How Broadcasting Quotas Harm Program Diversity

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January 2010

Online at http://mpra.ub.uni-muenchen.de/22549/
MPRA Paper No. 22549, posted 10. May 2010 12:51 UTC
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This version: May 2010

Abstract

Broadcasting quotas of domestic contents are commonplace in developed countries. The core argument for them is to promote diversity by making more room for domestic content and hence foster a more diverse production. However, this intuitive reasoning ignores the trade-off between repetition (broadcasting more of the same) and new program diffusion. If each consumer cares only about a small fraction on the total contents of the program, a broadcaster confronted to a quota will find optimal to compensate for the reduction of foreign programming by increasing the number of diffusions of substitutable domestic programs. Total broadcasting time being limited, this will force the broadcaster to slash marginal (less popular) types of programming, whereby reducing program diversity. This mechanism applies both in a monopoly and an imperfectly competitive setting. It thus undermines one of the main rationales for quotas of domestic content.

Keywords: radio, broadcasting, cultural quotas, diversity

JEL: L59, L82, Z10

1 Introduction

1.1 Quotas Everywhere

Import quotas generally have a bad name. Being a substitute for tariffs with adversarial effects on the country that imposes them, they are increasingly shunned by policymakers and globally dismantled by free-trade agreements. In the realm of cultural policy however, quotas of domestically-produced contents or contents in the domestic language are pervasive. One common rationale of those quotas is that cultural goods, most notably books, movies and music, have strong public good dimensions. They convey moods, values and states of mind that are constitutive of national culture and national identity. As such, they are reputed to have positive externalities, helping defuse social tensions, thereby fostering social cohesion. Contrary to the rampant anti-Americanism that goes with such discussion in most countries, this argument is

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not a trademark of minority, endangered or elitist cultures. Such quotas are also in existence or English-speaking countries, much notably Australia or Canada. As they are implemented, those quotas focus on two narrower policy objectives: granting domestic contents access to broadcasting and fostering diversity of contents broadcast.

In Europe for example, the 1989 Directive “Television Without Frontiers” compelled member states to enact laws that ensured that at least 50% of transmission time of TV stations (excluding news, sports events, games, advertising and teletext and teleshopping services) should be reserved for European works. In addition, member states have the opportunity to set more stringent rules if they see fit. The European Commission justifies these quotas in the name of cultural diversity. By providing domestic producers of cultural contents, it argues, this directive guarantees the public expression and exposure of vernacular cultural forms that would otherwise be denied access to broadcasting and financing.

The underlying economic argument is that the production of cultural contents is characterized by high sunk costs: the costs of manufacturing a CD are dwarfed by the costs of the first recording, that adds up rehearsal and studio time, editing and advertisement. All those costs are borne, and sunk, before it can be known whether the album will be a hit or not. Contents with a large domestic markets (that is, American contents) benefit from a larger potential demand that makes it easier for them to bear those costs. Thus, for demographic reasons, contents from smaller countries are initially at a competitive disadvantage since they incur approximately the same initial costs as those from larger countries and face a smaller potential demand, if only because of language barriers. This argument lies at the core of advocates of protectionism in the domain if cultural goods (see Acheson and Maule (2006) for an overview and François and van Ypersele (2002) for an application to the movies industry).

In the realm of music however, a rule of thumb is that almost any song that manages to reach radios’ playlist triggers enough sales to recoup its production costs even in fairly small markets. Broadcasting quotas, the argument goes, make more room for domestic songs and allow more of them to become profitable. This higher profitability in turn fosters entry (in the form of the production of more domestic contents) and, arguably, diversity.

While the argument stands to reason, this paper argues that it oversees the limits of the analogy between material goods and immaterial ones. With an inelastic demand, a quota for a material good will entail a one-to-one substitution between foreign and domestic goods once the quota is reached. In the case of programs (say, a song), the broadcaster faces a trade-off between buying new domestic songs and broadcast those he already has more often. Granted, the quota mechanically increases the share of domestic music. But in the same time, it can also decrease

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1 See Richardson (2004a) for examples. For Canada, quotas apply to Canadian-produced content and do not spin from the bilingualism issue.


3 See Caves (2002) for a full characterization the cost structure of cultural goods.
the number of domestic songs broadcast.

In the French case, a quota of 60% of French-speaking music was imposed in 1996, with at least 20% of new songs. The law was amended in 2000 to accommodate other mix between old and new songs, but the core constraint remains at 60%. Studying how diversity could be measured, Ranaivoson (2007) found that the share of French-speaking music did increase between 1997 and 2005, but shows that the number of French-speaking titles has actually decreased. No data on the number of titles exists for the years before 2003, but Ranaivoson notices that the rotation rate (the mean on the number of diffusions of a given song over a week) jumped from 3.3 in 1997 and 6.6 times in 2005. This increase is comparatively larger than French songs’ gain in broadcasting share, which implies that the number of French titles must have decreased. As Ranaivoson notices, the actual jump in rotations may even be higher, since the panel of radios (which represents 95% of the audience) comprises generalist and public-service radios for which music does not represent the core of their programs. In addition, the use of a mean rotation rate may not accurately summarize the actual distribution of rotations since broadcasting appears to be very skewed: 2.4% of the titles (1575 out 64774 individual titles broadcast in 2008) represent 74.6% and the top 40 titles represent 44% of broadcastings, and more than 60% for radios targeted at a teenage audience.

These figures thus suggest that following the introduction of a broadcasting quota, radios reacted by increasing the rotation rate of French-speaking songs up to a point that they excluded marginally popular ones they previously broadcasted. This happened at a time when the French music scene was fairly active, not in a dearth of broadcastable songs. Such a reduction of the number of titles broadcast goes against one of the main motivations of the quota, which was to foster diversity.

The aim of this paper is to shed some light on the forces that spurred such a counter-intuitive response to broadcasting quotas. The main finding is that whenever (i) programs can be ranked according to popularity, (ii) foreign and domestic programs are not perfect substitutes and (iii) broadcasters want to maximize audience (e.g. because ad receipts are increasing with audience), the broadcaster will respond to a quota by increasing the diffusion of the more popular segments of domestic contents and cutting the less popular types of programs. The intuition behind this result is that under the previous assumptions, a quota amounts to a reduction of broadcasting time. A broadcaster will thus optimally respond to a quota by cutting programming for less popular genres in order to keep serving denser parts of the demand. Thus, those quotas are bound to be somehow self-defeating in their attempt to increase both the share of domestic programs and the diversity of the program schedule.

This paper moreover shows that competition between broadcasters leads to a lower scope of available programs than what a similarly-endowed monopolist would do, and that competition increases the diversity reduction effect of quotas. The intuition is that competition for the denser segments of the audience leads one radio to increase programming targeted at these segments,
abandonning the rest of the audience to its competitor.

The rest of this paper is organized as follows: the remaining of this section reviews the existing literature on the topic, section 2 presents the model, section 3 deals with the case of a monopoly broadcaster, section 4 shows how competition interacts with the effect of a quota, section 5 provides an illustration with a specific case of the model, and section 7 concludes.

1.2 Related literature

This paper was initially spurred by the empirical evidence provided by Ranaivoson (2007) on the French music case. The theoretical possibility of a decrease in diversity in response to a quota is hinted at in Richardson (2004a), but not fully pursued in that paper nor in its companion Richardson (2004b). The author of these two papers focuses on the share of domestic music, which a quota impacts directly, and on two-sided aspects rather than diversity.

As the three papers above, the present paper relates to the wider litterature about the effect of competition on choices and mix between heterogeneous types of contents in the broadcasting industry. Two important starting points are Steiner (1952) and Beebe (1977) who, under the expressions of Principle of Minimum Differentiation and Lowest Common Denominator, set up the core idea that competition between content providers leads to a concentration on the more popular genres (a kind of “dumbing down”), thus reducing diversity. A significant number of papers have documented, theoretically or empirically, this effect, while more recently some have advocated the idea that competition forces broadcasters to respond adequately to consumer demand (see van der Wurff (2005) and citations therein).

Another strain in the literature deals with the effect of competition for advertising revenues on program differentiation and contents. Anderson and Gabszewicz (2006) provides a detailed review and theoretical summary of that literature. In these papers (notably Dukes and Gal-Or (2003), the papers by Gabszewicz, Laussel and Sonnac and Anderson and Coate (2005)) diversity is a problem of locational choice on a Hotelling segment, reflecting either a discrete choice (broadcasting one type of programs or two) or a choice in the mix between the two types. As Anderson and Gabszewicz (2006) states, that litterature was coming to terms with the problems of cross-externalities between listeners and advertisers, problems that inspired the more general literature on two-sided markets (with Rochet and Tirole (2002) and Caillaud and Jullien (2003) as seminal papers). Therefore, the issue of more complex dimensions of differentiation and diversity have been pushed in the background for the sake of understanding the effect of two-sidedness on differentiation. The gist of this paper being on understanding a type of response to a quota, the mecanism of this paper aims at being simpler in order to illustrate the core trade-off between share and diversity in domestic contents. I therefore set aside two-sided dimensions to focus on programming choices.

4Gabszewicz et al. (1999), Gabszewicz et al. (2001), Gabszewicz et al. (2002) and Gabszewicz et al. (2004).
2 The Model

We consider media companies that want to maximize audience. To simplify the exposition of the model, we will use the setup of music radio stations: media companies (platforms) are radios, contents are music titles and content providers are record labels.

2.1 Content and consumers

**Music** We represent music by a continuum of music “genres”. Genre, or alternatively class of titles, are ranked along $[0, +\infty[$ by a decreasing popularity index $\pi$. The closer $\pi$ is to 0, the greater the number of consumers interested by this genre. Titles are provided by record labels, who fall into two categories: $D$ (domestic) and $F$ (foreign) music. In each genre, there is one $D$ title and one $F$ title. A *musical program* is then given by $\Pi \subset \mathbb{R}_+$ the set of genres broadcast by a given radio, $m_D(.)$ the number of broadcasts of the $D$ title for each $\pi \in \Pi$ and $m_F(.)$ the same for $F$ titles.

**Consumers** There is a unit mass of consumers, and each consumer is interested by one and only one genre of music. Hence, decreasing popularity means that consumers can also be indexed by $\pi$ (the genre they like) over $[0, +\infty[$ following a decreasing distribution function $F$. Each consumer is hence a kind of “buff” of her favourite music and music genres with a lower $\pi$ have more fans than those with an higher $\pi$.

In order to model a trade-off between foreign and domestic music, I assume that foreign and domestic music consumption feature some sort of complementarity. More specifically, I assume that a consumer listening to a couple $m_D, m_F$ of her favourite genre enjoys utility $u(m_D, m_F)$, with:

$$
\forall i, j \in \{1, 2\}, i \neq j, \forall (m_D, m_F) \in \mathbb{R}_+^2, \\
u_i'(m_D, m_F) > 0, \\
u_i''(m_D, m_F) < 0 \\
u_{ij}'(m_D, m_F) > u_i''(m_D, m_F), \\
u_{ij}'(m_D, m_F) > u_{jj}''(m_D, m_F)
$$

where $u_i'$ denotes the derivative of $u$ with respect to its $i$-th argument.\(^5\) Utility is thus increasing and concave in each of its arguments. Although standard in general literature, this assumption is not completely straightforward for cultural goods. Some of them feature increasing marginal returns of consumption over time. Since this model is of static nature however, diminishing

\(^5\)For example, the standard Dixit-Sitglity utility with two goods $u(m_D, m_F) = \left(m_D^\rho + m_F^\rho\right)^{\frac{1}{\rho}}, 0 < \rho < 1$ features these properties.
2.2 Media companies

The assumption of the relative values of second- and cross-derivatives reflects the idea that as the level of one type of music increases, the marginal utility of hearing more music of the same type decreases more quickly than hearing more music of the other type. Notice that cross-derivatives need not be positive. This reflects a form of taste for diversity since above a threshold, consumers get sooner tired of hearing one type of music than of hearing a more balanced mix of domestic and foreign music. This assumption is key to the results of this paper, since it lays the foundations of a trade-off between foreign and domestic music.

While listening to the radio entails no direct cost, it means foregone opportunities to do other things: listening to recorded music, watching a film, and so on. Thus, I assume the opportunity cost of listening to the radio is \( \gamma \in \mathbb{R}_+ \), constant across consumers. I assume that a potential listener who is indifferent between listening and not listening will listen to the radio. A radio must hence guarantee its listeners at least a utility level equal to \( \gamma \).

In a nutshell, the utility of a consumer \( \pi \) exposed to a musical program characterized by \( m_D(\pi), m_F(\pi) \) is:

\[
U_\pi(m_D(\cdot), m_F(\cdot), \mu(\cdot)) = u(m_D(\cdot), m_F(\cdot))
\]  

and that consumers listen to the radio if his utility to do so is larger than \( \gamma \).

2.2 Media companies

Media companies (here, radios) provide content to consumers and sell audience to advertisers. Since a full-fledged modelisation of the advertising sector would be beyond the scope of this model, I assume that radios simply want to maximize audience. A radio thus chooses the genres it broadcasts and a level of broadcasts of domestic and foreign music for each genre selected. As a simplifying assumption, I restrict programming to be compact subsets of \( \mathbb{R}_+ \). This means a radio chooses a segment \([a, b] \subset \mathbb{R}_+\) and for each \( x \in [a, b] \) the programming levels \( m_D(x), m_F(x) \). A programming strategy thus sums up to the choice of \( a, b, m_D(\cdot), m_F(\cdot) \).

The program is submitted to a total time constraint \( T \), with one broadcast of any title taking one unit of time.

Let \( W_i \) be the set of consumers listening to a given radio \( R_i \). Since consumers of a given type \( \pi \) are identical, either all of them listen to the same radio, or none of them listens to any radio or, when several radios provides the same level of utility, they split evenly between those radios. For the sake of clarity, let us assume for the moment that the third possibility is ruled out. Let then \( I \) be the set of radios, and let \( u_{\pi,i} \) denote the utility of a consumer of type \( \pi \) listening to radio \( i \in I \). Then:

\[
W_i = \left\{ \pi \mid \forall j \in I, j \neq i, u_{\pi,i} \geq \max_{i,j} \{\gamma, u_{\pi,j}\} \right\}
\]
Hence, the total audience of $R_i$ is given by the total number of consumers in $W_i$, that is:

$$\sigma(W_i) = \int_{W_i} f(u) du$$

which is well-defined since $f$ is integrable and $W_i$ is a subset of $f$’s support. Hence, the profit-maximizing program of a radio is given by:

$$\max_{(a,b,m_D(.),m_F(.))} \left\{ \int_{W_i} f(u) du \right\}$$

s.t. $W_i = \left\{ \pi \left| \forall j \in I, j \neq i, u_{\pi,i} \geq \max_{I \setminus i} \left\{ \gamma, u_{\pi,j} \right\} \right. \right\}$

$$\int_0^{+\infty} (m_iD(u) + m_iF(u)) du \leq T$$

In this model, there is no marginal cost for a radio to broadcast more music, or a new genre of music. This stems from the cost structure of the industry and the type of contracts linking media companies and music producers. On the radio side, programming decisions are made weekly or monthly (see Caves (2002)), while operating costs (studio time, salaries, etc) are sunk for the whole year or more. On the production side, broadcasters do not contract directly with music production firms. They acquire broadcasting rights from copyright collectives, such as the RIAA, in the form of blanket licences that cover a wide spectrum of artists and genre. Hence, the expense for broadcasting rights can be also considered as sunk when programming decisions are made (see Connoly and Krueger (2006) for an overview). I thus consider that radio costs are summed up by a sunk cost $K$ normalized to zero.

**Diversity** As stated above, the main focus of this paper is how competition and regulation affect media companies’ programming. In this setup, the main measure of program diversity is the measure of the musical genres that eventually get broadcast by a radio or another. Usually, the measure of diversity used is the share of domestic content broadcast at the equilibrium in a setup where consumers have different preferences over the ideal share. Indeed, the share of domestic content was the instrument chosen by the regulators when they set broadcasting quotas. The point of this article being to show that this is a poor, and indeed potentially misleading, approach to diversity I use a still simple metric of the usual sense of diversity in programming: programs appealing to different listeners.

\footnote{See Richardson (2004a), Richardson (2004b), Doyle (1998), Anderson and Coate (2005), Anderson and Gab- szewicz (2006).}
3 Monopoly radio

In this section, I assume there is only one radio, which holds a monopoly position in the broadcasting market. This section presents the core intuitions on the optimal programming schemes and the response to a quota of domestic music. Proposition 3.1 states that a monopoly faced with a quota broadcasts a lower measure of genres, cutting programming of less popular genres.

3.1 Monopoly programming

Let us first consider the optimal programming when there is no quota constraint:

$$\max_{(a,b,m,D,F)} \{F(b) - F(a)\}$$

subject to:

$$\int_a^b [m_D(\pi) + m_F(\pi)] \, d\pi \leq T$$

$$\forall \pi \in [a,b], u(m_D(\pi), m_F(\pi)) \geq \gamma$$

The radio aims to cover as many listeners as possible within its time constraint $T$. Since there is no competition other than the outside option, which provides a utility level $\gamma$, the optimal programming strategy entails providing exactly utility $\gamma$ to each listener with as little programming as possible.

Lemma 3.1. A monopoly radio broadcasts on $[0, \pi^*]$ a constant level $m^*$ of music, with:

$$m^* = \min_{m = m_D + m_F} \{m | u(m_D, m_F) = \gamma\}$$

$$\pi^* = \frac{T}{m^*}$$

Proof. With no other constraint than a total broadcasting time, a monopoly radio locates on the denser part of the demand and just saturates listeners’ participation constraint. This is done by choosing $m^*$:

$$m^* = \min_{m = m_D + m_F} \{m | u(m_D, m_F) = \gamma\}$$

Each listener is thus served $m^*$, which allows the monopoly to capture all consumers between 0 and $T/m^* = \pi^*$. \qed

While that result is almost immediate, it is useful for what follows to consider how $m^*$ is composed in terms of domestic and foreign music.

Let us consider a radio willing to provide a level of utility $\gamma$ to some listener and starting its programming from scratch. The radio will first program whichever type of music (domestic or foreign) provides more utility to the consumer. Assume this is foreign music. Since $u''_2 < u''_1$, marginal utility of foreign music $u'_2$ decreases more rapidly than marginal utility of domestic
3.2 Monopoly quota programming

Assume now that a regulator imposes a ceiling $Q$ on the time devoted to type $F$ titles. For things to be interesting, the quota needs to be biting, that is $\pi^* m^*_F > Q$. The radio must now maximize its audience under an additional constraint:

$$\max_{(a,b; m^*_F; m^*_D)} \{ F(b) - F(a) \} \quad (5)$$

subject to:

$$\int_a^b [m_D(\pi) + m_F(\pi)] d\pi \leq T \quad (6)$$

$$\int_a^b [m_F(\pi)] d\pi \leq Q \quad (7)$$

$$\forall \pi \in [a,b], u(m_D(\pi), m_F(\pi)) \geq \gamma \quad (8)$$

The mechanic of the optimal quota programming remains akin to that without the quota. Lemma 3.2 states that formally.

Lemma 3.2. With a quota, the optimal programming features a constant level of domestic and foreign music across genres covered. These levels verify:

$$m^*_F = \frac{Q}{T-Q} m^*_D \quad (9)$$

$$u \left( m^*_D, \frac{Q}{T-Q} m^*_D \right) = \gamma \quad (10)$$

Proof. See Appendix A.1

The intuition of the proof is as follows. Let us assume that the radio wants to cover a given segment $[0, \pi_Q]$. Because the cost in terms of time of capturing an interval $[\pi, \pi + \varepsilon]$ does not depend on $\pi$, $m_D(.)$ and $m_F(.)$ will be constant over the segment, and we can reason point-wise. For a given $\pi$, the radio provides the most favoured music type until the marginal utility of listening the other type becomes larger. For any $\pi < \pi^*$, the quota limit will bite at some point,
and the radio will make up the rest of programming only with domestic music. This naturally leads to proposition 3.1.

**Proposition 3.1.** The interval of genres $[0, \pi^{**}]$ broadcast by a monopoly radio under a quota constraint is increasing in $Q$ and is smaller than $[0, \pi^*]$.

**Proof.** Proposition 3.1 states that $\pi^{**} < \pi^*$ and that $\frac{\partial \pi^{**}}{\partial Q} < 0$.

Let us start by the first relation. Since $\pi^{**}$ is the result of the same maximization program as $\pi^*$ with an additional (biting) constraint, it is immediate that $\pi^{**} \leq \pi^*$, with a strict inequality when the constraint is strictly binding.

Since the constraint is biting, also, the Lagrange multiplier associated with constraint (7) is positive. faced with a tightening of the constraint, that is $Q' < Q$, the radio will optimally not change $a$, since $f$ is decreasing. Since (8) is saturated in both cases, it cannot reduce neither $m_D$ nor $m_F$. Thus, it must reduce $b$, that is $\pi' < \pi^{**}$, or equivalently $\frac{\partial \pi^{**}}{\partial Q} < 0$.

The intuition behind proposition 3.1 is simply that the quota forces the radio to program domestic music where foreign music would provide more utility to listeners. In order to reach the cutoff utility $\gamma$ the radio thus needs to provide more programming to each genre it covers. Under a constant total time constraint $T$, this implies to cut programming for the less popular genres in order to increase programming for the more popular ones. This means the less popular genres get excluded, which lowers the diversity of music broadcast domestic as well as foreign. If we take the view that there are only a limited number of worthy songs in each genre, the rotation rate (the number of times a song is broadcast over a given period) of all domestic broadcasting songs will increase but the number of different domestic songs that are broadcast at equilibrium is lower under a quota. The key assumptions for these results are that listeners care only for a limited number of genres and that domestic and foreign music are imperfect substitutes of each other in the sense that there exists some form of complementarity between the two types of music in the utility function.

## 4 Radio competition

In this section I consider two radios $R_1$ and $R_2$ competing for audience. In the light of the monopoly case and in order to make competition more tractable, I restrict a priori the radios’ programming strategies be be constant levels on compact sets.

**Assumption 1.** The radios programs take the form of compact sets $[a, b] \subset \mathbb{R}_+$. On those sets,

$$\forall \pi \in [a, b], \quad m_D(\pi) = m_D$$

$$m_F(\pi) = m_F$$

$$\forall \pi \not\in [a, b], \quad m_D(\pi) = m_F(\pi) = 0$$
that is the radios broadcast a constant level of domestic and foreign music respectively.

Radio strategy can then be described by the four choice parameters \((m_i^F, m_i^D) \in \mathbb{R}^2_+\), its programming for any given genre covered and \((b^i, c^i)\) defining the segment \([b^i, c^i]\) of genres covered by this radio. The four parameters are bound together by the total time constraint and a quota if one exists. The best-response program of radio \(i\) is thus:

\[
\max \left\{ a^i, b^i, m_i^D, m_i^F \right\} \left\{ \int_{a^i}^{b^i} f(u)\mathbb{1}_{[u(m_i^D(u), m_i^F(u))] \geq \max\{\gamma, u \in (m_i^D, m_i^F)\}] du \right\}
\]

s. t. \((a^i - b^i)(m_i^D + m_i^F) \leq T\)

Even with the restriction above, the simultaneous-move competition game admits no pure strategies equilibrium (see Appendix A.2 for a proof). I therefore adopt a framework of sequential competition one of the radios acting as a Stackelberg leader.

\[\text{4.1 Sequential competition}\]

From this section on, I will assume that Radio 1 is an incumbent. It chooses its programming before Radio 2 (the entrant) does. Radio 1 thus acts as a Stackelberg-leader in the competition game.

At first blush, two strategies are possible for radio 1:

- **Popular incumbent**: Radio 1 can settle on the most popular genres, broadcasting all genres between 0 and some \(\pi^*_1\), radio 2 catering to the \([\pi^*_1, \pi^*_1 + \frac{T}{m^*}]\) segment.

- **Niche incumbent**: Radio 1 can also settle further down the popularity scale on some \([a^*_1, b^*_1]\) segment, letting Radio 2 broadcast on \([0, a^*_1]\) choosing the bounds so that radio 2 prefers to broadcast on \([0, a^*_1]\) rather than competing for 1’s leftmost listeners.

The following proposition states that the first strategy always dominates the second.

**Proposition 4.1.** Radio 1 always settle on a \([0, \pi^*_1]\) segment and Radio 2 on \([\pi^*_1, \pi^*_1 + \frac{T}{m^*}]\), with \(\pi^*_1\) such that

\[ F(\pi^*_1) = F\left(\pi^*_1 + \frac{T}{m^*}\right) - F(\pi^*_1)\]

and \(m^*\) the optimal level of programming derived in the monopoly case.

**Proof.** See Appendix A.3 for a full proof. The main features are given below. \[\square\]
The idea of the proof hinges on two features of that game. Firstly, by locating on the same segment as Radio 1 (minus some \( \varepsilon \)), Radio 2 can always do as well as Radio 1 in terms of audience. Due to that second-mover advantage, Radio 1 will ensure that at equilibrium, Radio 2 enjoys an audience at least equal as its own. Secondly, allowing an overlap between the program of the two radios is always (sometimes weakly) dominated for both radios. Thus, at equilibrium, the supports of the two programming will be disjoint.

**Popular incumbent** Assume first that Radio 1 follows a “Popular incumbent” strategy and settles on a \([0, \pi_1]\) segment. Radio 2 best response is either to compete for audience or accommodate and settle on some \([a^2, b^2]\) segment with \( a^2 \geq \pi_1 \).

If Radio 2 chooses to accommodate, its program is:

\[
\max(a^2, b^2, m_D^2, m_F^2) \left\{ F(b^2 + a^2) - F(a^2) \right\}
\]

\[
s.t. (a^2 - b^2)(m_D^2 + m_F^2) \leq T
\]

\[
\forall u \in [a^2, b^2], u(m_D^2(u), m_F^2(u)) \geq \gamma
\]

\[
a^2 \geq \pi_1
\]

This program is identical to that of a monopoly radio constrained by \( a^2 \geq \pi_1 \). Radio 2 thus optimally behaves as a monopolist on its audience, and its location if of the form \([a^2, a^2 + \frac{T}{m^*}]\), where \( m^* \) the optimal level of programming derived in the monopoly case. Since the density of its audience \( F(a^2 + \frac{T}{m^*}) - F(a^2) \) is strictly decreasing in \( a^2 \), it optimally locates on \([\pi_1, \pi_1 + \frac{T}{m^*}]\).

If Radio 2 chooses to compete for audience, it must serve its listeners a level of music strictly higher than the \( \frac{T}{\pi_1} \) that Radio 1 provides. It will thus cover a segment \([a^2, a^2 + \pi_1]\). Once again, the audience on that segment is decreasing with \( a^2 \), so Radio 2 will compete head-to-head with Radio 1 and settle on \([0, \pi_1 - \varepsilon]\).

Since competition leaves Radio 1 with an infinitesimal audience, it must ensure that Radio 2 will prefer accommodating. It must thus choose \( \pi_1 \) such that the audience on \([\pi_1, \pi_1 + \frac{T}{m^*}]\) is equal to its own audience, \( F(\pi_1) \). Hence the characterization of the optimal cut-off \( \pi_1^* \):

\[
F(\pi_1^*) = F(\pi_1^* + \frac{T}{m^*}) - F(\pi_1^*)
\]

The proof in the appendix show that this cutoff is unique if \( f \) is strictly decreasing.

**Niche incumbent** If Radio 1 follows a “Niche incumbent” strategy and settles on a \([b_1^r, c_1^r]\) segment, the setup is basically the same. Radio 2 either accommodates, broadcasting on \([0, a_1^r]\) or competes, which means locating on \([0, \frac{T}{b^r_1 - a^r_1}]\). Radio 1 always prefer to avoid competition. Assume that \( b^2_r = \frac{T}{b^r_1 - a^r_1} > a^r_1 \). Then, there exists an overlap between the two programs, and Radio 1 would have been better off locating on \([b^r_1, b^r_2 + (a^r_1 - b^r_2)]\): it would have made more
4.1 Sequential competition

audience, and the reaction of Radio 2 would have been identical. Radio 1 program is thus:

$$\max(a^1_r, b^1_r, m^1_D, m^1_F) \{ F(b^2 + a^2) - F(a^2) \}$$

s.t.:

$$(a^1_r - b^1_r)(m^1_D + m^1_F) \leq T$$

$$\forall u \in [a^1_r, b^1_r], u(m^1_D(u), m^1_F(u)) \geq \gamma$$

$$F(a^1_r) \geq F\left(\frac{T}{b^1_r - a^1_r}\right)$$

This set is non-empty, and admits a smallest elements in terms of $a^1_r$. Let $a^*_r$ denote that element and $b^*_r$ the associated bound.

Once the payoffs of the two strategies spelt out, the intuition behind the result is that Radio 1 must ensure that Radio 2 gets an audience at least as large as its own (with that being an equality in the popular incumbent case). If it prefers strictly the niche strategy, this means that it makes more audience that way, and consequently that Radio 2 also makes more audience than what Radio 1 would with a popular incumbent. This implies that $a^*$ is larger than $\pi^*_1$. However, this means that the audience made on the $[a^*_r, b^*_r]$ is lower than what could be made on $[\pi^*_1, \pi^*_1 + \frac{T}{m^*_F}]$, which is equal to the audience on $[0, \pi^*_1]$ that Radio 1 would make if it choose the popular incumbent strategy, a contradiction.

An interesting feature of that equilibrium is that listeners of the incumbent enjoy a utility that is strictly greater than $\gamma$ while listeners of the other radio get a utility just equal to $\gamma$. This result is consistent with an increase of the rotation rate following entry by a new radio.

The outcome of the competition game readily compares with the monopoly outcome. In order to have a proper benchmark, “monopoly” will here refer to a single radio with no competitor and endowed with $2T$ broadcasting time.

**Proposition 4.2.** At the competitive equilibrium, the diversity of genres broadcast is lower than with a monopoly.

**Proof.** This result is immediate. Since the monopoly exactly saturates listeners’ participation constraint, any other programming strategy that is compatible with consumer participation over all its genres will entail less diversity.

The result of proposition 4.2 are in line with some of the common arguments about program diversity. The competitive equilibrium in this model features both a reduction of diversity and an increase in the broadcasting of the most popular genres. If one wishes to translate “most popular” by “low brow” and “less popular” by “high brow”, this mirrors the argument of a dumbing down of programming compared to what a monopoly would do. It can also be noted that both effect are more pronounced when $f$ decreases steeply.
4.2 Competition with quotas

Assume now that both radios are held to a quota $Q$ of music of type $F$. Section 3.2 showed how saturating the quota uniformly across genres is the more efficient way to allocate $D$ and $F$ music. In section 4.1, I explained why competition between radios in my framework led to a concentration of programming and increased the surplus to listener of the most popular titles. Competition with quotas will combine those two insights.

Lemma 4.1. At the competitive equilibrium, Radio 1 always settle on a $[0, \pi_1^{**}]$ segment and Radio 2 on $[\pi_1^{**}, \pi_1^{**} + \frac{T}{m^{**}(Q)}]$, with $\pi_1^{**}$ such that

$$F (\pi_1^{**}) = F \left( \pi_1^{**} + \frac{T}{m^{**}(Q)} \right) - F (\pi_1^{**})$$

and $m^{**}(Q)$ the optimal level of programming derived in the monopoly case with a quota.

Proof. The principle of this proof is to show that the presence of quota does not affect the logic of the proof 4.1, that is neither radio can make a strategical use of the existence of a quota.

To see that quotas are not used strategically at equilibrium, assume first that one of the radio serves a mix such that on an interval of non-zero measure, $m_F \neq \frac{Q}{T} m_D$, that is $t$ chooses to deviate from the optimal reaction to a quota derived in the monopoly case. From the proof of lemma 3.2, we know that that radio could offer the same level of surplus to its listeners while using less programming type. Therefore, a mix of this kind if not optimal. At the equilibrium with quotas, both radio thus serve a mix of domestic and foreign music such that $m_F = \frac{Q}{T} m_D$.

Now, consider the proof of proposition 4.2. Let $m^{**} = m_D^{**} + m_F^{**}$ denote the quantity of music of any genre played at the optimal mix under a given quota. The proof of equilibrium selection is then identical, replacing $m^*$ by $m^{**}$.

Thus, the equilibrium with quotas features both radios offering a mix such that $m_F = \frac{Q}{T} m_D$ and Radio 1 following a “popular incumbent” strategy.

The equilibrium of competition with quotas is thus similar to competition without quotas. Each radio responds to the quota in the same way a monopoly does, that is by cutting programming on less popular titles in order to compensate listeners of more popular titles for the lower utility of a sub-optimal mix. It is therefore natural that the diversity-reducing effect of a quota that we saw with a monopoly radio carries out to the competition case.

Proposition 4.3. At the competitive equilibrium with a binding quota, the measure of genres broadcast is lower than without a quota.

Proof. From the lemma 4.1, we know that the equilibrium with quotas is of the form $[0, \pi_1], [\pi_1, \pi_1 + \frac{T}{m^{**}}]$ with the incumbent (radio 1) located on the first segment and the entrant (radio 2) on the
second one. Remember that the equilibrium condition is that audiences are equal in both segments, that is $F(\pi_1) = F(\pi_1 + \frac{T}{m^{**}}) - F(\pi_1)$.

Let us first consider the effect of a quota on the entrant’s audience. Instead of covering a segment on length $\frac{T}{m^{}}$, it serves a segment of length $\frac{T}{m^{**}} < \frac{T}{m^{}}$. For the $\pi_1^{*}$ corresponding to the case without quotas, audiences of both radios are such that:

$$F(\pi_1^{*}) = F\left(\pi_1^{*} + \frac{T}{m^{}}\right) - F(\pi_1) > F\left(\pi_1 + \frac{T}{m^{**}}\right) - F(\pi_1)$$

the equilibrium constraint is thus breached, since radio 2 would increase its audience by relocating on $[0, \pi_1^{*} - \varepsilon]$.

The equilibrium with quotas thus features a cutoff $\pi_1^{**}$ between the two radios such that:

$$F(\pi_1^{**}) = F\left(\pi_1 + \frac{T}{m^{**}}\right) - F(\pi_1) < F(\pi_1)$$

which imply $\pi_1^{**} < \pi_1^{*}$ since $F$ is increasing.

The total measure of genres broadcast at the equilibrium with quotas is thus $\pi_1^{**} + \frac{T}{m^{**}}$ with $\pi_1^{**} < \pi_1^{*}$ and $\frac{T}{m^{**}} < \frac{T}{m^{}}$. Therefore, $\pi_1^{**} + \frac{T}{m^{**}} < \pi_1^{*} + \frac{T}{m^{}}$. $\square$

Quotas with competing radios will thus have the same impact than with a monopoly radio: less popular genres will be evicted while more popular domestic titles will be repeated more often. The counter-productive effect of broadcasting quotas also exists when radios compete.

5 An Illustration

This section presents an illustration of the results above for a particular form of utility function and popularity distribution. This exercise allows me to give an idea of the scope of the effects I underlined. It also enables me to do some comparative statics on welfare in order to show how consumer surplus depends on the value of the quota and on the shape of the popularity distribution function.

Assumption 2. Throughout this section, I assume that:

- Utility is Dixit-Stiglitz with a elasticity of substitution $\frac{1}{2}$:
  $$u(m_D, m_F) = (\sqrt{m_D} + \sqrt{m_F})^2$$

- The distribution of popularity is an exponential distribution function of parameter $\lambda$:
  $$f(\pi) = \lambda \exp^{-\lambda \pi}$$
  $$F(\pi) = 1 - \exp^{-\lambda \pi}$$
This form of the utility function allows simple closed-form solutions of the maximization program and has the interesting feature that barring a constraint, the share of domestic and foreign music are equal at an optimum. In the absence of good data on peoples’ preferences and substitution elasticities on that matter, it is difficult to gauge the relevance of that particular form of utility: the subject does beg for more empirical research. Notice however that the quantitative results obtained are in the ballpark suggested by French data.

When it comes to popularity, the pattern of observed distributions of sales of French CDs seems to follow something akin to an exponential or to a Pareto distribution\(^8\), the former being more tractable.

5.1 Monopoly

With a Dixit-Stiglitz utility function, the unconstrained optimal mix of programming is straightforward, with an equal share of domestic and foreign music, that is here:

\[ m_D^* = m_F^* = \frac{7}{4} \]

With a ceiling \( Q < T/2 \) on foreign music, the radio cannot offer that mix to all listeners. From section 3.2, we know that it will offer the same mix to all its listeners, with \( m_F = \frac{Q}{T/2} m_D \).

\(^8\) Although they do not directly try to estimate the precise distribution function, figures presented by Benghozi and Benhamou (2008) suggest distributions of those families. Data truncation issues does not allow a formal test.
which gives here:

\[ m_D^{**} = \frac{\gamma(T - Q)}{(\sqrt{Q} + \sqrt{T-Q})^2} \]

\[ m_F^{**} = \frac{\gamma Q}{(\sqrt{Q} + \sqrt{T-Q})^2} \]

Let \( m^* = m_D^* + m_F^* \) and \( m^{**} = m_D^{**} + m_F^{**} \) denote the optimal level of programming without and with a quota respectively. Figure 1 illustrates how a quota leads to an increase in programming on the most popular genres and a cut in the less popular ones.

The distortive effect of a quota on programming can be evaluated by the ration between \( m^* \) and \( m^{**} \), the relative amount of music a radio has to add for the listeners it wants to keep after a quota is introduced. In other words, it shows the increase of the rotation rate of foreign songs relative to that of domestic songs. That ratio depends in fact only on the ratio between total broadcasting time and the ratio \( Q/T \) which is an alternate measure of the quota:

\[ \frac{m^{**}}{m^*} = \frac{2}{(\sqrt{Q/T} + \sqrt{1-Q/T})^2} \]

Figure 2 represents how this ratio evolves with respect to \( Q/T \). With the utility function used here, the effect of a quota, starting from \( 1/2 \) and going to a tighter bound, is initially small but increases sharply. At the limit, when no foreign music is allowed (\( Q/T = 0 \)), the radio has to broadcast twice as much music as without a quota in order to reach the same level of utility for its listeners.

For other values of the substitution elasticity \( \rho \) in a Dixit-Stiglitz utility function\(^9\), this curve

\(^9\)That is: \( u(m_D, m_F) = (m_D^\rho + m_F^\rho)^{1/\rho} \).
depends also on \( \gamma \), although only as a scale parameter. Compared with \( \rho = 1/2 \), the increase of the rotation rate as a function of the quota is larger for \( \rho < 1/2 \) and lower for \( \rho > 1/2 \), since substitution becomes more difficult (resp. easier) between domestic and foreign songs.

Since the total measure of genres broadcast is \( \pi^* = \frac{T}{m^*} \) without a quota and \( \pi^{**} = \frac{T}{m^{**}} \) with a quota, the ration between the two measures is equal to that between the two levels: \( \frac{\pi^*}{\pi^{**}} = \frac{m^{**}}{m^*} \). The relative decrease in variety (measure of genres) is thus directly proportional to the relative increase in rotation rate.

5.2 Competition

Moving to the competition case, the specified framework allows to get a quantitative idea of the effect that competition has on diversity and of how a quota interacts with competition.

5.2.1 Pure competition

Here, the choice of the exponential distribution means that popularity decreases rather quickly. From the incumbent radio perspective, a higher concentration on popularity means it must concentrate more on the most popular titles in order to accommodate enough audience on lower popularity genres for the entrant. Figure 3 illustrates that sharp decrease. Here, \( \pi_C \) stands for the total measure of genres covered at the competitive equilibrium and \( \pi_M \) stands for what a monopoly endowed with \( 2T \) time would do. As \( \lambda \) increases, concentration increases, and the ratio \( \frac{\pi_C}{\pi_M} \) (bottom line) falls. That ratio tends to a limit of \( 1/2 \), and its decrease is sharper as \( T \) grows large and \( \gamma \) gets small. On the other hand, the ratio between monopoly and competition audiences (top line) also falls for small values of \( \lambda \), but then growths back to one. Because a larger \( \lambda \) means a more concentrated popularity, the audience of the genres that get dropped in

\[ \text{Figure 3: Ratios of genres covered and audience for } T = 10 \text{ and } \gamma = 8 \]
the competitive setting relative to the monopoly setting becomes negligible for larger \( \lambda \). The initial drop is narrower and lower for large values of \( T \) and low values of \( \gamma \).

This graphic illustrates a trade-off that was not apparent in the first place: if demand is highly concentrated, the lowering of diversity due to competition may have a negligible impact on welfare, since it drops genres with a very low audience. In this setup, consumer welfare comparison are a bit trickier that audience comparison since they also depend on \( T \) and \( \gamma \) in non-trivial ways. Figure 4 plots the consumer welfare effect of competition. For this comparison, I take gross welfare, that is the utility provided by the radios to their listeners, without taking into account the outside option. The plotted lines are the ratio between the welfare of consumers in competition \( W_c \) divided by the welfare with a monopoly endowed with \( 2T \) broadcasting time \( W_m \).

As \( \lambda \) increases, the measure of genres excluded increases, but the density of listeners of those genres also decreases, with an \textit{a priori} ambiguous effect on welfare. On the other hand, the density of listeners of the most popular titles increases, as well as their utility level, thus increasing overall consumer welfare. With the specification chosen here, the latter effect dominates, and welfare under competition is larger that what a monopoly would provide.

### 5.2.2 Competition with a quota

The generic case showed that a quota mechanically degrades consumer surplus by adding a binding constraint to radios’ programming. How bad is that effect? Figure 5 provides some insight with this particular specification. It plots the ratio between gross consumer welfare (as defined in the previous section) without and with a quota (\( W_c/W_Q \)). For any value of the parameters, the effect has the same shape. The behaviour of that ratio is not monotonic in \( \gamma \). The worst welfare lost is achieved with \( \gamma = T \), with the loss decreasing between low values of \( \gamma \) and \( T \), and increasing afterwards.
6 Quotas and advertisement

Using a specific form of the utility function, this section extends the model to the case where the radios do not maximize audience, but advertisement revenues that are proportional to audience.

Utility throughout this section, I will use the Dixit-Stiglitz utility function of section 5, augmented by a term of consumer aversion to advertising. Consistently with the literature on broadcasting, I assume that advertisements on a flux media, being unavoidable (except by changing stations of switching off the radio), decrease the utility of listeners. in the analysis, this term replaces the outside option \( \gamma \) used up to this point. The utility of a listener exposed to a mix \((m_D, m_F)\) and a level of advertising \(a\) is thus:

\[
\begin{align*}
\mathcal{U}(m_D, m_F, a) &= (\sqrt{m_D} + \sqrt{m_F})^2 - \gamma(a)
\end{align*}
\]

Advertisement To keep things as simple as possible, I make two assumptions of advertisement. The first is that a radio is not able to target any subgroup of its audience, that is it broadcasts a unique level of advertising \(a\) across its whole range of programming (this assumption is in line with the one on constant programming schedule made in section 4). The second is that advertisement revenues are simply proportional to the level of advertisement and the audience reached. A way of interpreting that is that each advertiser has a unit demand and that his returns on advertising are proportional to the number of persons reached. A radio covering the
segment $[b_i, c_i]$ thus has the following profit maximization objective:

$$\max_{(b_i, c_i, m^D_i, m^F_i, a_i)} \left\{ \int_{W_i} f(u) du \right\}$$

s. t. $W = \{ \pi | u(m^D_i, m^F_i, a_i) \geq \max\{0, u_j \in J \setminus \{i\}(m^D_j, m^F_j)\} \}$

$$(c_i - b_i)(m^D_i + m^F_i) \leq T$$

7 Conclusion

In this paper, I set out to show how the trade-off between catering to the denser part of the audience and conquering listeners further away from the most popular genres entailed a trade-off between repetition (of popular songs) and diversity. I showed how, with consumers liking only one genre and with complementarity between domestic and foreign music, that trade-off made broadcasting quotas of domestic contents counter-productive in terms of diversity. Because the meaning of what a “genre” , “domestic” and “foreign” mean is somewhat fuzzy, the same reasoning applies to various setups of the broadcasting industries. The choice between more songs of a popular genre or some of a less-popular one is akin to that between a new season or a ripoff of an established series and making a completely new one, without an existing fan base.

This paper makes the assumption that broadcasters seek to maximize their audience. This is, of course, only an interim objective, since commercial broadcasters want to maximize advertising revenues. If listeners’ utility is negatively affected by the presence of ads, the level of advertisement becomes part of the trade-off. In such case, the competition game becomes even more intricate, since radios compete both in programming and in ads levels.

References


REFERENCES


A Appendix

A.1 Monopoly quota programming

The radio audience maximization program is:

\[
\begin{align*}
\max_{(a,b,m_F, m_D)} & \quad \{F(b) - F(a)\} \\
\int_a^b & m_F(\pi)d\pi \leq Q \\
\int_a^b & m_D(\pi)d\pi \leq T - Q \\
\forall \pi \in [a,b] & , u(m_D(\pi), m_F(\pi)) \geq \gamma
\end{align*}
\]

(13)

(14)

(15)

(16)

From the case without quota, we know that \(a = 0\), and than \(b\) will be the optimal \(\pi^{**}\).

Since we assume the quota to be a real constraint, we know that (14) is biting. Since at any level, marginal utility is positive, the monopoly radio can always increase its audience by broadcasting more music, hence (15) is also biting, and finally, audience maximization means that (16) is biting.

Firstly, let \(m_D(m_F)\) denote:

\[
m_D(m_F) = \arg \min_{m_D} \{m_D\}
\]

\[
s.t. \quad u(m_D, m_F) \geq \gamma
\]

From what we have seen in construction the optimal, non-quota, programming, we know that \(m_D(m_F)\) is a well-defined, monotonously decreasing mapping. Secondly, since \(F\) is monotonously increasing in \(\pi\), maximizing \(F(\pi)\) is equivalent to maximizing \(\pi\) itself. This allows to reduce the above problem to a standart minimum-time optimal control problem, where \(\pi\) is the target, \(m_D(m_F)\) links the two controls \(m_F(\cdot), m_D(\cdot)\) and the constraints (14) and (15) provide the evolution and the transversality conditions of the problem.

I rewrite the problem as a canonical maximum-time problem:

\[
\max_{m_F(.)} \left\{ \int_0^\pi 1du \right\}
\]

\[
\dot{q}(u) = m_F(u), \quad \dot{t}(u) = m_D(m_F(u))
\]

\[
q(0) = t(0) = 0, \quad q(\pi) = Q, \quad t(\pi) = T
\]

The Hamiltonian associated with this maximum-time problem is:

\[
\mathcal{H} = p_0 + p_1(u)m_F(u) + p_2(u)m_D(m_F(u))
\]
The necessary conditions are:

\[
\begin{align*}
\frac{\partial H}{\partial q} &= \frac{\partial p_1}{\partial u} = 0 \quad (17) \\
\frac{\partial H}{\partial t} &= \frac{\partial p_2}{\partial u} = 0 \quad (18) \\
\frac{\partial H}{\partial m_F} &= p_1 + p_2 \frac{\partial m_D}{\partial m_F} = 0 \quad (19)
\end{align*}
\]

Equations (17) and (18) tell us that \( p_1 \) and \( p_2 \) are constants. Equation (19) means that at the optimum there is a affine relation between \( m_F \) and \( m_D \), that is:

\[
m_D = -\frac{p_1}{p_2} m_F + k_1 \quad (20)
\]

Now, consider the transversality conditions. At the optimum, the second condition is fulfilled when the radio uses up its whole quota \( Q \). Since we assumed that the quota has some bite, it follows that the radio is always willing to do so, since it allows it to provide more utility to its listeners. For the same reason, the radio is always willing to use its whole time endowment \( T \). Then, the second transversality condition rewrites as:

\[
k_1 \pi - \frac{p_1}{p_2} \int_0^\pi m_F(u) du = T - Q \quad (21)
\]

which allows to simplify the expression of \( m_D(m_F) \) as:

\[
m_D = \frac{T - Q}{Q} m_F + k_1 \quad (22)
\]

Since we want both \( m_D(x) \) and \( m_F(x) \) to be nil for any \( x > \pi^{**} \) it is necessary that \( k_1 = 0 \). Hence, for all \( u \in [0, \pi^{**}] \),

\[
m_D(x) = m_F(x) \frac{T - Q}{Q} \quad (23)
\]

Equation 23 defines a function \( \tilde{m}_D(m_F) \) strictly increasing in \( m_F \). Since \( m_D(m_F) \) defined above is a decreasing function, the optimal programming level of foreign music \( m_F^{**} \) is the one that simultaneously satisfies (16) and (23), which is unique. Because both conditions do not depend on \( x \), that optimum \( m_F^{**} \) does not depend on \( x \). According constraint (16), this value is given by:

\[
u \left( \frac{T - Q}{Q} m_F^{**}, m_F^{**} \right) = \gamma \quad (24)
\]

or equivalently, \( m_D^{**} \) such that:

\[
u \left( m_D^{**}, \frac{Q}{T - Q} m_D^{**} \right) = \gamma \quad (25)
\]
A.2 No pure simultaneous-move equilibrium

This section shows that the simultaneous-move competition game admits no Nash equilibrium. The outline of this demonstration is as follows: in a first part, I show that the strategy space can be reduced to the choice of the segment of genres covered. I then show that there exists no symmetric equilibrium and then that asymmetric equilibria cannot exist either.

**Lemma A.1.** The strategy of a radio can be fully expressed by the segment $[\pi_i, \Pi_i]$ covered by its programming.

*Proof.* The strategy of radio $i$ is defined by its programming mix $(m_{id}, m_{id}^F)$ and the segment of genres it covers $[\pi_i, \Pi_i]$. The monopoly case shows that the strict concavity of $u$ entails that the couple $(m_{id}, m_{id}^F)$ maximizing utility on a given $[\pi_i, \Pi_i]$ is unique.

It is straightforward that strategies using a different mix are strictly dominated: if a radio chooses a different mix, the other one can offer to the segment of listeners the same level of utility with less total programming, and still have some spare time to capture listeners outside of the other radio’s audience. Such strategies will thus never be part of an equilibrium neither be a credible threat. \hfill \square

A strategy is given by $(\pi_i, \Pi_i)$. In what follows, results are clearer when one bears in mind that the lower the size $\Pi_i - \pi_i$ of the segment, the higher the utility of agents in that segment. Using that property, it is possible to delineate some characteristic of a potential equilibrium. Since the market can never be totally covered, the equilibrium profits are positive. In any case, a radio can move to the free tail of the types’ distribution and make some profit there. Moreover, I argue there exists no symmetric, pure-strategies equilibrium, but there can be an asymmetric equilibrium. To assert that, I first show that a candidate equilibrium cannot be symmetric, that the program schedules must have disjoint support and that consumer surplus must be equal to zero. This allows me to show that if consumer density $f$ is strictly decreasing, there is no pure-strategies, simultaneous-move equilibrium.

**Lemma A.2** (No symmetric equilibrium). The competition game admits no pure-strategy, symmetric equilibrium.

*Proof.* Assume there exists a pure-strategies, symmetric equilibrium $E$ with $\pi_i = \pi_j = \pi$ and $\Pi_i = \Pi_j = \Pi$. In such case, radios share half the audience on this segment. However, radio $i$ can reduce its $\Pi_i$ by a small $\varepsilon$. It would then provide a strictly higher utility to listeners over the $[\pi, \Pi - \varepsilon]$ segment, thereby capturing all the audience. Such a deviation is profitable when $F(\Pi - \varepsilon) - F(\pi) > \frac{1}{2}(F(\Pi) - F(\pi))$, which is true for some $\varepsilon$ on any non-degenerate segment $[\pi, \Pi]$. A symmetric situation thus cannot be an equilibrium. \hfill \square

Thus, if an equilibrium exists, it is asymmetric in at least one of the choice parameters.
Lemma A.3 (Disjoint support). If an equilibrium exists, the two programs have disjoint support, that is either $\Pi_i \geq \pi_j$ or $\Pi_j \geq \pi_i$.

Proof. First, notice that if at equilibrium there is an overlap between the two supports, then the two radios provide the same consumer surplus on the overlap (and hence on all the covered section of the market). If it were not the case, the radio with the lower consumer surplus gets no audience from the overlap and finds profitable to serve an uncovered part of the audience.

Next, since both radios offer the same surplus over the overlap, each captures half the audience there. This can be an equilibrium strategy only if the radio “on the left” (i.e. serving the denser part of consumers) is already serving all the market between 0 and the overlap region. Otherwise, abandoning the overlap region to serve an audience closer to 0 (and hence more numerous) would always be profitable. Then, an equilibrium with an overlap will always have the form $[0, \Pi_i], [\pi_j, \Pi_j]$ with $\Pi_i \geq \pi_j$ (that is an overlap on $[\pi_j, \Pi_i]$).

Since on the two radios provide the same utility to their listeners (see first paragraph of this proof), it is then profitable for radio j to relocate on $[0, \Pi_j - pi_j - \epsilon]$ and capture all demand: up to the small $\epsilon$, the width of the segment $[0, \Pi_j - pi_j - \epsilon]$ is equal to that of the segment $[\pi_j, \Pi_j]$ but located on a denser part of the market, which means a larger audience.

Thus, an equilibrium cannot feature an overlap in programs’ supports.

Lemma A.4 (Zero consumer surplus). If an equilibrium exists, then the consumer surplus of the listeners of both radios is equal to zero.

Proof. From lemmas A.2 and A.3, we know that an equilibrium is of the form : $(0, \Pi_i], [\pi_j, \Pi_j])$. Let $s_l = u(mD_l, mF_l) - \gamma$ be the net surplus for a consumer listening to radio $l \in \{i, j\}$. Obviously, $s_i$ and $s_j$ are positive, else the radios would have no listeners.

If $s_j > 0$, radio j’s listeners enjoy a positive surplus. Radio j can then reduce its level of programming until $s_j = 0$ in order to serve a positive measure of consumers located between $\Pi_j$ and $\Pi_j + \epsilon$, thus increasing its advertising revenues. Hence, $s_j = 0$.

For the same reasons, if $s_i > s_j$, radio i can cut in its programming in order to capture consumers between $\Pi_i$ and $\Pi_i + \nu$, encroaching on radio j’s public. Thus, at equilibrium, $s_i \leq s_j$.

Putting all conditions together gives: $0 \leq s_i \leq s_j = 0$.

The last two steps are to show that equilibrium, both profits must be equal and that consumer density $j$ strictly decreasing implies that radio j’s profit are always lower than radio i’s under the other equilibrium conditions.

Proposition A.1 (No simultaneous equilibrium). The symmetric simultaneous-move competition game admits no Nash equilibrium.

Proof. Let $A_i$ denote radio i’s audience at a candidate equilibrium and $A_j$ radio j’s. If $A_i > A_j$, radio j can profitably take radio i’s customer with a slightly narrower support of genres and increase its profit. Hence at equilibrium, $A_i = A_j$. 

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From the preceding lemmas, we know that at a candidate equilibrium both radio broadcast the same level $m^*$ of music of each genre they cover, with $m^* = m^*_D + m^*_F$ such that $(m^*_D, m^*_F)$ is the more efficient mix to reach $u(m^*_D, m^*_F) = \gamma$. In that case, the two radios have the following audiences:

$$A_i = \left\{ F\left(\frac{T}{m^*}\right) \right\}$$

$$A_j = \left\{ F\left(\frac{2T}{m^*}\right) - F\left(\frac{T}{m^*}\right) \right\}$$

Because $f$ is strictly decreasing and the length of the interval covered are the same and $f$ decreasing means that the density of consumers on $[0, \frac{T}{m^*}]$ is greater that on $[\frac{T}{m^*}, \frac{2T}{m^*}]$. Then, $V_i > V_j$, which is not compatible with an equilibrium since radio $j$ will want to take radio $i$’s location. \qed

Since no such equilibrium exist, one would be tempted to see what happens when mixed strategies are allowed. However, the strategy space (the set of closed intervals $[a,b]$ of $\mathbb{R}_+$ such that $b-a \leq T/m^*$) is cumbersome and the actual meaning of mixed programming strategies not extremely clear. For that reason, I prefer to consider a sequential game where one radio acts as a Stackelberg leader, choosing its location first.

### A.3 Strategy selection

In order to show that the “popular incumbent” strategy is a dominant strategy for the incumbent, I need to show that both strategies may lead to equilibrium candidates and then show the popular one is preferred by the incumbent.

#### A.3.1 Best responses and outcomes

In what follows, I show that the popular strategy always leads to a single equilibrium candidate, while allowing entry on the most popular genres also leads to an equilibrium candidate for any distribution of probability.

**Popular incumbent** In that type of equilibrium, the incumbent broadcasts on a $[0, \pi_1]$ segment, and sets a surplus $s_1$ such that radio 2 is better off by capturing monopoly profits on a $[b_2, c_2], b_2 > \pi_1$ segment.

**Lemma A.5.** For any decreasing distribution, there exists a unique “popular” strategy that makes radio 2 indifferent between entering on less popular genres and competing for more popular ones. Programs are then on the segments $[0, \pi_1], [\pi_1, \pi_1 + \frac{T}{m^*}]$.

**Proof.** The popular strategy means that radio 1 locates on a $[0, x]$ segment. Radio 2 best response is then of of the two:
(i) Locating at \([x, x + \frac{T}{m^*}]\), thus making monopoly profit on that (less popular) part of the audience.

(ii) Competing with 1 for the more popular genres, locating on \([0, x - \varepsilon]\), thus offering a slightly larger utility to listeners.

Since the second response entails (near)-zero profit for radio 1, it selects the greatest \(x\) such that radio 2 chooses the first response, that is:

\[
\max_x \{ F(x) \}
\]

s.t. \( F(x) \leq F \left( x + \frac{T}{m^*} \right) - F(x) \)

Now, I want to show that the maximum argument \(x^*\) of this program exists and in unique. Firstly, notice that

\[
\frac{\partial}{\partial x} \left[ F \left( x + \frac{T}{m^*} \right) - F(x) \right] = f \left( x + \frac{T}{m^*} \right) - f(x)
\]

Since \(f\) is monotonously decreasing and \(m^*\) does not depend on the location, \(f \left( x + \frac{T}{m^*} \right) < f(x)\), which means \(F \left( x + \frac{T}{m^*} \right) - F(x)\) is also monotonously decreasing in \(x\). For \(x = 0\), it is trivial that \(F \left( \frac{T}{m^*} \right) > F(0) = 0\), and because of \(f\) decreasing, it is straightforward that

\[
F \left( \frac{T}{m^*} \right) > F \left( \frac{2T}{m^*} \right) - F \left( \frac{T}{m^*} \right)
\]

The intermediate-values theorem then implies that there exists one unique \(x\) such that \(F(x) = F \left( x + \frac{T}{m^*} \right) - F(x)\) and that it is the greatest \(x\) such that \(F(x) \leq F \left( x + \frac{T}{m^*} \right) - F(x)\). \(\square\)

**Niche incumbent** This strategy is almost the mirror image of the previous one. Here, the incumbent uses consumer surplus in order to “squeeze” the entrant on the left of the demand. The entrant thus captures the most popular titles, but is constrained on its right by the incumbent.

**Lemma A.6.** For any decreasing distribution, there exists a unique strategy \([y^*, z^*]\) that maximizes Radio 1 audience and has Radio 2 prefer serving the \([0, y^*]\) segment to competing for Radio 1’s listeners.

**Proof.** A niche strategy is a segment \([y, z] \subset \mathbb{R}_+\) chosen as location by Radio 1. Radio 2’s response can then be:

i Settle on \([0, y]\) for an audience \(F(y)\)

ii Serve all listeners on \([0, y]\) and compete for some on \([y, z]\).
In order to fully characterize the second option, let $u_1$ denote the utility of consumers listening to Radio 1, $\tilde{m}(s) = (\tilde{m}_D(s), \tilde{m}_F(s))$ the optimal mix corresponding to a segment of length $s$, and $x_2$:

$$x_2 = \arg \max_{x_j} \{F(x_j)\}$$

s. t.: $u\left(\tilde{m}\left(\frac{T}{x_j}\right)\right) \geq u_1$

that is the larger audience that Radio 2 can get while providing a utility as least equal to that provided by Radio 1.

In order to maximize its audience, Radio 1 must then ensure that $F(y) \geq F(x_2)$. Notice that this also implies that $F(y) \geq F(z) - F(y)$ (since $f$ is decreasing). Radio 1 problem is thus:

$$\max_{y \leq z} \{F(z) - F(y)\}$$

s. t.: $F(y) \geq F(x_2)$

It is clear that for $y = \frac{T}{m}$, the constraint is fulfilled. The set of optimal strategies with entry on the most popular titles is thus non-empty and admits a larger element in terms of audience for Radio 1.

Now, let us compare the payoffs of the optimal strategies of each kind. I show that those of the niche strategy are always larger than those of the other strategy.

Let $l$ refer to the popular strategy and $r$ denote niche strategy. Let $A^h_k$ denote the audience of radio $k$ in equilibrium candidate $h \in \{l, r\}$. Let $[0, x]$ denote the optimum location of Radio 1 with the popular strategy and $[0, y], [y, z]$ denote the optimum location of Radios 2 and 1 respectively in the other case.

From the first conditions for best-response from Radio 2, we know that Radio 2 must has an audience at least as large as Radio 1 in either case, that is:

$$A^l_2 \geq A^l_1 \quad \text{and} \quad A^r_2 \geq A^r_1$$

Assume now that Radio 1 strictly prefers the niche strategy, that is $A^r_1 > A^l_1$. This implies:

$$A^r_2 > A^r_1 \geq A^l_1$$

$$A^r_2 > A^l_1 \Rightarrow F(y) > F(x)$$

$$\Leftrightarrow y > x \text{ since } F \text{ is strictly increasing}$$
Thus,

\[ A_r^1 = F(z) - F(y) \]
\[ \leq F \left( \frac{T}{m^*} + y \right) - F(y) \text{ since } z - y < \frac{T}{m^*} \]
\[ \leq F \left( \frac{T}{m^*} + x \right) - F(x) \text{ since } x < y, \text{ move to a denser part of the audience} \]
\[ \leq A_2^i \]
\[ A_r^1 \leq A_1^i \text{ since } A_1^i = A_2^i \]

Thus, \( A_r^1 > A_1^i \Rightarrow A_r^1 \preceq A_1^i \), which is obviously contradictory. Thus, \( A_1^i \succeq A_r^1 \), which means that radio 1 always weakly prefers the entry on the less popular genres.