Eventologically multivariate extensions of probability theory's limit theorems

Vorobyev, Oleg Yu. and Golovkov, Lavrentyi S.

Siberian Federal University, Institute of Mathematics

23 April 2009

Eventologically multivariate extensions of probability theory's limit theorems

Oleg Yu. Vorobyev
Institute of Mathematics
Siberian Federal University
Krasnoyarsk State
Trade Economic Institute
Krasnoyarsk
vorob@akadem.ru
www.eventology-theory.com

Lavrentiy S. Golovkov
Krasnoyarsk State
Trade Economic Institute
Krasnoyarsk
lavrentiy.golovkov@gmail.com

Abstract. *Eventologically multivariate extensions of probability theory's limit theorems are proposed. Eventologically multivariate version of limit theorems extends its classical probabilistic interpretation and involves into its structure of dependencies of arbitrary set of events which appears in sequence of independent tests.*

Keywords. *Event, probability, set of events, Bernoulli univariate test, Bernoulli multivariate test, eventological distribution, multivariate discrete distribution, limit theorem.*

In the paper there are eventological multivariate extensions of probability theory's limit theorems, which are continuing direction of our research of multivariate eventological distributions started at papers [4, 5], where eventological extensions of multivariate discrete distributions (Binomial and Poisson) are suggested and Poisson limit theorem is proofed.

At first univariate limit theorems (which are written on eventological language) are considered, and then — eventological extensions of multivariate analogs of these limit theorems. In univariate version eventological notation hasn't any advantages of classical and is given only as acquaintance with features of eventological notations. In multivariate version usage of eventological language is allowing to extend probabilistic interpretation of limit theorems by including into it structure of dependencies of arbitrary[1] set of events which appears in sequence of independent tests.

Eventological space $(\Omega, \mathcal{F}, \mathbf{P})$ and selected from algebra of events $\mathcal{F}$ finite set of events $\mathfrak{X} \subseteq \mathcal{F}$ are considered. Elements of this set are events $x \in \mathfrak{X}$. Let $N = |\mathfrak{X}|$ is a power of $\mathfrak{X}$. Although at $\mathfrak{X}$ there are no order, sometimes formal presentation: $\mathfrak{X} = \{x_1, \ldots, x_N\}$ is used in exclusive cases, which doesn't indicates any order.

Set of probabilities $p = \{p(X),\ X \subseteq \mathfrak{X}\}$, which consist of 2^N probabilities

$$p(X) = \mathbf{P}\left(\mathrm{ter}(X)\right), \quad X \subseteq \mathfrak{X},$$

событий-террасок[2]

$$\mathrm{ter}(X) = \bigcap_{x \in X} x \bigcap_{x \in X^x} x^c \in \mathcal{F}, \tag{1}$$

is called *eventological distribution (E-distribution)* of set of events $\mathfrak{X}$:

$$\sum_{X \subseteq \mathfrak{X}} p(X) = 1.$$

Let define

$$m = \{m(X),\ X \subseteq \mathfrak{X}\}$$

as integer numbers family, in which $m(X)$ — number of comes of terrace-event $\mathrm{ter}(X)$ in multivariate scheme based on n independent tests of set of events $\mathfrak{X}$: $\sum_{X \subseteq \mathfrak{X}} m(X) = n$. Besides, let denote

$$\widehat{p} = \{\widehat{p}(X) = m(X)/n,\ X \subseteq \mathfrak{X}\}$$

— compiled from probabilities $\widehat{p}(X)$ of terrace-events $\mathrm{ter}(X)$ E-distribution:

$$\sum_{X \subseteq \mathfrak{X}} \widehat{p}(X) = 1.$$

We denote

$$\mathcal{H}_{\widehat{p}/p} = \sum_{X \subseteq \mathfrak{X}} \widehat{p}(X) \ln \frac{\widehat{p}(X)}{p(X)} \tag{2}$$

relative entropy of E-distribution $\widehat{p}$ by relation to E-distribution p, which serves known measure of deviation one E-distribution from another and it is equal to zero for the equal E-distributions.

[1] In classical statement in independent tests narrow special set figures (which has structure of dependencies of disjoint events), but not arbitrary set.

[2] Terrace-events forms partition of elementary outcomes space: $\Omega = \sum_{X \subseteq \mathfrak{X}} \mathrm{ter}(X)$.

1 Bernoulli's schemes and binomial probabilities

Scheme of univariate Bernoulli tests is a sequence of independent tests, in each of which one fixed event can occur or cannot occur.

Eventological scheme of multivariate Bernoulli tests is a sequence of independent tests, in each of which *finite set of events* with fixed eventological distribution is tested, other words, in each test some events can occur and some cannot.

Polynomial scheme of multivariate Bernoulli tests. In classical probability theory polynomial scheme of tests particular case of defined above general multivariate scheme is commonly understood, when *finite set of disjoint events* is tested, other words, in each test only one event from fixed set can occur or nothing is occurs.

Obviously, any scheme of multivariate of arbitrary finite set of events formally can be reduced to polynomial scheme of multivariate tests of finite set of disjoint events, which has higher power. Nevertheless, this is possible only by loss ability scheme of multivariate tests to show arbitrary structure of dependence of set of events in its completeness.

In eventology scheme of multivariate tests common type is considered, that allows to describe multivariate tests in eventological theory in language of arbitrary sets of events, which can express and operate complete set of characteristics of arbitrary structures of dependencies of events for detail description of relations between subsets of events defined by its distributions. While terminology and notions in polynomial scheme of multivariate tests are based only on narrow special structure of dependencies of disjoint events and aren't suited to expressing singularities of dependencies in arbitrary structures of events.

1.1 Scheme of univariate tests

Scheme of univariate Bernoulli tests is considered: series of n independent tests, in each of which *fixed event* $x \in \mathfrak{X}$ can occur with probability $p_x = \mathbf{P}(x)$. Let ν_x is a number of occurs of fixed event $x \in \mathfrak{X}$ in series of n tests. Then probability of that in series of n tests fixed event $x \in \mathfrak{X}$ occurs exactly $m_x = 0, \dots, n$ times, is equal to *binomial probability*

$$B_n^{n_x}(p_x) = \mathbf{P}(\nu_x = m_x) = C_n^{m_x} p_x^{m_x}(1-p_x)^{n-m_x}. \quad (3)$$

1.2 Schemes of multivariate tests

1.2.1 General scheme of multivariate tests

General scheme of multivariate Bernoulli tests is considered: series of n independent tests, in each of which finite set of events $\mathfrak{X}$ with fixed E-distribution $p = \{p(X),\ X \subseteq \mathfrak{X}\}$ and therefore only one of terrace-events $\mathrm{ter}(X) \in \mathcal{F}$ can occur with probability $p(X)$. Let $\nu(X)$ — number of occurs of terrace-event $\mathrm{ter}(X)$ in series of n tests. Given all together these numbers form set $\nu = \{\nu(X),\ X \subseteq \mathfrak{X}\}$. Then probability of that in series of n tests terrace-events $\{\mathrm{ter}(X), X \subseteq \mathfrak{X}\}$ occurs times, which form set $m = \{m(X), X \subseteq \mathfrak{X}\}$, $\sum_{X\subseteq\mathfrak{X}} m(X) = n$, is equal to *multivariate binomial probability*

$$B_n^m(p) = \mathbf{P}(\nu = m) = \frac{n!}{\prod\limits_{X\subseteq\mathfrak{X}} m(X)!} \prod_{X\subseteq\mathfrak{X}} p(X)^{m(X)}. \quad (4)$$

1.2.2 Polynomial scheme of multivariate tests

Particular scheme of multivariate Bernoulli tests: series of n independent tests, in each of which fixed set of *disjoint* events $\mathfrak{X}$ with fixed E-distribution $p = \{p(\emptyset), p(x),\ x \in \mathfrak{X}\}$ is tested and therefore only one of events $\bigcap_{x\in\mathfrak{X}} x^c \in \mathcal{F};\ x \in \mathfrak{X}$ with appropriate probabilities $p(\emptyset) = 1 - \sum_{x\in\mathfrak{X}} p(x); p(x) = \mathbf{P}(x), x \in \mathfrak{X}$ can occur. Let $\nu(\emptyset), \nu(x), x \in \mathfrak{X}$ — numbers of occur appropriate enumerated disjoint events in series of n tests. Given all together these numbers form set $\nu = \{\nu(\emptyset), \nu(x),\ x \in \mathfrak{X}\}$. Then probability of that in series of n tests disjoint events $\{\mathrm{ter}(\emptyset), \mathrm{ter}(x) \in \mathfrak{X}\}$ occur times, which form set $m = \{m(\emptyset), m(x), x \in \mathfrak{X}\}$, $m(\emptyset) + \sum_{x\in\mathfrak{X}} m(x) = n$, is equal to known *polynomial probability*

$$B_n^m(p) = \mathbf{P}(\nu = m) =$$

$$= \frac{n!}{m(\emptyset)! \prod\limits_{x\in\mathfrak{X}} m(x)!} p(\emptyset)^{m(\emptyset)} \prod_{x\in\mathfrak{X}} p(x)^{m(x)}, \quad (5)$$

which is particular case of multivariate binomial probability (4) for the set $\mathfrak{X}$, which has structure of disjoint events. Indeed, if all events from $\mathfrak{X}$ are disjoint, then for each X for which $|X| > 1$ terrace-events are equal to impossible event: $\mathrm{ter}(X) = \varnothing$.

When number of tests n is large, calculating of binomial as in univariate and in multivariate versions may be a trouble. Therefore search of asymptotic formulas for these probabilities when $n \to \infty$ is important task. Solutions of these problems give limit theorems of probability theory and its eventological extensions.

2 Eventological formulation of limit theorems of probability theory [3, 1] and its eventological extension

We expound limit theorems on eventological language, because in "translation" we're meet with singularities of eventological notions[3]. Eventological "translation" of limit theorems in scheme of univariate tests can be considered only as training "exercise" in eventology, which doesn't involve nothing new, but in scheme of multivariate tests eventological "translation" of limit theorems allows us to give new eventologically extended formulation and proof, which doesn't appear in probability theory before now.

2.1 Local limit theorem

2.1.1 Scheme of univariate tests

Theorem 1. *При* $m_x \to \infty,\ n - m_x \to \infty$

$$B_n^{m_x}(p_x) = \mathbf{P}(\nu_x = m_x) = \mathbf{P}\left(\frac{\nu_x}{n} = \widehat{p}_x\right) \sim$$

$$\sim \frac{1}{\sqrt{2\pi n \widehat{p}_x(1-\widehat{p}_x)}} \exp\{-n\mathcal{H}_{\widehat{p}_x/p_x}\}, \qquad (6)$$

где $\widehat{p}_x = m_x/n$,

$$\mathcal{H}_{\widehat{p}_x/p_x} = \widehat{p}_x \ln\frac{\widehat{p}_x}{p_x} + (1-\widehat{p}_x)\ln\frac{1-\widehat{p}_x}{1-p_x} \qquad (6')$$

— relative entropy of eventological distribution $\widehat{p}$ in relation to eventological distribution p; and symbol $a_n \sim b_n$ for two numeric sequences $\{a_n\}$ и $\{b_n\}$ means that $a_n/b_n \to 1$ when $n \to \infty$.

Consequence 1. If $\widehat{p}_x = m_x/n$ is close to p_x, then for the right side in (6) other form of notion can be found, which is more interest commonly. The fact that relative entropy (2) is analytical function $H(s_x) = \mathcal{H}_{s_x/p_x}$ in interval $(0,1)$. Because

$$H'(s_x) = \ln\frac{s_x}{p_x} - \ln\frac{1-s_x}{1-p_x}, \quad H''(s_x) = \frac{1}{s_x} + \frac{1}{1-s_x},$$

then $H(p_x) = H'(p_x) = 0$ and when $\widehat{p}_x - p_x \to 0$

$$H(\widehat{p}_x) = \frac{1}{2}\left(\frac{1}{p_x} + \frac{1}{1-p_x}\right) + O(|\widehat{p}_x - p_x|^3).$$

Therefore if $\widehat{p}_x \sim p_x$ and $(\widehat{p}_x - p_x)^3 \to 0$, then

$$B_n^{m_x}(p_x) \sim$$

$$\sim \frac{1}{\sqrt{2\pi n p_x(1-p_x)}} \exp\left\{-\frac{n}{2p_x(1-p_x)}(\widehat{p}_x - p_x)^2\right\}.$$

Consequence 2. *If $\Delta_x = 1/\sqrt{np_x(1-p_x)}$, $\varphi(s) = \frac{1}{\sqrt{2\pi}}e^{-s^2/2}$ — density of standard normal distribution and $s_x = m_x - np_x = o(n^{2/3})$, then formula*

$$B_n^{m_x}(p_x) = \mathbf{P}(\nu_x - np_x = s_x) \sim \varphi(s_x\Delta_x)\,\Delta_x \qquad (7)$$

allows to get estimation of probability of events like $\{\nu_x < m_x\}$ in integral limit theorem[4].

Formula (6) in theorem 1 can be easy clarified by estimate error, which occurs in this formula by know estimation of error in Sterling's formula $n! = \sqrt{2\pi n}\; n^n e^{-n+\theta(n)}$. This evaluation follows from inequalities:

$$\frac{1}{12n+1} < \theta(n) < \frac{1}{12n}.$$

Theorem 2.

$$B_n^{m_x}(p_x) =$$

$$= \frac{1}{\sqrt{2\pi n \widehat{p}_x(1-\widehat{p}_x)}} \exp\{-n\mathcal{H}_{\widehat{p}_x/p_x} + \theta(m_x, n)\}, \qquad (8)$$

where

$$|\theta(m_x,n)| = |\theta(n) - \theta(m_x) - \theta(n-m_x)| <$$

$$< \frac{1}{12m_x} + \frac{1}{12(n-m_x)} = \frac{1}{12n\widehat{p}_x(1-\widehat{p}_x)}.$$

Formula (7) from consequence 2 can be clarified by the following way:

Theorem 3. *If $s_x = m_x - np_x$, а $\sigma_x^2 = p_x(1-p_x)$, then for each m_x for which*

$$\left|\frac{m_x}{n} - p\right| \le \frac{1}{2}\min\{p_x, 1-p_x\}$$

formula

$$B_n^{m_x}(p_x) \sim \frac{1}{\sqrt{2\pi n\sigma_x^2}} e^{-\frac{s_x^2}{2n\sigma_x^2}}(1 + \varepsilon(m_x, n)), \qquad (9)$$

is valid, where

$$1 + \varepsilon(m_x, n) = \exp\left\{\left(\frac{|s_x|^3\Delta^4}{3} + (|s_x| + 1/6)\Delta^2\right)\alpha\right\},$$

and modulus of numeric parameter α strictly lower than one: $|\alpha| < 1$.

Boundaries for α can be constricted, if we consider lower deviations $|\widehat{p}_x - p_x|$, for example, down to value $\gamma \cdot \min\{p_x, 1-p_x\}$, where $\gamma < 1/2$.

These theorems and consequences for binomial probability are local limit theorems for *univariate Bernoulli scheme.*

2.1.2 Schemes of multivariate tests

Polynomial scheme. Main asymptotic formula for binomial distribution from theorem 1 allows generalization to polynomial distribution $B_n^m(p)$, $p =$

[3]Easiest thing is to translate limit theorems from [3, p. 69] and [1, стр. 116, 120], for example. There its main formulation and proof as in univariate and in multivariate versions are based on notion of *relative entropy*, and then from it other more popular in probability theory formulations are inferred, which commonly are interested for most of readers.

[4]See below

$\{p(\emptyset), p_x, x \in \mathfrak{X}\}, m = \{m(\emptyset), m(x), x \in \mathfrak{X}\}$, when in sequence of n independent tests not one of two, but one of $N+1$ events $x \in \mathfrak{X}$ and $\bigcap_{x \in \mathfrak{X}} x^c$ occur, probabilities of which are accordingly equal to $p(x) > 0,\ x \in \mathfrak{X}$ and $p(\emptyset) = 1 - \sum_{x \in \mathfrak{X}} p(x) > 0$. Let $m(x)$ is number of occurs of event $x \in \mathfrak{X}$ in n tests and $m(\emptyset)$ is a number of tests, when nothing occur from $\mathfrak{X}$,

$$\nu = \{\nu(\emptyset), \nu(x), x \in \mathfrak{X}\},\ m = \{m(\emptyset), m(x), x \in \mathfrak{X}\},$$

$$\widehat{p} = \frac{m}{n} = \left\{\widehat{p}(\emptyset) = \frac{m(\emptyset)}{n},\ \widehat{p}(x) = \frac{m(x)}{n},\ x \in \mathfrak{X}\right\}.$$

Assuming that

$$\mathcal{H}_{\widehat{p}/p} = \widehat{p}(\emptyset) \ln \frac{\widehat{p}(\emptyset)}{p(\emptyset)} + \sum_{x \in \mathfrak{X}} \widehat{p}(x) \ln \frac{\widehat{p}(x)}{p(x)} \qquad (2')$$

— relative entropy of eventological distribution $\widehat{p}$ of set of disjoint events $\mathfrak{X}$ in relation to eventological distribution p.

Theorem 4. *If each of $N+1$ variables $\{\nu(\emptyset), \nu(x), x \in \mathfrak{X}\}$ is equal to zero or tends to ∞ when $n \to \infty$, then*

$$B_n^m(p) = \mathbf{P}(\nu = m) = \mathbf{P}\left(\frac{\nu}{n} = \widehat{p}\right) \sim$$

$$\sim (2\pi n)^{\frac{1-k_+}{2}} \left(p(\emptyset) \prod_{x \in \mathfrak{X}} p(x)\right)^{-1/2} \exp\{-n\mathcal{H}_{\widehat{p}/p}\}, \quad (10)$$

where k_+ — number of variables from $\{m(\emptyset), m(x), x \in \mathfrak{X}\}$, which isn't equal to zero.

General scheme. Main asymptotic formula for binomial distribution from theorem 1 allows not only generalization to polynomial, but to general multivariate binomial distribution $B_n^m(p)$, $p = \{p(X), X \subseteq \mathfrak{X}\}, m = \{m(X), X \subseteq \mathfrak{X}\}$, when in sequence of n independent tests in each test not one of $N+1$, but one of 2^N terrace-events $\text{ter}(X) \in \mathfrak{F}$ occurs, probability of which are accordingly equal to $p(X),\ X \subseteq \mathfrak{X}$. Let $m(X)$ is a number of occur of terrace-event $\text{ter}(X) \in \mathfrak{F}$ in n tests,

$$\nu = \{\nu(X),\ X \subseteq \mathfrak{X}\},$$

$$\widehat{p} = \frac{m}{n} = \left\{\widehat{p}(X) = \frac{m(X)}{n},\ X \subseteq \mathfrak{F}\right\}.$$

Assuming that

$$\mathcal{H}_{\widehat{p}/p} = \sum_{X \subseteq \mathfrak{X}} \widehat{p}(X) \ln \frac{\widehat{p}(X)}{p(X)}$$

— relative entropy of eventological distributions $\widehat{p}$ and p of set of arbitrary events $\mathfrak{X}$.

Theorem E4. *If each of 2^N variables $\{\nu(X), X \subseteq \mathfrak{X}\}$ is equal to zero or tends to ∞ when $n \to \infty$, then*

$$B_n^m(p) = \mathbf{P}(\nu = m) = \mathbf{P}\left(\frac{\nu}{n} = \widehat{p}\right) \sim$$

$$\sim (2\pi n)^{\frac{1-k_+}{2}} \left(\prod_{\substack{X \subseteq \mathfrak{X} \\ p(X) > 0}} p(X)\right)^{-1/2} \exp\{-n\mathcal{H}_{\widehat{p}/p}\}, \quad (E10)$$

where $k_+ = 2^N - \sum_{X \subseteq \mathfrak{X}} \delta(m(X), 0)$ — is a number of variables from $\{m(X), X \subseteq \mathfrak{X}\}$, which are not equal to zero.

Consequence 4. Change of variables: centering and normalization.

Theorems 4 and E4 for polynomial and multivariate binomial probability are local limit theorems for *multivariate Bernoulli scheme.*

2.2 Integral limit theorem

2.2.1 Scheme of univariate tests

Let a and b — fixed numbers, $u_x = \nu_x - np_x$. Then

$$\mathbf{P}(a < u_x < b) = \sum_{a/\Delta_x < s_x < b/\Delta_x} \mathbf{P}(u_x = s_x).$$

If here instead of $\mathbf{P}(u_x = s_x)$ we substitute formula (7) from consequence 2, then we have integral sum

$$\sum_{a < s_x\Delta_x < b} \varphi\left(s_x \Delta_x\right) \Delta_x,$$

which is approximately fits integral $\int_a^b \varphi(s)ds$ from standard normal density, which in calculated through known integral of probabilities $\Phi(s)$. Other words, consequence 2 allows the next theorem to be plausible:

Theorem 5 (integral limit theorem of Moivre – Laplace).

$$\lim_{n \to \infty} \mathbf{P}(a < u_x < b) = \int_a^b \varphi(s)ds = \Phi(b) - \Phi(a).$$

2.2.2 Schemes of multivariate tests: simple (non-entropic) formulation

Polynomial scheme [2, p.77]. We expound in eventological language polynomial scheme of independent tests, in each of which one of N events $x \in \mathfrak{X}$ ($|\mathfrak{X}| = N$) can occur with possibilities $p_x = \mathbf{P}(x)$, at that $0 < p_x < 1$, $p(\emptyset) = 1 - \sum_{x \in \mathfrak{X}} p_x > 0$. Let $m(x)$ — number of occurs of event $x \in \mathfrak{X}$ in series of n tests, $\eta_x(n) = (m(x) - np_x)/\sqrt{np_x(1-p_x)}$ — normalized deviation from mean number of occurs of event $x \in \mathfrak{X}$ in series of n tests, $\eta(n) = \{\eta_x(n),\ x \in \mathfrak{X}\}$ — vector of normalized deviations, components of which are dependent arbitrary variables, $w = \{w_x,\ x \in \mathfrak{X}\}$ — N-dimensional vector of numbers,

$$F_n(w) = \mathbf{P}\left(\bigcap_{x \in \mathfrak{X}} \{\eta_x(n) < w_x\}\right).$$

Theorem 6[5].

$$\lim_{n\to\infty} F_n(w) = \frac{1}{\sqrt{(2\pi)^N \det \mathrm{C}}} \times$$

$$\times \int_{-\infty}^{w_x} \cdots \Big|_{x\in\mathfrak{X}} \exp\left\{-\frac{1}{2}\sum_{x\in\mathfrak{X}}\sum_{y\in\mathfrak{X}} \mathrm{C}_{xy}^{-1} v_x v_y\right\} \prod_{x\in\mathfrak{X}} dv_x, \tag{11}$$

where

$$\mathrm{C} = \{\mathrm{C}_{xy},\ x, y \in \mathfrak{X}\}$$

— matrix of covariations of vector $\eta(n)$;

$$\mathrm{C}_{xy} = \mathbf{E}\eta_x(n)\eta_y(n) = \begin{cases} 1, & x = y, \\ -\sqrt{\frac{p_x p_y}{(1-p_x)(1-p_y)}}, & x \neq y; \end{cases}$$

$$\det \mathrm{C} = \frac{p(\emptyset)}{\prod_{x\in\mathfrak{X}} (1-p_x)} \neq 0;$$

$$\mathrm{C}_{xy}^{-1} = \begin{cases} \frac{(1-p_x)(p_x+p(\emptyset))}{p(\emptyset)}, & x = y, \\ \frac{\sqrt{p_x p_y (1-p_x)(1-p_y)}}{p(\emptyset)}, & x \neq y \end{cases}$$

— (x, y)-th element of inverse matrix C^{-1}.

General scheme. We consider general eventological scheme of independent tests of arbitrary finite set of events $\mathfrak{X}$ with fixed E-distribution $p = \{p(X),\ X \subseteq \mathfrak{X}\}$, in each of which one of 2^N terrace-events $\mathrm{ter}(X) \in \mathcal{F}$ can occur with probabilities $p(X)$, at that $0 < p(X) < 1$. Let $m(X)$ — number of occurs of terrace-event $\mathrm{ter}(X)$ in series of n tests, $\eta_n(X) = (m(X) - np(X))/\sqrt{np(X)(1-p(X))}$ — normalized deviation from mean number of occurs of terrace-event $\mathrm{ter}(X)$ in series of n tests, $\eta_n = \{\eta_n(X),\ X \subseteq \mathfrak{X}\}$ — vector of normalized deviations, components of which are dependent arbitrary variables, $w = \{w(X), X \subseteq \mathfrak{X}\}$ — 2^N-dimensional vector of numbers,

$$F_n(w) = \mathbf{P}\left(\bigcap_{\substack{X\subseteq\mathfrak{X}\\ X\neq\emptyset}} \{\eta_n(X) < w(X)\}\right).$$

Theorem E6.

$$\lim_{n\to\infty} F_n(w) = \frac{1}{\sqrt{(2\pi)^{2^N-1}\det \mathrm{C}}} \int_{-\infty}^{w(X)} \cdots \Big|_{\substack{X\subseteq\mathfrak{X}\\ X\neq\emptyset}}$$

$$\exp\left\{-\frac{1}{2}\sum_{\substack{X\subseteq\mathfrak{X}\\ X\neq\emptyset}}\sum_{\substack{Y\subseteq\mathfrak{X}\\ Y\neq\emptyset}} \mathrm{C}_{XY}^{-1} v(X) v(Y)\right\} \prod_{\substack{X\subseteq\mathfrak{X}\\ X\neq\emptyset}} dv(X), \tag{E11}$$

where

$$\mathrm{C} = \{\mathrm{C}_{XY},\ X, Y \subseteq \mathfrak{X}, X \neq \emptyset, Y \neq \emptyset\}$$

— matrix of covariations of vector η_n;

$$\mathrm{C}_{XY} = \mathbf{E}\eta_n(X)\eta_n(Y) =$$

$$= \begin{cases} 1, & X = Y, \\ -\sqrt{\frac{p(X)p(Y)}{(1-p(X))(1-p(Y))}}, & X \neq Y; \end{cases}$$

$$\det \mathrm{C} = \frac{p(\emptyset)}{\prod_{\substack{X\subseteq\mathfrak{X}\\ X\neq\emptyset}} (1-p(X))} \neq 0;$$

$$\mathrm{C}_{XY}^{-1} = \begin{cases} \frac{(1-p(X))(p(X)+p(\emptyset))}{p(\emptyset)}, & X = Y, \\ \frac{\sqrt{p(X)p(Y)(1-p(X))(1-p(Y))}}{p(\emptyset)}, & X \neq Y \end{cases}$$

— (X, Y)-th element of inverse matrix C^{-1}.

3 Discussion

This paper summarized in interim results in direction of eventological multivariate extensions of local and integral limit theorems of probability theory. Eventological multivariate extension of integral limit theorem in entropy formulation has became unsolved — the same formulation as it stated in eventological multivariate extension of local limit theorem E4.

In subsequent papers it's planned to continue research of entropy approach in eventological multivariate extension of limit theorems and in theory of eventological distributions. Also it's targeted to publish detailed proofs of obtained results.

References

[1] A. A. Borovkov. *Probability Theory.* Nauka, Moscow, 1986 (in Russian).

[2] Korolyuk, S., Portenko, N. I., Skorokhod, A. V., and Turbin, A. F. *Handbook on Probability Theory and Mathematical Statistics.* Nauka, Moscow (in Russian), 1985.

[3] A. N. Shiryaev. *Probability.* Nauka, Moscow, 1980 (in Russian).

[4] O. Yu. Vorobyev. Multivariate discrete distributions: eventological extension. In Oleg Vorobyev, editor, *Proc. of the VI All-Russian FAM Conf. on Financial and Actuarial Mathematics and Related Fields*, volume 1, pages 72–97. Siberian Federal University, Krasnoyarsk, Russia, 2007.

[5] O. Yu. Vorobyev and L. S. Golovkov. Multivariate discrete distributions. *Journal of Siberian Federal University. Mathematics & Physics*, 1:68–77, 2008.

[5] In formulation of theorems 6 и E6 we're using the following slightly unusual notion for repeated integral, in which order of variables of integration isn't fixed:

$$\int_{-\infty}^{w_x} \cdots \Big|_{x\in\mathfrak{X}} = \int_{-\infty}^{w_{x_1}} \cdots \int_{-\infty}^{w_{x_N}},$$

Without this theorem 6 can be stated and proved by enumeration somehow elements of $\mathfrak{X}$, but in theorem E6, where (2^N-1)-repeated integral appears, which consist of 2^N-1 integrals $\int_{-\infty}^{w(X)}$ when $X \subseteq \mathfrak{X}, X \neq \emptyset$, too much clauses should stated.