity to efficiency, (ii) sensitivity to equity. We adopt this logical scheme to compare our results with others from the literature. In order to better fulfil this aim we add another criteria, namely (0) the structure of the set of utility streams, viz., if it is any non-trivial setting, $[0,1]^{\mathbb{N}}$, l_∞, or the like. Two tables gather our taxonomical attempt. Table 1 helps to compare results when the Pigou-Dalton transfer principle is assumed. In Table 2 other equity principles are imposed under representability.

In Table 1, HSSX1 means Hara et al. (2008), Theorem 1 (see also comment in page 185). It concerns upper/lower semicontinuous relations with respect to the sup topology, and l_+^∞ means the bounded sequences with non-negative values in every component. BSS1 means Bossert et al. (2007), Theorem 1. In fact this Theorem identifies all the complete preorders that verify the properties in Criteria (3). A5 and A7 are Propositions 5 and 7 here. With respect

Table 1
Results relating to inequality aversion. I holds for Impossibility, P holds for Possibility.

	(0)	(1)	(2)	(3)
HSSX1: I	$l_+^\infty \subseteq \mathbf{X}$, $\mathbf{X} + l_+^\infty \subseteq \mathbf{X}$	Acyclicity	Usc/Lsc wrt sup topol.	(i) None (ii) PDT
BSS1: P	$\mathbb{R}^\mathbb{N}$	Compl. preorder	None	(i) SP (ii) PDT, AN
A5: P	$[0,1]^\mathbb{N}$	Representability	None	(i) WD (ii) PDT, AN
A6: I	$[0,1]^\mathbb{N}$	Representability	None	(i) WP (ii) PDT
A7: I	$[0,1]^\mathbb{N}$	Representability	None	(i) WP (ii) VWIA
A8: I	$[0,1]^\mathbb{N}$	Representability	None	(i) WP (ii) HE

to BSS1, A5 states that we can gain representability at the cost of efficiency. A6, i.e., Proposition 6 here, clarifies that WD can not be replaced by WP (or improved to PP) in Proposition 5 even if we forgo AN. A7/Proposition 7, resp. A8/Proposition 8, specify that A6 remains true if distributive fairness is implemented via VWIA, resp. HE, instead of via PDT.

In Table 2, B1 is Banerjee (2006), Theorem 1. AG1 is Alcantud and García-Sanz (2010a), Theorem 1. By comparison it proves that in its setting, possibility is reached with better performance both of efficiency and equity. A comparison can be made between B1 and A1, that is, Proposition 1 here. By Proposition 1 (b), impossibility is preserved when HEF is reinforced to mRNS and WD is weakened to WD_n if $n \leqslant m$, the other case yielding trivial compatibility with the SP^n axiom (Proposition 1 (a)). A3 (i.e., Proposition 3) furnishes the information that generations are not treated equally in the representations that case A1 allows for. A2 (i.e., Proposition 2) completes the analysis: it adds the information that anonymity can not obtain in the SP^n representations that case A1 allows for, even if mRNS is dropped and the setting is the simplest non-trivial one.

Table 2
Results for other equity axioms under representability. I holds for Impossibility, P holds for Possibility.

	(0)	(1)	(2)	(3)
B1: I	$[0,1]^{\mathbb{N}}$	Representab.	None	(i) WD (ii) HEF
AG1: P	$\mathbf{X} = Y^{\mathbb{N}}$, $Y \subseteq \mathbb{N}^*$	Representab.	None	(i) PP (ii) RNS, AN
A1: I $(n \leqslant m)$ P $(n > m)$	l_∞ $(n \leqslant m)$ $[0,1]^{\mathbb{N}}$ $(n > m)$	Representab.	None	(i) WD_n (SP^n if $n > m$) (ii) mRNS
A3: I	$[0,1]^{\mathbb{N}}$	Representab.	None	(i) WD_n (ii) mRNS, AN
A2: I	Non-trivial	Representab.	None	(i) SP^n (ii) AN

6 Bibliography

Alcantud, J.C.R. and García-Sanz, M.D. (2010a): Paretian evaluation of infinite utility streams: an egalitarian criterion. Economics Letters 106, 209-211.

Alcantud, J.C.R. and García-Sanz, M.D. (2010b): Evaluations of infinite utility streams: Pareto-efficient and egalitarian axiomatics. MPRA Paper No. 20133, http://mpra.ub.uni-muenchen.de/20133/

Banerjee, K. (2006): On the equity-efficiency trade off in aggregating infinite utility streams. Economics Letters 93, 63-67.

Basu, K. and Mitra, T. (2003): Aggregating infinite utility streams with intergenerational equity: the impossibility of being paretian. Econometrica 71, 1557-1563.

Basu, K. and Mitra, T. (2007): Possibility theorems for aggregating infinite utility streams equitably. In: Roemer, J., Suzumura, K. (Eds.), Intergenerational Equity and Sustainability: Conference Proceedings of the IWEA Roundtable Meeting on Intergenerational Equity (Palgrave).

Bossert, W., Sprumont, Y. and Suzumura, K. (2007): Ordering infinite utility streams. Journal of Economic Theory 135, 579-589.

Hara, C., Shinotsuka, T., Suzumura, K. and Xu, Y. (2008): Continuity and egalitarianism in the evaluation of infinite utility streams. Social Choice and Welfare 31, 179-191.

Sakai, T. (2006): Equitable intergenerational preferences on restricted domains. Social Choice and Welfare 27, 41-54.