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Abstract

We consider a market with sequential consumer search in which firms can distinguish potential customers visiting for the first time from returning visitors. We show that firms often have an incentive to make it costly for its visitors to return after investigating rivals, either by making an "exploding offer" (which permits no return once the consumer leaves) or by offering a "buy-now discount" (which makes the price paid by first-time visitors lower than that for returning visitors). Prices often increase when return costs are artificially increased in this manner, and this harms consumers and market performance. If firms cannot commit to their buy-later price the outcome depends on whether there is an intrinsic cost of returning to a firm: if the intrinsic return cost is zero, it is often an equilibrium for firms not to offer any buy-now discount; if the return cost is positive, firms are forced to make exploding offers.

Keywords: Consumer search, oligopoly, price discrimination, high-pressure selling, exploding offers, buy-now discounts, costly recall.

1 Introduction

In markets in which consumers sequentially search through available options, it is common for a consumer to return to buy from a previously sampled seller only after investigating other sellers.[1] In some circumstances, a seller may be able to distinguish potential

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[1] De los Santos (2008) presents a rare empirical study of consumer search behaviour prior to making a purchase, using data from online book purchases. De los Santos (2008, section 4) finds that three-quarters of consumers search only one retailer before making their purchase. Of the remaining consumers who

customers who come to the store for the first time from those who have returned after a previous visit. A sales assistant may tell from a potential customer's questions or demeanor whether she has paid a previous visit or not, or may simply recognize her face. In online markets, a retailer using tracking software may be able to tell if a visitor has visited the site before. Sometimes—as with job offers, tailored financial products, medical insurance, or home improvements—a consumer needs to interact with a seller to discuss specific requirements, and this process reveals the consumer's identity. In these situations where sellers can distinguish new from returning visitors, we argue that firms often have an incentive to discriminate against returning visitors, either by using so-called "exploding offers", which force the consumer to buy immediately or not at all, or by using "buy-now discounts", which offer first-time visitors a lower price than return visitors.

Because they are often used somewhat informally or furtively, it is hard to produce evidence of exploding offers or buy-now discounts in consumer markets. One of the authors encountered an in-home salesman of financial products, and when he said he wished to think about the offer and get back to the salesman, the salesman claimed it was his last day in his current job. The use of buy-now discounts is plausible in retailing situations where sales people have authority to offer discretionary discounts. For instance, one can imagine a sales assistant in an electronics store offering a customer a 10% discount if the sale is made immediately (e.g., before the assistant "leaves for the day"). When searching for air-tickets online, a consumer may find a quote on one website, go on to investigate a rival seller, only to return to the original website to find the price has mysteriously risen. One of the most notorious examples of high-pressure selling involves time-share vacation homes, where potential customers are lured (often with promise of a gift) to listen to a lengthy presentation about the properties, and then told they must buy immediately or not at all (or offered a discount off the list price if they sign immediately).

There are potentially two broad reasons why a firm may wish to make it costly, or impossible, for its first-time visitors to return. First, there is a *strategic* reason, which is to deter a potential consumer from going on to investigate rival—and perhaps superior—offers. If a consumer cannot return to a seller once she leaves, this increases the opportunity cost of onward search, as the consumer then has fewer options remaining relative to situation in which return is costless.[2] Second, the observation that a consumer has come back to a seller after sampling other options reveals relevant *information* about a consumer's tastes, and this may be a profitable basis for price discrimination. A seller may wish to charge a higher price to those consumers who have already investigated other sellers, because

search at least twice, approximately two-thirds buy from the final firm searched and one-third go back to a firm searched earlier. De los Santos also finds that the initial search is non-random, and one firm (*Amazon.com*) was sampled first by about two-thirds of all consumers making a purchase.

[2]Another strategic reason why a seller might try to force immediate sale is to prevent the consumer having time to evaluate the *current* product adequately, rather than preventing the evaluation of rival offers. We discuss this alternative rationale in section 4.

their decision to return indicates they are unsatisfied with rival products.[3] However, this incentive is tempered by the fact that returning consumers also do not have a strong taste for the firm's own product, for otherwise they would have purchased immediately instead of going on to investigate alternative sellers.

Our underlying framework is a sequential search model with horizontally differentiated products in which consumers search both for price and product fitness, as introduced by Wolinsky (1986). Each firm has two sources of demand: consumers who buy its product on their first visit to the firm ("fresh demand"), and consumers who sample the firm, go on to sample rival products, but eventually come back to buy ("returning demand"). In the standard search model, firms cannot distinguish between these two groups and so must treat all visitors equally, while in this paper firms are able to discriminate between the two groups. Using this basic market framework, we present two related models.

First, in section 2, we suppose that firms can employ one of just two return policies: consumers can freely return after leaving the firm (and buy at the same price), or exploding offers are used and first-time visitors are forced to buy immediately or never. We derive the equilibrium price when all firms use exploding offers, and show that typically it is higher than the corresponding price with free recall. The use of exploding offers also leads to inefficient matching between products and consumers. When a firm uses an exploding offer, this makes those consumers with strong tastes for the firm's product more likely to buy immediately, but it prevents consumers with moderate tastes from returning after they find nothing better elsewhere. We show that firms wish to use exploding offers when the density for match utility is increasing, while when this density decreases firms choose to allow free recall. In this model, only the strategic reason to make return costly is present, as by construction firms make no sales to returning visitors.

Second, in section 3, we assume firms have a richer set of return policies to choose from, and rather than simply banning return they can charge returning visitors a higher price; that is, they can offer first-time visitors a buy-now discount. Starting from a situation in which all firms treat fresh and returning consumers equally, we show under relatively mild conditions that a firm has an incentive to offer a buy-now discount. Compared to the case with exploding offers, a firm has a greater incentive to introduce these "tariff-intermediated" search frictions, because of the extra revenue generated from returning buyers. In the specific example of duopoly and a uniform distribution for match utility, we calculate the equilibrium prices for immediate and returning purchase, and find that the buy-now discount is largest when intrinsic search frictions are small. Because of the extra search frictions introduced by the buy-now discount, even the discounted buy-now price is

[3] This contrasts with the substantial literature about how firms can use the information of consumer *purchase* history to refine their prices. (See, for instance, Hart and Tirole (1988), Chen (1997), Fudenberg and Tirole (2000), and Acquisti and Varian (2005).) These models often predict that a firm will price low to a customer who previously purchased from a rival (or consumed the outside option in the case of monopoly), since such a customer has revealed she has only a weak preference for the firm's product.

higher than the non-discriminatory price. As such, this form of price discrimination lowers both consumer surplus and total welfare.

In section 3.3 we relax the assumption that firms commit to their buy-later price when consumers make their first visit. The outcome without commitment depends sensitively on whether or not consumers face an intrinsic (as opposed to artificially inflated) cost of returning to a previous firm. If there is no such cost, we show that it is often an equilibrium for firms to offer uniform prices, i.e., the fact that a consumer has come back to a firm after sampling other sellers may give no *ex post* incentive for a firm to raise its price. This implies that the informational incentive to set higher prices to returning customers is often non-existent, and it is the strategic impact on a consumer's incentive to buy immediately which is the dominant factor when a firm decides to make return costly. However, for reasons akin to Diamond's (1971) famous paradox, when consumers do incur a positive exogenous return cost (no matter how small), the unique credible outcome is that firms make exploding offers.

Our paper relates to several strands of the industrial organization literature. It is complementary to the model of ordered search in Armstrong, Vickers, and Zhou (2009). The two papers use the same market model and focus on the same distinction between fresh and returning demand, but there are two major differences.[4] First, Armstrong, Vickers, and Zhou (2009) suppose that firms know something about their place in the consumer search order and can set their price accordingly, while for the most part in this paper we assume random search whereby firms do not know where they are in a consumer's search process.[5] Second, Armstrong, Vickers, and Zhou (2009) assume that firms cannot directly distinguish between fresh and returning demand and must treat both sets of consumers equally, while this ability to distinguish between new and returning visitors lies at the heart of the current analysis. In Armstrong, Vickers, and Zhou (2009), a firm which is more "prominent" is predicted to set a lower price than its less prominent rivals. (If a firm is far back in the search order, it knows that any consumer who reaches it must not care for the products of its rivals, and so this firm has monopoly power over its consumers and sets its price accordingly.) This reflects the informational motive to set high prices to consumers who have already sampled, and rejected, rival products.

Our analysis is related to models of search with (exogenous) costly recall. Janssen and Parakhonyak (2010) study the optimal stopping rule when consumers care only about price and must incur a cost to return to a previous firm. This stopping rule is significantly more complicated than when return is costless. When there are more than two firms, a consumer's stopping rule is non-stationary and her reservation surplus level depends on her previous offers. They show that equilibrium prices do not depend on the recall cost

[4] A third difference is that the earlier paper relies heavily on an assumption that match utilities are uniformly distributed, whereas here most of the analysis is more general.

[5] We discuss the impact of having one firm more prominent in section 4 below.

(unlike our model, where prices are sensitive to the endogenously generated recall costs).[6]

Firms often benefit from the reduction of consumer search intensity, since this usually softens price competition. In our model, the buy-now discount or exploding offer serves this purpose. Alternatively, Ellison and Wolitzky (2008) present a model with homogenous products in which a consumer's incremental search cost increases with her cumulative search effort. If a firm increases its in-store search cost (say, by making its tariff harder to comprehend), this will make further search less attractive. They show that if the exogenous component of search costs falls, firms will unilaterally increase their self-determined element of search costs, with the result that equilibrium prices are unchanged. Though otherwise very different, the two models study how search frictions are determined endogenously: even if intrinsic search frictions are negligible, a market may suffer from substantial search frictions—and high prices—in equilibrium.

Our analysis of buy-now discounts is also somewhat related to the emerging literature on auctions with a "buy now" price (see Reynolds and Wooders, 2009, for instance). Online auctions sometimes offer bidders the option to buy the item immediately at a specified price rather than enter an auction against other bidders. In these situations, a seller has one item to sell to a number of potential bidders, and so a bidder needs to pay a *high* buy-now price in order to induce the seller from going on to search for other bidders by running an auction, whereas our model involves sellers offering a *low* buy-now price so as to induce a buyer from going on to search for other sellers. Common rationales for buy-now prices in auctions are impatience or risk-aversion on the part of bidders, neither of which is needed in our framework with costly search.

As far as we know, our paper is the first to study the use of exploding offers in consumer markets. In the alternative setting of matching markets, however, there are a number of studies in which exploding offers play a role. Exploding offers are often used in specialized labor markets, such as those for law clerks, sports players, medical staff, and student college allocations. When exploding offers are used, these markets have a tendency to "unravel", and employers compete to make earlier and earlier offers. The result can be significant inefficiency.[7] Niederle and Roth (2009) run an experiment to measure the impact of a policy which bans the use of exploding offers in a laboratory matching market. They find that firms do tend to use exploding offers when they are permitted to do so, and the result is that matching occurs inefficiently early and match quality is poor, relative to

[6] Daughety and Reinganum (1992) make the point that the extent of consumer recall may be endogenously determined by firms' equilibrium strategies. In their model, the instrument that a firm can use to influence consumer recall is the length of time that it will hold the good for consumers at the quoted price. In contrast to our assumption that a consumer can discover a seller's return policy only after investigating that seller, Daughety and Reinganum suppose that sellers can announce their recall policies to the population of consumers before search begins.

[7] Roth and Xing (1994, page 1001) document some examples of high-pressure job offers. For instance, in the market for judicial clerkships, some judges use exploding offers which would be withdrawn if they are not accepted in some very short time, or even during the telephone conversation itself.

the situation in which exploding offers cannot be used (or when applicants can renege on previous agreements).

2 Exploding Offers

Our underlying model of the market is based on Wolinsky (1986). (See Anderson and Renault (1999) for a further development of Wolinsky's model.) There are $n \geq 2$ firms in the market, each supplying a horizontally differentiated product at zero production cost. A consumer's valuation of product i, u_i, is a random draw from some common distribution with support $[0, u_{\max}]$ and with cumulative distribution function $F(\cdot)$ and density $f(\cdot)$. We suppose that the realization of match utility is independent across consumers and products. In particular, there are no systematic quality differences across the products. Each consumer wishes to buy one item, provided an item can be found with a positive surplus. Both firms and consumers are assumed to be risk neutral.

Consumers initially have imperfect information about the deals available in the market. They gather this information through a sequential search process, and by incurring a search cost $s \geq 0$, a consumer can visit a firm and find out its price, its return policy, and match value.[8] In this section, the two return policies available to a firm are to use an exploding offer or to allow free recall. (If a firm allows free recall, it sets the same price to first-time visitors and returning visitors.) After sampling one firm, a consumer can choose to buy at this firm immediately or to investigate another firm. If permitted, she can costlessly return to a previous firm after sampling subsequent firms. To implement an exploding offer, firms are assumed to be able to distinguish first-time visitors from returning customers. We focus on symmetric situations with random search, so that a consumer is equally likely to investigate any of the remaining unsampled firms when they search.

For expositional convenience, we introduce a piece of notation which summarizes the distribution of match utilities and the extent of search frictions:

$$V(p) \equiv \int_p^{u_{\max}} (u - p)\, dF(u) - s \ . \tag{1}$$

Thus, $V(p)$ is the expected surplus of sampling a product if a consumer expects that the price is p, the cost of sampling the product is s, and this is the only product available. Note that $V(p)$ is decreasing but $p + V(p)$ is increasing in p. Throughout this paper we assume that the search cost s is relatively small, so that

$$V(\bar{p}) > 0 \ , \tag{2}$$

where $\bar{p}$ is the monopoly price, i.e., $\bar{p}$ maximizes $p[1 - F(p)]$.[9] This condition means that consumers are willing to sample a product sold even at the monopoly price. In the example

[8] If the search cost is zero, we require that consumers nevertheless consider products sequentially.

[9] Under regularity conditions (e.g., F has an increasing hazard rate), $\bar{p}$ solves the first-order condition $\bar{p} = \frac{1-F(\bar{p})}{f(\bar{p})}$ uniquely.

where u is uniformly distributed on $[0, 1]$, which we use for illustration at several points in the following analysis, condition (2) requires $s < \frac{1}{8}$.

2.1 The free-recall benchmark

If all firms allow free recall, the situation is as in Wolinsky (1986). For reference later, in this section we recapitulate part of his analysis. Wolinsky shows that in a symmetric equilibrium in which all firms set the same price p_0, consumers have a stationary stopping rule whereby they buy a product immediately if they obtain a match utility u greater than a threshold a, and if no product yields that level of utility, the consumer samples all products and buys from the best of the n options provided that one option generates a positive surplus. Here, the reservation utility a is determined by the formula

$$V(a) = 0 \ . \tag{3}$$

The expression $\int_a^{u_{\max}} (u - a)\, dF(u)$ in $V(a)$ is just the incremental benefit of engaging in one more search if the best current utility is a and the consumer can freely return to this best offer if the next product does not yield higher surplus. So the optimal threshold makes the consumer indifferent between searching on, which incurs the cost s, and purchasing this product with utility a. Since $V(\cdot)$ is a decreasing function, (3) has a unique solution and a decreases with s. The search cost condition (2) is therefore equivalent to $a > \bar{p}$.

Given that the other firms are charging the equilibrium price p_0, if firm i deviates and charges $\tilde{p}$, its demand is

$$Q = \frac{1 - F(a)^n}{n(1 - F(a))} \left[1 - F(a - p_0 + \tilde{p})\right] + \int_{p_0}^{a} F(u)^{n-1} f(u - p_0 + \tilde{p}) du \ . \tag{4}$$

To understand this expression, consider the two sources of firm i's demand. Suppose firm i is in the k_{th} position in a consumer's search order. Then to reach the firm, the consumer must have sampled, and rejected, $k - 1$ firms first, which occurs with probability $F(a)^{k-1}$ (since a consumer will buy immediately if $u_j \geq a$). If $k < n$, the consumer will buy immediately at firm i if $u_i - \tilde{p} \geq a - p_0$, which occurs with probability $1 - F(a - p_0 + \tilde{p})$. If the firm is in the final search position (i.e., $k = n$), then she will surely buy from firm i if $u_i - \tilde{p} \geq a - p_0$, since then her surplus $u_i - \tilde{p}$ is positive and higher than all other firms. Since firm i is equally likely to be in any of the search positions, the firm's demand from this source is $[1 - F(a - p_0 + \tilde{p})] \times \frac{1}{n}[1 + F(a) + F(a)^2 + \cdots + F(a)^{n-1}]$, which yields the first term in (4). The second source of demand comes from the scenario in which the consumer searches all sellers and does not find any product with net surplus greater than $a - p_0$. This consumer will then buy from the firm with the greatest net surplus, if this surplus is positive. The fraction of consumers for whom this happens and then go on to buy from firm i is

$$\Pr(\max_{j \neq i}\{0, u_j - p_0\} < u_i - \tilde{p} < a - p_0) = \int_{\tilde{p}}^{a - p_0 + \tilde{p}} F(u_i - \tilde{p} + p_0)^{n-1} dF(u_i) \ ,$$

which equals the second term in (4) by changing variables from u_i to $u = u_i + p_0 - \tilde{p}$.

In equilibrium, firm i maximizes $\tilde{p}Q$ by choosing $\tilde{p} = p_0$, and so expression (4) implies the first-order condition for p_0 to be the equilibrium price is[10]

$$\frac{1 - F(p_0)^n}{p_0} = f(a)\frac{1 - F(a)^n}{1 - F(a)} - n \int_{p_0}^{a} F(u)^{n-1} f'(u)du \ . \tag{5}$$

Assuming a strictly increasing hazard rate for the match utility (i.e., $1 - F$ is strictly logconcave), a finite number of firms, and condition (2), one can show that in the relevant interval $0 \leq p_0 \leq a$, expression (5) has a unique solution, and this lies in the range

$$\frac{1 - F(a)}{f(a)} < p_0 < \bar{p} \ . \tag{6}$$

As the number of firms becomes infinite, the equilibrium price converges to $p_0 = \frac{1-F(a)}{f(a)}$. As the search cost tends to its upper bound in (2) (i.e., as a tends to $\bar{p}$), consumers stop searching whenever they find a product with positive surplus and each firm acts as a monopolist, so the equilibrium price converges to $p_0 = \bar{p}$ (which then also equals $\frac{1-F(a)}{f(a)}$).

In the remainder of section 2, we extend this model to allow firms to use the additional instrument of exploding offers; that is to say, firms can require first-time visitors to buy their product immediately or not all. We discuss this issue in two stages: first, we analyze equilibrium prices under an assumption that all firms use exploding offers, and second, we discuss when firms do indeed have an incentive to use this high-pressure sales tactic.

2.2 Equilibrium prices with exploding offers

Suppose now that the n firms force their first-time visitors to buy immediately or not at all. Suppose consumers anticipate that each firm sets the same price p. What is a consumer's optimal search strategy? As we will show, and as is intuitive, consumers become less choosy as they run out of options, and their reservation utility for purchasing decreases the more firms they have already sampled. Indeed, if they reach the final firm they will have to accept any offer which leaves them non-negative surplus.[11]

Given the anticipated price p, let a_m denote a consumer's utility threshold when she has $0 \leq m \leq n - 1$ unsampled products remaining; that is, she will buy if her current match utility satisfies $u \geq a_m$ when she has m options remaining. Therefore, $a_m - p$

[10] Anderson and Renault (1999) show that, if $1 - F$ is logconcave, the equilibrium price is increasing in the search cost s and decreasing in the number of firms (see their Proposition 1). (However, Anderson and Renault assume that all consumers buy one product, i.e., there is no outside option, and this affects the first-order condition for the equilibrium price.) It is a subtle issue in this model whether second-order conditions are satisfied in this candidate equilibrium. For discussion, see Proposition B2 in Anderson and Renault (1999). However, a sufficient condition is that the density function f be weakly increasing.

[11] The stopping rule we derive in the following is discussed further in pages 166-171 in Lippman and McCall (1976).

is a consumer's expected surplus from participating in a no-recall search market with m products each sold at price p. Clearly, $a_0 = p$. Recursively, when facing $m + 1$ unsampled products, if the consumer searches on and if the next product has utility greater than a_m, then she will buy the next product, while if the next product's utility is below a_m, she will continue to search and so obtain expected surplus $a_m - p$. Hence,

$$a_{m+1} - p = \int_{a_m}^{u_{\max}} (u - p)dF(u) + (a_m - p)F(a_m) - s \ ,$$

which simplifies to

$$a_{m+1} = a_m + V(a_m) \ . \tag{7}$$

This is a first-order difference equation which governs how a consumer's optimal stopping rule evolves as she has more products remaining unsampled. The right-hand side of (7) increases with a_m. Note that $a_1 > a_0 = p$ whenever $V(p) > 0$, i.e., when $p < a$. In this case, it follows from (7) that $a_{m+1} > a_m$ for all $m \geq 0$, so that a consumer is willing to accept a less suitable product as she nears the end of the search process.[12] In particular, it is possible that a consumer will end up purchasing a product with lower match utility than a product she previously rejected. It also follows from (7) that the difference $a_{m+1} - a_m$ decreases with m. Unlike the case with free recall, each a_m depends on price p since the starting value a_0 does so. Provided the sequence a_m converges as $m \to \infty$ (which it always will do if $s > 0$ or if u has bounded support), it will converge to the free-recall threshold a in expression (3).

This analysis has taken as given the market price p, and we next derive the symmetric equilibrium price. Suppose $n - 1$ firms set the price p and one firm is considering its choice of price, say $\tilde{p}$. (Of course, when choosing their search strategy consumers anticipate that this firm has set the equilibrium price p.) Suppose this deviating firm happens to be in the k_{th} position of a consumer's search process, so there are $n - k$ firms remaining unsampled. Then the probability that the consumer will visit this firm is $h_1 \equiv 1$ if $k = 1$, and if $k > 1$ this probability is

$$h_k \equiv \prod_{i=1}^{k-1} F(a_{n-i}) \ . \tag{8}$$

She will then buy at this firm if $u - \tilde{p} > a_{n-k} - p$, which has probability $1 - F(a_{n-k} - p + \tilde{p})$, and so the firm's demand given it is in a consumer's k_{th} search position is

$$h_k[1 - F(a_{n-k} - p + \tilde{p})] \ . \tag{9}$$

[12] With free recall, the optimal stopping rule is stationary, and $a_m \equiv a$ given in formula (3). Thus, in this situation consumers do not become less choosy as they near the end of the search process. In the alternative setting of matching markets, an applicant for a job (say) may also be reluctant to search for long because the desirable vacancies may quickly be filled.

Since the firm is in any position $1 \leq k \leq n$ with equal probability, its total demand with price $\tilde{p}$ when all other firms are expected to set price p is

$$Q = \frac{1}{n} \sum_{k=1}^{n} h_k [1 - F(a_{n-k} - p + \tilde{p})] \ ,$$

and its profit is $\tilde{p}Q$. The firm's profit is concave in its price $\tilde{p}$ if (but not only if) each function $\tilde{p}[1 - F(a_{n-k} - p + \tilde{p})]$ is concave in $\tilde{p}$. A sufficient condition for this is that the density $f(u)$ weakly increases with u.

Therefore, the first-order condition for p to be the equilibrium price is

$$p = \frac{\sum_{k=1}^{n} h_k [1 - F(a_{n-k})]}{\sum_{k=1}^{n} h_k f(a_{n-k})} \ ,$$

which can be simplified to

$$p = \frac{1 - \prod_{k=1}^{n} F(a_{n-k})}{\sum_{k=1}^{n} h_k f(a_{n-k})} \ . \tag{10}$$

Since each a_{n-k} depends on p, this equation defines p only implicitly. Note that the numerator in (10) is equilibrium industry demand,[13] while $\sum_{k=1}^{n} h_k$ is the expected number of searches performed by a consumer. As with the free-recall case, if $1 - F$ is strictly logconcave, the number of firms is finite and condition (2) holds, expression (10) has a solution in the range

$$\frac{1 - F(a)}{f(a)} < p < \bar{p} \ .$$

In particular, assumption (2) implies that consumers are willing to participate in the market. It can be shown that as the number of firms tends to infinity, this equilibrium price converges to the same lower bound $\frac{1-F(a)}{f(a)}$ as in the free-recall case. Intuitively, when the number of firms is unlimited, a consumer would never choose to return to a previously sampled firm, even if she could freely do so, and so the use of exploding offers then has no effect on the equilibrium price. It is also clear that as the search cost tends to its upper bound (i.e., as a tends to $\bar{p}$), p_0 converges to the monopoly price $\bar{p}$.

At this level of generality, it is hard to compare market performance with and without the use of exploding offers, and the comparison between the prices in (5) and in (10) is opaque. Following Wolinsky, to gain further insights consider the case of a uniform distribution for match utility. (In section 2.3, we will show that with the uniform distribution it is an equilibrium for all firms to use exploding offers.)

Uniform example: If u is uniformly distributed on $[0, 1]$, then (7) implies

$$a_{m+1} = \frac{1}{2}(a_m^2 + 1) - s$$

[13] A consumer will leave the market without buying anything if she searched through all products and the final one has a utility lower than the price. The probability of that is $\prod_{k=1}^{n} F(a_{n-k})$.

starting with $a_0 = p$. This difference equation appears to have no analytical solution. It converges as m becomes large to $a = 1 - \sqrt{2s}$, the free-recall threshold. Except when n is small, equation (10) has no analytical solution, but it can be solved numerically. The solid curve in Figure 1a depicts how the equilibrium price p varies with the number of firms when $s = 0$. The dashed curve represents the corresponding price (5) in the free-recall market. Both prices converge to zero for large n, but it seems that prices with exploding offers are approximately double those which prevail with free recall. (This figure includes the monopoly case $n = 1$, in which case the monopolist charges the price $\bar{p} = \frac{1}{2}$ and the use of exploding offers has no impact since consumers have only one option in any event.) The difference between the two prices is greatest for an intermediate numbers of firms. In the same example, Figure 1b shows that the exploding-offer equilibrium has a higher profit level than the free-recall equilibrium except when $n = 2$.[14] Numerical calculations suggest that as the search cost gets larger, the difference between the exploding-offer and free-recall prices decreases (and if $s = \frac{1}{8}$, the difference vanishes). However, for any positive $s < \frac{1}{8}$, a similar pattern holds, except that the exploding-offer equilibrium more likely leads to a lower profit than the free-recall equilibrium when s is larger (for example, when $s = \frac{1}{20}$ profits are lower with exploding offers when $n \leq 4$).

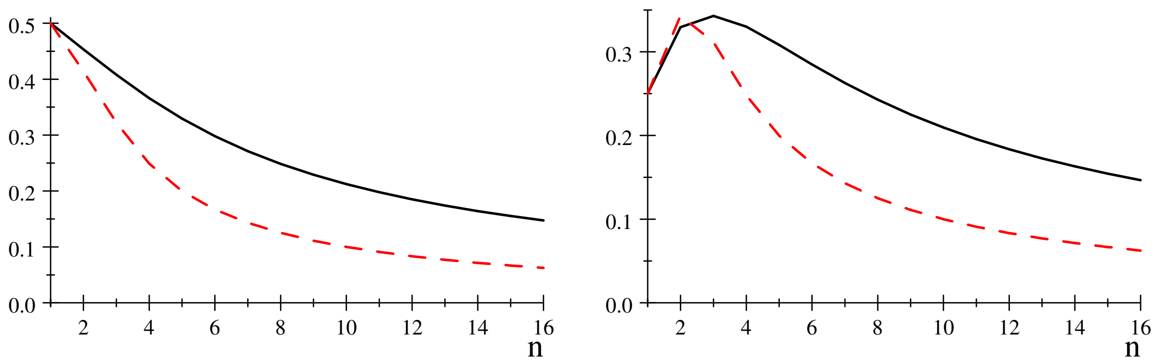

Figure 1a: Prices with exploding offers   Figure 1b: Profits with exploding offers

In this uniform example, aggregate consumer surplus and total welfare (measured by the sum of consumer surplus and profit) fall when firms use exploding offers. Consumer surplus falls since the price rises compared to the free-recall situation *and* consumers are prevented from returning to a product which yields positive surplus. (Even if $p = p_0$, i.e., if using exploding offers did not change the market price, consumers would obtain lower surplus in the exploding-offer case due to the no-return restriction. The resulting higher price $p > p_0$ only adds to their loss.) As far as total welfare is concerned, relative to

[14]The reason why industry profits increase with n for small n is that with few suppliers many consumers will not find a product which yields them positive surplus. With monopoly, for instance, half of consumers are excluded from the market, while with many firms almost all consumers will eventually find a suitable product. But with more firms profits fall with n, as the price reduction effect outweighs this market expansion effect.

the free-recall situation, the use of exploding offers not only induces suboptimal consumer search (i.e., consumers on average cease their search too early due to "buy now or never" requirement, resulting in sub-optimal matching), but also excludes more consumers from the market, both of which harm efficiency.

Exponential example: To illustrate how the use of exploding offers need *not* increase equilibrium prices, consider a second example in which $F(u) = 1 - e^{-u/\mu}$, where μ is the expected value of match utility. The special feature of this distribution is that a monopoly firm facing this population of consumers, where each consumer has an outside option with utility $z \geq 0$, will choose the same price $p = \mu$ regardless of z.[15] When firms use exploding offers, this immediately implies that each firm will choose $p = \mu$, regardless of the number of firms and the search cost (as long as s is relatively small such that consumers are willing to enter the market). One can also show that the same price is chosen when there is free recall, so that $p_0 = \mu$ solves expression (5) in this example for all n and a. Thus, the use of exploding offers has no impact on equilibrium prices. Nevertheless, in this example this sales technique harms both consumers and firms, as demand and match quality are artificially restricted by the requirement that consumers cannot return to a firm. We will see in the next section that firms will not choose to use exploding offers in this example.[16]

2.3 Incentives to use an exploding offer

Here we discuss when the behaviour discussed in the previous section is in fact an equilibrium. That is, if all its rivals set the price p in (10) and make exploding offers, does a firm have an incentive to deviate and allow free recall (and, possibly, set a different price as well)? Before pursuing the analysis in detail, consider this simple duopoly example with fixed prices which yields the main insight.

Suppose there are two firms, both of which set the exogenous price $p < a$. Is a firm's demand boosted or reduced if it decides to force its first-time visitors to buy immediately or not at all? First, for those consumers who first sample its rival, firm i's decision whether or not to use an exploding offer has no impact on its demand. Therefore, the only impact on the firm's demand comes from that half of the consumer population who sample it first. If firm i allows free recall, a consumer will buy from it immediately whenever $u_i > a$, and a consumer will return to buy from it whenever $p < u_i < a$ and $u_i > u_j$. This pattern of demand is depicted in Figure 2a below. If, instead, firm i uses an exploding offer,

[15] This is the "memoryless" property of the exponential distribution. With price p, the monopolist will sell to a consumer if $u - p \geq z$, and so will choose p to maximize $pe^{-(p+z)/\mu}$, a choice which does not depend on z.

[16] While we have been unable to make progress in comparing prices with and without exploding offers with general distributions for match utility, numerical simulations confirm that for a wide range of distributions prices are higher when exploding offers are employed. (We conjecture that this is true provided $1 - F$ is strictly logconcave.)

expression (7) implies that a consumer will buy from it if and only if $u_i > a_1 = p + V(p)$. This pattern of demand is depicted in Figure 2b.

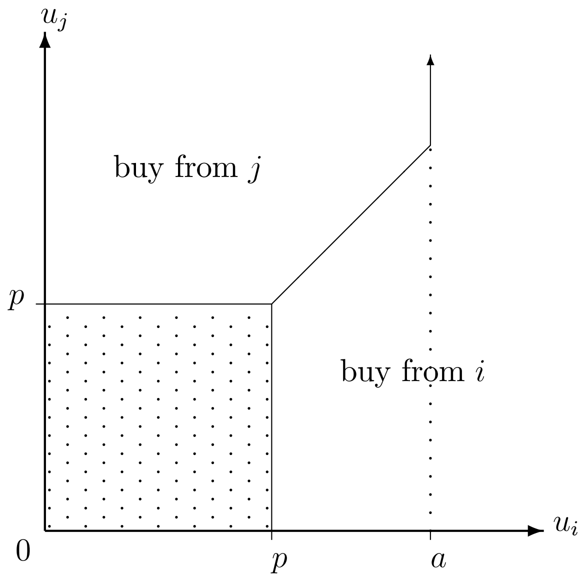

Figure 2a: Demand with free recall

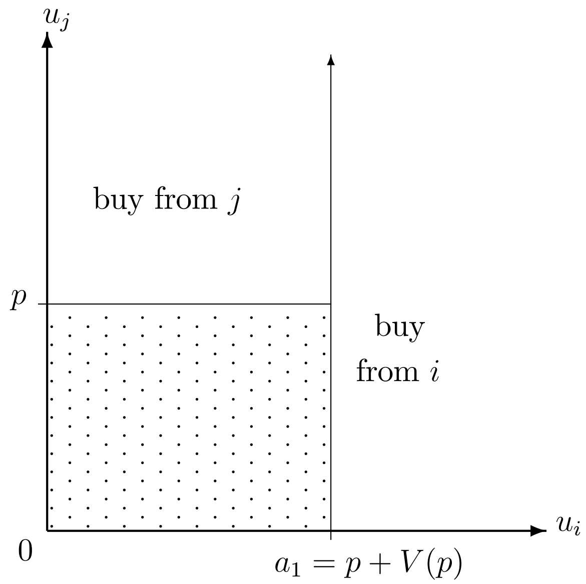

Figure 2b: Demand with exploding offer

As discussed in section 2.2, $a_1 \in (p, a)$ and so the use of an exploding offer makes a consumer more likely to buy immediately, but it eliminates all the returning demand. One can calculate that when u is uniformly distributed on $[0, 1]$, firm i's demand in the two figures is identical, and when a firm forces immediate sale this has no net impact on its demand. More generally, the impact of using an exploding offer is to eliminate the firm's demand from "low u_i" consumers, who have match utility close to price p and might otherwise come back, and to boost its demand from "high u_i" consumers, who do not wish to risk losing the existing desirable option by going on to sample the rival. If u has an increasing density, the latter effect dominates the former, and the net impact of forcing immediate sale is to boost a firm's demand. Similarly, if the density decreases, then the former effect dominates and demand is reduced when an exploding offer is used.

The next result proves that this insight is valid with an arbitrary finite number of firms.[17]

Proposition 1 *Suppose the number of firms is $1 < n < \infty$.*
(i) If the density f is strictly increasing then the only symmetric equilibrium involves firms using exploding offers;

[17] Note that if there were unlimited firms in the market ($n = \infty$), banning return or artificially raising the cost of return has no impact on a firm's profit. This is because, as is well known, with unlimited options, consumers would not choose to return to a previously sampled option even if it was free for them to do so. As such, both equilibria with exploding offers and with free recall can exist for any match utility distribution.

(ii) If the density f is strictly decreasing then the only symmetric equilibrium involves firms allowing free recall;
(iii) If u is uniformly distributed then an equilibrium with exploding offers and an equilibrium with free recall both exist.

(All omitted proofs can be found in the appendix.)

Thus, we see there are plausible cases when exploding offers are used in equilibrium, as well as other plausible cases (such as the exponential distribution considered above) when a firm prefers to let consumers return freely after sampling rival products. In the uniform example at least (see Figure 1a), the use of exploding offers leads to higher prices being chosen in equilibrium. In these situations, firms may choose to use exploding offers and yet consumers are harmed by the practice.[18]

Nevertheless, our analysis covers only situations with monotonic densities. The reason why results are so clear-cut with monotonic densities is that the impact of exploding offers on a firm's demand is unambiguous, regardless of the prevailing price. With a non-monotonic density function, whether exploding offers are an equilibrium sales technique may depend on price. In particular, it may depend both on the number of firms in the market and the size of the search cost. A second factor which could come into play with non-monotonic densities is that firms may choose intermediate return policies, which make return costly for their first-time visitors but not prohibitively so.[19] (With a monotonic density, a firm wishes either to make return impossible or free, even if it could impose intermediate returning costs.) As can be seen from the proof of Proposition 2 below, when we start from the free-recall equilibrium with price p_0, introducing a small return cost boosts a firm's demand if

$$\int_{p_0}^{a} F(u)^{n-1} f'(u)du > 0 \ . \tag{11}$$

Whether this condition holds for non-monotonic densities depends both on the number of firms and the search cost. Consider for example a Weibull distribution with $F(u) = 1-e^{-u^3}$ defined on $[0,\infty)$, which has a hump-shaped density with mode around 0.87. If the search cost is high enough that a is smaller than the mode, then (11) always holds. With a low search cost such that $a = 2$, then condition (11) always fails and free recall is the equilibrium outcome. However, if the search cost is moderate so that $a = 1$, then condition (11) holds for $n = 2, 3$ but fails for $n \geq 4$. In this case, an oligopoly with few firms has an incentive to make return costly, while a more competitive market will allow free recall.

[18] The use of exploding offers could be prohibited by mandating a "cooling off period", so that consumers have the right to return a product in some specified time after agreeing to purchase. (They could then return a product if they subsequently find a preferred option.) Many jurisdictions impose cooling off periods for some products, especially those sold in the home.

[19] For example, online sellers can ask customers to log on to their accounts or input information again; firms can ask consumers to queue again or make another appointment if they want to come back.

Finally, this analysis relies on a firm's ability to *commit* to an exploding offer. If a consumer does come back to a firm after sampling a rival, the firm will have an incentive to sell to that consumer. This credibility problem is enhanced by the fact that consumers often will wish to return to previous firms, since their stopping rule is such that their remaining option may have lower utility than previously rejected options. This commitment problem could sometimes be solved in a dynamic environment, where sellers gain a reputation for sticking to exploding offers. (In labour market settings, for instance, some employers may be known to keep their word.) Alternatively, in our next model in which firms set higher prices to returning visitors rather than banning their return, we show in section 3.3 that if firms cannot commit to their "buy-later" price then exploding offers are the only credible equilibrium whenever consumers face a positive intrinsic cost of returning to a firm. This argument is akin to the "Diamond paradox", and holds for arbitrary distributions of the match utility (including those with decreasing densities).

3 Buy-Now Discounts

An alternative framework allows a firm to charge a higher price to returning visitors instead of the drastic measure of banning return. Consider the same model as before, except that instead of choosing the extreme policies of either allowing free return or no return, each firm can choose two distinct prices: $\hat{p}$ is the price for returning customers and p is the price for first-time visitors. Whenever $\hat{p} > p$, returning to a previous firm is costly.[20] Indeed, when $\hat{p}$ is sufficiently high, the firm in effect uses exploding offers. One interpretation of this discriminatory pricing is that each firm sets a regular (or "buy-later") price $\hat{p}$ and offers the first-time visitors a "buy-now" discount $\tau \equiv \hat{p} - p$. We assume for now that a firm can commit to $\hat{p}$ when it offers new visitors the buy-now price p. (We discuss the impact of more limited commitment later in section 3.3.)

3.1 Incentives to offer a buy-now discount

In this section we analyze when a firm unilaterally has an incentive to offer a buy-now discount τ, starting from the situation in which all firms offer the equilibrium uniform price p_0 in expression (5). As a preliminary result, we observe that the impact of offering a small buy-now discount on a firm's profit is just as if the firm levies a small buy-later premium:

[20] If $\hat{p} < p$, then a consumer has an incentive to leave a firm and then return, even if she has no intention of investigating other firms. If this kind of consumer arbitrage behavior—of stepping out the door and then back in again—cannot be prevented, then setting $\hat{p} < p$ is equivalent to setting a uniform price $\hat{p}$, and so without loss of generality we assume firms are constrained to set $\hat{p} \geq p$.

Lemma 1 *Starting from the situation in which all firms offer the equilibrium uniform price p_0 in (5), the impact on a firm's profit of offering a small buy-now discount τ (so its buy-now price is $p_0 - \tau$ and its buy-later price is p_0) is equal to the impact of levying a buy-later premium τ (so its buy-now price is p_0 and its buy-later price is $p_0 + \tau$).*

Proof. Suppose all but one firm choose the uniform price p_0 in (5). If the remaining firm offers the buy-now price p and buy-later price $p+\tau$, denote this firm's profit by $\pi(p, \tau)$. If $p \approx p_0$ and $\tau \approx 0$ we have the first-order approximation

$$\begin{aligned} \pi(p,\tau) &\approx \pi(p_0,0) + (p-p_0)\pi_p(p_0,0) + \tau\pi_\tau(p_0,0) \\ &= \pi(p_0,0) + \tau\pi_\tau(p_0,0) \ , \end{aligned} \tag{12}$$

where the equality follows from the assumption that p_0 is the equilibrium uniform price and subscripts denote partial derivatives. It follows that the impact on the firm's profit is captured by the term $\tau\pi_\tau(p_0, 0)$, which implies the result. ■

Intuitively, the fact that p_0 is the equilibrium uniform price implies that a firm's profit is not affected by small changes in its uniform price, and the only impact on a firm's profit comes from its buy-now discount τ (regardless of whether this is interpreted as a discount for immediate purchase relative to the buy-later price p_0, or as a premium for later purchase relative to the buy-now price p_0).

To illustrate the pros and cons of offering a discount most transparently, consider the case of duopoly. It is somewhat more straightforward to consider the incentive to set a buy-later premium, and then to invoke Lemma 1. If firm i introduces a buy-later premium, this has no impact on its demand and profit from those consumers who first sample the rival given they hold equilibrium beliefs, and so we can restrict attention to that portion of consumers who sample firm i first. A buy-later premium not only discourages consumers from searching on, as the exploding offer did in the earlier analysis, but also generates extra revenue from returning consumers.

How exactly does τ affect a consumer's decision whether to buy immediately from firm i? Denote by $a(\tau)$ the reservation utility which leads the consumer to buy immediately, i.e., if she finds match utility $u_i \geq a(\tau)$ at the firm she will buy without investigating the rival. Clearly if no premium is levied ($\tau = 0$) then $a(0) = a$, the free-recall reservation level in (3). By definition, if a consumer discovers utility $u_i = a(\tau)$ at firm i she is indifferent between buying immediately (thus obtaining surplus $a(\tau) - p_0$) and going on to investigate firm j, which yields expected utility

$$\underbrace{\int_{a(\tau)-\tau}^{u_{\max}} (u_j - p_0) dF(u_j)}_{\text{utility when she buys from } j} + \underbrace{F(a(\tau)-\tau)[a(\tau)-p_0-\tau]}_{\text{utility when she returns to buy from } i} - s \ . \tag{13}$$

To understand expression (13), note that if the consumer finds utility u_j at the rival, she will buy from that firm if $u_j - p_0 \geq a(\tau) - p_0 - \tau$, and otherwise she will return to buy from firm i (but at the higher price $p_0 + \tau$). Equating $a(\tau) - p_0$ with expression (13) yields the following formula for $a(\tau)$ given τ:

$$V(a(\tau) - \tau) = \tau \ . \tag{14}$$

(Remember $V(\cdot)$ is defined in (1), and given τ this equation has a unique solution $a(\tau)$.) The resulting pattern of demand for those consumers who first sample firm i is illustrated in Figure 3.[21] Note that $a(\tau)$ decreases with τ, and by differentiating (14) we obtain

$$a'(\tau) = \frac{-F(a(\tau) - \tau)}{1 - F(a(\tau) - \tau)} \ . \tag{15}$$

This is intuitive, as raising the cost of returning makes a consumer more likely to buy immediately (just as in the extreme case of exploding offers).

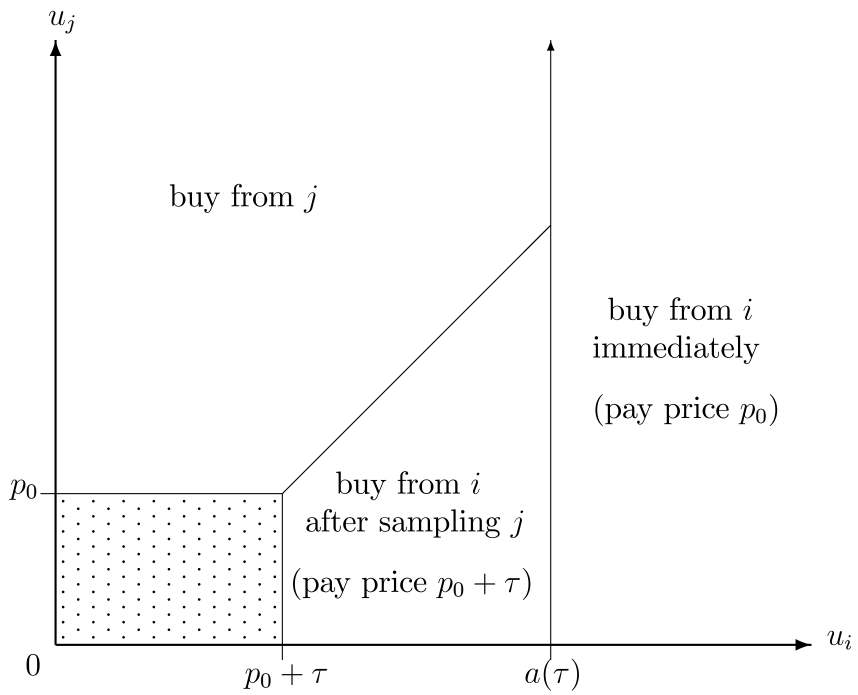

Figure 3: Pattern of demand when firm i levies buy-later premium τ

Using Figure 3, from those consumers who sample firm i first the fraction who buy from the firm is

$$1 - F(a(\tau)) + \int_{p_0+\tau}^{a(\tau)} F(u - \tau) f(u) du \ .$$

[21] This analysis and Figure 3 presume that some consumers do return to firm i after sampling firm j, which requires that the premium τ is not too large. By examining the figure, one sees that the exact condition is $a(\tau) > p_0 + \tau$. From (14), and noting that $V(\cdot)$ is a decreasing function, this is equivalent to $\tau < V(p_0)$. This is possible for sufficiently small τ as long as $V(p_0) > 0$, which is true given (2). When the discount exceeds $V(p_0)$, the returning cost is so great that the consumer never returns to a firm once she leaves it (i.e., the firm in effect uses an exploding offer).

By using (15), the derivative of firm i's demand with respect to τ is equal to

$$\int_{p_0+\tau}^{a(\tau)} F(u-\tau)f'(u)du \ . \tag{16}$$

In particular, the firm's demand is boosted with a buy-later premium whenever the density is increasing, as we saw earlier when we discussed exploding offers in section 2.3.

Firm i makes revenue p_0 from each of its customers, and an additional τ from each of its returning customers. It follows that the derivative of firm i's profits with respect to τ evaluated at $\tau = 0$ is

$$\int_{p_0}^{a} F(u)\left[f(u) + p_0 f'(u)\right] du \ . \tag{17}$$

Here, $\int_{p_0}^{a} F f du$ is the extra revenue generated from the returning customers while $\int_{p_0}^{a} F f' du$ is the extra (maybe negative) demand generated by increasing the cost of return.

From (17) and Lemma 1, the firm has an incentive to introduce a buy-now discount if the density f is increasing. But it has an incentive to introduce a discount much more generally, and the incentive is present whenever p_0 in (5) is strictly above $\frac{1-F(a)}{f(a)}$. To see this, use (5) to obtain

$$\begin{aligned} p_0 \int_{p_0}^{a} F(u)f'(u)du &= \frac{1}{2}\left[\frac{p_0 f(a)}{1-F(a)}(1-F(a)^2) - (1-F(p_0)^2)\right] \\ &> -\frac{1}{2}\left[F(a)^2 - F(p_0)^2\right] \\ &= -\int_{p_0}^{a} F(u)f(u)du \ , \end{aligned}$$

where the inequality follows from the assumption that $p_0 > \frac{1-F(a)}{f(a)}$. Thus, expression (17) is positive and a firm has a unilateral incentive to offer a buy-now discount.

This result holds for arbitrary (but finite) numbers of firms:

Proposition 2 *Starting from the free-recall equilibrium with uniform price p_0 in (5), a firm has a unilateral incentive to offer first-time visitors a buy-now discount if $p_0 > \frac{1-F(a)}{f(a)}$.*

As discussed in section 2, a sufficient condition to ensure $p_0 > \frac{1-F(a)}{f(a)}$ is that the hazard rate for the match utility is strictly increasing and that the number of firms is finite.

Proposition 2 indicates that a seller (a sales assistant in an electronics store, say) typically has an incentive to offer a first-time visitor a discount on the regular price if the consumer buys immediately. The intuition for this result is as follows. As Lemma 1 shows, the impact of a small buy-now discount is the same as a small buy-later premium. A small buy-later premium has two effects: the extra revenue effect—every returning consumer now pays a premium, and the demand effect—the first-time visitors become more likely to buy immediately, but those potential returning consumers become less likely to come back. The second effect is similar to the demand effect caused by exploding offers, and as we have shown whether it is positive or negative depends on the shape of f. However, the first revenue effect must be positive. Proposition 2 shows that this first effect is powerful enough that the overall effect becomes positive under a mild hazard-rate condition.

From the proof of the result one can also see that if $p_0 = \frac{1-F(a)}{f(a)}$, then a firm has no local incentive to introduce a buy-now discount. For instance, in the exponential example discussed in section 2.2, we have $p_0 = \frac{1-F(a)}{f(a)}$. Similarly, if a firm acts as a monopoly (e.g., when the search cost is high so that (2) is binding) or if there are infinitely many firms, then $p_0 = \frac{1-F(a)}{f(a)}$. In all such cases, a firm has no (local) incentive to offer a buy-now discount. We will also see in section 3.3 below that if the firm cannot commit to raising its price to returning visitors, uniform pricing without buy-now discounts may often emerge as an equilibrium outcome.

3.2 Equilibrium discounts in a duopoly example

The previous result indicated that firms have an incentive to offer a buy-now discount, provided a hazard rate condition was satisfied. In this section we derive the *equilibrium* discount and price in a duopoly setting, and compare this outcome to the situation with uniform prices.[22]

For convenience, we analyze the model in terms of the buy-now price p and the buy-now discount $\tau = \hat{p} - p$ (rather than in terms of p and $\hat{p}$). Let the symmetric equilibrium outcome be (p, τ), and suppose firm i deviates and offers an alternative tariff (p_i, τ_i). Similarly to Figure 3 above, firm i's demand from those consumers who sample it first is as depicted on Figure 4a. (Recall that $a(\cdot)$ is defined above in (14).) Firm i's demand from those consumers who first encounter the rival is shown on Figure 4b.

As discussed earlier, these figures presume that $\tau, \tau_i \leq V(p)$. In equilibrium we will indeed have $\tau < V(p)$ so that some consumers do return to a firm after sampling the rival. And it is without loss of generality that we consider deviations restricted to $\tau_i \leq V(p)$.[23] When firm i unilaterally deviates to (p_i, τ_i), with $\tau_i \leq V(p)$, its profit is

$$p_i Q_T + \tau_i Q_R \ , \tag{18}$$

where Q_T is firm i's total demand and Q_R is the portion of demand from its returning customers. (The firm obtains revenue p_i from each of its customers, plus the incremental revenue τ_i from each of its returning customers.)

[22]When there are more than two firms, the consumer stopping rule with buy-now discounts depends on the history of realized match utilities, and this makes the equilibrium analysis very complex. (When exploding offers are used, by contrast, the stopping rule does not depend on previous offers, since the consumer has no ability to return.)

[23]When $\tau_i > V(p)$, returning demand disappears and the firm's profit is independent of τ_i. Hence, our restriction to $\tau_i \leq V(p)$ is without loss of generality.

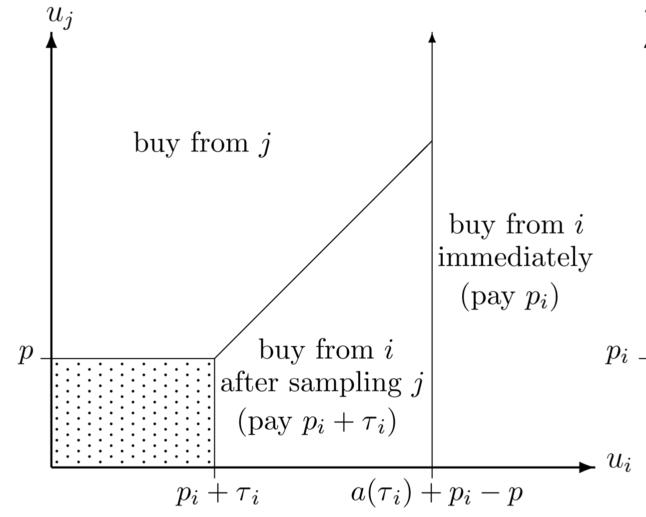

Figure 4a: Firm i's demand when it is sampled first

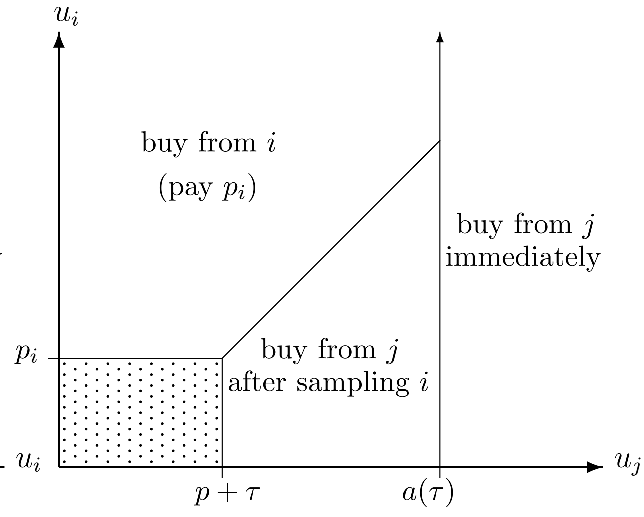

Figure 4b: Firm i's demand when it is sampled second

For simplicity, from now on we focus on the example in which match utility u_i is uniformly distributed on $[0, 1]$, so that expression (14) becomes

$$a(\tau) = 1 + \tau - \sqrt{2(s + \tau)}\ . \tag{19}$$

This greatly simplifies the algebra and enables us to check second-order conditions for the candidate equilibrium. (See Table 2 below for illustrations of the equilibrium when the distribution of match utility is non-uniform.) To ensure an active market we assume $s < \frac{1}{8}$.

The demand functions in (18) can be derived by calculating the areas of the various regions in Figure 4 to yield

$$2Q_T = \underbrace{1 - (a(\tau_i) + p_i - p)}_{\text{buy immediately from } i} + \underbrace{a(\tau)(1 - p_i) - \frac{1}{2}(a(\tau) - \tau - p)^2}_{\text{buy from } i \text{ after first sampling } j} + \underbrace{\frac{1}{2}(a(\tau_i) - \tau_i)^2 - \frac{1}{2}p^2}_{\text{returning demand } 2Q_R}\ , \tag{20}$$

where $a(\tau)$ is given in (19). Note that the firm's returning demand does not depend on its buy-now price p_i over the relevant range in this uniform example. (By examining Figure 4a, we see that varying p_i simply shifts the region of returning demand uniformly to the left or right.) Note also that the firm's total demand Q_T does not depend on its buy-now discount τ_i. (This is a special case of expression (16) above, when $f \equiv 1$.) Thus, firm i's profit in (18) is additively separable in its buy-now price p_i and its buy-now discount τ_i.

In particular, firm i will choose its buy-now discount τ_i to maximize $\tau_i Q_R$, the extra revenue from its returning consumers, which has first-order condition[24]

[24] We can show that $\tau_i Q_R$ is concave in τ_i for $\tau_i \leq 1/8$ and decreasing in τ_i for $\tau_i > 1/8$. So the first-order condition is also sufficient.

$$[a(\tau) - \tau]^2 - p^2 - \frac{2\tau[a(\tau) - \tau]}{1 - [a(\tau) - \tau]} = 0 \ . \tag{21}$$

(Here, we used expression (15).) Note that the left-hand side of (21) is strictly positive when $\tau = 0$ (provided that $s < \frac{1}{8}$ and $p < \frac{1}{2}$, and we will show shortly that $p < \frac{1}{2}$). It follows that the equilibrium discount is positive, as was already indicated by Proposition 2.

Turning to the equilibrium buy-now price p, note that firm i's total demand in (20) is linear in p_i, and so its profit is concave in p_i. Therefore, the first-order condition for p_i to be optimal is sufficient. Each firm's equilibrium total demand is $\frac{1}{2}[1 - p(p + \tau)]$. That is, a consumer will leave the market without buying anything if and only if she neither buys at the second firm nor wants to go back to the first one. Using this fact, the first-order condition for the equilibrium buy-now price p, given τ, is

$$\frac{1}{p} - p = 1 + a(\tau) + \tau \ . \tag{22}$$

The right-hand side of (22) is greater than $\frac{3}{2}$.[25] Since the left-hand side of (22) is decreasing in p, it follows that the solution to this first-order condition satisfies $p < \frac{1}{2}$. Moreover, for $0 \leq \tau \leq \frac{1}{8} - s$, which will turn out to be the relevant range of τ, the right-hand side of (22) is decreasing in τ, and so the buy-now price p in (22) is an increasing function of τ. Intuitively, a buy-now discount increases search frictions in the market, which in turn allows firms to charge a higher price.

The equilibrium strategy (p, τ) is then found by solving the pair of nonlinear equations (21)–(22), which can typically be done only numerically.[26] For instance, when $s = 0$, solving these equations shows that $p \approx 0.45$ and $\tau \approx 0.06$ and hence a buy-later price of $\hat{p} \approx 0.51$ (which is actually slightly above the monopoly price of $\bar{p} = 0.5$). In this example, although the market has no intrinsic search frictions, firms in equilibrium impose "tariff intermediated" search frictions on consumers via the buy-now discount, which here is about 12% of the buy-later price. By contrast, in a market with $s = \frac{1}{8}$, which is the highest intrinsic search cost which induces consumers to participate, one can check that the (exact) solution to this pair of equations is $p = \bar{p} = \frac{1}{2}$ and $\tau = 0$, so that there is no buy-now discount. (When $s = \frac{1}{8}$, search costs are so high that consumers will accept the first offer which yields them a non-negative surplus. In particular, there are no returning consumers even with costless recall.)

More generally, the equilibrium buy-now discount τ decreases with the search cost s. That is, the higher is the intrinsic search cost, the less incentive firms have to deter

[25] Note that $a(\tau) + \tau$ is a convex function which is minimized by setting $\tau = 1/8 - s$, which makes the right-hand side of (22) equal to $7/4 - 2s$. Since $s < 1/8$, the claim follows.

[26] Given $s < \frac{1}{8}$, we can show that the system of (21)–(22) has a solution $(p, \tau) \in (0, \frac{1}{2}) \times (0, \frac{1}{8} - s)$, and $p < a(\tau) - \tau$ or $\tau < V(p)$ (so there do exist returning consumers in equilibrium). See our previous working paper Armstrong and Zhou (2010) for more details.

consumers from searching on. This can be seen from Figure 5a below, which depicts how the buy-later price $\hat{p} = p + \tau$ (the upper solid curve) and the buy-now price p (the middle solid curve) vary with s. As is expected, the buy-now price increases with the search cost. Less expected is the observation that the buy-later price depends non-monotonically on s (and is always above the monopoly price $\bar{p} = \frac{1}{2}$ in this example).

We next compare this outcome with the situation in which firms must offer uniform prices to their fresh and returning customers. The equilibrium uniform price p_0 is given by expression (5), or equivalently it is given by (22) after setting $\tau = 0$. Recall that the price which solves (22) increases with the buy-now discount τ over the relevant range. Since the equilibrium buy-now discount is positive, we deduce the following result:

Proposition 3 *In the uniform-duopoly case with $s < \frac{1}{8}$, the use of buy-now discounts leads to higher prices, i.e., $p_0 < p < \hat{p}$.*

That is, even the discounted buy-now price in the discriminatory case is higher than the uniform price, and the ability to offer discounts for immediate purchase drives up *both* prices.[27] The intuition is that the buy-now discount adds to the intrinsic search frictions in the market, and this allows firms to charge a higher price. (Relative to the uniform-price case, consumers become less willing to search on, and so the firms' demand is less price elastic.) Figure 5a depicts the three prices, where from the bottom up the three curves represent p_0, p and $\hat{p}$, respectively. As we have already mentioned, when $s = \frac{1}{8}$ the search cost is so high that no firms have incentive to offer buy-now discounts, and so all three prices coincide.

Since both prices rise, the buy-now discount equilibrium excludes more consumers from the market. In addition, one can show that the use of buy-now discounts boosts fresh demand (the sum of the first two terms in (20)) and reduces returning demand. This is illustrated for the case $s = 0$ in Table 1 (including for reference the case where exploding offers are used).

	p	$\hat{p}$	fresh	returning	excluded
no discount	0.41	0.41	41%	41%	17%
with discount	0.45	0.51	66%	11%	23%
exploding offer	0.45	n/a	73%	0%	27%

Table 1: The impact on prices and demand of buy-now discounts and exploding offers

However, whether the use of buy-now discounts leads to higher profit depends on the magnitude of the search cost. Figure 5b shows how industry profits with uniform pricing (the dashed curve) and profits with buy-now discounts (the solid curve) vary with the

[27]It is not unusual that the ability to price discriminate in oligopoly leads to falls in all prices, but cases where all prices rise are less familiar.

search cost s. We see that price discrimination leads to higher profit only if the search cost is relatively small. When the search cost is relatively high, price discrimination leads to prices which exclude too many consumers. In these cases, firms are engaged in a prisoner's dilemma: when feasible an individual firm wishes to offer a buy-now discount, but when both do so industry profits fall.[28] Nevertheless, as was seen in the exploding offer analysis in Figure 1b above, when there are more than two firms we anticipate that profits will rise when buy-now discounts are used, since the price-increasing effect will then outweigh the market participation effect. (When there are many firms, most consumers will eventually find a product they buy.)

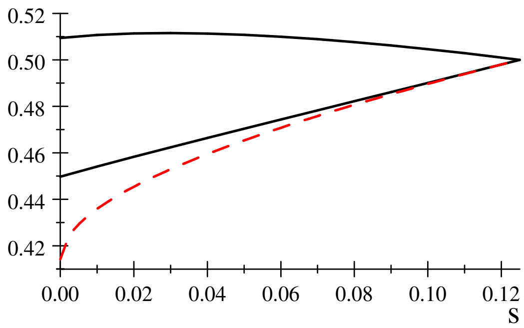

Figure 5a: Prices and search cost

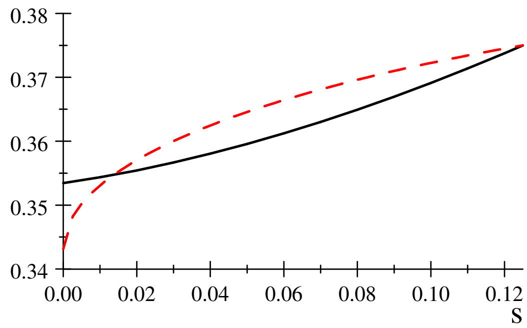

Figure 5b: Profits and search cost

Finally, we observe in this example that aggregate consumer surplus and total welfare (measured by the sum of consumer surplus and profit) fall when firms use buy-now discounts.

Our analysis in this section so far has assumed that the match utility is uniformly distributed. It is possible to derive equilibrium prices in non-uniform examples by calculating the measure, rather than simply the area, of the regions in Figure 4. We report numerical calculations for the equilibrium tariff in examples where the density function f is linear rather than constant. Specifically, suppose that the density takes the form $f(u) = 2\beta u + 1 - \beta$, where $u \in [0, 1]$ and $\beta \in [-1, 1]$, so that the density function is a straight line with slope 2β passing through the point $(\frac{1}{2}, 1)$. All such distributions have an increasing hazard rate, and so Proposition 2 indicates that firms will set a positive buy-now discount. Table 2 reports the equilibrium prices for various values of β, assuming that the search cost is zero.

[28] This bears some similarities to situations with competitive bundling. There, a firm often has a unilateral incentive to offer consumers a discount for buying two products rather than one, and when all firms do this industry profits fall. However, in contrast to the current case where the discount relaxes competition and drives prices up, with bundling the discount intensifies competition and drives prices down. For instance, see Armstrong and Vickers (2010) for more details.

Here, the first row reports the equilibrium uniform price, the second and third rows report the buy-now price and buy-now discount, while the final row reports the buy-now discount as a proportion of the buy-later price. The central column with $\beta = 0$ is the uniform case already discussed. As expected, when the density is increasing, the incentive to set a buy-now discount is reinforced by an additional strategic effect: as seen in (16), with an increasing density a firm's total demand is boosted if it makes it costly to return. Thus, we see that the size of the buy-now discount increases with β, both in absolute terms and as a proportion of the buy-later price. Notice also that all prices are higher with price discrimination than without discrimination, even when the density is decreasing. In particular, consumers are worse off with this form of price discrimination.

	$\beta = -1$	$\beta = -0.5$	$\beta = 0$	$\beta = 0.5$	$\beta = 1$
p_0	0.312	0.382	0.414	0.405	0.360
p	0.313	0.392	0.450	0.474	0.470
τ	0.017	0.032	0.060	0.091	0.124
$\tau/(p+\tau)$	0.05	0.075	0.12	0.16	0.21

Table 2: Equilibrium prices with a linear density function ($s = 0$)

3.3 Buy-now discounts without commitment

We discuss next whether buy-now discounts can emerge as an equilibrium outcome if we relax the assumption that a firm can commit to its buy-later price when consumers first visit. In this case, a consumer can discover a firm's actual buy-later price only after she returns to the firm.

First, consider a situation with partial commitment, by which we mean that firms can commit to a returning purchase price *cap* (but cannot commit to a specific price). This case could apply in situations where firms can post a "regular" price. For example, the price printed on the price label in a store usually has this kind of commitment power. Here, there is an equilibrium with the same outcome as in the full commitment case. Specifically, in this equilibrium, firms charge a buy-now price p, commit to a buy-later price cap $\hat{p}$, and actually charge returning consumers $\hat{p}$, where both p and $\hat{p}$ take the same values as the equilibrium prices in the commitment case. To sustain this equilibrium, we assume that all consumers believe that for any (maybe off-equilibrium) committed price cap $\hat{p}_i$ the firm's actual buy-later price will be $\hat{p}_i$. To see that this is an equilibrium, observe that when consumers hold the above beliefs, they will return to a previously visited firm, say firm i, only if $u_i \geq \hat{p}_i$, where $\hat{p}_i$ is the anticipated firm i's buy-later price. This implies that firm i can have no incentive to charge them a price below $\hat{p}_i$.[29] (It may have an incentive to raise the price above $\hat{p}_i$, but that is not permitted given that the firm commits to this

[29] Note that charging a returning price below $\hat{p}_i$ is a private deviation, so it will not increase the number of consumers who return to this firm. Hence, such a deviation does not bring the firm any benefit.

cap.) This in turn fulfills consumer beliefs. Thus, a buy-later price cap can be used as a full commitment device.[30]

In other situations, firms may not be able to make any commitments about their buy-later price when consumers first visit. Here, and unlike the rest of this paper, it makes an important difference whether or not consumers face an *intrinsic* returning cost when they come back to a previously-visited firm. We discuss the two cases in turn.

No intrinsic return cost: If consumers face no such cost (as we assumed for simplicity in the rest of the paper), there are usually multiple equilibria. For example, it is a trivial equilibrium that firms charge a sufficiently high returning price such that consumers never return (i.e., all firms actually use exploding offers). Given there are no returning consumers, firms have no incentive to decrease the returning price. It is also possible (at least in the uniform-duopoly example of the previous section) to construct equilibria of the constant markup form which are qualitatively similar to the commitment prices: consumers anticipate that a firm will set a return price $\hat{p}_i = p_i + \tau$ if that firm offers the buy-now price p_i, and given these expectations firms have no incentive to set a different buy-later price. (There is a continuum of such credible τ.)

However, there is often an equilibrium in which uniform pricing is a credible strategy, so that no buy-now discount is offered. That is to say, (i) consumers do not anticipate that they will face a higher price if they return to buy from a previously sampled firm and plan their search strategy accordingly, and (ii) when a consumer does return to a firm, that firm has no *ex post* incentive to "surprise" the consumer with an unexpected price hike.

First of all, this is easy to understand in the extreme case with $s = 0$. The reason is that when search costs are zero, consumers sample all firms before they purchase (given their belief that there is no returning purchase surcharge), and so all buyers are returning customers. Thus, we are just in the situation of Wolinsky model with zero search costs, and the incentive to set the price to returning consumers is exactly the same as the incentive to set the uniform price p_0 in (5).

Consider next cases with $s > 0$. For simplicity, focus on the case of duopoly. Suppose firm i sets a slightly different buy-now price p and surprises the returning consumers with a small premium $\tau \geq 0$.[31] Consumers still hold the equilibrium beliefs that if they come back to firm i, they will only pay p instead of $p + \tau$. Let $\Pi(p, \tau)$ be firm i's deviation profit

[30] There may exist other equilibria involving different consumer beliefs.

[31] We consider in this discussion only local deviations. Given that firm i has no profitable local deviation, it also has no profitable global deviation if its profit function is quasiconcave in p and τ, for instance. Although in our search model it is hard to derive more primitive conditions, we can show that it is true at least for a uniform distribution for match utility.

Note that setting $\tau < 0$ will only reduce each returning consumer's payment but not increase the returning demand since consumers observe this deviation only after they come back to the firm and all of them value the product at $u_i \geq p$. Thus, the firm will never choose to surprise a returning visitor with a price reduction.

when it offers this alternative tariff. When p is close to p_0 and τ is close to zero, as with expression (12) above we have

$$\Pi(p,\tau) \approx \Pi(p_0,0) + \tau \Pi_\tau(p_0,0) \ .$$

Thus, this deviation is unprofitable if $\Pi_\tau(p_0,0) < 0$. This implies that we need only consider the deviation with an unchanged buy-now price p_0 and a small buy-later premium τ.

Notice the (unexpected) buy-later premium τ will only affect firm i's returning demand, which is depicted on Figure 6. The sole difference between Figures 3 and 6 is that the threshold for buying immediately at firm i is lower in Figure 3 than in Figure 6—it is $a(\tau)$ instead of a—due to the fact that the consumer anticipated the buy-later premium in Figure 3 but not in Figure 6.

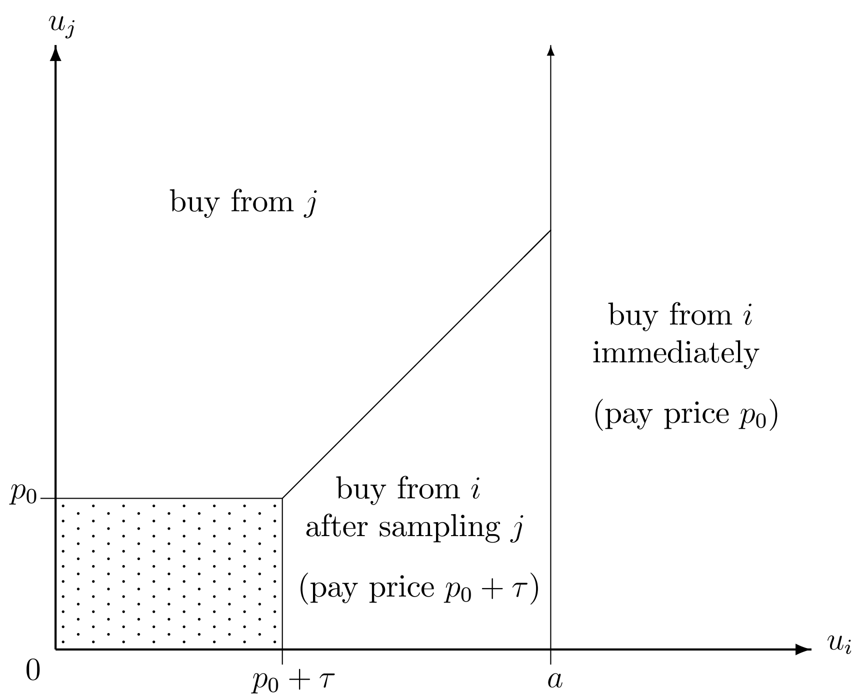

Figure 6: Pattern of demand when firm i surprises return consumers with price increase τ

From the figure it follows that firm i's profit from the returning customers when it increases the buy-later price by τ is

$$(p_0 + \tau) \int_{p_0+\tau}^{a} F(u-\tau) f(u) du \ .$$

Hence, we have

$$\begin{aligned} \Pi_\tau(p_0,0) &= \int_{p_0}^{a} F(u) f(u) du - p_0 \left\{ F(p_0) f(p_0) + \int_{p_0}^{a} f(u)^2 du \right\} \\ &= \int_{p_0}^{a} F(u) f(u) du - p_0 \left\{ F(a) f(a) - \int_{p_0}^{a} F(u) f'(u) du \right\} \ . \end{aligned}$$

(The second step follows after integrating by parts.) The first term is just firm i's returning demand in equilibrium, so it reflects the marginal benefit from each returning consumer paying the premium τ. The second term is the loss due to the reduction of returning demand caused by the unexpected premium τ (some returning consumers leave the market and some go back to buy from other firms). Compared to (17), which describes the incentive to raise returning price in the case with commitment, the current incentive is reduced by $p_0 F(a)f(a)$. This is because in the commitment case, committing to a higher return price can induce more consumers to buy immediately and so has an extra (strategic) demand benefit, which is absent in the no-commitment case.

Using the expression for the equilibrium uniform price (5), we obtain

$$\Pi_\tau(p_0, 0) = \frac{1}{2}(1 - F(a))f(a)\left\{p_0 - \frac{1 + F(a)}{f(a)}\right\} . \tag{23}$$

When the search cost is high and close to the limit in (2) (so $a \approx \bar{p} \approx p_0$), then $p_0 \approx \frac{1-F(a)}{f(a)}$ and so (23) is negative. More generally, expression (23) reveals that the firm has no incentive to deviate from uniform pricing if

$$p_0 < \frac{1 + F(a)}{f(a)} . \tag{24}$$

A sufficient condition for this is that

$$\frac{u}{1 + F(u)} \text{ increases with } u . \tag{25}$$

(Condition (25) implies that $1 + F(u) \geq uf(u)$, and so $p_0 < a \leq \frac{1+F(a)}{f(a)}$.) For example, (25) holds for any distribution with a weakly decreasing density (given the lower bound of the match utility is zero as assumed in our model).[32] In particular, it holds with a uniform distribution for match utility.

It appears to be relatively hard to find distributions for the match utility such that (24) is violated.[33] This provides one possible explanation for why in many markets uniform prices offered to first-time and returning visitors is the norm.[34] The precise reason why a firm typically, but not always, has no incentive to raise price to returning consumers is not transparent. The fact that a consumer has rejected the rival's product suggests that a firm should raise its price, since it has some monopoly power over this consumer. (This is the reason why less prominent firms set higher prices in Armstrong, Vickers, and Zhou (2009).)

[32] One can show that a weakly decreasing density implies that uniform price is a credible equilibrium for an arbitrary number of firms, not just for duopoly.

[33] One example which violates (24) is $F(u) = \frac{9}{10}u + \frac{1}{10}u^{100}$ with support $0 \leq u \leq 1$ when $s \approx 0$. When $s \approx 0$, $a \approx 1$ and condition (24) requires that $p_0 < \frac{2}{10.9}$. However, (5) implies that $p_0 \approx 0.31$. Note that this distribution has an increasing hazard rate, so this example demonstrates that the standard increasing hazard rate condition cannot guarantee (24).

[34] Another possible reason why in many cases firms do not surcharge their returning customers is consumers' antagonism to an unexpected price rise.

But set against this is the fact that the consumer has also rejected the firm's product on a first visit, which tends to make the firm want to set a low price to the returning consumer. The net impact of these two forces renders the informational motive to set high prices to returning consumers weak or non-existent.

Positive intrinsic return cost: The preceding discussion depends on the assumption that consumers had no cost of returning to a previously visited firm. If there is even a small intrinsic returning cost, say $r > 0$, the firm has an incentive to "surprise" returning consumers with a price rise of at least r. (Returning customers all have match utility $u_i > p_0+r$, otherwise they would not return.) It turns out that the only credible equilibrium when $r > 0$ involves exploding offers. To see this, suppose in some equilibrium that each consumer forecasts that a firm's buy-later price is $\hat{p}(p_i)$ when its buy-now price is p_i, where $\hat{p}(\cdot)$ can take any form. Suppose that the buy-now price in this equilibrium is p^*, say, and suppose—contrary to the claim—there is some returning demand in this equilibrium. But if a consumer returns to firm i after sampling other firms, her match utility must satisfy $u_i \geq \hat{p}(p^*) + r$, since the consumer needs to pay the returning cost $r > 0$. Since all its returning customers have match utility at least as great as $\hat{p}(p^*) + r$, the firm's optimal price for these customers must be at least $\hat{p}(p^*) + r$,[35] which contradicts the assumption that $\hat{p}(p^*)$ was the correctly anticipated buy-later price.

Thus, when there is an intrinsic returning cost, no matter how small, rational consumers anticipate that buy-later prices will be so high that it is never worthwhile to return to a previous firm after leaving it. In effect, firms are forced to make exploding offers, and consumers have just one chance to buy from any firm. (The equilibrium price is then as described in section 2.2.) This result is analogous to Diamond's (1971) paradox, showing how a small search cost can cause a market to shut down. Diamond's result relies on consumers knowing their match utility in advance, and a central advantage of Wolinsky's formulation with *ex ante* unknown match utilities is that this paradox can be avoided. But even in our Wolinsky-type framework, the *returning* consumers know their match utility, and so the returning market fails for the same reason as the primary market failed in Diamond's framework.

4 Extensions

This paper has explored the incentives firms have to make it costly for consumers to return after investigating rival sellers. The use of exploding offers can be individually profitable for firms under certain conditions, such as when the density for match utility is increasing. A less extreme policy is to offer first-time visitors a buy-now discount, and firms have an

[35] Note that surprising returning consumers by charging them $\hat{p}(p^*) + r$ will not induce any of them to leave this firm again and buy from others, since going back to any other firm also involves a return cost r.

incentive to offer such discounts under relatively mild conditions. Either selling technique tends to raise market prices and lower both consumer surplus and total welfare. If firms cannot commit to their buy-later price the outcome depends on whether there is an intrinsic cost of returning to a firm: if the intrinsic return cost is zero, it is often an equilibrium for firms not to offer a buy-now discount; if the return cost is positive, firms are forced to make exploding offers.

Several extensions to this analysis are worthwhile, including the following:

The impact of prominence: In some markets consumers are known to search in a non-random order. For example, when one seller is more prominent than others, more consumers may sample it first. (Recall that De los Santos (2008) showed how one seller in the online book market attracted a greatly disproportionate share of initial searches.)

In the model of exploding offers, prominence does not affect a firm's incentive to adopt exploding offers if the utility density is monotonic. It can be understood by looking at Figure 2 for the duopoly case. The decision about whether or not to use an exploding offer only affects a firm's demand from consumers who sample it first, and this demand effect is positive (negative) if the density is increasing (decreasing), independent of the proportion of such consumers. However, prominence does affect the equilibrium price when exploding offers prevail. Intuitively, firms placed earlier positions in the consumer search order should have incentive to charge lower prices than their rivals. This is because consumers who face exploding offers are more choosy at the start of their search process, and so the demand faced by prominent firms is more price elastic.[36] This result is akin to the prominence model with free recall analyzed in Armstrong, Vickers, and Zhou (2009).

In the model of buy-now discounts, the situation is more complicated, because firms also care about the extra revenue generated by the high-price returning customers. In the duopoly example analyzed in section 3.2, one can show numerically that when $s = 0$ the prominent firm's buy-now discount decreases from 0.06 to 0.053 as its share of first-time visitors increases from 50% to 100%, while the less prominent firm's buy-now discount increases from 0.06 to 0.063. That is, the less prominent firm will actually offer a deeper buy-now discount than its rival.

More ornate schemes: In the buy-now discount model, sellers may be able to extract more surplus from buyers by offering them an additional option—namely, buyers can pay a deposit d for the option to return and buy at a specified price q.[37] With this new option, more consumers may opt to search on, and among the consumers who do search

[36] This implies that if consumers can choose their search orders freely, the no-recall model with *ex ante* symmetric firms also has *asymmetric* equilibria (in addition to the symmetric equilibrium discussed in section 2.2) in which consumers sample a certain firm first and this firm then provides better deals.

[37] For example, some business schools demand a deposit from applicants who want to keep the admission offer for a longer time. It is also sometimes used in business-to-business transactions.

on, those having relatively high valuations of the first product will buy the deposit contract while others having relatively low valuations will not since they rarely come back. In the uniform-duopoly example, one can show that: (i) starting from the buy-now discount equilibrium in section 3.2, each firm has an incentive to introduce a deposit contract; (ii) the purchase price in the deposit contract is even lower than the buy-now price (i.e., $q < p$) but the consumers who buy the deposit contract and eventually come back pay more than fresh consumers (i.e., $d + q > p$); (iii) with the new instrument firms earn lower profit in equilibrium. This extension could be extended further, so that firms offer a *menu* of deposit contracts (a bigger deposit would grant the right to come back and buy at a lower price). Nevertheless, the informal and perhaps furtive nature of buy-now discounts will often make the use of these ornate contracts implausible in practice.

Other forms of search-based discrimination: In this paper we assumed that firms can distinguish first-time from returning visitors. In some situations, firms may be able to distinguish more finely between consumers, and can further identify whether a first-time visitor has previously sampled other sellers or not. (For instance, an online firm may be able to track whether a consumer has previously paid it a visit *and* whether she has already visited some rivals.) In the duopoly case, one can show that each firm will charge the same price to returning consumers and its first-time visitors who have sampled the rival firm first, and charge a lower price to the first-time visitors who sample it first. Thus, in equilibrium a firm discriminates equally against all consumers who have investigated the rival seller, regardless of whether or not the consumer first sampled the rival.[38]

Consumers' incentives to conceal/reveal their search history: Notice that from an *ex ante* perspective, in both of our models a consumer will be better off if she can conceal her identity as a returning consumer.[39] Thus, if it is costless to pretend to be a new visitor (e.g., by deleting cookies on your computer), all consumers will do this, and the market will operate as a standard search market with uniform prices as in Wolinsky (1986). But if there are some costs involved in concealing search history, or if some consumers do not think to do so, there will remain an incentive to condition prices on observed search history.

Consumers may also have incentive to (selectively) reveal their search history. For instance, they may want to force the current seller to offer a better deal by providing hard information of a previous price offer (if this is possible).[40] Investigating how such a

[38] See our previous working paper Armstrong and Zhou (2010) for more details.

[39] This is somewhat related to models of sequential bargaining. If one buyer is negotiating with a sequence of sellers, then the buyer may gain from keeping the order (and the outcome) of negotiations secret from sellers. Noe and Wang (2004) present such a model, and find that when the objects sold are complements for the buyer, then the buyer obtains greater surplus if he randomizes and conceals the order in which he approaches the sellers.

[40] In Daughety and Reinganum (1992), if a consumer makes contact with two sellers, she can force the

possibility could affect price competition and market performance is an interesting topic for future investigation.

High-pressure selling to conceal information about match utility: A focus of this paper has been on a seller's strategic incentive to prevent a consumer from acquiring information about rival offerings. By making it hard to return to a firm, a consumer is reluctant to go on to investigate other deals. An alternative form of high-pressure selling is to force a potential customer to buy quickly, before she has had a chance to evaluate the *current* product adequately. (This seems to be a reason for the sales techniques used by time-share companies, for instance.) If a seller forces consumers to decide quickly (or offers a discount if they buy quickly), a consumer might have to decide whether or not to purchase before she has worked out how much she actually wants the product. Without accurate information about the realized match utility, suppose that a consumer bases her purchase decision on the expected match utility, which is $\bar{u}$, say.

This setting can be analyzed within a monopoly framework (unlike our main model). Suppose the monopolist has marginal cost c for supplying the product. If the seller gives the consumer time to calculate her (privately observed) match utility u, the seller's profit with price p is $(p-c)(1-F(p))$, and the optimal price maximizes this expression. If instead the seller forces the consumer to buy immediately or never knowing only her expected utility, the seller can charge $p=\bar{u}$ and obtain profit $\bar{u}-c$. Since $\bar{u} > p(1-F(p))$ for all p, it follows that the latter strategy is more profitable whenever c is sufficiently close to zero. By contrast, if c is sufficiently large (above $\bar{u}$, for instance), then the monopolist prefers to give consumers enough time to understand the realized match utility.[41]

One can also consider a search version of this problem. Consider the Wolinsky model, but suppose a consumer's initial search is costless so that all consumers are willing to participate in the market. When marginal production cost c is small enough, it turns out to be an equilibrium for all firms to force sales before the consumer discovers her utility and to fully extract expected utility with the monopoly price $p=\bar{u}$. (Suppose all other firms do so. Then when a consumer arrives at a seller, she will never search further. So the seller acts as a monopolist and, as we have seen, its most profitable strategy is then to force a quick sale to conceal match-specific information.[42]) Even with very small search costs, then, all firms engage in this form of high-pressure selling, with undesirable results: consumers are left with no surplus; even low-u consumers buy, despite the costs

sellers to compete in Bertrand fashion and force her price down to marginal cost.

[41] For further details of the monopolist's incentives to reveal or conceal match-specific information, see Lewis and Sappington (1994). They show that the monopolist typically will choose to reveal all information or none. Anderson and Renault (2009) discuss when a firm wishes to disclose match-specific information to consumers about a *rival*'s product.

[42] It is therefore clear that if c is large enough (above $\bar{u}$, say), this hihg-pressure selling equilibrium cannot be sustained.

of serving them, and consumers are randomly matched with sellers rather than buying the most suitable product. Thus, even in a search market with differentiated products as in Wolinsky (1986), if firms have the ability to conceal match-specific information by means of high-pressure sales techniques they will often choose to do so, and the Diamond Paradox emerges once again.

APPENDIX

Proof of Proposition 1: Part (i): Our proof consists of two steps. First, we show that if the match utility density f is strictly increasing, then all firms using exploding offers is an equilibrium. Second, we exclude the possibility that all firms allowing free recall is also an equilibrium.

The hypothesis is that all firms choose to use exploding offers and to set the price p in (10). Suppose a deviating firm chooses price $\tilde{p}$ and allows free recall, while other firms follow the proposed equilibrium strategy. Suppose that the deviating firm is in the k_{th} position of a consumer's search process and $k < n$. (If $k = n$ then allowing free recall or not does not affect the firm's demand.) Then the probability that this consumer will visit the firm is still h_k in (8), since consumers hold equilibrium beliefs. However, her incentive to search beyond the firm is now altered. Since she can return to this firm whenever she wants, she becomes more willing to continue searching. If at the deviating firm she finds utility u such that $u - \tilde{p} \leq 0$, she will never buy from the firm (either immediately or later). So consider the situation where $u - \tilde{p} > 0$. Then if she leaves the deviating firm, she will enter a no-recall search market with $n - k$ products each being sold at price p, but now with an outside option $u - \tilde{p}$. To calculate the consumer's stopping rule in this situation, we need to calculate her expected surplus from entering such a search market.

Denote by $W_m(z)$ the expected surplus from a no-recall search market with m unsampled products with price p and outside option $z \geq 0$. It is difficult to derive an explicit expression for $W_m(z)$, and instead we use an indirect method.[43] Let $r_m(z)$ be the probability that the consumer will eventually consume the outside option. By standard envelope reasoning we have the following result.[44]

Claim 1 *$W_m(z)$ is convex and $W_m'(z) = r_m(z)$ almost everywhere.*

[43]By contrast, it is straightforward to derive an explicit expression for consumer surplus in the case of free recall—see expression (30) below.

[44]A sketch of a proof goes as follows. Let Θ_m be the set of all possible stopping rules in the no-recall search market with m products and outside option z. If the consumer uses $\theta \in \Theta_m$, her expected surplus is $zR(\theta) + U(\theta)$, where $R(\theta)$ is the probability that the consumer will opt for z given the stopping rule θ, and $U(\theta)$ is the surplus from buying other products (including the expected search costs). Thus, $W_m(z) = \max_{\theta \in \Theta_m}[zR(\theta) + U(\theta)]$ and $r_m(z) = R(\theta(z))$, where $\theta(z)$ is the optimal stopping rule given z. $W_m(z)$ is convex since the objective function is linear in z, and its derivative is $r_m(z)$ almost everywhere.

Notice that $W_m(0)$, the expected surplus from a no-recall search market with a zero outside option, is just $a_m - p$. Thus, we have

$$W_m(z) = a_m - p + \int_0^z r_m(x)dx \ .$$

Since $0 < r_m(z) \le 1$, $W_m(z)$ is an increasing function with slope no greater than one. In addition, since the consumer can always consume the outside option without searching at all, we have $W_m(z) \ge z$. In particular, for a sufficiently large z (e.g., $z \ge u_{\max} - p$), the consumer will consume the outside option immediately, so $W_m(z) = z$. Hence, we can deduce that $W_m(z) = z$ and $r_m(z) = 1$ for $z \ge z_m$, where $z_m = \inf\{z : W_m(z) = z\}$ and $z_m \in (a_m - p, u_{\max} - p)$. For $z < z_m$, the consumer will search and so $r_m(z) < 1$.

When the deviating firm occupies the k_{th} position in a consumer's search order, the consumer will buy from it immediately if and only if $u - \tilde{p} \ge W_{n-k}(u - \tilde{p})$, i.e., if $u - \tilde{p} \ge z_{n-k}$, where z_{n-k}, according to its definition, satisfies

$$z_{n-k} = a_{n-k} - p + \int_0^{z_{n-k}} r_{n-k}(x)dx \ . \tag{26}$$

Thus, the firm's demand when it is in the k_{th} position, charges price $\tilde{p}$ and permits free return, is

$$\begin{aligned} & h_k \left[1 - F(z_{n-k} + \tilde{p}) + \int_{\tilde{p}}^{z_{n-k}+\tilde{p}} r_{n-k}(u - \tilde{p})f(u)du\right] \\ & \quad = h_k \left[1 - F(z_{n-k} + \tilde{p}) + \int_0^{z_{n-k}} r_{n-k}(u)f(u + \tilde{p})du\right] \ , \end{aligned} \tag{27}$$

where the equality follows after changing variables in the integral. Compared to the demand generated with an exploding offer given in (9), it now has reduced immediate demand since $z_{n-k} > a_{n-k} - p$, but has positive returning demand comprised of the integral term.

Claim 2 *Demand in (27) is smaller than that in (9) if f is strictly increasing.*

Proof. We need to show

$$\int_0^{z_{n-k}} r_{n-k}(u)f(u + \tilde{p})du < F(z_{n-k} + \tilde{p}) - F(a_{n-k} - p + \tilde{p}) \ . \tag{28}$$

Define

$$\phi(u) \equiv z_{n-k} + \tilde{p} - \int_u^{z_{n-k}} r_{n-k}(x)dx \ .$$

Note that $\phi'(u) = r_{n-k}(u)$, $\phi(z_{n-k}) = z_{n-k} + \tilde{p}$, and $\phi(0) = a_{n-k} - p + \tilde{p}$ (which follows from (26)). Then the right-hand side of (28) can be written as

$$\int_0^{z_{n-k}} r_{n-k}(u)f(\phi(u))du \ .$$

Since $\phi(u) > u + \tilde{p}$ (because $r_{n-k}(x) < 1$ for $x < z_{n-k}$), expression (28) holds if f is an increasing function. ■

Therefore, for any price $\tilde{p}$, unilaterally allowing free recall causes the deviating firm's demand (and hence profit) to fall when f is increasing. (This is true regardless of the firm's position in a consumer's search order, except when it is in the final position in which case the use of exploding offers makes no difference to the firm's demand.) It follows that an equilibrium in which all firms use exploding offers exists.

The second step is to exclude the possibility of a free-recall equilibrium when f is strictly increasing. We show that, starting from the hypothetical free-recall equilibrium with price p_0, each firm has a unilateral incentive to use an exploding offer no matter what position it is in the consumer's search process (except when it is in the final position).

As in expression (4), a firm's demand, if it is in the k_{th} position of the consumer's search process with $k < n$, is

$$F(a)^{k-1}[1 - F(a)] + \int_{p_0}^{a} F(u)^{n-1} f(u) du \ . \tag{29}$$

The first term is demand when the consumer buys the firm's product immediately, and the second term is demand when the consumer first leaves the firm but eventually comes back. Suppose now that the firm unilaterally uses an exploding offer but still charges the price p_0. (We will show the firm's profits increase with this deviation, and hence the hypothetical equilibrium is not valid. The firm's profits would increase still further if it altered its price as well.) Define $\delta \equiv \max\{0, u_1 - p_0, \cdots, u_{k-1} - p_0\}$. Then the consumer will visit the firm if and only if $\delta < a - p_0$. If she finds match utility u at the firm, she will buy (immediately) if $u - p_0$ is greater than the expected surplus from searching further.

Denote by $V_m(z)$ the expected surplus from participating in a free-recall search market with m products offered at price p_0 and an outside option $z < a - p_0$. Then[45]

$$V_m(z) = z + \int_{z+p_0}^{a} [1 - F(u)^m] du \ . \tag{30}$$

One can check that $z \leq V_m(z) < a - p_0$.

[45] The consumer will stop searching before she runs out of options if and only if she finds a product with match utility greater than a. (This is true regardless of z provided that $z < a - p_0$.) Her expected surplus is therefore

$$V_m(z) = [1 - F(a)^m] \cdot [E(u|u \geq a) - p_0] + \Pr(u^* < a) \cdot E[\max\{u^* - p_0, z\}|u^* < a] - sT,$$

where $u^* = \max\{u_1, \cdots, u_m\}$ and $T = [1 - F(a)^m] / [1 - F(a)]$ is the expected number of searches. The first term is the surplus when the consumer ends up buying a product with match utility higher than a, and the second term is the surplus when she ends up sampling all firms. From the definition of the reservation utility a in (3), we have $s = [1 - F(a)][E(u|u \geq a) - a]$. Substituting this into $V_m(z)$ yields the formula.

The consumer will buy from firm i if and only if $u - p_0 \geq V_{n-k}(\delta)$. Here, δ is the consumer's outside option if the consumer leaves the firm and continues searching (since the firm is using an exploding offer). The c.d.f. of δ defined on $[0, u_{\max} - p_0]$ is $G(\delta) \equiv F(\delta + p_0)^{k-1}$, which has a mass point at zero. Therefore, the deviating firm's demand when it is in the k_{th} position is

$$\begin{aligned} & \Pr\left(\delta < a - p_0 \text{ and } u - p_0 > V_{n-k}(\delta)\right) \qquad (31)\\ = \; & G(0)[1 - F(p_0 + V_{n-k}(0))] + \int_0^{a-p_0} [1 - F(p_0 + V_{n-k}(\delta))] \frac{dG(\delta)}{d\delta} d\delta \\ = \; & F(a)^{k-1}[1 - F(p_0 + V_{n-k}(a - p_0))] + \int_{p_0}^{a} f(p_0 + V_{n-k}(x - p_0)) V'_{n-k}(x - p_0) F(x)^{k-1} dx \;, \end{aligned}$$

where the second equality follows after integrating by parts and changing the integral variable. According to the definition of $V_m(\cdot)$ in (30), we have $V_{n-k}(a - p_0) = a - p_0$ and

$$V_{n-k}(x - p_0) = x - p_0 + \int_x^a [1 - F(u)^{n-k}] du \;, \; V'_{n-k}(x - p_0) = F(x)^{n-k} \;.$$

Substituting these into (31) shows that the firm's demand is

$$F(a)^{k-1}[1 - F(a)] + \int_{p_0}^{a} F(x)^{n-1} f\left(a - \int_x^a F(u)^{n-k} du\right) dx \;. \qquad (32)$$

Since $a - \int_x^a F(u)^{n-k} du > x$ for $x < a$, one can see that if f is strictly increasing (we actually only need f to be strictly increasing on $[p_0, a]$), demand in (32) is strictly greater than demand in (29). Therefore, the firm does have an incentive to deviate from the supposed free-recall equilibrium. This completes the proof of part (i). Parts (ii) and (iii) can be proved in a similar manner.

Proof of Proposition 2: We will show that a firm has an incentive to introduce a small buy-later premium, and then invoke Lemma 1 to show that the firm also has an incentive to offer a small buy-now discount. Compared to the duopoly case analyzed in the main text, the additional analysis needed for the general n-firm case involves the extra complexity of a consumer's stopping rule. In particular, the consumer's stopping rule at a firm which offers a buy-later premium will depend on the history of offers she sees before she encounters the firm, and this feature is absent in the duopoly analysis.

Let p_0 be the price in the free-recall equilibrium defined by (5). Assumption (2) implies that $p_0 < a$. We first consider this hypothetical search problem:

A search problem: *Suppose the consumer encounters firm i first, and is offered match utility u_i, the buy-now price p_0, and a buy-later premium $\tau > 0$ (so the buy-later price at firm i is $\hat{p} = p_0 + \tau$.) Suppose she expects that all m remaining firms charge price $p_0 < a$ and allow free recall, and suppose the consumer has an outside option $\delta < a - p_0$. What is her optimal stopping rule at firm i?*

It is clear that (a) if $u_i \geq a$, the consumer will surely stop searching and buy at firm i immediately (this is even true when $\tau = 0$); and (b) if $u_i - p_0 \leq \delta$, then firm i's offer is dominated by the outside option and the consumer will not buy from the firm (either immediately or later), and she will keep searching since $\delta < a - p_0$.

Now consider the intermediate case with $u_i - p_0 \in (\delta, a - p_0)$. If the consumer buys immediately at firm i, her payoff is $u_i - p_0$. If she leaves firm i, she will begin a free-recall search process with m firms and an outside option

$$z = \max\{\delta, u_i - \hat{p}\} < a - p_0 \ .$$

(Recall she will pay the higher price $\hat{p} > p_0$ if she returns to buy from firm i.) As before, the expected surplus $V_m(z)$ from entering this search market is given by (30). Given δ, z is a function of u_i and we can therefore regard $V_m(z)$ as a function of u_i: it is flat until u_i reaches $\delta + \hat{p}$ and then increases with u_i with slope less than one. (Note that we are considering the case with $u_i < a$, so the slope cannot be equal to one.) Recall from (30) that for $z < a - p_0$, $z < V_m(z) < a - p_0$.

Clearly, the consumer will buy immediately from firm i if and only if

$$u_i - p_0 \geq V_m(\max\{\delta, u_i - \hat{p}\}) \ . \tag{33}$$

Given the properties of $V_m(\cdot)$, the equality of (33) has a unique solution $a_m(\tau) \in (\delta + p_0, a)$. We conclude that the consumer will buy immediately from firm i if and only if $u_i \geq a_m(\tau)$.

There are then two cases, depending on the size of the premium τ:

(i) If $u_i - p_0$ crosses $V_m(z)$ at the flat portion, which occurs when $\delta + \hat{p} - p_0 > V_m(\delta)$ or $\tau > V_m(\delta) - \delta$, then

$$a_m(\tau) = p_0 + V_m(\delta) \ , \tag{34}$$

which does not depend on τ. In this case, the consumer will never return to firm i once she leaves because $u_i - \hat{p}$ is dominated by δ. Therefore, τ is so large that firm i has no returning demand.

(ii) If $u_i - p$ crosses $V_m(z)$ at the increasing portion, which occurs when $\tau \leq V_m(\delta) - \delta$, then $a_m(\tau)$ is implicitly determined by $a_m(\tau) - p_0 = V_m(a_m(\tau) - p_0 - \tau)$, which from (30) implies $a_m(\tau)$ satisfies

$$\tau = \int_{a_m(\tau)-\tau}^{a} [1 - F(u)^m] du \ , \tag{35}$$

which does not depend on p_0 or δ. In particular, $a_m(0) = a$. Expression (35) is the generalization beyond duopoly of our earlier formula (14). In this case, the consumer will initially reject firm i's offer if $u_i < a_m(\tau)$, but will come back to the firm after sampling the remaining m firms if $u_i - \hat{p} > \max_{1 \leq j \leq m}\{\delta, u_j - p_0\}$.[46] Note that the assumption $\delta < a - p_0$ implies that $V_m(\delta) - \delta > 0$, and so case (ii) is relevant for all sufficiently small $\tau > 0$.

In sum, we deduce the following result:

[46] Note that once the consumer leaves firm i, she has the outside option $z < a - p_0$ and so she will never come back before sampling all the remaining m firms.

Claim 3 *In this hypothetical search problem, the consumer will buy from firm i immediately if and only if $u_i \geq a_m(\tau)$, where $a_m(\tau)$ is defined in (34) if $\tau > V_m(\delta) - \delta$ and otherwise $a_m(\tau)$ is defined in (35).*

Finally, since $V_m(\delta) - \delta$ is decreasing in δ, the condition $\tau > V_m(\delta) - \delta$ is equivalent to $\delta \in (\delta_\tau, a - p_0)$, where δ_τ solves

$$\tau = V_m(\delta_\tau) - \delta_\tau = \int_{\delta_\tau + p_0}^{a} [1 - F(u)^m] du \tag{36}$$

if $\tau < V_m(0)$, and $\delta_\tau = 0$ otherwise. In particular, $V_m(\delta_0) = \delta_0 = a - p_0$.

We now prove Proposition 2. Starting from the free-recall equilibrium with price p_0, suppose firm i unilaterally introduces a returning purchase premium $\tau > 0$ but keeps the buy-now price unchanged at p_0. Suppose firm i happens to be in the k_{th} position of the consumer's search process. If $k = n$, then τ has no impact on firm i's profit. In the following, we show that for any $k < n$, introducing a small premium $\tau > 0$ is profitable for the firm.

As in the proof of Proposition 1, let $\delta \equiv \max\{0, u_1 - p_0, \cdots, u_{k-1} - p_0\}$ be the best offer from the previous $k - 1$ firms. A consumer will visit firm i if $\delta < a - p_0$. If the consumer arrives at firm i and discovers match utility u_i and the buy-later premium τ (but still holds the equilibrium belief about the remaining $n - k$ firms' policies), she faces the search problem we have just analyzed with $m = n - k$, and her stopping rule will depend on her best previous offer δ. Let us focus on a relatively small τ such that $\tau < V_{n-k}(0)$ and define δ_τ as in (36) with $m = n - k$. Then if $\delta \in (\delta_\tau, a - p_0)$, the reservation utility according to (34) is $a_{n-k}(\tau) = p_0 + V_{n-k}(\delta)$. In this case, the consumer will buy immediately if $u_i \geq a_{n-k}(\tau)$, and otherwise she will keep searching and never come back. Alternatively, if $\delta \leq \delta_\tau$ the reservation utility $a_{n-k}(\tau)$ is as given in (35) with $m = n - k$. In this case, even if the consumer leaves firm i first (i.e., if $u_i < a_{n-k}(\tau)$), she will eventually come back after sampling all remaining firms if $u_i - p_0 - \tau$ is greater than their offered surplus *and* the outside option δ which represents the best offer among the previous $k - 1$ firms. Explicitly, firm i's returning demand in this case is

$$\begin{aligned}
&\Pr(\max_{j>k}\{\delta, u_j - p_0\} < u_i - p_0 - \tau < a_{n-k}(\tau) - p_0 - \tau) \\
= &\int_{p_0+\tau}^{a_{n-k}(\tau)} F(u_i - \tau)^{n-1} dF(u_i) = \int_{p_0}^{a_{n-k}(\tau)-\tau} F(u) f(u + \tau) du \; .
\end{aligned}$$

(Note δ is also a random variable with c.d.f. $G(\delta) = F(\delta + p_0)^{k-1}$, and the second step follows after changing the integral variable.) Therefore, firm i's profit if it is in the k_{th} search position and charges the buy-later premium τ is

$$p_0 \int_{\delta_\tau}^{a-p_0} [1 - F(p_0 + V_{n-k}(\delta))] \, dG(\delta) + p_0 G(\delta_\tau)[1 - F(a_{n-k}(\tau))]$$

$$+ (p_0 + \tau) \int_{p_0}^{a_{n-k}(\tau)-\tau} F(u)^{n-1} f(u+\tau) du \ . \quad (37)$$

Note from (35) that

$$(1 - a'_{n-k}(0))(1 - F(a)^{n-k}) = 1 \ . \quad (38)$$

By using the observations $V_{n-k}(\delta_0) = \delta_0 = a - p_0$ and (38), the derivative with respect to τ of firm i's profit in (37) when it is in the k_{th} position (with $k < n$), evaluated at $\tau = 0$, is

$$\int_{p_0}^{a} F(u)^{n-1}[f(u) + p_0 f'(u)]du \ , \quad (39)$$

which generalizes the duopoly expression (17). Here, $\int_{p_0}^{a} F^{n-1} f du$ is the extra revenue generated from the returning customers, while $\int_{p_0}^{a} F^{n-1} f' du$ is the extra demand generated by increasing the cost of return. That (39) is positive when $p_0 > \frac{1-F(a)}{f(a)}$ follows the argument given in the main text for duopoly. Since (39) is positive (and the same) for all $k < n$, the proof is complete.

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