Wages and Higher Education Participation

Eleftheriou, Konstantinos and Athanasiou, George and Petrakis, Panagiotis

University of Piraeus, National and Kapodistrian University of Athens

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Wages and Higher Education Participation

Konstantinos Eleftheriou

Department of Economics, University of Piraeus, 80 Karaoli & Dimitriou Str., 185 34, Piraeus, Greece

George Athanasiou

Department of Economics, University of Athens, 5 Stadiou Str., 105 62, Athens, Greece.

Panagiotis Petrakis

Department of Economics, University of Athens, 5 Stadiou Str., 105 62, Athens, Greece

Abstract

The paper develops a model for the screening mechanism for higher education, within an adverse selection framework. Specifically it examines the effect of wage earned by high school graduates on higher education participation. The model pinpoints a positive relation between the “high school” wage and the number of candidates entered in higher education with positive influences on the quality of selection mechanism. An empirical examination is conducted, using U.S. data, in order to investigate the validity of our analytical results.

JEL classification: I28; J24; J39

Keywords: Admissions; High school wage; Higher education; Quality

1 Corresponding author. Email: kostasel@otenet.gr
1. Introduction

The admission of students to higher education is a challenging matter for educational authorities. This is a result of the fact that both parts face a dilemma. On the one hand, students must decide whether they will continue their education in order to acquire more qualifications or they will drop out early in order to enter the labor market. Their decision depends on a number of factors (i.e., wage, unemployment, abilities, etc.) prevailing at that time. On the other hand, the educational authority (usually the university) must decide on the number of entrants, based on the quality of the candidates and the cost per admitted student. The admission policy of top universities in the U.S.A. and Europe initiated a long debate among the academic community.

The literature which studies the relation between labor remuneration and participation in tertiary education is large and focuses mainly on minimum wage. One of the early studies on the effects of minimum wages on the youth labor market is that of Ragan (1977). Ragan empirically tested the hypothesis that minimum wage legislation reduces the fraction of youths employed and increases youth unemployment rates. However, this study focuses only on the employment effects of the minimum wage and ignores the important interaction between schooling, employment and the minimum wage. Neumark and Wascher (1995), examined the impact of minimum wage on employment and school enrollment for teenagers. By estimating a conditional logit model using state-year observations for the period 1977 to 1989, they concluded that there is a negative influence of minimum wages on school enrollment and a positive impact on the teenage idleness. A study similar to that of Neumark and Wascher is that of Landon (1997). Landon used Canadian provincial-level data and showed again that there is a strong negative relationship between minimum wage and school enrollment. Moreover, he argued that this effect seems to be relatively persistent since an increase in education spending (e.g. better paid teachers, administrative spending on instructional supplies, other school board operating expenditures etc.) have no systematic effect on enrollment rates. More recently Pacheco and Cruickshank (2007) and Chaplin et al. (2003) reinforced the argument of the positive correlation between minimum wage and school dropouts.

Dickerson and Jones (2004, preliminary draft) presented a model where
individuals are heterogeneous in their educational abilities and they face an ex-ante uncertainty regarding their probability of success in higher education. The main finding of their paper is that the effect of the introduction of a minimum wage on the decision to work or continue in education is small. The main reason for that result is that the educational decision of individuals is mainly based on their anticipated probability of success and/or the rewarding wage premiums on successful completion of study. Hence, the impact of minimum wage is limited. In their work, Lipowski and Ferreira (2005) presented a multi-agent evolutionary model of student’s dilemma. They assumed that agents are heterogeneous regarding their ability and therefore their expected probability of success. One of their main results is that when the give-up payoff is high enough then only a part of the population aims at the university education. More specifically, only the high ability individuals (with high probability of success) decide to take the exams so as to enter higher education. As far as our knowledge is concerned, until now there has not been an attempt to study the effect of high school wage on university enrollment rates. Thus, we consider that approaching the specific subject might shed new light in our understanding of the decision mechanism of the would-be participants in the labor market.

In our analysis, we assume:

- the ex-ante existence of a wage (high school wage) received by the individuals who either fail the examinations (and therefore not admitted in universities) or decide not to continue in full-time higher education.

- a public sector which finances high schools and universities. The universities in turn conduct the examinations in order to select their students.

According to our analysis, an increase (decrease) in high school wage, under certain assumptions, will increase (decrease) the number of admitted university candidates and the level of quality of education provided by high schools.

This paper is organized as follows. Section 2 presents the model and derives the results. Section 3 is devoted to the quantitative analysis of the model. Section 4 concludes.
2. The Model

2.1 Environment

We assume a continuum of risk neutral agents (university candidates), normalized to unity. Agents are of two types: either ‘good’ (type-\(g\)) or ‘bad’ (type-\(b\)). We consider that the ‘good’ candidates have certain characteristics (e.g., IQ abilities), which differentiate them from the ‘bad’ agents. Moreover, we assume a public authority which a) finances universities which are responsible for the examination procedure leading to the admission of agents to higher education and b) finances high schools in order to provide a certain quality level of education. The probability of success for an agent of type-\(i\) is \(P_i\) (\(i = g, b\)) where \(P_g > P_b\) and this probability is a function of the quality of high school education (denoted by \(q\)) and the maximum affordable number of university entrants (denoted by \(\eta\)). The cost for participating in the examinations for an agent of type-\(i\) is \(C_i\) where \(C_i\) is a function of \(q\) and \(\eta\), whereas the cost of not participating is equal to zero. If an agent passes the exams, he is admitted to the university and he gets a net wage \((1 - \tau)W_u\) after his graduation, where \(W_u\) is the gross wage and \(\tau\) is the tax rate. On the other hand, if he fails, he gets the wage for high school graduates \(W_h\), which we assume that is not taxed. Moreover, we assume that \((1 - \tau)W_u > W_h\). The public authority faces the following costs\(^2\): the cost for the organization of the examinations denoted by \(k(\eta)\), and the cost for the provision of a certain quality level of high school education, denoted by \(c(q)\). For the purpose of our analysis, we will assume that the fraction of ‘good’ individuals in the total population is equal\(^3\) to \(p(q)\).

The properties of the aforementioned functions are stated in the following table:

\[\textbf{[Table 1]}\]

\(^2\)Indirectly, since these costs are paid by universities and high schools.
\(^3\)\(p\) is also a function of a number of factors such as inherited characteristics, family environment etc. which will not concern as here.
2.2 The Problem

Public authority faces the following adverse selection (screening) problem: since there is imperfect information regarding the abilities of the individuals, she must design a mechanism such that the type-$g$ agents decide to participate the examinations, while the type-$b$ to drop out and enter the labor market. If type-$g$ agents enter the university then resources are allocated more efficient and growth perspectives (due to a future increase in human capital) are improved. The algebraic form of the above problem is:

\[
F = \arg \max_{\eta, q, h} \left[ \eta W_u + (1-\eta)W_h - k(\eta) - c(q) - p(q)C_g(q, \eta) \right] \quad (1)
\]

under the following constraints:

\[
\eta \leq P_g(q, \eta) p(q) \quad (2)
\]

\[
P_g(q, \eta)[(1-\tau)W_u - C_g(q, \eta)] + (1 - P_g(q, \eta))[W_h - C_g(q, \eta)] \geq W_h \quad (3)
\]

\[
P_b(q, \eta)[(1-\tau)W_u - C_b(q, \eta)] + (1 - P_b(q, \eta))[W_h - C_b(q, \eta)] \leq W_h \quad (4)
\]

where \(\eta W_u + (1-\eta)W_h - k(\eta) - c(q) - p(q)C_g(q, \eta)\) is the objective function of the public authority, which is consisted by the total income minus the cost (private and public). Inequality (2) determines the affordable number of entrants to the universities (i.e., the capacity of the universities). Inequalities (3), (4) are self-selection constraints for type-$g$ and type-$b$ individuals, respectively. Inequality (4) ensures that type-$b$ individuals will not participate in the exams, while inequality (3) ensures that type-$g$ will follow the opposite direction. The key feature in our analysis which differentiates it from the rest of the literature is the fact that we incorporate in our model the cost and the probability of entering into higher education and we implicitly relate them with the level of high school wage.

By separating the endogenous from the exogenous variables inequality constraints (2), (3), (4) take the following form:

\[
\eta - P_g(q, \eta) p(q) \leq 0 \quad (5)
\]
In the Appendix we present the mathematical analysis, i.e. the comparative statics of the model under consideration. Our main result is that an increase (decrease) in high school wage, increases (decreases) the number of students admitted in higher education and the quality of secondary education.

Let’s try now to provide some rationale for the aforementioned results. An increase in the high school wages under the assumption that \( \frac{\partial P_g}{\partial \eta} < 1 \) (see appendix), will decrease the right-hand side of the self selection constraints for type-\( g \) and type-\( b \) individuals. This will permit an increase in \( \eta \) without violating the self selection constraints (it can be easily shown that \( \frac{C_g(q,\eta)}{P_g(q,\eta)} \), \( \frac{C_b(q,\eta)}{P_b(q,\eta)} \) are decreasing in \( \eta \)). The increase in \( \eta \) is desirable for the public authority since it increases its objective function (the higher public cost, \( k(\eta) \) due to an increase in \( \eta \) can be offset by the increase in the total income and the decrease of the private cost, \( p(q)C_g \)). Thus, under this assumption, the derivatives show that an increase (decrease) in high school wage, increases (decreases) the number of students admitted in higher education and the quality of secondary education. The analysis is the same for the impact of high school wage on \( q \).

3. Quantitative Analysis

In order to verify the findings of our model, we used U.S. data over the period 1973 to 2004 for the following variables: Total first time entrants in public higher education (denoted as \( \text{pubs} \)), real hourly wage for high school graduates in 2005 dollars (denoted as \( \text{whs} \)), expenditures of elementary and secondary schools as a percentage of Gross Domestic Product (denoted as \( \text{exgdp} \)), and mean Scholastic
Assessment Test (SAT) scores (denoted as $sat$). We use the SAT test scores as an index of the quality of high school education. Moreover, we assume that $exgdp$ has an impact on the quality of high school education. In the rest of our analysis, we will use the natural logarithm (ln) of the above variables. The first step of our analysis is to test whether $\ln(pubs)$, $\ln(whs)$, $\ln(sat)$ and $\ln(exgdp)$ are stationary. Table 2 reports unit root test statistics of the augmented Dickey and Fuller (ADF) test (1981) and Phillips-Perron (PP) test (1988). The results in Table 2, indicate that all series are non-stationary and contain a unit root. In order to examine whether they are integrated of order one, $I(1)$, we perform the augmented Dickey-Fuller/Phillips-Perron test on first differences. The results suggest that all variables are stationary in first differences.

[Table 2]

We examine the validity of our comparative statics results by:

- Regressing $\ln(pubs)$ on $\ln(whs)$ (including constant and trend), using Ordinary Least Squares (OLS), so as to check the relation between high school wage and the number of candidates admitted in higher education.
- Regressing $\ln(sat)$ on $\ln(whs)$ and $\ln(exgdp)$ (including only constant), using Ordinary Least Squares (OLS), so as to check the relation between high school wage and the quality of secondary education.

Since, all our variables are $I(1)$, we perform the relevant cointegration tests by making use of Engle and Granger methodology (1987) so as to avoid generating spurious results. The results of Engle - Granger cointegration test are illustrated in Table 3 and indicate the existence of cointegrating relations.

[Table 3]

By performing the appropriate tests, we get strong evidence of serial correlation in the residuals. Therefore, we use the following autoregressive (AR) specifications in order to eliminate this problem.

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4The time series were obtained from the following sources: Economic Policy Institute: www.epi.org; National Center for Education Statistics: www.nces.ed.gov; College Board: www.collegeboard.com.
5If the residuals of an OLS regression between $I(1)$ variables are integrated of order zero ($I(0)$), then these variables are cointegrated.
\[
\ln(pubs_t) = \beta_0 + \beta_1 t + \beta_2 \ln(whs_t) + u_t
\]
\[
u_t = \rho u_{t-1} + \varepsilon_t
\]  \hspace{1cm} (8)

\[
\ln(sat_t) = a_0 + a_1 \ln(whs_t) + a_2 \ln(exgdp_t) + \nu_t
\]
\[
u_t = \xi_1 v_{t-1} + \xi_2 v_{t-2} + \ldots + \xi_{12} v_{t-12} + \omega_t
\]  \hspace{1cm} (9)

where \( u_t, \nu_t \) are the disturbance terms and \( \varepsilon_t, \omega_t \) are the corresponding innovations in the disturbances\(^*\).

Table 4 and 5 present the results of the Ordinary Least Squares regressions of (8) and (9), which in turn indicate that empirical evidence are consistent with the prediction of our model, namely the impact of the wage earned by high school graduates on the number of admitted candidates in higher education and on the quality of secondary education offered is positive.

[Table 4]

[Table 5]

Moreover, we check the specification of our estimated models by performing various diagnostic tests. These tests are reported in Table 6. Our results indicate that our model seems to be fairly well specified and free from specification error.

[Table 6]

4. Conclusion

The present paper investigated a selection mechanism for higher education. To this purpose, we studied the impact of high school wage on: 1) the number of students admitted in higher education and 2) the quality of high school education.

The main result of our analysis is that there is a positive relationship between the variables under consideration. More specifically, an increase in the wage earned by individuals with low qualifications will create an incentive for not continuing in tertiary education. This development will further discourage low ability individuals

\(^*\)The AR(12) specification in equation (9), may attributed to ‘intragenerational’ effects.
from trying to enter higher education. Therefore, the effectiveness of the screening mechanism in allocating resources more efficiently (low ability individuals enter labor market, high ability individuals enter higher education) and consequently increasing productivity, will be enhanced. On the other hand, in order to avoid educational ‘leakages’ from the group of high ability candidates due to the decrease in the wage premium, public authority should mitigate this effect with an increase in the number of admissions in higher education up to the level where the aforementioned disincentive for low ability individuals will be preserved. At the end of the day, the entry of more high ability individuals in higher, education will increase the stock of human capital in the society, the return to it and the tax revenues in a faster rate. This in turn can increase the expenditures in secondary education and therefore the quality of the educational system and so on. Thus, policies aiming at the increase of high school wage can induce economic growth. The robustness of our analytical results was tested against empirical evidence from U.S.. Finally, we consider that further research in this field is required in order to decode the educational decision patterns of individuals and their interrelation with economic activity.

**Appendix A. Comparative Statics Analysis**

The first order conditions of the problem are:

\[
\ell = \Pi(q, \eta) + \lambda[P_s(q, \eta)p(q) - \eta] + \mu[(1 - \tau)W_u - W_h - C_s(q, \eta) + \frac{C_s(q, \eta)}{P_s(q, \eta)}] + \nu[\lambda - (1 - \tau)W_u + \frac{C_s(q, \eta)}{P_h(q, \eta)}] \quad (A.1)
\]

where \( \Pi(q, \eta) = \eta W_u + (1 - \eta)W_h - k(\eta) - c(q) - p(q)C_s(q, \eta) \)

\[
\frac{\partial \ell}{\partial q} = 0 \Rightarrow -\frac{\partial c(q)}{\partial q} - \frac{\partial p(q)}{\partial q} C_s(q, \eta) - p(q)\frac{\partial C_s(q, \eta)}{\partial q} + \lambda\left[\frac{\partial P_s(q, \eta)}{\partial q} p(q) + P_s(q, \eta)\frac{\partial p(q)}{\partial q}\right]
\]

\[
\frac{\partial C_s(q, \eta)}{\partial q} \left[\frac{P_s(q, \eta) - C_s(q, \eta)}{(P_s(q, \eta))^2}\right] + \nu\left[\frac{\partial P_s(q, \eta)}{\partial q} P_s(q, \eta) - C_s(q, \eta)\frac{\partial P_s(q, \eta)}{\partial q}\right] = 0 \quad (A.2)
\]
Assume that two restrictions bind; the restriction of the maximum affordable number of university entrants and the self-selection constraint for type-\(b\) individuals.

Our guess that in the optimum solution \(\frac{C_b(q, \eta)}{P_b(q, \eta)} = (1 - \tau)W_u - W_h\), can be justified as follows:

\[
\frac{C_b(q, \eta)}{P_b(q, \eta)} = (1 - \tau)W_u - W_h \quad \text{describes an indifference curve which is above}
\]

\[
\frac{C_s(q, \eta)}{P_s(q, \eta)} = (1 - \tau)W_u - W_h, \quad \text{since} \quad \frac{C_b(q, \eta)}{P_b(q, \eta)} > \frac{C_s(q, \eta)}{P_s(q, \eta)} \quad \text{by assumption and}
\]

\[
\frac{\partial(C_s(q, \eta))}{\partial q}, \frac{\partial(C_b(q, \eta))}{\partial q}, \frac{\partial(C_s(q, \eta))}{\partial \eta}, \frac{\partial(C_b(q, \eta))}{\partial \eta} < 0.
\]

Hence,

\[
\frac{C_s(q, \eta)}{P_s(q, \eta)} < (1 - \tau)W_u - W_h \quad \text{is described by all the indifference curves which are above}
\]

\[
\frac{C_s(q, \eta)}{P_s(q, \eta)} = (1 - \tau)W_u - W_h \quad \text{and} \quad \frac{C_b(q, \eta)}{P_b(q, \eta)}>(1 - \tau)W_u - W_h \quad \text{is described by all the}
\]

indifference curves which are below \(\frac{C_b(q, \eta)}{P_b(q, \eta)} = (1 - \tau)W_u - W_h\). If we assume that the objective function of the public authority is increasing in \(q, \eta\) then the self-selection constraint for type-\(b\) individuals should bind. Moreover, we assume that we do not have a corner solution.

Under this assumption in order to have a maximum [If the last \(h - (e + z) - \text{where} \ h \text{are the unknown variables,} \ e \text{are the constraints that bind and} \ z \text{are the equality constraints - leading principal minors alternate in sign with the sign of the determinant of the largest matrix the same as the sign of} \ (-1)^h, \text{then we have a strict local constraint maximum}]:

\[
\frac{\partial^l}{\partial \eta} = 0 \Rightarrow W_u - W_h - \frac{\partial k(\eta)}{\partial \eta} - p(q) \frac{\partial C_s(q, \eta)}{\partial \eta} + \lambda\left[\frac{\partial P_s(q, \eta)}{\partial \eta} \right] p(q) - 1
\]

\[
- \lambda\left(\frac{\partial C_s(q, \eta)}{\partial \eta} P_s(q, \eta) - C_s(q, \eta) \frac{\partial P_s(q, \eta)}{\partial \eta}\right) \left(\frac{P_s(q, \eta)}{P_b(q, \eta)}\right)^2 \right] + \nu\left(\frac{\partial C_s(q, \eta)}{\partial \eta} P_s(q, \eta) - C_s(q, \eta) \frac{\partial P_b(q, \eta)}{\partial \eta}\right) \left(\frac{P_b(q, \eta)}{P_b(q, \eta)}\right)^2 = 0 \quad (A3)
\]
\[D = \det \begin{bmatrix}
0 & 0 & \frac{\partial R_1}{\partial q} & \frac{\partial R_1}{\partial \eta} \\
0 & 0 & \frac{\partial R_1}{\partial R_3} & \frac{\partial R_3}{\partial \eta} \\
\frac{\partial R_1}{\partial q} & \frac{\partial R_1}{\partial \eta} & \frac{\partial q \partial \eta}{\partial R_1} & \frac{\partial \eta^2}{\partial R_1} \\
\frac{\partial R_3}{\partial q} & \frac{\partial R_3}{\partial \eta} & \frac{\partial q \partial \eta}{\partial R_3} & \frac{\partial \eta^2}{\partial R_3}
\end{bmatrix} > 0 \quad (A.4)\]

where \( R_1 = \eta - P_s(q, \eta)p(q) \) and \( R_3 = (1 - \tau)W_u - W_h - \frac{C_s(q, \eta)}{P_s(q, \eta)} \).

It can be easily shown that:

\[\text{det} \begin{bmatrix}
0 & 0 & \frac{\partial R_1}{\partial q} & \frac{\partial R_1}{\partial \eta} \\
0 & 0 & \frac{\partial R_1}{\partial R_3} & \frac{\partial R_3}{\partial \eta} \\
\frac{\partial R_1}{\partial q} & \frac{\partial R_1}{\partial \eta} & \frac{\partial q \partial \eta}{\partial R_1} & \frac{\partial \eta^2}{\partial R_1} \\
\frac{\partial R_3}{\partial q} & \frac{\partial R_3}{\partial \eta} & \frac{\partial q \partial \eta}{\partial R_3} & \frac{\partial \eta^2}{\partial R_3}
\end{bmatrix} = \text{det} \begin{bmatrix}
\frac{\partial^2 \ell}{\partial q^2} & \frac{\partial^2 \ell}{\partial q \partial \eta} & -\frac{\partial R_1}{\partial q} & -\frac{\partial R_1}{\partial \eta} \\
\frac{\partial^2 \ell}{\partial R_1^2} & \frac{\partial^2 \ell}{\partial R_1 \partial \eta} & -\frac{\partial R_3}{\partial q} & -\frac{\partial R_3}{\partial \eta} \\
\frac{\partial R_1}{\partial q} & \frac{\partial R_1}{\partial \eta} & \frac{\partial q \partial \eta}{\partial R_1} & \frac{\partial \eta^2}{\partial R_1} \\
\frac{\partial R_3}{\partial q} & \frac{\partial R_3}{\partial \eta} & \frac{\partial q \partial \eta}{\partial R_3} & \frac{\partial \eta^2}{\partial R_3}
\end{bmatrix} > 0 \quad (A.5)\]

Under the assumption that the constraint of ‘good’ does not bind, we get that in the optimum \( \mu = 0 \).

By total differentiating \( \frac{\partial \ell}{\partial q}, \frac{\partial \ell}{\partial \eta}, R_1 \) and \( R_3 \) with respect to \( W_h \), we get (in a matrix form):
\[
\begin{bmatrix}
\frac{\partial^2 \ell}{\partial q^2} & \frac{\partial^2 \ell}{\partial q \partial \eta} & -\frac{\partial R_1}{\partial q} & \frac{\partial^2 \ell}{\partial \eta^2} & -\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_3}{\partial \eta} \\
\frac{\partial^2 \ell}{\partial \eta \partial \eta} & \frac{\partial R_1}{\partial \eta} & \frac{\partial R_1}{\partial \eta} & 0 & 0 \\
-\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_3}{\partial \eta} & 0 & 0 \\
\frac{\partial q}{\partial \eta} & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\frac{\partial q}{\partial W_h} \\
\frac{\partial \eta}{\partial W_h} \\
\frac{\partial \lambda}{\partial W_h} \\
\frac{\partial \nu}{\partial W_h} \\
\frac{\partial W_h}{\partial W_h}
\end{bmatrix} = \begin{bmatrix}
0 \\
1 \\
0 \\
1
\end{bmatrix}
\] (A.6)

Hence, the derivative of $\eta$ with respect to $W_h$ will be equal to:

\[
\frac{\partial \eta}{\partial W_h} = \frac{1}{D} \det \begin{bmatrix}
\frac{\partial^2 \ell}{\partial q^2} & -\frac{\partial R_1}{\partial q} & -\frac{\partial R_3}{\partial q} \\
\frac{\partial^2 \ell}{\partial \eta \partial q} & \frac{\partial R_1}{\partial \eta} & \frac{\partial R_1}{\partial \eta} \\
-\frac{\partial R_1}{\partial q} & -\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_3}{\partial \eta} \\
\frac{\partial q}{\partial \eta} & 0 & 0
\end{bmatrix} + \frac{1}{D} \begin{bmatrix}
\frac{\partial^2 \ell}{\partial \eta^2} & -\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_3}{\partial \eta} \\
\frac{\partial^2 \ell}{\partial \eta \partial \eta} & \frac{\partial R_1}{\partial \eta} & \frac{\partial R_1}{\partial \eta} \\
-\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_3}{\partial \eta} \\
\frac{\partial q}{\partial \eta} & 0 & 0
\end{bmatrix} > 0 \quad \text{(if} \quad \frac{\partial P_g}{\partial \eta} < 1) \).

The derivative of $q$ with respect to $W_h$ will be equal to:

\[
\frac{\partial q}{\partial W_h} = -\frac{1}{D} \det \begin{bmatrix}
\frac{\partial^2 \ell}{\partial q^2} & -\frac{\partial R_1}{\partial q} & -\frac{\partial R_3}{\partial q} \\
\frac{\partial^2 \ell}{\partial \eta \partial q} & \frac{\partial R_1}{\partial \eta} & \frac{\partial R_1}{\partial \eta} \\
-\frac{\partial R_1}{\partial q} & -\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_3}{\partial \eta} \\
\frac{\partial q}{\partial \eta} & 0 & 0
\end{bmatrix} - \frac{1}{D} \begin{bmatrix}
\frac{\partial^2 \ell}{\partial \eta^2} & -\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_3}{\partial \eta} \\
\frac{\partial^2 \ell}{\partial \eta \partial \eta} & \frac{\partial R_1}{\partial \eta} & \frac{\partial R_1}{\partial \eta} \\
-\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_1}{\partial \eta} & -\frac{\partial R_3}{\partial \eta} \\
\frac{\partial q}{\partial \eta} & 0 & 0
\end{bmatrix} > 0 \quad \text{(if} \quad \frac{\partial P_g}{\partial \eta} < 1) \).
References


### Tables

#### Table 1: Assumptions about the Derivatives of the Main Variables

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$\frac{\partial P_s(q,\eta)}{\partial q}, \frac{\partial P_b(q,\eta)}{\partial q} &gt; 0$</td>
<td>$\frac{\partial P_s(q,\eta)}{\partial \eta}, \frac{\partial P_b(q,\eta)}{\partial \eta} &gt; 0$</td>
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<tr>
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<td>$\frac{\partial P^2_s(q,\eta)}{\partial \eta^2} &gt; 0, \frac{\partial P^2_b(q,\eta)}{\partial \eta^2} &lt; 0$</td>
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</tr>
<tr>
<td>$\frac{\partial c(q)}{\partial q} &gt; 0, \frac{\partial c^2(q)}{\partial q^2} &lt; 0$</td>
<td>$\frac{\partial k(\eta)}{\partial \eta} &gt; 0, \frac{\partial k^2(\eta)}{\partial \eta^2} &lt; 0$</td>
</tr>
</tbody>
</table>

#### Table 2: Stationarity Tests

<table>
<thead>
<tr>
<th>Variables in levels</th>
<th>ADF test (lags)</th>
<th>PP test (bandwidth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(pubs)</td>
<td>-1.649 (0)</td>
<td>-1.912 (2)</td>
</tr>
<tr>
<td>ln(whs)</td>
<td>-1.078 (0)</td>
<td>-1.236 (1)</td>
</tr>
<tr>
<td>ln(sat)</td>
<td>-1.96 (1)</td>
<td>-2.003 (4)</td>
</tr>
<tr>
<td>ln(exgdp)</td>
<td>-1.573 (0)</td>
<td>-1.586 (2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables in first difference</th>
<th>ADF test (lags)</th>
<th>PP test (bandwidth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(pubs)</td>
<td>-5.165*** (0)</td>
<td>-5.198*** (3)</td>
</tr>
<tr>
<td>ln(whs)</td>
<td>-4.646*** (0)</td>
<td>-4.646*** (0)</td>
</tr>
<tr>
<td>ln(sat)</td>
<td>-3.504** (0)</td>
<td>-3.398** (2)</td>
</tr>
<tr>
<td>ln(exgdp)</td>
<td>-5.052*** (0)</td>
<td>-5.052*** (3)</td>
</tr>
</tbody>
</table>

Notes: Boldface values denote sampling evidence in favour of unit roots. *** and ** signify rejection of the unit root hypothesis at the 1% and 5% level of significance respectively. The numbers in parentheses for the Augmented Dickey – Fuller (ADF) test are the optimal numbers of lagged difference terms, which are determined using AIC (Akaike Information Criterion). The numbers in parentheses for the Phillips – Perron (PP) test are the Newey – West bandwidth parameters of the Kernel – based estimator of the residual spectrum at frequency zero. Trend and constant were included in the test equation for ln(pubs), ln(whs) and ln(exgdp), whereas only constant was included in the test equation for ln(sat).
### Table 3: Engle – Granger cointegration test

<table>
<thead>
<tr>
<th>Residuals of regression of ( \ln(\text{pubs}) ) on ( \ln(\text{whs}) ) (including constant and trend)</th>
<th>ADF test (lags)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residuals of regression of ( \ln(\text{sat}) ) on ( \ln(\text{whs}) ) and ( \ln(\text{exgdp}) ) (including constant)</td>
<td>-3.849*** (0)</td>
</tr>
<tr>
<td>Residuals of regression of ( \ln(\text{sat}) ) on ( \ln(\text{whs}) ) and ( \ln(\text{exgdp}) ) (including constant)</td>
<td>-3.225*** (1)</td>
</tr>
</tbody>
</table>

Notes: *** signifies rejection of the unit root hypothesis at the 1% level of significance. The numbers in parentheses for the Augmented Dickey – Fuller (ADF) test are the optimal numbers of lagged difference terms, which are determined using AIC (Akaike Information Criterion). The test was performed by the use of MacKinnon (1996) one–sided p-values.

### Table 4: OLS Results of Equation (8)

Dependent Variable: \( \ln(\text{pubs}) \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>4.678***</td>
<td>1.078</td>
<td>4.341</td>
<td>0.000</td>
</tr>
<tr>
<td>trend</td>
<td>-0.004**</td>
<td>0.002</td>
<td>-2.388</td>
<td>0.024</td>
</tr>
<tr>
<td>( \ln(\text{whs}) )</td>
<td>1.107**</td>
<td>0.411</td>
<td>2.691</td>
<td>0.012</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.491***</td>
<td>0.148</td>
<td>3.323</td>
<td>0.003</td>
</tr>
</tbody>
</table>

R-squared 0.707, Mean dependent var. 7.515, Adjusted R-squared 0.674, S.D. dependent var. 0.069, S.E. of regression 0.039, Akaike info criterion -3.521, Sum squared resid. 0.041, Schwarz criterion -3.336, Log likelihood 58.576, F-statistic 21.686, Durbin-Watson stat. 1.497, Prob(F-statistic) 0.000

Note: *** and ** denote statistical significance at 1% and 5%, respectively.
### Table 5: OLS Results of Equation (9)
Dependent Variable: \( \ln(sat) \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>6.783***</td>
<td>0.066</td>
<td>102.079</td>
<td>0.000</td>
</tr>
<tr>
<td>( \ln(whs) )</td>
<td>0.141***</td>
<td>0.019</td>
<td>7.446</td>
<td>0.000</td>
</tr>
<tr>
<td>( \ln(exgdp) )</td>
<td>0.073***</td>
<td>0.010</td>
<td>7.037</td>
<td>0.000</td>
</tr>
<tr>
<td>AR(12)</td>
<td>0.516***</td>
<td>0.102</td>
<td>5.048</td>
<td>0.000</td>
</tr>
</tbody>
</table>

R-squared 0.882    Mean dependent var. 6.919
Adjusted R-squared 0.860    S.D. dependent var. 0.008
S.E. of regression 0.0001    Akaike info criterion -8.566
Sum squared resid. 0.0001    Schwarz criterion -8.367
Log likelihood 89.660    F-statistic 40.016
Durbin-Watson stat. 1.679    Prob(F-statistic) 0.000

Note: *** and ** denote statistical significance at 1% and 5%, respectively.

### Table 6: Diagnostic tests

<table>
<thead>
<tr>
<th>Equation (8)</th>
<th>Value of test statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( JB )</td>
<td>0.864</td>
<td>[0.649]</td>
</tr>
<tr>
<td>Reset test</td>
<td>2.490</td>
<td>[0.127]</td>
</tr>
<tr>
<td>( LM_1/LM_2 ) test</td>
<td>3.346/1.943</td>
<td>[0.079/0.164]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation (9)</th>
<th>Value of test statistic</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( JB )</td>
<td>1.116</td>
<td>[0.572]</td>
</tr>
<tr>
<td>Reset test</td>
<td>5.116</td>
<td>[0.039]</td>
</tr>
<tr>
<td>( LM_1/LM_2 ) test</td>
<td>0.357/0.295</td>
<td>[0.559/0.749]</td>
</tr>
</tbody>
</table>

Note: Figures in brackets represent asymptotic P-values associated with the tests. \( JB \) denotes the Jarque-Bera normality test of errors. The Reset test tests the null hypothesis of functional form misspecification. \( LM_1/LM_2 \) is the Lagrange multiplier test for first and second order serial correlation (under the null there is no serial correlation in the residuals up to the specified order).