Costly Information, Planning Complementarity and the New Keynesian Phillips Curve

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14 April 2010

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This Version: 13 April 2010, Preliminary

Abstract

I show that in a setting with costly information processing, strategic complementarity in pricing, by generating planning complementatrities, results in the aggregate price responding slowly to nominal shocks even though individual firm prices change by large amounts in response to idiosyncratic shocks. Klenow and Kryvtsov (2008) conclude that none of the commonly used pricing models is capable of matching all the facts from micro data and at the same time generate a large and persistent response to monetary policy. Unlike the standard state dependent pricing models which rely on physical costs of changing prices to generate unresponsiveness of prices, I instead focus on costs of planning and processing information, a channel which researchers have found empirically more important than physical costs of changing prices in determining pricing decisions of firms. The model is able to match all the features of micro pricing data and at the same time generates a sluggish response of aggregate price to monetary policy, thus predicting a short run Phillips curve. Also, the model generates firms behavior in which they set price plans rather than prices and also shows that firms may choose to index prices to long run inflation optimally as is often assumed in New-Keynesian models. The paper highlights the fact that to explain non-neutrality in the short run, prices need not be sticky, it is just that they do not contain all the information in the short run but become informationally efficient in the long run resulting in a long run neutrality result.

Keywords: Planning Complementarity, Price Rigidity, Costly Information Acquisition, Real Effects of Nominal Shocks, Forecasting, Strategic Complementarity

1 Introduction

One of the oldest questions in macroeconomic theory is whether nominal rigidities are important. Prices and wages do not immediately adjust proportionally in the *short run* to changes in nominal expenditures and hence the issue of why prices are sticky (or if they are *sticky* at all) has been a hugely debated issue. A vast theoretical literature tries to explain the existence of sticky prices. *Sticky* prices are given so much importance because it is believed that the monetary transmission mechanism operates through aggregate price not responding fully and immediately to increases in money supply, thus creating increased economic activity in the short run. Much recent research has studied the frequency of price changes and whether *sticky* prices are important for the dynamics of output.

The answer to whether *sticky* prices is the correct channel through which monetary policy has real effects in the short run, is not very clear. Recent microeconomic pricing studies such as Bils and Klenow (2004) show that the median non-housing consumer price changes at a frequency less than once every 4.3 months. This is slightly more than a quarter and suggests that prices are not very sticky. Further micro level pricing evidence from Klenow and Kryvtsov (2008) shows that conditional on a price change, the size of a price change is large in absolute terms, at about 13 percent if one includes sales or 8.5 percent if sales are removed. Thus, micro level pricing behavior does not support the *sticky* price story. However, many studies at the macro level do support a role for sticky prices in the monetary transmission mechanism. As Christiano et al. (1999) point out, while many different identification schemes have been used to identify monetary policy shocks, there is considerable agreement over the qualitative effects of a monetary policy shock:

> *"The nature of this agreement is as follows: after a contractionary monetary policy shock, short term interest rates rise, aggregate output, employment, profits and various monetary aggregates fall, <u>the aggregate price level responds very slowly</u>, and various measures of wages fall, albeit by very modest amounts." - pg 7,Christiano et al. (1999)*

Other research, such as Uhlig (2005), provides evidence of sluggish adjustment of aggregate prices. According to Uhlig (2005), only about 25 percent of the long-run response of the U.S. GDP price deflator to a monetary policy shock occurs within the first year after the shock, hence indicating a sluggish response of aggregate prices to monetary policy shocks.

Existing standard models in macroeconomics cannot match both macro and micro facts convincingly. The standard time dependent, in the tradition of Calvo (1983) and Yun (1996) can explain the sluggish price level only if firms change prices infrequently and by *small* amounts. These requirements are not satisfied in micro data as documented above. The other popular alternative is the state dependent *menu* cost models such as Dotsey et al. (1999) and more recently Midrigan (2008) and Golosov and Lucas Jr. (2007). Golosov and Lucas Jr. (2007) calibrated using micro pricing data, find that monetary shocks do not induce large or persistent real responses. Thus, standard macroeconomic models of pricing behavior are incapable of matching simultaneously both micro and macro facts from the data. Less than a handful of new research is able to account for volatile prices at the firm level and at the same time sluggish aggregate price so that the sticky price hypothesis has some bite and at the same time the micro level pricing evidence is satisfied.

In this paper, I explore the idea that price stickiness is due to informational constraints for a firm and show that in a setting where there is imperfect information, firms will optimally choose not to change prices *fully* in response to aggregate shocks such as monetary expansions. This makes the response of aggregate prices to aggregate shocks sluggish and this results in in a hump shaped response of real aggregate output. The idea that monetary policy is non-neutral in the short to medium run due to imperfect information is not new and goes as far back as Phelps (1970) and Lucas Jr. (1973). Lucas Jr. (1973) suggests that lack of information about monetary policy shocks explains why prices are slow to adjust their prices to these shocks. A common criticism of Lucas Jr. (1973) is that news about monetary policy shocks is available with little delay, and so the Lucas model cannot explain persistent real effects of monetary policy shocks. Thus, an explanation was required why firms choose not to observe publicly available information. Recent work following Sims (1998) introduces the concept of *Rational Inattention* to incorporate agents with limited information processing capabilities into standard models. Because these agents have a limited information processing capacity, they rationally ignore some information that is publicly available. This idea has been extended in work by Woodford (2002), who presents a model in which strategic complementarity in price setting can generate large and persistent real effects of a nominal shock. Recently, Mackowiak and Wiederholt (2009) show that it is optimal for agents to pay less attention to aggregate shocks than to idiosyncratic shocks and use this mechanism to explain why prices at the micro level may be volatile even while aggregate prices respond sluggishly to nominal aggregate demand shocks resulting in real effects on output. This paper adopts the approach in which the firm does not know the true realization of the monetary shock because it chooses not to observe it rather than assuming that firms do not have access to this information.

Another branch of literature which invokes imperfect information to explain the behavior of prices is the *sticky information* literature (Mankiw and Reis (2002) and Reis (2007)), which builds on an idea similar to Calvo (1983) by assuming that only a fraction of firms get perfect information every period. As a result of this *sticky* information, only a fraction of firms have full information when making their pricing decision and this results in prices adjusting slowly to shocks. However, this literature cannot explain differential price adjustment to aggregate and idiosyncratic shocks.

The idea that the explanation for the slow response of prices to nominal shocks lying in information has a lot of support. As Radner (1992) points out, a large proportion of the workforce employed in American firms is employed for the purpose of information processing. Also, direct support that firms do not change prices due to the costs of processing information continuously can be seen in the work by Zbaracki et al. (2004). The authors identify and measure three types of managerial costs: information gathering, decision making and communication costs and find that the managerial costs are quantitatively much more important than *physical* costs of changing prices. These facts suggest that menu cost models attribute a misplaced importance to the physical costs of changing prices and ignore the more important costs related to gathering and processing information to come up with new prices. In this paper I formally try to incorporate this idea that firms have to incur a cost to acquire and process information to come up with a new price. To capture this idea, I assume that each firm faces a fixed cost if they decide to *plan*. Thus, the model can be thought of as providing micro foundations to the menu costs model. By possibly being able to fit data better than standard menu cost models such as Golosov and Lucas Jr. (2007), who find that on calibrating the model to match the micro pricing facts, the model predicts a small and transitory response to monetary shocks, this model suggests that the current state dependent models might be looking at the wrong place for costs of changing prices.

This paper builds on all the above mentioned literature and can be thought of being closest to the *sticky information*[1] augmented with strategic motives in acquiring information. The combination of endogenous information choice with strategic complementarity in pricing results in strategic complementarity in information acquisition (Hellwig and Veldkamp (2009)) and I exploit this feature to be able to not only generate a smaller response in magnitude of the price change to the same size of the aggregate monetary shock as compared to an idiosyncratic shock. In addition I am able to also endogenize to a certain extent the frequency of price changes: firms optimally choose to incorporate information about the aggregate shock into prices less often than they incorporate idiosyncratic shocks into prices and hence, aggregate price move in a sluggish manner. Thus, the model generates a short

[1] Most of the sticky information literature does not model information choice endogenously. Reis (2007) is an exception which models the information choice endogenously and shows that this constitutes a micro-foundation to the usual sticky information arrival assumption.

run Phillips curve allowing for inflation in the short run to result in higher economic activity but no such trade-off in the long run.

The main mechanism through which I am able to explain a differential adjustment of price changes, both in terms of magnitude and frequency, in response to aggregate nominal and idiosyncratic shocks is that there is a strategic complementarity associated in acquiring information about the aggregate state but not so about the idiosyncratic state. When a firm i gets new information about a positive nominal aggregate shock, its response in terms of a price change is not only dependent on the true aggregate state, it is also depend on how other firms react to the nominal aggregate shock. If other firms do not adjust their price then it is not be optimal for firm i to do so as increasing the price would actually make its price above the price of the others and hence this results in a loss in profits due to loss in demand. This can be seen in terms of the fact that the monetary shock is a public signal and so on observing a public signal entails not just determining how to respond to this information but also how other are going to respond to it. This is exactly what distinguishes a firms response to an idiosyncratic shock. Information about the firm's own idiosyncratic state is a private signal and so since no one else observes it, the firms only needs to concentrate on how to respond to it individually and not worry about how other respond to it. I solve for a staggered equilibrium[2], i.e. one in which all firms optimally choose not to update their information at the same time. At each instant only a fixed fraction of firms choose to update their information sets. In such a staggered setting, a new aggregate shock is not observed by all firms when it occurs and hence the firms that do observe it are forced to temper their price change to incorporate the fact that a large fraction of firms is still uninformed about this shock and their prices will not respond to this. This results in less than proportional increases in price on impact of the monetary shock. Prices catch up eventually when every one updates their information sets and is informed about the shock. On the other hand, an idiosyncratic shock is met by a proportional change as the response of others to a shock that they don't observe is not a concern, and so even by changing prices in response to idiosyncratic shocks is not going to move it way out of line with the average price. The frequency aspect can be thought of in the same way; firms can delay their decision to update their information about the aggregate state because at any instant a large fraction of firms is uninformed about the true aggregate state and hence the private loss from being uninformed is not big. This motive is not present in the case of updating in formation about the idiosyncratic state as the loss from being uninformed about the idiosyncratic state does not depend on how well informed others are about it. Thus, firms tend to update their information about the idiosyncratic state more often than about the aggregate state and hence, prices of firms incorporate

[2] I discuss the possibility of other equilibria in the following section.

new information about the idiosyncratic shock more often. This allows prices to change frequently and because information about the aggregate shock is incorporated slowly into prices, the response of prices to nominal shocks can be sluggish. The reason why firms do not update their information sets at all times is because of the non-convex costs of updating the information set. This, as in the large literature on menu costs, results in firms only updating their information infrequently.

The model presented in this paper is one of the few which can reconcile the seemingly contradictory macro and micro level pricing evidence. The model predicts the presence of a trade-off between inflation and output in the *short run* but not in the *long run*, hence it predicts a perfectly vertical *Long Run Phillips Curve* but not *short run Phillips curve* which is not perfectly vertical. To the extent of my knowledge, this is one of the few papers which tries to endogenously model the frequency of price changes. The model is consistent with evidence from various studies. By allowing for frequently changing and volatile prices it matches evidence presented in Bils and Klenow (2004) and Klenow and Kryvtsov (2008). At the same time, by being able to generate a sluggish response of aggregate price to monetary shocks, it is in good standing with a large macro literature which claims that the sluggish response of prices in the short run results in real effects of monetary shocks in the short run. The relevance of this literature had come into question with the micro level pricing evidence which suggested that prices were volatile and changed frequently. In addition, the mechanism presented in this paper, unlike other papers such as Mackowiak and Wiederholt (2009), does not require that idiosyncratic shock are relatively more volatile than aggregate shocks. In fact, I show that, even if the *cost of planning* for and the volatility of the aggregate and idiosyncratic states is the same, firms will optimally choose to update their information set about the idiosyncratic shock more often and hence, prices will reflect new information about the idiosyncratic shock more frequently and hence change frequently in response to idiosyncratic shocks but respond sluggishly to monetary policy shocks.

Standard New Keynesian macro models which model price stickiness using time dependent models such as Gali and Gertler (19990) often assume that the non-adjusting firms prices are indexed to lagged or average inflation. Kryvtsov and Kichian (2008) state that adding these features is motivated by methodological convenience as the model then fits the data better. This paper provides micro foundations for such an assumption. In Section 4, I show that when the average long run inflation is positive, the firms that are not updating their information set, choose to endogenously index their price to this long run inflation.

Also, this paper is capable of matching some important regularities seen in price data as laid out in Mankiw and Reis (2010). Mankiw and Reis (2010) state that a few facts from data stand out: that firms change prices *all the time*, firms set price schedules over time rather than prices at each instant and sometimes theses schedules are flat. This paper is able to capture all these aspects. The paper is also consistent to a certain extent with studies such as Blinder et al. (1998) and Zbaracki et al. (2004) who used interviews with firm managers to determine what the reasons were for firms pricing behavior. Blinder et al. (1998) find evidence that suggests that managers set price plans rather than attempt to determine the optimal price at each instant. Extensions of the basic setup are capable of accounting for firms setting price plans over time rather than prices at an instant. This is similar to Burstein (2006) except that this paper does not have to rely on physical costs of changing price schedules like that paper.

This paper is a full dynamic stochastic general equilibrium model in the sticky information tradition and follows others like Mankiw and Reis (2006) and Mankiw and Reis (2007). Also, unlike previous attempts to incorporate sticky information into a DSGE setting as in Knotek-II (2006), I do not assume that firms face a cost in changing prices. In the basic model presented, firms can change prices at any instant but choose not to do so optimally. The paper is arranged as follows. In the next section, I present the basic model which allows me to derive closed form or analytical solutions for most of the results. In section 3, I discuss the results from the previous section. In section 4, I use some other specifications to make some additional points. I conclude in Section 5.

2 Model

The model combines features from Golosov and Lucas Jr. (2007), Hellwig and Veldkamp (2009) and Reis (2007). Time is continuous[3]. At each instant, the economy consists of continuum of identical consumers and a unit mass of monopolistically competitive firms indexed by $i \in [0, 1]$ which produce differentiated goods. Each of the households are infinitely lived and consume goods produced by each monopolistically competitive firm. I do not model entry or exit of firms and hence, the mass of firms remains constant over time.

The economy is subject to two kinds of shocks: a monetary shock and a firm specific idiosyncratic shock. The log of nominal money supply is assumed to follow a Brownian motion with drift μ and variance σ_m^2.

$$d \ln M(t) = \mu dt + \sigma_m dW(t) \tag{1}$$

where $W(t)$ is a standard Brownian motion.

[3]The use of continuous time is only to avoid a integer problem in the choice of the optimal planning horizons which enables one to find unique planning horizons. A working paper of Hellwig and Veldkamp (2009) shows how a discrete time setup may result in multiple equilibria. The use of continuous time helps one get a unique staggered equilibrium.

Firm specific productivity shocks $Z_i(t)$ are assumed to be independent and identical across firms and follows the mean reverting Ornstein-Uhlenbeck[4] process with 0 drift, η the rate of mean reversion and variance σ_z^2 as in Golosov and Lucas Jr. (2007).

$$d\ln Z_i(t) = -\eta \ln Z_i(t)dt + \sigma_z B_i(t) \tag{2}$$

where $B_i(t)$ is a standard Brownian motion such that for $j \neq j'$, B_j and $B_{j'}$ are independent and also B_j is independent of W, $\forall j \in [0,1]$.[5]

2.1 Representative Household's Problem

Each consumer enjoys utility from consumption of a final good, leisure and from holding real balances. I introduce the dis-utility from labor entering the utility in a linear fashion as in Hansen (1985).The representative household's problem can be written as choosing the sequence $\left\{C(t), n(t), \frac{M^D(t)}{P(t)}\right\}_{t=0}^{\infty}$ to maximize

$$E_0\left\{\int_0^{\infty} e^{-\rho t}\left[\frac{C(t)^{1-\gamma}}{1-\gamma} - \alpha n(t) + \ln\left(\frac{M^D(t)}{P(t)}\right)\right]dt\right\}$$

subject to its budget constraint

$$M(0) \geq E_0\left\{\int_0^{\infty} Q(t)[P(t)c(t) + R(t)M^D(t) - \omega(t)n(t) - \Pi(t)]dt\right\} \tag{3}$$

where $Q(t)$ is the shadow price of nominal cash flows, $\Pi(t)$ the nominal profits from firms and lump sum transfers. $R(t)$ is the nominal interest rate and satisfies the following

$$Q(t) = e^{R(t)dt}E_t\{Q(t+dt)\}$$

Also, $C(t)$ is the *Dixit-Stiglitz* consumption aggregator

$$C(t) = \left[\int_0^1 c_i(t)^{\frac{\epsilon-1}{\epsilon}}di\right]^{\frac{\epsilon}{\epsilon-1}}$$

which aggregates consumption of a continuum of goods indexed $i \in [0,1]$, each produced by one of the continuum of monopolistically competitive firms which operate in the market.

The first-order conditions with respect to $C(t)$, $n(t)$ and $M^D(t)$ can be written as

$$e^{-\rho t}C(t)^{-\gamma} = \lambda Q(t)P(t) \tag{4}$$

$$e^{-\rho t}\alpha = \lambda Q(t)\omega(t) \tag{5}$$

$$e^{-\rho t}\frac{1}{M^D(t)} = \lambda Q(t)R(t) \tag{6}$$

[4] $\eta \geq 0$ is the rate of mean reversion and so for $\eta = 0$, the process is a Brownian motion with 0 drift and variance σ_z^2.

[5] Earlier work such as Lach and Tsiddon (1992) and Golosov and Lucas Jr. (2007) argue for the need of an idiosyncratic shock to be able to match the characteristics of price changes seen in firm level pricing data.

where λ is the multiplier on (3) and is independent of time. Equations (5) and (6) imply the following relationship between equilibrium the wage rate $\omega(t)$ and money supply $M(t)$:

$$\omega(t) = \alpha R(t)M(t) \tag{7}$$

Thus, $\ln \omega(t)$ is also a Brownian motion with variance σ_m^2. Also, equations (4) and (6) imply that the level of consumption each period is given by

$$C(t) = \left(\frac{R(t)M(t)}{P(t)}\right)^{\frac{1}{\gamma}} \tag{8}$$

Given (8), the demand for the i-th consumption good an be written as:

$$c_i(t) = C(t)\left(\frac{P_i(t)}{P(t)}\right)^{-\epsilon} \tag{9}$$

where

$$P(t) = \left[\int_0^1 P_i(t)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} \tag{10}$$

It is shown in Appendix A.1 that an equilibrium with a constant nominal interest rate exists. In equilibrium, the constant nominal interest rate is given by

$$R(t) = \rho + \mu - \frac{\sigma_m^2}{2} \text{ ,}\forall t \tag{11}$$

2.2 Firm's Problem

Each monopolistically firm i's production technology can be described by a decreasing returns to scale production function

$$y_i(t) = AZ_i(t)L_i(t)^{\theta} \tag{12}$$

where $A > 0$ is a constant and $\theta \in (0, 1]$. $Z_i(t)$ is the firm specific idiosyncratic productivity shock and $L_i(t)$ is the amount of labor the firm hires labor in an economy-wide labor market from households at a wage rate $\omega(t)$ at date t. Firm i's nominal profit at date t can be written as

$$\pi_i(t) = P_i(t)C_i(t) - \omega(t)L(t)i = P_i(t)C_i(t) - \omega(t)\left(\frac{C_i(t)}{AZ_i(t)}\right)^{\frac{1}{\theta}}$$

Zbaracki et al. (2004) find that costs of acquiring information and planning to incorporate this information into the pricing decision are very important in a firms decision not to change prices often. The authors find that such costs are quite large and quantitatively more important than the physical costs associated with changing prices, and thus, have an impact on the frequency with which firms change their prices. To capture this idea, I assume that each firm faces a fixed labor cost F_m , if they decide to "plan" about the aggregate state/monetary shock and F_z if they "plan" about their idiosyncratic state. The firm then chooses its process of prices $\{P_i(t)\}_{t=0}^{\infty}$, and a process of planning dates $D_i^a(t)$, where $dD_i^a(t) = 1$ if the firm decides to plan about the aggregate state at date t, and $dD_i^a(t) = 0$ otherwise, and a process of planning dates $D_i^z(t)$ where $dD_i^z(t)$ is defined in the same way, so as to maximize its expected discounted profits[6]:

$$E_0^i \left\{ \int_0^\infty Q(t)\pi_i(t)dt - F_m \int_0^\infty Q(t)\omega(t)dD_i^a(t) - F_z \int_0^\infty Q(t)\omega(t)dD_i^z(t) \right\}$$

taking as given $\{P(t), Q(t), \omega(t), c(t)\}_{t=0}^{\infty}$ and its information set at date 0. This Since,

$$Q(t)\pi_i(t) = \frac{e^{-\rho t}}{\lambda} \left[\left(\frac{RM(t)}{P(t)}\right)^{\frac{1}{\gamma}-1} \left(\frac{P_i(t)}{P(t)}\right)^{1-\epsilon} - \frac{\alpha}{[AZ_i(t)]^{\frac{1}{\theta}}} \left(\frac{RM(t)}{P(t)}\right)^{\frac{1}{\gamma\theta}} \left(\frac{P_i(t)}{P(t)}\right)^{-\frac{\epsilon}{\theta}} \right]$$

the firm's objective can be rewritten as

$$E_0^i \left\{ \frac{1}{\lambda} \int_0^\infty e^{-\rho t} \left[\left(\frac{RM(t)}{P(t)}\right)^{\frac{1}{\gamma}-1} \left(\frac{P_i(t)}{P(t)}\right)^{1-\epsilon} - \frac{\alpha}{[AZ_i(t)]^{\frac{1}{\theta}}} \left(\frac{RM(t)}{P(t)}\right)^{\frac{1}{\gamma\theta}} \left(\frac{P_i(t)}{P(t)}\right)^{-\frac{\epsilon}{\theta}} \right] dt \right.$$
$$\left. -\alpha \left[F_m \int_0^\infty e^{-\rho t} dD_i^a(t)dt + F_z \int_0^\infty e^{-\rho t} dD_i^z(t)dt \right] \right\} \quad (13)$$

2.2.1 Costless Information

This is the case when $C_m = C_z = 0$. Thus, each firm updates its information set at each instant and hence matches the target price exactly.

Proposition 1. *In the Full Information case, prices track nominal money balances.*

$$\ln P_i^f(t) = \zeta \ln Z_i(t) + r \ln P^f(t) + (1-r) \ln M(t) \quad (14)$$

where $\zeta = \frac{-1}{\theta(1-\epsilon)+\epsilon}$ *and* $r = 1 - \frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}$ *and*

$$\ln P^f(t) = \ln M(t) \quad (15)$$

Proof. See Appendix A.2. □

Thus, under costless information processing, firms update their information set at each instant. This results in prices reflecting up to date information about the aggregate and idiosyncratic state at each instant. Thus, prices adjust proportionally to changes in money supple and so nominal expenditure shocks do not affect real output even in the short run. This is the benchmark case in which both the long and short run Phillips curves are perfectly vertical.

[6]When a firm i incurs the fixed cost to obtain information about the idiosyncratic state, it only receives information pertaining to its own idiosyncratic shock, not about other firms. Since the idiosyncratic state for firm i follows an independent Brownian motion, firm i's best guess about other firms idiosyncratic shock is 0

2.2.2 Costly Information

In the costly information case, C_m and C_z are positive. Since the firm faces a non-convex cost to update its information set, it is natural to think of a solution to the firms decision as being one in which the firm does not continuously update its information but does so only periodically. In fact, the firms decision of whether to update or not can be seen as a threshold rule on the variance of the forecasted loss from not updating its information about each of the states. I explain this in greater detail later. The expected lifetime loss of the firm from not updating its information set at each instant can be written as:

$$\mathcal{L} = E_0^i \left\{ \int_0^\infty Q(t) \left[\pi(P_i^f(t); P(t), M(t), Z_i(t)) - \pi(P_i(t); P(t), M(t), Z_i(t)) \right] dt \right. \\ \left. + F_m \int_0^\infty Q(t)\omega(t) dD_a^i(t) + F_z \int_0^\infty Q(t)\omega(t) dD_z^i(t) \right\} \tag{16}$$

Maximizing the objective in equation (13) is equivalent to minimizing equation (16) as it is derived by subtracting the objective from the flow of profits under full information which is a constant. It can be shown that the second order Taylor approximation of the function above is:

$$\mathcal{L} \approx E_0^i \left\{ \int_0^\infty e^{-\rho t} [\ln P_i(t) - \ln P^f(t)]^2 dt + C_m \int_0^\infty e^{-\rho t} dD_i^a(t) dt + C_z \int_0^\infty e^{-\rho t} dD_i^z(t) dt \right\} \tag{17}$$

where $C_m = \frac{\alpha(\epsilon-1)F_m}{\epsilon}\left[\frac{\theta-\theta\epsilon+\epsilon}{\theta}\right]$ and $C_z = \frac{\alpha(\epsilon-1)F_z}{\epsilon}\left[\frac{\theta-\theta\epsilon+\epsilon}{\theta}\right]$. C_k, $k = m, z$ can be interpreted as the cost in terms of labor of acquiring and processing information about the state k.

At anytime t, the economy has a cross sectional distribution of firms planning dates about the aggregate state $\Gamma_t^a(\tau_a)$ and the idiosyncratic state $\Gamma_t^z(\tau_z)$. $\Gamma_t^a(\tau_a) \in [0, 1]$ is the fraction of firms (at time t) that acquired information about the aggregate state prior to date τ_a. Similarly, $\Gamma_t^z(\tau_z)$ is the fraction of firms that last acquired information about the idiosyncratic information prior to date τ_z. $d\Gamma_t^a(\tau)$ and $d\Gamma_t^z(\tau)$ is the density of firms (at date t) that acquired information about the aggregate state and the idiosyncratic state respectively exactly at date τ. In other words, the fraction $1 - \Gamma_t^a(\tau_a)$ is the fraction of firms that know all the realizations of the aggregate state up to date τ : $\{q_s\}_{s \leq \tau}$ and $1 - \Gamma_t^z(\tau_z)$ has the analogous interpretation for the idiosyncratic state[7]. The evolution of $\Gamma_t^K(\tau)$, $k = a, z$ can be written recursively as

$$\Gamma_{t+dt}^k(\tau) = \Gamma_t^k(\tau) - \int_{-\infty}^{\tau} \mathcal{P}_t^k(s) d\Gamma_t^k(s) \text{ ,} \forall t \leq \tau \text{ and } k = a, z \tag{18}$$

where $\mathcal{P}_t^k(s)$ is the probability that a firm that acquired information about the aggregate (idiosyncratic)state at date s will acquire information about the aggregate (idiosyncratic) state again at date t.

[7] As mentioned earlier firm i only finds out the realization of its own idiosyncratic shock.

Solving the Model[8]

The firm's decision depends on the entire lag of realizations of the aggregate and idiosyncratic state. This means that the dimensionality of the firms problem is infinite dimensional. I use the method of undetermined coefficients to find an analytical solution to the firms problem. This can be found in Appendix A.3.

Define $p_i(t) = \ln P_i(t)$, $p(t) = \ln P(t)$, $z_i(t) = \ln Z_i(t)$, $m(t) = \ln M(t)$ and $p^*(t) = rp(t) + (1 - r)m(t)$. Firm i that last planned at $(\hat{\tau}_a, \hat{\tau}_z)$ will set price[9]

$$p_i(t) = E\{p^*(t) \mid \mathcal{I}_{\hat{\tau}_a}\} + \zeta E\{z_i(t) \mid \mathcal{I}^i_{\hat{\tau}_z}\} \tag{19}$$

where $\mathcal{I}_{\hat{\tau}_a} = \{m(s)\}_{s \leq \hat{\tau}_a}$ and $\mathcal{I}_{\hat{\tau}_a} = \{m(s)\}_{s \leq \hat{\tau}_a}$ and $\mathcal{I}^i_{\hat{\tau}_z} = \{z_i(s)\}_{s \leq \hat{\tau}_z}$.

Proposition 2. *In equilibrium, the following are true:*

1. *Aggregate (log) price $p(t)$ follows the following process*

$$p(t) = \sigma_m \int_{-\infty}^{t} \frac{[1 - \Gamma^a_t(\tau)](1 - r)}{1 - r + r\Gamma^a_t(\tau)} dW(\tau) \tag{20}$$

2.

$$p^*(t) = \sigma_q \int_{0}^{t} \frac{1 - r}{1 - r + r\Gamma^a_t(\tau)} dW(\tau) \tag{21}$$

3.

$$E\{p^*(t) \mid \mathcal{I}_{\hat{\tau}_m}\} = \sigma_q \int_{0}^{\hat{\tau}_m} \frac{1 - r}{1 - r + r\Gamma^a_t(\tau)} dW(\tau) \tag{22}$$

Proof. See Appendix A.3 □

Thus, the evolution of the target price of firm i can be written as:

$$p^f_i(t) = \sigma_q \int_{-\infty}^{t} \frac{1 - r}{1 - r + r\Gamma^q_t(\tau)} dW(\tau) + \zeta\sigma_z \int_{-\infty}^{t} dB_i(\tau) \tag{23}$$

For a firm that last planned about the aggregate state at $\hat{\tau}_a$ and about the idiosyncratic state at $\hat{\tau}_z$, the expected loss in profits from being imperfectly informed at date t can be written as

$$L(t, \hat{\tau}_m, \hat{\tau}_z) = E\left\{ \left(p_i(t) - p^f_i(t)\right)^2 \middle| \mathcal{I}_{\hat{\tau}_m}, \mathcal{I}^i_{\hat{\tau}_z} \right\} \tag{24}$$

[8] For ease of exposition and solving the baseline model, I set $\mu = 0$ and $\eta = 0$ for the rest of this section. I relax these assumptions in Section 4.

[9] This is the expected value of the log of the full information price specified in equation (14) given the firms information set.

Such a firm would set a price $p_i(t)$ which satisfies equation (19)

$$\begin{aligned} p_i(t) &= E\{p_i^f(t) \mid \mathcal{I}_{\hat{\tau}_m}, \mathcal{I}^i_{\hat{\tau}_z}\} \\ &= \sigma_m \int_{-\infty}^{\hat{\tau}_m} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \zeta\sigma_z \int_{-\infty}^{\hat{\tau}_z} dB_i(\tau) \end{aligned} \tag{25}$$

Also, $p_i(t) - p_i^f(t)$ can be written as

$$p_i(t) - p_i^f(t) = \sigma_m \int_{\hat{\tau}_m}^{t} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \zeta\sigma_z \int_{\hat{\tau}_z}^{t} dB_i(\tau)$$

Since the $W(t)$ and $B_i(t)$ are standard Brownian motion with unit variance, the expected instantaneous loss from remaining uninformed can be written as

$$L(t, \hat{\tau}_m, \hat{\tau}_z) = \sigma_m^2 \int_{\hat{\tau}_m}^{t} \left(\frac{1-r}{1-r+r\Gamma_t^q(\tau)} \right)^2 d\tau + \zeta^2\sigma_z^2 \int_{\hat{\tau}_z}^{t} d\tau \equiv L_1(t, \hat{\tau}_q) + L_2(t, \hat{\tau}_z)$$

Since the loss function is can be separated into a purely aggregate part and a purely idiosyncratic part, the problem of the firm can be broken into two separate problems. This enables me to write the choice of the planning horizon for acquiring information about the aggregate state and about the idiosyncratic as two separate dynamic programs.

In such a setup, two structures of equilibria arise naturally: synchronized and staggered. The synchronized equilibrium is one where all firms choose to update their information about state k, $k = m, z$ at the same date. The staggered equilibrium is one where all firms do not plan and change prices at the same time. I solve for a stationary staggered equilibrium in which a fixed fraction of firms *plans* about the aggregate and idiosyncratic state at each date. Lach and Tsiddon (1992) looking at price distributions in Israel find that price changes are not synchronized. Thus, the more interesting equilibrium structure is the staggered equilibrium and I focus on this in the paper.[10]I concentrate on the pricing problem of firm i. Assume that all other firms acquire information about the aggregate state every T_m periods and about their idiosyncratic state every T_z periods. Thus, the proportion of firms acquiring information about the aggregate state over any interval is given $\frac{1}{T_m}dt$ and about the idiosyncratic state is $\frac{1}{T_z}dt$.As a result, in the staggered equilibrium

$$\Gamma_t^k(\tau) = \begin{cases} 0 & \text{if } \tau < t - T_k \\ 1 - \frac{t-\tau}{T_k} & \text{if } t - T_k < \tau < t \end{cases}$$

for $k = m, z$.

A firm's problem of choosing when to update its information set pertaining to the aggregate state, given that it updated last at $\hat{\tau}_m$(today) can be written as

$$\mathcal{L}_1(\hat{\tau}_m) = \min_{\tau_m' \geq \hat{\tau}_m} \int_{\hat{\tau}_m}^{\tau_m'} e^{-\rho(s-\hat{\tau}_m)} L_1(s, \hat{\tau}_m) ds + e^{-\rho(\tau_m' - \hat{\tau}_m)} [C_m + \mathcal{L}_1(\tau_m')] \tag{26}$$

[10]I discuss some aspects of the synchronized equilibrium in the next section

where

$$L_1(t,\hat{\tau}) = \begin{cases} \sigma_m^2 \int_{\hat{\tau}}^t \frac{(1-r)^2}{(1-r\frac{(t-\tau)}{T_m})^2} d\tau & \text{if } \hat{\tau} \geq t - T_m \\ \sigma_q^2 \int_{t-T_m}^t \frac{(1-r)^2}{(1-r\frac{(t-\tau)}{t})^2} ds + \sigma_q^2(t - T_m - \hat{\tau}) & \text{if } \hat{\tau} > t - T_m \end{cases}$$

Similarly the problem to choose when to next plan for the idiosyncratic state given that the firm planned today about the idiosyncratic state can be written as:

$$\mathcal{L}_2(\hat{\tau}_z) = \min_{\tau_z' \geq \hat{\tau}_z} \int_{\hat{\tau}_z}^{\tau_z'} e^{-\rho(s-\hat{\tau}_z)} L_2(s,\hat{\tau}_z) ds + e^{-\rho(\tau_z'-\hat{\tau}_z)} [C_z + \mathcal{L}_1(\tau_z')] \tag{27}$$

$$L_2(t,\hat{\tau}) = \zeta^2 \sigma_z^2 (t-\tau)$$

Interpreting the decision problem of the firms as it is written above seems daunting but in fact the solution is a simple threshold rule as discussed. $\sigma_m \int_{\hat{\tau}_m}^t \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau)$ is the error difference between the the forecasted aggregate component of the target price $p_i(t)$ and the aggregate component of the actual target price $p^f(t)$ given that forecasts are formed with respect to the information set $\mathcal{I}_{\hat{\tau}_m}$. Similarly $\zeta\sigma_z \int_{\hat{\tau}_z}^t dB_i(\tau)$ is the error difference between the forecasted idiosyncratic component of the target and the actual target price where the forecasts are made with respect to the information set $\mathcal{I}^I_{\hat{\tau}_z}$. Thus, the loss function can be seen as a variance of this error difference which grows linearly with time in this basic case. Thus, the solution to the firms problem can be seen as a threshold for these error variances. Once the error variance for state k state, $k = m, z$ is reached, the firms chooses to incur the fixed cost to update that part of its information set and resets the price error variance to zero. It then makes forecasts on the basis of this newly expanded information set. Then it waits again till the variance again grows large enough to warrant new information to sharpen its forecasts. The threshold is chosen such that if the firm did not incur the fixed cost to update its information set, its losses from a poorly forecasted target price would result in larger losses than incurring the cost and reducing the forecast error.

The problem can be reformulated with the time since when last information was acquired. Define $\delta_m = t - \hat{\tau}_m$ and $\delta_z = t - \hat{\tau}_z$ as the time since firm i last acquired information about the aggregate state and about the idiosyncratic state respectively. Thus, the two Bellman equation above can be rewritten as

$$\mathbb{L}_1(\delta_m) = \min_{\delta_m' \geq \delta_m} \int_0^{\delta_m'-\delta_m} e^{-\rho s} L_1(s) ds + e^{-\rho(\delta_m'-\delta_m)} [C_q + \mathbb{L}_1(0)] \tag{28}$$

$$\mathbb{L}_2(\delta_z) = \min_{\delta_z' \geq \delta_z} \int_0^{\delta_z'-\delta_z} e^{-\rho s} L_2(s) ds + e^{-\rho(\delta_z'-\delta_z)} [C_z + \mathbb{L}_2(0)] \tag{29}$$

where

$$L_1(\delta_m) = \begin{cases} \sigma_m^2 \int_0^{\delta_m} \frac{(1-r)^2}{(1-r\frac{s}{T_m})^2} ds & \text{if } \delta_m \leq T_m \\ \sigma_m^2 \int_0^T \frac{(1-r)^2}{(1-r\frac{s}{T_m})^2} ds + \sigma_m^2(\delta_m - T_m) & \text{if } \delta_m > T_m \end{cases}$$

and

$$L_2(\delta_z) = \zeta^2 \sigma_z^2 \int_0^{\delta_z} ds = \sigma_z^2 \delta_z$$

The solution to the first Bellman equation is characterized by the optimal planning horizon (for aggregate money shocks) T_m^*, iff $t > s + T_m^*$. Taking the first order with respect to $\hat{\delta}_q$ for the problem described in equation (28) and using the fact that the optimal horizon is T_m^*, we can write

$$L_1(T_m^*) = \rho[C_q + \mathbb{L}_1(0)] \tag{30}$$

where

$$\mathbb{L}_1(0) = \frac{\int_0^{T_m^*} e^{-\rho s} L_1(s) ds + e^{-\rho T_m^*} C_m}{1 - e^{-\rho T_m^*}} \tag{31}$$

Proposition 3. *Optimal Planning Horizon T_m^**

1. *The optimal planning horizon for planning about the aggregate monetary shock, T_m^* is implicitly defined by*

$$C_m = \sigma_m^2 T_m^* \int_0^{T_m^*} e^{-\rho s} \frac{T_m^* - s}{T_m^* - rs} ds \tag{32}$$

2. *and is unique.*

Proof. See Appendix A.4. □

Thus, each firm chooses to update its information set about the nominal demand shock every T_m^* periods. Note that for $C_m > 0$, $T_m = 0$ does not solve the equation above and hence, it is never optimal for a firm to update its information about the aggregate state at each instant unless doing so is costless. Similarly for a finite C_m, $T_m^* < \infty$ and hence each firm will update its information set in a finite amount of time and thus, all firms in the long run will incorporate all the information about he monetary shock which leads to a vertical long run Phillips curve. Similarly, one can solve for the unique optimal planning horizon for the idiosyncratic state T_z^* which is implicitly defined by

$$C_z = \zeta^2 \sigma_z^2 \int_0^{T_z^*} e^{-\rho\delta} (T_z^* - \delta) d\delta \tag{33}$$

As was argued above, for $C_z \in (0, \infty)$, $T_z^* \in (0, \infty)$, i.e., a firm will never find it optimal to update its information about the idiosyncratic state to incorporate this into prices each instant. Neither will it choose never to do so as long as the cost of doing so is positive and finite.

3 Results and Discussion

Note that under the basic specification of the previous section firms change prices only when they update their information even though there is no fixed cost attached to changing prices. With the specification in the last section, a firm optimally chooses not to change prices if it is not "planning". This is because the firm's expected target price does not change[11] if it does not updating its information set. Thus, the setup generates a firms behavior where they do not change prices in response to every change in the state variable, in fact they change prices in response to aggregate and idiosyncratic shocks at fixed and possibly different intervals.

A very interesting result that emerges from the analysis in the previous section is that even if the cost of *planning* about the aggregate and idiosyncratic shock is the same, i.e. $C_m = C_z$ and both of them are equally volatile $\sigma_m = \sigma_z$, $T_q \neq T_z$ in general. Thus, the model is capable of explaining differential adjustment of prices in response to aggregate and idiosyncratic shocks. The model presents a scenario under which firms update their information about their idiosyncratic state more often than about the aggregate state.

Proposition 4. *T^*_m is increasing in the strength of the strategic complementarity which is measured by r.*

Proof. This can be shown using the Implicit function theorem on equation (32). □

Proposition 5. *For $r \in (0, 1)$ and normalizing $\zeta = 1$, if $\sigma_m = \sigma_z = \sigma$ and $C_m = C_z = C$, then $T_m > T_z$*

Proof. The only difference in the form of equations (32) and (33) (with ζ normalized to zero) is that equation (33) is the same as equation (32) with r set to 0. Thus, from the previous proposition, it must be that $T^*_z > T^*_m$. □

Proposition 5 implies that firms change prices in response to idiosyncratic shocks much more often than in response to aggregate shocks. This feature of the model enables one to match the facts from micro pricing data (prices change often) and also the evidence that the aggregate prices move in a sluggish manner in response to monetary shocks. Reis (2007) uses a similar setup but, by not incorporating the strategic complementarity in pricing and idiosyncratic shocks, cannot capture these

[11]This is because the stochastic processes for the money supply and idiosyncratic productivity shock are assumed to follow Brownian motions with 0 drift in this section since $\mu = \eta = 0$ in this section. This is not the case in the following section. Since firms expected target price in general will not stay fixed over time, firms set price plans as will be shown in the next section.

features. As opposed to earlier models, this paper can explain a differential frequency of price changes to idiosyncratic and aggregate shocks. The difference in this model which enables one to match this is the recognition of the fact strategic complementarity in pricing spills over into information acquisition decisions about the aggregate state and causes a *delay* in information acquisition acquisition about the aggregate state. This can be seen as a combination of two forces:

- When a firm gets new information about the aggregate state, it realizes that only a fraction of firms are making their pricing decisions based on current information. All other firms are setting prices based on old information and hence firms that get new information that warrants a large price change under full information, in a setting of incomplete information temper their action to account for all the firms which are making their decisions based on old information about the aggregate state[12], and

- The loss from being uninformed is increasing in the fraction of other firms that are informed. It can be shown that

$$\frac{\partial L_1(t,\hat{\tau})}{\partial(1-\Gamma_t^q(\tau))} > 0 \ ,\forall \tau \in (\hat{\tau}, t] \text{ iff } r > 0$$

  The staggered nature of information acquisition[13] The synchronized equilibrium also has similar properties. However, instead of a unique optimal planning horizon for the monetary state, there exists a closed interval on the real line of optimal planning horizons, out of which anyone can be the equilibrium. This multiplicity of equilibria is due to the complementarity $r \in (0, 1]$. The

[12] This is similar to the older literature on strategic complementarity such as Haltiwanger and Waldman (1985)

[13] The synchronized equilibrium also has similar properties. However, instead of a unique optimal planning horizon for the monetary state, there exists a closed interval on the real line of optimal planning horizons, out of which anyone can be the equilibrium. This multiplicity of equilibria is due to the complementarity $r \in (0, 1]$. The optimal planning horizon for the idiosyncratic state remains unique as there is no complementarity associated with that aspect of pricing. For more details on the multiplicity of equilibria, see Hellwig and Veldkamp (2009). In any case, it can be argued that the optimal planning horizon for planning about the idiosyncratic case is still shorter than that for the aggregate state even in the synchronized equilibria. Thus, the result still goes through. In a synchronized equilibrium, all firms adjust prices simultaneously and hence when firms choose to update their information, prices reflect this information fully unlike in the staggered equilibrium, where a prices only gradually adjust because firms who observe the new information have to temper their response to it to account for those who have not acquired it yet. Thus, if there is a monetary policy shock at a time between two planning dates, the output stays at the high level and does not decline till the next planning date at which it then falls to the natural level as the aggregate price adjusts proportionally to the change in the monetary policy. Thus, even in this setup, monetary policy is effective in the stimulating the economy in the short run but not in the long run. However, the synchronized equilibria is not an interesting and realistic equilibrium as empirical evidence rules out the existence of synchronized price changes. Thus, for the rest of the paper, I focus exclusively on the staggered equilibrium.

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implies that the smaller fraction of firms is much better informed than firm i. As a result, the firm is able to delay the decision to acquire costly information about the aggregate state without losing too much profit.

Thus, even if planning about the aggregate and idiosyncratic states is equally costly and the two shocks are equally volatile, firms will optimally choose to update their information about the idiosyncratic state more often than about the aggregate state and hence prices will incorporate new information about the idiosyncratic at shorter intervals than about the aggregate state. This is because of the beauty contest nature of the price setting problem. Firms want to set prices as close to the average action and the true state. In setting price in response to aggregate shocks, the firm also has to consider how other firms will respond to the aggregate shock and how that affects the average action. This is because the same information about the monetary shock is observed by many firms and so each firm not only has to set its price to incorporate this new information but also take care of how other interpret this information and set prices. This is not the case when it comes to incorporating information about their own idiosyncratic state. The firms price response to its own idiosyncratic productivity shock is not influenced by the actions of others as each firm is small enough so that it alone cannot affect the average price, but the price response to aggregate shocks depends on how other respond to it as it affects the average action which each firm wants to track in addition to the true state so that they do not set a price too high or too low compared to the average and hence either lose a lot of demand or have too much demand so that it is forced to produce beyond the profit maximizing level. Also, as explained earlier, the staggered nature of the equilibrium results in a large fraction of firms being uninformed about the current true state. Thus, the firms that know the true state need to temper their response to this new information. Thus, the immediate response to a monetary shock is a less than proportional increase in prices whereas idiosyncratic shocks result in large changes. Figure 1 plots the response of aggregate price and real output to a positive monetary shock. Real output increases at impact and then gradually decreases as more firms update their prices to incorporate the shock to monetary policy into their prices. Once all firms update their information sets such that each of them has incorporated the shock to monetary policy into their prices, the aggregate price adjusts *fully* to the shock and output goes back to the *natural* level. Thus, the model displays a trade-off between inflation and economic activity in the short run but no such trade-off in the long run and hence generates a Phillips curve which is vertical in the long run but not so in the short run.

At the same time, the model is capable of matching both the *large* and frequent price changes seen in micro data. The setup thus, offers a different result than Golosov and Lucas Jr. (2007) who find that once they calibrate their model to micro pricing facts as in Klenow and Kryvtsov (2008), nominal shocks do not have persistent effects. However, the current model is able to match the micro pricing facts as well as providing for persistent effects of nominal shocks as in the standard time-dependent pricing macro models ala Calvo (1983). This model can be seen as further micro-foundations of menu cost models of pricing. If one looks at behavior of firms in this model; every time a firm incurs a fixed cost, it changes price. This would be what would be seen in the data if the model is a menu-cost model. However, this model gives an interpretation of menu costs as managerial costs of planning which have been found to be very important in a firms decision not to change prices often by Zbaracki et al. (2004).

4 Extensions

In this section, I talk about how this model is also consistent with other features of micro data. Nakamura and Steinsson (2008) find that the frequency of price increases co-varies strongly with inflation but not price decreases. In the setup μ is the constant rate of long run wage inflation. In the case with $\mu > 0$,[14] the price firm i sets at time t when it last updated its information sets at the dates

[14] Assuming that $\eta = 0$. This will be relaxed in subsequent paragraphs.

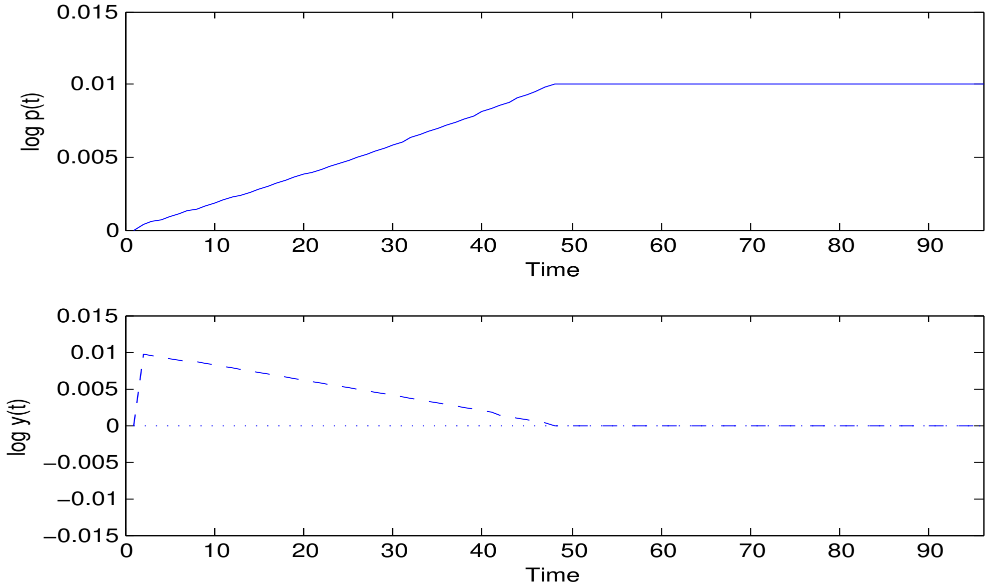

Figure 1: Response of aggregate price and real output to a monetary shock

$(\hat{\tau}_m, \hat{\tau}_z)$ can be seen as an appropriately altered version of equation (25):

$$\begin{aligned} p_i(t) &= E\{p_i^f(t) \mid \mathcal{I}_{\hat{\tau}_m}, \mathcal{I}^i_{\hat{\tau}_z}\} \\ &= \sigma_m \int_{-\infty}^{\hat{\tau}_m} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \mu t + \zeta \sigma_z \int_{-\infty}^{\hat{\tau}_z} dB_i(\tau) \end{aligned}$$

In a setting with positive long run inflation, firms set a price schedule where prices are indexed to the level of inflation. Thus, a higher level of average long run inflation results in prices increasing by larger amounts as firms adjust for the higher rate of inflation. At the same time, the frequency of updating information about the idiosyncratic state remains the same. As a result, the frequency with which firms reduce price stays the same. At the same time, to adjust for higher inflation firms increase prices each period by indexing prices to the long run rate of inflation. This is consistent with the finding that prices increase more frequently than prices decrease when inflation is higher.

In such a setting, even though prices change all the time, prices only incorporate new information at discrete intervals. Thus, rather than setting prices, firms are setting price schedules. Looking at the previous section in this light, one realizes that firms were setting perfectly flat price schedules when $\mu = 0$. Now since inflation is a positive, firms set a price schedule where they index their prices to long run inflation and at discrete intervals, update their information about the monetary state.

A large portion of the literature on the New Keynesian Phillips curve which make use of the Calvo-Yun type sticky price setup assume that the non-adjusting firms index their prices to past lags of inflation or average inflation. This helps the New Keynesian Phillips curve fit data better (Kryvtsov and Kichian, 2008). The way this is often motivated is by imposing rule-of-thumb backward looking behavior on the non-adjusting firms. This model gives micro foundations to such an assumption. IN the current setup, firms that do not update their information set about the aggregate state at time t take into account the average long run level of inflation and set a price plan in which the prices are indexed to this long run level of inflation. This inflation indexation is an attempt by the firm to keep the forecast error to the minimum possible. Unlike in Section 3, here the firms forecast of the target price is not constant over time because of the positive rate of inflation. Thus, unlike the last section, firms here do not set flat price plans (unchanging prices). In fact firms set increasing price plans, they raise prices every period by the level of long run average inflation. Even though prices are indexed to long run inflation and hence change all the time, monetary policy is still effective in the short run. This is because even though prices change often, new information is only incorporated into prices gradually which implies that aggregate price still moves sluggishly in response to a monetary policy shock.

I now set $\mu = 0$ again and instead set $\eta > 0$. This allows me to study price paths for firms in more detail. Mankiw and Reis (2010) point out three features which are prominent when analyzing price paths.

1. Prices change all the time, on average every three to four months.

2. Many of the price changes follow what seem like predetermined patterns that follow simple algorithms and actual resetting of price plans based on new information seems less frequent.

3. There are many horizontal segments, reflecting short-lived intervals when nominal prices are unchanged.

The current setup is consistent with all the above features. Evidence such as in Klenow and Kryvtsov (2008) suggests that idiosyncratic shocks are much more volatile and hence firms would tend to update their information set about idiosyncratic shocks quite frequently and hence prices would change frequently. The model can be calibrated to have prices change every 3-4 months. The present setup is also capable of matching the second fact: generating price schedules that follow simple algorithms. In such a setup, prices would change all the time but only incorporate new information at discrete intervals. To look at such an equilibrium, set $\eta > 0$ in the basic model in section 3. Most of the argument follows from before and it can be shown that firm i that last updated its information set at $(\hat{\tau}_m, \hat{\tau}_z)$ will set a price[15]:

$$\begin{aligned} p_i(t) &= E\{p_i^f(t) \mid \mathcal{I}_{\hat{\tau}_m}, \mathcal{I}^i_{\hat{\tau}_z}\} \\ &= \sigma_m \int_{-\infty}^{\hat{\tau}_m} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \zeta z_i(\hat{\tau}_z) e^{-\eta(t-\hat{\tau}_z)} \end{aligned} \tag{34}$$

Thus,

$$p_i(t) - p_i^f(t) = \sigma_m \int_{\hat{\tau}_m}^{t} \frac{1-r}{1-r+r\Gamma_t^m(\tau)} dW(\tau) + \zeta \left[z_i(\hat{\tau}_z) e^{-\eta(t-\hat{\tau}_z)} - z_i(t) \right]$$

and the loss function can be written as

$$L(t; \hat{\tau}_m, \hat{\tau}_z) = \sigma_m^2 \int_{\hat{\tau}_m}^{t} \left(\frac{1-r}{1-r+r\Gamma_t^m(\tau)} \right)^2 d\tau + \frac{\sigma_z^2}{2\eta}(1 - e^{-2\eta(t-\hat{\tau}_z)})$$

As was the case before, the loss function can be separated into two parts, the first dealing solely with the loss from being uninformed about the aggregate state and the second the loss from being uninformed about the aggregate state. The optimal planning horizon T_m^{**} and T_z^{**} in this case can be found the same way as in Section 3. Assume that there are no monetary shocks. This allows a clearer exposition of what a price path will look like. Suppose firm i *plans* about the idiosyncratic

[15] See Dixit and Pindyck (1993) for more on expectations and variances of Ornstein-Uhlenbeck processes.

state today at $t = t_0$ and observes that $z(t_0) = z_0 > 0$. Then, it will choose to update its information set at $t' = t_0 + T_z^{**}$. In between t_0 and $t_0 + T_z^{**}$, the firm changes prices so that

$$p_i(t) = z_0 e^{-\eta t} \text{for } t_0 < t \leq T_z^{**}$$

Thus, the firm sets a price plan according to which it reduces prices each period till $t_0 + T_z^{**}$. At $t_0 + T_z^{**}$, the firm *plans* again and observes $z(t_0 + T_z^{**}) = z_1$, sets price so that

$$p(t_0 + T_z^{**} + dt) = z_1 e^{-\eta dt} \text{ for } dt < T_z^{**}$$

As was the case in Section 3, the firm's decision of when to update their information set can be seen as a threshold rule for the variance of the forecast error where the forecast is made conditional on the information set of the firm at time t which is given by $\mathcal{I}_t^i = \mathcal{I}_{\tau_m} \times \mathcal{I}_{\tau_z}^i$ since the firm last updated its information set regarding the aggregate state at tau_m and the idiosyncratic state at τ_z. Here the difference from Section 3 is that the error variance of the forecast for the idiosyncratic shock no longer grows in a linear fashion over time. This is because the process for the idiosyncratic shock is no longer described by a Brownian motion. With $\eta > 0$, the process defining the idiosyncratic productivity shock is the mean reverting Ornstein-Uhlenbeck process. As Golosov and Lucas Jr. (2007) point out, defining a stationary stochastic process for the idiosyncratic shock is essential if a model is to match the features seen in actual data.

The firm sets simple pricing plans in periods it is not updating its information so as to track the target price as closely as possible and the price jumps to a different plan every time the firm plans again. Thus, in this setting, firms change prices all the time but only update their pricing plan infrequently. This is consistent with the findings of Blinder et al. (1998) who find possible evidence that managers were adjusting their price plans. In terms of the model the flat parts of the price path could be those price plans which are chosen when at the planning date the level of $z(t_0) = 0$; then the price plan is just not to change prices till the next time the firm plans. These can be frequent events if the volatility of the productivity shocks σ_z^2 is low or if the tendency to be around the mean is high (high η). Thus, this model is capable of all the notable features of data which are highlighted by Mankiw and Reis (2010). Figure 2 plots an example of a price path in response only to idiosyncratic productivity shocks. The bottom panel of the figure plots the inverse of the realization of the idiosyncratic shock. This is done to make the two panels respond in the same direction. A higher idiosyncratic productivity shock would reduce the marginal cost and hence lower price while an adverse productivity shock will warrant a rise in prices. For the particular parametrization used, the firm *plans* every 10th week and sets a price plan for the next 10 weeks. The triangles in the figure mark the dates at which the firm updates its information set about the idiosyncratic state. Notice that the triangles correspond to discrete jumps in the price. These discrete jumps in price are indicative of new information being incorporated into prices. Even though prices are changing between successive triangles, they do not contain any new information compared to when the pricing plan was set. There are dips in prices and then prices gradually go up. These episodes could be interpreted as *sales* episodes which are observed in the data[16]. Also, there is an extended period where prices do not change at all, for example the duration between weeks 25 and 32.

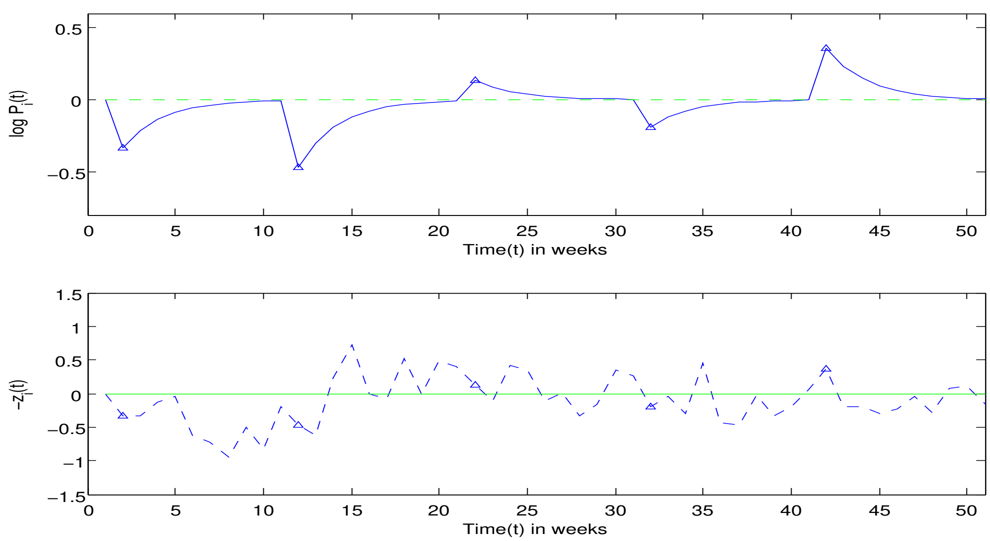

Figure 2: Price path of firm i in response to idiosyncratic productivity shocks

[16]This model in the current form is maybe too simplistic to analyze sales but this could be a possible direction to extend the model in.

5 Conclusion and further work

The model presented in this paper is one of the few papers which can reconcile the seemingly contradictory macro and micro level pricing evidence. The model predicts the presence of a trade-off between inflation and output in the *short run* but not in the *long run*, hence it predicts a perfectly vertical *Long Run Phillips Curve* but not *short run Phillips curve* which is not perfectly vertical. To the extent of my knowledge, this is one of the few papers which tries to model the frequency of price changes endogenously. The model is consistent with evidence from various studies. By allowing for frequently changing and volatile prices it matches micro-pricing evidence presented in Bils and Klenow (2004) and Klenow and Kryvtsov (2008) and at the same time predicts a sluggish response of aggregate price to nominal shocks, hence allowing for real effects of nominal shocks in the short run. This model is capable of explaining both the differential adjustment of prices (both in terms of frequency of price change and magnitude) in response to aggregate and idiosyncratic shocks. Earlier literature such as the sticky information literature was unable to explain this differential adjustment. Mackowiak and Wiederholt (2009) explained the differential adjustment in magnitude but the frequency of price changes is not modeled in their setup. By incorporating continuous time, I find it easier to model the frequency aspect of price changes and show that the strategic complementarity in pricing decisions combined with endogenous information acquisition decisions results in complementarity associated with planning about the aggregate state and this causes a firm to delay the decision to acquire information about the aggregate state and hence, aggregate price is based on old information for a longer time and this manifests itself as sluggish response of aggregate price to monetary shocks.

This paper is also capable of matching some important regularities seen in price data as laid out in Mankiw and Reis (2010). The paper is also consistent to a certain extent with studies such as Blinder et al. (1998) and Zbaracki et al. (2004) who used interviews with firm managers to determine what the reasons were for firms pricing behavior. As mentioned before, Blinder et al. (1998) find evidence that suggests that managers set price plans rather than attempt to determine the optimal price at each instant. This is consistent with the model as it shows that firm's optimally choose to update their information at discrete intervals and in the mean time set prices according to some price schedule which may or may not be a constant price. The fact that firms only plan periodically and so prices incorporate new information only periodically is supported by the findings of both Blinder et al. (1998) and Zbaracki et al. (2004). The fact that the model is able to match these additional features, which was not the main aim of the paper, suggests that the setup is robust.

The model can be thought of as a micro foundations to the menu costs model. In the model firms incur a fixed cost every time they try to incorporate new information into prices. Unlike standard menu models such as Golosov and Lucas Jr. (2007) who find that on calibrating these menu cost models to match the micro pricing facts, these models predict a small and transitory response to monetary shocks. The current model by virtue of being able to match micro-level pricing facts and at the same time generating sluggish response of aggregate prices to nominal shocks, seems to suggest that the current state dependent models might be looking at the wrong place for costs of changing prices. This finding is consistent with that of Zbaracki et al. (2004) who find that costs associated with acquiring and processing information are much more important than purely physical costs of changing prices in determining the frequency of price changes.

The paper also provides micro-foundations for the commonly made methodological assumption in the New Keynesian Phillips Curve literature, where in a Calvo-Yun type of sticky price environment, non-adjusting firms are assigned rule of thumb behavior according to which they index their prices to lagged or average inflation. I show that in a setting with positive long run average inflation, firms on dates in which they do not update their information sets about the monetary state, optimally choose to index their price to the long run inflation level. This results in firms setting increasing price plans over time.

The paper suggests that nominal rigidities are important in explaining the short run Phillips curve trade-off. Unlike the standard models used in macroeconomics, the current paper looks to explain the existence of nominal rigidities as arising out of costly information acquisition and processing. This setup by virtue of being consistent with both micro-pricing facts and the macro literature on sluggish adjustment of prices to monetary shocks, appears to be a better description of the firms price setting problem than the popular time-dependent or state-dependent menu-cost models which rely on the physical costs of changing prices to generate these rigidities. The next step is to calibrate this model to be able to generate quantitative predictions from this model so as to compare this model with the standard models quantitatively.

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A Appendix: Proofs

This section contains proofs of the propositions in the main text.

A.1 Existence of a constant equilibrium nominal interest rate

By definition

$$Q(t) = e^{R(t)dt} E_t\{Q(t+dt)\}$$

From equation (6):

$$E_t\left(\frac{Q(t+dt)}{Q(t)}\right) = e^{-\rho dt} E_t\left(\frac{R(t)M^D(t)}{R(t+dt)M^D(t+dt)}\right)$$

Impose $R(t) = R(t+dt) = R$:

$$E_t\left(\frac{Q(t+dt)}{Q(t)}\right) = e^{-\rho dt} E_t\left(\frac{M^D(t)}{M^D(t+dt)}\right) = e^{\left(-\rho-\mu+\frac{\sigma_m^2}{2}\right)dt}$$

Thus,

$$R = \rho + \mu - \frac{\sigma_m^2}{2}$$

A.2 Proof of Proposition 1

The Full-Information case is when $F_m = F_z = 0$. All firms adjust prices in response to all shocks at every instant. Each firm sets $P_i(t)$ so as to maximize (13):

$$P_i^f(t) = \left[\frac{\epsilon\alpha}{\theta(\epsilon-1)} \frac{R^{\frac{1+\theta\gamma-\theta}{\gamma\theta}}}{A^{\frac{1}{\theta}}}\right]^{\frac{\theta}{\theta(1-\epsilon)+\epsilon}} Z_i(t)^{-\frac{1}{\theta(1-\epsilon)+\epsilon}} M(t)^{\frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}} P(t)^{1-\frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}}$$

Define $A = \left[\frac{\epsilon\alpha}{\theta(\epsilon-1)} R^{\frac{1+\gamma\theta-\theta}{\gamma\theta}}\right]^{\theta}$ so that the initial constant term goes to 1. Thus,

$$P_i^f(t) = Z_i(t)^{-\frac{1}{\theta(1-\epsilon)+\epsilon}} M(t)^{\frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}} P(t)^{1-\frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}}$$

Taking logs on both sides

$$\ln P_i^f(t) = \zeta \ln Z_i(t) + r \ln P^f(t) + (1-r)\ln M(t)$$

where $\zeta = \frac{-1}{\theta(1-\epsilon)+\epsilon}$ and $r = 1 - \frac{1+\gamma\theta-\theta}{\gamma(\theta(1-\epsilon)+\epsilon)}$.

The price index defined in equation (10) can be approximated by

$$\ln P(t) = \int_0^1 \ln P_i(t) di$$

and so integrating equation (14) over $i \in [0, 1]$ yields:

$$\begin{aligned}
\ln P^f(t) &= \int_0^1 \ln P_i^f(t) di = \zeta \int_0^1 \ln Z_i(t) di + r \ln P^f(t) + (1-r) \ln M(t) \\
&= r \ln P^f(t) + (1-r) \ln M(t) \\
&= \ln M(t)
\end{aligned}$$

A.3 Proof of Proposition 2

Guess that $p(t)$ follows the path

$$p(t) = \sigma_m \int_{-\infty}^t g_t(\tau) dW(\tau) + h_t(\tau)t$$

Plugging this guess into the expression for $p^*(t)$ yields

$$p^*(t) = \sigma_m \int_{-\infty}^t [1 - r + r g_t(\tau)] dW(\tau) + (1-r)\mu t + r h_t(\tau)t$$

Note that

$$E\{p^*(t) \mid \mathcal{I}_{\hat{\tau}_a}\} = \sigma_m \int_{-\infty}^{\hat{\tau}_a} [1 - r + r g_t(\tau)] dW(\tau) + (1-r)\mu t + r h_t(\tau)t$$

and

$$E\{z_i(t) \mid \mathcal{I}^i_{\hat{\tau}_z}\} = \sigma_z \int_{-\infty}^{\hat{\tau}_z} dB_i(\tau)$$

Thus, from equation (19), firm i with the information set $\mathcal{I}^i_t = \mathcal{I}_{\hat{\tau}_a} \cup \mathcal{I}^i_{\hat{\tau}_z}$ is

$$p_i(t) = \sigma_m \int_{-\infty}^{\hat{\tau}_m} [1 - r + r g_t(\tau)] dW(\tau) + (1-r)\mu t + r h_t(\tau)t + \zeta\sigma_z \int_{-\infty}^{\hat{\tau}_z} dB_i(\tau)$$

The aggregate (log) price can be derived by integrating over the two distributions Γ^m_t and Γ^z_t.

$$p(t) = \sigma_q \int_{-\infty}^t [1 - \Gamma^m_t(\tau)][1 - r + r g_t(\tau)] dW(\tau) + (1-r)\mu t + r h_t(\tau)t$$

Using the method of undetermined coefficients yields

$$g_t(\tau) = \frac{[1 - \Gamma^a_t(\tau)](1-r)}{1 - r + r\Gamma^a_t(\tau)} \text{ and } h_t(\tau) = \mu$$

Thus,

$$p(t) = \int_{-\infty}^t \frac{(1-r)(1-\Gamma^m_t(\tau))}{1 - r + r\Gamma^m_t(\tau)} dW(\tau) + \mu t$$

Plugging this into the expression for

$$
\begin{aligned}
p^*(t) &= r\sigma_m \int_{-\infty}^{t} \frac{([1-\Gamma_t^a(\tau)]1-r)}{1-r+r\Gamma_t^a(\tau)} dW(\tau) + r\mu t + (1-r)\sigma_m \int_{-\infty}^{t} dW(\tau) + (1-r)\mu t \\
&= \sigma_m \int_{-\infty}^{t} \frac{1-r}{1-r+r\Gamma_t^a(\tau)} dW(\tau) + \mu t
\end{aligned}
$$

The next claim follows from the fact that $W(t)$ is a standard Brownian motion.

A.4 Proof of Proposition 3

Following equations (30) and (31), in equilibrium since $T_m = T_m^*$, it must be the case that

$$L_1(T_m^*) = \frac{\rho}{1-e^{-\rho T_m^*}} \left(\int_0^{T_m^*} e^{-\rho s} L_1(s) ds + C_m \right)$$

or

$$C_m = \int_0^{T_m^*} e^{-\rho\delta} [L_1(T_m^*) - L_1(\delta)] d\delta$$

Recall that

$$L_1(\delta) = \sigma_m^2 \int_0^{\delta} \frac{(1-r)^2}{\left(1 - r\frac{s}{T_m}\right)^2} ds$$

if $\delta \leq T_m$. Define $\Theta = \frac{s}{T_m^*}$, $L_1(\delta)$ can be written as

$$L_1(\delta) = \sigma_m^2 T_m^* \int_0^{\delta/T_m^*} \left(\frac{1-r}{1-r\Theta} \right)^2 d\Theta = \sigma_m^2 (1-r)^2 T_m^* \frac{\delta}{T_m^* - r\delta}$$

which can be used to write

$$L_1(T_m^*) - L_1(\delta) = (1-r) T_m^* \frac{T_m^* - \delta}{T_m^* - r\delta}$$

Therefore, T_m^* is implicitly defined by

$$F_m(\sigma_m, r, C_m, T_m^*) = 0$$

where

$$F_m(\sigma_m, r, C_m, T_m) = C_m - \sigma_m^2 T_m \int_0^{T_m} e^{-\rho s} \frac{T_m - s}{T_m - rs} ds$$

Note that

$$\frac{\partial F_m}{\partial T_m^*} = -\sigma_m^2 (1-r) \int_0^{T_m^*} \frac{T_m^* - s}{T_m^* - rs} ds - \sigma_m^2 (1-r) T_m^* \int_0^{T_m^*} \frac{s(1-r)}{(T_m^* - rs)^2} ds < 0 \text{ for } 0 < r < 1$$

Since, F_m crosses zero only once, T_m^* is unique.