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Abstract

In this paper I will introduce a new political economy model, where there exists a competition amongst two political candidates, which aim to set a policy which enables them to win elections, maximising the probability of winning. I will show that, if taxes necessary to repay the debt are not lump sum but proportional to income, we have dramatic distorting effect on the labour supply. The problems is exacerbate once we take into account that the Government set taxes in order to favour the most influencing social group. As a consequence, effective marginal tax rates are differentiated amongst social groups and thus the burden of public debt is not equally borne.

1 Introduction

In this paper I will introduce a new political economy model, where there exists a competition amongst two political candidates, which aim to set a policy which enables them to win elections, maximising the probability of winning. I will show that, if taxes necessary to repay the debt are not lump sum but proportional to income, we have dramatic distorting effect on the labour supply. The problems is exacerbate once we take into account that the Government set taxes in order to favour the most influencing social group. As a consequence, effective marginal tax rates are differentiated amongst social groups and thus the burden of public debt is not equally borne. Unlike Neoclassical Theory, in a political economy framework an issue of debt has dramatic effects on society welfare: first of all it distorts individual choices about the amount of labour so supply and this has consequences both on consumption and on production. But most importantly, the necessity to set taxes to repay the debt has redistribution effects amongst different groups, since the most powerful groups receive a tax benefit from the fiscal system, whilst the less powerful groups must borne the entire burden of taxation. Summarizing, the issue of public debt has real economic consequences both in terms of efficiency and in terms of equity.
2 Debt Issue and Ricardian Equivalence

The Ricardian Equivalence is one of the most debated theory in the history of Economics. Originally proposed to be rejected by the 19-th century English economist David Ricardo [40], the theory lived its peak of success under Robert Barro’s works [4], [5]. It was Barro himself that in the paper of 1979 On the Determination of the Public Debt described the Ricardian equivalence theorem as

the proposition that shifts between debt and tax finance for a given amount of public expenditure would have no first-order effect on the real interest rate, volume of private investment, etc.

Secondly, in the famous paper of 1974 Are Government Bonds Net Wealth? he wrote in his conclusions

The basic conclusion is that there is no persuasive theoretical case for treating government debt, at the margin, as a net component of perceived household wealth

and that

in the case where the marginal net-wealth effect of government bonds is close to zero (...) fiscal effects involving changes in the relative amounts of tax and debt finance for a given amount of public expenditure would have no effect on aggregate demand, interest rates, and capital formation.

Especially this last conclusion was a negation of the Keynesian theory which stated that fiscal policies are effective according to the Keynesian multiplier mechanism. Furthermore, Barro’s theory contradicted the theory of the public debt which was developing in the 70’s by prominent economists such as Buchanan, Modigliani and Diamond. Also economist who were working on a theory of social security systems, such as Feldstein felt attached. In 1976 Feldstein replied to Barro demonstrating that, even in the presence of an economy characterized by the existence of bequests, government debt reduces private savings and the equilibrium rate of capital to labour if, unlike Barro’s assumptions, the economy has not a constant population and the growth of rate is different from zero. Under the condition that the rate of interest on public debt is lower than the growth of rate Feldstein argued that the first generation will not increase the amount of bequest, knowing that no future generation must borne the burden of the debt. But, what if the rate of interest on public debt exceeds the rate of growth of the economy? Even in this case, if the difference between the two greatesses is small, Feldstein demonstrated that the first generation increases its bequest by less than the value of the debt. Another source of critique came from Buchanan [8] who accused Barro to have misread Ricardo’s original statement and even though he was himself accused by O’Driscoll [?]
not to have understood that the actual Ricardo’s position was against rather than pro-Ricardian equivalence. At the beginning of the ’80s first econometric studies to discover whether the Ricardian Equivalence held was conducted by Feldstein [18] who concluded that changes in government spending or taxes can have substantial effects on aggregate demand, denying the equivalence. Otherwise, the monetarist theory spoused the Ricardian Equivalence; in 1984 McCallum [31] validated the monetarist hypothesis according to which a constant, positive budget deficit can be maintained permanently and without inflation, if it financed by government bonds rather than taxation. Nevertheless, as himself recognized, uncertainty, distribution effects and multiple interest rates are ignored by Barro’s theory but that this did not represent a problem since “the same is true of most policy-oriented theoretical analyses of macroeconomic phenomena ”, then “there is no apparent reason why the issue at hand requires a different type of treatment ” and “it would seem satisfactory to neglect them there, as elsewhere ”. We will see later on that this reason not only exists but it is fundamental to understand the effect of public debt on the welfare of society. Some years later Feldstein [19] demonstrated that the Barro’s theory did not hold even in the presence of non-negative bequests, just considering individuals who are uncertain about their future incomes and, as a consequence, modify their consumption path, raising consumption at present time following, for instance, a tax cut. Hence, the reaction to a change in fiscal policies is non-neutral but consumption modifies according to the sign of the policy.

3 The basic model

I consider a four-period model with overlapping generations, where each generation lives only for two periods, the youth and old age. At any period of time, the generation of youths coexists with the generation of the elderly. At the beginning of the next period, the elderly die, the youths become elderly and a new generation of youths is born. As a consequence, there are two overlapping generations of people living at any one time. Generations are unlinked, meaning that there is no possibility to leave any bequest. Individuals consume all the available income earned at a given period of time; thus, it is not possible neither to save nor to borrow money.

Then, at time $t$, let a population of size one be partitioned into two groups of workers, the young, representing the generation born at time $t$ and denoted by $\tau$, and the old, representing the generation born at time $t - 1$ and who denoted by $\tau - 1$. I will use capital letters to indicate the group and small letters to indicate single individuals belonging to a group. The size of a group does not change over time.

Each worker has to decide how to divide his total amount of time $\bar{t}$ between work and leisure (denoted by $l$). If the level of leisure reach 100%, I assume that the worker retires and gets a benefit equal to $p_{t-1}^{-1}$.

Furthermore, I introduce the core assumptions of the model. I assume that the old and the young are identical in every respect except two: first of all the
The intrinsic value of the old workers for leisure is assumed to be strictly greater than the same value of the young workers. That is, $\psi^{\tau - 1} \gg \psi^\tau$, where Greek letter \(\psi\) denotes the intrinsic value for leisure. Thus, the two social groups have different preferences with respect to the choice between work and leisure. Secondly, the wage of the old is higher than the wage of the young. That is: $w^{\tau - 1} > w^\tau$. The difference in wages is due to the existence of a labour market imperfection, according to which firms praise the older workers (insiders) and penalizes the younger (outsiders). As a consequence, the old can be seen as the richest member of society and the young the poorests. The wage rate does not change over time.

This assumption is supported by the empirical evidence. For a review on differences in preferences for leisure, see Canegrati. For the evidence on the differences in wage rates amongst cohorts... (to be finished). Old workers’ preferences can be represented by a quasi-linear utility function\(^1\).

### 3.1 Timing of the game

The game takes place on time horizon which runs from time \(t - 1\) to time \(t+2\). At every period, elections between two candidates, say A and B, wish to maximize their number of votes to win elections\(^2\). Both of them have an ideological label (for instance they are seen as “Democrats” or “Republicans”). I assume that this label is exogenously given.

In the first stage of the game, the two candidates, simultaneously and independently, announce (and commit to) a policy vector, \(?^A\) and \(?^B\).

In the second stage elections take place. A candidate wins elections if and only if it obtains the majority of votes; in the case of a tie a coin is tossed in order to choose the Government which will come to power. Furthermore, I assume that each party prefers to stay out from the competition than to enter and lose, that prefers to tie than stay out and it prefers to win than to tie.

Finally, in the third stage, workers choose their work and leisure, given the level of credits chosen by the Government.

### 3.2 Utility functions and budget constraints

A representative worker of generation \(\tau - 2\) at time \(t - 1\) has the following lifetime utility function:

$$U^{\tau - 2} = c_{t-1}^{\tau - 2} + \psi^{\tau - 2} \log l_{t-1}^{\tau - 2}$$

\(\forall \ \tau - 1 \in T - 2\)

where \(c_{t-1}^{\tau - 2}\) is consumption at time \(t - 1\), \(l_{t-1}^{\tau - 2}\) is leisure at time \(t - 1\) and \(\psi^{\tau - 2}\) is a parameter representing the intrinsic preference of the worker for leisure \((\psi^{\tau - 2} \in [0, 1])\).

\(^1\)A quasi-linear utility function entails the non existence of the income effect

\(^2\)Lindbeck and Weibull 1987 and Dixit and Londregan 1996 demonstrated that the Nash equilibrium obtained if candidates maximize their vote share is identical to that obtained when candidates maximize their probability of winning.
The worker consumes all his income:

\[ c_{t-1}^{\tau-2} = (w^{\tau-2}(1 - \tau_L) + a_{t-1}^{\tau-2} L_1)(\overline{I} - l_{t-1}^{\tau-2}) \]  

(2)

where \( w^{\tau-2} \) is the unitary wage per hour worked, \( \tau_L \) is the tax rate on labour income (equal for every group and constant over time) and \( a_{t-1}^{\tau-2} \) the tax credit.

Similarly, preferences of a representative worker of generation \( \tau - 1 \) at time \( t - 1 \) are given by the following lifetime utility function:

\[ U^{\tau-1} = c_t^{\tau-1} + \psi^{\tau-1} \log l_t^{\tau-1} + \beta(c^{\tau-1}_{t+1} + \psi^{\tau-1} \log l_{t+1}^{\tau-1}) \]  

(3)

\( \forall \tau - 1 \in T - 1 \)

where \( c_t^{\tau-1} \) and \( c^{\tau-1}_{t+1} \) represent consumption at time \( t - 1 \) and \( t \), \( l_t^{\tau-1} \) and \( l_{t+1}^{\tau-1} \) leisure at time \( t - 1 \) and \( t \), and \( \psi^{\tau-1} \) is the intrinsic preference of the worker for leisure \( (\psi^{\tau-1} \in [0, 1]) \). Since at time \( t - 1 \) a worker of generation \( \tau - 1 \) knows that at time \( t \) will be old, their utility function includes the leisure of the next period, weighted by a discount factor \( \beta \in [0, 1] \).

The worker’s inter temporal budget constraint is given by:

\[ c_t^\tau + \beta c_{t+1}^\tau = (w_t^\tau(1 - \tau_L) + a_t^\tau L_t)(\overline{I} - l_t^\tau - b_t) \]  

(4)

\( \beta = (w_t^\tau(1 - \tau_L) + a_t^\tau L_t)(\overline{I} - l_t^\tau - b_t) \)

where \( b_t \) represents the per capita public debt.

A representative worker of generation \( \tau \) at time \( t \) has the following utility function:

\[ U^\tau = c_t^\tau + \psi^\tau \log l_t^\tau + \beta(c_{t+1}^{\tau+1} + \psi^\tau \log l_{t+1}^{\tau+1}) \]  

(5)

\( \forall \tau \in T \) under the budget constraint

\[ c_t^\tau + \beta c_{t+1}^{\tau+1} = (w_t^\tau(1 - \tau_L) + a_t^\tau L_t)(\overline{I} - l_t^\tau - b_t) \]  

(6)

\( + \beta((w_{t+1}^\tau(1 - \tau_L) + a_{t+1}^{\tau+1} L_{t+1})(\overline{I} - l_{t+1}^{\tau+1} + (1 + r)b_t) \)

where \( r \) represents the interest rate paid on public debt.

Finally, a representative worker of generation \( \tau + 1 \) at time \( t + 1 \) has the following utility function:

\[ U^{\tau+1} = c_{t+1}^{\tau+1} + \psi^{\tau+1} \log l_{t+1}^{\tau+1} + \beta(c_{t+2}^{\tau+2} + \psi^{\tau+1} \log l_{t+2}^{\tau+2}) \]  

(7)

\( \forall \tau + 1 \in T + 1 \) under the budget constraint

\[ c_{t+1}^{\tau+1} + \beta c_{t+2}^{\tau+2} = (w_{t+1}^{\tau+1}(1 - \tau_L) + a_{t+1}^{\tau+1} L_{t+1})(\overline{I} - l_{t+1}^{\tau+1}) \]  

(8)

\( + \beta((w_{t+2}^{\tau+2}(1 - \tau_L) + a_{t+2}^{\tau+2} L_{t+2})(\overline{I} - l_{t+2}^{\tau+2})) \)

where \( r \) represents the interest rate paid on public debt.
3.3 The Government

The literature has used different formulations for the Government’s objective function. A typical normative approach considers a benevolent Government which aims to maximize a Social Utility Function by choosing the optimal tax rate on labour, subject to a budget constraint where public expenditures are financed either by current taxation or by public debt issue. For instance, Barro [5] used a type of budget equation where, in each period, interest payments during period $t$ are assumed to apply to the stock of debt outstanding at the beginning of the period (see equation 1). Furthermore, Barro consider an overall budget constraint where present value of government expenditure (aside from interest payments) added to the initial amount of debt is equated to the present value of taxes over an infinite horizon of time (see equation 2). Unfortunately, the Barro’s idea about the existence of an infinitely living Government which chooses tax rates for every period solving an optimization problem seems to be rather unrealistic and of course at odds with reality where, instead, usually Governments are short-lived and remain in charge only for few years. Things worsen once we consider that many times Governments are not benevolent but are rent-seekers, utterly involved in the quest for election win. In such an environment, it is clear that Barro’s theory may do not hold anymore. In this paper, I will use a classical political economy model characterized by the presence of a Probabilistic Voting Model with single-minded groups (see [?]) where politicians act in order to maximize the probability of being re-elected.

The policy vector the Government has to choose is given by:

$$\hat{q} = (a_{t-1}^{r-2}, a_{t-1}^{r-1}, a_{t}^{r-1}, a_{t+1}^{r}, a_{t+1}^{r+1})$$

encompassing tax credits at time $t-1$ just before the debt issue, tax credits at time $t$ when the debt is issued and tax credits at time $t+1$ when the debt is paid back. I will assume that from period $t + 2$ on, tax credits turn back to be the same as at $t - 1$.

Hence, I introduce government’s budget equations.

3.3.1 time $t - 1$

$$n^{r-2} r_L (\bar{t} - l_{t-1}^{r-2}) (w_{t-1}^{r-2} - a_{t-1}^{r-2}) + n^{r-1} r_L (\bar{t} - l_{t-1}^{r-1}) (w_{t-1}^{r-1} - a_{t-1}^{r-1}) = 0 \quad (9)$$

where $n^{r-2} r_L (\bar{t} - l_{t-1}^{r-2}) (w_{t-1}^{r-2} - a_{t-1}^{r-2})$ represents total revenues generated by the taxation of generation $\tau - 2$ at time $t - 1$, whilst $n^{r-1} r_L (\bar{t} - l_{t-1}^{r-1}) (w_{t-1}^{r-1} - a_{t-1}^{r-1})$ the total revenues generated by the taxation of generation $\tau - 1$.

Since revenues are proportional to the amount of labour supplied, the taxation entails inefficiencies, since it distorts workers’ decisions on the amount of labour supplied. I also assume that a contingent budget surplus is entirely used to pay pensions to the retirees.
3.3.2 time $t$

At time $t$, the Government issues an amount of debt equal to $B_t$, seen as an one-period, single-coupon bonds and issued at par allocated amongst all the voters who get exactly the same amount debt, equal to $b_t$. Hence, the new government’s budget equations is:

$$n^{\tau-1} \tau_L (\bar{t} - l^\tau_{t-1}) (w^\tau_t - a^\tau_t) + n^{\tau} \tau_L (\bar{t} - l^\tau_t) (w^\tau_t - a^\tau_t) + B_t = 0 \quad (10)$$

3.3.3 time $t + 1$

At time $t + 1$ the Government pays an amount of real interest, $rB_t$, and the principal. Thus the government’s budget equations may be written as:

$$n^{\tau} \tau_L (\bar{t} - l^\tau_{t+1}) (w^\tau_{t+1} - a^\tau_{t+1}) + n^{\tau+1} \tau_L (\bar{t} - l^\tau_{t+1}) (w^\tau_{t+1} - a^\tau_{t+1}) = (1 + r)B_t \quad (11)$$

3.4 The equilibrium

I solve the game, starting from period $t - 1$ where a representative worker of generation $\tau - 2$ solves the following optimization problem:

$$\max U^{\tau-2} = c^{\tau-2}_t + \psi^{\tau-2} \log l^{\tau-2}_{t-1}$$

s.t. $c^{\tau-2}_t = ((1 - \tau_L) + a^{\tau-2}_{t-1} \tau_L) (\bar{t} - l^{\tau-2}_{t-1})$

Solving with respect to $l^{\tau-2}_{t-1}$ I obtain an expression for the optimal amount of leisure:

$$l^{\tau-2*}_{t-1} = \frac{\psi^{\tau-2}}{w^{\tau-2}((1 - \tau_L) + a^{\tau-2}_{t-1} \tau_L)} \quad (12)$$

and substituting into (1) I obtain an expression for the Indirect Utility Function:

$$V^{\tau-2} = \bar{t} w^{\tau-2}((1 - \tau_L) + a^{\tau-2}_{t-1} \tau_L) - \psi^{\tau-2} + \psi^{\tau-2} \log \psi^{\tau-2} - \psi^{\tau-2} \log (w^{\tau-2}((1 - \tau_L) + a^{\tau-2}_{t-1} \tau_L)) \quad (13)$$

I do the same for the representative worker of generation $\tau - 1$:

$$\max U^{\tau-1} = c^{\tau-1}_t + \psi^{\tau-1} \log l^{\tau-1}_{t-1} + \beta(c^{\tau-1}_t + \psi^{\tau-1} \log l^{\tau-1}_{t-1})$$

s.t. $c^{\tau-1}_t + \beta c^{\tau-1}_t = ((1 - \tau_L) + a^{\tau-1}_{t-1} \tau_L) (\bar{t} - l^{\tau-1}_{t-1}) + \beta((\bar{t} - l^{\tau-1}_{t-1})((1 - \tau_L) + a^{\tau-1}_{t-1} \tau_L) - b_t)$

$$l^{\tau-1*}_{t-1} = \frac{\psi^{\tau-1}}{w^{\tau-1}((1 - \tau_L) + a^{\tau-1}_{t-1} \tau_L)} \quad (14)$$
\[ l^{\tau-1}_t = \frac{\psi^{\tau-1}}{w^{\tau-1}((1 - \tau L) + a^{\tau-1}_t \tau L)} \] (15)

\[ V^{\tau-1} = \tilde{w}^{\tau-1}((1 - \tau L) + a^{\tau-1}_t \tau L) - \psi^{\tau-1} + \psi^{\tau-1} \log \psi^{\tau-1} - \psi^{\tau-1} \log w^{\tau-1} (((1 - \tau L) + a^{\tau-1}_t \tau L)) + a^{\tau-1}_t \tau L + b \tilde{L}((1 - \tau L) + a^{\tau-1}_t \tau L) + \beta \log(\psi^{\tau-1}) - \psi^{\tau-1} \log(w^{\tau-1}((1 - \tau L) + a^{\tau-1}_t \tau L) - b) \] (16)

The same for the representative worker of generation \( \tau \)

\[
\max \ U^{\tau} = c^{\tau}_t + \psi \log l^*_t + \beta(c^{\tau+1}_t + \psi \log l^*_t) \\
\text{s.t.} \ c^{\tau}_t + \beta c^{\tau+1}_t = ((1 - \tau L) + a^{\tau}_t \tau L)((\tilde{I} - l^*_t) - b_t + \beta((\tilde{I} - l^*_t)((1 - \tau L) + a^{\tau+1}_t \tau L) + rb_t) \\
\]

\[ l^{\tau*}_t = \frac{\psi^{\tau}}{w^{\tau}((1 - \tau L) + a^{\tau}_t \tau L)} \] (17)

\[ l^{\tau*+1}_t = \frac{\psi^{\tau+1}}{w^{\tau+1}((1 - \tau L) + a^{\tau+1}_t \tau L)} \] (18)

\[ V^{\tau} = \tilde{w}^{\tau}((1 - \tau L) + a^{\tau}_t \tau L) - \psi^{\tau} + \psi^{\tau} \log \psi^{\tau} - \psi^{\tau} \log w^{\tau} (((1 - \tau L) + a^{\tau}_t \tau L)) + a^{\tau}_t \tau L - b_t + \beta((1 - \tau L) + a^{\tau+1}_t \tau L) + \beta \log(\psi^{\tau}) - \psi^{\tau} \log(w^{\tau}((1 - \tau L) + a^{\tau+1}_t \tau L) + rb_t) \] (19)

and for a representative worker of generation \( \tau + 1 \)

\[
\max \ U^{\tau+1} = c^{\tau+1}_t + \psi^{\tau+1} \log l^*_t + \beta(c^{\tau+2}_t + \psi^{\tau+1} \log l^*_t) \\
\text{s.t.} \ c^{\tau+1}_t + \beta c^{\tau+2}_t = ((1 - \tau L) + a^{\tau+1}_t \tau L)((\tilde{I} - l^*_t)((1 - \tau L) + a^{\tau+1}_t \tau L) + rb_t + 1) \\
\]

\[ l^{\tau+1*}_t = \frac{\psi^{\tau+1}}{w^{\tau+1}((1 - \tau L) + a^{\tau+1}_t \tau L)} \] (20)

\[ l^{\tau+2*}_t = \frac{\psi^{\tau+1}}{w^{\tau+2}((1 - \tau L) + a^{\tau+2}_t \tau L)} \] (21)

\[ V^{\tau+1} = \tilde{w}^{\tau+1}((1 - \tau L) + a^{\tau+1}_t \tau L) - \psi^{\tau+1} + \psi^{\tau+1} \log \psi^{\tau+1} - \psi^{\tau+1} \log w^{\tau+1} (((1 - \tau L) + a^{\tau+1}_t \tau L)) + a^{\tau+1}_t \tau L + \beta((1 - \tau L) + a^{\tau+2}_t \tau L + \beta \log(\psi^{\tau+1}) - \psi^{\tau+1} \log(w^{\tau+1}((1 - \tau L) + a^{\tau+2}_t \tau L)) \] (22)

In the second stage of the game elections take place. It is easy to verify that the elections’ outcome is a tie. The proof arises as an obvious consequence of the resolution of the first stage, where it will be demonstrated that in equilibrium, both parties choose an identical policy vector.
equivalently, the probability of winning. The resolution is made for candidate A, but it also holds for candidate B.

$$\begin{align*}
\text{max } & \pi^A = \frac{1}{2} + \frac{h}{s} \sum_{t=\{T-1,T\}} n^I s^I [V^I(\bar{q}^A) - V^I(\bar{q}^B)] \\
& n^T \tau_L (\bar{t} - l^T i^T - 1)(w^T i^T - a^T i^T) + n^T \tau_L (\bar{t} - l^T i^T)(w^T i^T - a^T i^T) = 0
\end{align*}$$

I provide a complete resolution to the problem in the Appendix.

**Proposition 1** In equilibrium both candidates’ policy vectors converge to the same platform; that is $\bar{q}^A = \bar{q}^B = \bar{q}^*$. 

**Proof**: $\bar{q}^*$ represents the policy which captures the highest number of swing voters. Instead, suppose there exists other two policies $\bar{q}'$ and $\bar{q}''$; in moving from $\bar{q}^*$ to $\bar{q}'$ (or $\bar{q}''$) a candidate loses more swing voters than those it is able to gain. Thus, suppose a starting point where candidate A chooses $\bar{q}'$ and candidate B chooses $\bar{q}''$ such that in choosing $\bar{q}'$ and $\bar{q}''$ the elections outcome is a tie. If one candidate moved toward $\bar{q}^*$, it would be able to gain more swing voters than those it loses and thus, it would win the elections. So, choosing any policy but $\bar{q}^*$ cannot be an optimal answer. The only one policy which represents a Nash Equilibrium is $\bar{q}^*$ since it is the intersection between the optimal answers of the two candidates and no one candidate has an incentive to deviate. Since each candidate maximizes its share of votes, in equilibrium the two candidates receive both one half of votes; if one candidate should receive less than one half of votes it would always have the possibility to adopt the platform chosen by the other candidate and get the same number of votes. Notice that what we found here is the multidimensional analogue of Hotelling’s principle of minimum differentiation.

**Corollary 1** The utility levels reached by workers are the same; that is: $V^1A = V^1B$.

**Proposition 2** The marginal tax rate on labour is equal for both groups but the tax credit is more beneficial for the elder generation.

**Proof**: obtained via numerical simulations. See Tables 1-3 in Appendix 1.

**Proposition 3** Optimal allowances are a function of the numerosity and density of both groups, of the marginal tax rate, of the total endowment of time and of the parameters representing preferences of groups for leisure. That is: $a^I = a(s^I, s^{-I}, n^I, n^{-I}, \tau_L, \bar{t}, \psi^I, \psi^{-I})$.

**Proof**: see Appendix 1.

Thus, the political economy framework suggests that effective tax rates should be differentiated amongst cohorts, as Proposition 2 states. Indeed, if
a traditional and normative approach suggests that a benevolent Governments should tax less the poorest social groups, this political economy approach suggests that in a real world vote-seeker Governments tax groups according to their ability to threat politicians in the electoral competition.

**Proposition 4** In equilibrium, the amount of leisure for the older generation is higher than the amount of leisure for the younger generation.

*Proof*: see Appendix 1.

**Corollary 2** Tax revenues collected via labour taxation on the younger generation are positive; tax revenues generated via the labour taxation on labour taxation on the older generation are negative.

*Proof*: It derives from Proposition 2 and 4.

Thus, the fiscal system described by the model, is compatible with the decision of the old to early retire. As a consequence, revenues collected from the taxation of the old are equal to zero, whilst revenues collected with the taxation of the young are positive and equal to the amount of pensions that the old receive. The fact that tax revenues are negative for the old and positive for the young means that the old get a transfer financed with revenues of the young.

**Corollary 3** The old are more single-minded than the young. That is: \((s_{1}^{\tau-1} > s_{\tau})\).

*Proof*: The result derives from the assumption that the density function is monotonically increasing in leisure \((s = s(l))\). Since the old obtain more leisure in equilibrium, the density is higher and, by definition, the group is more single-minded.

Finally, the Lagrange multiplier has a political meaning: it represents the increase in the probability of winning for a candidate, if it had an additional dollar available to spend on redistribution.

4 Numerical Simulations

Numerical simulations was made since an analytical solution for the system is hard to compute. Indeed, to get the optimal policy vector we have to solve several systems of three equations with three unknowns (the two tax credits and the Lagrange multiplier). Nevertheless, also this process suffers from some problems. First of all, the simultaneity. We have assumed that the density function is endogenous in leisure; this implies that we are able to know the value of the density only after having calculated the optimal level of leisure which depends on the level of taxation but which is exactly what we want to evaluate! A possible way out for this problem is to guess a numerical value for the density function and verify afterwards that the level of leisure calculated is compatible with the initial guess; in other words, if we assume that the density is monotonically increasing in leisure, we should expect to find higher
levels of leisure for the group of the old for which we have guessed an higher
density. Should not this happen, it would mean that our guess is wrong and the
SMT fails. Furthermore, we also know that in equilibrium optimal leisure is an
increasing function of $\psi$. As a consequence, the density function should be an
increasing function of $\psi$ as well. This is why I will use a Logit Density Function
of $\psi$ as a proxy for the actual Logit Density Function of leisure. Secondly, the
value of exogenous variable should be realistic, but unfortunately it is difficult
to attribute a real value to some parameters such as the preferences of workers
for leisure and the tax allowances should assume values between -1 and 1 which
does not always happen.

4.1 Main Findings

Main results are reported in tables 1-3. Tables 1.a, 2.a and 3.a report the matrix
of inputs, whilst tables 1.b, 2.b and 3.b the matrix of outcomes. Tables 1.a and
1.b refer to period $t-1$, tables 2.a and 2.b refer to period $t$ and table 3.a and
3.b refer to period $t+1$. We may notice some very interesting results. First
of all, tax credits granted to the elder generation are sistematically higher than
those granted to the younger generation. This happens at every period of time.
Secondly, over time, there exists differences in trends amongst generations. In
fact, the trend of the old generation systematically increases over time, whilst
the latter increases only from period $t-1$ to time $t$, and it dramatically decreases
from period $t$ to time $t+1$. Hence, a first conclusion may be drawn: at time
when debt is issued, both groups a reduction in the effective marginal tax rate:
the debt acts as a substitute of taxation. Otherwise, at time when the debt is
paid off, we realize that the burden is borne only by the younger generation,
whilst the elder gets an even greater reduction in the effective marginal tax
rate. Another interesting result refers to the labour supply; we may note that
the elder generation tends to work (rest) less (more) than the younger, with only
one exception at time $t+1$ for simulations obtained with a nominal tax rate
equal to 0.3 and 0.6. Nevertheless, notice that tax credits granted to the two
groups are nearly the same. Furthermore, the labour supply increases over the
three periods for both groups. As a consequence, also the total labour supply
increases steadily over time. Tax revenues are always positive for the younger
generation and always negative for the older generation, meaning that the former
is a net taker, whilst the second is a net payer. Notice also that the tax benefit
for the older generation tends to increase over time and that tax revenues for
the younger decreases when the debt is issued but dramatically increases once
the debt must be repaid. This again underlies the asymmetry in the allocation
of the burden: the young borne the cost of repayment. The last result refers to
the level of production. For numerical simulations I used a production function
equal to $Y = 10\sqrt{L} - w^{-1}(1 - (1-a^{-1})T)(T-l^{-1}) + w^{-1}(1 - (1-a^{-1})T)(T-l^{-1})$, 
where $L = T - l^{-1} + T - l^{-1}$. It can be seen that at the period when the debt is
issued the production decreases, whilst it increases at its highest level at time
when the debt is paid off.
5 A model with altruism

I consider now a model where the old workers care of their offspring’s wealth. A classical altruistic model considers that households can be represented by a dynasty who is willing to perpetuate forever. As a consequence, the old internalize the utility function of the young and thus the new utility function of the old may be written as:

\[ U^{\tau-1} = c^{\tau-1} + \psi^{\tau-1} \log l^{\tau-1} + \sigma U^{\tau} \]  \hspace{1cm} (23)

\( \forall \tau \in T - 1 \)

where \( \sigma \in [0, 1] \) is a parameter which captures the degree of altruism of the old for the young; the higher \( \sigma \) the more the old attach a greater importance to the young’s wealth. In Appendix 3 i will provide a complete resolution to the problem.

**Proposition 5** The introduction of altruism in the model entails higher tax credits for the group of the young. The magnitude of this shift is higher, the higher is \( \sigma \).

**Proof**: see Appendix 2. As we can see by numerical simulation reported in Appendix 3, the introduction of altruism turns previous results around. This time the group of the young gain higher tax credits than the group of the old which is higher the higher is the parameter representing altruism. Furthermore, the taxation on the old is positive whilst that on the young is negative, meaning that they get a fiscal benefit whose burden is carried by the old. Nevertheless, the old still reach higher level of leisure. Results are very easy to interpret: the higher the old care of the young’s welfare, the higher they are willing to accept to carry the burden of the social security system. This is a great achievement for the welfare of society: the old get satisfaction both from leisure and from being generous, whilst the young get satisfaction by working more and receiving a fiscal benefit; since Leviathan politicians want to satisfy the electorate, they are more than happy to set the new pro-younger policy.

6 Conclusions

In this paper I will introduce a new political economy model, where there exists a competition amongst two political candidates, which aim to set a policy which enables them to win elections, maximising the probability of winning. I will show that, if taxes necessary to repay the debt are not lump sum but proportional to income, we have dramatic distorting effect on the labour supply. The problems is exacerbate once we take into account that the Government set taxes in order to favour the most influencing social group. As a consequence, effective marginal tax rates are differentiated amongst social groups and thus the burden of public debt is not equally borne.
Acknowledgments

I am particularly grateful to Massimo Bordignon, Federico Etro, Torsten Persson, Ronnie Razin, Jorgen Weibull, Micael Castanheira and Rachel Ngai for the very helpful comments and Santino Piazza who introduced me in the Latex and Mathematica world; to all the participants at the WISER Conference in Warsaw, in particular to Maksymilian Kwiek and Anna Ruzik; to DEFAP at the Universita’ Cattolica del Sacro Cuore, Universita’ degli Studi di Milano and Universita’ di Milano - Bicocca, for the financial support and to the London School of Economics where the SMT was developed. All the remaining errors are mine.
8 Appendix 1

In this Appendix I provide a complete resolution to the candidates’ problem. The two candidates face exactly the same optimization problem; they maximize their share of votes or, equivalently, the probability of winning. The resolution is made for candidate A, but it also holds for candidate B.

\[
\max \pi_A = \frac{1}{2} + \frac{h}{s} \sum_{I=1 \rightarrow T} n^I s^I [V^I(q^A) - V^I(q^B)]
\]

\[
T_1 = n^r-2 \tau_L (t - t^r-2) (1 - a_t^r-2) + n^r-1 \tau_L (t - t^r-1) (1 - a_t^r-1) = 0
\]
at time \(t - 1\)

\[
T_1 bis = n^r-1 \tau_L (t - t^r-1) (1 - a_t^r-1) + n^r \tau_L (t - t^r) (1 - a_t^r) + B_t = 0
\]
at time \(t\)

\[
T_1 tris = n^r \tau_L (t - t^r) (1 - a_t^r) + n^r+1 \tau_L (t - t^{r+1}) (1 - a_t^{r+1}) = (1 + r)B_t
\]
at time \(t + 1\)

I write the Lagrangian function for a given budget constraint BC:

\[
\ell = \frac{1}{2} + \frac{h}{s} \sum_{I=1 \rightarrow T} n^I s^I [V^I(q^A) - V^I(q^B)] + \lambda (BC)
\]

I obtain the following first order conditions which may be seen as a modified version of the original Lindbeck and Weibull first order conditions, at time \(t\):

(1) \[\frac{\partial \ell}{\partial a_t^A} \equiv n^r-2 \frac{\partial \tau_L}{\partial a_t^A} (V^r-1A - V^r-1B) + n^r-1 \frac{\partial \tau_L}{\partial a_t^A} = \lambda \frac{\partial \ell}{\partial a_t^A}\]

(2) \[\frac{\partial \ell}{\partial a_t^B} \equiv n^r \frac{\partial \tau_L}{\partial a_t^B} (V^rA - V^rB) + n^r+1 \frac{\partial \tau_L}{\partial a_t^B} = \lambda \frac{\partial \ell}{\partial a_t^B}\]

(3) \(BC = 0\)

According to the result stated in Corollary 1, FOC’s can be re-written in the following manner:

(1) \[\frac{\partial L}{\partial a_t^A} \equiv n^r-1 s^r-1 \frac{\partial \tau_L}{\partial a_t^A} = \lambda \frac{\partial \ell}{\partial a_t^A}\]

(2) \[\frac{\partial L}{\partial a_t^B} \equiv n^r s^r \frac{\partial \tau_L}{\partial a_t^B} = \lambda \frac{\partial \ell}{\partial a_t^B}\]

(3) \(BC = 0\)

and after some easy calculations, I obtain:

(1) \[n^r-1 s^r-1 (\bar{T}_L + \frac{\psi^{r-1} \tau_L}{(1 - (1 - a_t^r)^\psi \tau_L)}) - \lambda (1 - a_t^r)^2 \frac{\psi^{r-1} \tau_L}{(1 - (1 - a_t^r)^\psi \tau_L)^2} - n^r-1 \tau_L (\bar{T} - \frac{\psi^{r-1}}{1 - (1 - a_t^r)^\psi \tau_L}) = 0\]

(2) \[n^r s^r (\bar{T}_L + \frac{\psi \tau_L}{1 - (1 - a_t^r)^\psi \tau_L}) - \lambda (1 - a_t^r)^2 \frac{\psi^{r-1} \tau_L}{(1 - (1 - a_t^r)^\psi \tau_L)^2} - n^r \tau_L (\bar{T} - \frac{\psi^{r}}{1 - (1 - a_t^r)^\psi \tau_L}) = 0\]

(3) \(BC = 0\)
From (1) and (2) obtain:

\[
\lambda = \frac{n^{\tau-1}s^{\tau-1}(I + \frac{\psi^{\tau-1}}{(1-(1-a^\tau_L)\psi^\tau)}}}{(1-a^\tau_L^2)/(1-(1-a^\tau_L)\psi^\tau})} - n^{\tau-1}(I - \frac{\psi^{\tau-1}}{(1-(1-a^\tau_L)\psi^\tau)}}{(1-(1-a^\tau_L^2)/(1-(1-a^\tau_L)\psi^\tau)}}}
\]

\[
= n^{\tau} s^{\tau}(I + \frac{\psi^\tau}{(1-(1-a^\tau_L^2)/(1-(1-a^\tau_L)\psi^\tau)}}}
\]

Solving this system of equations analytically is a very difficult task. This is why I performed some numerical simulations instead. In the following tables the main results are reported.
Table 1.a - Main results obtained via numerical simulations with *Mathematica 5.2* - Input Matrix - Period $t-1$

<table>
<thead>
<tr>
<th>$n^{t-2}$</th>
<th>$n^{t-1}$</th>
<th>$\tau_{k}$</th>
<th>$I$</th>
<th>$\psi^{t-2}$</th>
<th>$\psi^{t-1}$</th>
<th>$s^{t-2}$</th>
<th>$s^{t-1}$</th>
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<th>$r$</th>
<th>$B$</th>
</tr>
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<tbody>
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<td>0.5</td>
<td>0.5</td>
<td>0.3</td>
<td>0.9</td>
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<td>0.689</td>
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<td>1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
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<td>0.5</td>
<td>0.4</td>
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<td>0.8</td>
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<td>1</td>
<td>0.1</td>
<td>0.5</td>
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<tr>
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<td>0.5</td>
<td>0.4</td>
<td>0.9</td>
<td>0.8</td>
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<td>1</td>
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<td>0.3</td>
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</table>

Table 1.b - Main results obtained via numerical simulations with *Mathematica 5.2* - Output Matrix - Period $t-1$

<table>
<thead>
<tr>
<th>$a_{12}^{t-2}$</th>
<th>$a_{12}^{t-1}$</th>
<th>$I - I_{t-1}^{t-1}$</th>
<th>$I - I_{t-1}^{t-1}$</th>
<th>$T_{t-1}^{t-2}$</th>
<th>$T_{t-1}^{t-1}$</th>
<th>$L_{t-1}$</th>
<th>$Y_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.478</td>
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<td>0.550</td>
<td>0.633</td>
<td>--0.078</td>
<td>0.078</td>
<td>1.183</td>
<td>9.146</td>
</tr>
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<td>1.358</td>
<td>0.377</td>
<td>0.550</td>
<td>0.633</td>
<td>--0.078</td>
<td>0.078</td>
<td>1.183</td>
<td>9.146</td>
</tr>
<tr>
<td>1.239</td>
<td>0.584</td>
<td>0.550</td>
<td>0.633</td>
<td>--0.078</td>
<td>0.078</td>
<td>1.183</td>
<td>9.146</td>
</tr>
<tr>
<td>1.358</td>
<td>0.377</td>
<td>0.550</td>
<td>0.633</td>
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<td>0.078</td>
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<td>9.146</td>
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<tr>
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<td>0.584</td>
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<td>0.633</td>
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<td>0.078</td>
<td>1.183</td>
<td>9.146</td>
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Table 2.a - Main results obtained via numerical simulations with Mathematica 5.2 - Input Matrix - Period $t$

<table>
<thead>
<tr>
<th>$n^{-1}$</th>
<th>$n^{+}$</th>
<th>$\tau_I$</th>
<th>$\tilde{f}$</th>
<th>$\psi^{-1}$</th>
<th>$\psi^{+}$</th>
<th>$\omega$</th>
<th>$r$</th>
<th>$B$</th>
</tr>
</thead>
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<td>0.3</td>
<td>0.9</td>
<td>0.8</td>
<td>0.2</td>
<td>0.689</td>
<td>0.549</td>
<td>2</td>
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<tr>
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<td>0.5</td>
<td>0.4</td>
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<td>0.8</td>
<td>0.2</td>
<td>0.689</td>
<td>0.549</td>
<td>2</td>
</tr>
<tr>
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<td>0.5</td>
<td>0.4</td>
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<td>0.8</td>
<td>0.2</td>
<td>0.689</td>
<td>0.549</td>
<td>2</td>
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<td>0.5</td>
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<td>0.9</td>
<td>0.8</td>
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</table>

Table 2.b - Main results obtained via numerical simulations with Mathematica 5.2 - Output Matrix - Period $t$

<table>
<thead>
<tr>
<th>$a_{t-1}^{-1}$</th>
<th>$a_t^{+}$</th>
<th>$\tilde{f} - \tilde{f}_{t-1}^{-1}$</th>
<th>$\tilde{f} - \tilde{f}_{t-1}^{+}$</th>
<th>$\tilde{f}_{t-1}^{-1}$</th>
<th>$\tilde{f}_{t-1}^{+}$</th>
<th>$L_{t-1}$</th>
<th>$Y_{t-1}$</th>
</tr>
</thead>
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<tr>
<td>3.748</td>
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<td>0.061</td>
<td>1.334</td>
<td>8.536</td>
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<td>6.713</td>
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<td>Not</td>
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Table 3.a - Main results obtained via numerical simulations with Mathematica 5.2 - Input Matrix - Period $t+1$

<table>
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<tr>
<th>$n^t$</th>
<th>$n^{t+1}$</th>
<th>$\tau_L$</th>
<th>$r$</th>
<th>$\psi^t$</th>
<th>$\psi^{t+1}$</th>
<th>$s^t$</th>
<th>$s^{t+1}$</th>
<th>$w$</th>
<th>$\omega$</th>
<th>$r$</th>
<th>$B$</th>
</tr>
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<td>0.6</td>
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<td>0.8</td>
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<td>0.8</td>
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Table 3.b - Main results obtained via numerical simulations with Mathematica 5.2 - Output Matrix - Period $t+1$

<table>
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<tr>
<th>$a_x^t$</th>
<th>$a_x^{t+1}$</th>
<th>$\bar{r} - \bar{T}_{t-1}^{i}$</th>
<th>$\bar{r} - \bar{T}_{t-1}^{t+1}$</th>
<th>$T_t^{r}$</th>
<th>$T_{t+1}^{r}$</th>
<th>$L_{t-1}$</th>
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<td>1.680</td>
<td>1.647</td>
<td>11.849</td>
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<td>0.9</td>
<td>-1.217</td>
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<td>1.647</td>
<td>11.849</td>
</tr>
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References


